SHOCK AND VIBRATION RESPONSE SPECTRA COURSE
Unit 8. Transmissibility Function for Acceleration

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Introduction

Certain systems are subjected to a base excitation vibration. Examples include:

1. A building during an earthquake
2. An automobile traveling down a washboard road
3. An avionics component on a rocket vehicle bulkhead during powered flight

The purpose of this unit is to determine the steady state response of a single-degree-of-freedom system to sinusoidal base excitation. The transmissibility function is the ratio of the response to the input.

Model

Consider the single-degree-of-freedom system subjected to base excitation shown in Figure 1. The free-body diagram is shown in Figure 2.

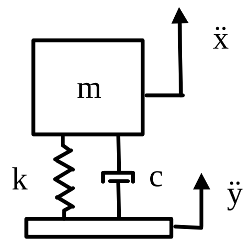

Figure 1. Single-degree-of-freedom System

The variables are

m = mass,
c = viscous damping coefficient,
k = stiffness,
x = absolute displacement of the mass,
y = base input displacement.

The double-dot notation indicates acceleration

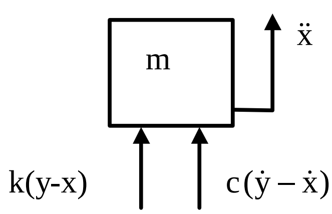

Figure 2. Free-body Diagram

Summation of forces in the vertical direction

$$\sum F = m\ddot{x} \tag{1}$$

$$m\ddot{x} = c(\dot{y} - \dot{x}) + k(y - x) \tag{2}$$

Let z = x - y. The variable z is thus the relative displacement.

Substituting the relative displacement into equation (2) yields

$$m(\ddot{z} + \ddot{y}) = -c\dot{z} - kz \tag{3}$$

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \tag{4}$$

Dividing through by mass yields

$$\ddot{z} + (c/m)\dot{z} + (k/m)z = -\ddot{y} \tag{5}$$

By convention,

$$(c/m) = 2\xi\omega_n \tag{6}$$

$$(k/m) = \omega_n{}^2 \tag{7}$$

where ω_n is the natural frequency in (radians/sec), and ξ is the damping ratio.

Substituting the convention terms into equation (5),

$$\ddot{z} + 2\xi\omega_n\dot{z} + \omega_n{}^2 z = -\ddot{y} \tag{8}$$

Either Laplace or Fourier transforms may be used to derive the steady state transmissibility function for the absolute response acceleration, as shown in Reference 1. After many steps, the resulting magnitude function is

$$\left| \frac{\ddot{x}}{\ddot{y}} \right| = \sqrt{\frac{1 + (2\xi\rho)^2}{\left(1 - \rho^2\right)^2 + (2\xi\rho)^2}} ,$$

$$\rho = f / f_n \tag{9}$$

where f is the base excitation frequency and fn is the natural frequency.

Recall that the damping is often represented in terms of the quality factor Q.

$$Q = \frac{1}{2\,\xi} \tag{10}$$

The transmissibility function is plotted for several Q values in Figure 3.

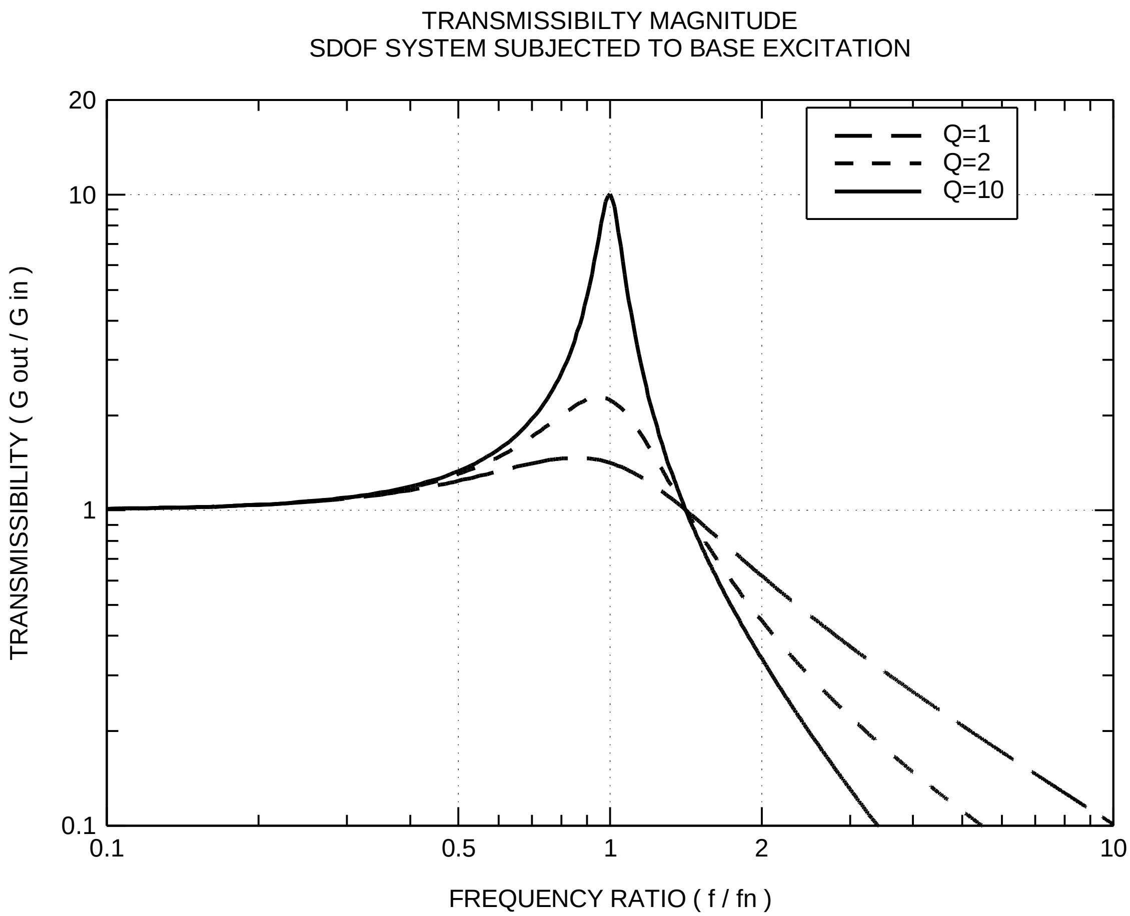

Figure 3.

Note that f is the base excitation frequency in Figure 3.

The transmissibility curves in Figure 3 have several important features:

1. The response amplitude is independent of Q for f << fn.
2. The response is approximately equal to the input for f << fn.
3. Resonance occurs when f $\cong$ fn.
4. The peak transmissibility is approximately equal to Q for f = fn and Q $\geq$ 2.
5. The transmissibility ratio is 1.0 for f = $\sqrt{2}$ fn regardless of Q.
6. Isolation is achieved for f >> fn.

The curves in Figure 3 are particularly useful for designing isolation systems.

Example 1

As a review, the natural frequency of a single-degree-of-freedom system is

$$fn = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{11}$$

Now consider an avionics component which has a natural frequency fn = 200 Hz and an amplification factor Q = 10.

The component is hard-mounted to a bulkhead in a rocket vehicle. Assume that the component will be subjected to a 200 Hz sinusoidal oscillation with a base input amplitude of 10 G during powered flight. The Q = 10 curve in Figure 3 shows that the response will be 100 G, which is severe. How can the response be reduced?

The Q = 10 curve in Figure 3 shows that the response can be reduced to approximately 3.2 G if f = 2 fn. The excitation frequency f is fixed, however. Thus, fn must be reduced.

Recall that fn = 200 Hz. Thus, change the mounting design so that fn=100 Hz. This can be achieved by mounting the avionics component with the appropriate isolator grommets. The grommets will act as a spring in series with the component. The grommets thus reduce the natural frequency by reducing the overall stiffness.

In reality, the grommets will decrease the Q value, thus changing the calculation somewhat. Nevertheless, the main effect is the natural frequency reduction.

Example 2

Grommets are typically made from some rubber or plastic material. They are effective when they break "metal-to-metal contact" between the component and the mounting surface.

Suppose isolation grommets cannot be used in the previous example. There could be several reasons. One might be that the component must be hard-mounted because the mounting surface serves as a thermal ground plane. How can the response be reduced in this case?

The answer is to use the opposite approach as was used in Example 1. In this case, the goal should be f = 0.5 fn. The response would thus be reduced to about 1.3 G. The natural frequency goal would thus be 400 Hz, since the base excitation frequency is fixed at 200 Hz. The frequency increase could be achieved by increasing the stiffness or by decreasing the mass. In most cases, increasing the stiffness would be the practical choice.

Further Notes

An alternative for both examples would be to decrease the Q value while leaving the natural frequency fixed. Thus, the damping ratio would increase per equation (10).

Nevertheless, the stiffness is usually the easiest parameter to modify for practical design purposes.

Homework

1. Consider a system with a natural frequency fn = 100 Hz and amplification factor Q=10. The system is subjected to a sinusoidal base input with a variable frequency. The input amplitude is 1 G. Use program steady.exe to complete the following table.

Excitation Frequency (Hz)	Response (G)
50	
60	
70	
80	
90	
95	
99	
100	
101	
105	
110	
120	
130	
140	
150	
160	
180	
200	

2. Plot the Response (G) versus Excitation Frequency (Hz).

3. How does damping effect the response of a system subjected to base excitation?

4. Optional. Procure a slinky. Hold the top of the slinky as shown in Figure 4.

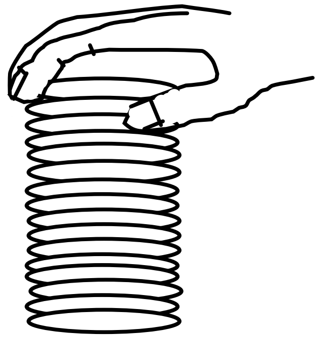

Figure 4.

Very, very slowly move your hand up-and-down in a sinusoidal manner. How does the free end of the slinky respond?

Now find the natural frequency of the slinky by trial-and-error experiment. Do this in a qualitative sense. Excite the natural frequency by moving your hand at the natural frequency but with a small amplitude. How does the free end of the slinky respond?

Now excite the slinky at a frequency much higher than its natural frequency. Well, the slinky may undergo some chaotic motion, but how would it respond if it were an ideal single-degree-of-freedom system?

References

1. T. Irvine, An Introduction to the Vibration Response Spectrum, Vibrationdata.com Publications, 1999.