

# SHOCK AND VIBRATION RESPONSE SPECTRA COURSE

## Unit 1B. Damping

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### Introduction

Recall the homework assignment from Unit 1A. The data.txt time history represented a rocket vehicle dropped from an aircraft. The rocket vehicle oscillated as a free-free beam during the interval between release and motor ignition. A beam is a continuous structure with numerous vibration modes. The fundamental frequency, however, dominated the response. Thus, the rocket vehicle could be modeled as a single-degree-of-freedom system with a natural frequency of 9.6 Hz.

The oscillation of the rocket vehicle was the result of initial displacement. The vehicle bowed downward due to gravity while attached to the aircraft. Thus, there was initial potential energy in the form of strain energy.

During free vibration, the oscillation amplitude decreases with time due to damping effects. The damping of the rocket of vehicle is due to several effects: internal material friction, friction at joints, and air resistance.

In addition, structures may lose energy by sound radiation and by friction at boundaries.

### Modal Analysis

A multi-degree-of-freedom system is composed of numerous vibration modes. Each mode has an independent natural frequency and damping ratio. "Modal analysis and testing" is carried out to resolve the natural frequencies and damping ratios.

### Free Vibration Equation

Return to the single-degree-of-freedom example. The acceleration time history  $\ddot{y}(t)$  for free vibration can be approximated by the following formula

$$\ddot{y}(t) = A \exp(-\xi \omega_n t) \sin(\omega_d t + \phi) \quad (1)$$

where

- A is the acceleration amplitude
- $\xi$  is the damping ratio or fraction of critical damping
- $\omega_n$  is the natural frequency
- $\omega_d$  is the damped natural frequency
- $\phi$  is the phase angle

Note that the damping ratio is sometimes represented by  $\zeta$  in other references.

The phase angle is determined by the initial conditions, as explained in Reference 1.

The damped natural frequency is

$$\omega_d = \omega_n \sqrt{1 - \xi^2} \quad (2)$$

### Lightly-Damped Systems

For many systems, the damping ratio  $\xi < 0.25$ . Thus,

$$\omega_d \approx \omega_n \quad (3)$$

Equation (1) simplifies to

$$\ddot{y}(t) \approx A \exp(-\xi \omega_n t) \sin(\omega_n t + \phi) \quad (4)$$

### Measuring Damping

Consider the analytical time history function in Figure 1. The natural frequency is 10 Hz, as determined by inspection. The task is to determine the damping ratio.

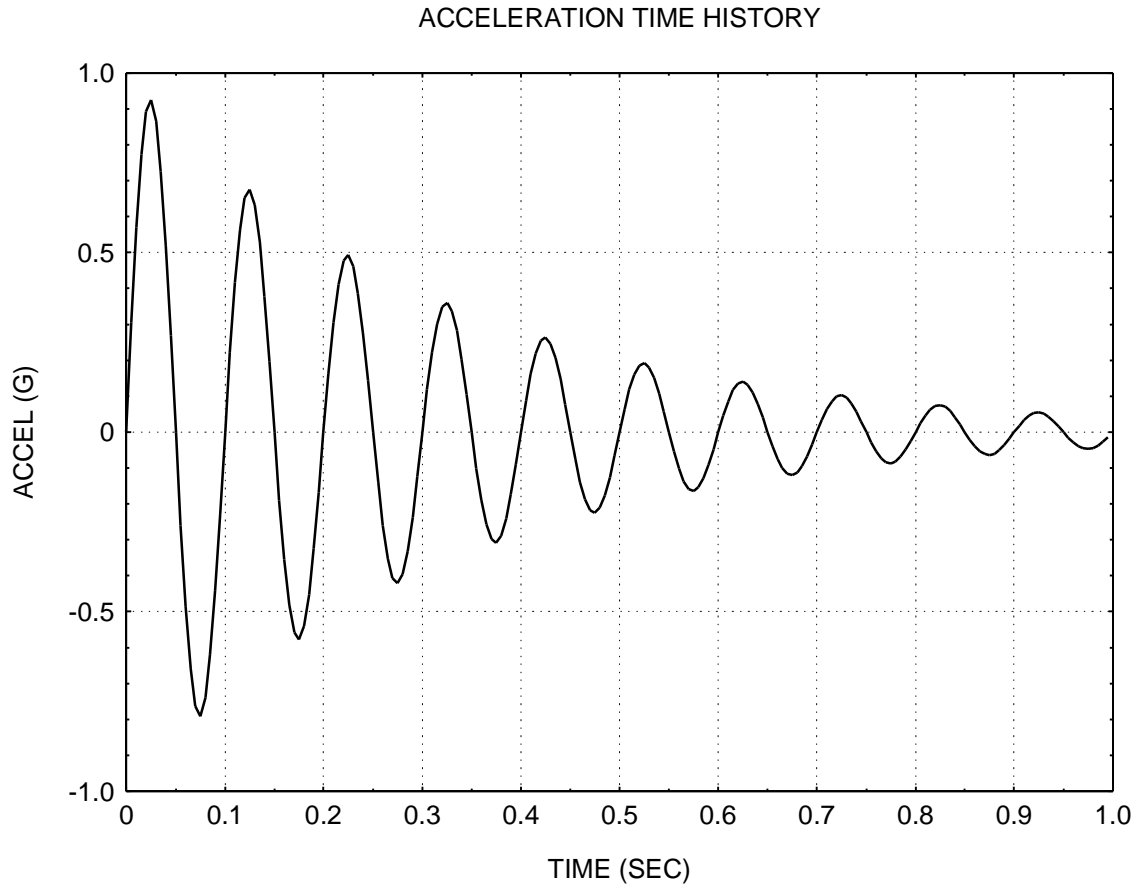


Figure 1.

Thomson gives a "log decrement method" for determining the damping ratio in Reference 2. This method uses the ratio of peak amplitudes between two consecutive peaks to determine the damping ratio. Furthermore, the method can also be used for two peaks separated by n cycles.

Thomson's log decrement method has merit. Nevertheless, test data often has slight non-linearities and other effects which pose an obstacle for this method.

A better method for test data is to perform a trial-and-error curve-fit using the exponential term in equation (4). Let E(t) be the envelope function.

$$E(t) = A \exp(-\xi \omega_n t) \tag{5}$$

The sample time history is shown again in Figure 2 along with an envelope curve. The envelope curve was obtained by trial-and-error inspection.

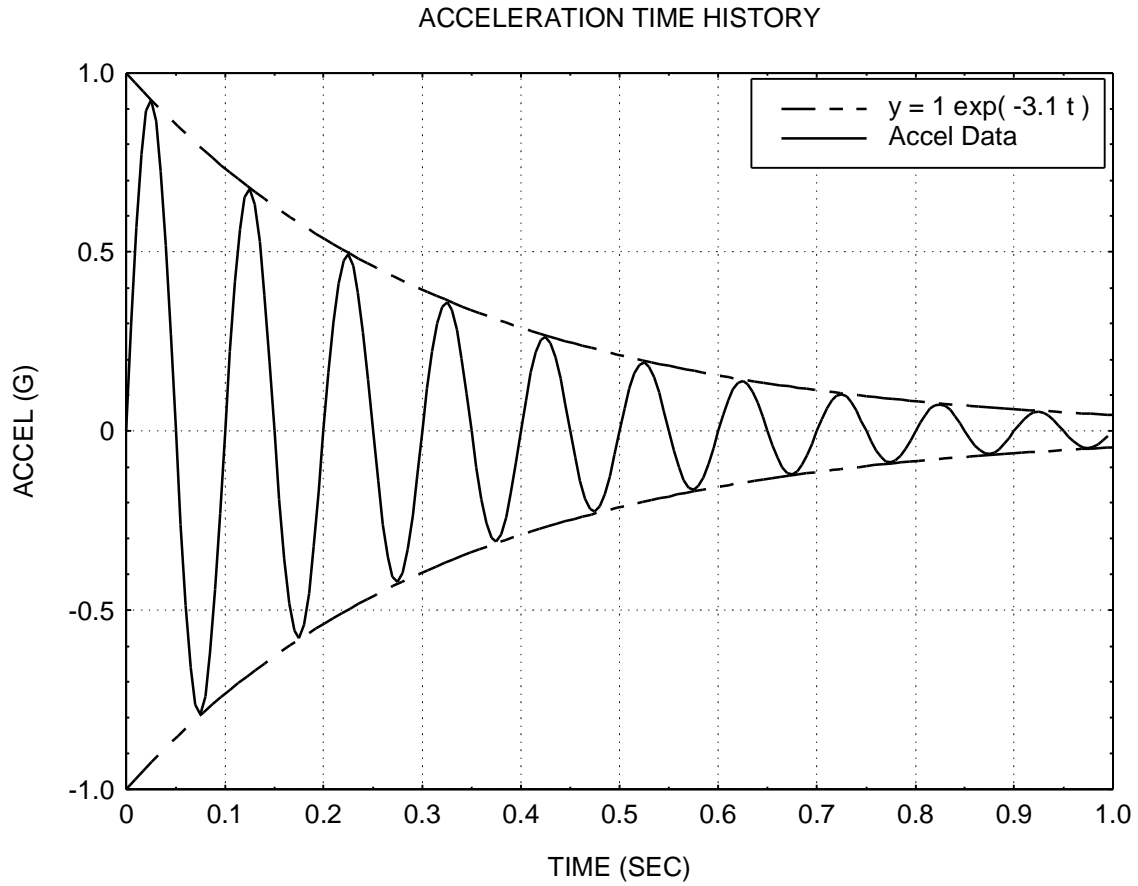


Figure 2.

The envelope equation in Figure 2 is

$$E(t) = 1 \exp(-3.1 t) \quad (6)$$

Compare equation (5) with (6).

$$\xi \omega_n = 3.1 \quad (7)$$

$$\xi = \frac{3.1}{\omega_n} \quad (8)$$

$$\xi = \frac{3.1}{2\pi f_n} \quad (9)$$

The natural frequency is 10 Hz.

$$\xi = \frac{3.1}{2\pi(10)} \quad (10)$$

$$\xi = \frac{3.1}{2\pi(10)} \quad (11)$$

$$\xi = 0.049 \quad (12)$$

Or

$$\xi = 4.9\% \quad (13)$$

The exact value is 5%. The envelope method thus yields a very good estimate.

Note that damping is represented in a variety of forms. For example, the amplification factor Q is

$$Q = \frac{1}{2\xi} \quad (14)$$

The sample problem thus has an amplification factor of  $Q = 10$ .

### Homework

1. Fun assignment. The Rex2launch.qt file is a public domain file of a Pegasus launch in QuickTime format. View the video. The video covers the period from drop through early stage 1 burn. The body-bending motion is not visible in the video, but nevertheless is measured by the accelerometers. Again, the drop.txt file is taken from a Pegasus launch, but the amplitude has been modified by an undisclosed scale factor for proprietary reasons. Furthermore, the drop.txt data is taken from a flight other than the one shown in the video. The drop transient data is fully disclosed to the payload engineers, however. In fact, the mass of the payload affects the frequency of the drop transient. Thus, the launch vehicle provider and the payload engineers must perform a "coupled-loads" analysis to determine how the launch vehicle will affect the payload and vice-versa. This is carried out using a finite element model.
2. Recall the drop.txt file from Unit 1A. Use the envelope method to estimate the damping ratio. Program dratio.exe can be used to generate sample envelope curves. Hint: the time history takes one or two cycle to work itself into a pure bending oscillation. Thus ignore the first few peaks. Take the third or fourth peak coordinate as a reference for program dratio.exe. An alternate method is to generate the curves within the graphics program itself. Note: perfect envelope fit is impossible due to non-linearities. Try to come up with reasonable fit.

3. What is the amplification factor for drop.txt time history?
4. Review tutorials damping.pdf and dcross.pdf.

#### References

1. T. Irvine, Free Vibration of a Single-Degree-of-Freedom System, Vibrationdata Publications, 1999.
2. W. Thomson, Theory of Vibration with Applications, Second Edition, Prentice-Hall, New Jersey, 1981.