SHOCK AND VIBRATION RESPONSE SPECTRA COURSE
Unit 6F. Inverse Fourier Fast Transform

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Introduction

Recall that the Fourier transform F_k for a discrete time series x_n can be expressed as

$$F_k = \frac{1}{N}\sum_{n=0}^{N-1}\left\{x_n \exp\left(-j\frac{2\pi}{N}nk\right)\right\}, \text{ for } k = 0,1,..., N-1 \quad (1)$$

The corresponding inverse transform is

$$x_n = \sum_{k=0}^{N-1}\left\{F_k \exp\left(+j\frac{2\pi}{N}nk\right)\right\}, \text{ for } n = 0,1, ..., N-1 \quad (2)$$

Note that F_k has dimensions of [amplitude].

Points

Here are some important points about the Fourier transform and its inverse:

1. The Fourier transform converts a time history to the frequency domain. The inverse Fourier transform converts the frequency domain function back to a time history.
2. In some cases, an intermediate calculation may be performed on the Fourier transform prior to taking its inverse. This calculation might involve a transfer function. This will be covered in future units.
3. The Fourier transform and its inverse must be a matched pair. This is an absolute requirement since there are many different types of Fourier transforms.
4. The main difference between the two transforms is the polarity of the argument in the exponential function. In addition, the Fourier transform has a scale factor of 1/N.
5. A measured time history consists only of a real amplitude. The Fourier transform converts this to a complex function. The inverse Fourier transform converts this complex function to a complex time history, but the resulting imaginary component should be zero.
6. An exception to point 5 could occur if some intermediate calculation were performed prior to taking the inverse Fourier transform.

7. Theoretically, the input time history could be complex with a non-zero imaginary component. For practical purposes, this never occurs.
8. A Hanning window may be applied in the Fourier transform, but it is never applied to the inverse calculation.
9. An inverse Fourier transform can be performed as an "inverse Fast Fourier transform" if the number of points is an integer power of 2.

Homework

1. Plot the time history file panel.txt from Unit6E.
2. Take the FFT of panel.txt. Use program fft.exe with a rectangular window.
3. Take the inverse FFT using program invfft.exe. The input file is fft.out from step 2.
4. Program invfft.exe will generate two files: real.out and complex.out. Compare real.out with panel.txt.
5. Verify that file complex.out has an approximately zero imaginary component.