SHOCK AND VIBRATION RESPONSE SPECTRA COURSE
Unit 7C. Overall GRMS Value of a PSD Specification

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Introduction

Certain components must be designed and tested to withstand random vibration environments. Typically, the test specification is in the form of a power spectral density function.

A power spectral density specification is usually represented as follows:

1. The specification is represented as a series of piecewise continuous segments.
2. Each segment is a straight line on a log-log plot.

An example is shown in Figure 1.

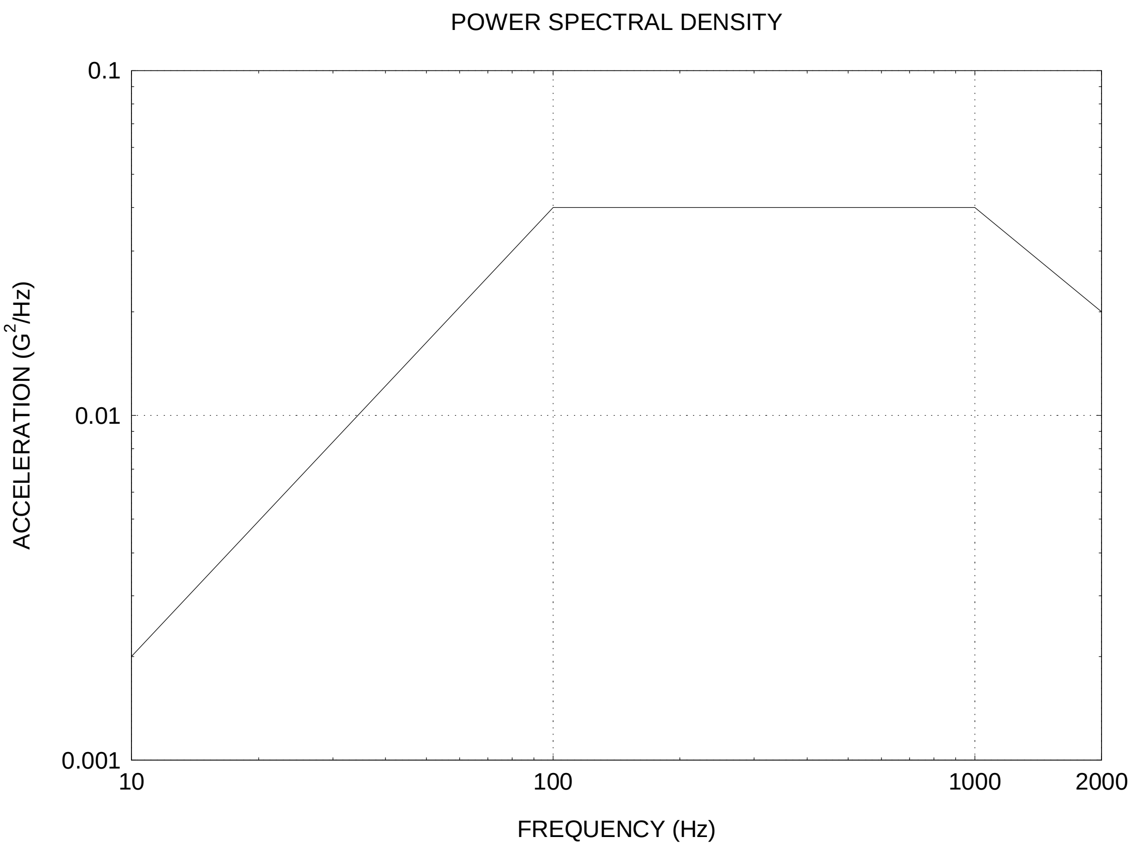

Figure 1.

The specifications are almost always given in the form of an acceleration power spectral density. The main reason is that acceleration is easier to measure than velocity or displacement, in a vibration sense.

Note that the power spectral density amplitude is represented in units of (G^2/Hz). This is an abbreviated notation. The actual unit is (G_{RMS}^2/Hz).

<u>Derivation</u>

The goal is to calculate the overall GRMS value. Use of the trapezoidal rule would require tedious interpolation. Instead, a direct integration approach is available. Special rules must be followed due to the log-log format.

The equation for each segment is

$$y(f) = \left[\frac{y_1}{f_1^{\,n}}\right] f^{\,n} \tag{1}$$

The starting coordinate is (f_1, y_1).

The exponent n is a real number which represents the slope. The slope between two coordinates (f_1, y_1) and (f_2, y_2) is

$$n = \frac{\log\left(\frac{y_2}{y_1}\right)}{\log\left(\frac{f_2}{f_1}\right)} \tag{2}$$

The area a_1 under segment 1 is

$$a_1 = \int_{f_1}^{f_2} \left[\frac{y_1}{f_1^{\,n}}\right] f^{\,n} df \tag{3}$$

There are two cases depending on the exponent n.

The first case is

$$a_1 = \left[\frac{y_1}{f_1^{\,n}}\right]\left[\frac{1}{n+1}\right] f^{\,n+1}\Big|_{f_1}^{f_2} \quad , \quad \text{for } n \neq -1 \tag{4}$$

$$a_1 = \left[\frac{y_1}{f_1^{\,n}}\right]\left[\frac{1}{n+1}\right]\left[f_2^{\,n+1} - f_1^{\,n+1}\right], \quad \text{for } n \neq -1 \tag{5}$$

The second case is

$$a_1 = \int_{f_1}^{f_2}\left[\frac{y_1}{f_1^{\,-1}}\right] f^{-1}\, df\,, \quad \text{for } n = -1 \tag{6}$$

$$a_1 = \int_{f_1}^{f_2}\left[y_1 f_1\right]\frac{df}{f}\,, \quad \text{for } n = -1 \tag{7}$$

$$a_1 = \left[y_1 f_1\right]\ln\left(f\right)\Big|_{f_1}^{f_2}, \quad \text{for } n = -1 \tag{8}$$

$$a_1 = \left[y_1 f_1\right]\left[\ln\left(f_2\right) - \ln\left(f_1\right)\right], \quad \text{for } n = -1 \tag{9}$$

$$a_1 = \left[y_1 f_1\right]\left[\ln\left(\frac{f_2}{f_1}\right)\right], \quad \text{for } n = -1 \tag{10}$$

In summary, the area under segment i is

$$a_i = \begin{cases} \left[\dfrac{y_i}{f_i^{\,n}}\right]\left[\dfrac{1}{n+1}\right]\left[f_{i+1}^{\,n+1} - f_i^{\,n+1}\right], & \text{for } n \neq -1 \\[2ex] \left[y_i\, f_i\right]\left[\ln\left(\dfrac{f_{i+1}}{f_i}\right)\right], & \text{for } n = -1 \end{cases} \tag{11}$$

The overall level L is

$$L = \sqrt{\sum_{i=1}^{m} a_i} \tag{12}$$

where m is the total number of segments.

Example

Consider the power spectral density function in Figure 1. The breakpoints are given in Table 1.

Table 1. Power Spectral Density	
Freq (Hz)	Level (G^2/Hz)
10	0.002
100	0.04
1000	0.04
2000	0.02

Consider the first pair of coordinates:

f_1 = 10 Hz	y_1 = 0.002 G^2/Hz
f_2 = 100 Hz	y_2 = 0.04 G^2/Hz

Calculate the slope.

$$n = \frac{\log\left(\frac{0.04}{0.002}\right)}{\log\left(\frac{100}{10}\right)} \tag{13}$$

$$n = 1.3 \tag{14}$$

Substitute into equation (11).

$$a_1 = \left[\frac{0.002}{10^{1.3}}\right]\left[\frac{1}{1.3+1}\right]\left[100^{1.3+1} - 10^{1.3+1}\right] \tag{15}$$

$$a_1 = \left[\frac{0.002}{10^{1.3}}\right]\left[\frac{1}{2.3}\right]\left[100^{2.3} - 10^{2.3}\right] \tag{16}$$

$$a_1 = 1.726 \ \ G^2 \tag{17}$$

Consider the second pair:

f_2 = 100 Hz	y_2 = 0.04 G^2/Hz
f_3 = 1000 Hz	y_3 = 0.04 G^2/Hz

Calculate the slope.

$$n = \frac{\log\left(\frac{0.04}{0.04}\right)}{\log\left(\frac{1000}{100}\right)} \tag{18}$$

$$n = 0. \tag{19}$$

Substitute into equation (11).

$$a_2 = \left[\frac{0.04}{100^0}\right]\left[\frac{1}{0+1}\right]\left[1000^{0+1} - 100^{0+1}\right] \tag{20}$$

$$a_2 = \left[\frac{0.04}{1}\right]\left[\frac{1}{1}\right]\left[1000^1 - 100^1\right] \tag{21}$$

$$a_2 = 36.000 \;\; G^2 \tag{22}$$

Consider the third pair:

f_3 = 1000 Hz	y_3 = 0.04 G^2/Hz
f_4 = 2000 Hz	y_4 = 0.02 G^2/Hz

Calculate the slope.

$$n = \frac{\log\left(\frac{0.02}{0.04}\right)}{\log\left(\frac{2000}{1000}\right)} \tag{23}$$

$$n = -1. \tag{24}$$

Substitute into equation (11).

$$a_3 = [(0.04)(1000)]\left[\ln\left(\frac{2000}{1000}\right)\right] \tag{25}$$

$$a_3 = 27.726 \tag{26}$$

Now substitute the individual area values into equation (12).

$$L = \sqrt{(1.726 + 36.000 + 27.726)\,G^2} \tag{27}$$

The overall level is

$$L = 8.09\ G_{RMS} \tag{28}$$

Homework

1. Determine the overall level of the power spectral density in Table 2. Use hand calculations.

Table 2. Power Spectral Density	
Freq (Hz)	Level (G^2/Hz)
10	0.001
200	0.08
500	0.08
2000	0.02

2. Verify the overall GRMS value for Table 2 using program psdint.exe.

3. Why do random vibration specifications typically begin at a frequency $\geq$ 10 Hz?