SHOCK AND VIBRATION RESPONSE SPECTRA COURSE
Unit 6E. Fast Fourier Transform (FFT)

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Introduction

The discrete Fourier transform requires a tremendous amount of calculations. A time history with M coordinates would require M^2 complex multiplication steps.

The discrete Fourier transform can be carried out by a Fast Fourier transform method, however. The method is based on a time series with a number of points equal to 2^N, where N is an integer.

The FFT requires $M \log_2 M$ complex multiplication steps, where $M = 2^N$.

The details of the FFT algorithm are given in Reference 1.

Example

Now consider a time history with 1,000,000 points. A regular Fourier transform would require 10^{12} complex multiplication steps. On the other hand, an FFT would only require approximately $2(10^7)$ steps. Thus, the FFT achieves the calculation in $1/50{,}000^{th}$ of the time.

Limitation of the FFT

The above example is not quite correct. Again, the FFT is based on a time series with 2^N coordinates. Note that

$$2^{19} = 524{,}288$$

and

$$2^{20} = 1{,}048{,}576$$

Unfortunately, a time history with 1,000,000 points falls between these two cases.

There are two options for dealing with a time history that is not an integer power of 2.

One option is to truncate the time history. This should be acceptable if the data is stationary. In the above example, the time history would thus be truncated to 524,288 points.

The second option is to pad the time history with trailing zeroes to bring its length to an integer power of 2. A problem with this option is that it artificially reduces the amplitude of the Fourier transform spectral lines.

Truncation, rather than zero-padding, is the preferred method in this course.

Homework

1. Plot the accelerometer time history in file panel.txt. The file has two columns: time(sec) and accel(G). The data was measured on the front panel of a big-rig trailer, as it was driven over a test course. The data has 8192 points, which is conveniently an integer power of 2. In many cases, data acquisition systems are set-up to measure data segments which are an integer power of 2.
2. Calculate the Fourier transform of panel.txt using program fourier.exe. Use the Hanning window. Plot the full.out magnitude file.
3. Calculate the FFT of panel.txt using program FFT.exe. Use the mean removal and Hanning window options. Plot the fft_full.out magnitude file.
4. Compare the results of steps 2 and 3. Are the results similar or different?
5. Aside from the interactive data entry, what was the difference in processing time between the fourier.exe program and the FFT.exe program? You may give a qualitative answer.
6. What were the two dominant frequencies in the Fourier transform magnitude?

Reference

1. T. Irvine, The Fast Fourier Transform (FFT), Vibrationdata Publications, 1998.