SHOCK AND VIBRATION RESPONSE SPECTRA COURSE Unit 20
Force Shock: Arbitrary Pulse

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Introduction

Vibration is typically measured in terms of acceleration. An accelerometer can usually be placed at any location on an object's surface. Note that acceleration is analogous to voltage. The voltage between any two points of a circuit is usually easy to measure.

Force, on the other hand, is analogous to electrical current. Note that an electrical circuit must be broken in order to measure current. The current meter itself becomes part of the circuit. A "mechanical circuit" is often impractical to break. Thus, dynamic force measurements are seldom made.

Nevertheless, there are some exceptions.

Consider a rocket motor mounted horizontally in a test stand for a static fire test. A force transducer might be placed between the forward end of the motor and the test stand. The force transducer measures the thrust of the motor. The thrust versus time curve typically has a rectangular shape. An oscillation may be superimposed on this shape, however.

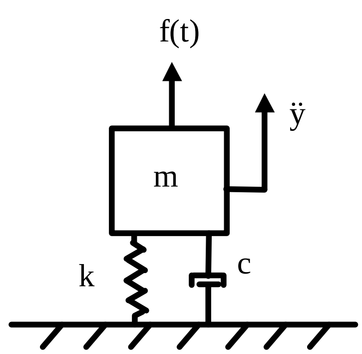
Furthermore, wind and turbulent airflow are typically modeled in terms of pressure, which is proportional to force.

The purpose of this Unit is to consider a single-degree-of-freedom system subjected to an arbitrary force shock.

The maximum response of a system with a variable natural frequency is plotted as a force shock response spectrum. The response may be in terms of displacement, velocity, or acceleration.

Derivation of Equations

Consider a single-degree-of-freedom system.



where

m is the mass,
c is the viscous damping coefficient,
k is the stiffness,
y is the absolute displacement of the mass,
f(t) is the applied force.

Note that the double-dot denotes acceleration.

$$m\ddot{y} + c\dot{y} + ky = f(t) \tag{1}$$

$$\ddot{y} + 2\xi\omega_n \dot{y} + \omega_n^2 y = \frac{1}{m} f(t) \tag{2}$$

where

ω_n is the natural frequency in (radians/sec),
ξ is the damping ratio.

The intermediate steps of the derivation are given in Reference 1. The response can be represented in terms of a digital recursive filtering relationship, where the index represents time.

The displacement x at index i is

$$\begin{aligned} x_i = &\\ &+ 2\exp[-\xi\omega_n\Delta t]\cos[\omega_d\Delta t]\, x_{i-1} \\ &- \exp[-2\xi\omega_n\Delta t] x_{i-2} \\ &- \left\{\frac{\Delta t}{m\omega_d}\right\}\exp[-\xi\omega_n\Delta t]\{\sin[\omega_d\Delta t]\} f_{i-1} \end{aligned} \tag{3}$$

where Δt is the time step,

x_i is the response at time t,

x_{i-1} is the response at time t-Δt,

x_{i-2} is the response at time t-2Δt,

f_{i-1} is the force input at time t-Δt,

The acceleration $\ddot{x}$ at index i requires a two-step calculation. The first step is

$$\ddot{w}_i =$$

$$+2\exp[-\xi\omega_n\Delta t]\cos[\omega_d\Delta t]\,x_{i-1}$$

$$-\exp[-2\xi\omega_n\Delta t]x_{i-2}$$

$$+\left\{\frac{-2\xi\omega_n\Delta t}{m}\right\}f_i$$

$$-\Delta t\exp[-\xi\omega_n\Delta t]\left\{\left[\frac{\omega_n^{\,2}\left(-1+2\xi^2\right)}{m\,\omega_d}\right]\sin[\omega_d\Delta t]+\left[\frac{2\xi\omega_n}{m}\right]\cos[\omega_d\Delta t]\right\}f_{i-1} \quad (4)$$

The second step is

$$\ddot{x}_i = \ddot{w}_i + [f_i/m] \quad (5)$$

<u>Example</u>

Consider a single-degree-of-freedom system subjected to a sinusoidal force input as shown in Figure 1. The system has a natural frequency of 100 Hz and a damping ratio of 0.05, equivalent to Q=10. The system's mass is 1 kg.

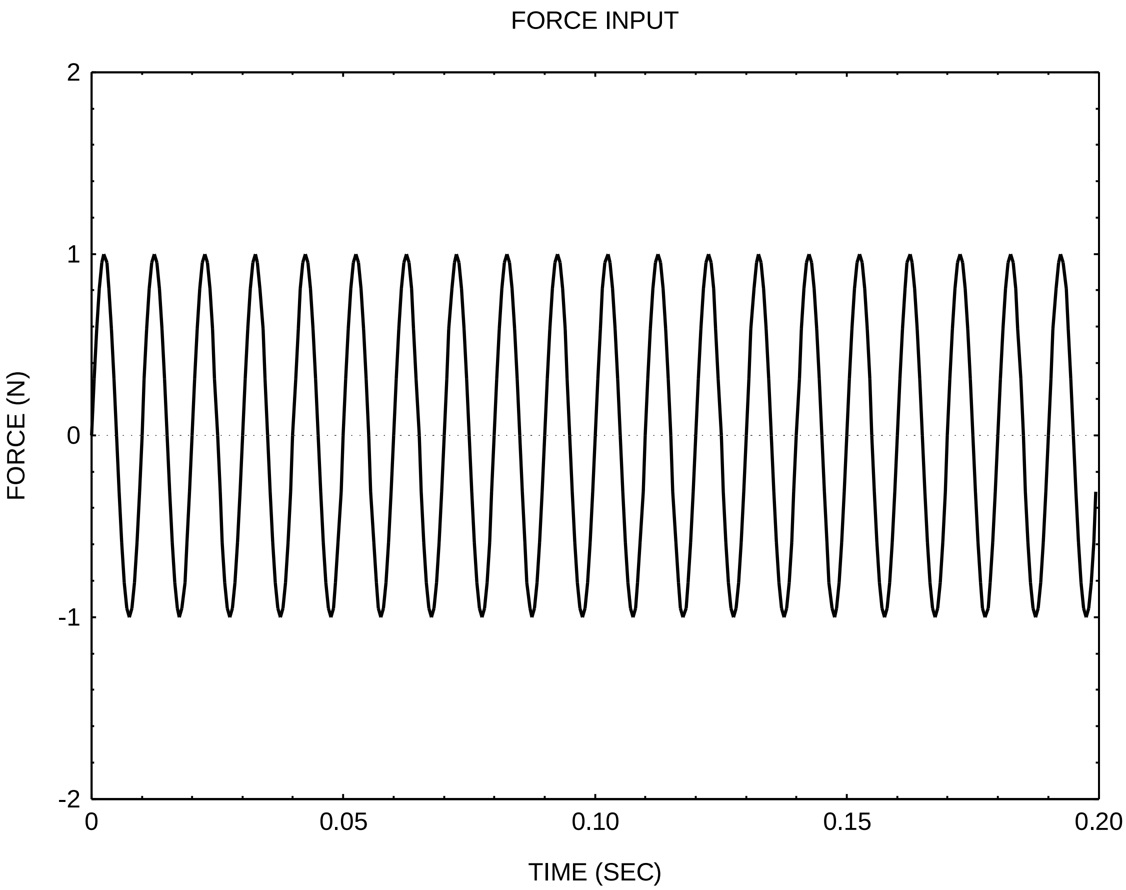


Figure 1.

The corresponding response displacement and acceleration are shown in Figures 2 and 3, respectively. These responses are calculated using equations (3) through (5). These equations are implemented using program arbit_f.exe.

The shock response spectrum is shown in Figure 4. It was calculated using program fsrs.exe.

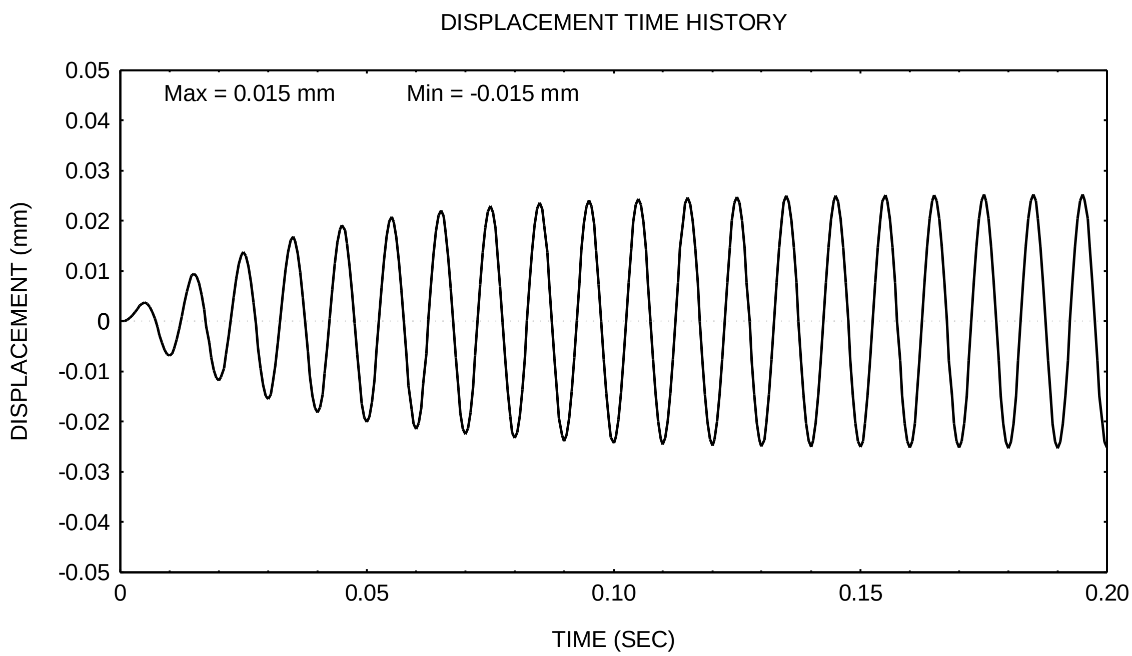


Figure 2.

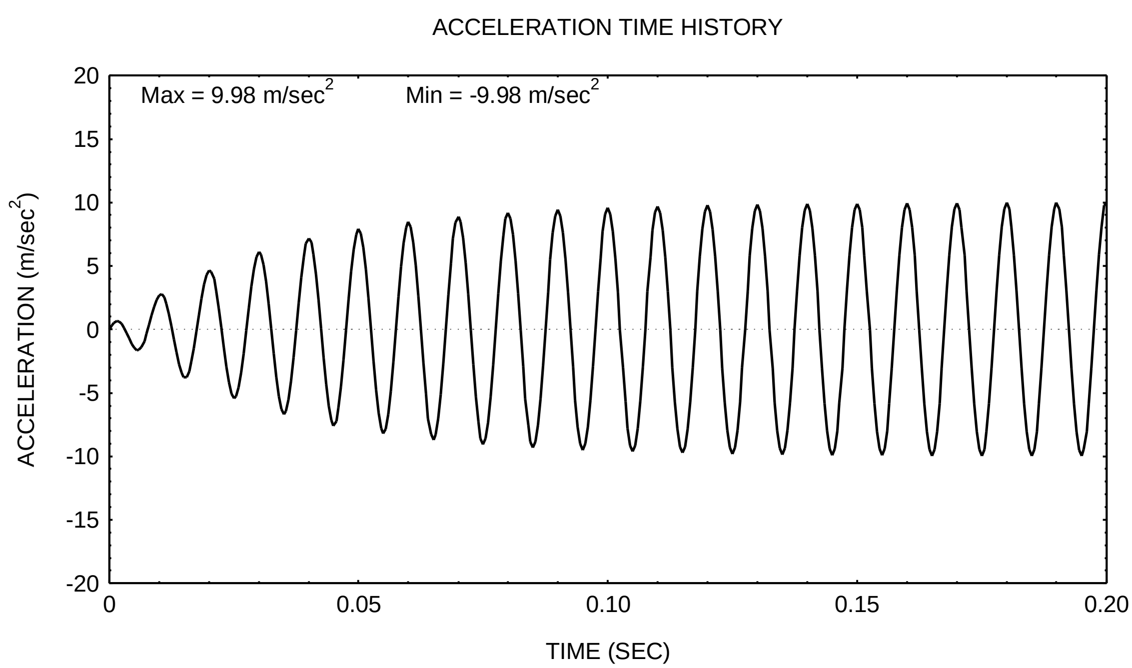


Figure 3.

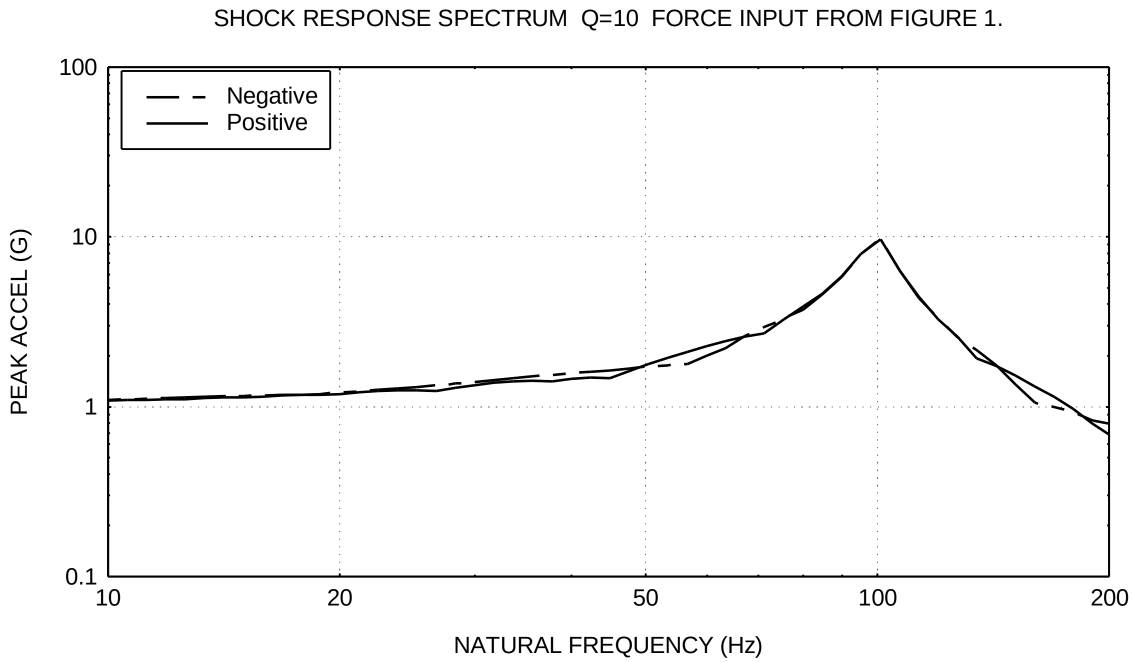


Figure 4.

Note that the exact solution can be determined either by Laplace transforms or by other methods. Thus, this problem presents a good opportunity to judge the accuracy of the equations presented in this Unit. Assume steady-state behavior. The comparison is shown in Table 1.

Table 1. Peak Response Values

Parameter	Digital Recursive Filter	Steady-State
Displacement (mm)	±0.015	±0.016
Acceleration (m/sec^2)	±9.98	±10.00

Recall that the steady-state method for a sinusoidal force input was given in Unit 9.

The comparison results are good. Higher accuracy could be achieved by using a higher sample rate. The sample rate was 2000 samples/second. In addition, the input consisted of 20 cycles. Additional cycles would be required to achieve the steady-state ideal.

Reference

1. T. Irvine, An Introduction to the Shock Response Spectrum, Vibrationdata Publications, 2000.

Homework

1. Assume a single-degree-of-freedom system with a natural frequency of 70 Hz and an amplification factor Q=10. Assume mass = 1 kg. Calculate the acceleration response to a 1 N, 0.010 symmetric sawtooth force pulse, as given in file: ssaw.txt. Use program arbit_f.exe
2. Calculate the shock response spectra for ssaw.txt using program fsrs.exe.

Note that the symmetric sawtooth pulse is a classical pulse. The homework problems could thus be solved by Laplace transforms. Nevertheless, a classical pulse is a special example of an arbitrary pulse.