Chapter 2

TRUSS ANALYSIS

2.1 Introduction

Truss structures are characterized by members whose primary forces are axial tension or compression and these forces alone are sufficient to equilibrate the applied node loads. The truss has members connecting its nodes in such a way that the complete structure forms a rigid body. Rigid that is, except for very small deformations of the members due to elastic strains produced by the axial forces. A simple truss may be constructed from a basic triangle (1-2-3) in Figure 2.1 and then connecting each additional node (4) with two members (4,5). For the initial triangle it is seen that,
number of members = number of nodes = 3 . For each additional node 2 members are required, so that if J= number of nodes and M= mumber of members,

$$(M-3) = 2(J-3) \quad \text{that is} \quad 2J = M+3 \tag{2.1}$$

A general test for determinacy of a truss is carried out from the equilibrium equations of the nodes. The analysis of a truss is based on the very simple concept of expressing all forces in terms of their global components (see also section 1.2.1). That is, it is an exercise in coordinate transformation of components of forces from local to global axes. The basis of this was given in Chapter 1, equation (1.2).

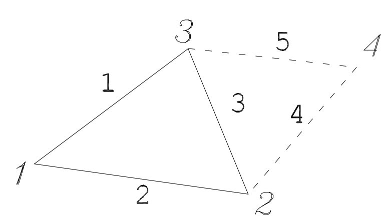

Figure 2.1: Truss constructed from triangle (1-2-3).

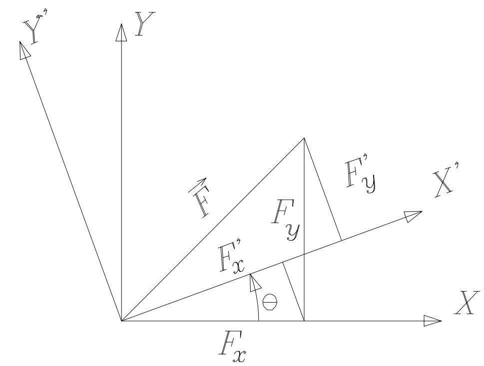

Figure 2.2: Rotation of coordinate axes.

It will be seen from the theory developed herein that the basic ideas are all derived from the equilibrium of the forces acting either on individual members or on the nodes of the assemblage of members.

The basic transformation required is for the components of a vector (F_x, F_y) in the $X-Y$ Cartesian coordinate system. Its components (F'_x, F'_y) in the $(X'-Y')$ coordinates obtained from the $(X-Y)$ coordinates by the rotation θ, are calculated in Figure 2.2 by simple projection. Then either,

$$\left\{ \begin{array}{c} F'_x \\ F'_y \end{array} \right\} = \left[\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right] \left\{ \begin{array}{c} F_x \\ F_y \end{array} \right\} \tag{2.2}$$

or the inverse transformation,

$$\left\{ \begin{array}{c} F_x \\ F_y \end{array} \right\} = \left[\begin{array}{cc} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{array} \right] \left\{ \begin{array}{c} F'_x \\ F'_y \end{array} \right\} \tag{2.3}$$

These two equations are sufficient theory to allow the setting up of the joint equilibrium equations of a truss structure composed of bar members connected together at nodes by pins that are assumed to be frictionless.

2.2 Theory for planar truss analysis

2.2.1 Member forces, reactions and equilibrium equations

A two dimensional truss is an idealization of a planar structure whose members predominantly carry axial forces (tension or compression). The nodes to which forces and reactions are applied are connected by bar members and are considered to be frictionless pins, although in reality pins are rarely used in practice. In the plane, a node (or pin), can be

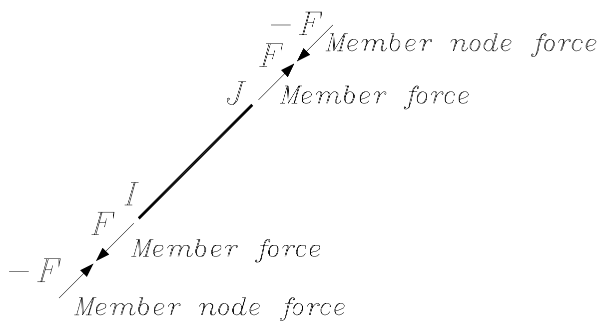

Figure 2.3: Member forces and member node forces.

idealized as a point to which the two equilibrium equations for the forces acting there apply,

$$\sum F_x = 0 \;; \quad \sum F_y = 0 \tag{2.4}$$

The forces acting on a node are possible from three sources,

1. applied loads.
2. reactions from the supports.
3. forces applied to the nodes from the members.

From Figure 2.3, it is seen that the force applied to the node from a member is equal and opposite to the force applied to the member, so that the general relationship applies,

$$\text{member } \textbf{\textit{node}} \text{ force} = -\text{member force} \tag{2.5}$$

The truss to be analysed must be statically determinate and stable if the member forces are to be calculated by statics alone. To fix a simple truss in the plane, a minimum of three reactions must be provided, because in addition to the two equilibrium equations, equation (2.4), a third equation (2.6), of moment equilibrium of forces about any point in the plane must apply. This equation is expressed as,

$$\sum M_z = 0 \tag{2.6}$$

If the truss has the following properties,

J = number of joints
M = number of members
R = number of reaction components, (R=3 for a simple truss)

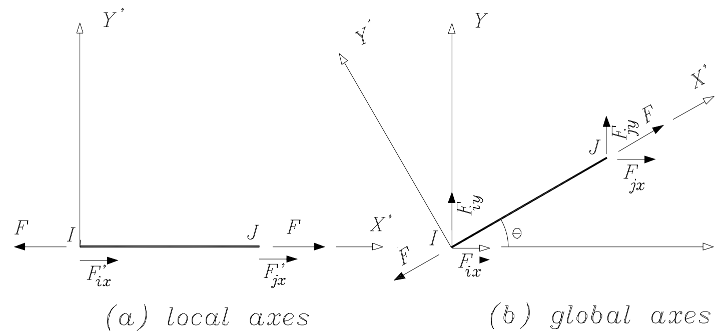

Figure 2.4: Truss member, local and global coordinates.

then because two equations are required for each joint, for statical determinacy,

$$2\text{J} = \text{M} + \text{R} \tag{2.7}$$

In general, for any truss constructed without regard to equation (2.7), the equality will not hold. Then the following conditions should be checked:

If 2J $>$ M+R, the truss will be statically unstable
If 2J $<$ M+R, the truss will be statically indeterminate
If 2J $=$ M+R, the truss should be stable and determinate

The STATICS-2020 returns an error message if the greater than condition occurs. If the statically indeterminate condition is encountered then the message is given that the stiffness method of analysis must be used. Details on how to accomplish this are found in this text, see Section (2.2.6), or in the STATICS-2020 help command. The analysis of a determinate structure is carried out by solving all joint equilibrium equations simultaneously.

In order to setup these equations, a single member connecting the nodes $I - J$ of the truss at its ends is considered first with an axial force F taken to be positive when tensile, see Figure 2.4(a). In Figure 2.4(a), the member is shown in its own local $X' - Y'$ axes and in Figure 2.4(b) in relation to the global $X - Y$ axes. For the truss member in Figure 2.4(a), the forces on the ends of the member, in the member coordinate system, are given by,

$$\begin{Bmatrix} F'_{Ix} \\ F'_{Iy} \\ F'_{Jx} \\ F'_{Jy} \end{Bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} F \tag{2.8}$$

and in the global coordinate system Figure 2.4(b), with c = $\cos\theta$ and s = $\sin\theta$,

$$\begin{Bmatrix} F_{Ix} \\ F_{Iy} \\ F_{Jx} \\ F_{Jy} \end{Bmatrix} = \begin{bmatrix} -c \\ -s \\ c \\ s \end{bmatrix} F \tag{2.9}$$

The components in equation (2.9) are member forces. The forces on the nodes, the member *node* forces, are equal to these in magnitude but of opposite sign. At any node then, the following equilibrium equation applies for the components in each of the global directions:

$$\text{applied force} + \text{reaction} + \text{member } \textbf{\textit{node}} \text{ force} = 0 \tag{2.10}$$

If the condition in equation (2.7) is satisfied then there will be sufficient equilibrium equations to solve for the member forces and reactions. It should be seen that four sets of information are required to set up the equilibrium equations. These are:

1. Nodes must be numbered and their coordinates known.
2. Members must be labelled and their appropriate $(I - J)$ node numbers made available.
3. Points of application of reactions must be identified.
4. Points of application and magnitudes of loads must be given.

A simple truss labelled with this information is shown in Figure 2.5.

2.2.2 Reactions

A reaction is a force applied by the supports to a node of a truss and may be parallel to the X or Y axes, or inclined at an angle β as shown in Figure 2.6, (a), (b) and (c). These three situations can all be combined in the single expression for the $X - Y$ components of the reaction as,

$$\begin{Bmatrix} S_{Rx} \\ S_{Ry} \end{Bmatrix} = \begin{bmatrix} -\sin\beta \\ \cos\beta \end{bmatrix} S'_R \tag{2.11}$$

Thus in Figure 2.6, the values of β are:

Case(a),	$\beta =$	0.0
Case(b),	$\beta =$	−90.0
Case(c),	$\beta =$	30.0

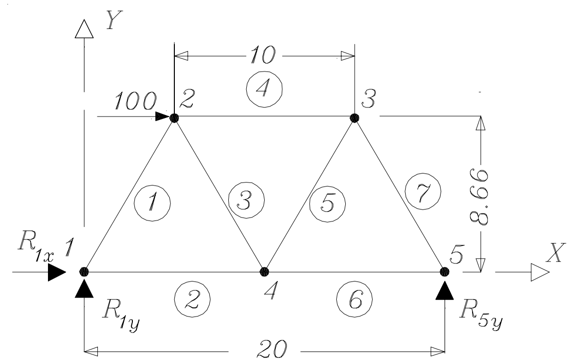

Figure 2.5: Truss node and member numbering, reaction identification.

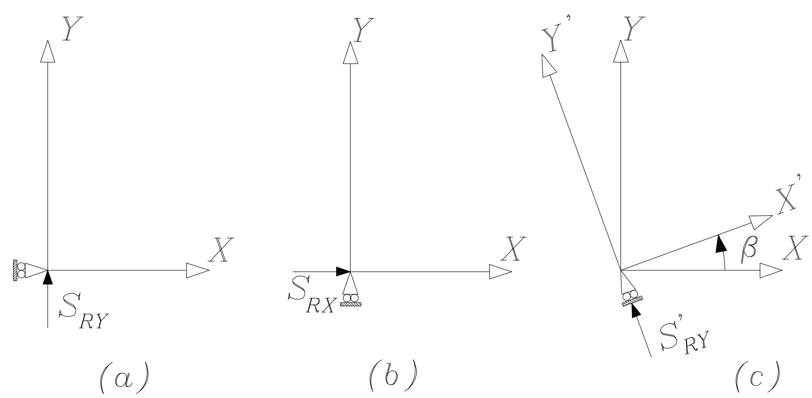

Figure 2.6: Reaction components and directions.

When all the equations for all nodes are written down, they take the form in equation (2.12).

$$[A_{SM}|A_{SR}]\left\{\begin{array}{c} S_M \\ S_R \end{array}\right\} = \{R\} \tag{2.12}$$

in which the coefficient matrix on the left hand side is a square non-singular matrix, if 2J=M+3.

2.2.3 Determinate trusses-solution of the equilibrium equations

The equation (2.12) is written in equation (2.13) with the member forces and reactions grouped together as $\{S\}$ such that now,

$$[A]\{S\} = \{R\} \tag{2.13}$$

Solving for $\{S\}$, using the matrix inverse,

$$\{S\} = [A]^{-1}\{R\} = [b]\{R\} \tag{2.14}$$

The matrix $[b]$ will be called the member force transformation matrix of the structure. Remember the partitioning of $\{S\}$,

$$\{S\} = \left\{\begin{array}{c} S_M \\ S_R \end{array}\right\} \tag{2.15}$$

so that equation (2.14) gives both member forces and reactions. The computer software provides four routines (commands) to study the analysis of determinate trusses. An additional command is available to call up pre-programmed data for 12 trusses, see Figures 2.8 to 2.12 of which the first 7 are statically determinate. See also Exercises on the main menu window of STATICS-2020. All exercises with particular dimensions and loads can also be run using the SUBMIT command and the data file DATN.DAT.
The commands available are:

1. TRUSS A B C D
2. JOINEQ A B C R S N = ?
3. TRUSLD A L R
4. TOPOL A B B1
5. TRUSEX E = ? D=?,?

The first command TRUSS is called to set up the joint equilibrium equations D and it requires information to have been stored in the three matrices A, B and C, as follows:

1. Matrix A, (real array), stores $X - Y$ coordinates by rows, one row for each node.
2. Matrix B, (integer array), stores the member number and member connectivities. In each row are stored, member number, node-I, node-J.
3. Matrix C, (real array), stores the reaction nodes and corresponding angles.

4. The equilibrium equations are in D and matrix B1, (integer array), contains the generated topology matrix.

For example, for the truss shown in Figure 2.5, using the numbering in the figure:

$$[A] = \begin{bmatrix} 0.0 & 0.0 \\ 5.0 & 8.66 \\ 15.0 & 8.66 \\ 10.0 & 0.0 \\ 20.0 & 0.0 \end{bmatrix} \tag{2.16}$$

$$[B] = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 4 \\ 3 & 2 & 4 \\ 4 & 2 & 3 \\ 5 & 3 & 4 \\ 6 & 4 & 5 \\ 7 & 3 & 5 \end{bmatrix} \tag{2.17}$$

$$[C] = \begin{bmatrix} 1 & 0.0 \\ 1 & -90.0 \\ 5 & 0.0 \end{bmatrix} \tag{2.18}$$

The truss command to generate the equilibrium equations, is then,

TRUSS A B C D

The matrix [D] is where the equilibrium matrix is stored and the analysis sequence to calculate member forces and reactions is:

LOADR	A	R=5 C=2
(nodal coordinates)		
LOADI	B	R=7 C=3
(member node numbers,[integers])		
LOADR	C	R=3 C=2
(reaction information)		
TRUSS	A	B C D
INVERT	D	T=2
LOADR	R	R=10 C=1
(load information)		
MULT	D	R S
PRINT	S	

Note that the equilibrium matrix is unsymmetric so that the option, T=2 is used with INVERT which can be used for either, symmetric (T=1), or unsymmetric (T=2), matrices. The routine TOPOL is provided and can be used simply by giving the command,

TOPOL A B B1

The topology matrix shows the number of member connections to each node and the connections between nodes. If only one or two nodes of the truss have loads applied then the command,

TRUSLD A L R

may be used. This command puts the individual node loads given in L into their correct locations in the total load vector R. In the command TRUSLD,

A is the matrix of nodal coordinates
L is a matrix of 3 columns, with as many rows as node loads,
  Column 1 is the node number
  Column 2 is the X load
  Column 3 is the Y load on the node
R is the combined load vector of all nodes

The command sequence to complete the analysis for member forces having inverted the equilibrium matrix and using the TRUSLD command is:

```
LOADR    L   R=? C=3
(load data)
TRUSLD   A   L R
MULT     D   R S
PRINT    S
```

STATICS-2020 provides 12 truss examples that may be called with the command,

TRUSEX E=? D=?,?

where E=1 to 7, or 11, 12 for the statically determinate trusses provided. The data D gives the two parameters needed to generate the truss geometry. This command returns the data matrices A, B and C. The user then proceeds as for any truss input.

2.2.4 Plot commands for viewing truss geometry

Two simple commands are available for viewing trusses analysed by STATICS-2020. The first of these plots the truss members and reaction locations showing the node and member numbers. If the analysis has been completed it is possible to show tension members(blue) and the compression members(red).
The general command is:

PLTRUS A B C [S] [R or V] N=?

If N=1 the member force vectors [S] and [R/V] are not required in this command and the members only are plotted in a single colour. On the other hand if N=2, the force vectors [S] (member forces) and [R] (nodal forces) must be given and the members are colour coded according to tension or compression and the nodal force components drawn. If N=3 then [S] and [V] (nodal displacements) must be given and the deflected position of the truss is drawn. The displacements have been magnified to 1/10 th of the maximum dimension of the truss. In this case the members are drawn with the same colour coding as for N=2. The second plot command is used in conjunction with the joint equilibrium check command,

JOINEQ A B C R S S1 N=?

The command JOINEQ identifies all forces acting at joint N and calculates their $X - Y$ components, these are stored in S1, the last row of which contains their sums that should be zero for equilibruim.

The command:

PLTJEQ A S1 N=?

is called after JOINEQ and draws the polygon of forces using the components of all the joint forces for the node N stored in S1. This is used to illustrate that a joint is in equilibrium with all the forces acting on it and thus the polygon of forces closes. These two commands (JOINEQ and PLTJEQ) may repeated for several joints.

2.2.5 Calculation of truss deflections

For any determinate structure calculation of deflections can be made via the contragredient law, see section 1.3.3, since the member force transformation is known,

$$\{S\} = [b]\{R\} \tag{2.19}$$

The corresponding nodal deflections $\{r\}$, are obtained from the member distortions $\{v\}$, by

$$\{r\} = [b]^T\{v\} \tag{2.20}$$

If the member distortions $\{v\}$ are known in terms of the member forces by the flexibility relationship, for member i, for example,

$$\{v\}_i = [f]_i\{S\}_i \tag{2.21}$$

all these values can be combined together in the single expression as,

$$\{v\} = [f]\{S\} \tag{2.22}$$

If reactions are included in $\{S\}$, their flexibilities can be set to zero. Combining the equations (2.19-2.20) and (2.22),

$$\{R\} = [b]^T[f][b]\{R\} \tag{2.23}$$

and,

$$[F] = [b]^T[f][b] \tag{2.24}$$

is the structure flexibility matrix. For the truss structure, $[f]$ is particularly easy to obtain, being for the member i, simply,

$$f_i = \frac{l_i}{EA_i} \tag{2.25}$$

In this equation, l_i = member node to node length, A_i = area of cross section,E = Young's modulus of elasticity.

In STATICS-2020, the tansformation matrix $[b]$ has already been calculated and node loads used to calculate member forces. The Young's modulus and areas will be read into an array, $(1 \times$ number of members $+ 1)$, designated here by AR, with AR(1) being the Young's modulus, assumed to be the same for all truss members. The commands for statically determinate structure flexibility matrix (FL) generation and the calculation of the nodal deflections are thus:

TRUSFL A B AR D FL
MULT FL R V

Deflections are in V. The same theory applies for statically indeterminate structures. However the flexibility matrix has already been calculated in the user defined matrix K as the inverse of the stiffness matrix (see the theory of indeterminate trusses) and so the deflections are then calculated,

MULT K R V

The deflected shape is plotted with the command,

PLTRUS A B C S V N=3

2.2.6 Displacements of supports

The displacements of the nodes of a determinate structure caused by the displacements of the support points can be obtained from equation (2.20). For a determinate structure these will be rigid body displacements. To give a positive displacement at the reaction a negative value of the displacement must be supplied. This is because the equation of equilibrium for a reaction (R) and an applied force (F) is,

$$R + F = 0 \text{ that is } \qquad R = -F \tag{2.26}$$

Then contragredience gives,

$$r_F = -r_R \tag{2.27}$$

Thence a negative value for r_R in equation (2.20) gives a positive (r_F). An exercise has been given in (**T8**) for the calculation of the node deflections for a reaction displacement. The transpose multiplication TMULT is available and the support displacements can be input one at a time and stored in the appropriate location of the member displacement vector S1 using STOSM. The sequence of commands for the vertical deflection of node 1 of the truss in problem (5)-A5, are :

ZERO	S1	R=20	C=1	member displacement vector
LOADR	L	R=1	C=1	load displacement data in L
-1.0				(displacement data)
STOSM	S1	L	L=18,1	set up displacement vector S1
TMULT	D	S1	V1	calculate displacements($b^T$is in D)
PRINT	V1			print node displacements

The deflected position is displayed using the command,
PLTRUS A B C S S1 N=3

2.2.7 Statically indeterminate planar trusses

In STATICS-2020, when the equilibrium equations are set up in D with the command,
TRUSS A B C D
a check is made on the dimensions of [D] (NR,NC), rows = NR, columns = NC. If the condition is encountered,

NR < NC "too many unknowns, indeterminate analysis necessary" is printed.

In this case (NR < NC), the analysis can still proceed, and the $[b]$ matrix in the equation,

$$\{S\} = [b]\{R\} \tag{2.28}$$

calculated using the compatibility conditions of the deformations of the members. Use is made of the contragredient principle. Starting from the equilibrium equations,

$$[A]\{S\} = \{R\} \tag{2.29}$$

the contragredient principle, (see equations(1.7) and (1.9)), shows that the corresponding displacement transformation connecting member deformations to nodal deflections is given,

$$\{v\} = [A]^T\{r\} \tag{2.30}$$

In equation (2.30), $\{v\}$ are the changes in length of the truss members and support defelctions, and $\{r\}$ are the nodal displacements corresponding to $\{R\}$. In the stiffness method of analysis it is necessary to know the relationship between the member forces $\{S\}$ and the member distortions, $\{v\}$. This relationship is easily established for an elastic truss member (i) with an area of cross section A_i, length L_i and Young's modulus of elasticity E, by the expression,

$$S_i = E(\frac{A_i}{L_i})v_i = [k_i]\{v_i\} \tag{2.31}$$

and for all members and reactions, with the diagonal matrix $[k]$,

$$\{S\} = [k]\{v\} \tag{2.32}$$

For the reaction values the corresponding diagonal stiffness terms in $[k]$ are set to a large stiffness number, for example 10^{20}. Combining equations (2.29), (2.30) and (2.32) gives

$$[A][k][A]^T\{r\} = [K]\{r\} = \{R\} \tag{2.33}$$

The structure stiffness matrix $[K]$ is now a non-singular, square, symmetric matrix and it follows that the displacements $\{r\}$ may be obtained by inversion. That is,

$$\{r\} = [K]^{-1}\{R\} \tag{2.34}$$

Combining equations (2.30), (2.32) and (2.34) gives the member forces,

$$\{S\} = [k][A]^T[K]^{-1}\{R\} = [b]\{R\} \tag{2.35}$$

That is, the force transformation matrix $[b]$ in the statically indeterminate structure is simply,

$$[b] = [k][A]^T[K]^{-1} \tag{2.36}$$

In STATICS-2020 two commands are made available to calculate the $[b]$ matrix in equation (2.36). Firstly the areas of the truss members, together with the Young's modulus of elasticity are read into a row array, labelled AR in the following text. The value of Young's modulus may simply taken as unity if only member forces are being calculated. That is,

AR(1)	=	Young's modulus
AR(2) ...	=	area of members 1
to AR(no. of members)	=	to number of members

The sequence is then,

TRUSS	A	B	C	D	
LOADR	AR	R=1	C=?		
(Young's	modulus	and	member	stiffness)	
TRMSTF	A	B	C	AR	MS

The command TRMSTF(=TRuss Member STiFness), is used to calculate the member stiffness matrix, together with the reaction stiffness values in MS. To calculate $[b]$, the global stiffness matrix $[K]$ is formed and inverted and the result, in $[K]$ is premultiplied by $[k][A]^T$. The $[b]$ matrix is then stored in the array with the name KA in the STATICS-2020 software. The command, (TRuss Global STiFness) to produce this result is:

```
TRGSTF D MS K
```

Remember that the matrix of the equilibrium equations has been generated in D. Loads are calculated via the command,

```
TRUSLD A L R
```

Then member forces are calculated in S deflections in V by the matrix multiplications and printed,

```
MULT     KA R S
MULT K R V
PRINT    S
PRINT    V
```

Sequence of commands for an indeterminate truss analysis:

LOADR (coordinate data)	A R=? C=2	coordinates of nodes (X, Y)
LOADI (member data)	B R=? C=3	member node numbers, (number),I,J
(reaction data)	C R=? C=2	node number,type 1, 2-(X, Y), θ angle to X axis
TRUSS	A B C D	forms equilibrium matrix, tests for solvability

LOADR (load data)	L R=? C=3	node loads
TRUSLD	A L R	generate node load vector R
LOADR (member data)	AR R=? C=1	Young's modulus, member area array AR
TRMSTF	A B C AR MS	member and reaction stiffnesses MS
TRGSTF	D MS K	global stiffness, calculates $[b]$ matrix
MULT	KA R S	calculate member forces and reactions
PRINT	S	print member forces and reactions
(PLTRUS	A B C S R N=2	plot truss, sense of member forces shown)

The truss members may be plotted with the command,

PLTRUS A B C N=1

The sense of member forces, tension or compression can be shown by the plot command

PLTRUS A B C S R N=2

Alternatively one of the exercises may be undertaken using the TRUSEX command to generate the data.

2.2.8 Example of statically indeterminate truss.

Examples of indeterminate trusses are to be found in the exercises numbers 8 to 12 Figures 2.8, 2.11, 2.12. That is, using the command for the truss Example 8 shown in Figure 2.7, and a load shown on node 3 with 100 units of force in the positive X direction.

Command sequence

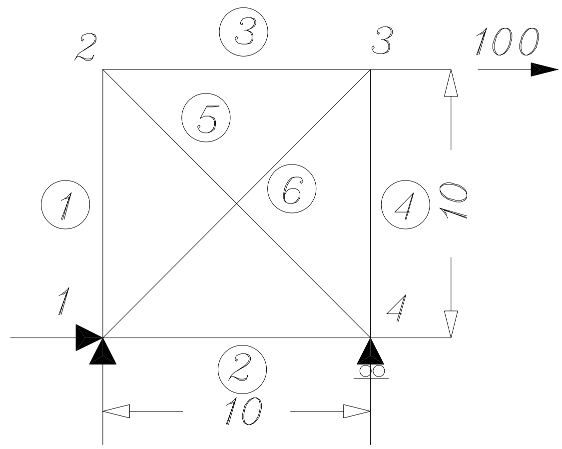

Figure 2.7: Exercise 8, statically indeterminate truss

TRUSEX	E=8	D=1.0,1.0			
PLTRUS	A	B	C	N=1	
TRUSS	A	B	C	D	
LOADR	L	R=1	C=3		
3	100.0	0.0			
TRUSLD	A	L	R		
LOADR	AR		R=7,	C=1	
10.0	1.0	1.0	1.0	1.0	1.0 1.0
TRMSTF	A	B	C AR	MS	
TRGSTF	D	MS	K		
MULT	KA	RS			
PRINT	S				
PLTRUS	A	B	C	S	R N=2

2.3 Truss study module

2.3.1 Study programme in truss analysis

The purpose of the first session of this course is to teach the students studying engineering the theory and application of the statics of trusses. In the second session topics in the strength of materials are studied. The objective of the course is to teach the material in an arithmetic free environment. It achieves this objective by providing the software package STATICS-2020 to undertake the computations required in each analysis module. In a purely simplistic approach it would be possible to undertake this course using only a simple matrix interpretative program, of which there are many excellant examples available. In fact the core of STATICS-2020 is a matrix interpretative program suggested by CAL86 written by Professor E. Wilson of the University of California, Berkeley. However the simple matrix interpretative program suffers from some severe limitations both from the student's and the teacher's points of view. For the student there will be too much data preparation and too many programming steps that make the class exercises slow and liable to error. From the teacher's perspective, too few problems can be solved and incorrect solutions mean that the student's confidence in the theory is not adequately developed. Thus the rationalé behind the development of STATICS-2020 is to produce a number of modules each of which has a command structure oriented towards the particular topic to be studied. These modules then form the basis of the individual units of the course and they may be studied independently although there is a necessary progression from simple topics to the more difficult. There will be a module associated with each theory topic these have been listed in Chapter 1 section(1.1).

In the examples of statically determinate trusses, the dimensions are relative. For practical cases in either system of units, imperial or metric, the dimensions given in the exercises may be scaled to give realistic sized structures. For example for the trusses in Figure 2.9, the panel size is 10 units. This may be 10 feet with a truss span of 60 feet. If the matrix system is used the truss span would be 60 metres which may require scaling by 1/3 to make a realistic span.

2.3.2 Statically determinate truss analysis exercises

From the lecture notes the course starts with a discussion of the equilibrium of forces at a point and the necessary and sufficient conditions being,

$$\sum F_x = 0 \qquad \sum F_y = 0 \tag{2.37}$$

See Sections 2.2.1 to 2.2.3 for the theory of node equilibrium of trusses, setting up of the equilibrium equations etc.. Having developed the theory, to undertake truss analysis, using STATICS-2020, seven commands are used of which only two are particular to the theory of trusses. See the lecture notes, Chapter 9, for examples and explanations. The summary of these commands is given below.

1. LOADR, LOADI read in real or integer arrays of information
2. PRINT A print array A
3. LIST list arrays that have been used in the problem
4. TRUSS A B C D set up node equilibrium equations in D
5. INVERT D T=2 invert unsymmetric equilibrium matrix D
6. MULT D R S member forces, reactions from load vector R

7. PRINT S print member forces and reactions
8. PLTRUS A B C [S/R/V] N=1,2 or 3 plot truss

Three additional commands may be used in the truss exercises. These are:

1. TOPOL A B B1 calculates connectivity matrix B1 for the nodes and members
2. JOINEQ A B C R S N=? undertakes equilibrium check of all forces acting on node
3. PLTJEQ draws vector diagram of the forces obtained in the JOINEQ command

Note: the command: TRUSEX E=? D=?,? generates the A B C data matrices for the twelve trusses given in the exercises, scaled by the values D, (see the Figures (2.8) to (2.12) for these examples).

The lecture notes should be read to cover the theory and the command functions, (see also the online HELP command).

The purpose of the examples is to:

1. Illustrate the principles of joint and free body equilibrium.
2. Show the flow of forces throughout a truss structure, noting those members in tension and compression.
3. Study the general principles of truss action, for example, chord members carry the bending moments and web members the shear.
4. The use of influence lines for member forces to calculate maximum values for uniformly distributed load and point load patterns.
5. Calculate truss deflections.
6. Study the analysis of determinate trusses and influence of indeterminacy on member forces.

The class exercises are all based on the examples for determinate truss analysis given in Figures 2.8, 2.9 and 2.10. Use of these examples will ensure that students start the exercises with the correct data. Only the load vectors have to be input by the students and these can be varied if tutorials are to have individual problems for each student. It is hoped that the exercises given herein are self , see also course details in Chapter 9.

2.3.3 Exercises for determinate truss analysis

(T1) For the truss (1) in Figure 2.8, apply a vertical load of -100 in the Y direction at node 3.
Answer the following:

1. What are the signs of the forces in members (1), (2) and (4).
2. What are the magnitudes of the forces in members (3), (6) – to – (9).
3. Analyse the truss using STATICS-2020 to verify your answers in (1) and (2).
4. Now apply a load of -100 in the Y direction on node 5 and examine the members forces. From first principles check the reaction values at (1) and (2).
5. Use the JOINEQ and PLTJEQ commands to check equilibrium of node 5 in load case (4).

(T2) For the truss (2) in Figure 2.8, apply a load of 100 units in the Y direction of node 6. Answer the following:

1. Which members can you say have zero force simply from inspection. Hint: start at nodes 4 and 7.
2. Analyse the truss using STATICS-2020 and prove your result. Taking moments about node 6 of the reaction at node (11), check the magnitude of the force in member 10.
3. Now apply load components $(+100, -100)$ to node 6 and analyse the truss. Use JOINEQ and PLTJEQ commands to prove that node 6 is in equilibrium.
4. Apply a vertical load at node 4 and obtain member forces using the already calculated $[b]$ matrix. Discuss the results.

(T3) Trusses (3) and (4) in Figure 2.9 have vertical loads of 100 units applied at node 7. Answer the following:

1. From first principles, what is the sense of the forces in the diagonal members in the two cases, and can you calculate their magnitudes?
2. In truss (3) what is the magnitude of the force in member (13), and in truss (4) magnitudes of forces in members (1), (4), (13), (24), and (25).
3. Analyse both trusses using STATICS-2020 to verify your results.

(T4) A force of -100 is applied to node (11) of truss (6) in Figure 2.9.
Answer the following:

1. From first principles what are the signs of forces in members (16), (17), (1) and (6)?
2. Analyse the truss using STATICS-2020 and use JOINEQ and PLTJEQ to verify equilibrium of nodes 10 and 11.
3. What is the relationship between the forces in members (21) and (22) for the same load? Examne node 13 using JOINEQ and PLTJEQ to verify the results.

(T5) A force of -100 units is applied to the truss (5) in Figure 2.9.
For the two load cases answer the following:

1. Load applied to node 5, and
2. Load applied to node 6.
   Analyse the two cases and examine the results. Explain the difference.
3. What are the differences between truss (5) and trusses (3) and (4).

(T6) The truss (7) in Figure 2.10 is to be analysed.
Answer the following:

1. Setup A,B and C matrices using TRUSEX command.
2. Use TOPOL to examine the joint connectivity matrix. What do you notice about the number of members incident on each joint.
3. Analyse the truss for load cases of (-100) units in the Y direction on nodes 3 and 4. Discuss both of the results.

2.3.4 Influence lines

A force influence line gives the value of a particular force (in the present case a truss member force or a support reaction), as a unit load traverses the nodes of the truss. The values of the influence line for a particular member force are thus obtained from the row of the $[b]$ matrix corresponding to the force. Influence lines are useful for determining the maximum value of a force due to a pattern load or a uniformly distributed load that may be applied over all or part of the structure. For the trusses shown in Figure 2.9 it is possible to obtain the influence line for a particular member force as a vertical load traverses the lower chord of the truss. For truss (3) in Figure 2.9, this would involve unit loads at nodes 1,3,5 ... 13. A web member for example (19) will have the sign of the force in the member change when the unit load moves from node 9 to node 11. The exercise (T7) is to give practice in the use of influence lines.

(T7) From the $[b]$ matrix obtained for the truss (3) in Figure 2.9, draw the influence lines for,

1. member force (14)
2. member force (19).
3. A load system (20,20,10) in the Y direction at equal spacings of 4 moves across the lower chord. Position the load group so that maximum forces (tension and compression) are obtained in members (14) and (19).
4. If a uniformly distributed load of 2 /unit length in the Y direction can have any length, what is the maximum tensile, (compressive) force in member (19)?
5. From the $[b]$ matrix plot the influence line for the reaction at 13. Check the result.

2.3.5 Calculation of truss node deflections

Determinate trusses
When the truss analysis module has been used, the inversion of the equilibrium coefficient matrix D produces the member force transformation matrix $[b]$. The theory for the calculation of the truss flexibility matrix F and from that the nodal deflections for any set of node forces is given in section 2.2.5. That is, once the $[b]$ matrix has been calculated, it is necessary only to read in the matrix of member flexibilities and to calculate,

$$[F] = [b]^T[f][b] \tag{2.38}$$

Thus after calculating $[b]$, proceed as follows:

LOADR	AR		R=1	C=?		read in Young's modulus/member areas
(				)		member Young's modulus/areas
TRUSFL	A	B	AR	D	FL	generate truss flexibility matrix FL
MULT	FL	R	U			calculate node deflections in U
PRINT	U					print node deflections

Indeterminate trusses

For indeterminate trusses the same theory can be used as for determinate trusses if the force transformation matrix [b] given in KA is used rather than the inverted equilibrium matrix [D] However the flexibility matrix has already been calculated in the inverted stiffness matrix [K] so that deflections are calculated by the matrix multiplication,

MULT K R V

Node displacements are in V and the deflected shape is plotted,

PLTTRS A B C V N=3

2.3.6 Truss exercises for deflection calculations

(T8) The truss (1) in Figure 2.8 has been analysed in Exercise (**T1**) for a load of -100 kN on node 5. Assume all members have an area of $200mm^2$ and Young's modulus of $20 \times 10^9 N/mm^2$.

1. Calculate the node deflections of the truss if the base dimension between nodes is 2 metres.

(T9) The truss (3) in Figure 2.9 has the following properties:

area top chord	$36 \times 10^4 mm^2$
area bottom chord	$24 \times 10^4 mm^2$
area web members	$18 \times 10^4 mm^2$
Young's modulus	$200 \times 10^3 MPa$

1. Calculate the deflection for a load of 10kN is applied in turn at node 3, 5 and 7.

2. The span of the truss in Figure 2.9 is 60 metres. What is the deflection if the span is 15 metres?

3. The truss support at node(13) settles an amount of 10 mm. Calculate the displacement of the nodes of the truss, (See theory in section 2.2.5).

2.4 Statically indeterminate truss analysis

When a truss is statically indeterminate and the command TRUSS is used to set up the equilibrium equations D, the coefficient matrix has more columns than rows. That is,
2J > M + R
See the theory in section 2.2.1 for the conditions for stability, determinacy and indeterminancy of trusses and also section 2.2.5 for indeterminate truss analysis theory. When the indeterminate condition is detected by the TRUSS command, a message is displayed to this effect and the user is able to use the stiffness method to calculate the $[b]$ matrix. The theory is given in section 2.2.5. Additional information giving the material Young's modulus of elasticity and the areas of the member cross sections must first be supplied. Two STATICS-2020 commands are available to undertake the analysis. These are:

TRMSTF	A	B	C	AR	MS calculate matrix of member stiffnesses in MS
TRGSTF	D	MS	K		calculate nodal stiffness matrix, invert, form $[b]$

The matrix $[b]$ is stored in the array called KA, so that if the load vector has been generated in R, member forces and reactions are calculated by the command,

MULT KA R S

An example is given in section 2.2.6 of the analysis of a simple truss with one redundant member. A number of indeterminate truss exercises are given in Figures 2.11 and 2.12. The trusses 8 to 11 have a basic bay size of 10 units. Using the SCALE command this dimension can be changed to suit either the problem being solved, or the system of measurement, imperial or metric, being used. This scaling will be applied uniformly to both the X and Y dimensions. Repeating section 2.2.5, the commands for analysis of truss, Example 8, Figure 2.11, are

TRUSEX	E=8	Sets up A,B,C matrices
TRUSS	A B C D	Generates D matrix
LOADR	AR R=7 C=1	Young's modulus, member area
TRMSTF	A B C AR MS	generate member stiffnesses MS
TRGSTF	D MS K	generates $[b]$ in KA

See section 2.2.5 for the complete list of commands. Remember that the commands JOINEQ and PLTJEQ can still be used. Class exercises are given below in (**T10**) to (**T15**). With indeterminate structures, the relative member areas (stiffnesses) affect the magnitudes of the member forces, whereas in statically determinate structures their areas have no effect on member forces although they influence truss deflections.

2.5 Exercises in indeterminate truss analysis

(T10) The truss (8) in Figure 2.11 is to be analysed for a horizontal load of 100kN in the positive X direction on node 4 and the dimension 10m is to be scaled to 4m using the SCALE command after the coordinates have been generated in the matrix A. Young's modulus is $200 \times 10^3 MPa$ for all members. Use the PLTRUS command to display the truss.

1. Calculate member forces if all members have the same area of $2000mm^2$.
2. Compare results for analyses with the area of member (5) reduced to 1000, 500, $100mm^2$ respectively. Plot your results.

(T11) The node (3) of the truss (8) in Figure 2.11 has a roller support. How can you use STATIC-2020 to provide a fixed support at 3? (Hint: Move matrix C into another matrix CC with an additional row and then put the extra support condition into this row using the MODIFY command. Repeat the analyses in (**T10**).

(T12) Analyse the truss (11) in Figure 2.11 for a 100kN horizontal force at node (6). First use PLTRUS to display the truss. Use the same member areas as in (**T10**).

1. Analyse the truss (11) for a 100 kN horizontal load on node (4).
2. Now analyse the truss for the combined loads of 100 kN on node (4) and 100kN on node (6). Prove that superposition applies by adding the first two analyses and comparing them with the third.
3. Repeat the process in (**T11**) of fixing node (2) in the horizontal direction.

(T13) The truss (10) in Figure 2.11 is to be analysed for a load of 100kN vertically downwards on node 9. All members have the same area.
Answer the following questions:

1. In what proportion do you expect the shear to be carried by members (8) and (9).
2. Analyse the truss to prove your assumptions
3. If members 9 and and11 have 1/2 the area of the other members, how are the member forces changed?

(T14) Repeat the analyses in (**T13**) for the following cases and answer the questions given.

1. Loads of 100kN downwards on each of nodes 3 and 5.
2. Compare forces in members 11 and 12.
3. Reduce areas of members 12 and 15 by 1/2 and compare results of the new analyses with (2).

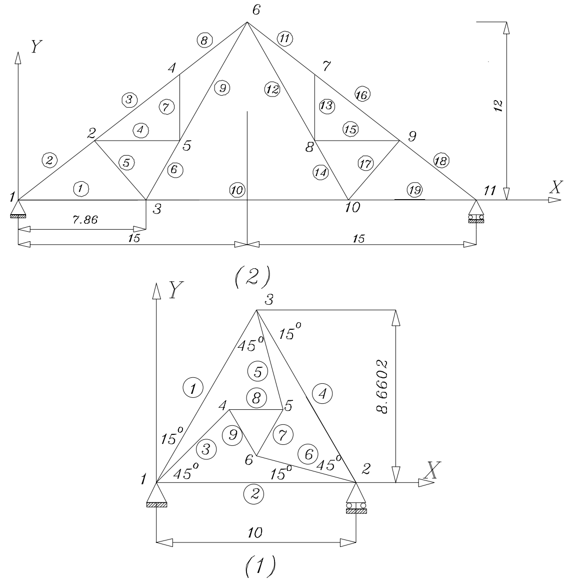

Figure 2.8: Truss examples 1 and 2

(T15) Use the command TRUSEX E=12 to generate the data for the truss (12) in Figure 2.12.

Answer the following questions, assuming that all members have equal area:

1. Analyse the truss for member forces, with loads of 10 units vertically downwards on nodes 2,4,6,10,12,14.
2. For the load case (1), record the magnitudes of the member forces, 12,23,27 and 17,21,25,29.
3. Modify the coordinate data giving node 9 an upwards repositioning of +2 m. How are member forces now calculated changed from those in (1).
4. Now return node 9 to its original position and modify the coordinate of node 8 repositioning the node -2 m downwards. Analyse the truss and determine how member forces are now modified from those of the two previous analyses.
5. From 3 and 4 is it possible to estimate a truss shape, either by upper or lower chord modification so that member forces in the top and bottom chord are essentially constant for the load case (1)?

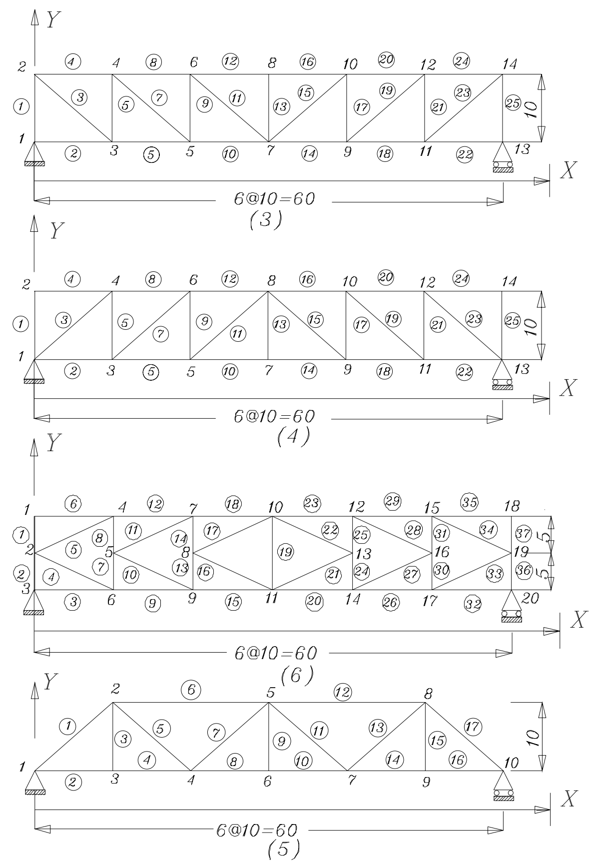

Figure 2.9: Truss examples 3 to 6

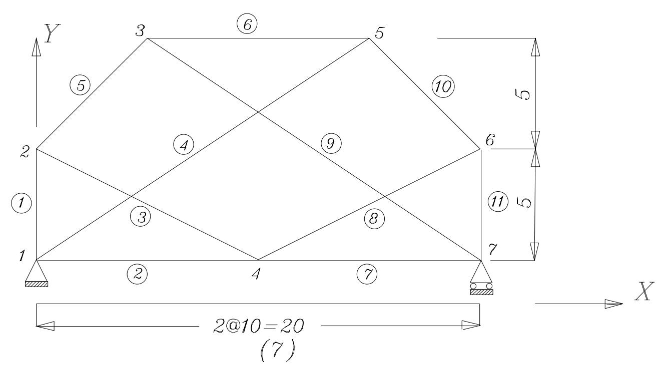

Figure 2.10: Truss example 7

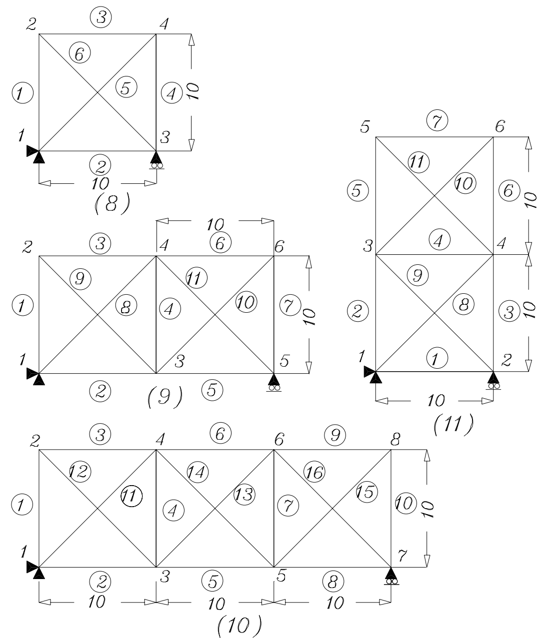

Figure 2.11: Indeterminate truss examples 8 to 11

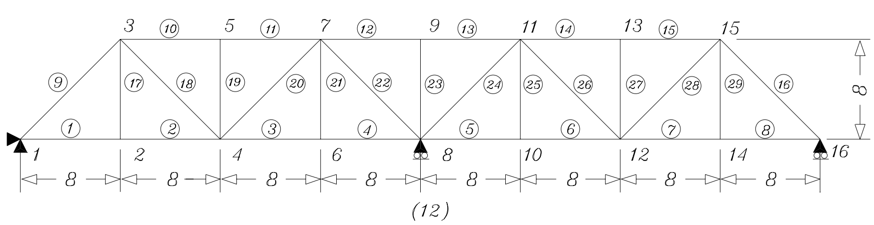

Figure 2.12: Indeterminate truss example 12

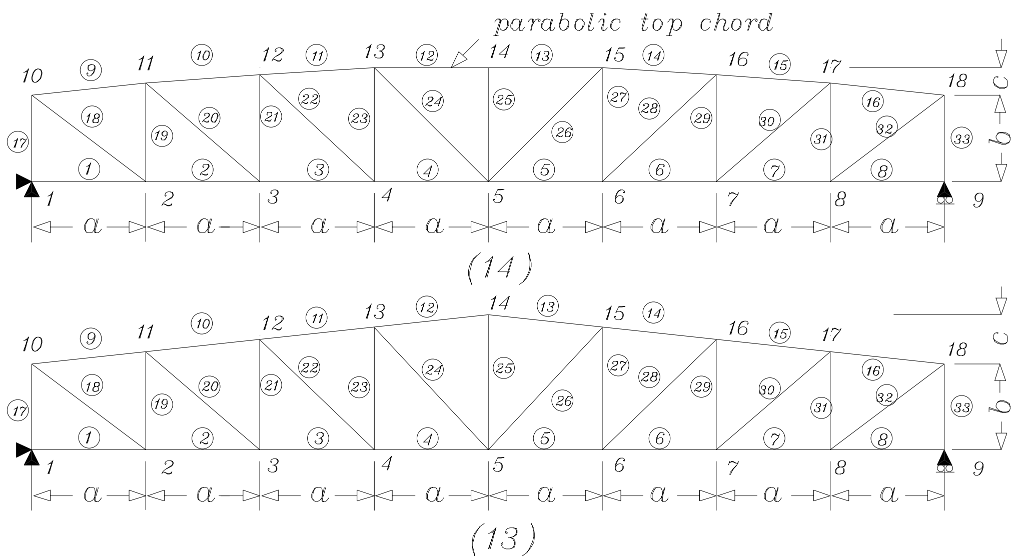

Figure 2.13: Determinate trusses examples 13, 14, variable height top chord