SHOCK AND VIBRATION RESPONSE SPECTRA COURSE Unit 16.
Sample Rate Criteria and the Nyquist Rule

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Introduction

Engineers collect accelerometer data in a variety of settings. Examples include:

1. Aerospace vehicle flight data
2. Automotive proving grounds
3. Machinery condition monitoring
4. Building response to seismic excitation
5. Modal testing of structures

The accelerometers measure the data in analog form. The analog signal is sent through a signal conditioner. The signal conditioner may have an analog lowpass filter. Filtering will be covered more extensively in Unit 18.

An understanding of sample rate criteria requires some preliminary consideration of filtering, however. Lowpass filtering of the *analog* signal is necessary to prevent an error source called aliasing. Aliasing is covered in Unit 17.

Eventually, the accelerometer data is passed through an analog-to-digital converter. The proper sampling rate must be selected to ensure that the digitized data is accurate.

This Unit gives guidelines for choosing the sampling rate. It also briefly covers amplitude resolution.

Sampling Rate, First Requirement

The first requirement is that the sampling rate must be greater than the maximum analysis frequency. Industry has established guidelines for this requirement, as discussed in Reference 1. These guidelines are summarized in Table 1.

Table 1. Sampling Rate First Requirement

(minimum sampling rate) $\geq$ (N)(maximum analysis frequency)

Analysis Type	N
Frequency Domain	2
Time Domain	10

Fourier transforms and power spectral density functions are used in frequency domain analysis.

The shock response spectrum (SRS) is an example of a time domain analysis. The shock response spectrum will be covered in Units 19 through 22.

Frequency Domain

The frequency domain requirement in Table 1 is based on the fact that at least two time-domain coordinates per cycle are required to resolve a sine wave for analytical purposes.

The Nyquist frequency is equal to one-half the sampling rate. The frequency domain analysis thus extends up to the Nyquist frequency.

Note that some conservative sources specify a value of N = 2.5 for frequency domain analysis.

Time Domain

Reference 1 gives the following guideline:

> Unlike other spectral quantities evolving from the discrete Fourier transform computations, the SRS is essentially a time domain quantity. Hence, the digital sampling rate given by Rs=1/(delta t), introduces errors beyond those associated with aliasing about the Nyquist frequency. Thus, Rs must be high enough to accurately describe the response of the SRS oscillators. To minimize potential error, it is recommended that the SRS computations be performed with a sampling rate of Rs $\geq$ 10 fh, where fh is the highest natural frequency of the SRS computation.

A sampling rate of 100,000 samples per second is thus required for a shock response spectrum analysis extending to 10,000 Hz per this guideline. Again, the shock response spectrum is calculated in the time domain.

Sampling Rate, Second Requirement

The second requirement is that the sampling rate must be greater than the maximum frequency present in the source energy at the measurement location. This requirement is necessary to prevent aliasing.

The guidelines for the second requirement are summarized in Table 2

Table 2. Sample Rate Second Requirement (minimum sampling rate) $\geq$ (M)(maximum frequency in source energy)	
Analysis Type	M
Frequency Domain	2
Time Domain	10

Note the similarity between Tables 1 and 2.

Shannon's sampling theorem states that a sampled time signal must not contain components at frequencies above the Nyquist frequency, from Reference 2. Again, the Nyquist frequency is equal to one-half the sampling rate. Shannon's theorem applies to frequency domain analysis.

Lowpass Filtering

In many cases, the maximum expected frequency is unknown. Thus, lowpass filtering can be used as a precaution to ensure compliance with the requirement in Table 2. Filtering will be covered in Units 17 and 18.

Summary

Note that the maximum source energy frequency may be independent of the maximum analysis frequency. Thus, the first and second requirements may be independent.

A common example of this independence occurs in rocket vehicle vibration testing.

Avionics components are typically subjected to power spectral density specifications which are defined up to 2000 Hz. The test specifications assume that the components are immune to vibration above 2000 Hz. The same specifications, however, assume that the components must be tested up to 2000 Hz to verify their integrity, even if the expected flight levels occur at a lower frequency domain.

The component test specifications are derived, in part, from measured or predicted flight levels. Note that a rocket vehicle is excited by aerodynamic turbulence and motor pressure oscillations during its powered flight. The content of this energy may be well below, or perhaps above, 2000 Hz.

An engineer designing a telemetry system must thus considered the expected flight vibration environments as well as post-flight analytical needs.

Amplitude Resolution

Amplitude resolution is considered in this Unit as a supplementary topic.

Analog-to-digital conversion systems have amplitude resolution, which is measured in bits.

The amplitude resolution is one part in 2^(number of bits). Thus, a 12-bit system has a resolution of one part in 4096.

Consider a 12-bit system set up to measure a full scale amplitude of 10 V peak-to-peak. The resolution is (10 V / 4096) = 2.4 mV. This example is shown in Table 3, along with two other cases.

Table 3. Example for 10 V peak-to-peak

Bit Resolution	Voltage Resolution (mV)
8	39.1
12	2.4
16	0.15

Note that telemetry data is sometimes given in terms of bits, where the bits are in integer format. The user must apply a scale factor to the bit values. The scale factor might convert the bit values to volts, or to some engineering unit.

The voltage resolution is proportional to the G level for an accelerometer. For example, consider the following configuration:

1. An accelerometer has a 10 V peak-to-peak range.
2. The accelerometer sensitivity is 10 G/volt (0.010 G /mV).
3. The accelerometer signal is applied to a 12 bit acquisition system

In this case, the accelerometer data would have an amplitude bit resolution of 0.024 G. This would cause a measured sine wave to have a "stair-step" appearance if the peak amplitude were below, say, 0.2 G.

The bit resolution for a data acquisition system is usually fixed. The user can manipulate the resolution by choosing an accelerometer with a particular sensitivity. The user may also have control over the full-scale voltage value.

References

1. IES Handbook for Dynamic Data Acquisition and Analysis, Institute of Environmental Sciences, Illinois. *Particularly, paragraphs 3.7.2 and 5.5.3.5.*
2. R. Randall, Frequency Analysis Third edition, Bruel & Kjaer, 1987.

Homework

1. A flight accelerometer must be mounted on a rocket motor for a static fire test. The motor will be mounted to fixed frame for this test. The resulting vibration data will be transformed into power spectral density curves with an upper frequency of 2000 Hz. The highest energy component in the data is expected to be 5000 Hz, however. What is the minimum sampling rate per the guidelines given in this Unit?

2. The same data in problem 1 will be converted to a shock response spectrum. A time domain calculated is required to form this spectrum. The spectrum will have an upper frequency of 10,000 Hz. What is the recommended minimum sampling rate?

3. Some actual flight telemetry data is given in file: t_flight.txt. The data has two columns: time (sec) and amplitude (bits). The amplitude resolution is: 0.2 G/bit. Convert the amplitude column to G units and plot the data. You may use a spreadsheet to perform the conversion. Or you may use program A_scale.exe.

4. What are the dominant frequencies of the accelerometer signal in problem 3? Note that it represents a transient event which occurred during the burnout of a solid, upper-stage motor.