SHOCK AND VIBRATION RESPONSE SPECTRA COURSE
Unit 6D. Hanning Window

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Introduction

A Fourier transform may have a leakage error, as discussed in Unit 6C. The leakage error can be reduced by subjecting the time history to a window, as discussed in References 1 through 6.

Two common types of windows are the rectangular window and the Hanning window.

Rectangular Window

The rectangular, or flat, window leaves the time history data unmodified. Thus, a rectangular window is equivalent to no window at all. A rectangular window is appropriate for transient data or nonstationary data. Ideally, the time history includes some data during the "quiet" periods both before and after the event. An example of a transient event is shown in Figure 1.

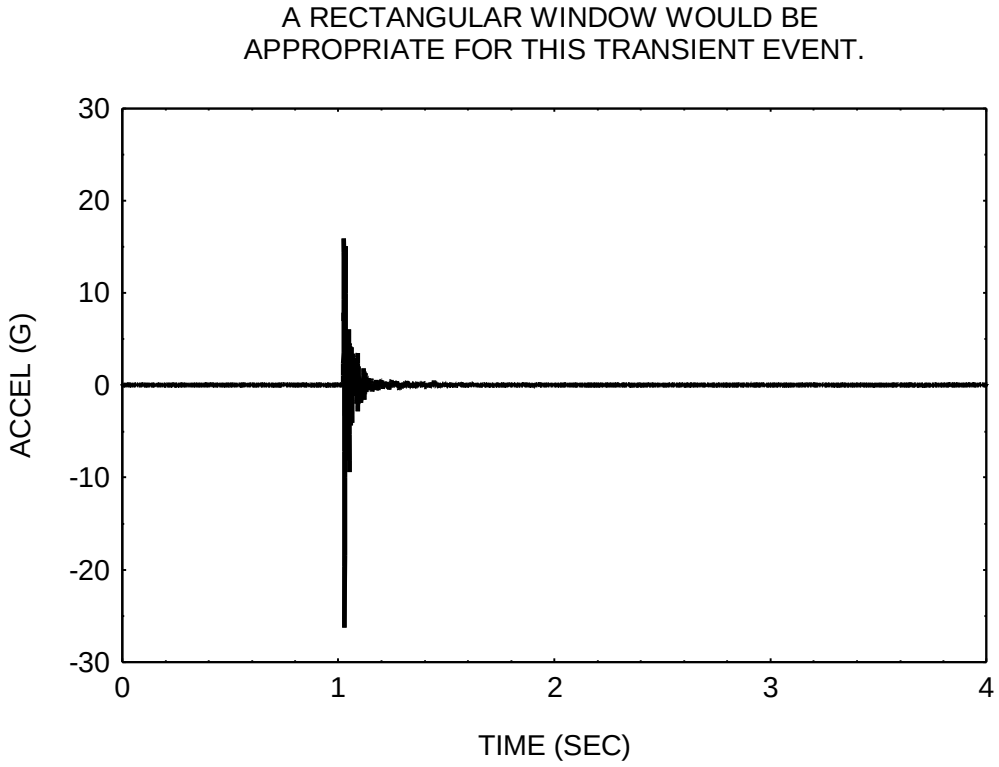

Figure 1.

Hanning Window

One of the most common windows is the Hanning window, or the cosine squared window. It is appropriate for stationary vibration.

This window tapers the time history data so that the amplitude envelope decreases to zero at both the beginning and end of the time segment. The Hanning window w(t) can be defined as

$$w(t) = \begin{cases} 1 - \cos^2\left[\pi \dfrac{t}{T}\right], & 0 \le t \le T \\ 0, \quad \text{elsewhere} \end{cases} \qquad (1)$$

Equation (1) is plotted in Figure 2.

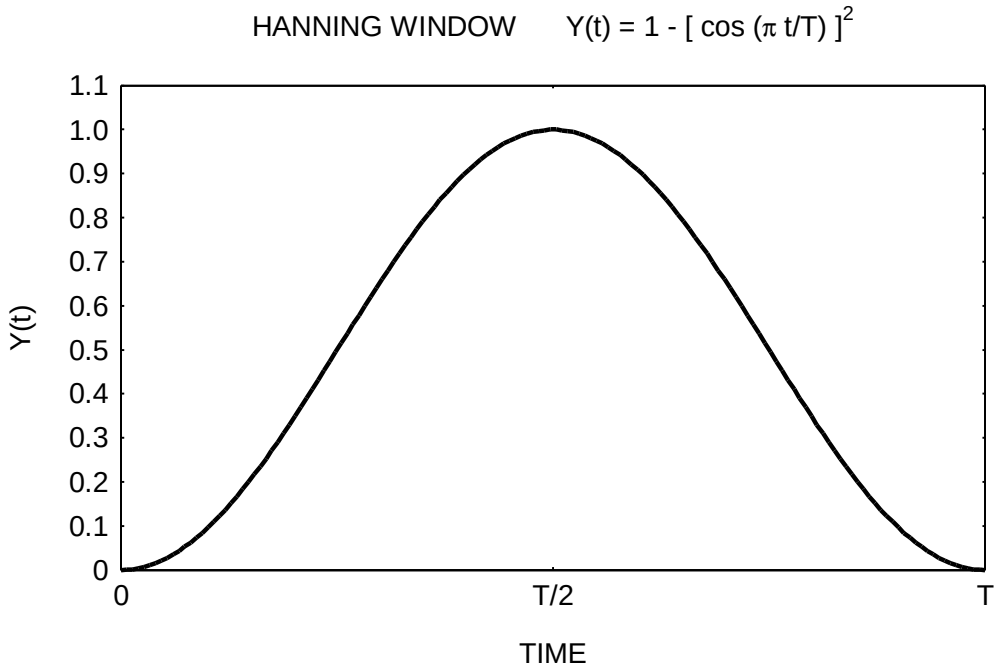

Figure 2.

Furthermore, a normalization factor of $\sqrt{8/3}$ may be applied to the Hanned data to compensate for the lost energy, from Reference 6.

Example

A 1 Hz sine function is shown in Figure 3. The same function is shown after a normalized Hanning window is applied in Figure 4.

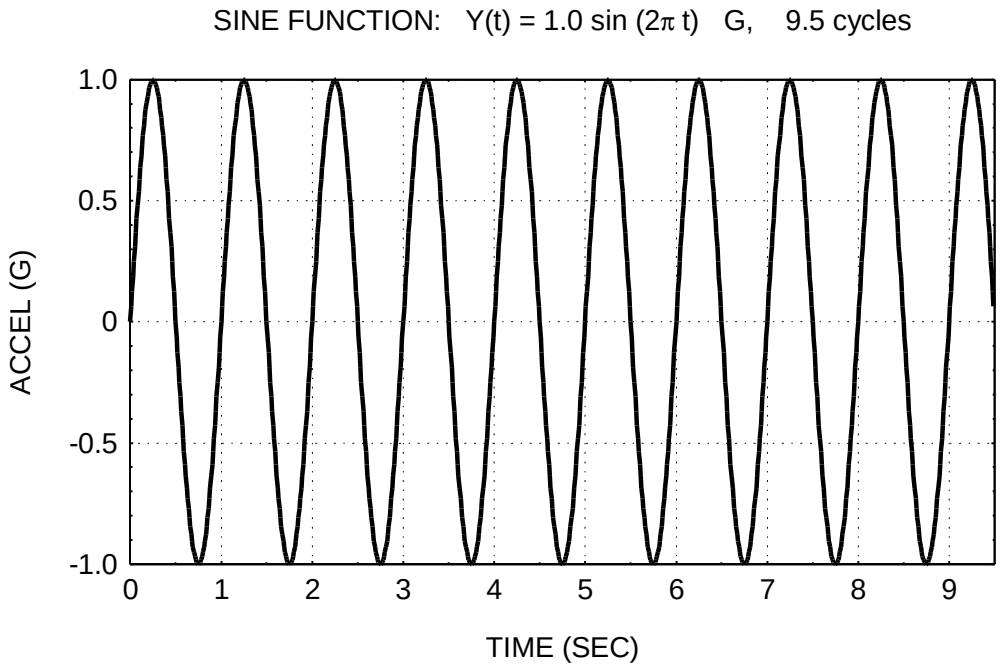

Figure 3.

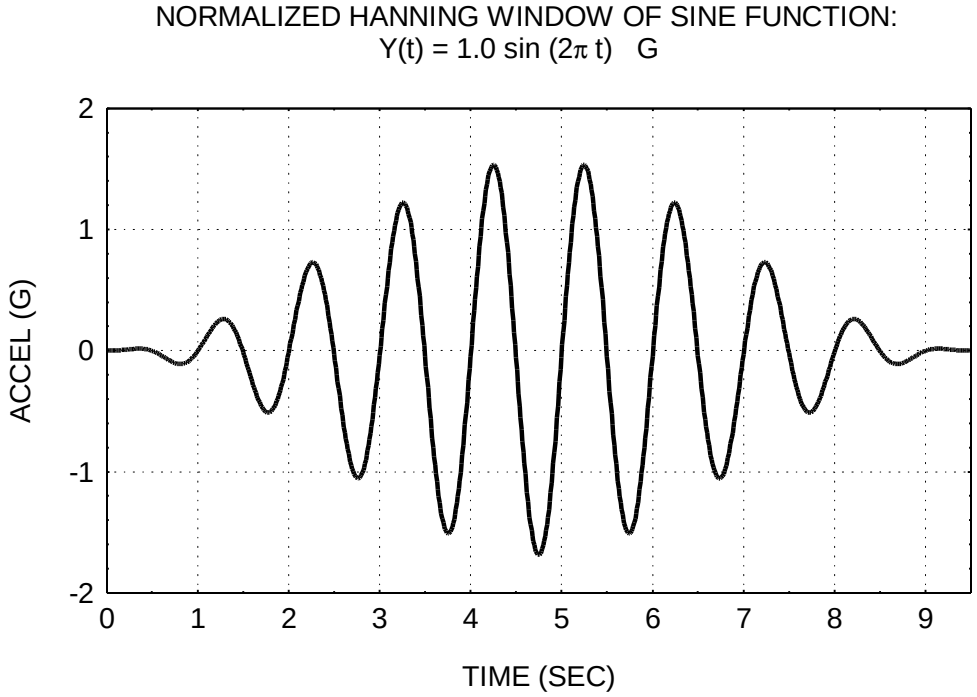

Figure 4.

The Fourier transforms of two time histories are shown together in Figure 5.

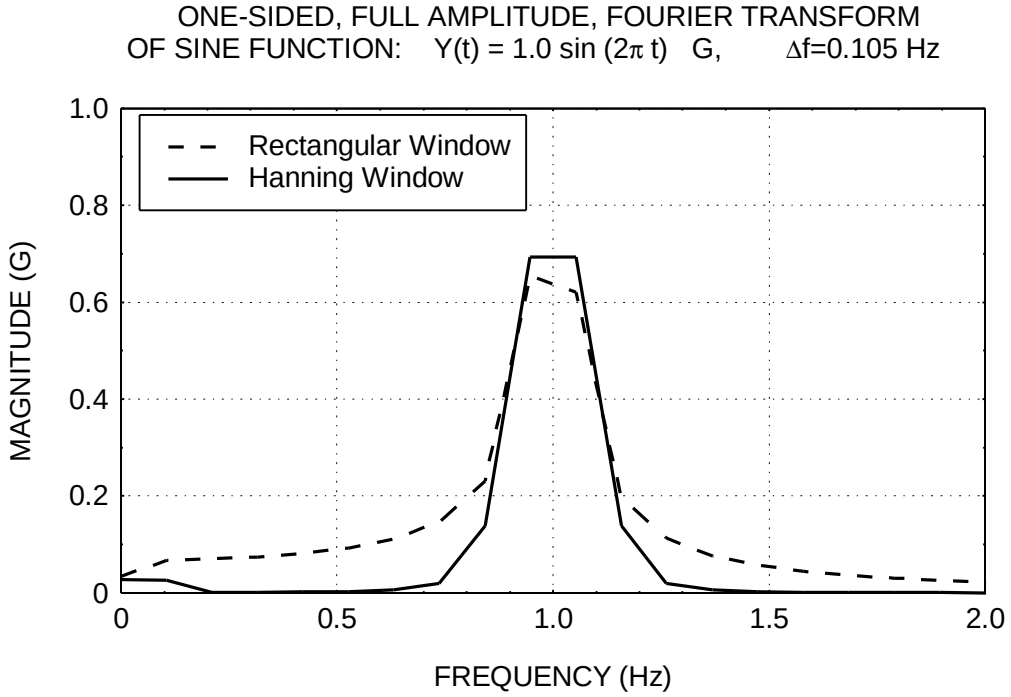

Figure 5.

Ideally, the Fourier transform would have a single, discrete line at 1 Hz with an amplitude of 1 G.

Both the rectangular and Hanning Fourier transforms have some leakage error, however. The rectangular window produces more leakage error than the Hanning window. Thus, the Hanning window is recommended for stationary data.

Homework

1. This unit includes the fourier.exe program, version 1.3. This version has options for both the rectangular and Hanning windows. Use this program to recreate the examples shown in this unit.

References

1. C. Harris, editor; Shock and Vibration Handbook, 3rd edition; R. Randall, "Chapter 13 Vibration Measurement Equipment and Signal Analyzers," McGraw-Hill, New York, 1988.

2. MAC/RAN Applications Manual Revision 2, University Software Systems, Los Angeles, CA, 1991.

3. Vibration Testing, Introduction to Vibration Testing, Section 9 (I), Scientific-Atlanta, Spectral Dynamics Division, San Diego, CA, Rev 5/72.

4. R. Randall, Frequency Analysis, Bruel & Kjaer, Denmark, 1987.

5. TSL25, Time Series Language for 2500-Series Systems, GenRad, Santa Clara, CA, 1981.

6. F. Harris, Trigonometric Transforms, Scientific-Atlanta, Spectral Dynamics Division, Technical Publication DSP-005 (8-81), San Diego, CA.