SHOCK AND VIBRATION RESPONSE SPECTRA COURSE Unit 14.
Integration of an Acceleration Time History to Determine Displacement and Velocity

By Tom Irvine
Email: tomirvine@aol.com

Introduction

Mechanical vibration is usually characterized in terms of acceleration. The main reason is that acceleration is easier to measure than velocity or displacement.

Acceleration can be measured with either a piezoelectric or piezoresistive accelerometer.

Velocity measurements require a Doppler laser or a geophone. The laser is expensive, and it requires a direct line of sight. The geophone is bulky and is intended for seismology measurements.

Dynamic displacement can be measured by a linear variable displacement transducer (LVDT). Unfortunately, the frequency response is only suited for low-frequency measurements.

If velocity is required, the typical method is to integrate the acceleration signal. If displacement is required, the typical method is to double-integrate the acceleration signal. The integration can be performed either in the time domain or frequency domain.

This tutorial focuses on time domain integration of a digital signal.

Motivation

There are at least two situations in which velocity time history measurement would be required. The first occurs in pyrotechnic shock measurement. The second regards design criteria.

Pyrotechnic Shock

Pyrotechnic shock is almost always measured in terms of acceleration by piezoelectric accelerometers. The shock pulse can cause the piezoelectric crystal to saturate, however, thereby degrading or corrupting the accelerometer signal. Specifically, the acceleration time history may have a spurious offset due to the saturation effect. This symptom is usually apparent in the acceleration time history but becomes exaggerated in the velocity time history. Thus, velocity time histories are used to judge the validity of acceleration time histories measured during pyrotechnic events. Reference 1 gives further information on this technique.

In addition, some experts consider that the damage potential of pyrotechnic shock has a stronger correlation with velocity than with acceleration.

Design Criteria

Furthermore, certain specifications and criteria are given in terms of velocity rather than acceleration. Velocity seems to be the “metric” of choice for building vibration limits. For example, Colin Gordon has established a generic vibration criteria for floor vibration in terms of velocity. His paper is included in this Unit as homework assignment 1.

Gordon actually gives the floor limits in terms of one-third octave band velocity spectra. Thus, the floor vibration could be measured by an accelerometer. The acceleration signal could then be integrated to determine the velocity time history. The next step would be to calculate the velocity Fourier transform magnitude for each band.

Integration Method

The integration of a digital signal is typically performed using the trapezoidal rule. The trapezoidal rule for a continuous function f(x) over $a \leq x \leq b$ is

$$\int_a^b f(x)dx = \frac{h}{2}\left[f(x_o) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n) \right] \quad (1)$$

where

h = (b-a)/n
n = the number of intervals.

Note that the integration of a time history is carried out on a "running sum" basis. Let the acceleration time history be represented by a_1, a_2, a_3, . . . , a_n. The velocity time history is calculated as follows.

$$v_1 = \frac{\Delta t}{2}\left[a_1 \right] \quad (2)$$

$$v_2 = v_1 + \Delta t \left[a_2 \right] \quad (3)$$

$$v_3 = v_2 + \Delta t \left[a_2 \right] \quad (4)$$

In general, the velocity at points inside the limits is given by

$$v_j = v_{j-1} + \Delta t \left\lfloor a_j \right\rfloor , \quad \text{for } 1 < j < n \quad (5)$$

Finally,

$$v_n = v_{n-1} + \frac{\Delta t}{2}\left[a_n \right] \quad (6)$$

Note that Δt is the time step.

More accurate methods, such as Simpson's rule, would yield better accuracy. Nevertheless, the challenge in vibration analysis is dealing with spurious offsets rather than achieving high degrees of numerical precision.

Preprocessing and Postprocessing

Typically, the acceleration signal must be highpass filtered prior to integration in order to yield a stable velocity signal. Filtering will be covered in an upcoming Unit.

Another method is to perform a trend removal prior to integration. This method will be demonstrated in the following example.

Furthermore, postprocessing of the signal may also be required, as shown in the following example. The reason is that the integration procedure effectively calculates the change in velocity. Thus, the user must supply the initial velocity. This may be done via postprocessing.

Example

Consider the following acceleration time history, consisting of a sine function with an offset.

$$a(t) = 1 + 1 \sin\left[(2\pi)(1 \text{ Hz})\, t\right] \quad \text{G} \tag{7}$$

The function is shown in Figure 1 for duration of 10 seconds.

Assume that the 1 G offset is due to a spurious effect within the instrumentation.

Now integrate the acceleration time history to obtain the velocity time history. The resulting velocity signal is shown in Figure 2. The velocity signal is unstable because the integration procedure greatly exaggerates the offset by transforming it into a ramp.

Now remove the offset. Then repeat the integration. The resulting velocity signal is shown in Figure 3. The signal is now stable, but note that it has a mean value of 61.4 in/sec. This offset implies rigid-body motion. This offset must now be removed in order to obtain a velocity signal with zero mean.

The resulting velocity signal with zero mean is shown in Figure 4.

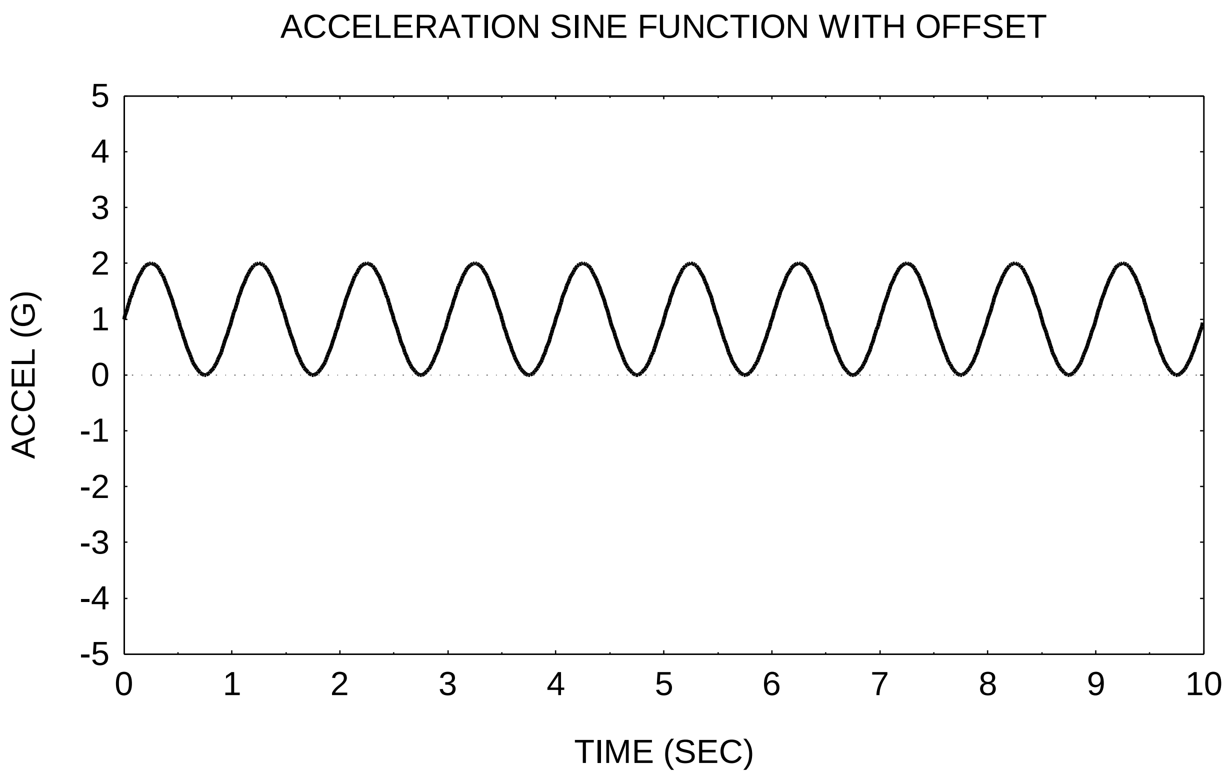

Figure 1.

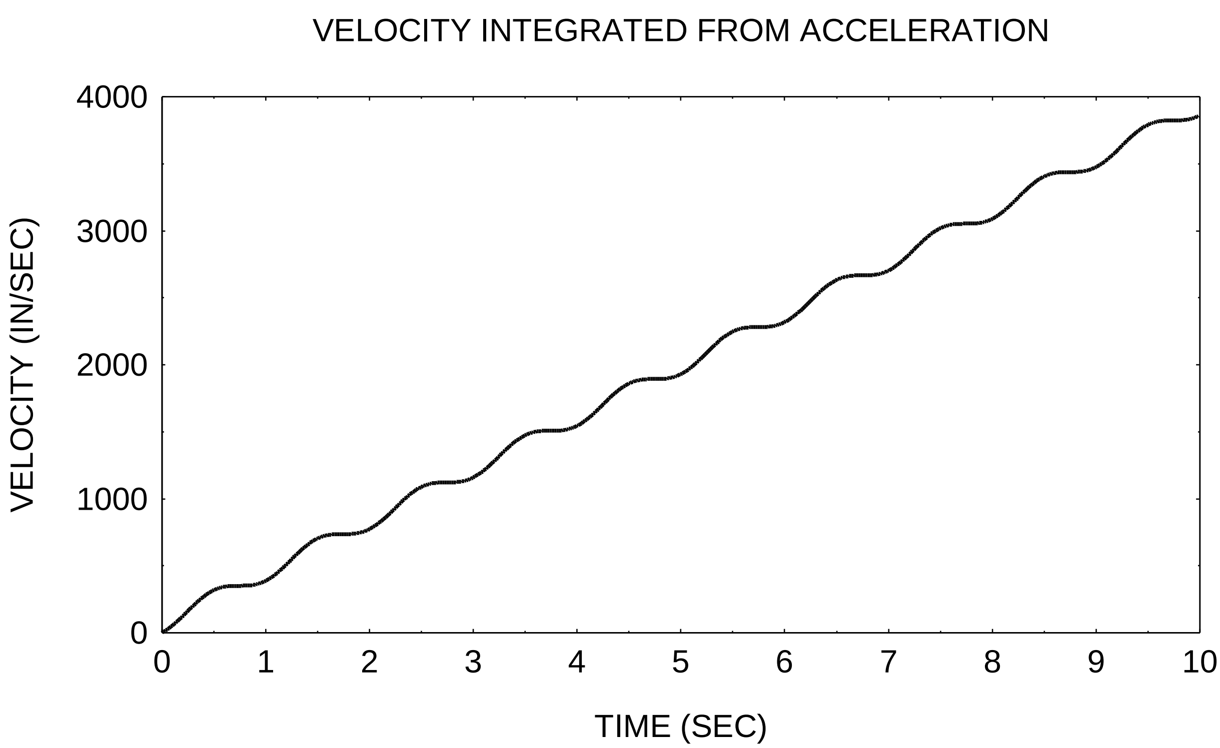

Figure 2.

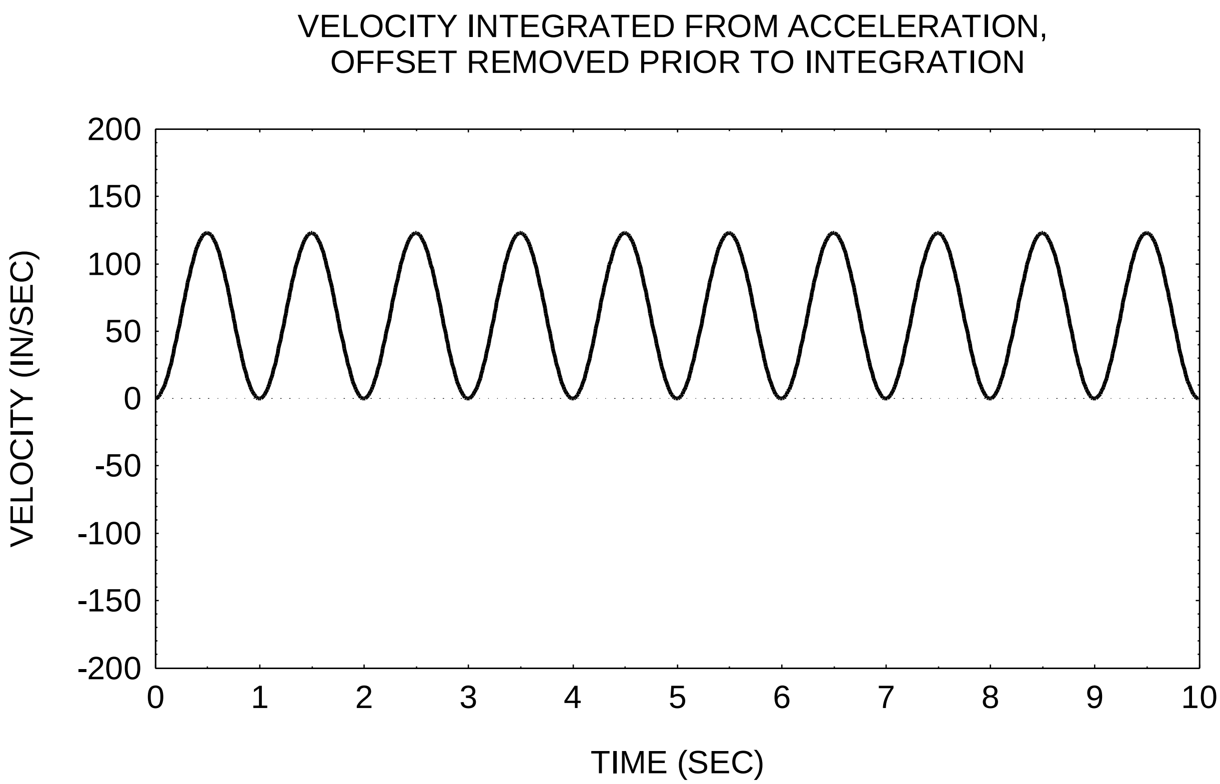

Figure 3.

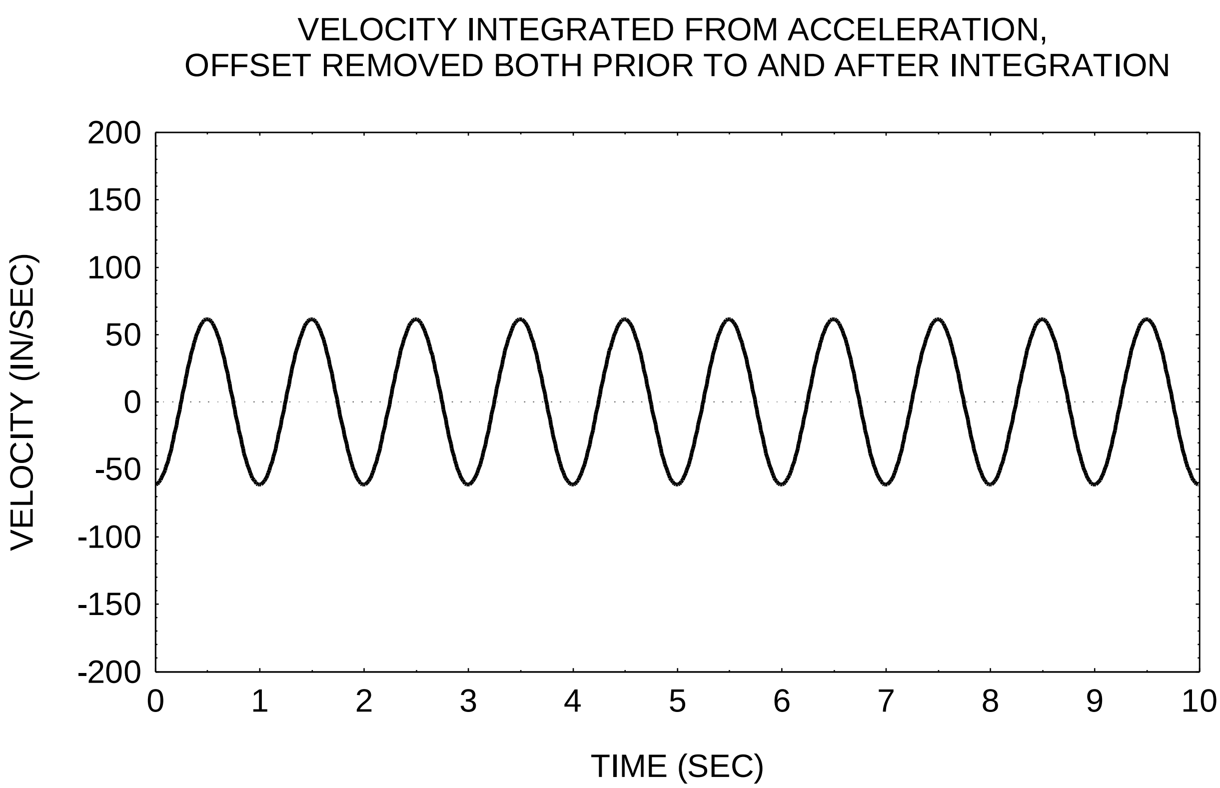

Figure 4.

Units Note

Acceleration is typically represented in units of G. The integrated signal has units of (G sec). The integrated signal must be multiplied by 386 to convert the units to in/sec.

Displacement

Double integrating a digital acceleration signal to obtain displacement is difficult because excessive highpass filtering and/or trend removal is often required.

A better method is to perform the double integration on the analog signal, prior to digitization. A signal conditioner with integration circuitry is required for this operation.

Reference

1. Himelblau, Piersol, et al., IES Recommended Practice 012.1: Handbook for Dynamic Data Acquisition and Analysis, Institute of Environmental Sciences and Technology, Mount Prospect, Illinois.

Homework

1. Read Colin Gordon's paper, "Generic Vibration Criteria for Vibration-Sensitive Equipment."
2. Plot the acceleration time history pyro.txt. The file has two columns: time(sec) and accel (G). This is actual data measured during a pyrotechnic shock test on a rocket vehicle.
3. Use program integ.exe to integrate pyro.txt
4. Use program mult2.exe to multiply the integrated time history by 386. The resulting file should have two columns: time(sec) and velocity (in/sec). Plot the velocity file.
5. The resulting velocity signal in step 4 has an unstable velocity. Use program zeromean.exe to remove the offset from pyro.txt. Call the output file: pyrozero.dat.
6. Repeat steps 3 and 4 using file pyrozero.dat.

Note that the resulting velocity signal in step 6 is an improvement. Nevertheless, it seems to still contain some questionable low-frequency energy. Actually, highpass filtering would be a better choice than simple mean removal for this case. Again, filtering will be taught in a future Unit. File pyro.txt will be revisited in that Unit.