

SHOCK AND VIBRATION RESPONSE SPECTRA COURSE  
Unit 7E. dB/octave Slopes

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Introduction

NAVMAT P-9492 gives the power spectral density specification shown in Figure 1.

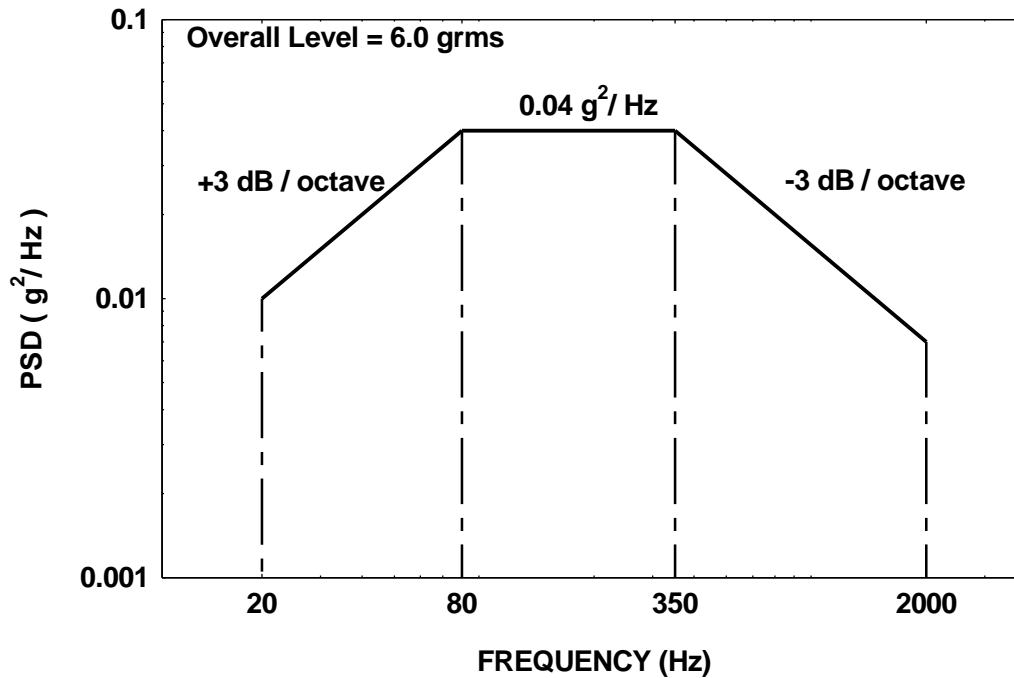


Figure 1.

The task is to determine the coordinates of the endpoints.

Derivation

Assume that  $a_1$  and  $a_2$  each has an amplitude in  $G^2/Hz$ . The difference in dB between  $a_1$  and  $a_2$  is

$$\Delta dB = 10 \log \left[ \frac{a_2}{a_1} \right] \quad (1)$$

Furthermore,

$$a_2 = a_1 \left[ 10^{\Delta \text{dB} / 10} \right] \quad (2)$$

Additional equations are needed.

The slope N between two coordinates  $(f_1, a_1)$  and  $(f_2, a_2)$  in a log-log plot is

$$N = \frac{\log \left[ \frac{a_2}{a_1} \right]}{\log \left[ \frac{f_2}{f_1} \right]} \quad (3)$$

Solve for  $a_2$  .

$$N \log \left[ \frac{f_2}{f_1} \right] = \log \left[ \frac{a_2}{a_1} \right] \quad (4)$$

$$\log \left\{ \left[ \frac{f_2}{f_1} \right]^N \right\} = \log \left[ \frac{a_2}{a_1} \right] \quad (5)$$

Take the anti-log.

$$\left[ \frac{f_2}{f_1} \right]^N = \left[ \frac{a_2}{a_1} \right] \quad (6)$$

$$\left[ \frac{a_2}{a_1} \right] = \left[ \frac{f_2}{f_1} \right]^N \quad (7)$$

Thus,

$$a_2 = a_1 \left[ \frac{f_2}{f_1} \right]^N \quad (8)$$

Now consider a one-octave frequency separation.

$$f_2 = 2f_1 \quad (9)$$

Substitute equation (9) into (3).

$$N = \frac{\log\left[\frac{a_2}{a_1}\right]}{\log[2]} \quad (10)$$

Substitute equation (1) into (10).

$$N = \frac{\Delta\text{dB}/10}{\log[2]} \quad (11)$$

Note that  $\Delta\text{dB}$  represents the dB/octave slope in equation (11). Again, equations (10) and (11) assume a one-octave frequency separation.

Now substitute equation (11) into (8).

$$a_2 = a_1 \left[ \frac{f_2}{f_1} \right] \left\{ \frac{\Delta\text{dB}/10}{\log[2]} \right\} \quad (12)$$

### Example

Calculate the amplitude at 2000 Hz for the power spectral density in Figure 1.

Note

$$f_1 = 350 \text{ Hz}$$

$$f_2 = 2000 \text{ Hz}$$

$$a_1 = 0.04 \text{ G}^2 / \text{Hz}$$

$$a_2 = 0.04 \text{ G}^2 / \text{Hz} \left[ \frac{2000 \text{ Hz}}{350 \text{ Hz}} \right] \left\{ \frac{-3 \text{ dB}/10}{\log[2]} \right\} \quad (13)$$

$$a_2 = 0.007 \text{ G}^2 / \text{Hz} \quad (14)$$

### Homework

1. Calculate the amplitude at 20 Hz for the power spectral density curve in Figure 1.
2. Calculate the overall GRMS value for the power spectral density curve in Figure 1. Use program psdint.exe. Give the answer to two decimal places.