SHOCK AND VIBRATION RESPONSE SPECTRA COURSE
Unit 10. Vibration response spectrum - Miles rule

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Introduction

Again, certain systems are subjected to a base excitation vibration. Examples include:

1. A building during an earthquake
2. An automobile traveling down a washboard road
3. An avionics component on a rocket vehicle bulkhead during powered flight

The purpose of this unit is to determine the response of a single-degree-of-freedom system to random base excitation.

Model

Consider the single-degree-of-freedom system subjected to base excitation shown in Figure 1. The free-body diagram is shown in Figure 2.

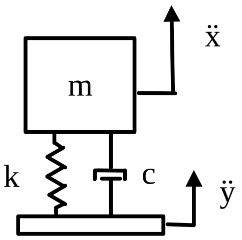

Figure 1. Single-degree-of-freedom System

The variables are

m = mass,
c = viscous damping coefficient,
k = stiffness,
x = absolute displacement of the mass,
y = base input displacement.

The double-dot notation indicates acceleration

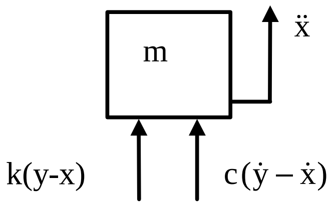

Figure 2. Free-body Diagram

The following equation of motion for the relative displacement z was derived in Unit 8.

$$\ddot{z} + 2\xi\omega_n\dot{z} + \omega_n{}^2 z = -\ddot{y} \tag{1}$$

Either Laplace or Fourier transforms may be used to derive the steady state transmissibility function for the absolute response acceleration.

Base Input

A sample base input power spectral density is shown in Figure 3.

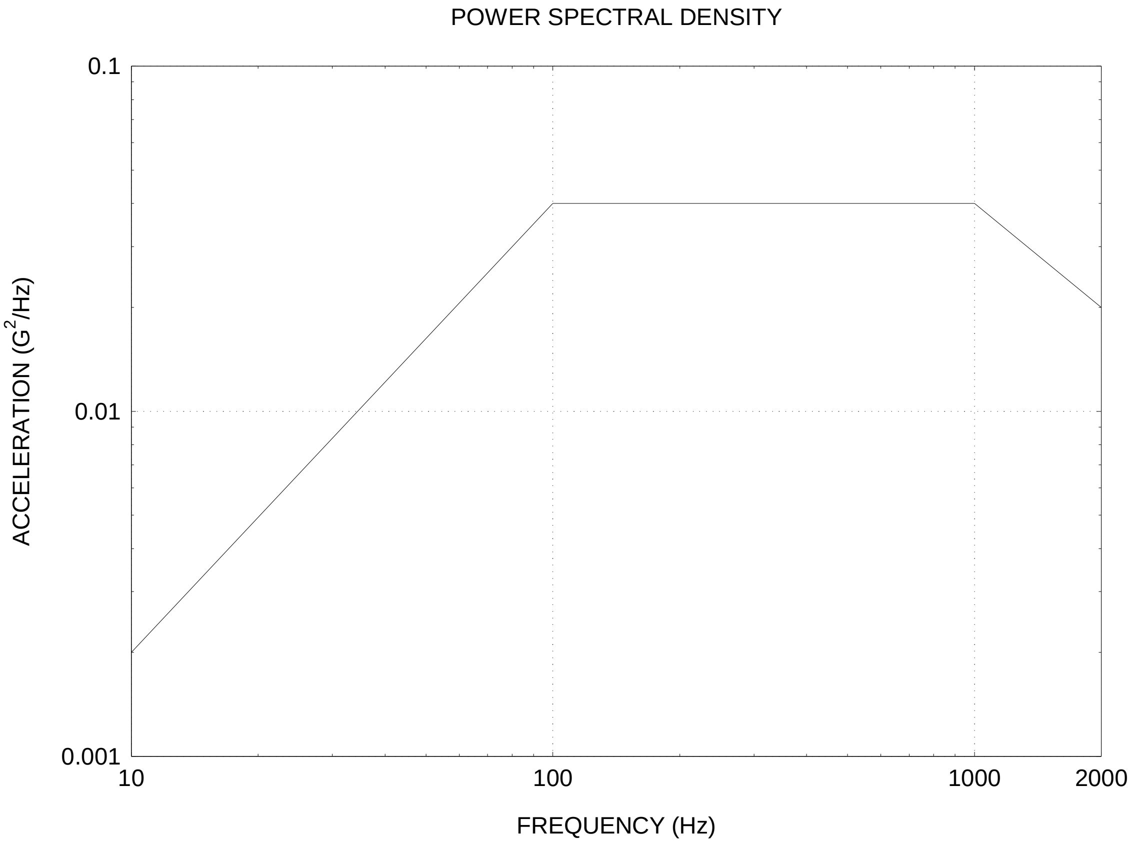

Figure 3

The breakpoints for the level in Figure are given in Table 1.

Table 1.
Power Spectral Density,
8.1 GRMS Overall Level

Freq (Hz)	Level (G^2/Hz)
10	0.002
100	0.04
1000	0.04
2000	0.02

The level in Figure 3 could represent an envelope of flight accelerometer data. A safety margin might be included in this level.

Assume that this level is the random vibration test level for an avionics component. The component will be tested on a shaker table. The component will be powered and monitored during the vibration test.

The next task is to determine the response of the component to the base input level.

Response

The Miles equation is a simplified method of calculating the response of a single-degree-of-freedom system to a random vibration base input, where the input is in the form of a power spectral density.

The Miles equation is

$$\ddot{x}_{GRMS} = \sqrt{\frac{\pi}{2} P f_n \left(\frac{1}{2\xi} \right)} \tag{2}$$

where

$\ddot{x}_{GRMS}$ = the overall response,
P = the power spectral density level at the natural frequency,
f_n = the natural frequency.

Note that the damping is often represented in terms of the quality factor Q.

$$Q = \frac{1}{2\xi} \tag{3}$$

Miles equation can thus be expressed as

$$\ddot{x}_{GRMS} = \sqrt{\frac{\pi}{2} P \ f_n Q} \tag{4}$$

This equation is given in Reference 1.

Miles equation must be calculated for each natural frequency system of interest. The damping term is typically held as a constant.

Furthermore, equation (2) is an approximate formula which assumes a flat power spectral density from zero to infinity Hz. As a rule-of-thumb, it may be used if the power spectral density is flat over at least two octaves centered at the natural frequency.

Response for Known Natural Frequency

A certain avionics component has a circuit board which can be idealized as a single-degree-of-freedom system. The natural frequency is fn = 200 Hz. The damping ratio is 0.05, equivalent to Q=10.

What is the response of the component to the base input shown in Figure 3?

Use Miles equation. Note that the base input level is 0.04 G^2/Hz at 200 Hz

$$\ddot{x}_{GRMS} = \sqrt{\frac{\pi}{2}\left[0.04 \ \frac{G^2}{Hz}\right][200 \ Hz][10]} \tag{5}$$

$$\ddot{x}_{GRMS} = \ 11.2 \quad GRMS \tag{6}$$

Note that the RMS value is equal to the 1σ value assuming a zero mean. Recall that the 1σ value is the standard deviation.

What is the peak response value?

The precise answer is unknown. It depends on the duration and on other factors. Often, the peak response is assumed to be 3σ.

$$\ddot{x}_{peak} = \ 33.6 \ G \quad (3\sigma) \tag{7}$$

The response time history has the following probability characteristics assuming a normal distribution.

Probability inside ±3σ Limits = 99.73%

Probability outside ±3σ Limits = 0.27%

Vibration Response Spectrum

Now consider that the natural frequency of the avionics component is unknown, but assume an amplification factor of Q=10. The base input is the level in Figure 3.

How can Miles equation be used for an unknown or variable natural frequency?

First, choose a sample of frequencies.

Next, determine the base input power spectral density at each frequency. This requires interpolation at certain frequencies. The interpolation can be done by hand calculations. A more efficient method is to use the dboct.exe program.

Finally, calculate the response at each frequency.

The result of these steps is shown in Table 2. The response level was calculated via an Excel spreadsheet.

Table 2.
Vibration Response Spectrum, Q=10

Natural Frequency (Hz)	Base Input Level (G^2/Hz)	Response Level (GRMS)
10	0.002	0.56
20	0.005	1.25
50	0.016	3.54
100	0.04	7.93
200	0.04	11.21
400	0.04	15.85
600	0.04	19.42
800	0.04	22.42
1000	0.04	25.07
1200	0.033	24.94
1600	0.025	25.07
2000	0.02	25.07

The vibration response spectrum is the response level versus natural frequency, as shown in Figure 4.

Note that the analysis in Table 2 violates the assumption that the input level is flat within one octave on either side of the natural frequency. Some error may occur as a result. An improved method which overcomes this assumption will be discussed in Unit 11.

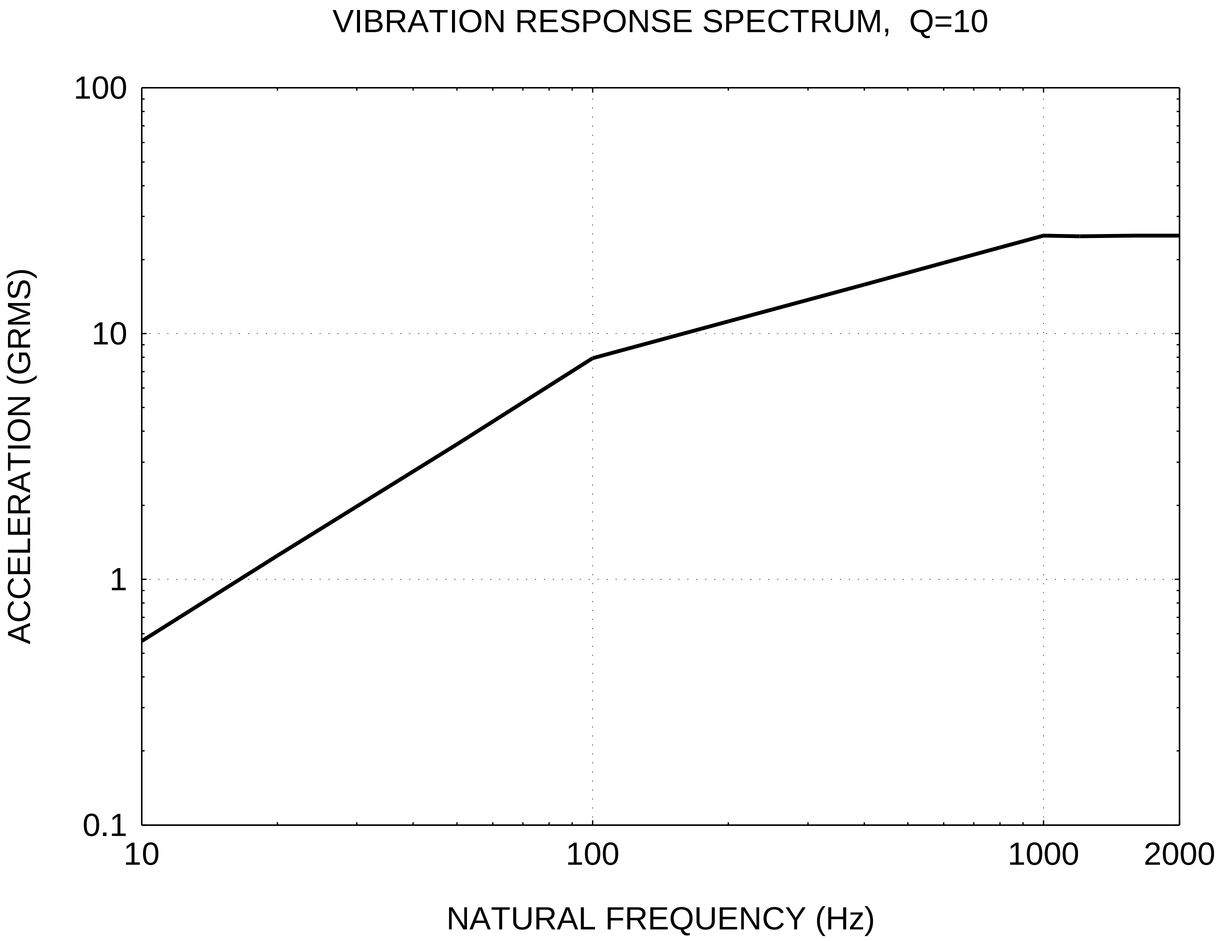

Figure 4.

The vibration response spectrum shows that reducing the natural frequency reduces the response level, with respect to the given base input.

The natural frequency reduction could be achieved by mounting the avionics component with isolator grommets.

For example, the response is 11.2 GRMS at 200 Hz. The response is 7.9 GRMS at 100 Hz, which is 3.0 dB lower than the response at 100 Hz.

<u>References</u>

1. Dave Steinberg, Vibration Analysis for Electronic Equipment, Wiley-Interscience, New York, 1988.

Homework

1. NAVMAT P-9492 gives the power spectral density specification shown in Figure A-1.

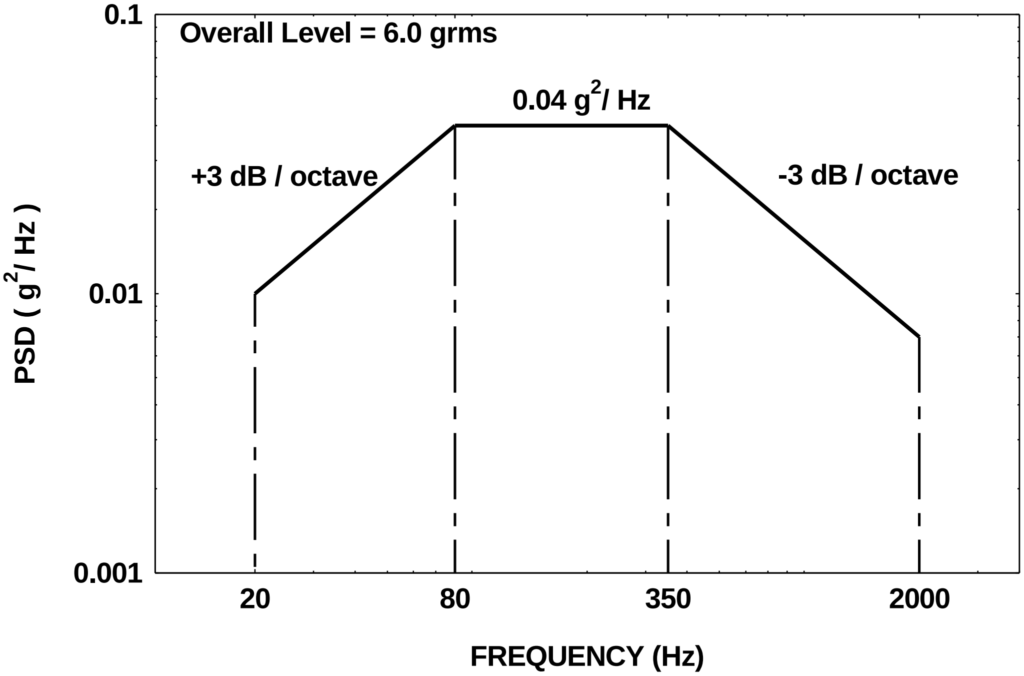

Figure A-1.

Calculate the vibration response spectrum using Miles equation for the level in Figure A-1. Assume an amplification factor of Q = 10. Use program dBoct.exe to calculate the input level at the ramp frequencies.

2. Plot the vibration response spectrum from problem 1. Superimpose the vibration response spectrum from Table 2 in the main text.

3. A component has a natural frequency of 90 Hz and an amplification factor of Q = 10. It has been successfully tested to the 6.0 GRMS level in Figure A-1. Next, the design requirements are changed. The component must now be tested to the 8.1 GRMS level in Figure 3. A program management goal is to avoid additional testing despite the new requirement. How could you justify that the original test is sufficient to meet the revised test specification? This is a very, very typical problem in the "real-world " sense.