

SHOCK AND VIBRATION RESPONSE SPECTRA COURSE Unit 18.

Filtering

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Introduction

Filtering is a tool for resolving signals. Filtering can be performed on either analog or digital signals.

Furthermore, filtering can be used for a number of purposes.

For example, analog signals are typically routed through a lowpass filter prior to analog-to-digital conversion. The lowpass filter in this case is designed to prevent an aliasing error. This is an error whereby high frequency spectral components are added to lower frequencies.

Another purpose of filtering is to clarify resonant behavior by attenuating the energy at frequencies away from the resonance.

Filtering theory is discussed in Reference 1. The student may read this reference at his or her own leisure.

This Unit is concerned with practical application and examples. It covers filtering in the time domain using a digital Butterworth filter. This filter is implemented using a digital recursive equation in the time domain.

Highpass and Lowpass Filters

A highpass filter is a filter which allows the high-frequency energy to pass through. It is thus used to remove low-frequency energy from a signal.

A lowpass filter is a filter which allows the low-frequency energy to pass through. It is thus used to remove high-frequency energy from a signal.

A bandpass filter may be constructed by using a highpass filter and lowpass filter in series.

Butterworth Filter Characteristics

A Butterworth filter is one of several common infinite impulse response (IIR) filters. Other filters in this group include Bessel and Chebyshev filters. In addition, these filters are classified as feedback filters.

The Butterworth filter can be used either for highpass, lowpass, or bandpass filtering.

A Butterworth filter is characterized by its cut-off frequency. The cut-off frequency is the frequency at which the corresponding transfer function magnitude is -3 dB, equivalent to 0.707.

A Butterworth filter is also characterized by its order. A sixth-order Butterworth filter is the filter of choice for this Unit. Further details on the significance of order are given in Reference 1.

A property of Butterworth filters is that the transfer magnitude is -3 dB at the cut-off frequency regardless of the order. Other filter types, such as Bessel, do not share this characteristic, however.

Consider a lowpass, sixth-order Butterworth filter with a cut-off frequency of 100 Hz. The corresponding transfer function magnitude is given in Figure 1.

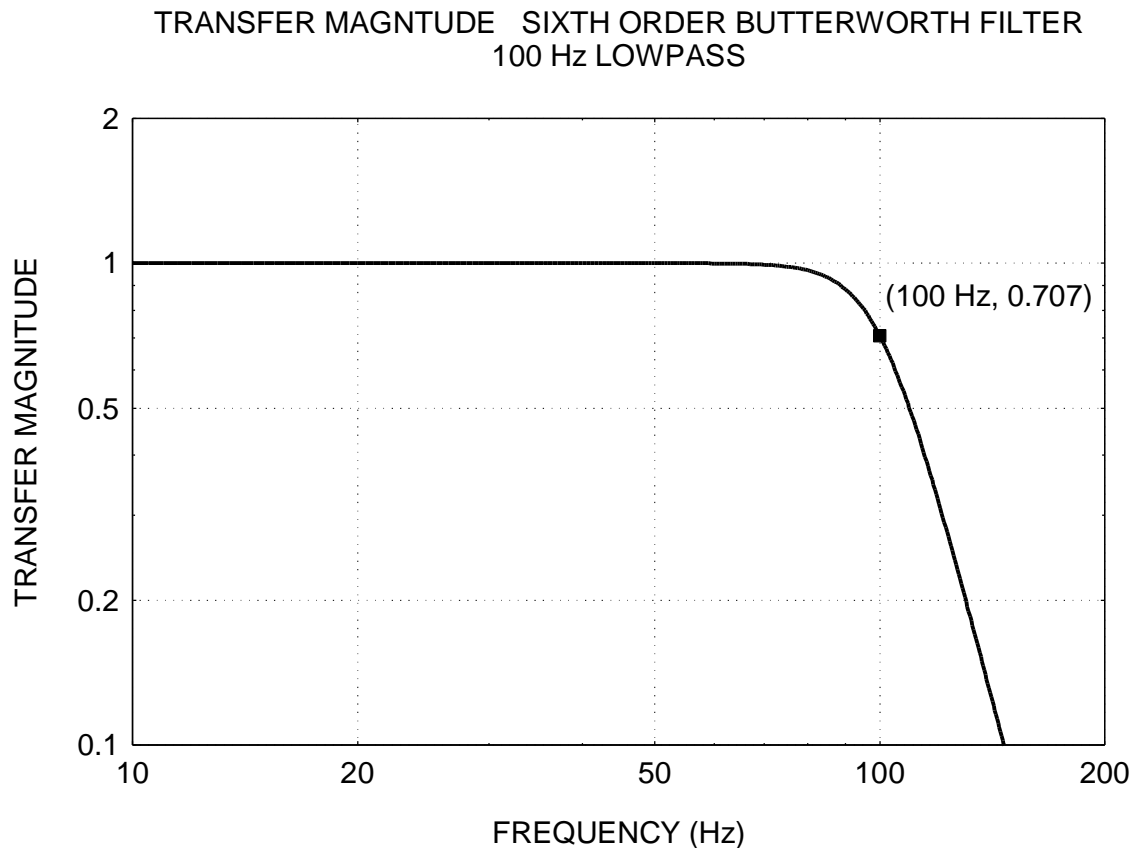


Figure 1.

Note that the curve in Figure 1 has a gradual roll-off beginning at about 70 Hz. Ideally, the transfer function would have a rectangular shape, with a corner at (100 Hz, 1.00). This ideal is never realized in practice, however. Thus, a compromise is usually required to select the cut-off frequency.

The transfer function in Figure 1 also has a corresponding phase relationship, but this is not shown. The transfer function could also be represented in terms of a complex function, with real and imaginary components.

A transfer function magnitude plot for a sixth-order Butterworth filter with a cut-off frequency of 100 Hz as shown in Figure 2.

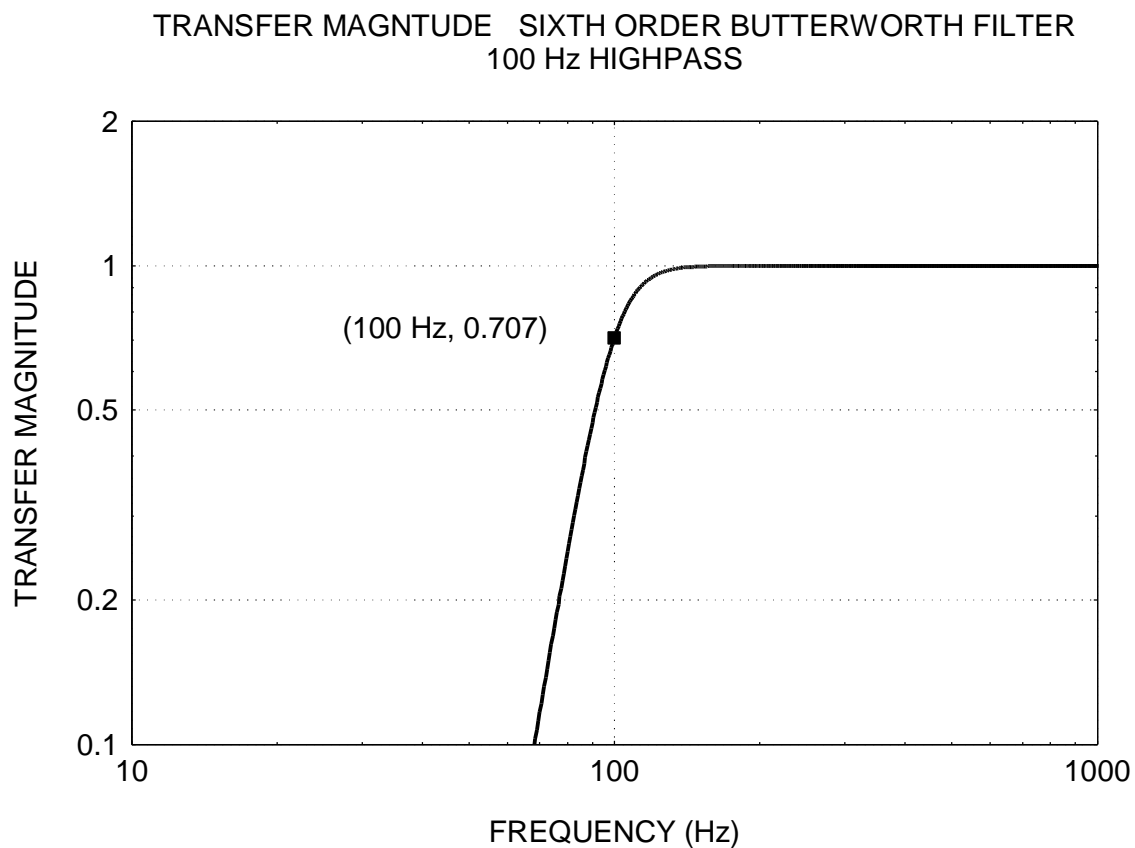


Figure 2.

Frequency Domain Implementation

The curves in Figures 1 and 2 suggests that filtering could be achieved as follows:

1. Take the Fourier transform of the input time history.
2. Multiply the Fourier transform by the filter transfer function, in complex form.
3. Take the inverse Fourier transform of the product.

The above frequency domain method is valid. Nevertheless, the filtering algorithm is usually implemented in the time domain for computational efficiency.

Time Domain Implementation

The transfer function can be represented by $H(\omega)$.

Digital filters are based on this transfer function, as shown in the block diagram in Figure 3. Note that x_k and y_k are the time domain input and output, respectively.

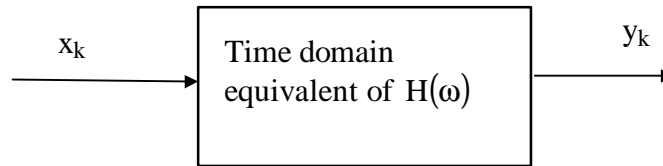


Figure 3. Filter Block Diagram

Phase Correction

Ideally, a filter should provide linear phase response. This is particularly desirable if shock response spectra calculations are required. Butterworth filters, however, do not have a linear phase response, for reasons discussed in Reference 2. Other IIR filters share this problem.

A number of methods are available, however, to correct the phase response. One method is based on time reversals and multiple filtering as shown in Figure 4.

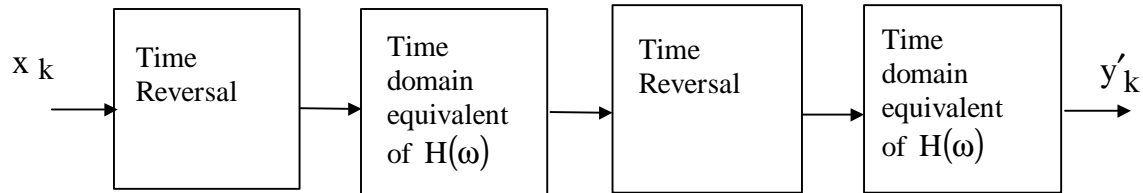


Figure 4. Phase Correction Method

Further information about refiltering is given in References 1 and 2.

An important note about refiltering is that it reduces the transfer function magnitude at the cut-off frequency to -6 dB.

Example 1

Consider the synthesized time history in Figure 5. The time history appears to be random, perhaps even white noise. The corresponding power spectral density function is shown in Figure 6.

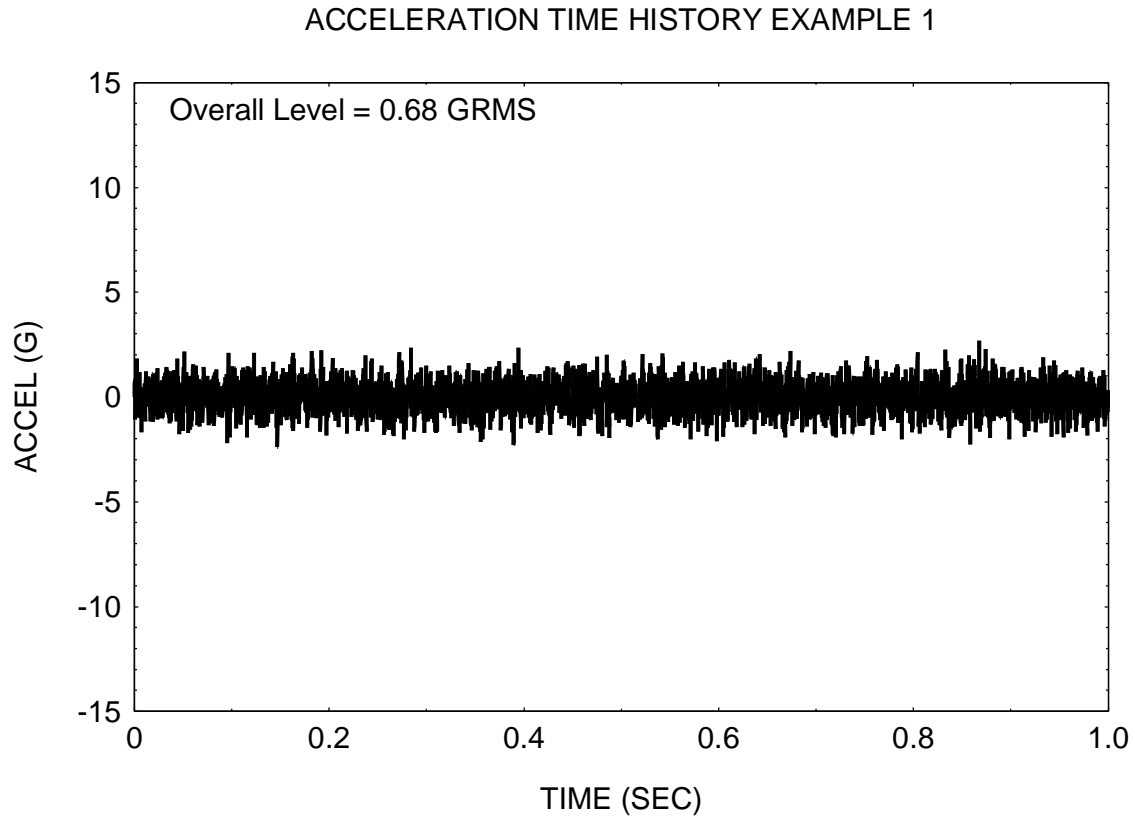


Figure 5.

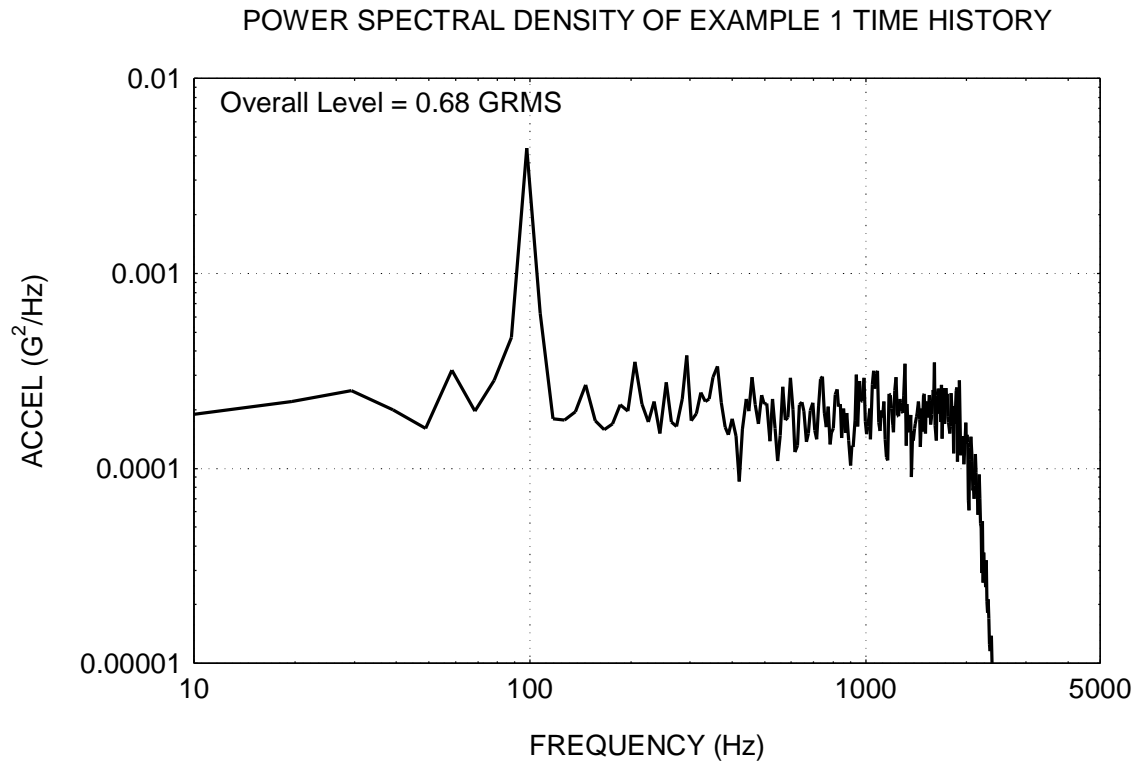


Figure 6.

The power spectral density displays some characteristics of white noise. Nevertheless, a distinct spectral peak occurs at 100 Hz. The signal is perhaps best described as “sine-on-random.”

The behavior of the 100 Hz signal can be clarified by bandpass filtering the time history in Figure 5. The time history is bandpass filtered from 50 Hz to 150 Hz in Figure 7. A close-up view of a 200 millisecond segment is shown in Figure 8.

ACCELERATION TIME HISTORY EXAMPLE 1
50 Hz to 150 Hz BP FILTERED

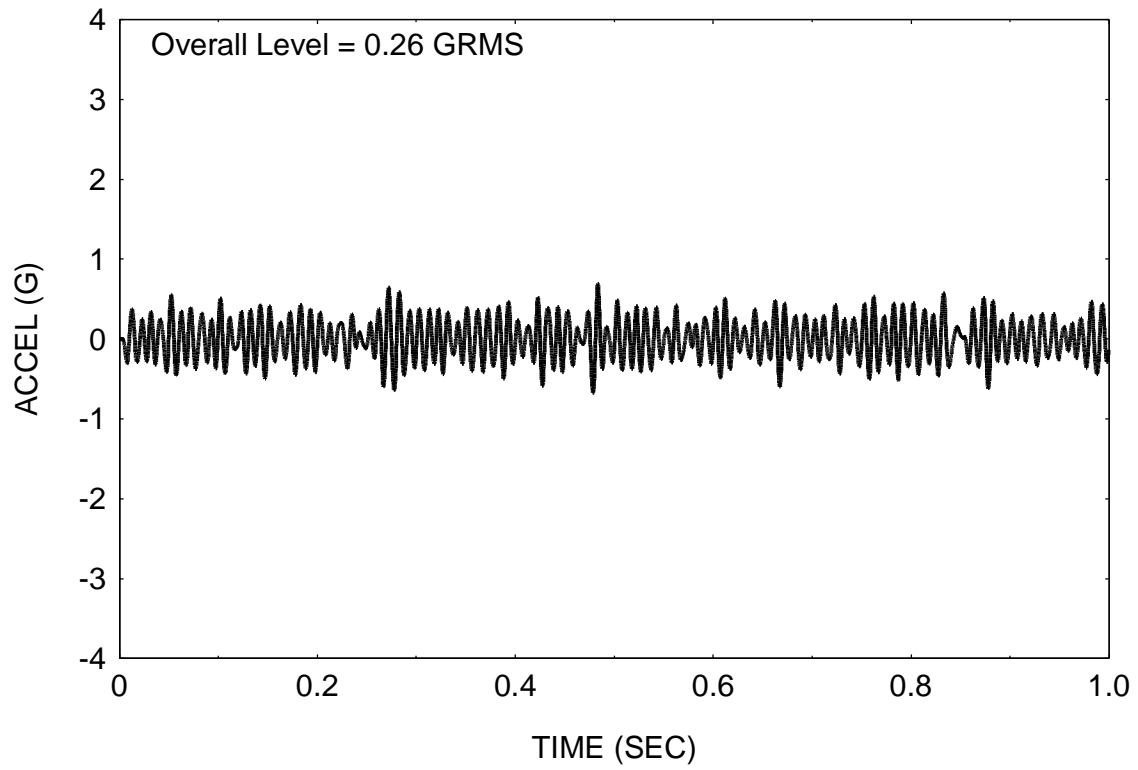


Figure 7.

ACCELERATION TIME HISTORY EXAMPLE 1
50 Hz to 150 Hz BP FILTERED

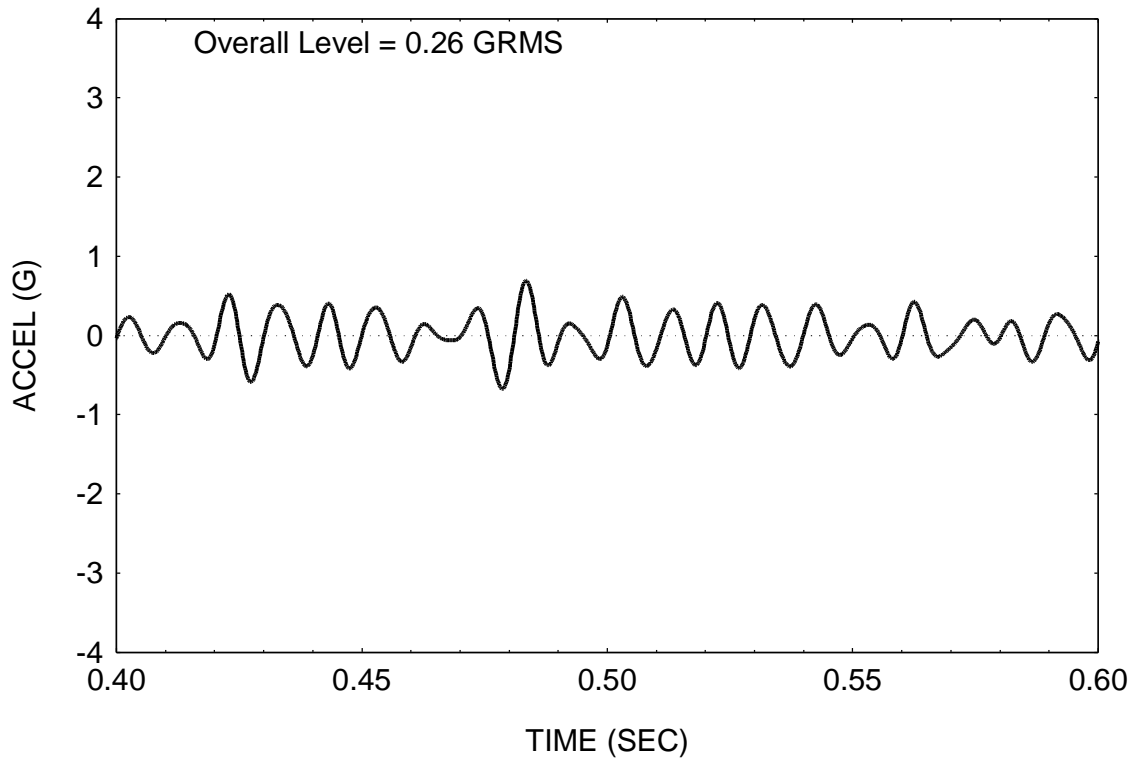


Figure 8.

The signal in Figure 8 is not pure sine, but it can be modeled as such for engineering purposes. The number of cycles is nearly 20. The duration is 200 milliseconds. The dominant frequency is thus

$$(20 \text{ cycles} / 0.200 \text{ seconds}) = 100 \text{ Hz}$$

(1)

This calculation confirms the observation of the 100 Hz peak in the power spectral density plot in Figure 6.

The sine function in Figures 7 and 8 tended to remain stable with time, in terms of amplitude and frequency. Either of these parameters, however, could have shifted with time. One of the purposes of filtering is to study this behavior.

Consider the solid rocket motor in Figure 9.

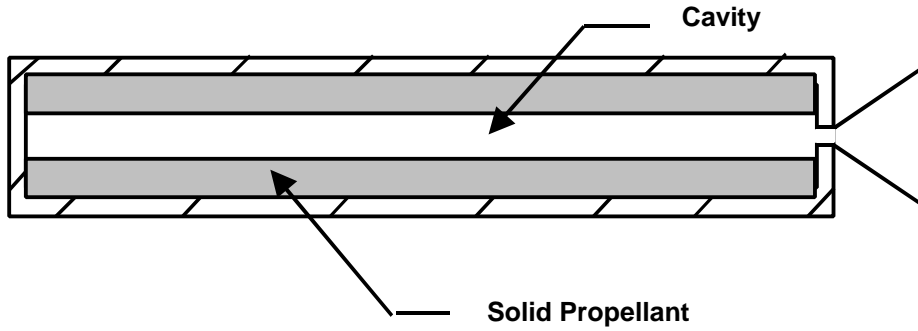


Figure 9.

The cavity has an acoustic pressure natural frequency. The cavity can be modeled as a closed pipe, because the nozzle throat diameter is very small. The propellant is expended during powered flight. The cavity volume increases as a result. The acoustic pressure natural frequency tends to decrease as the volume increases. Again, this behavior could be analyzed by filtering the data.

Example 2

Reconsider the time history in Figure 6. Bandpass filter the data from 10 Hz to 60 Hz. The resulting time history is shown in Figure 10.

The bandpass filtered data in Figure 10 is “narrowband random.” Note that the overall level is 0.090 GRMS. The frequency bandwidth is

$$60 \text{ Hz} - 10 \text{ Hz} = 50 \text{ Hz} \quad (2)$$

The power spectral density amplitude for this band can be calculated as follows

$$\text{PSD} = \frac{[0.090 \text{ GRMS}]^2}{50 \text{ Hz}} \quad (3)$$

$$\text{PSD} = 0.00016 \text{ GRMS}^2 / \text{Hz} \quad (4)$$

By convention, the unit is abbreviated as

$$\text{PSD} = 0.00016 \text{ G}^2 / \text{Hz} \quad (5)$$

ACCELERATION TIME HISTORY EXAMPLE 2
10 Hz to 60 Hz BP FILTERED

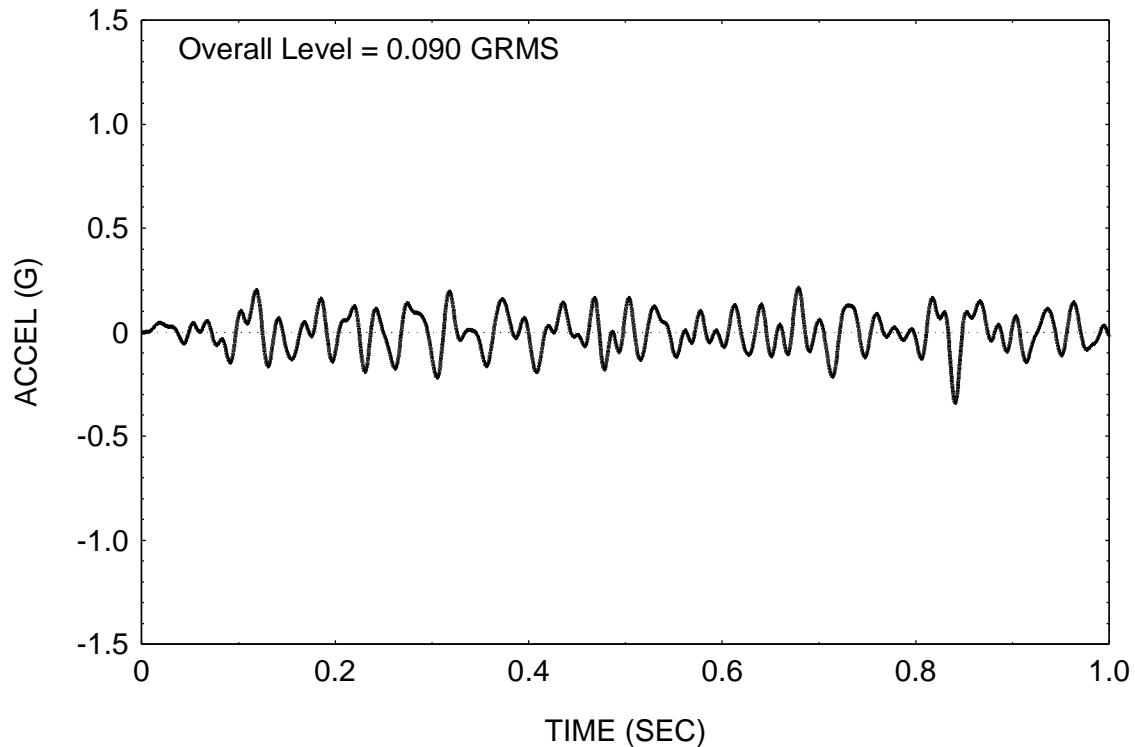


Figure 10.

Now compare the level in equation (5) with plot in Figure 6 over the domain from 10 Hz to 60 Hz. The plot amplitude tends to agree with the calculation from the bandpass filtering operation.

Power Spectral Density Summary

Example 2 demonstrates an important point: a complete power spectral density function can be constructed via bandpass filtering in successive bands.

For example, the first band could be taken from 0 Hz to 10 Hz; the second from 10 Hz to 20 Hz, the third from 20 Hz to 30 Hz; and so on in 10 Hz increments. The overall GRMS value is then calculated for each band. The level is then squared. The square is divided by the bandwidth, which is 10 Hz in this example. The power spectral density level is thus calculated for each band.

This bandpass method altogether bypasses the Fourier transform step.

The Fourier transform method remains more efficient for computational purposes. The bandpass filter method, however, is easier to understand.

References

1. T. Irvine, An Introduction to the Filtering of Digital Signals, Vibrationdata Publications, 2000.
2. Stearns and David, Signal Processing Algorithms in Fortran and C, Prentice Hall, Englewood Cliffs, New Jersey, 1993.

Homework

1. File shock.txt is a shock pulse which occurred during the flight of a rocket vehicle. The specific event was the tail-off, or burnout, of a rocket motor. The data is composed of several dominant frequencies. The dominant frequencies could be forcing frequencies, natural frequencies, or some combination of both. Use program poweri.exe to take a power spectral density function of the data. Use a rectangular window with 4096 samples per segment. Plot the resulting power spectral density function. List the frequencies of the dominant peaks that are below 200 Hz.
2. One of the dominant frequencies occurs near 120 Hz. The next goal is to filter the time history to clarify the behavior of this frequency. Use program filter.exe. Bandpass filter the data using frequency limits of 100 Hz and 150 Hz. Plot the results. Note the beat frequency effect, suggesting two closely-space frequencies.