SHOCK AND VIBRATION RESPONSE SPECTRA COURSE Unit 17. Aliasing

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Introduction

Again, engineers collect accelerometer data in a variety of settings. Examples include:

1. Aerospace vehicle flight data
2. Automotive proving grounds
3. Machinery condition monitoring
4. Building response to earthquake and seismic excitation
5. Modal testing

The accelerometers measure the data in analog form. The accelerometer may have an integral mechanical lowpass filter. Furthermore, the signal conditioning unit may have an analog lowpass filter.

Eventually, the accelerometer data is passed through an analog-to-digital converter. The proper sampling rate must be selected to ensure that the digitized data is accurate. Sampling rate guidelines were given in Unit 16.

Lowpass filtering of the analog signal is necessary to prevent an error called aliasing.

The purpose of this Unit is to discuss aliasing.

Filtering is briefly mentioned in this Unit. The details of filtering will be covered in Unit 18.

Aliasing Examples

The following examples show the consequences of failure to comply with the sampling rate guidelines in Unit 16. An aliasing error results.

Consider a sine wave sampled at 2000 samples per second. The Nyquist frequency is thus 1000 Hz. The Nyquist frequency is also the upper limit for a frequency domain calculation, per the Unit 16 guidelines.

The power spectral density function of a 200 Hz sine wave sampled at this rate is given in Figure 1. As expected, a spectral line appears at 200 Hz.

The power spectral density of an 1800 Hz sine wave is given in Figure 2. Note that aliasing occurs. The 1800 Hz signal is folded about the Nyquist frequency such that a spectral line appears at 200 Hz. The Nyquist frequency thus forms a line of symmetry.

The power spectral density of a 200 Hz sine wave appears to equal that of a 1800 Hz sine wave. Again, this error occurs due to inadequate sampling rate.

The time histories for each of these sine waves is given in Figure 3. Note that the 1800 Hz sine wave appears to equal a 200 Hz sine wave with a 180 degree phase difference.

The alias frequency is summarized in equation (1).

$$\text{Alias frequency} = S_f - E_f, \quad \text{if} \quad \frac{1}{2} S_f < E_f < S_f \tag{1}$$

where

S_f is the sample rate

E_f is the energy frequency

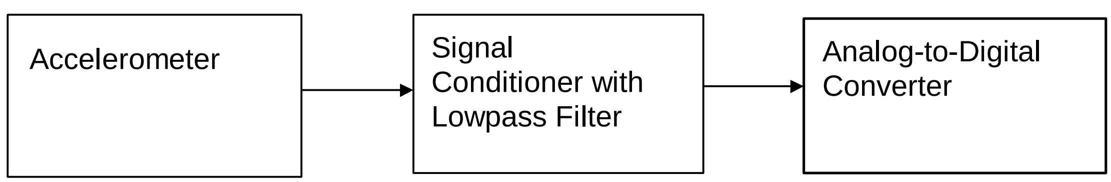
In addition, aliasing will occur if the energy frequency is above the sample rate. A separate formula is required, however.

<u>Lowpass Filtering</u>

Aliasing can be prevented by lowpass filtering the analog data.

Consider a stage separation test or a launch vehicle flight. The maximum expected frequency in the source energy is essentially unknown. Thus, there is no proper means to set the sampling rate, other than setting it at some exceedingly high value.

The simple solution is to pass the analog data through a lowpass filter as shown in the flowchart.



The lowpass filter removes the high-frequency energy from the signal. This filter is often called an "anti-aliasing" filter.

The filter can be part of the signal conditioning system. Typically, a Butterworth filter is used. The Butterworth filter has a roll-off which attenuates the signal by 3 dB at the cut-off frequency.

The cut-off frequency is typically set at, or slightly above, the maximum analysis frequency.

Recommended Filtering Parameters

Let f_C be the cutoff frequency.

Let f_N be the Nyquist frequency.

Reference 1 gives the following guidelines:

(1) A lowpass anti-aliasing filter with a cutoff rate of at least 60 dB/octave should be used for the analog-to-digital conversion of all dynamic data.

(2) With a 60 dB/octave cutoff rate, the half-power point cutoff frequency of the filter should be set at $f_C \leq 0.6 f_N$.

If the anti-aliasing filter has a more rapid cutoff rate, a higher cutoff frequency can be used, but the bound $f_C \leq 0.8 f_N$ should never be exceeded.

Telemetry Design Example

Ideally, the sampling rate could be chosen after the maximum excitation and analysis frequencies were identified. Practical considerations often require a reverse approach.

Consider a telemetry system for a launch vehicle. Several accelerometers will be mounted in the vehicle. The data will be digitized on-board the vehicle. The digitized signal will be sent via a radio link to a ground station.

The flight dynamic environments are unknown. The maximum sampling rate, however, is 4000 samples per second for each accelerometer channel. This sampling rate is constrained by the available radio link bandwidth and other considerations.

Given this constraint, choose an analog lowpass filter with a cut-off frequency at 2000 Hz. This frequency does not meet the strict guidelines in Reference 1, which would set the cut-off frequency at 1200 Hz. Some compromise is often required in telemetry system design, however. In this case, the cut-off frequency is set higher than the guidelines in order to capture additional data beyond 1200 Hz.

The lowpass filter is placed between the accelerometer and the vehicle's analog-to-digital converter.

Now consider that the vehicle has flown and the digital data has been received at the ground station.

Power spectral density functions of the flight data can be calculated up to 2000 Hz, per Table 1. Some roll-off may appear starting at about 1600 Hz depending on the filter characteristics, but this is a practical trade-off.

Table 1. Sampling Rate First Requirement

(minimum sampling rate) $\geq$ (N)(maximum analysis frequency)

Analysis Type	N
Frequency Domain	2
Time Domain	10

Recall that Fourier transforms and the power spectral density functions are used in frequency domain analysis.

On the other hand, the shock response spectrum is a time domain function.

Shock response spectra of the flight data can be calculated accurately up to 200 Hz, per Table 1. This frequency can be extended somewhat if greater error margins are allowed.

This telemetry system will thus yield usable vibration data.

On the other hand, the telemetry system will yield marginal shock data. The resulting shock data may be adequate to characterize motor ignition and launch shock, which are typically dominated by energy below 2000 Hz. Unfortunately, the telemetry data will be inadequate to characterize high-frequency pyrotechnic shock from stage separation events.

Stage separation shock must thus be measured during ground development tests prior to flight. Data acquisition systems with high sampling rates can be used during ground tests.

<u>Reference</u>

1. IES Handbook for Dynamic Data Acquisition and Analysis, Institute of Environmental Sciences, Illinois.

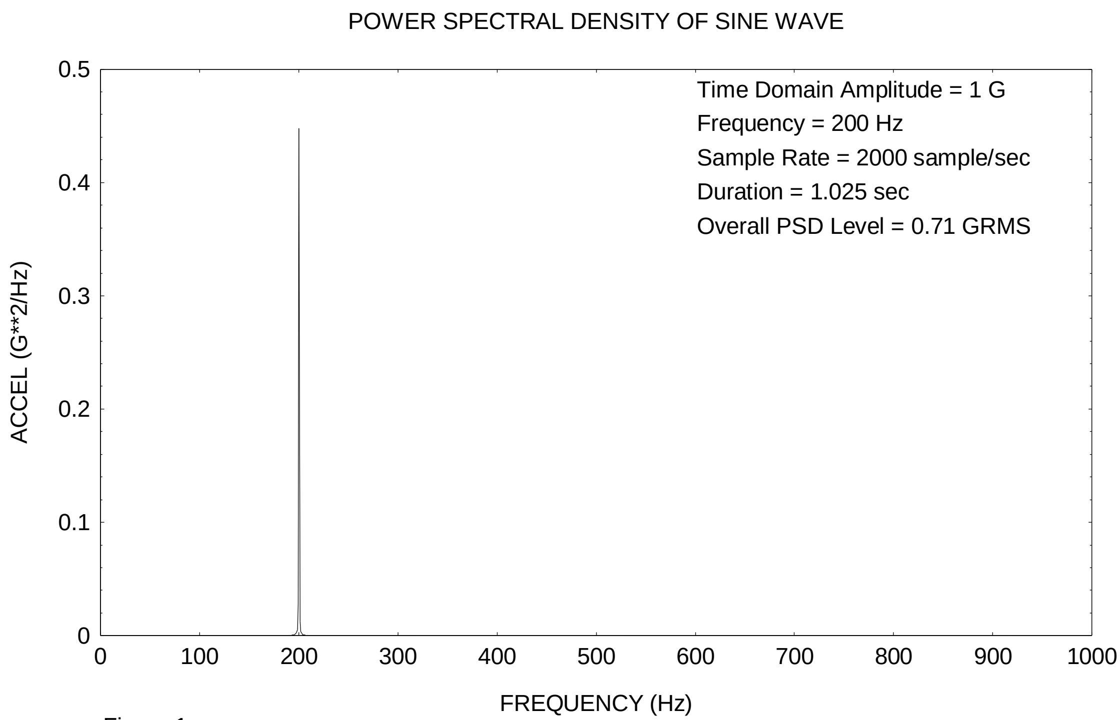


Figure 1.

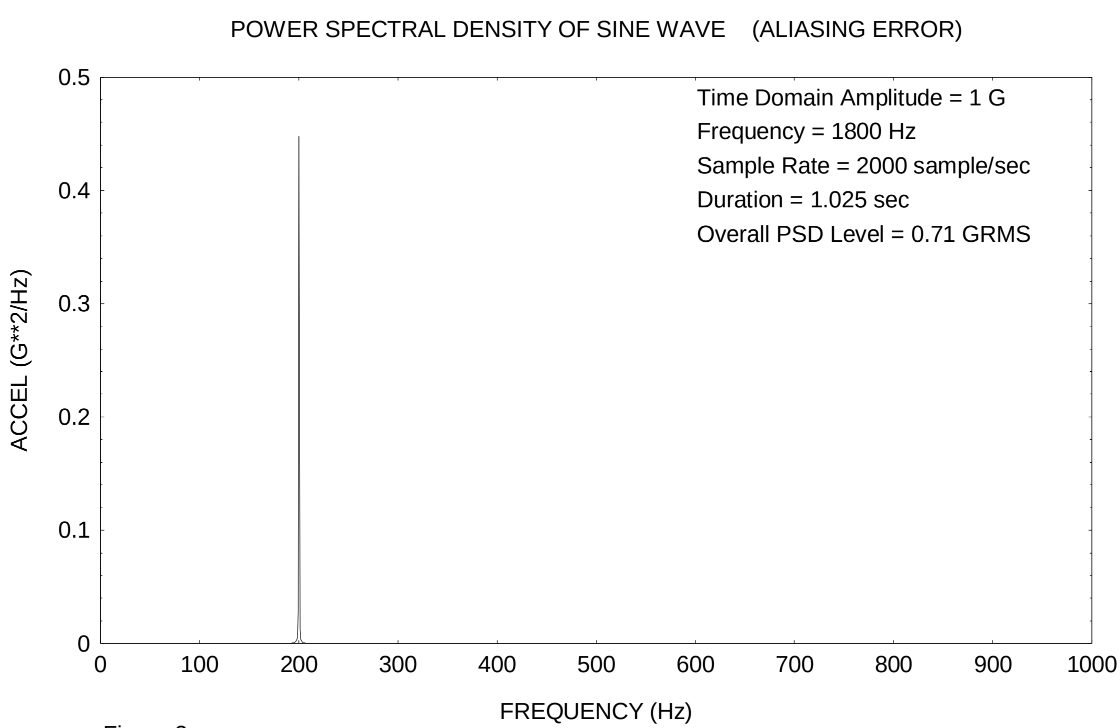


Figure 2.

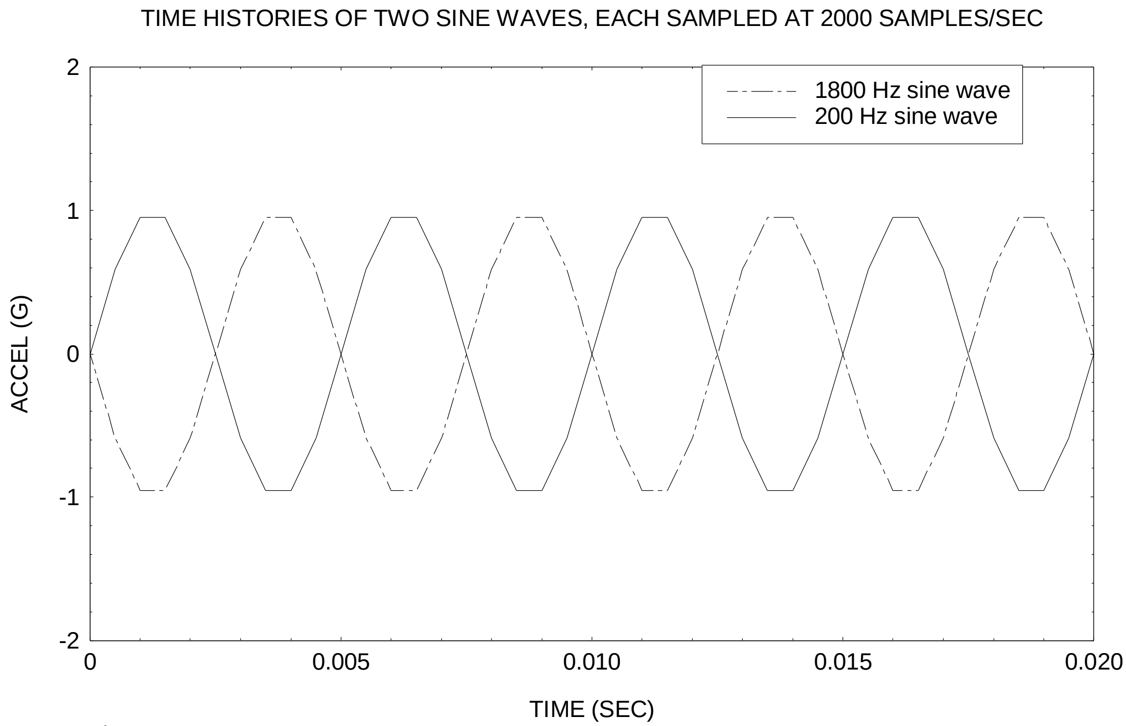


Figure 3.

Homework

1. A telemetry system has a sampling rate of 600 samples per second. It has no lowpass filtering. The system measures a sinusoidal time history with an apparent frequency of 150 Hz.

Assume that the signal may have an aliasing error. Assume that the true frequency would be greater than the Nyquist frequency but less than the sampling rate. What is the possible frequency of the true energy signal?

Note: discerning the true frequency in the above example may be impossible. Additional, data or analysis would be required to address the problem.

2. The student is highly encouraged to obtain a copy of Reference 1, available from:
   http://www.iest.org

3. Watch an old western movie. Notice whether the stagecoach wheels appear to rotate backwards. If so, this is a visual form of aliasing.

4. Set a ceiling fan at is lowest speed. Blink your eyes. Experiment with different blink rates. If the fan appears to rotate backwards, this is another form of aliasing.