SHOCK AND VIBRATION RESPONSE SPECTRA COURSE Unit 21
Base Excitation Shock: Classical Pulse

By Tom Irvine
Email: tomirvine@aol.com

Introduction

Consider a structure subjected to a base excitation shock pulse. Base excitation is also referred to as support motion.

The pulse is typically defined in terms of acceleration, although it could be defined in terms of displacement of velocity.

Examples of base excitation are

1. An automobile traveling down a washboard road.
2. A building subjected to an earthquake or seismic motion.
3. A crystal oscillator mounted on a circuit board subjected to shock.

For simplicity, the automobile, building, or crystal oscillator can be modeled as a single-degree-of-freedom system. The response of the object to a given base excitation can then be calculated given the natural frequency and the damping ratio. Note that the mass value is not explicitly required.

Typically, the peak response acceleration is the desired output parameter. The natural frequency of the object, however, may be unknown. Thus, the response calculation can be performed for a series of natural frequencies. The peak acceleration response can then be plotted against the natural frequency. The resulting function is a shock response spectrum.

The shock response spectrum is useful for evaluating the damage potential of the shock pulse.

The purpose of this Unit is to present the shock response spectrum for the case of a classical base input pulse, such as a half-sine pulse.

Derivation of Equations

Consider the single-degree-of-freedom system subjected to base excitation shown in Figure 1. The free-body diagram is shown in Figure 2.

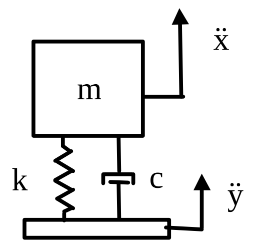

Figure 1. Single-degree-of-freedom System

The variables are

m = mass,
c = viscous damping coefficient,
k = stiffness,
x = absolute displacement of the mass,
y = base input displacement.

The double-dot notation indicates acceleration

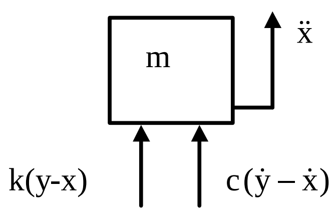

Figure 2. Free-body Diagram

Define a relative displacement z as follows.

Let
$$z = x - y$$
$$\dot{z} = \dot{x} - \dot{y}$$
$$\ddot{z} = \ddot{x} - \ddot{y}$$
$$\ddot{x} = \ddot{z} + \ddot{y}$$

The following equation of motion for the relative displacement z was derived in Unit 8.

$$\ddot{z} + 2\xi\omega_n\dot{z} + \omega_n{}^2 z = -\ddot{y} \tag{1}$$

Consider the half-sine pulse given by equation (2).

$$\ddot{y}(t) = \begin{cases} A \sin\left(\dfrac{\pi t}{T}\right), & 0 \le t \le T \\ 0\,, & t > T \end{cases} \tag{2}$$

The equation of motion becomes

$$\ddot{z} + 2\xi\omega_n \dot{z} + \omega_n{}^2 z = -A \sin\left(\frac{\pi t}{T}\right), \qquad 0 \le t \le T \tag{3}$$

Let

$$\omega = \frac{\pi}{T} \tag{4}$$

$$\ddot{z} + 2\xi\omega_n \dot{z} + \omega_n{}^2 z = -A \sin(\omega t), \qquad 0 \le t \le T \tag{5}$$

Equation (5) may be solved via Laplace transforms, as shown in Reference 1.

After many steps, the absolute acceleration during the pulse is

$$\ddot{x}(t) = -\omega_n \exp(-\xi\omega_n t)\left\{[\omega_n z(0) + 2\xi \dot{z}(0)]\cos(\omega_d t) + \frac{\omega_n}{\omega_d}\left[-\xi\omega_n z(0) + \left(1 - 2\xi^2\right)\dot{z}(0)\right]\sin(\omega_d t)\right\}$$

$$+ \frac{A\omega^2}{\left[\left(\omega^2 - \omega_n^2\right)^2 + (2\xi\omega\omega_n)^2\right]}\left[-\left(\omega^2 - \omega_n^2\right)\sin(\omega t) - (2\xi\omega\omega_n)\cos(\omega t)\right]$$

$$+ \frac{A\omega\omega_n\,[\exp(-\xi\omega_n t)]}{\left[\left(\omega^2 - \omega_n^2\right)^2 + (2\xi\omega\omega_n)^2\right]}\left\{\left\{2\xi\,\omega^2\right\}\cos(\omega_d t) + \left\{\frac{\omega_n}{\omega_d}\right\}\left\{-\omega_n{}^2 + \omega^2\left(1 - 2\xi^2\right)\right\}\sin(\omega_d t)\right\}$$

$$+ A \sin(\omega t)\,, \qquad \text{for} \quad 0 \le t \le T \tag{6}$$

The absolute acceleration after the pulse is

$$\ddot{x}(t) =$$

$$-\omega_n \exp(-\xi\omega_n(t-T))\{[\omega_n z(T) + 2\xi\dot{z}(T)]\cos(\omega_d(t-T))\}$$

$$-\omega_n \exp(-\xi\omega_n(t-T))\left\{\frac{\omega_n}{\omega_d}\left[-\xi\omega_n z(T) + \left(1-2\xi^2\right)\dot{z}(T)\right]\sin(\omega_d(t-T))\right\}, \quad \text{for } t > T$$

(7)

Example

Consider the example in Table 1. The base pulse is shown in Figure 3. The calculations were made using equations (6) and (7), as implemented in the ahsine.exe program.

Table 1.
Shock Response Spectra, Q=10,
Acceleration Base Input = 0.010 sec, 10 G, Half-sine Pulse

Natural Frequency (Hz)	Peak Positive Acceleration (G)	Peak Negative Acceleration (G)	Figure
10.00	3.69	-3.15	4
20.00	7.18	-6.13	-
30.00	10.26	-8.77	-
40.00	12.79	-10.93	-
50.00	14.62	-12.50	-
60.00	15.74	-13.42	-
70.00	16.31	-13.63	-
80.00	16.51	-13.18	5
90.00	16.44	-12.15	-
100.00	16.24	-10.65	-
110.00	15.90	-8.74	-
120.00	15.51	-6.60	6
130.00	15.10	-4.42	-
140.00	14.64	-2.45	-
150.00	14.21	-1.30	
160.00	13.76	-1.91	
170.00	13.33	-2.84	
180.00	12.92	-3.52	-
190.00	12.50	-3.84	-
200.00	12.12	-3.78	-

Note that only the peak positive and negative values are retained for each time history response. The peak values are found via a simple search method rather than a calculus method.

Furthermore, note that the peak response can occur either during or after the half-sine pulse.

The overall shock response spectrum is shown in Figure 7. It is constructed by plotting the peak positive and negative acceleration amplitudes versus natural frequency in (Hz).

The are other types of shock response spectra which could be plotted. For example, the absolute value acceleration response spectrum could be plotted, instead of the individual positive and negative spectra.

Furthermore, the peak displacement or peak velocity could be plotted versus the natural frequency.

In addition, this process could be repeated for other classical pulses, such as trapezoidal, sawtooth, and rectangular pulses.

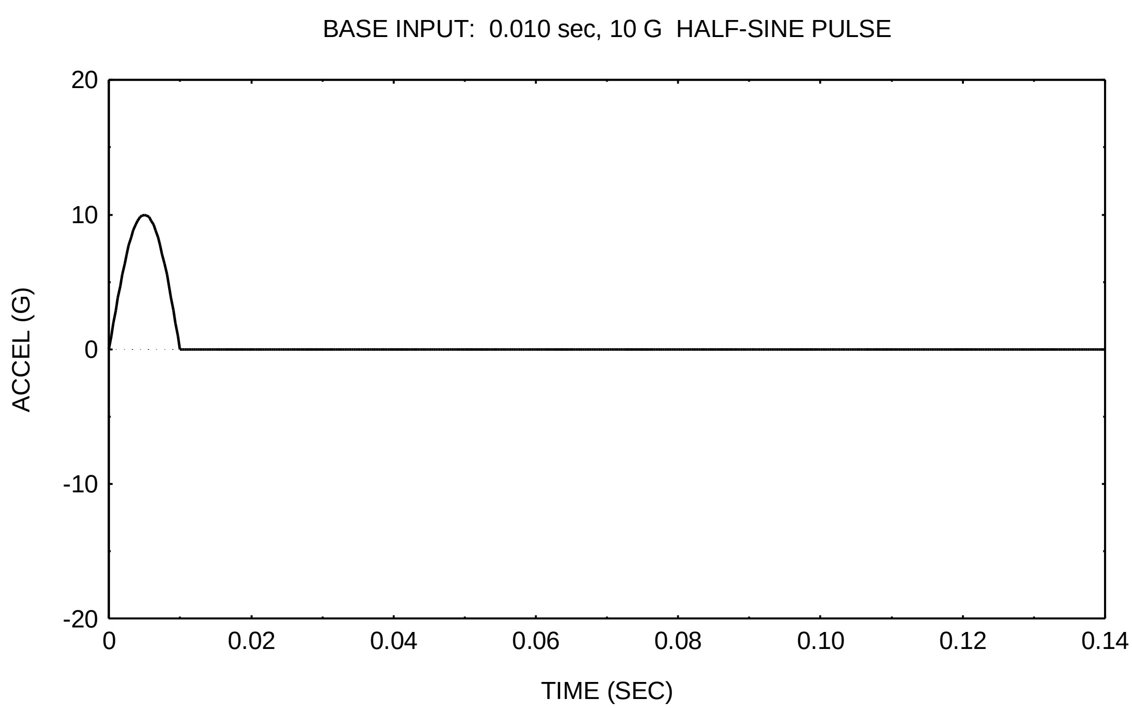

Figure 3.

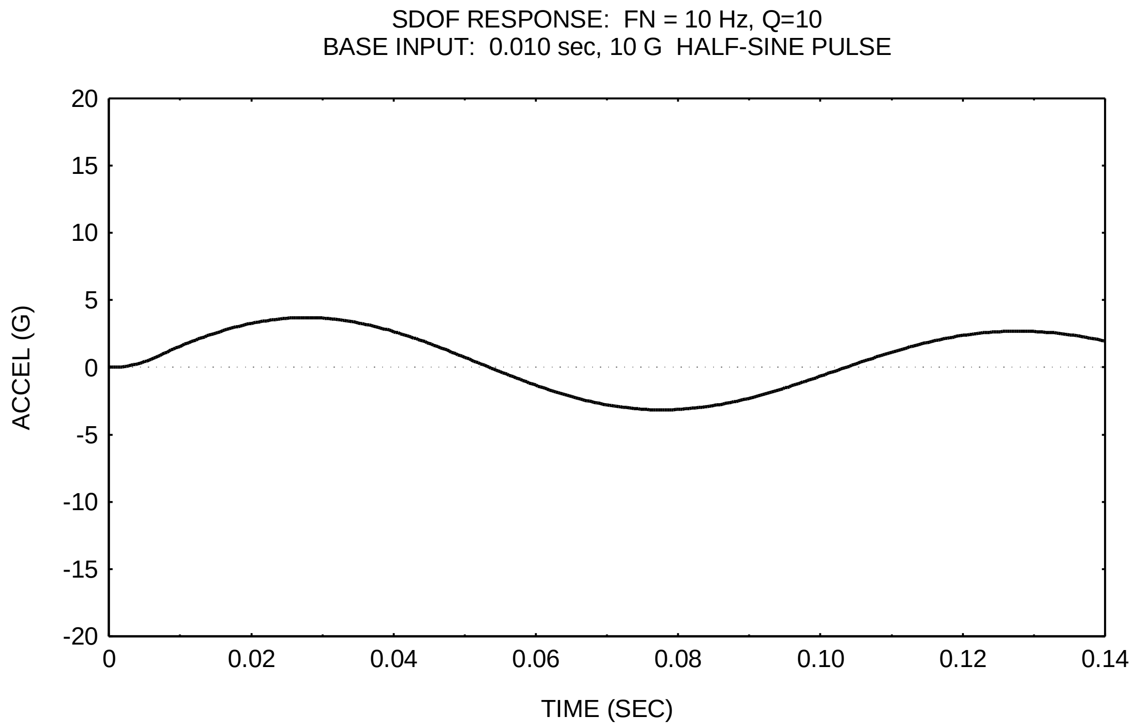

Figure 4.

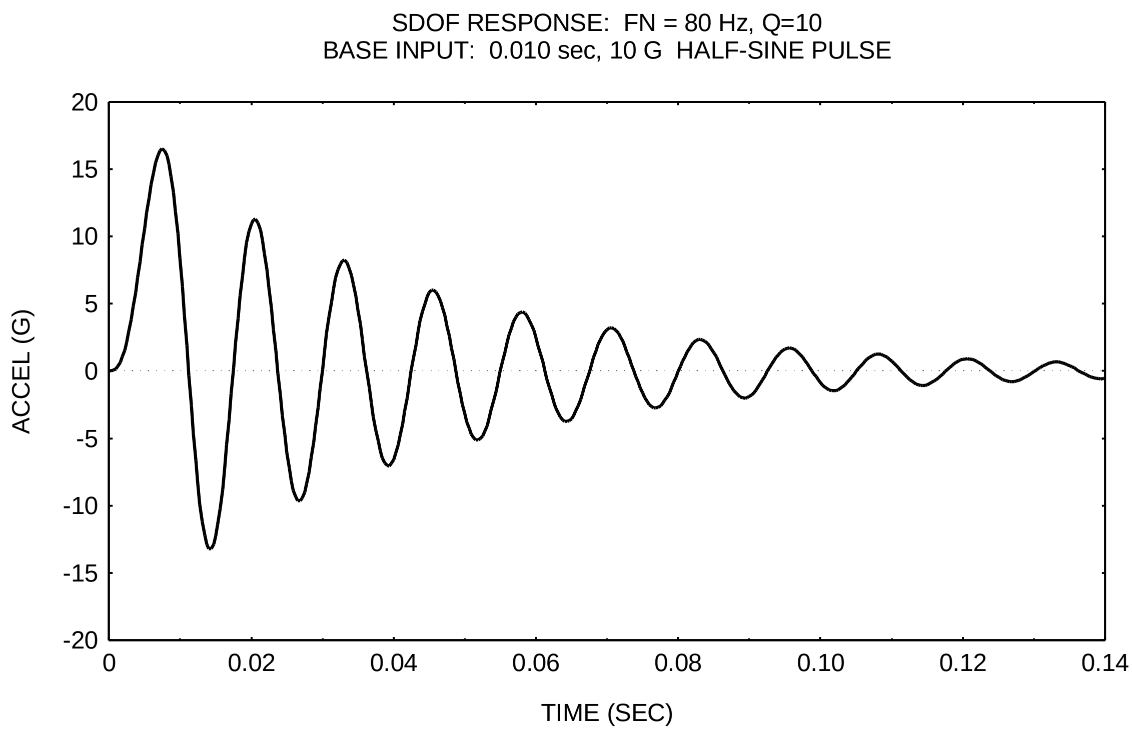

Figure 5.

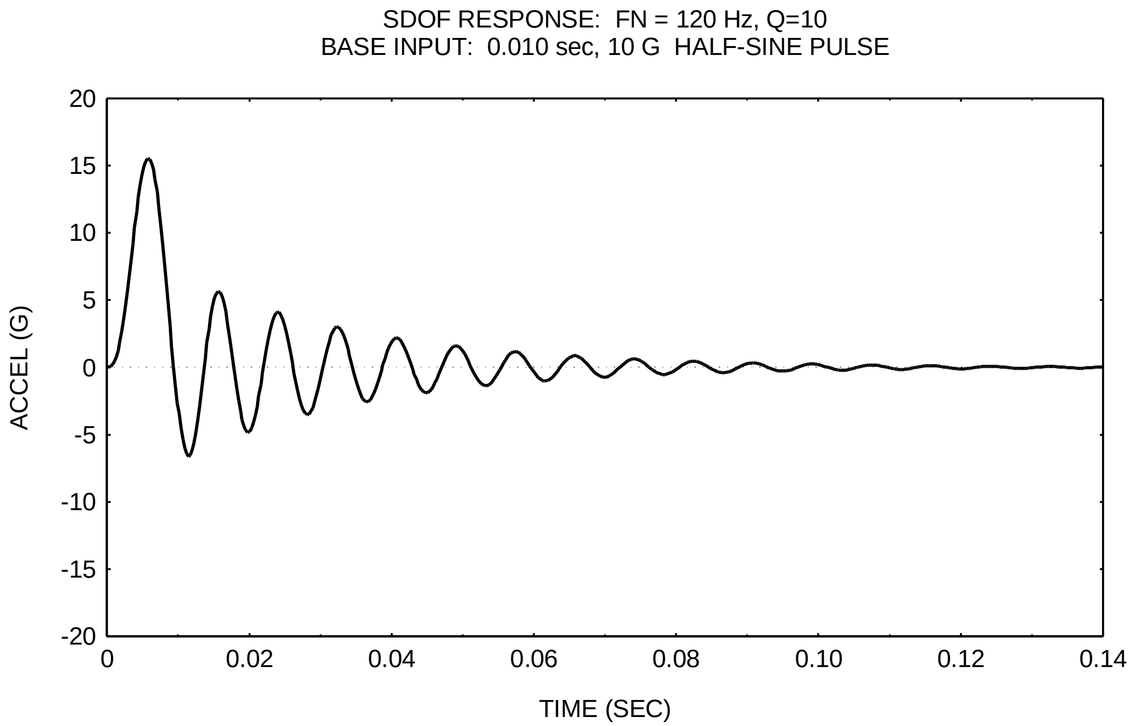

Figure 6.

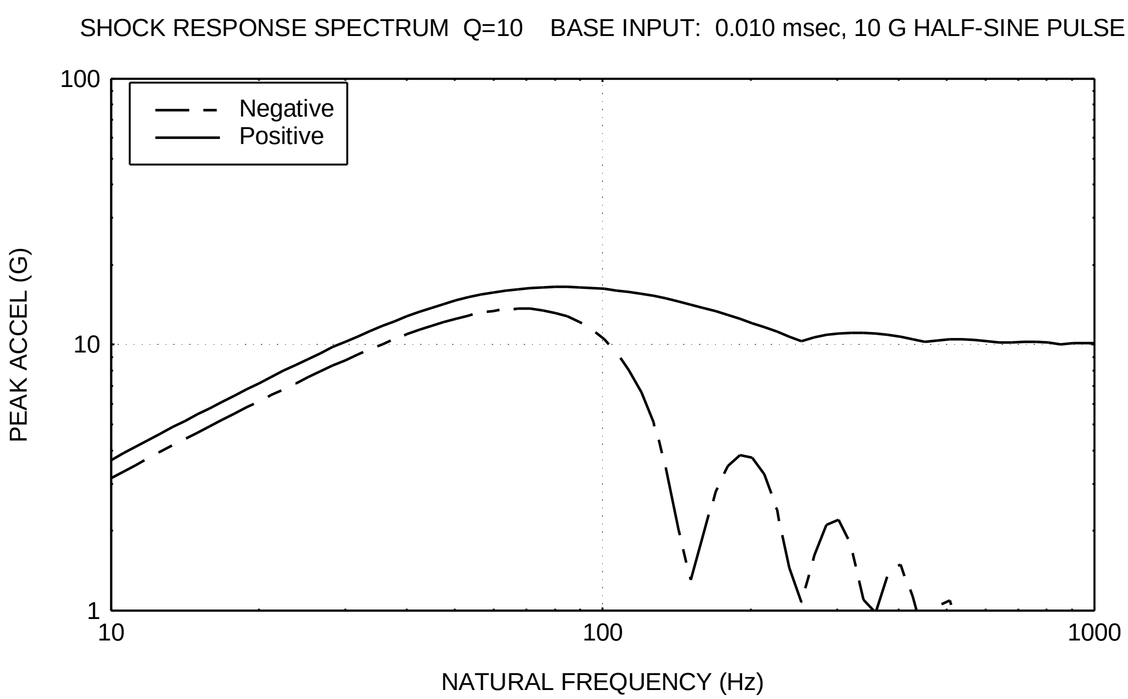

Figure 7.

Note that the positive shock response spectrum in Figure 7 converges to the peak input amplitude at higher natural frequencies, say above 200 Hz.

Component Shock Testing

Some shock tests are specified in terms of a base input half-sine pulse. The test may be performed on a drop tower or on a shaker table.

There is a significant difference between the positive and negative shock response spectra of a half-sine pulse, however, as shown in Figure 5. Thus, these specifications typically require that the test be performed in each direction of each axis.

Testing on shaker table presents a further challenge. The net displacement and net velocity of the shaker table must each be zero. Thus, pre and post pulses must be used to achieve these requirements.

References

1. T. Irvine, Response of a Single-degree-of-freedom System Subjected to a Classical Base Excitation, Vibrationdata.com Publications, 1999.

Homework

1. Repeat the example for a half-sine base input of 0.008 seconds, 10 G. Assume a damping value of Q = 10. Use program ahsine.exe for both the time history response and shock response spectra calculations.

2. What is the natural frequency which has the highest absolute value response for problem 1?

3. A component has already been tested to a 50 G, 0.011 sec base input half-sine pulse. A new requirement is that the component must withstand a 100 G, 0.002 sec half-sine pulse. Compare the specifications in terms of their corresponding shock response spectra. What conclusions can be made about the need for retesting?

4. Read tutorial srs_intr.pdf.