SHOCK AND VIBRATION RESPONSE SPECTRA COURSE
Unit 6C. Leakage Error in Fourier Transforms

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Introduction

There are a number of error sources associated with the Fourier transform. One error source is called "leakage."

Leakage is a smearing of energy throughout the frequency domain.

Leakage results when both of the following conditions are present:

1. The signal is taken over a finite duration.
2. The signal is "non-periodic" in the time record.

Both these conditions are usually present in engineering data. Thus, leakage usually occurs.

For example, leakage occurs if a Fourier transform is calculated for a non-integral number of sine function cycles.

Sine Function Example 1

Consider that a data acquisition system is used to monitor a continuous sine function. The sine function has an amplitude of 1 G and a frequency of 1 Hz, as shown in Figure 1. The sample rate is 32 samples per second.

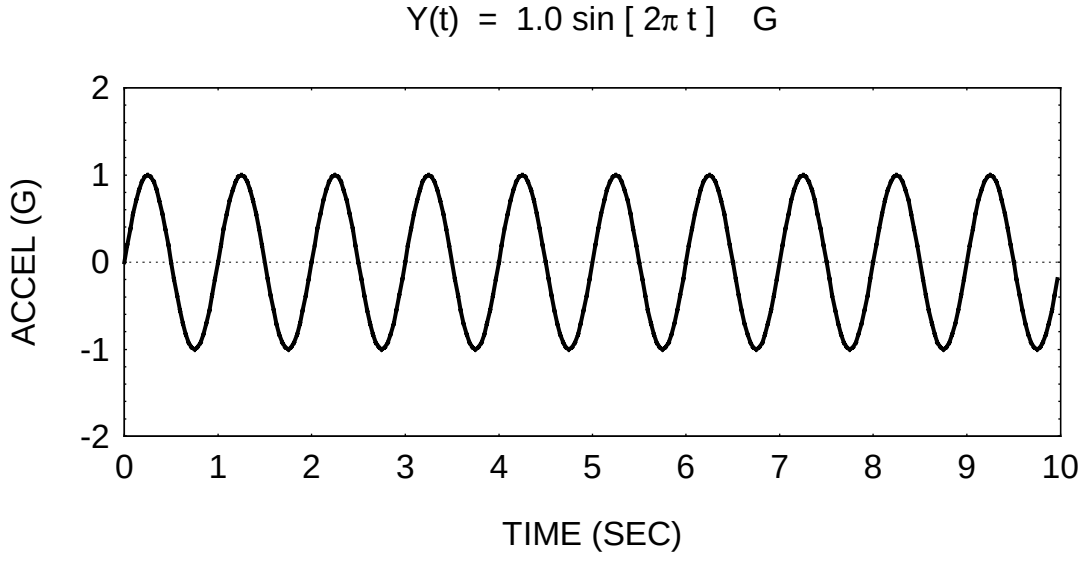

Figure 1.

Now consider that the data acquisition system measures three cycles as shown in Figure 2. Note that the time history amplitude is zero at the start and end of the record.

In essence, the Fourier transform will correctly assume that the original signal is a series of three-cycle segments as shown in the time history in Figure 3.

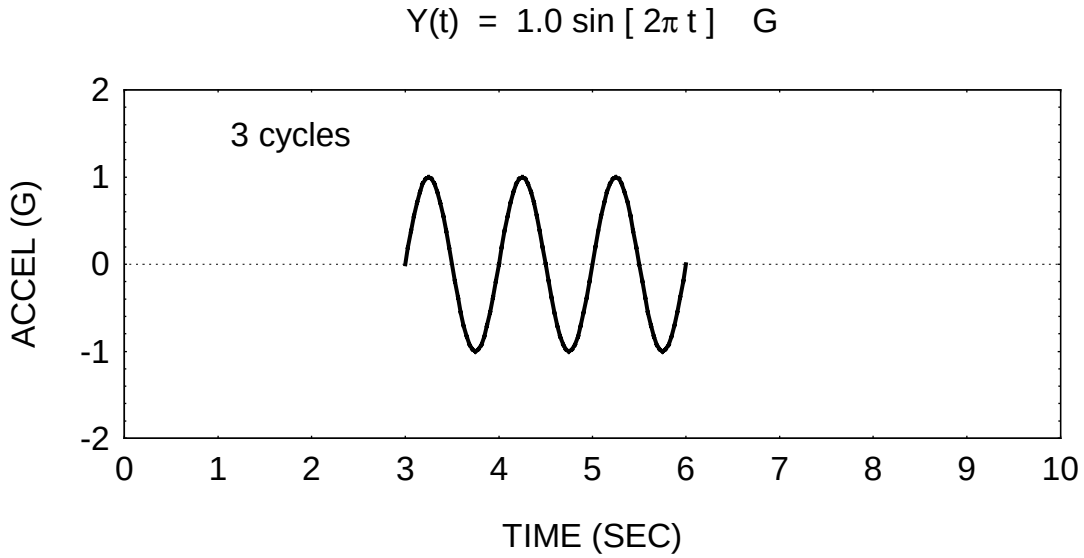

Figure 2.

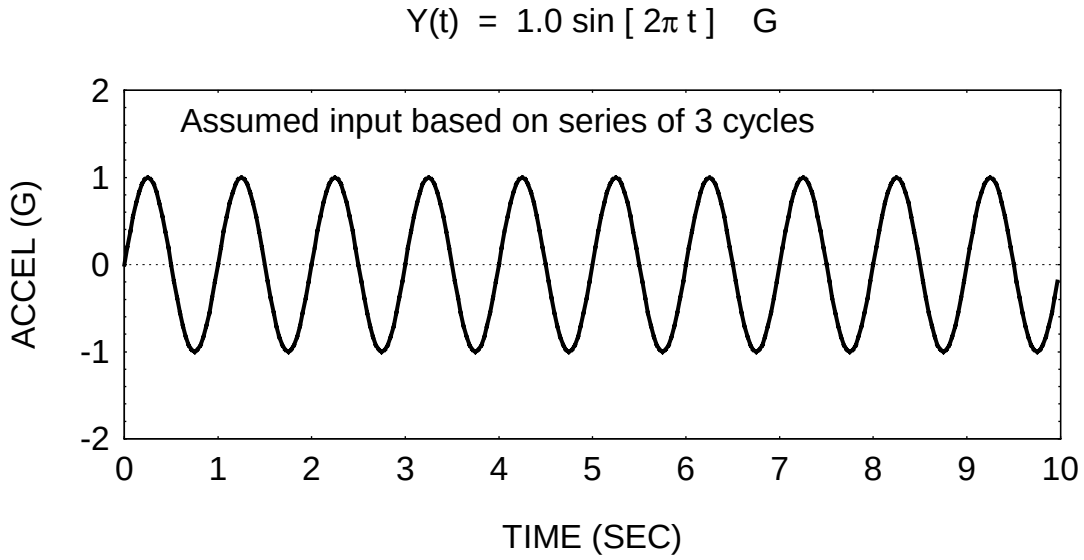

Figure 3.

The three-cycle sine function in Figure 2 is converted to a Fourier transform in Figure 4. As expected, a spectral line of 1 G appears at 1 Hz. Note that $\Delta f = 0.333$ Hz.

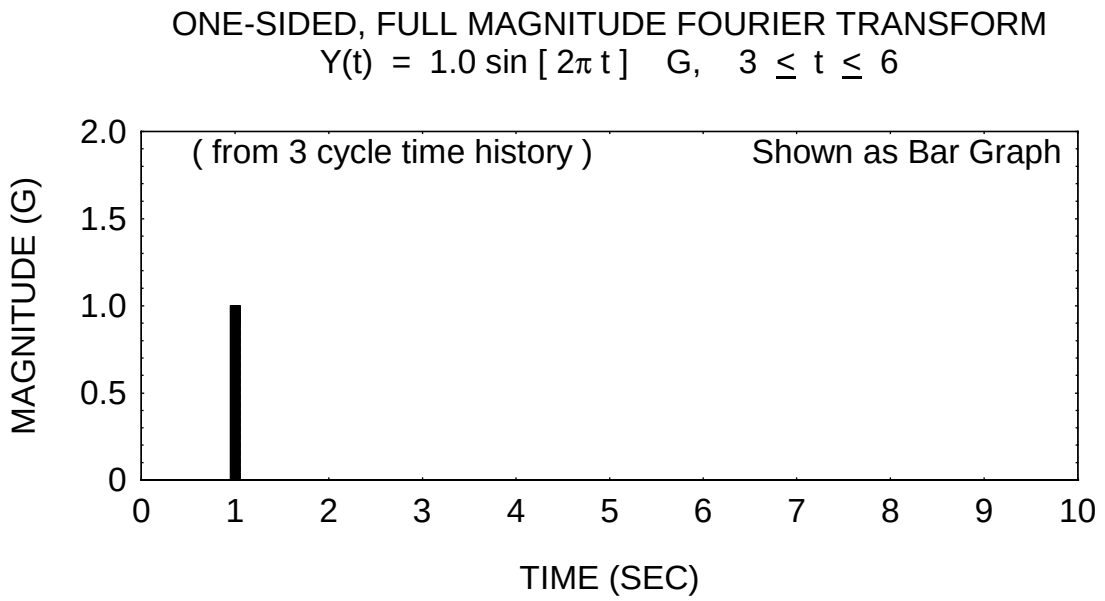

Figure 4.

Sine Function Example 2

Now assume that the data acquisition system has a limited memory buffer and is only able to capture $2\frac{1}{2}$ cycles of the sine function, as shown in Figure 5.

In essence, the Fourier transform will assume that the original signal is a series of $2\frac{1}{2}$ cycle segments as shown in Figure 6. Distortion is clearly visible in the time history in Figure 6. Specifically, the input signal is not periodic in the time record.

The $2\frac{1}{2}$ cycle sine function in Figure 5 is converted to a Fourier transform in Figure 7. Note that leakage occurs as shown by the smearing of energy across the frequency band.

A related problem is that $\Delta f = 0.4$ Hz. Thus, there are spectral lines at the following frequencies in Hz: 0, 0.4, 0.8, 1.2, …. There is no spectral line at 1 Hz, however, which is the frequency of the sine function.

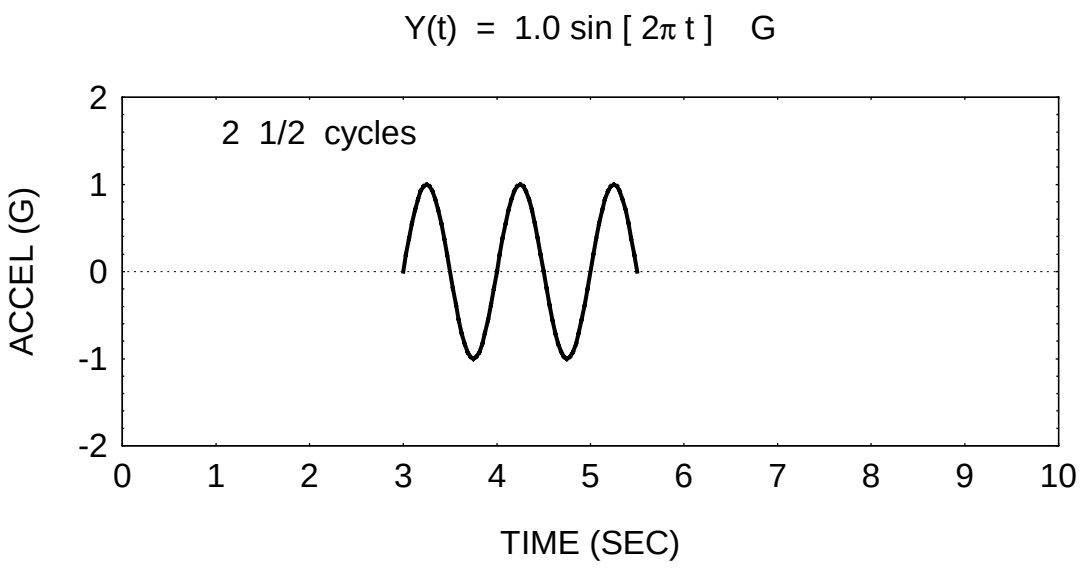

Figure 5.

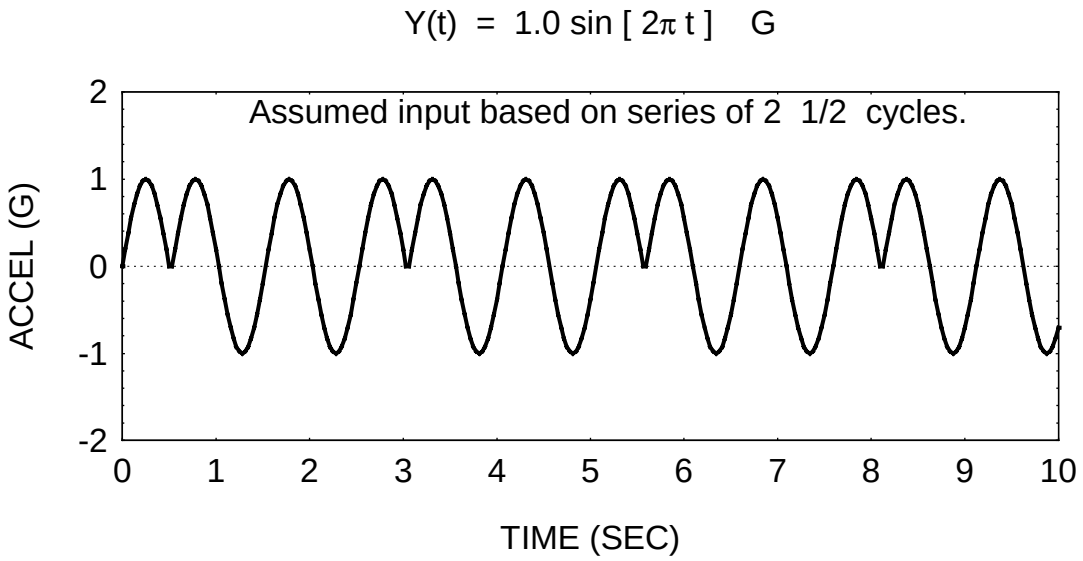

Figure 6.

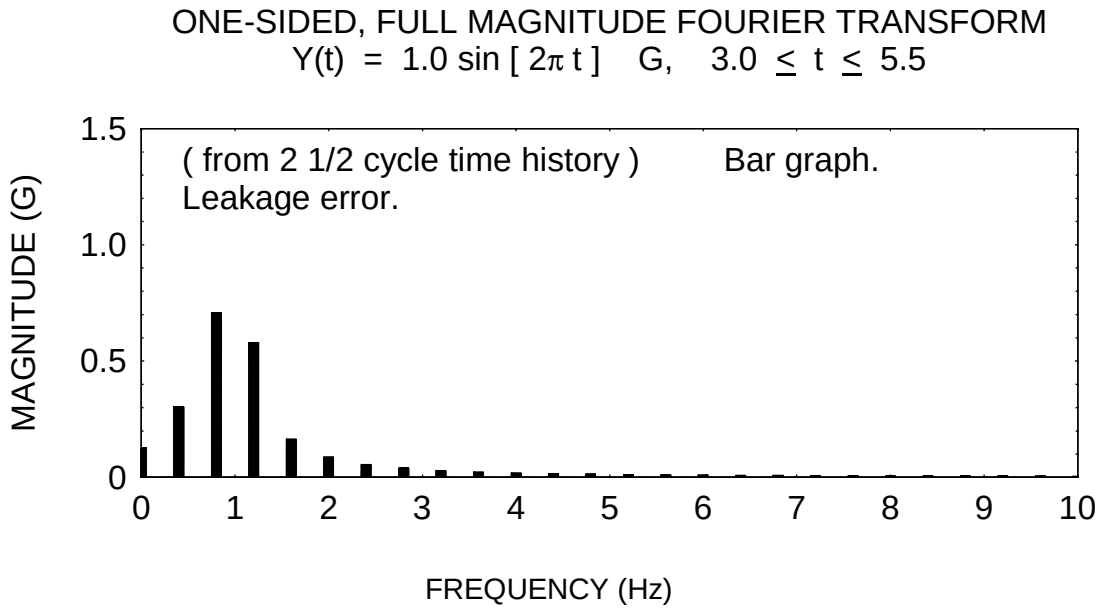

Figure 7.

Homework

There are no homework problems associated with this unit. As an option, the student may try experimenting with the generate.exe program and the Fourier.exe program to create examples of leakage.