SHOCK AND VIBRATION RESPONSE SPECTRA COURSE Unit 29.
Shock Response Spectrum Synthesis via Wavelets

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Introduction

Mechanical shock can cause electronic components to fail. Crystal oscillators may shatter, for example. Components such as DC-to-DC converters can detach from circuit boards. Housings and other mechanical parts may develop fatigue cracks, even those made from metal.

Mechanical shock can cause temporary malfunctions in addition to hard failures. Mechanical relays can experience chatter, for example. Computer hard disk drives may lock up, thereby requiring a re-boot.

Components should thus be subjected to shock tests in order to verify their design integrity.

There are several common shock test methods. One method is to mount the component on a shaker table and then subject it to a base excitation pulse.

The specification for a base excitation shock may be in the form of a shock response spectrum. If so, a time history must be synthesized to satisfy the specification.

The purpose of this report is to present a synthesis method using wavelets.

Wavelet Theory

Again, a shock response spectrum can be met using a series of wavelets. The wavelets are synthesized into a time history on a control computer. The control computer applies the time history to an electromagnetic shaker. The shaker then applies the shock pulse to the test item. The control computer then verifies the resulting shock pulse.

The equation for an individual wavelet is:

$$W_m(t) = \begin{cases} 0\,, \quad \text{for} \quad t < t_{dm} \\ A_m \sin\left[\dfrac{2\pi f_m}{N_m}\left(t - t_{dm}\right)\right] \sin\left[2\pi f_m\left(t - t_{dm}\right)\right]\,, \ \text{for} \ t_{dm} \le t \le \left[t_{dm} + \dfrac{N_m}{2 f_m}\right] \\ 0\,, \ \text{for} \ t > \left[t_{dm} + \dfrac{N_m}{2 f_m}\right] \end{cases}$$

where

$W_m(t)$	is the acceleration of wavelet m at time t,
A_m	is the wavelet acceleration amplitude,
f_m	is the wavelet frequency,
N_m	is the number of half-sines in the wavelet,
t_{dm}	is the wavelet time delay.

(1)

Note than N_m must be an odd integer and must be at least 3.

The total acceleration at any time t for a set of n wavelets is

$$\ddot{x}(t) = \sum_{m=1}^{n} W_m(t)$$

(2)

Selection of the proper wavelet parameters to fulfill a given shock response spectrum is a trial-and-error process. Prior experience is a valuable guideline. Note that the wavelet is designed to have zero net velocity and zero net displacement.

A time history of sample wavelet is shown in Figure 1.

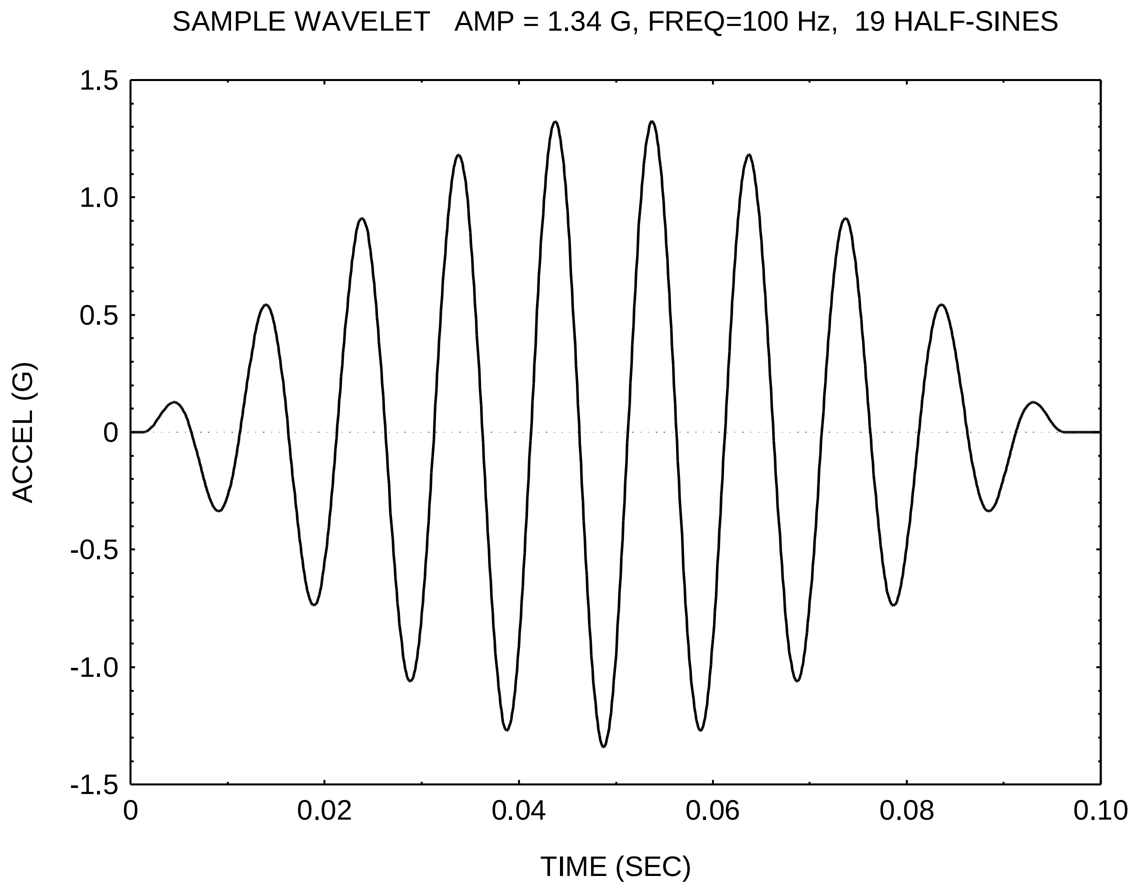
SAMPLE WAVELET AMP = 1.34 G, FREQ=100 Hz, 19 HALF-SINES
ACCEL (G)
TIME (SEC)
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
0
0.02
0.04
0.06
0.08
0.10

Figure 1.

Example

Consider the specification in Table 1.

Table 1. MIL-STD-810E Crash Hazard, SRS Q=10	
Natural Frequency (Hz)	Peak Acceleration (G)
10	9.4
80	75
2000	75

Notes:

1. Require that both the positive and negative spectral curves meet the specification.
2. Assume tolerance bands of ± 3 dB.
3. Use 1/6 octave spacing.
4. Allow a 0.400 second duration.

Synthesize an acceleration time history which satisfies the specification via wavelets. Optimize the time history to minimize the peak acceleration, velocity, and displacement values.

A synthesis can be performed using the approach in Table 2.

Table 2. Synthesis Steps	
Step	Description
1	Generate a random amplitude, delay, and half-sine number for each wavelet. Constrain the half-sine number to be odd. These parameters form a wavelet table.
2	Synthesize an acceleration time history from the wavelet table.
3	Calculate the shock response spectrum of the synthesis.
4	Compare the shock response spectrum of the synthesis to the specification. Form a scale factor for each frequency.
5	Scale the wavelet amplitudes.
6	Generate a revised acceleration time history.
7	Repeat steps 3 through 6 until the SRS error is minimized or until an iteration limit is reached.
8	Calculate the final shock response spectrum error. Also calculate the peak acceleration values. Integrate the signal to obtain velocity, and then again to obtain displacement. Calculate the peak velocity and displacement values.
9	Repeat steps 1 through 9 many times.
10	Choose the waveform which gives the lowest combination of SRS error, acceleration, velocity, and displacement.

The method in Table 2 is a rough outline.

For the example, the time delays are arranged in a "reverse sine sweep" manner. In other words, the highest frequency component has zero delay. Each successive wavelet moving downward in frequency has a progressively longer delay. The delay step is proportional to the wavelet frequency.

The two wavelet components having the lowest frequencies, however, are allowed to begin at time zero.

This synthesis method produced the wavelet series shown in Table 3. The peak time history values are shown in Table 4.

The acceleration, velocity, and displacement time histories are shown in Figures 2, 3, and 4, respectively. The shock response spectrum is shown in Figure 5. The spectral curves satisfy the tolerance bands.

Table 3. Wavelet Parameters			
Frequency (Hz)	Amplitude (G)	Half-sines	Delay (sec)
10.0	-1.84	7	0.000000
11.2	1.80	7	0.000000
12.6	0.51	5	0.155000
14.1	0.10	7	0.137990
15.9	0.44	7	0.122840
17.8	0.00	9	0.109350
20.0	4.85	11	0.097320
22.4	6.35	13	0.086610
25.2	4.23	15	0.077070
28.3	2.78	17	0.068570
31.7	2.40	17	0.060990
35.6	3.50	17	0.054240
40.0	4.11	17	0.048230
44.9	3.76	17	0.042870
50.4	4.81	17	0.038100
56.6	5.88	17	0.033850
63.5	5.93	17	0.030070
71.3	6.93	17	0.026690
80.0	6.91	17	0.023680
89.8	6.74	17	0.021010
100.8	7.34	17	0.018620
113.1	6.73	17	0.016500
127.0	7.23	17	0.014600
142.5	6.85	17	0.012910
160.0	6.52	17	0.011410
179.6	7.28	17	0.010070
201.6	6.72	17	0.008880
226.3	6.94	17	0.007820
254.0	7.06	17	0.006870
285.1	6.50	17	0.006030
320.0	7.33	17	0.005280
359.2	6.81	17	0.004610
403.2	6.79	17	0.004010
452.5	7.26	17	0.003480
508.0	6.50	17	0.003000

Table 3. Wavelet Parameters (continued)			
Frequency (Hz)	Amplitude (G)	Half-sines	Delay (sec)
570.2	7.24	17	0.002580
640.0	7.02	17	0.002210
718.4	6.52	17	0.001870
806.3	7.40	17	0.001570
905.1	6.63	17	0.001310
1015.9	7.01	17	0.001070
1140.4	7.25	17	0.000860
1280.0	6.44	17	0.000670
1436.8	7.21	17	0.000510
1612.7	6.58	17	0.000360
1810.2	4.73	17	0.000220
2031.9	6.67	17	0.000000

Table 4. Peak Values of Synthesized Time History		
Parameter	Maximum	Minimum
Acceleration (G)	13.2	-12.9
Velocity (in/sec)	25.5	-23.3
Displacement (inch)	0.344	-0.122

Note that the reverse sine sweep method tends to produce lower acceleration, velocity, and displacement values than a random delay method.

Minimization of these parameters is highly desirable due to shaker table limitations.

ACCELERATION TIME HISTORY
SYNTHESIS TO SATISFY MIL-STD-810E CRASH HAZARD SPECIFICATION

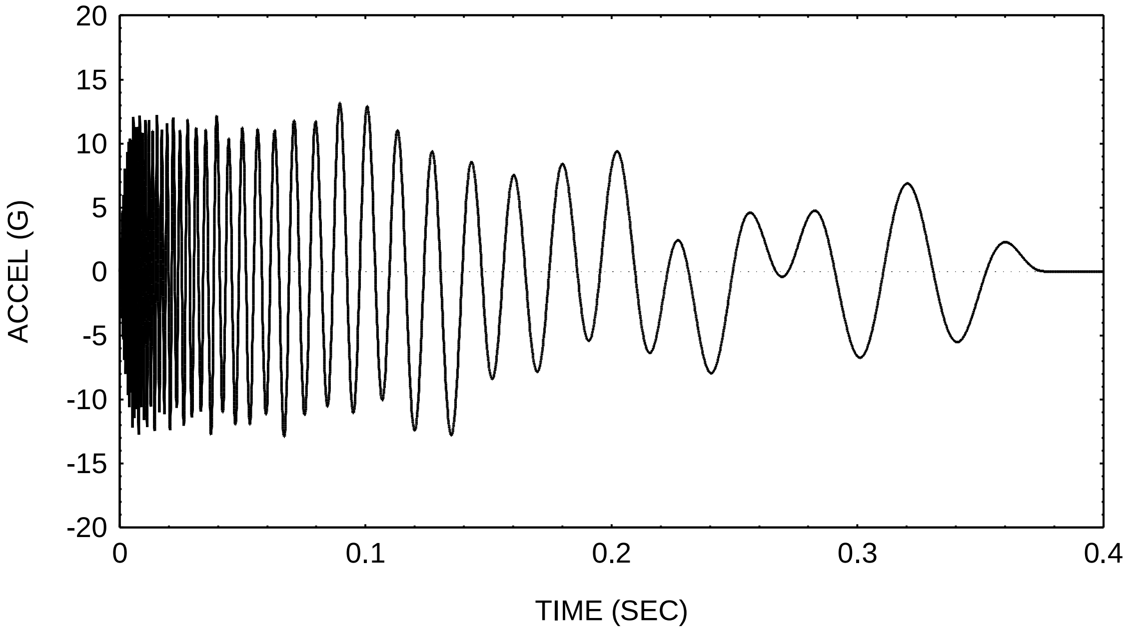

Figure 2.

VELOCITY TIME HISTORY
SYNTHESIS TO SATISFY MIL-STD-810E CRASH HAZARD SPECIFICATION

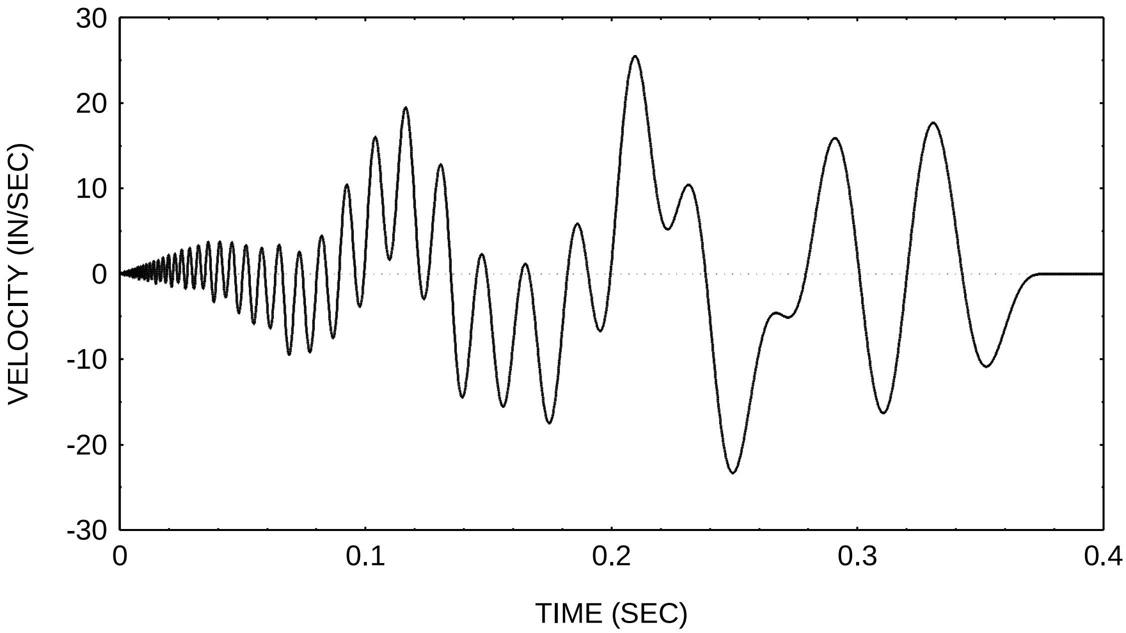

Figure 3.

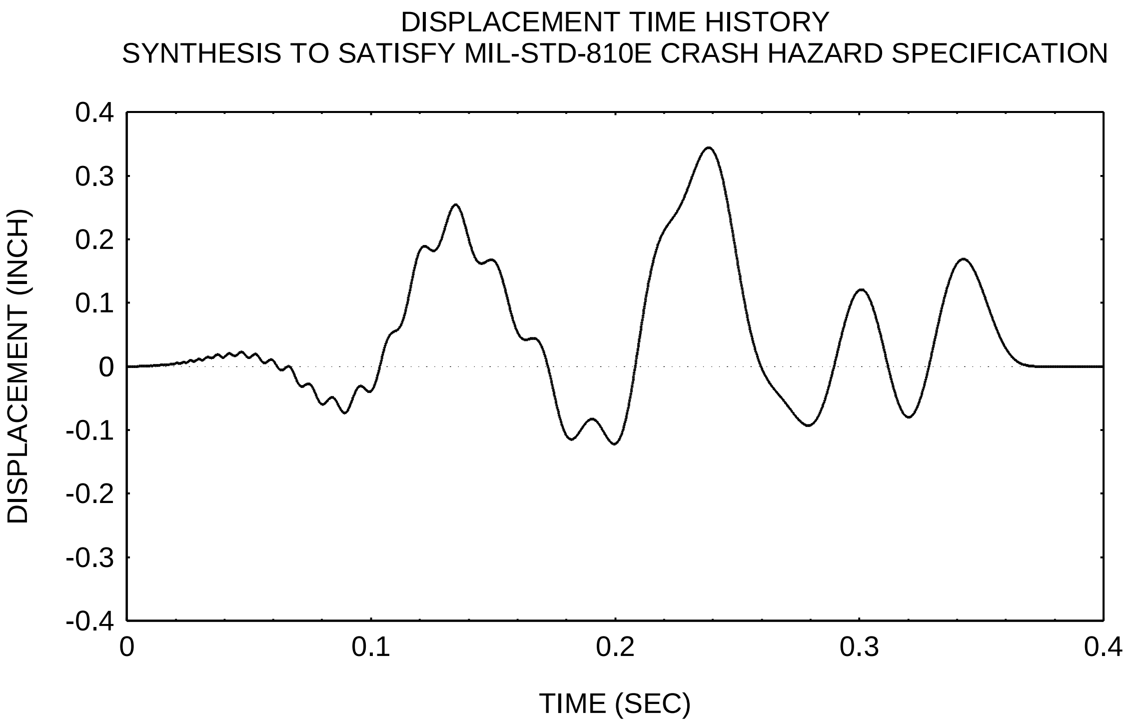

Figure 4.

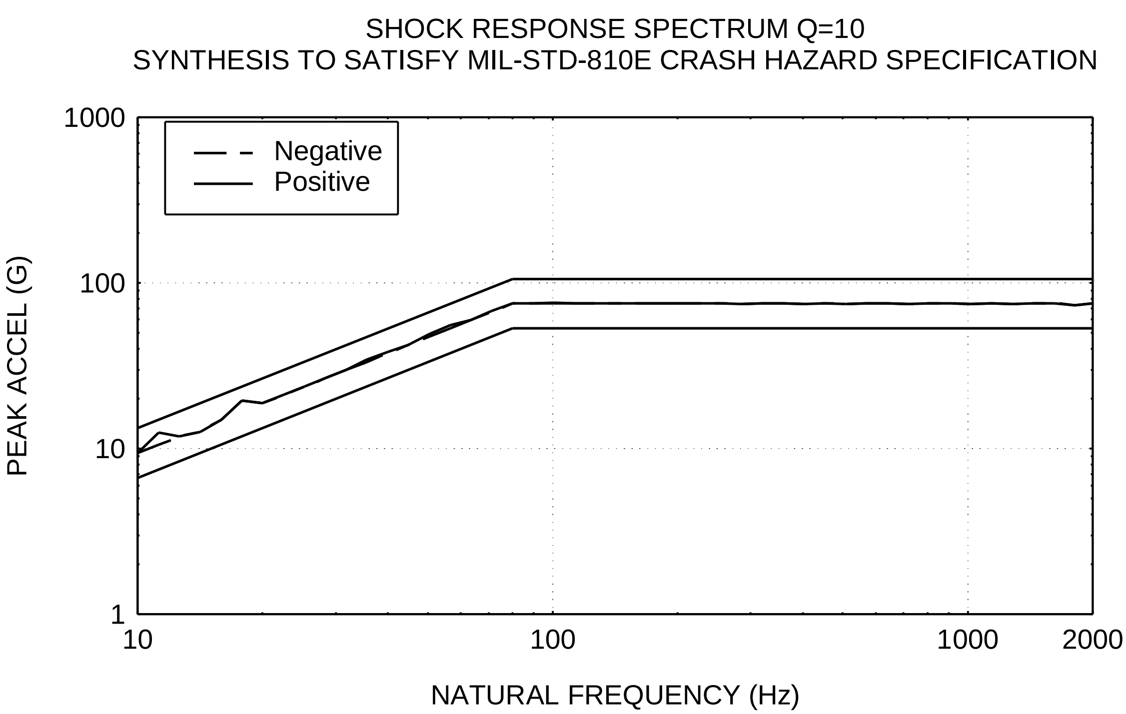

Figure 5.

Homework

Consider the specification in Table 2.

Table 2. SRS Q=10	
Natural Frequency (Hz)	Peak Acceleration (G)
10	10
50	50
500	50
1000	100
2000	100

Notes:

1. Require that both the positive and negative spectral curves meet the specification.
2. Assume tolerance bands of ± 3 dB.
3. Use 1/6 octave spacing.
4. Allow a 0.400 second duration.

Synthesize a time history pulse to satisfy the specification using program jsynth.exe. Plot the acceleration, velocity, displacement, and shock response spectra. Optimize the waveform by trying 300 cases.