SHOCK AND VIBRATION RESPONSE SPECTRA COURSE
Unit 6B. Notes on the Fourier Transform Magnitude

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Introduction

Fourier transforms, which were introduced in Unit 6A, have a number of potential error sources and other peculiar characteristics. The purpose of this unit is to discuss the Fourier transform magnitude, which must be interpreted with great care.

Sine Example

Consider a sine function with a 1 Hz frequency and 1 G amplitude. Let the period be 20 seconds, which is equivalent to 20 cycles. Thus, $\Delta f = 0.05$ Hz. The corresponding Fourier transform magnitude is shown in Figure 1.

Now define the same sine function over a period of 40 seconds. Thus, $\Delta f = 0.025$ Hz. The Fourier transform magnitude is shown in Figure 2.

The Fourier transform magnitude at 1 Hz is 1 G in each case, independent of the duration difference. Thus, the Fourier transform magnitude is shown to be a good tool for resolving sinusoidal amplitudes.

In each Fourier transform, there is a spectral line exactly at a frequency of 1 Hz. Otherwise, the acceleration amplitude would be smeared between frequencies adjacent to 1 Hz. This smearing effect is not a concern if the duration is sufficiently long and hence the frequency resolution is sufficiently narrow.

ONE-SIDED, FULL-AMPLITUDE FOURIER TRANSFORM OF
$Y(t) = 1.0 \sin [2\pi (1 \text{ Hz}) t]$ G, $0 \le t \le 20$ sec

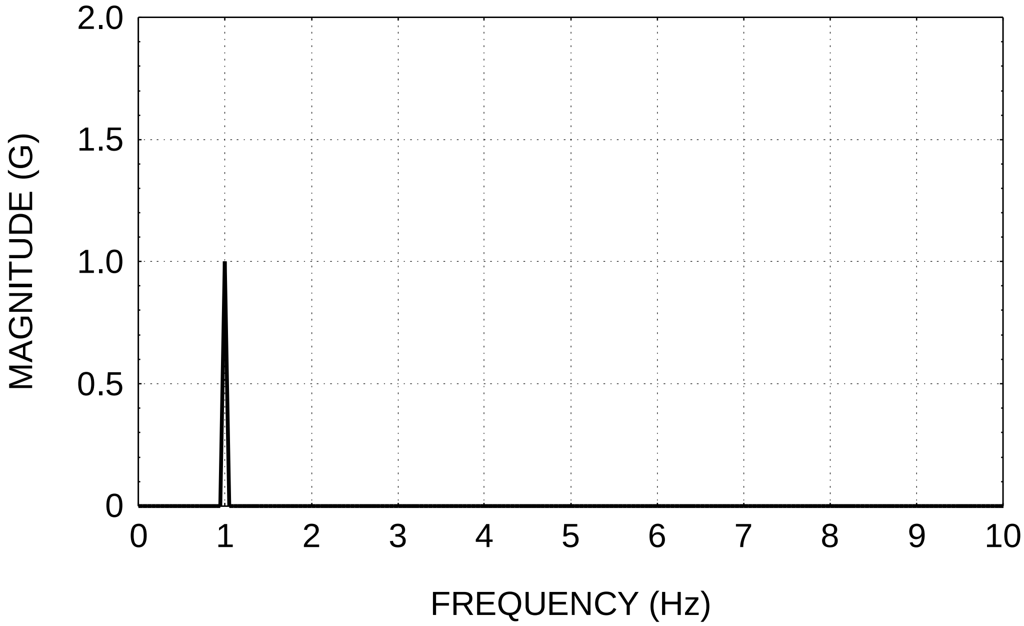

Figure 1.

ONE-SIDED, FULL-AMPLITUDE FOURIER TRANSFORM OF
$Y(t) = 1.0 \sin [2\pi (1 \text{ Hz}) t]$ G, $0 \le t \le 40$ sec

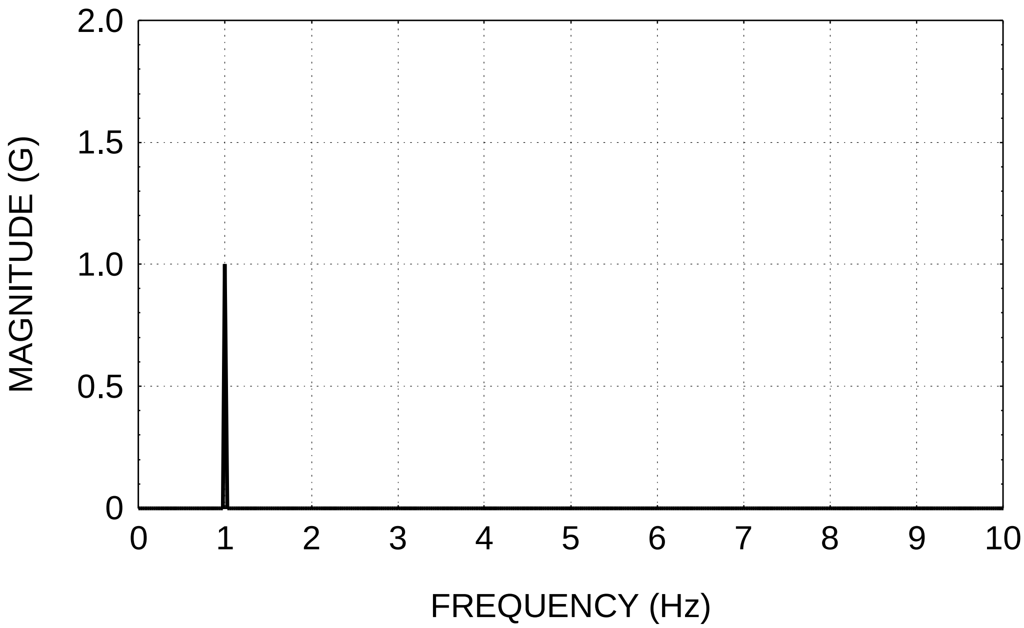

Figure 2.

White Noise Example

Consider the two white noise time histories in Figures 3 and 4. Each has a sample rate of 200 samples per second. Each has a standard deviation of 1 G. The overall level is 1 GRMS since the mean is zero, in each case.

The parameters for the Fourier transform calculation are given in Table 1.

Table 1. White Noise Parameters for Fourier Transform

Parameter	Figure 3	Figure 4
Overall Level	1 GRMS	1 GRMS
Duration	5 sec	10 sec
Δf	0.2 Hz	0.1 Hz
Sample Rate	200 sps	200 sps
Frequency Domain (Hz)	0 to 100 Hz	0 to 100 Hz
Number of Spectral Lines	500	1000

sps = samples per second.

Recall that the frequency resolution Δf is the inverse of the duration T.

$$\Delta f = 1/T \tag{1}$$

The frequency domain is taken from zero to one-half the sample rate.

The number of spectral lines N is equal to the maximum frequency divided by the frequency resolution.

$$N = \frac{F\max}{\Delta f} \tag{2}$$

The Fourier transforms of the respective white noise time histories are shown in Figures 5 and 6.

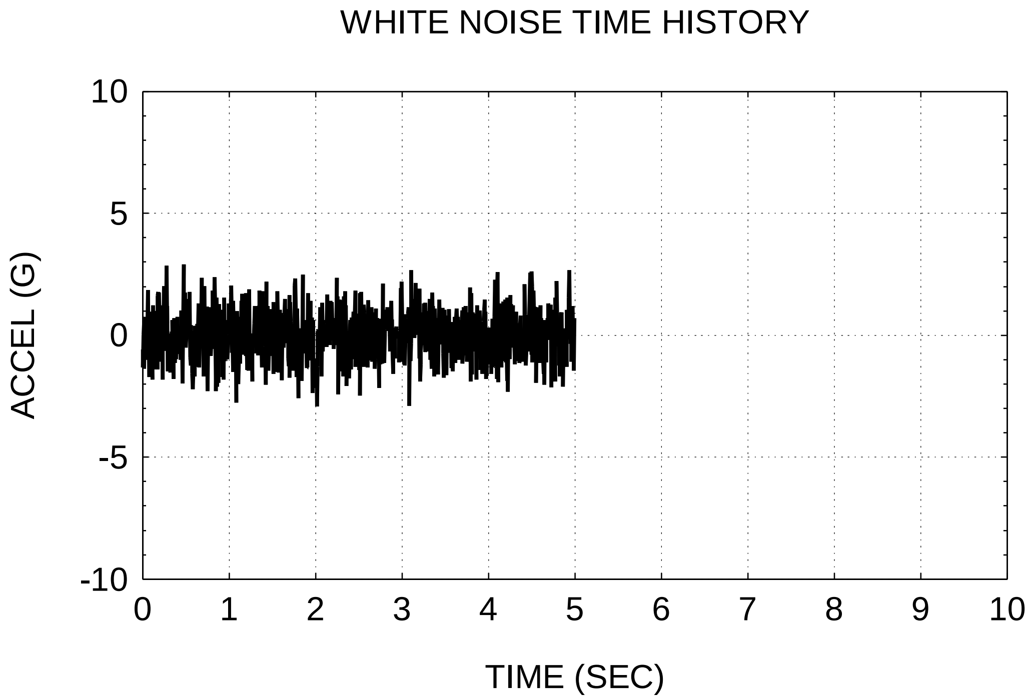

Figure 3.

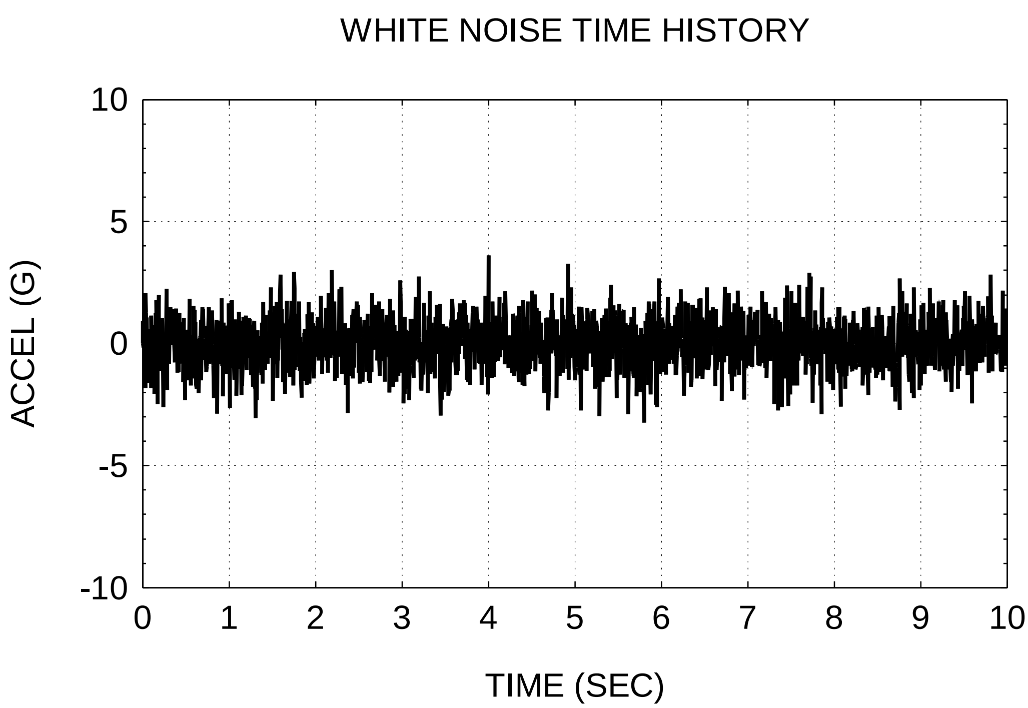

Figure 4.

ONE-SIDED, FULL-AMPLITUDE FOURIER TRANSFORM OF
5 SECOND DURATION WHITE NOISE TIME HISTORY

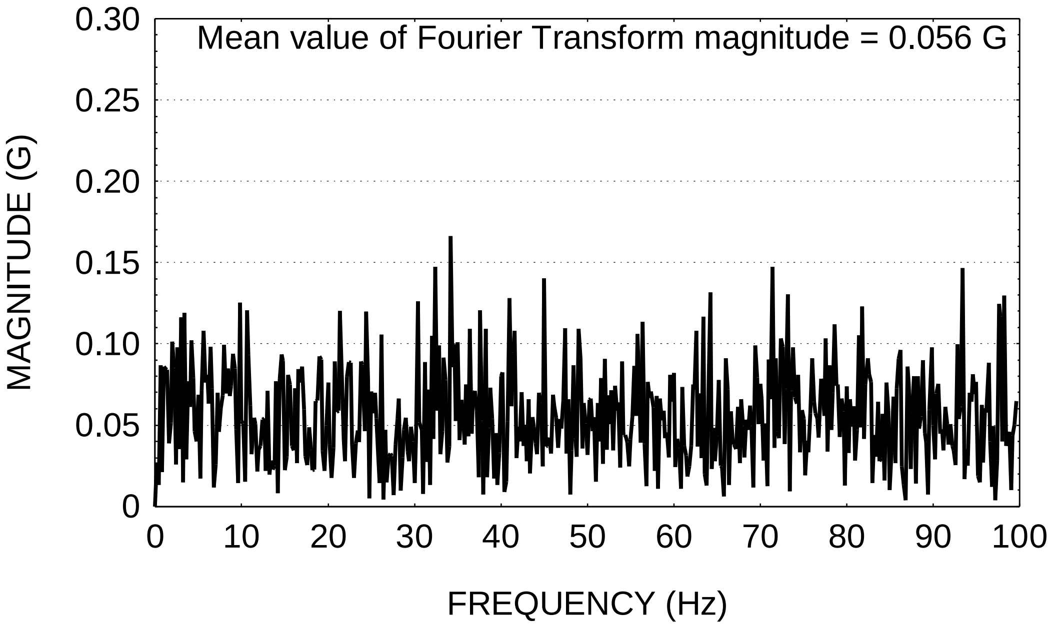

Figure 5.

ONE-SIDED, FULL-AMPLITUDE FOURIER TRANSFORM OF
10 SECOND DURATION WHITE NOISE TIME HISTORY

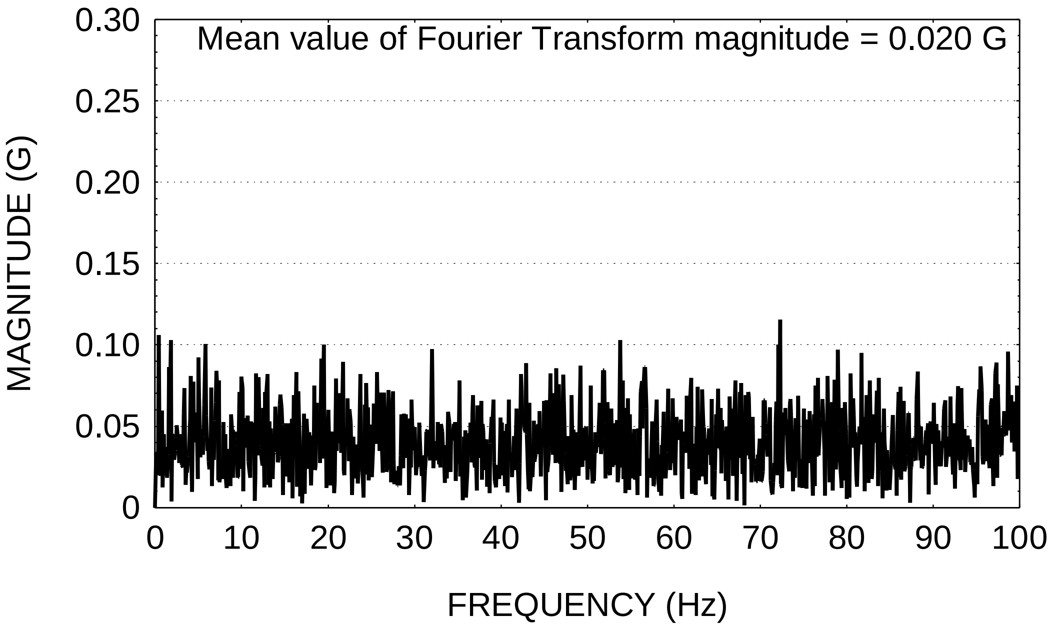

Figure 6.

Ideally, the "white noise" would have a constant Fourier transform magnitude with respect to frequency. The fact that there is some variation within Figures 5 and within Figure 6 is unimportant for this example. The pertinent point is that the mean magnitude decreases by about one-half, comparing the transform in Figure 6 to the transform in Figure 5.

The reason for the decrease is that the transform in Figure 6 has 1000 spectral lines compared to the 500 spectral lines in the Figure 5 transform. Thus, the "energy" is divided into a greater number of spectral lines in the Figure 6 transform.

Each transform, however, yields the same overall value of 1 GRMS. This is found as follows:

1. Divide each spectral magnitude by $\sqrt{2}$ to convert from peak to RMS.
2. Square each spectral RMS value to convert to mean square.
3. Sum the mean square values.
4. Take the square root of the sum.

Conclusion

For a random signal, the Fourier transform magnitude depends on the number of spectral lines.

This drawback is overcome by the power spectral density function, which is covered in a later unit.

Homework

1. Use the generate.exe program to synthesize white noise time histories similar to those in Figures 3 and 4. Then calculate and plot the respective Fourier magnitudes using the fourier.exe program.

2. Optional problem intended for avid Excel users. Call each Fourier transform magnitude into Excel. Calculate the overall RMS value using the four steps shown previously in the text. Verify that this agrees with the RMS value of the time history. Program maxfind.exe can be used to calculate the RMS value of the time history.

For reference, a sample Excel spreadsheet for problem 2 is given in file: Fourier.xls.

Students who prefer to use Matlab or some other software tool are welcome to do so.