

SHOCK AND VIBRATION RESPONSE SPECTRA COURSE

Unit 4. Random Vibration Characteristics

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Introduction

Random Forcing Function and Response

Consider a turbulent airflow passing over an aircraft wing. The turbulent airflow is a forcing function. Furthermore, the turbulent pressure at a particular location on the wing varies in a random manner with time.

For simplicity, consider the aircraft wing to be a single-degree-of-freedom system. The wing would vibrate in a sinusoidal manner if it were disturbed from its rest position and then allowed to vibrate freely. The turbulent airflow, however, forces the wing to undergo a random vibration response.

Random Base Excitation

As another example, consider earthquake motion. The ground vibrates in random manner during the transient duration.

Common Characteristics

One common characteristic of these examples is that the motion varies randomly with time. Thus, the amplitude cannot be expressed in terms of a "deterministic" mathematical function.

Dave Steinberg wrote in Reference 1:

The most obvious characteristic of random vibration is that it is nonperiodic. A knowledge of the past history of random motion is adequate to predict the probability of occurrence of various acceleration and displacement magnitudes, but it is not sufficient to predict the precise magnitude at a specific instant.

Frequency Content

Pure sinusoidal vibration is composed of a single frequency. On the other hand, random vibration is composed of a multitude of frequencies. In fact, random vibration is composed of a continuous spectrum of frequencies.

Random vibration is somewhat analogous to white light.. White light can be passed through a prism to reveal a continuous spectrum of colors. Likewise, random vibration can be passed through a spectrum analyzer to reveal a continuous spectrum of frequencies.

On the other hand, sinusoidal vibration is analogous to a laser beam, where the light wave is composed of a single frequency.

Statistics of a Random Vibration Sample

A sample random vibration time history is shown in Figure 1. This time history was "synthesized," or generated analytically. It has the descriptive statistics shown in Table 1.

| Table 1. Random Vibration Descriptive Statistics | |
|---|-----------|
| Parameter | Value |
| Duration | 4 sec |
| Samples | 4000 |
| Mean | 0.00 |
| Std dev | 0.59 G |
| RMS | 0.59 GRMS |
| Kurtosis | 3.04 |
| Maximum | 2.30 G |
| Minimum | -1.96 G |

The statistical parameters were calculated using the formulas in Unit 2A, as implemented in the maxfind.exe program.

Recall that pure sine vibration has a peak value that is $\sqrt{2}$ times its RMS value.

On the other hand, random vibration has no fixed ratio between its peak and RMS values. The ratio between the absolute peak and RMS values in Table 1 is

$$\frac{2.30 \text{ G peak}}{0.59 \text{ G RMS}} = 3.90 \quad (1)$$

Also recall, that the RMS value is equal to the standard deviation value if the mean is zero. The standard deviation is often represented by sigma, σ .

Thus, the sample in Figure 1 has a peak value of 3.90σ . A different random sample could have a higher or lower peak value in terms of its σ , however.

A typical assumption is that random vibration has a peak value of 3.0σ for design purposes. Again, the example in Figure 1 deviates from this assumption with its peak value of 3.90σ .

SAMPLE RANDOM VIBRATION

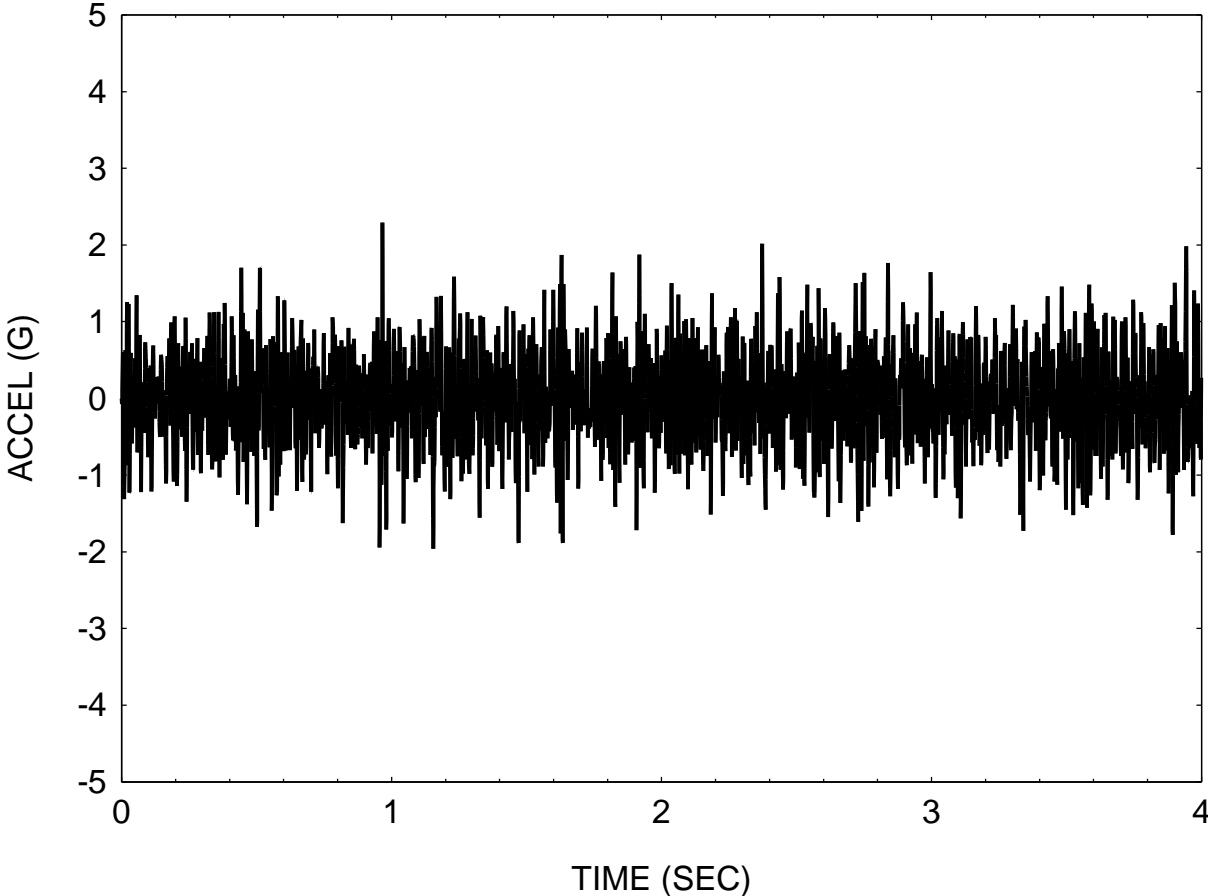


Figure 1.

The time history of Figure 1 is shown again in Figure 2. The amplitude in Figure 2 is scaled in terms of the σ value. According to theory, the amplitude should be within the $\pm 1\sigma$ limits 68.26% of the time.

SAMPLE RANDOM VIBRATION

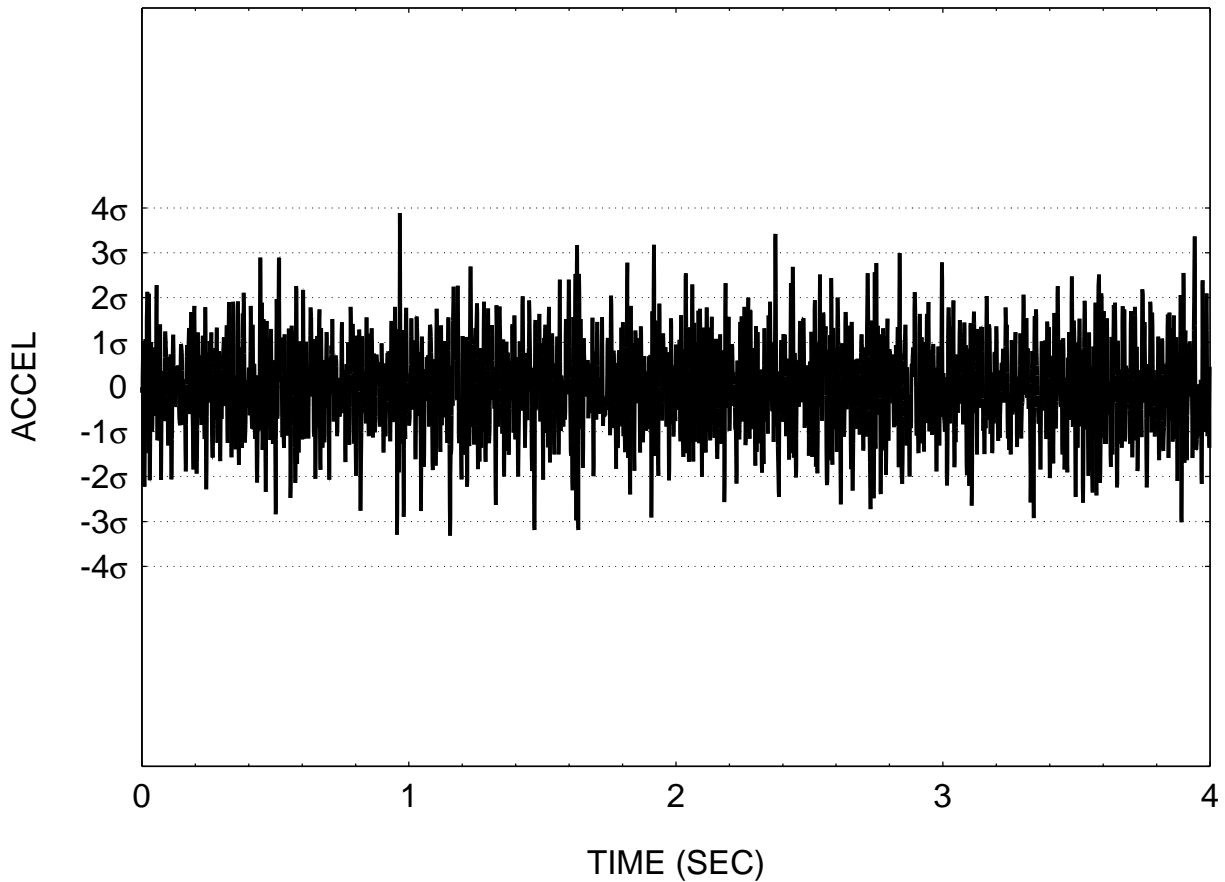


Figure 2.

Histogram

The histogram of the time history in Figure 1 is shown in Figure 3.

Note that the histogram of the random vibration sample has a "bell-shaped" curve. The histogram is an approximate example of a Gaussian or normal distribution.

The histogram shows that the random vibration signal has a tendency to remain near its mean value, which in this case is zero.

In contrast, recall the histogram of the sinusoidal time history in Unit 2A. The histogram of a sinusoidal signal has the shape of a bathtub. Sine vibration thus tends to remain at its positive and negative peak values.

For this and other reasons, sine and random are two very different forms of vibration. There really is no "equivalency" between the two forms, although many engineers have tried to derive a relation.

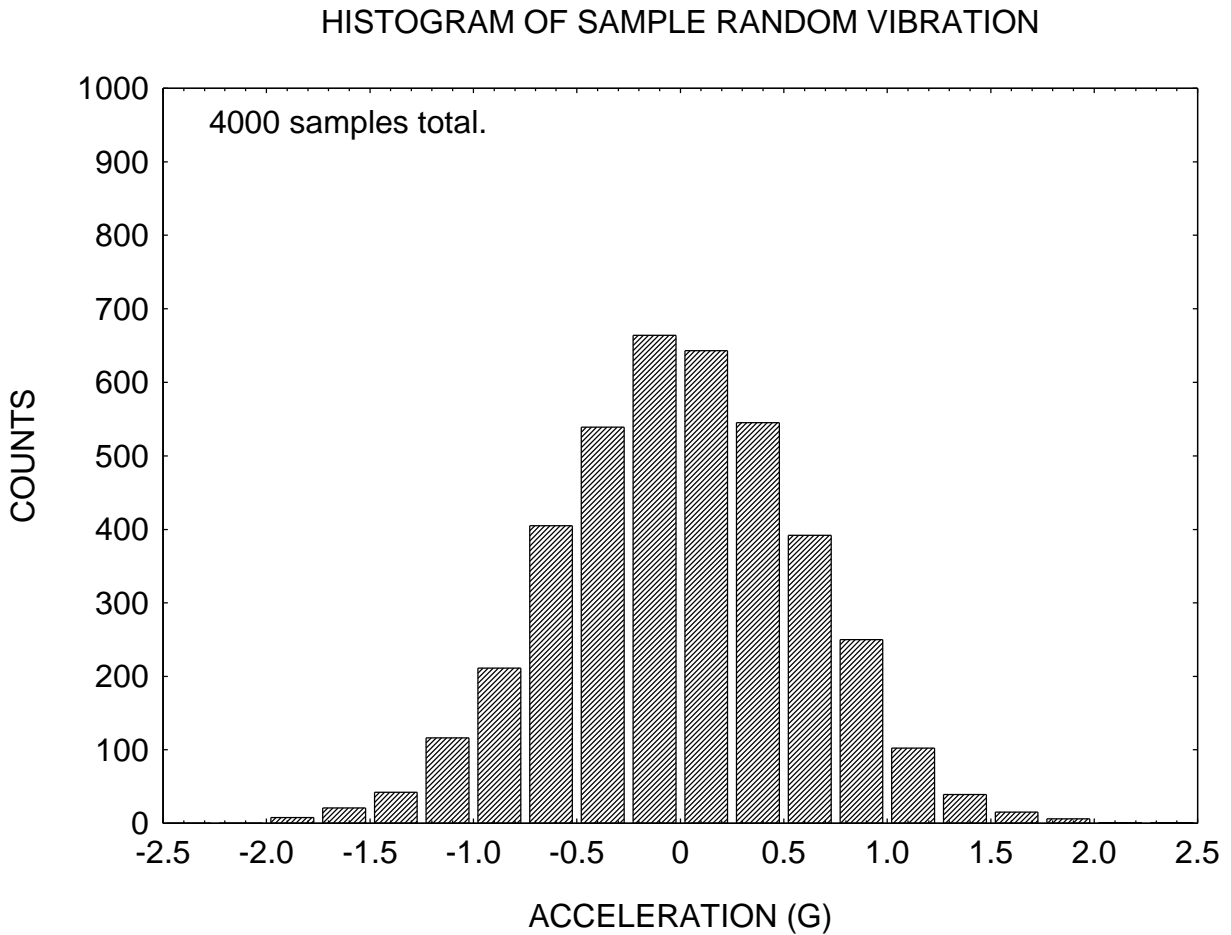


Figure 3.

Probability Density Function

The histogram in Figure 2 can be converted to a "probability density function." This would be done by dividing each bar by the total number of samples, 4000 in this case. The resulting function would be a probability density function. Furthermore, the amplitude along the X-axis could be represented in terms of σ . The resulting probability density function would then approximate a "normal probability density function."

The equation which characterizes the normal probability function is well-known. It is available in References 1 and 2. This equation can be integrated to determine the probability that an amplitude will occur inside or outside certain limits.

Normal Probability Values

Consider a random vibration time history $x(t)$. Again, the amplitude $x(t)$ cannot be calculated for a given time.

Nevertheless, the probability that $x(t)$ is inside or outside of certain limits can be expressed in terms of statistical theory.

The probability values for the amplitude are given in Tables 2a and 2b for selected levels in terms of the standard deviation or σ value.

| Statement | Probability Ratio | Percent Probability |
|---------------------------|-------------------|---------------------|
| $-\sigma < x < +\sigma$ | 0.6827 | 68.27% |
| $-2\sigma < x < +2\sigma$ | 0.9545 | 95.45% |
| $-3\sigma < x < +3\sigma$ | 0.9973 | 99.73% |

| Statement | Probability Ratio | Percent Probability |
|-----------------|-------------------|---------------------|
| $ x > \sigma$ | 0.3173 | 31.73% |
| $ x > 2\sigma$ | 0.0455 | 4.55% |
| $ x > 3\sigma$ | 0.0027 | 0.27% |

The probability tables can be used as follows. Suppose that a random vibration time history has a total duration of 60 seconds. For what amount of time will the amplitude exceed 1σ in terms of absolute value?

$$(60 \text{ sec})(0.3173) = 18.22 \text{ sec} \quad (2)$$

Suppose the same time history is digitized such that it consists of 200,000 points. How many points will exceed 2σ in terms of absolute value?

$$(200,000)(0.0455) = 9100 \text{ samples} \quad (3)$$

Types of Random Vibration

Random vibration can be broadband or narrow band. It can be stationary or non-stationary. In addition, white noise and pink noise are two special cases of random vibration. These distinctions are covered in upcoming units.

Sometimes, measured data has a "sine-on-random" characteristic. Recall, the rocket vehicle drop transient data from Units 1A and 1B. The raw data contained some random noise. Nevertheless, the decaying sinusoidal signal dominated the response. The data as given to the student, however, was "bandpass filtered" to clarify the decaying sinusoid. Filtering will be covered in a future unit.

Computer hard disk drives are an example of device where both sine and random vibration environments are a concern, as discussed in Appendix A, located after the homework section.

Furthermore, note that a great deal of analysis effort is spent searching vibration data for particular sinusoids which may be hidden inside a random signal.

References

1. D. Steinberg, "Vibration Analysis for Electronic Equipment," Wiley-Interscience, New York, 1988.
2. T. Irvine, Integration of the Normal Distribution Curve, Vibrationdata Publications, 1999.

Homework

1. Use program generate.exe to synthesize a white noise random vibration time history with a standard deviation value of 1, duration of 5 seconds, and sample rate of 1000 samples/sec. Plot the resulting time history. Use maxfind.exe to evaluate its statistical parameters. What is the ratio between the absolute peak value and the standard deviation value?
2.
 - a. Use the program stats.exe.
 - b. Select normal distribution.
 - c. Select "Calculate probability for a given Z value." Note that Z represents σ assuming a zero mean.
 - d. Click on -Z to +Z.
 - e. Input $Z = 3.2$, which is equivalent to 3.2σ .
 - f. What is the probability that a random vibration time history will have peaks in excess of 3.2σ in terms of absolute value?
 - g. Given a time history with 4000 points. How many will exceed 3.2σ in terms of absolute value?
3. Read NAVMAT P-9492. It may be downloaded from:
<http://www.vibrationdata.com/tutorials.htm>
4. Consider an avionics component. It is powered and monitored during a bench test. It passes this "functional test." Nevertheless, it may have some latent defects such as bad solder joints or bad parts. A decision is made to subject the component to a base excitation test on a shaker table to check for these defects. Which would be a more effective test: sine sweep or random vibration? Why?
5. Review question. What is the kurtosis value of pure sine vibration? What is the kurtosis value of broadband random vibration?

APPENDIX A

Hard Disk Drive Vibration

A hard disk is made from metal or some other rigid material. The disk is called a platter. It is coated with a magnetic material that is used to store data as transitions of magnetic polarity. Each polarity corresponds to a “1” or a “0.”

One or more platters are mounted on a single spindle shaft. The drive platters are divided into cylinders.

Each drive may have a spin rate somewhere between 3600 rpm and 10,000 rpm. This corresponds to the frequency domain between 60 Hz and 167 Hz. Future designs may have even higher spin rates.

A read/write head is mounted on the actuator arm. There is typically one head on each side of every platter.

The heads move in unison back and forth across the platter according to a control algorithm. The algorithm compensates for the flexibility of the actuator arm, platter vibration, and other disturbances.

Source Energy

Rotating imbalance, misalignment, and other defects may produce sinusoidal vibration at the spin rate or at integer multiples thereof.

Movement of the head assembly produces additional vibration. For example, random seek movement may cause rotational oscillations with a broadband random frequency content. This oscillation causes in-plane rotation of the drive and wrapper assembly.

Error and Failure Modes

Hard drive units have a number of error and failure modes related to vibration.

Impact

A particular concern is that the head or arm might impact against the surface of one or more disks, thereby creating voids in the recording film. This damage could cause data errors.

There are several means by which this impact could occur. For example, the vibration could come from an external source that propagates into the arm, possibly exciting a head/arm natural frequency. Another scenario is that excessive disk vibration causes the disk to impact against the head or arm.

Stack Shift

Stack shift is also a concern. This occurs when individual disks shift from their initial center of rotation. This shift could occur if the shock and vibration forces overcome the initial clamping forces. Stack shift may produce a sinusoidal position error signal with a frequency equal to two times (2X) the spin frequency.

Servo Control Algorithm

Vibration may interfere with the control algorithm. This is a particular concern during write operations. The algorithm is designed to prevent existing data from being inadvertently overwritten. Excessive vibration may cause a delay in the writing process. This condition is called latency.

Control algorithms vary from one hard drive model to the next. Most algorithms, however tend to be sensitive in the frequency domain from 500 Hz to 600 Hz.