

SHOCK AND VIBRATION RESPONSE SPECTRA COURSE Unit 12.
Synthesizing a Time History to Satisfy a Power Spectral Density using Sinusoids

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Introduction

Consider the power spectral density specification Table 1 in Figure 1. This level is taken from MIL-STD-1540C.

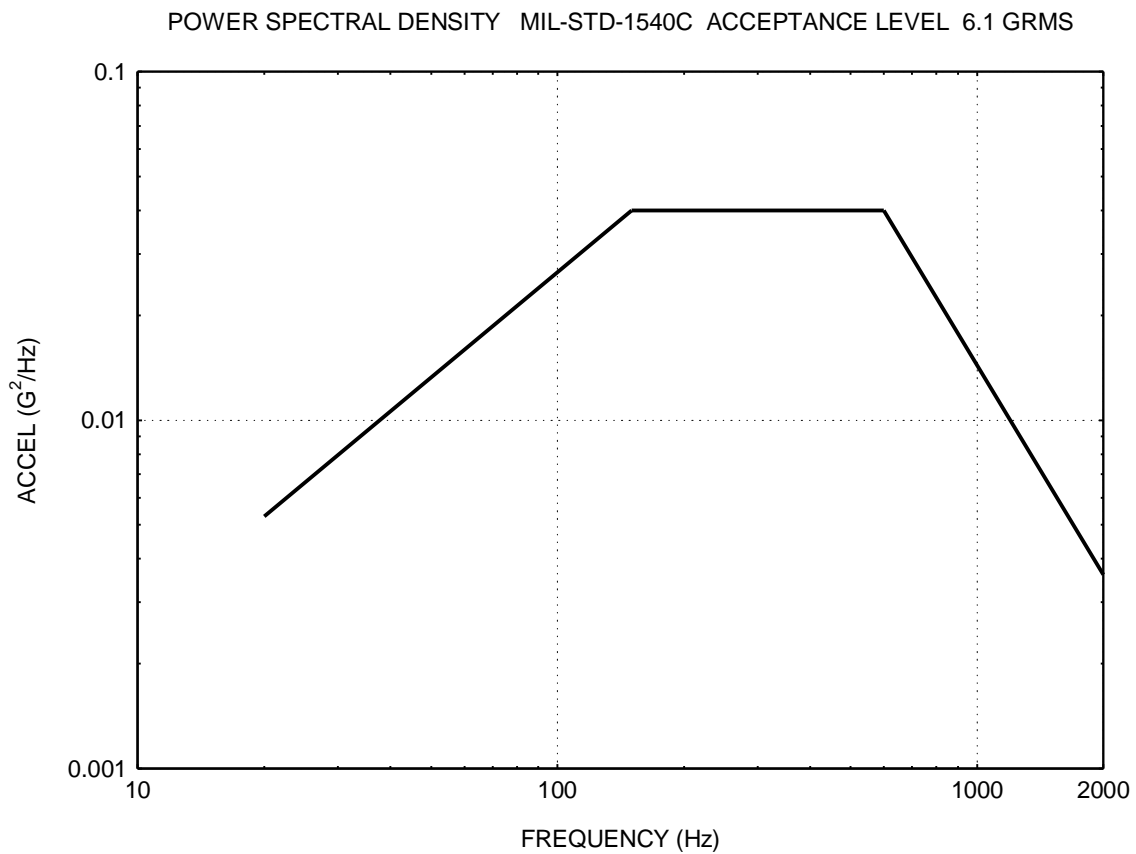


Figure 1.

Table 1. MIL-STD-1540C Acceptance Level, 6.1 GRMS Overall	
Frequency (Hz)	PSD (G^2/Hz)
20	0.0053
150	0.04
600	0.04
2000	0.0036

Previous Units gave methods for determining the response of a single-degree-of-freedom system to a base input power spectral density, such as Figure 1. The specific tools were the Miles equation and the general method.

Indeed, vibration analyses are often performed in the frequency domain. Nevertheless, there are certain situations that require analysis in the time domain. In these situations, the analyst may need to convert a power spectral density specification into an equivalent time history.

Furthermore, consider a closed-loop random vibration test where a computer controls a shaker to a specified power spectral density, such as Figure 1. The control software must synthesize a corresponding time history. Control computers thus have built-in synthesis algorithms

The purpose of this Unit is to give a method for synthesizing a time history to satisfy a power spectral density specification. The time history will be composed of a series of sinusoids.

Phase Angles

As an important note, a given time history has a unique power spectral density.

On the other hand, a given power spectral density does not have a unique time history. The reason is that the phase angles are discarded in the power spectral density calculation.

Synthesis Steps

The synthesis steps are outlined in Table 2.

This synthesis method yields a time history that is, in reality, periodic or deterministic. Nevertheless, the goal is to obtain a time history that reasonably resembles a random signal. Obviously, the time history must also satisfy the specified power spectral density.

Table 2. Synthesis Steps	
Step	Description
1	Choose the frequencies f_i . These frequencies can be chosen arbitrarily. For simplicity, constant frequency spacing is recommended.
2	Choose the phase angles ϕ_i . Typically, the phase angles are generated using a random number routine.
3	<p>Calculate the amplitudes A_i from the specified power spectral density. Let P_i represent the power spectral density amplitude at frequency index i. Note that the units of P_i are G^2/Hz. Furthermore, the G is actually GRMS.</p> $A_i = \sqrt{2} \sqrt{P_i \Delta f_i}$ <p>Δf_i is the frequency bandwidth associated with each frequency f_i. Again, a simple approach is to make the bandwidth a constant term.</p>
4	<p>Sum the individual spectral components for each time t of interest. Choose a constant sampling rate. The sampling rate should be ten times the highest spectral frequency. The acceleration is $Y(t)$ as given by the following formula.</p> $Y(t) = \sum_{i=1}^n A_i \sin(2\pi f_i t + \phi_i)$
5	Judge the quality of the time history. Take a histogram. The histogram should resemble a Gaussian bell-shaped curve. Also, take the kurtosis value, as explained in Appendix A. The kurtosis should be approximately 3.0.
6	Calculate the power spectral density of $Y(t)$ to verify compliance with the specification.

Synthesis with a Coarse Frequency Resolution

Recall the power spectral density specification from Figure 1. A proposed synthesized time history is shown in Figure 2a. The duration is 5 seconds. A close-up view of the time history is given in Figure 2b.

The time history was made using a frequency resolution of 10 Hz. Thus, there is a sinusoidal function defined at each 10 Hz increment.

Obviously, the signal has a periodic appearance.

SYNTHESIZED TIME HISTORY WITH $\Delta f = 10$ Hz

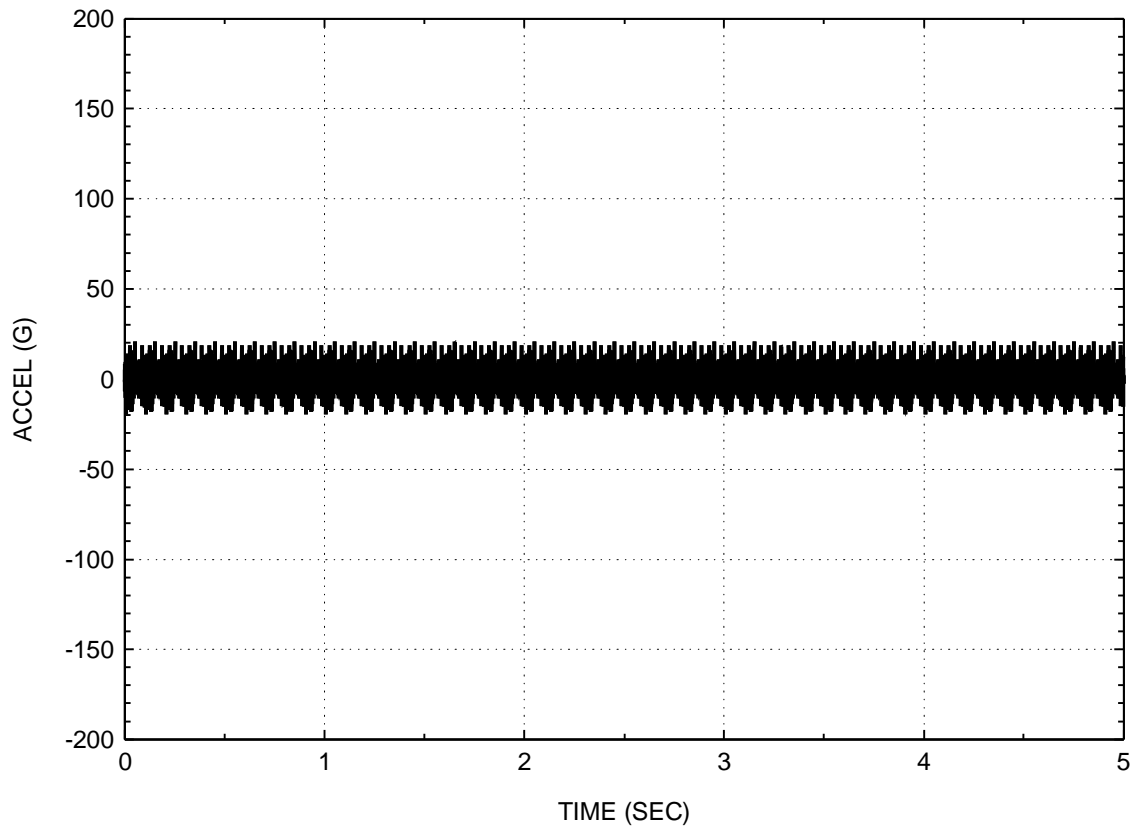


Figure 2a.

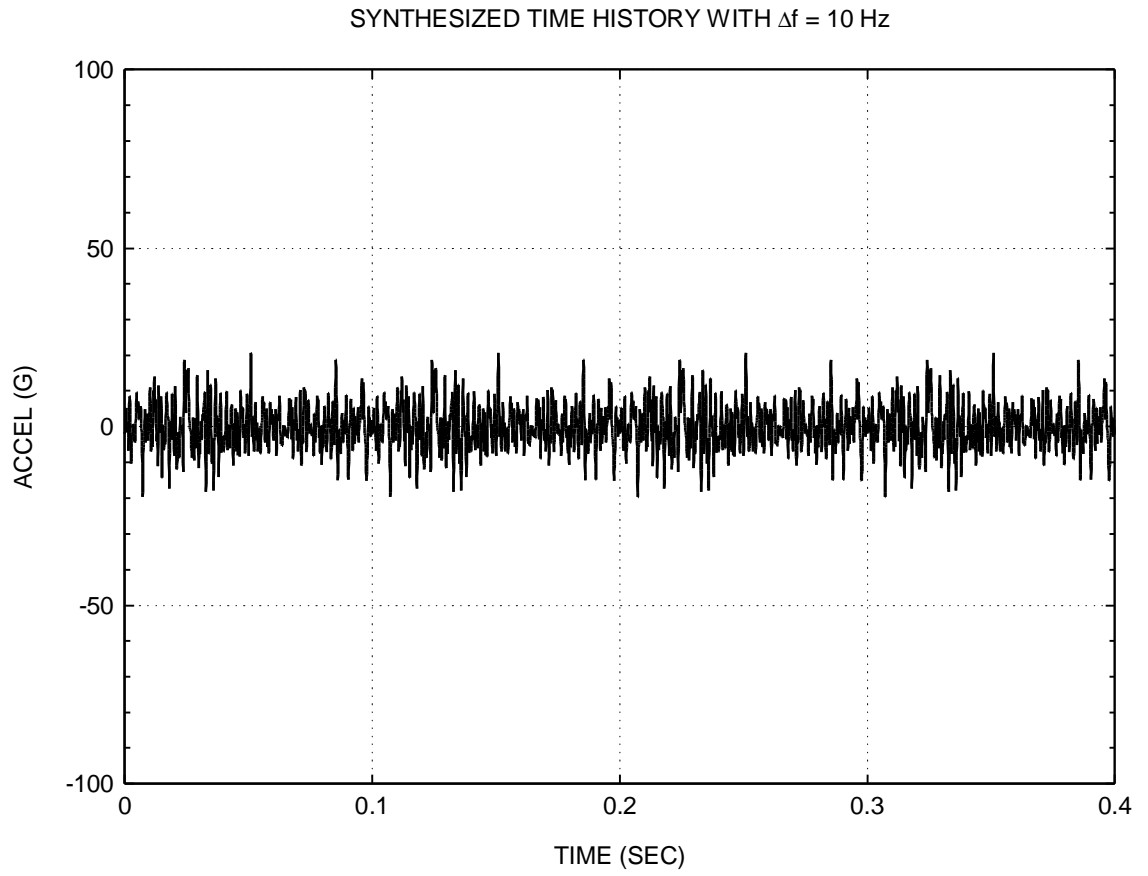


Figure 2b.

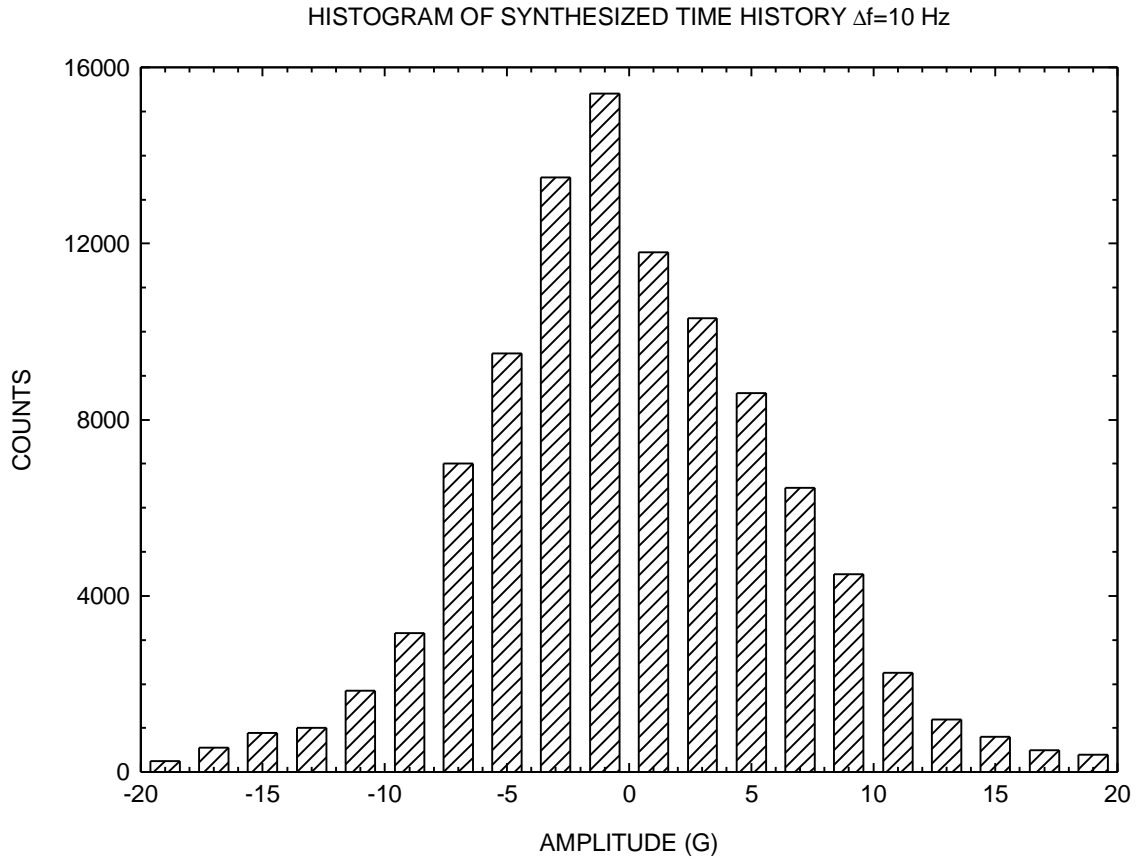


Figure 3.

The histogram of the synthesized time history is given in Figure 3. The function is somewhat skewed to the negative side. It thus departs somewhat from the bell-curve ideal.

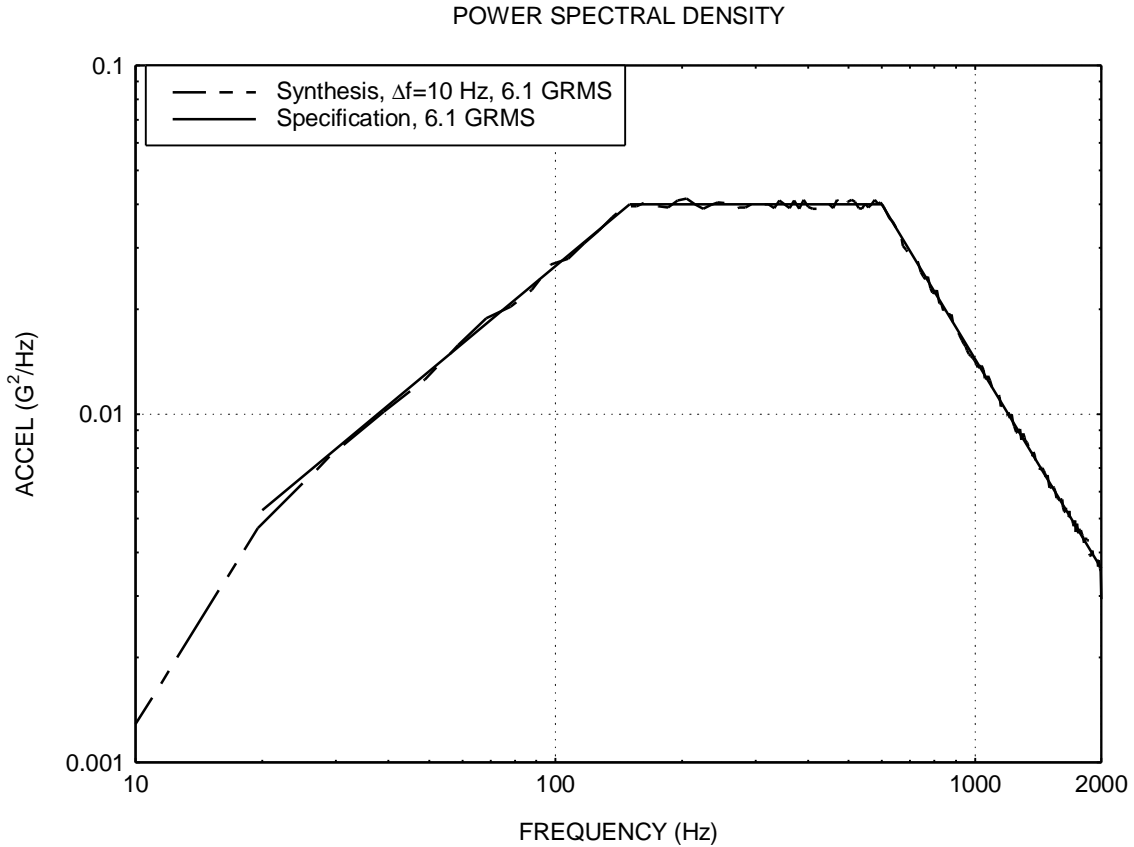


Figure 4.

The power spectral density function of the synthesized time history is given in Figure 4. It agrees well with the specification.

Note that the power spectral density function has a bandwidth of 9.77 Hz. On the other hand, the time history has sinusoids at each 10 Hz increment.

The descriptive statistics of the time history are given in Table 3.

Table 3. Synthesized Time History with $\Delta f = 10$ Hz	
Parameter	Value
Overall GRMS	6.1
Kurtosis	3.4

Again, the ideal kurtosis value for random vibration is 3.0.

Synthesis with a Fine Frequency Resolution

Another proposed time history is given in Figure 5a. A close-up view is given in Figure 5b. The frequency resolution is 0.1 Hz, which is 100 times finer than the previous example. A sinusoidal function is thus defined at each 0.1 Hz increment.

The time history has a random appearance.

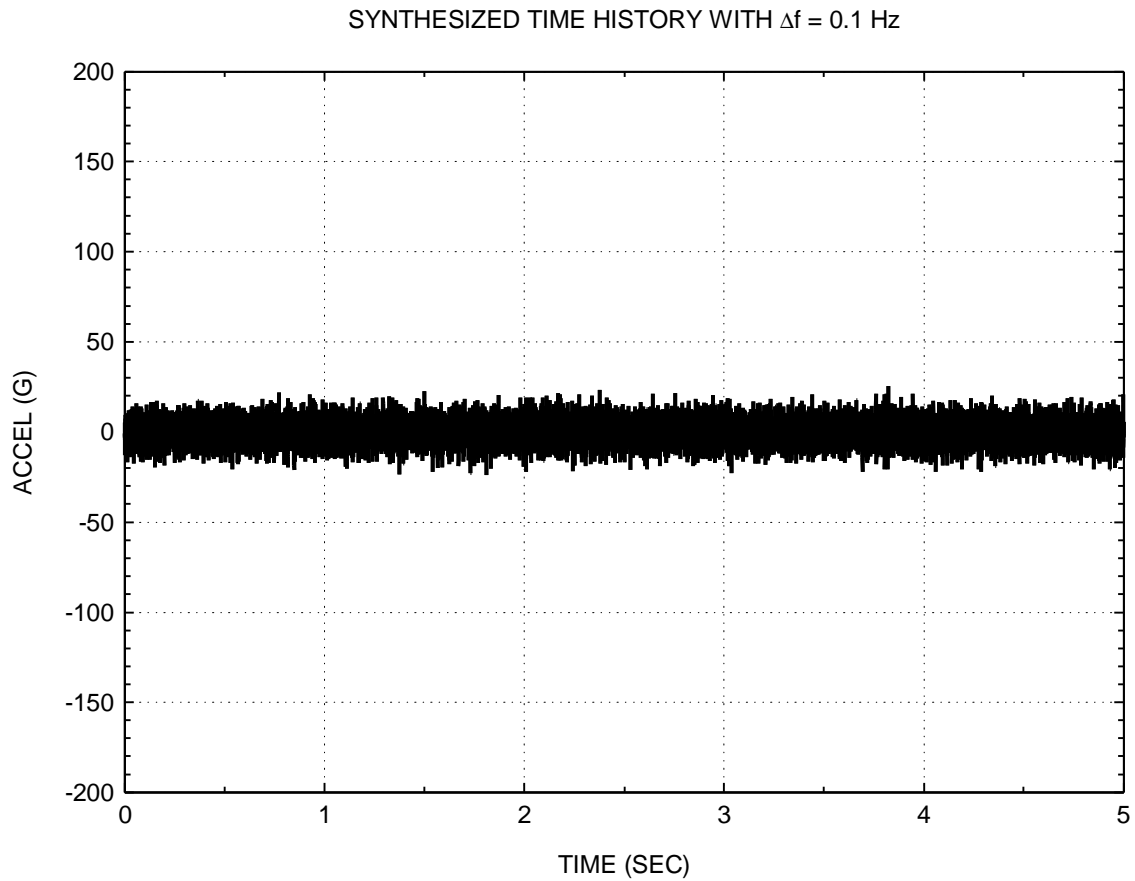


Figure 5a.

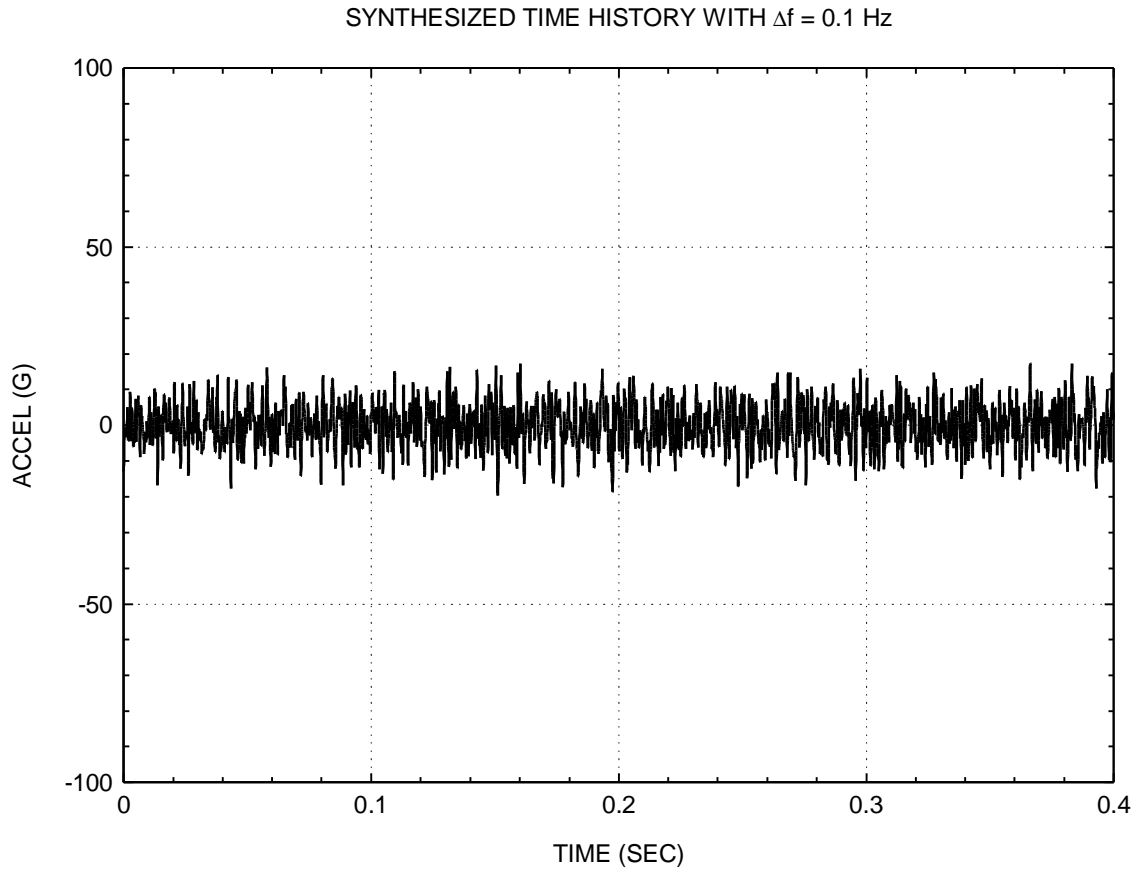


Figure 5b.

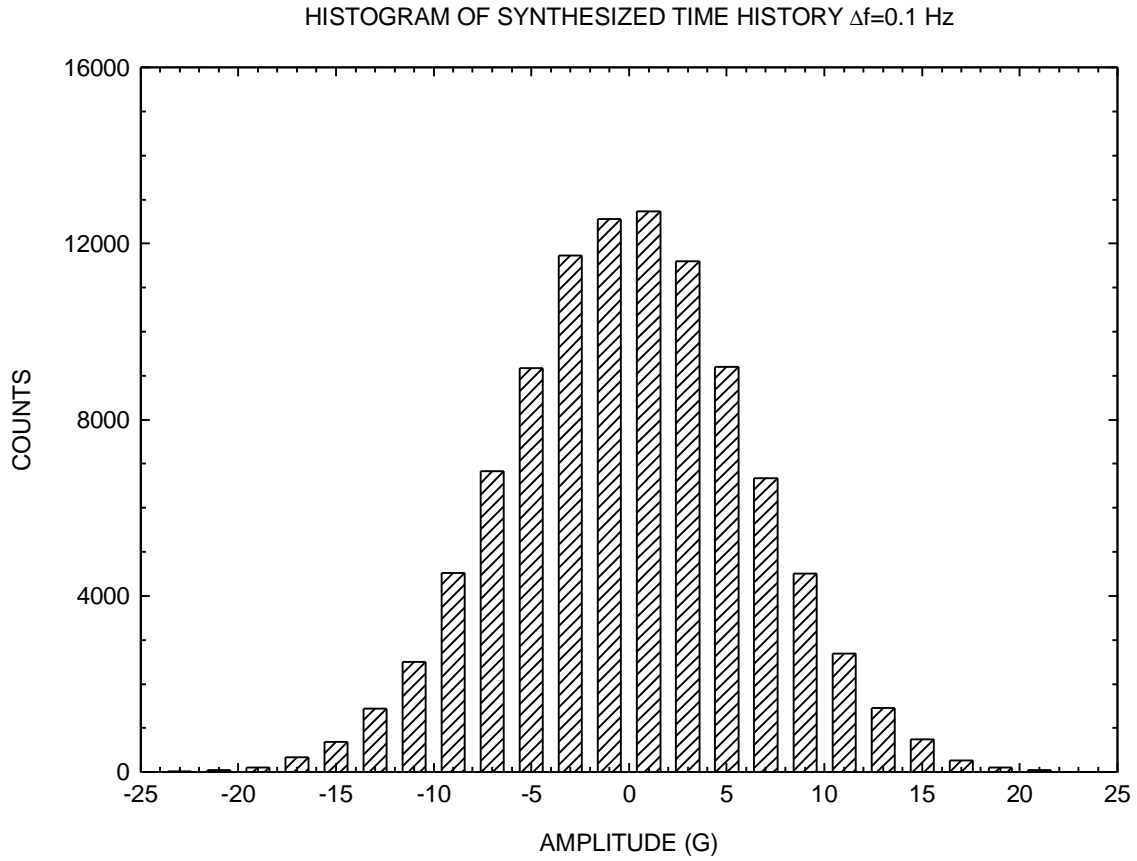


Figure 6.

The histogram of the 0.1 Hz synthesized time history is shown in Figure 6. The histogram has a bell-curve appearance. The histogram in Figure 6 is much closer to the bell-shape curve ideal than the histogram of the 10 Hz synthesis, as was shown in Figure 4.

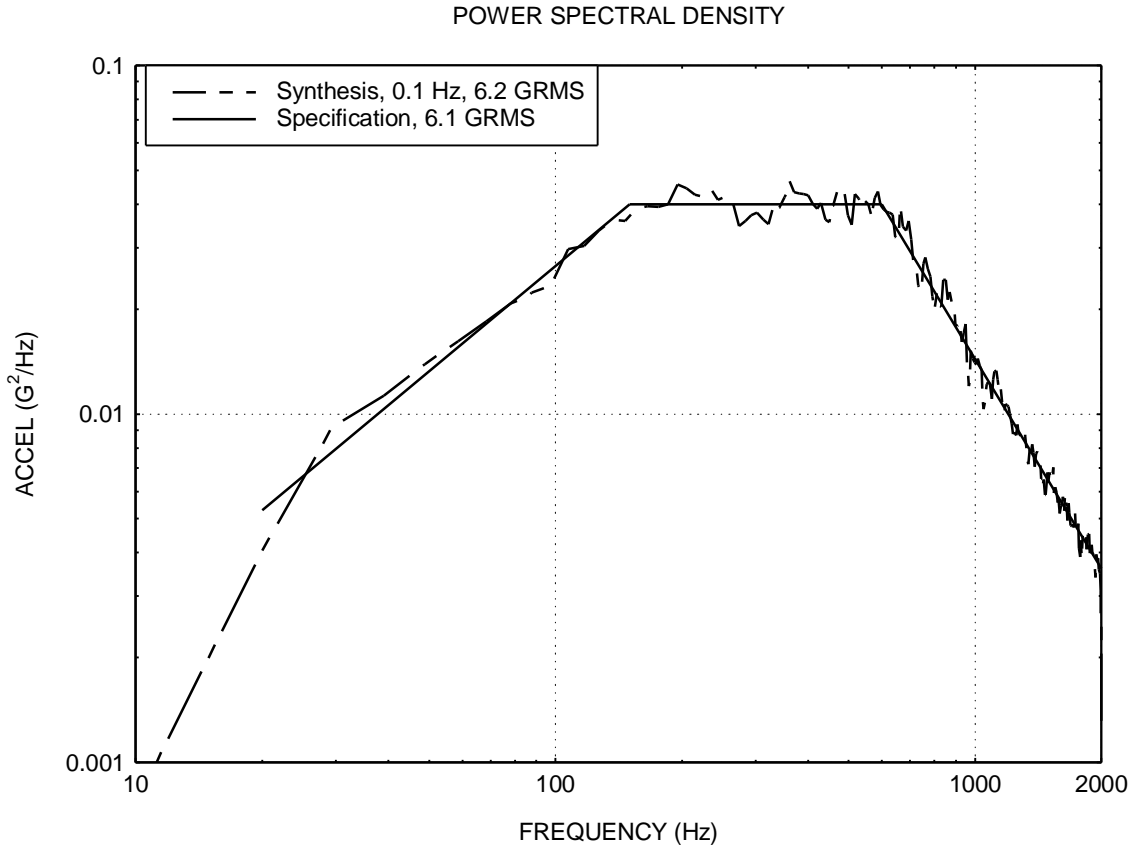


Figure 7.

The power spectral density is given in Figure 7. The 0.1 Hz synthesis has a good match with the specification, although the 10 Hz synthesis gave a closer match as shown in Figure 4.

The descriptive statistics of the 0.1 Hz synthesis are given in Table 4.

Table 4. Synthesized Time History with $\Delta f = 0.1$ Hz	
Parameter	Value
Overall GRMS	6.2
Kurtosis	2.998

The measured kurtosis value nearly meets the 3.0 ideal.

Overall, the 0.1 Hz synthesis is a very good approximation of a random signal, which meets the specified power spectral density.

Conclusion

This Unit has given a method for synthesizing a time history to meet a specified power spectral density function. The synthesized time history is periodic rather than random. Nevertheless, a periodic time history can be made to appear random by using a very fine frequency resolution.

The drawback to this method is excessive computational time. For the second example, nearly 2 billion data points were required in the computation.

$$\left[20,000 \frac{\text{samples}}{\text{sec}} \right] \left[5 \text{ sec} \right] \left[\frac{(2000 - 20) \text{ Hz}}{0.1 \text{ Hz}} \right] \approx 2 \text{ billion}$$

A more efficient algorithm will be considered in Unit 13.

Homework

1. Calculate the overall GRMS level of the power spectral density Table 5 using program psdint.exe. Plot the power spectral density.

Frequency (Hz)	Level (G ² /Hz)
10	0.001
40	0.2
60	0.2
100	0.01
200	0.01

2. Use program psd_sine.exe to synthesize a time history to satisfy the power spectral density in Table 5. Use the parameters in Table 6. Plot the time history.

Duration	4 seconds
Sampling Rate	2000 samples/sec
Frequency Resolution	0.1 Hz

3. Use program maxfind.exe to calculate the GRMS and kurtosis of the time history. Compare the GRMS value of the time history with the GRMS value of the power spectral density.
4. Calculate and plot the power spectral density of the synthesized time history. Use program poweri.exe. Superimpose this time history against the level in Table 1.
5. Calculate and plot the histogram of the synthesized time history using program histogra.exe. Use a delta x value of 2.

APPENDIX A

Kurtosis Theory

Kurtosis is a measurement of the “peakedness” of the data. A sine wave, for example, has a kurtosis value of 1.5. A pure Gaussian random signal has a kurtosis value of 3.0. A higher kurtosis value may indicate the presence of higher sigma peaks on the time history than would be expected from a Gaussian distribution. These peaks could, for example, represent a transient event. On the other hand, they may simply represent random vibration that has a probability density function that departs from the Gaussian ideal.

The equation is:

$$\text{kurtosis} = \frac{\sum (x - \bar{X})^4}{n s^4} ,$$

where n = number of samples,
 s = standard deviation,
 \bar{X} = mean value.

Note that the kurtosis value appears dimensionless.