SHOCK AND VIBRATION RESPONSE SPECTRA COURSE
Unit 2B. Sine Vibration Displacement, Velocity, and Acceleration

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Introduction

Again, consider a single-degree-of-freedom system undergoing sinusoidal excitation. The displacement amplitude x(t) is

$$x(t) = X \sin(\omega t) \tag{1}$$

where

X is the displacement
ω is the frequency (radians/time)

Note that each amplitude in this unit is taken as zero-to-peak.

The velocity $\dot{x}(t)$ is obtained by taking the derivative.

$$\dot{x}(t) = \omega X \cos(\omega t) \tag{2}$$

The acceleration $\ddot{x}(t)$ is obtained by taking the derivative of the velocity.

$$\ddot{x}(t) = -\omega^2 X \sin(\omega t) \tag{3}$$

Disregard phase. Take the peak absolute values of each parameter. The peak values are summarized in Table 1.

Table 1.
Peak Values Referenced to Peak Displacement.

Parameter	Equation
Displacement	$x_{peak} = X$
Velocity	$\dot{x}_{peak} = \omega X$
Acceleration	$\ddot{x}_{peak} = \omega^2 X$

Note that

$$\ddot{x}_{peak} = \omega^2 x_{peak} \tag{4}$$

Now let A be the peak acceleration. The relationships in Table 2 can be derived via algebra.

Table 2. Peak Values Referenced to Peak Acceleration	
Parameter	Equation
Displacement	$x_{peak} = A / \omega^2$
Velocity	$\dot{x}_{peak} = A / \omega$
Acceleration	$\ddot{x}_{peak} = A$

The equations in Table 2 have enormous consequence in vibration testing, as explained in the next section.

Shaker Table

A satellite is shown mounted to an electromagnetic shaker table in Figure 1. The shaker supplies a base excitation to the test item. These types of shakers are used to test a variety of components including small RF couplers, flight computers, and spacecraft.

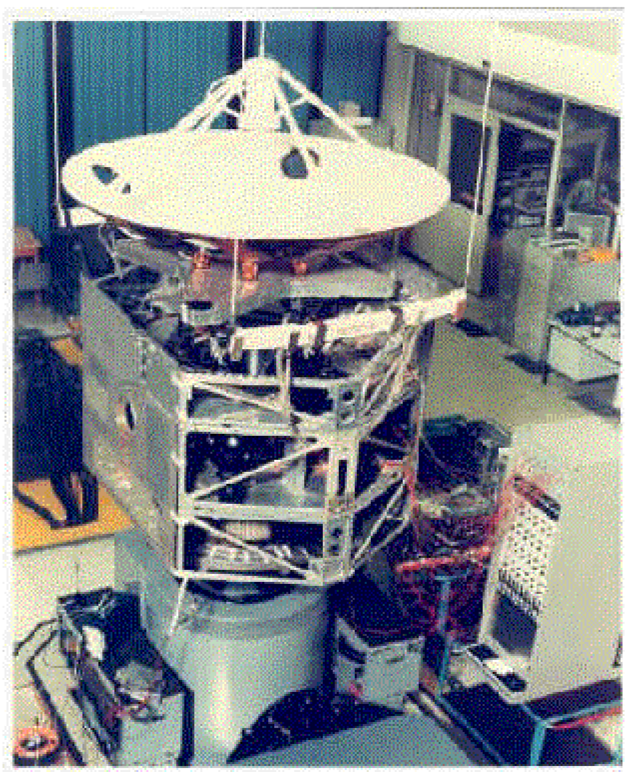

Figure 1.

This type of testing is used extensively in the aerospace industry. It is also used in the automotive and other industries.

The testing is typically performed in a closed-loop manner. A test specification is entered into the control computer. The computer outputs a signal to the shaker via a power amplifier. The shaker vibrates in response, applying a base excitation to the test item. A control accelerometer is placed on the shaker. This accelerometer is part of the feedback loop. The control computer monitors the accelerometer. It computes a system gain factor, thereby adjusting the output to achieve the test specification.

Specification Format

The test specification may come from a reference document such as MIL-STD-810E, MIL-STD-1540C, or NAVMAT P9492. Or a custom test specification may have been derived for the particular test item.

The test specification might be a random test, sine sweep, or some other format.

A test could be specified in terms of displacement, velocity, acceleration. The most common specification parameter is acceleration, however. The reason is that acceleration is easiest of the three parameters to measure.

Acceleration can be measured with a piezoelectric accelerometer or with a piezoresistive accelerometer.

Velocity measurements require a Doppler laser or a geophone. The laser is expensive. The geophone is bulky and is intended for seismology measurements.

Dynamic displacement can be measured by a linear variable displacement transducer (LVDT). Unfortunately, the frequency response is only suited for low-frequency measurements.

If velocity is required, the typical method is to integrate the acceleration signal. If displacement is required, the typical method is to double-integrate the acceleration signal.

Thus, acceleration is the amplitude parameter of choice for most vibration test specifications.

Recall the displacement equation from Table 2.

$$x_{peak} = A / \omega^2 \tag{5}$$

Now consider a sine sweep specification where a test item is to be subjected to a base excitation of 10 G amplitude. The frequency will be varied, starting with a low frequency and sweeping upward to a higher frequency. The amplitude will be held constant, however. The displacement at selected frequencies is shown in Table 3. The displacement is calculated per equation (5).

Table 2. Displacement for 10 G Sine Excitation	
Frequency (Hz)	Displacement (inches zero-to-peak)
0.1	9780
1	97.8
10	0.978
15	0.435
20	0.245
50	0.0391
100	0.00978
1000	9.78e-005

The shaker, however, has some displacement limit. A typical limit might be 1 inch peak-to-peak, or 0.5 inch zero-to-peak. Thus, 15 Hz would be the practical lower frequency limit for the 10 G sine test. The test could be performed at lower frequencies, but the acceleration amplitude would need to be reduced to the limit the displacement.

Most vibration test specifications thus begin at a frequency of 10 Hz or 20 Hz. Examples will be given in upcoming units.

<u>Homework</u>

1. File trailer.txt is measured acceleration data from a big-rig trailer tested at an automotive proving ground oval. The data is not purely sinusoidal, but is reasonably so. First, determine the frequency in Hertz. Then use equation (5) to estimate the peak displacement. Remember to convert G to in/sec^2, as part of the calculation. You may use a hand calculation or Excel.

2. Check your calculation from problem 1 using program sine.exe. Use the pure sine option in this program.

3. Use sine.exe for this problem. A shaker has a displacement limit of 1 inch peak-to-peak, or 0.5 inch zero-to-peak. What is the maximum velocity at a frequency of 10 Hz?