SHOCK AND VIBRATION RESPONSE SPECTRA COURSE Unit 22
Base Excitation Shock: Arbitrary Pulse

By Tom Irvine
Email: tomirvine@aol.com

Introduction

Unit 21 discussed a structure subjected to a base excitation shock pulse, where the pulse was a classical pulse such as a half-sine pulse.

The advantage of the classical pulse is that the response can be analyzed in terms of deterministic functions, derived via Laplace transforms.

Some real-world shock events indeed take the form of a half-sine pulse. Most pulses, however, have a complex, oscillating shape, as shown in the example in Figure 1.

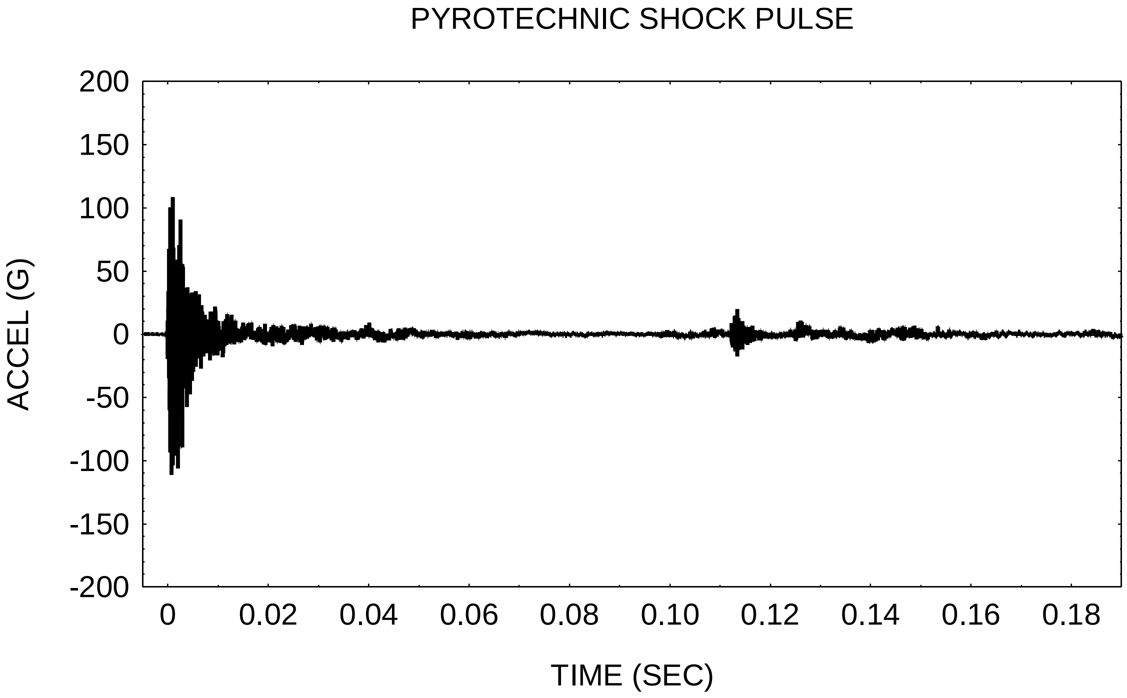


Figure 1.

The time history in Figure 1 is measured data from a rocket vehicle shroud separation test. The shroud is the nosecone of the vehicle. It opens up like a clamshell prior to the payload release.

The damage potential of the shock pulse in Figure 1 can be analyzed in terms of a shock response spectrum, as described in previous Units.

The purpose of this Unit is to present the shock response spectrum for an arbitrary base input pulse.

Derivation of Equations

As a review, consider the single-degree-of-freedom system subjected to base excitation shown in Figure 2. The free-body diagram is shown in Figure 3.

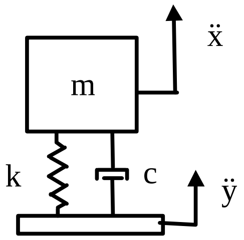


Figure 2. Single-degree-of-freedom System

The variables are

m = mass,
c = viscous damping coefficient,
k = stiffness,
x = absolute displacement of the mass,
y = base input displacement.

The double-dot notation indicates acceleration

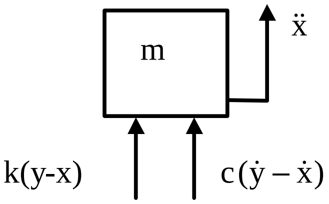


Figure 3. Free-body Diagram

The equation of motion is

$$m\ddot{x} = c(\dot{y}-\dot{x}) + k(y-x) \tag{1}$$

Define a relative displacement z. Let $z = x - y$.

The following equation of motion for the relative displacement z was derived in Unit 8.

$$\ddot{z} + 2\xi\omega_n \dot{z} + \omega_n{}^2 z = -\ddot{y} \tag{2}$$

Equation cannot be solved exactly for the case where the base input is an arbitrary pulse. A convolution integral approach is needed as described in Reference 1.

After many steps, the response can be represented in terms of a digital recursive filtering relationship, as derived in Reference 1.

$$\begin{aligned}
\ddot{x}_{\,i} = &\\
&+ 2\exp[-\xi\omega_n\Delta t]\cos[\omega_d\Delta t]\,\ddot{x}_{\,i-1}\\
&- \exp[-2\xi\omega_n\Delta t]\ddot{x}_{\,i-2}\\
&+ 2\xi\omega_n\Delta t\,\ddot{y}_{\,i}\\
&+ \omega_n\Delta t\,\exp[-\xi\omega_n\Delta t]\left\{\left[\frac{\omega_n}{\omega_d}\left(1-2\xi^2\right)\right]\sin[\omega_d\Delta t] - 2\xi\cos[\omega_d\Delta t]\right\}\ddot{y}_{i-1}
\end{aligned} \tag{3}$$

where

ξ is the damping ratio,

$\omega_d = \omega_n\sqrt{1-\ \xi^2}$,

Δt = time step,

$x_{\,i}$ is the response at time t,

$x_{\,i-1}$ is the response at time t-Δt,

$x_{\,i-2}$ is the response at time t-2Δt,

$y_{\,i}$ is the base input at time t,

$y_{\,i-1}$ is the base input at time t-Δt.

Example

Calculate the shock response spectrum of the pulse in Figure 1, using equation (3) as implemented in programs arbit.exe and qsrs.exe. Program arbit.exe generates a complete time history for a given natural frequency.

Program qsrs.exe generates a complete shock response spectrum for a family of natural frequencies.

Sample response values are shown in Table 1.

Table 1.
Shock Response Spectra of Pyrotechnic Shock Pulse, Q=10,

Natural Frequency (Hz)	Peak Positive Acceleration (G)	Peak Negative Acceleration (G)	Figure
50	6.5	-7.3	4
100	6.5	-5.6	-
150	5.0	-6.5	-
200	7.1	-6.2	-
250	13.2	-11.5	-
300	22.5	-20.5	5
350	17.0	-17.3	-
400	14.2	-16.4	-
450	14.4	-16.1	-
500	15.2	-14.2	6

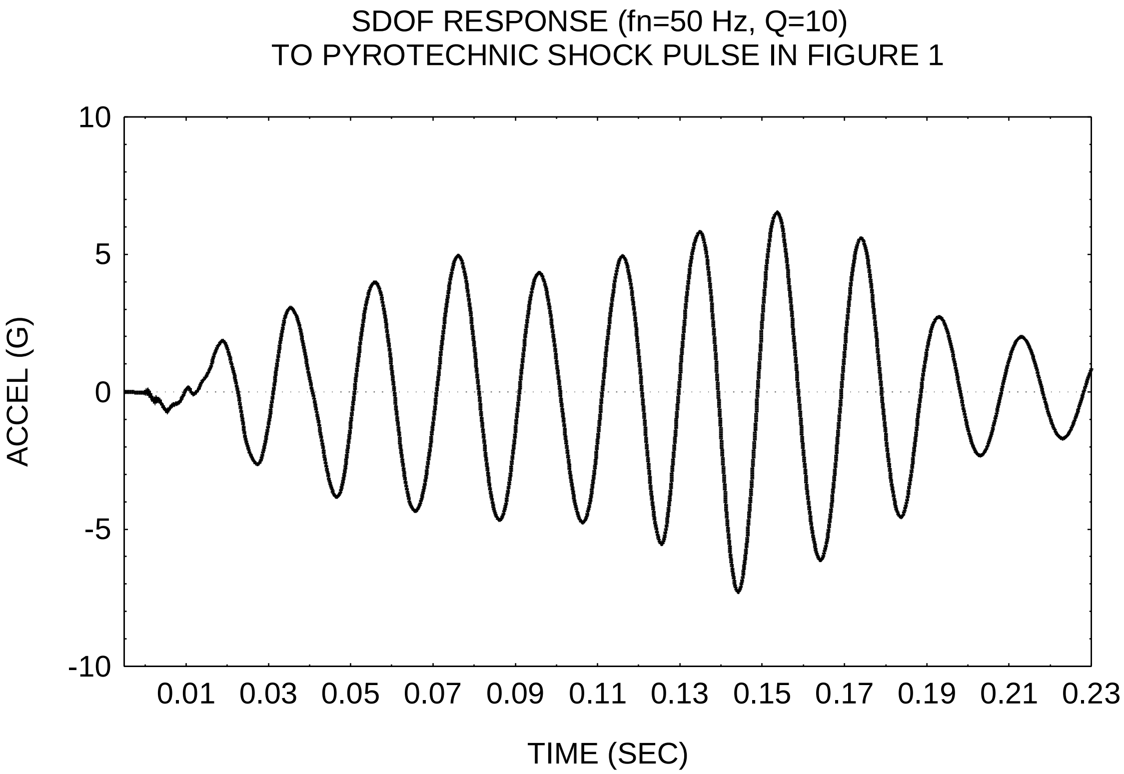
Note that the duration of the input pulse in Figure 1 extends to 0.190 seconds. The response of the 50 Hz system is extended to 0.230 seconds, however, to allow for a few cycles of free vibration.

Also note that only the peak positive and negative values are retained for each time history response. The peak values are found via a simple search method rather than a calculus method.

The overall shock response spectrum is shown in Figure 7. It is constructed by plotting the peak positive and negative acceleration amplitudes versus natural frequency in (Hz). Each of the coordinates in Table 1 is represented in Figure 7.

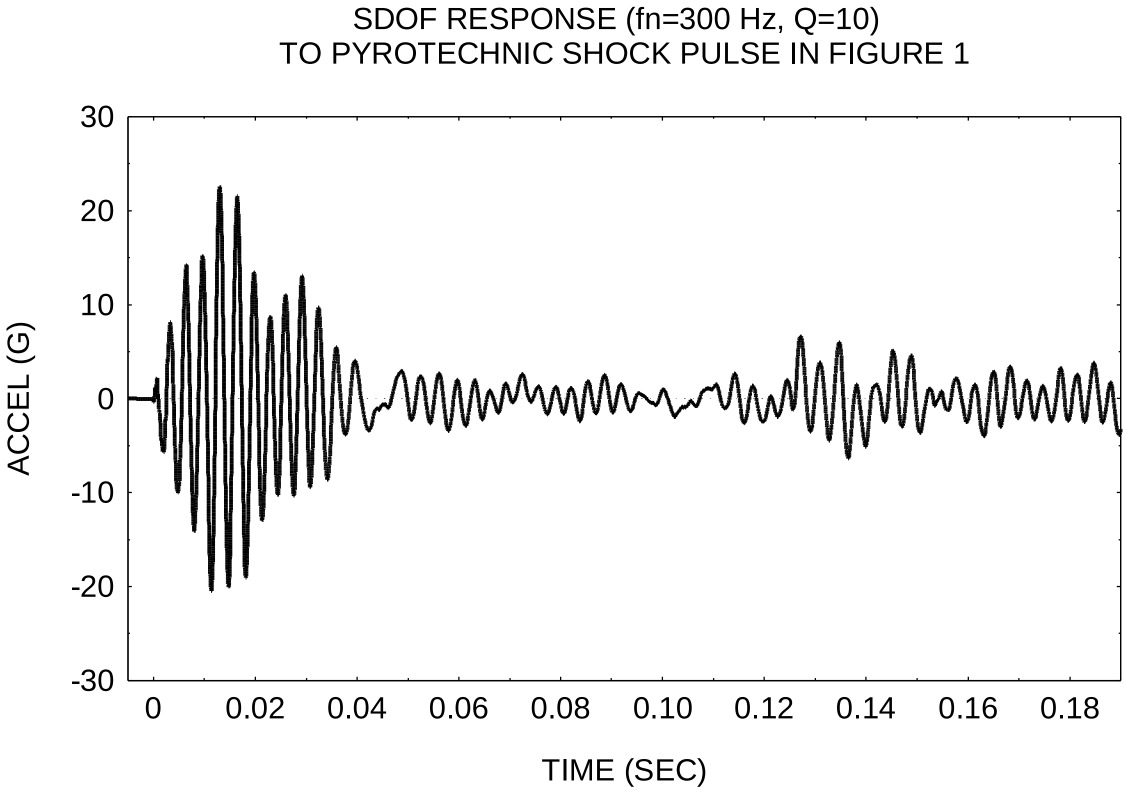
The positive and negative spectral curves in Figure 7 are nearly equal, suggesting that the data quality is good per the guidelines in Reference 1.

The absolute value shock response spectrum is shown in Figure 8. The absolute value curve envelops both the positive and negative curves.



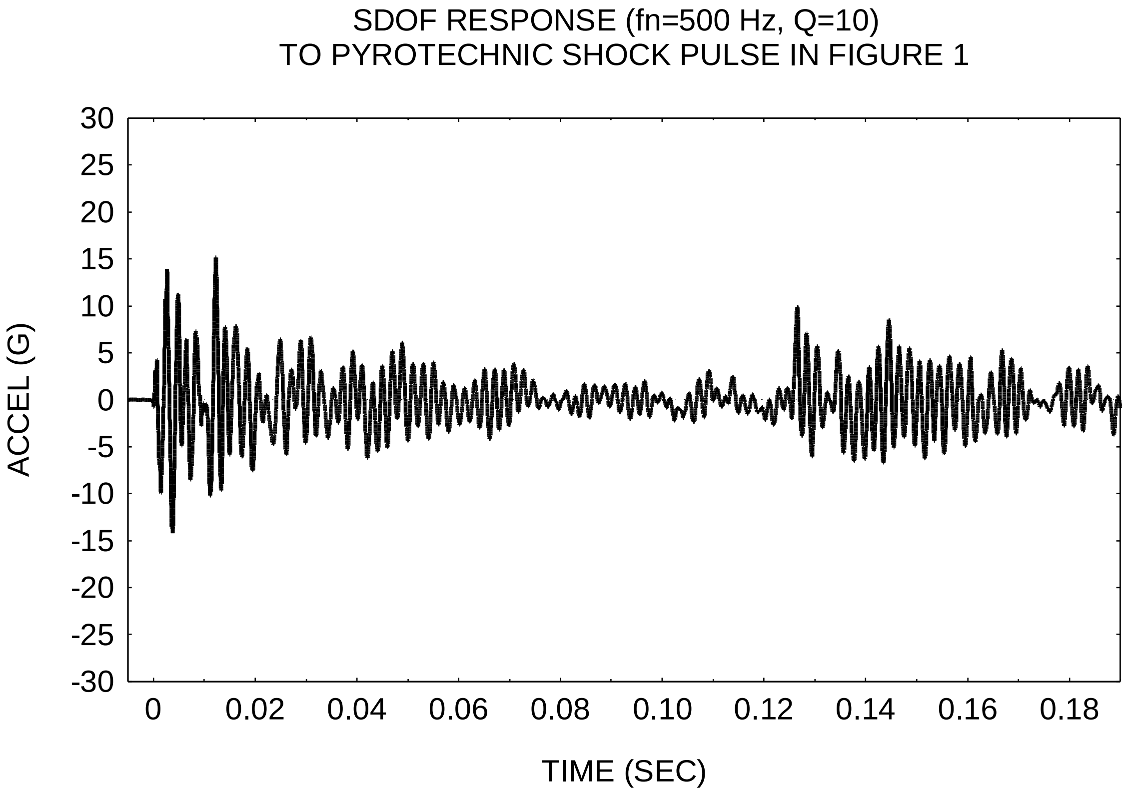
SDOF RESPONSE (fn=50 Hz, Q=10)
TO PYROTECHNIC SHOCK PULSE IN FIGURE 1

Figure 4.



SDOF RESPONSE (fn=300 Hz, Q=10)
TO PYROTECHNIC SHOCK PULSE IN FIGURE 1

Figure 5.



SDOF RESPONSE (fn=500 Hz, Q=10)
TO PYROTECHNIC SHOCK PULSE IN FIGURE 1

Figure 6.

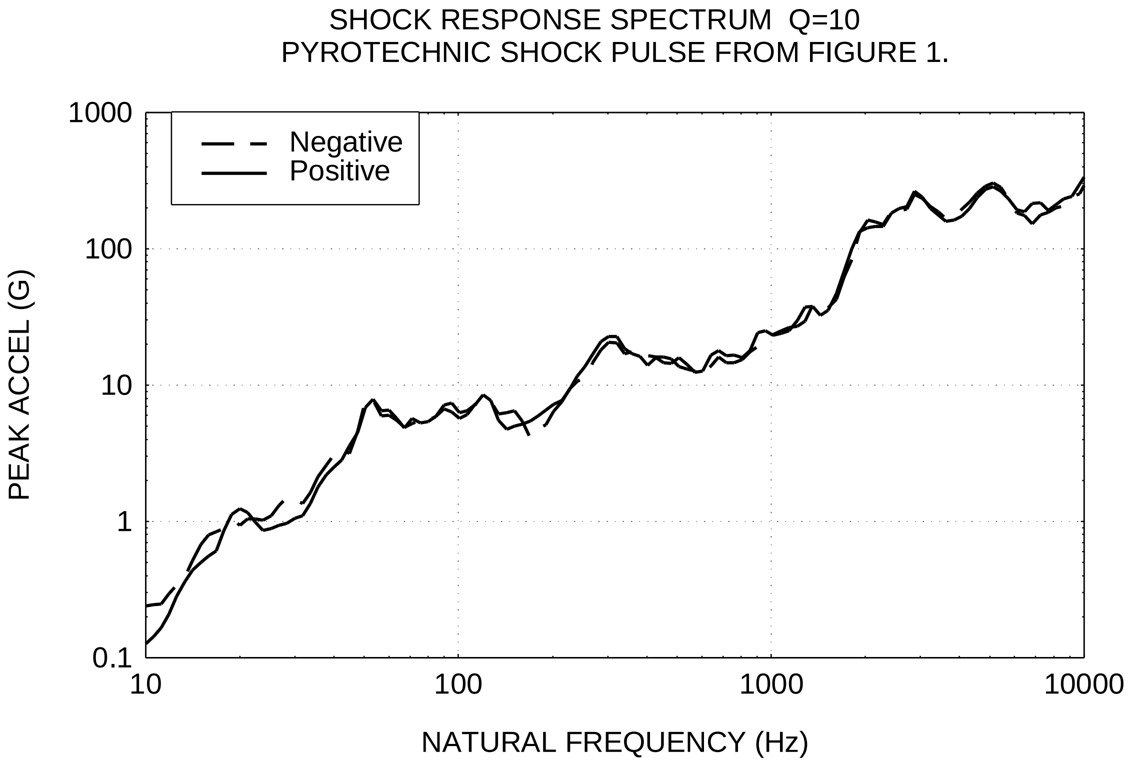


Figure 7.

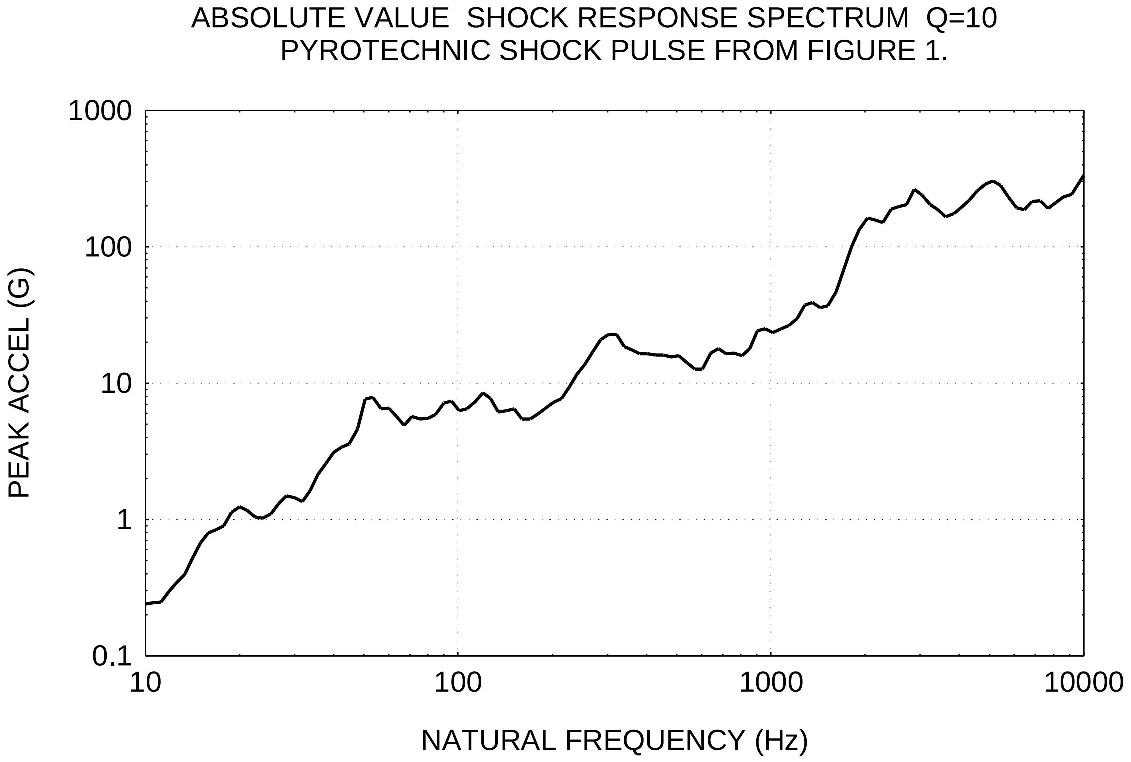


Figure 8.

The local peaks in Figure 8 may correspond to structural natural frequencies which were excited by the pyrotechnic shock. Further analysis and testing would be required to resolve this, however.

The spectra in Figures 7 and 8 show that the response amplitude tends to rise in some proportion to the natural frequency. This is typical of pyrotechnic shock.

Consider an avionics component that is to be mounted near the measurement location corresponding to the data in Figure 1. A design engineer may wish to design the component to have the smallest possible natural frequency in order to minimize the component's response to the shroud separation event in flight.

Other flight environments and design constraints must be considered, however, in the design process.

References

1. T. Irvine, An Introduction to the Shock Response Spectrum, Vibrationdata.com Publications, 2000.

Homework

1. Use program arbit.exe to calculate the time history response for file: pyro.txt. this file contains two columns: time (sec) and accel (G). This is actual measured data from the previously described shroud separation test. Assume a natural frequency of 50 Hz and Q=10. What are the peak positive and negative amplitudes? Program maxfind.exe can be used to determine the peaks.

2. Calculate the complete shock response spectrum of pyro.dat using program qsrs.exe. Again, assume Q=10. Plot the positive and negative spectra on one graph. Plot the absolute value spectrum on another.

3. How do the peak values in problem 1 relate to the spectral curves in problem 2?

4. Read the NASA pyrotechnic shock document, NasaPyro.zip, available at: http://www.vibrationdata.com/tutorials/.htm