Chapter 6

SECTION PROPERTIES

6.1 Introduction

In Chapters 2-5 metods are developed for the calculation of axial forces, bending moments and shear forces in members that form parts of truss, beam, frame or grid structures. Initially only statically determinate systems are considered (Lecture Course I-Chapter 9). In this Chapter 6, the theory is given for the calculation of bending stresses in prismatic beams of either solid or thin walled cross section. The process involves the calculation of the cross section area, location of its centroid and the evaluation of the integrals of x^2, y^2 and xy over the cross section area and about axes through the centroid, of the cross section. The approach used is in antipication of the finite element method of analysis of field problems in engineering science. To this end the theory of interpolation of functions of (x, y) over a triangle and their integration are first studied in Section 6.2.1, followed by the location of principal axes in Section 6.2.2. The calculation of bending stresses in beams is given in Section 6.2.3. In Section 6.2.4 the various commands that are used for cross section analysis and their application are given and illustrated. Section 6.3 deals with the approximations made for thin walled cross sections and 6.4 deals with the shear flow and location of centre of twist of thin walled sections.

6.2 Section properties of areas bounded by straight lines

A typical cross section is shown in Figure 6.1(a). The properties of the cross section bounded by the straight lines, numbered by the nodes, (1-2), (2-3) etc., traversing the boundary in a counter clockwise sense can be obtained by summing the properties of the triangles, such as that shown in Figure 6.1(b), in which node 1 is the origin of coordinates and the side (2-3) is one of the sides of the perimeter. Herein the elementary theory of polynomial interpolation over the area of a triangle is recapitulated and is used in the software to generate the section properties. The integration of polynomials over the triangle is central to the theory developed here. In Figure 6.1(b), the node numbers $(1, 2, 3)$ have the cyclic properties,

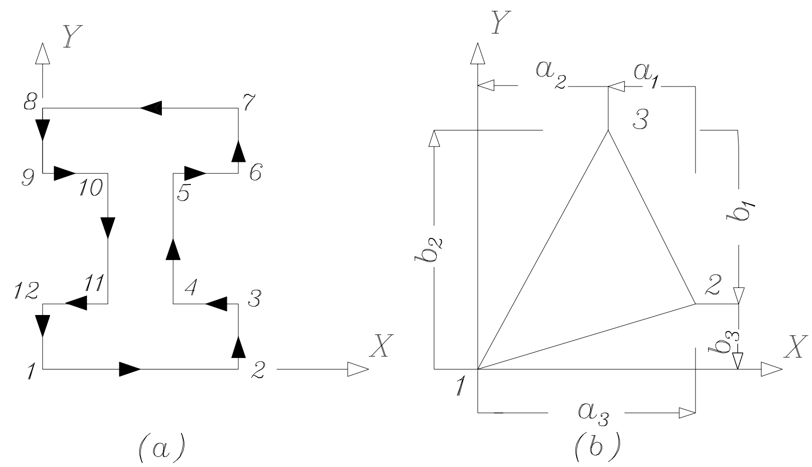

Figure 6.1: Beam cross section and basic triangle element.

$$\begin{array}{cccc} & i & j & k \\ i & 1 & 2 & 3 \\ j & 2 & 3 & 1 \\ k & 3 & 1 & 2 \end{array} \tag{6.1}$$

and the global dimensions of the triangle shown in Figure 6.1(b) are:

$$a_i = x_k - x_j\,; \qquad b_i = y_j - y_k \tag{6.2}$$

The triangle area is given by,

$$2A = a_i b_k - a_k b_i \tag{6.3}$$

For example,

$$2A = a_3 b_2 - a_2 b_3 \quad \text{etc.} \tag{6.4}$$

This expression is easily proven by taking the cross product of the vectors $\vec{12}(a_3, -b_3)$ and $\vec{13}(-a_2,\, b_2)$ and equating the result to $2A$. In Figure 6.1(b), area coordinates ζ_i are defined $\zeta_i = A_i/A$ and $\zeta_1 + \zeta_2 + \zeta_3 = 1$. Integration of polynomials of the area coordinates over the triangle area are given by $I_2,\ I_2 = \int \zeta_1^p \zeta_2^q \zeta_3^r\, dA = \dfrac{2A(p!\,q!\,r!)}{(p+q+r+2)!}$ For example the integrals required here in are : $I_2 = \int C\, dA = 2AC$; $I_2 = \int \zeta_i\, dA = A/3$; $I_2 = \int \zeta_i^2\, dA = A/6$; $I_2 = \int \zeta_i \zeta_j\, dA = A/12$

6.2.1 Interpolation of space within a triangle

The coordinates of a point (x, y) within the triangle are given,

$$[x, y] = N^T [X\ Y] \tag{6.5}$$

in which $[N]$ is the interpolation polynomial in area coordinates,

$$[N]^T = [\zeta_1 \zeta_2 \zeta_3] \tag{6.6}$$

and the global coordinates of the apex nodes are,

$$[X\ Y] = \begin{Bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{Bmatrix} \tag{6.7}$$

To calculate the location (x_T, y_T) of the triangle centroid equations (6.5-6.7), and the first moment of the area are used. Then,

$$\begin{aligned} Ax_T = \int_A x\,dA &= \int_A N^T X\,dA \\ &= 2A \int [\zeta_1\ \zeta_2\ \zeta_3] d\zeta X \\ &= \frac{A}{3}(x_1 + x_2 + x_3) \end{aligned} \tag{6.8}$$

That is,

$$x_T = \frac{1}{3}(x_1 + x_2 + x_3)\,; \qquad y_T = \frac{1}{3}(y_1 + y_2 + y_3) \tag{6.9}$$

In the same way, the second moment of area matrix, about the origin of coordinates is given,

$$\begin{aligned} I_{00} &= \int_A \begin{Bmatrix} x \\ y \end{Bmatrix} \{xy\} dA \\ &= \begin{bmatrix} X^T \\ Y^T \end{bmatrix} \int_A N N^T\,dA [X\ Y] \end{aligned} \tag{6.10}$$

Thence,

$$I_{00} = \frac{A}{12} \begin{bmatrix} X^T \Lambda X & X^T \Lambda Y \\ Y^T \Lambda X & Y^T \Lambda Y \end{bmatrix} = \begin{bmatrix} I_{00xx} & I_{00xy} \\ I_{00yx} & I_{00yy} \end{bmatrix} \tag{6.11}$$

In equation (6.11) $[\Lambda]$ is the (3×3) matrix,

$$\Lambda = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \tag{6.12}$$

Now shift the origin of the coordinates to the centroid, (x_T, y_T), of the triangle so that the coordinates $(\bar{x}, \bar{y})$ are,

$$\bar{x} = x - x_T\,; \qquad \bar{y} = y - y_T \tag{6.13}$$

Then the second moment of the area matrix referenced to the centroid is,

$$I_{cc} = \int \begin{Bmatrix} \bar{x} \\ \bar{y} \end{Bmatrix} [\bar{x}\bar{y}] dA = \int \begin{bmatrix} \bar{x}\bar{x} & \bar{x}\bar{y} \\ \bar{y}\bar{x} & \bar{y}\bar{y} \end{bmatrix} dA \tag{6.14}$$

From equations (6.5) and (6.13),

$$\bar{x} = N^T X - x_T \; ; \qquad \bar{y} = N^T Y - y_T \tag{6.15}$$

So that, for example,

$$\begin{aligned} \int \bar{x}\bar{x} dA &= \int (N^T X - x_T)(N^T X - x_T) dA \\ &= X^T \int N N^T dA X - x_T \int N^T X \, dA - x_T \int N^T X \, dA + A x_T^2 \\ &= I_{00xx} - A x_T^2 \end{aligned} \tag{6.16}$$

Finally making all these substitutions in equation (6.14),

$$I_{cc} = I_{00} - A \begin{bmatrix} x_T^2 & x_T y_T \\ x_T y_T & y_T^2 \end{bmatrix} \tag{6.17}$$

That is, with $\bar{X}^T = \{x_T, y_T\}$,

$$I_{cc} = I_{00} - A \bar{X} \bar{X}^T \tag{6.18}$$

Note! The location of the centroid of the whole cross section is obtained by summing for all triangles,

$$x_c = \frac{\sum A_T x_T}{\sum A_T} ; \qquad y_c = \frac{\sum A_T y_T}{\sum A_T} \tag{6.19}$$

and the total area A,

$$A = \sum A_T \tag{6.20}$$

and the second moment of area,

$$\sum I_{cc} = \sum I_{00} - \sum A \begin{bmatrix} x_T^2 & x_T y_T \\ x_T y_T & y_T^2 \end{bmatrix} \tag{6.21}$$

Then equation (6.21) can be used to shift I_{00} to the value about the centroid of the whole cross section.

6.2.2 Principal axes, principal directions

In the $x - y$ coordinate axes now referenced to the centroid of the cross section, the inertia tensor is given,

$$I = \int_A x x^T dA \tag{6.22}$$

In the principal axes x', y' the expression becomes,

$$\begin{aligned} I_P &= \int_A x' x'^T dA = L \int_A x x^T dA L^T \\ &= \begin{bmatrix} I'_{xx} & 0 \\ 0 & I'_{yy} \end{bmatrix} \end{aligned} \tag{6.23}$$

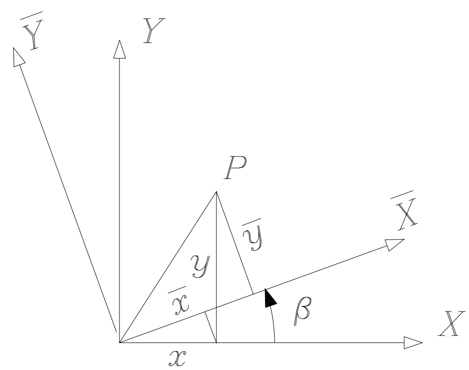

Figure 6.2: Principal axes

From Figure 6.2,

$$x' = Lx \; ; \qquad L = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \tag{6.24}$$

Expanding equation (6.23), the value of I_P is given,

$$\begin{aligned} I_P &= \begin{bmatrix} I_{Px'x'} & 0 \\ 0 & I_{Py'y'} \end{bmatrix} \\ &= \begin{bmatrix} c^2 I_{xx} + s^2 I_{yy} - 2cs I_{xy} & -sc(I_{xx} - I_{yy}) + (c^2 - s^2) I_{xy} \\ -sc(I_{xx} - I_{yy}) + (c^2 - s^2) I_{xy} & s^2 I_{xx} + c^2 I_{yy} + 2cs I_{xy} \end{bmatrix} \end{aligned} \tag{6.25}$$

Then the condition for the principal axes,

$$I_{x'y'} = 0 \tag{6.26}$$

gives the directions of the principal axes,

$$\tan 2\beta = \frac{2 I_{xy}}{I_{xx} - I_{yy}} \tag{6.27}$$

or,

$$\beta = \frac{1}{2} \tan^{-1}\left(\frac{2 I_{xy}}{I_{xx} - I_{yy}}\right) \tag{6.28}$$

Finally, from equation (6.23)

$$I_P = L \, I_{xy} L^T \tag{6.29}$$

This transformation may be used for any angle of rotation β of axes from the X axis. That is,

$$I_\beta = L \, I_{xy} L^T \tag{6.30}$$

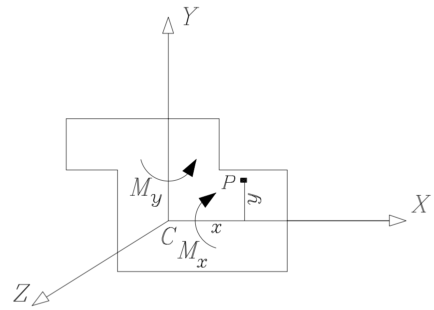

Figure 6.3: Cross section-bending moments.

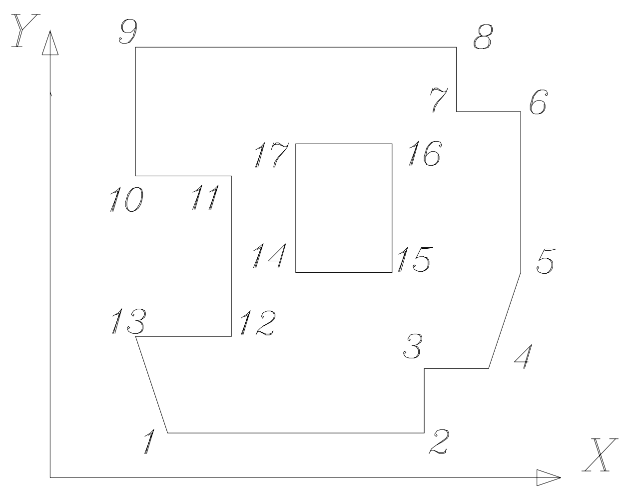

Figure 6.4: Numbering of cross section.

6.2.3 Calculation of axial stresses in the beam

The axial stress σ_z is expressed as a linear function of the coordinates (x, y) of the point P, measured from the centroidal axes (X, Y) see Figure 6.3. That is,

$$\sigma_z = a_0 + a_1 x + a_2 y = [1 \; x \; y] \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} \tag{6.31}$$

Then if F is the resultant axial force on the section and M_x, M_y the bending moments about X, Y, axes it follows that for σ_z acting on the infinitesimal area dA at P, the contributions to these values are,

$$\begin{aligned} \Delta F &= \sigma_z \\ \Delta M_y &= -\sigma_z x \, dA \\ \Delta M_x &= -\sigma_z y \, dA \end{aligned} \tag{6.32}$$

Integrating over the area of the cross section gives the resultants,

$$\begin{aligned} \begin{Bmatrix} F \\ M_y \\ M_z \end{Bmatrix} &= -\int \begin{Bmatrix} -1 \\ x \\ y \end{Bmatrix} \sigma_z dA \\ &= -\int \begin{Bmatrix} -1 \\ x \\ y \end{Bmatrix} [1 \; x \; y] \, dA \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} \\ &= -\int \begin{bmatrix} -1 & -x & -y \\ -x & x^2 & xy \\ -y & xy & y^2 \end{bmatrix} dA \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} \end{aligned} \tag{6.33}$$

For the centroidal axes, $\int x \, dA = \int y \, dA = 0$ and $\int dA = A$ and by definition,
$I_x = \int x^2 dA, \quad I_y = \int y^2 dA$ and $I_{xy} = \int xy \, dA$.
Substituting these values,

$$F = A a_0, \quad a_0 = \frac{F}{A} \tag{6.34}$$

$$\begin{Bmatrix} M_y \\ M_x \end{Bmatrix} = -\begin{bmatrix} I_x & I_{xy} \\ I_{xy} & I_y \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = -I_{cc} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} \tag{6.35}$$

$$\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = -I_{cc}^{-1} \begin{Bmatrix} M_y \\ M_x \end{Bmatrix} \tag{6.36}$$

From these equations the axial stress due to axial force and bending moments about the x and y axes is calculated as,

$$\sigma_x = \frac{F}{A} - [x \; y][I_{cc}]^{-1} \begin{bmatrix} M_y \\ M_x \end{bmatrix} \tag{6.37}$$

The signs of (M_x, M_y) positive are shown in Figure 6.3 or Figure 6.5.

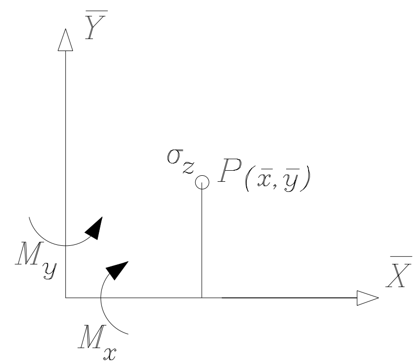

Figure 6.5: Moments applied to cross section.

6.2.4 Commands for calculating section properties

The perimeter nodes of the outside boundary and all inside boundaries are numbered sequentially in an anticlockwise direction as shown in Figure 6.4. Then the $x-y$ coordinates must be loaded into the incore data base with the LOADR command (given here for data for Figure 6.4),

LOADR A R=14 C=2

Each row gives the $x-y$ coordinate value of the corresponding node in the natural sequence. Then the call to PERIM is made once for the external boundary and once for each internal boundary,

PERIM A M=1 N=10 S=1 (outside)
PERIM A M=11 N=14 S=−1 (inside)

The numbers used are for Figure 6.4. Then section properties are calculated relative to the centroid, with the PROPER command.

PROPER A B C D

A gives the coordinates now referenced to the centroid of the section.
B gives the area.
C gives the coordinates of the centroid from the initial origin.
D contains the second moment of area matrix about x, y axes through the centroid.

Then the command,

PRINC E F

calculates the principal inertia tensor (in E) and the angle from the x axis to x', in F, in degrees.

Finally stresses may be calculated with the STRESS command,

STRESS A G F=? M=?,?

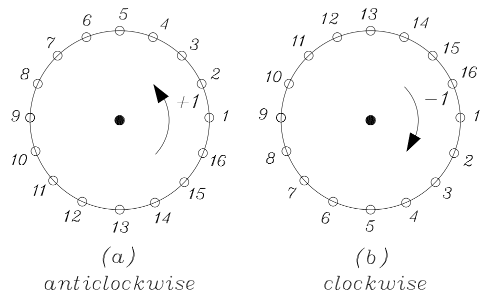

Figure 6.6: Node points generated on circumference of a circle.

This command calculates the axial stresses at the coordinate points A, and stores them in the array G. The axial load is F, and the moments are M_y, M_x, about the y and x axes respectively as shown in Figure 6.5. Note that the principal axes are not required for the stress calculations in equation (6.37).
The command,

CIRCOR AC R=?,?,?,? N=?

is provided to allow portions of a circular arc to be included in the perimeter coordinates. The parameters in R are radius, $x-y$ coordinates of the centre of the circle and the direction in which the circumference is traversed. The coordinates calculated are stored in AC, the user defined array, and depending on the 4th parameter chosen will be generated in a clockwise (-1) or an anticlockwise $(+1)$ sense, as shown in Figure 6.6. N gives the number of points on the circumference of the circle $N >= 32$. These coordinates or part thereof can be incorporated in the cross section perimeter coordinates using the DUPSM and STOSM commands. Note! The coordinates are generated with the starting node being given at both the beginning and end of the sequence.

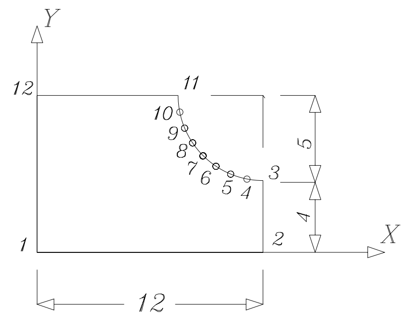

Figure 6.7: Rectangle with a quadrant cutout.

EXAMPLE: Section with quadrant cutout, See Figure 6.7

LOAD	A		R=12	C=2	
0 0					
12 0					
0 0					
0 0					
0 0					
0 0					
0 0					
0 0					
0 0					
0 0					
0 0					
0 9					
CIRCOR	AC		R=2,12,9,−1	N=32	
DUPSM	AC	A1	R=9	C=2	L=9,1
STOSM	A		A1	L=3,1	

STRESS A G F=1000.0 M=0.0,10000.0

6.2.5 Section property exercises

STATICS-2020 includes 5 examples of typical beam cross (see Figure 6.8) sections that are available to generate section properties using the standard section commands, PERIM, PROPER, etc.. To access these cross sections the command used:

SECTEX E=? (1 to 5) D=(b,d,t_1), t_2

The five section types included are:

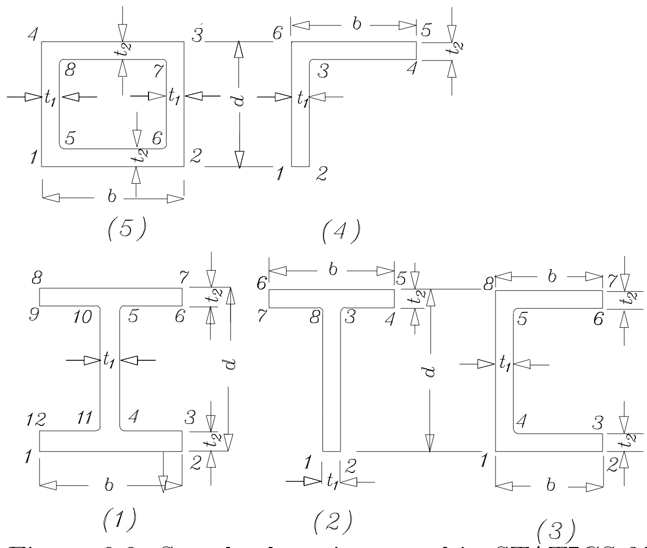

Figure 6.8: Standard sections used in STATICS-2020

(1) I-section
(2) Tee section
(3) Channel section
(4) Angle
(5) Hollow box section

Each of these sections is defined by the four parameters shown in Figure 6.8:

(1) b=breadth, d=depth
(2) t_1= web thickness, t_2= flange thickness

The command, SECTEX generates the matrix A which has the coordinates values (x, y) of the node points in the order given for each section in the Figure 6.8. Having used the command SECTEX, the PERIM command must be used before the section can be viewed with the command,

PLTSEC A N=?

also a perspective iew is given with the command

PLTSEC A G A=HA,VA,ZOOM N=4

which plots the axial stress distribution[G]in perspective viewed from the position defined by the horizonatl and vertical angles, (HA, VA), and scaled by the factor ZOOM. values of (40,30,1.5) for these three parameters are usually suitable. Exercises are given in the section (6.7) the properties tutotial module.

6.3 Shear stress in beams

In this discussion the beam will be assumed to have principal planes XY and XZ and to be loaded in the XY plane so that all deformations are in this plane, that is $M = M_Z$ is the only bending moment and the shear force V on the cross section is in the XY plane, see Figure 6.9. the section ABCD is parallel to the $X0Z$ plane and the bending stress σ_x at the distance y from the neutral axis is given,

$$\sigma_x = \frac{M_z y}{I_z} \tag{6.38}$$

Consider the equilibrium of the portion of the beam isolated by the plane section ABCD. Summing forces in the X direction

$$-\tau_{yx} b dx + \int_y^{y_t} \{-\sigma_x + (\sigma_x + \frac{d\sigma_x}{dx}\, dx)\}\, dA = 0 \tag{6.39}$$

Simplifying this equation,

$$-\tau_{yx} b + \int_y^{y_t} \frac{d\sigma_x}{dx}\, dA = 0 \tag{6.40}$$

Hence the horizontal shear stress is given,

$$\tau_{yx} = \frac{1}{bI_z} \int_y^{y_t} y \frac{dM_z}{dx}\, dA \tag{6.41}$$

and hence the vertical shear stress is,

$$\tau_{by} = \tau_{yx} = \frac{V_y}{bI_z} \int_y^{y_t} y\, dA = \frac{V_y Q_z}{bI_z} \tag{6.42}$$

In equation (6.42), Q_z is defined bas the integral,

$$Q_z = \int_y^{y_t} y\, dA \tag{6.43}$$

that is $Q_Z$is the static moment of the area cut off by the section CD(or AB) in Figure 6.9 about the $0Z$ axis. The equation (6.42) can be used whenever the shear V_y is given in one of the principal planes of the cross section(a plane of symmetry is always a principal plane).

6.4 Thin walled cross section

A thin walled member is composed of elements, here considered to be straight with dimensions of length much greater than thickness, $l >> t$, as shown in Figure 6.10.

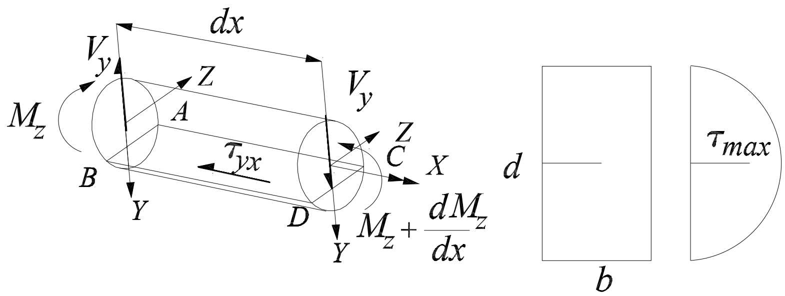

Figure 6.9: Shear-Beam section and internal forces

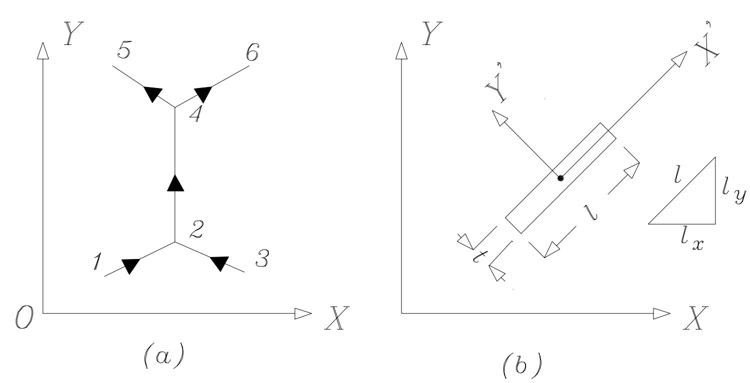

Figure 6.10: Thin walled cross section and typical element.

In the local x', y' axes the second moment of area matrix of the single element is written nelgecting the properties in the t direction,

$$[I'] = \begin{bmatrix} \frac{tl^3}{12} & 0 \\ 0 & 0 \end{bmatrix} \tag{6.44}$$

and that in the xy axes, by the $L^T I' L$ transformation,

$$I_c = L^T I' L = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \frac{tl^3}{12} \tag{6.45}$$

If (l_x, l_y) are the intercepts of l on the x, y axes respectively, then

$$sl = l_x\,; \qquad cl = l_y \tag{6.46}$$

and the equation (6.45) becomes,

$$I_c = \frac{tl}{12} \begin{bmatrix} l_x^2 & l_x l_y \\ l_x l_y & l_y^2 \end{bmatrix} \tag{6.47}$$

For the whole section in Figure 6.10(a), the section properties are calculated, firstly about the origin O and then shifted to the centroidal axes of the cross section, by using the shift theorem,

$$I_{cc} = I_{00} - A \begin{Bmatrix} x_c \\ y_c \end{Bmatrix} \{x_c y_c\} \tag{6.48}$$

The centroid is located simply by,

$$x_c = \frac{\sum l_i t_i x_i}{\sum l_i t_i}\,; \qquad y_c = \frac{\sum l_i t_i y_i}{\sum l_i t_i} \tag{6.49}$$

and the area,

$$A = \sum l_i t_i \tag{6.50}$$

In the STATICS-2020 software, the command,

THSECT A M

is used, in which A stores the (x, y) coordinates of all node points defining the cross section elements, and M stores (I,J,t), the node numbers of the ends of an element and its thickness. This routine calculates three quantities, namely, area, first moments of area about (X, Y) coordinate axes and the inertia matrix about the origin of the whole section. Then using the command, PROPER, centroid coordinates are calculated and the inertia matrix about the axes through the centroidal axes (see section 6.2.1).
That is,

PROPER A B C D

Then also,

PRINC E F

calculates the principal inertia tensor E and F the angle of the principal direction $\bar{X}$ to the X axis. The stresses at the node points are calculated from,

STRESS A G F=? M=?,?

The order of the moment components is shown in Figure 6.5 as (M_y, M_x).

6.4.1 Thin section property exercises

STATICS-2020 includes 5 examples of typical beam cross sections available for the THINEX command (see Figure 6.11), to generate coordinates that can be used with the section property commands to calculate properties using the thin section commands. To provide the data for these cross sections the command used is:

THINEX E=(1 to 5) D=b,d,t_1,t_2

There are five section types similar to the thick walled sections given previously. These five types are:

(1) I section
(2) Tee section
(3) Channel section
(4) Angle section
(5) Hollow box section

Each of these sections is defined by the four parameters (see Figure 6.11),

b	=	breadth	d	=	depth
t_1	=	web thickness	t_2	=	flange thickness

The command THINEX generates the two matrices A and M. The first of these contains the $x - y$ coordinates of the section nodes. The second contains the the node numbers of the segments that make up the cross section together with their corresponding thicknesses. Having used the command THINEX, the section may be then viewed using the command

PLTTHN A M N=(1 to 3)

After using THINEX, THSECT is used before PROPER that transfers all values to the centroidal axes. Also axial stress [G] distribution can be viewed in perspective, using,

PLTTHN A M G A=HA,VA,ZOOM N=4

Viewed from the position defined by the angles (HA,VA) and scaled by ZOOM, values of (40,30,1.5) are usually suitable for these parameters.

6.5 Shear flow in thin walled beams

The directions of positive shear and bending moment are shown in Figure 6.12. From equation (6.37), noting the direction of the moments in the Figure 6.12, the axial stress

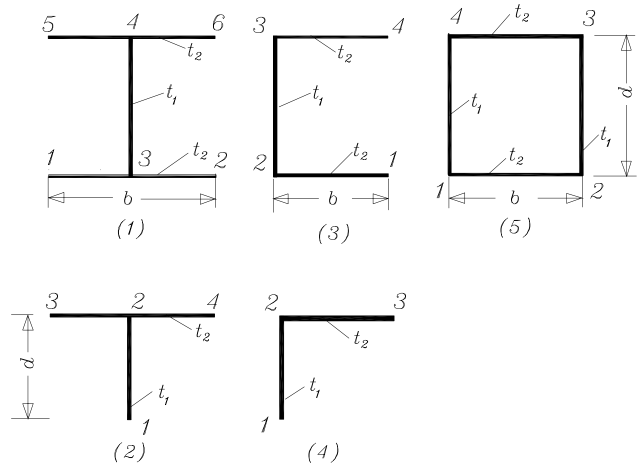

Figure 6.11: Standard thin section figures available for data preparation

σ_z, due to the bending moments (M_y, M_x) is given,

$$\sigma_z = [-x \;\; -y][F]\left\{ \begin{array}{c} M_y \\ M_x \end{array} \right\} \tag{6.51}$$

In which $[F]$ is defined as the inverse of the second moment of area matrix $[I]$ calculated about axes through the centroid of the cross section,

$$F = I^{-1} = \frac{1}{I_x I_y - I_{xy}} \begin{bmatrix} I_y & -I_{xy} \\ -I_{xy} & I_x \end{bmatrix} \tag{6.52}$$

Now the $X - Y$ shear forces are calculated from, (see Figure 6.12 and also Lecture 4 in Chapter 9),

$$\left\{ \begin{array}{c} V_x \\ V_y \end{array} \right\} = -\left\{ \begin{array}{c} \dfrac{dM_y}{dz} \\ \dfrac{dM_x}{dz} \end{array} \right\} \tag{6.53}$$

Differentiating equation (6.51) with respect to z, multiplying by a small distance ds along an element gives,

$$ds\frac{d\sigma_z}{dz} = [x \; y][F]\left\{ \begin{array}{c} V_x \\ V_y \end{array} \right\} \tag{6.54}$$

From Figure 6.13, for equilibrium of forces in the Z direction,

$$t\frac{d\tau}{ds}ds = -tds\frac{d\sigma_z}{dz} \tag{6.55}$$

Combining equation (6.55) with equation (6.54), and integrating from s_o, where $\tau = 0$,

$$t\tau_s = q_s = -\int_{s_o}^{s_1} t[x \; y]\, ds[F]\left\{ \begin{array}{c} V_x \\ V_y \end{array} \right\} \tag{6.56}$$

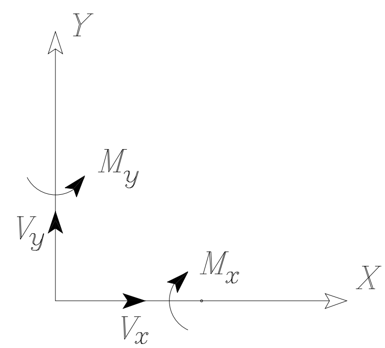

Figure 6.12: Positive sense of shears and moments.

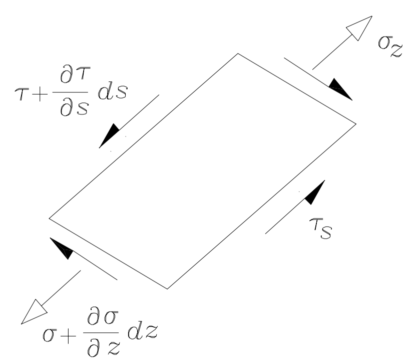

Figure 6.13: Stresses on an elemental arc of cross section.

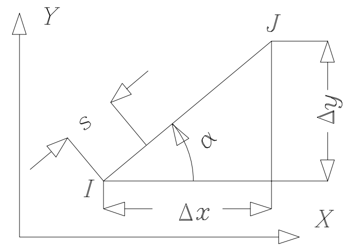

Figure 6.14: Element of cross section: x and y intercepts.

In equation (6.56), q_s is the shear flow intensity at s. The finite element concept is now introduced. The element of section $I-J$, is straight, and has a constant thickness t, as shown in Figure 6.14. Its length is l, and its $x-y$ intercepts are $(\Delta x, \Delta y)$, respectively. The segment length s is measured from the end I as shown. It is required to calculate the variation of q_s from end I to end J of the element. From equation (6.56), with $q_{s_o} = q_I$,

$$q_{sJ} = q_{sI} - \int_{s_I}^{s_J} t[x\ y]\, ds[F] \begin{Bmatrix} V_x \\ V_y \end{Bmatrix} \tag{6.57}$$

Note! $[x\ y]$ must be calculated from the centroid of the whole section, because of the equation (6.52). For a point s from I, the (x, y) coordinates are given,

$$\begin{aligned} x &= x_I + x_s = x_I + s\cos\alpha \\ y &= y_I + y_s = y_I + s\sin\alpha \end{aligned} \tag{6.58}$$

Then making these substitutions evaluate the integral in equation (6.57),

$$\begin{aligned} \int_{s_I}^{s_J} t[xy]ds &= t\int_o^l [(x_I + s\cos\alpha)(y_I + s\sin\alpha)]ds \\ &= t\int_o^l [x_I y_I]ds + t\int_o^l sds[\cos\alpha\sin\alpha] \\ &= lt[x_I y_I] + \frac{tl^2}{2}[\cos\alpha\sin\alpha] \end{aligned} \tag{6.59}$$

Thence,

$$q_{sJ} = q_{sI} - lt\{[x_I y_I] + \frac{l}{2}[\cos\alpha\sin\alpha]\}[F] \begin{Bmatrix} V_x \\ V_y \end{Bmatrix} \tag{6.60}$$

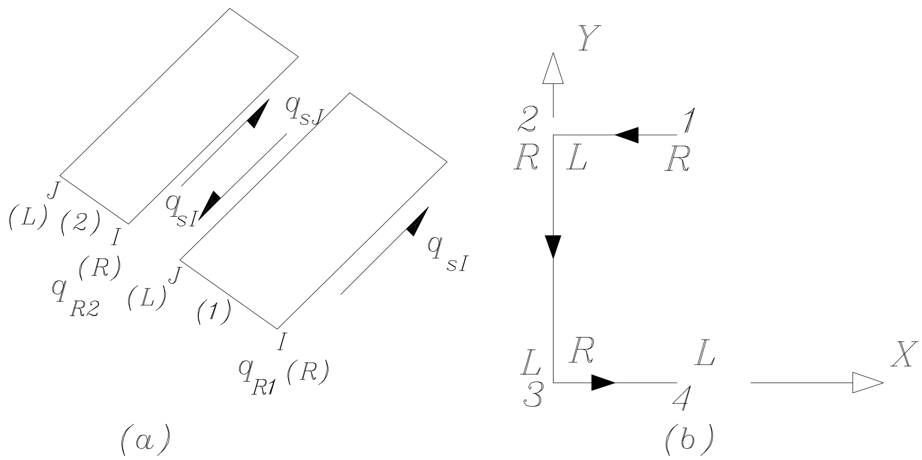

Figure 6.15: Channel section nodes and elements.

Finally since $x_J = x_I + (l \cos\alpha)$ etc.,

$$q_{sJ} - q_{sI} = -lt[\frac{1}{2}(x_I + x_J)\ \frac{1}{2}(y_I + y_J)][F]\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} \tag{6.61}$$

At any point s along the element from equations (6.59) and (6.60),

$$q_s = q_{sI} - st\{[x_I y_I] + \frac{s}{2}[\cos\alpha\ \ \sin\alpha]\}[F]\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} \tag{6.62}$$

$$q_s = q_{sI} - st\{\frac{1}{2}[x_I y_I] + \frac{1}{2}[x\ y]\}[F]\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} \tag{6.63}$$

Integrating equation (6.62), from 0 to l and calling the result V_s,

$$\begin{aligned} V_s &= q_{sI}l - \{\frac{tl^2}{2}[x_I y_I] + \frac{tl^3}{6}[\cos\alpha\ \ \sin\alpha]\}[F]\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} \\ &= q_{sI}l - \frac{tl^2}{6}[(2x_I + x_J)\ (2y_I + y_J)][F]\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} \end{aligned} \tag{6.64}$$

The strategy in the computer software is first calculate (q_{s_I}, q_{sJ}) for all members using equation (6.61) and then set up the topology of the joints. Then from equation (6.64), V_s can be calculated for all members as follows,

$$\text{calculate} \quad V_s, \text{ for } \quad V_x = 1, V_y = 0 \tag{6.65}$$

$$\text{and} \quad V_s, \text{ for } \quad V_x = 0, V_y = 1 \tag{6.66}$$

Then calculate the element $x - y$ components,

$$\left\{ \begin{array}{c} V_x \\ V_y \end{array} \right\} = \left\{ \begin{array}{c} \cos\alpha \\ \sin\alpha \end{array} \right\} V_s \tag{6.67}$$

The moments about the centroid for element i will be,

$$\begin{aligned} \{M_z\}_i &= [-y_i \; x_i] \left\{ \begin{array}{c} V_x \\ V_y \end{array} \right\} \\ &= [-y_i \; x_i] \left\{ \begin{array}{c} \cos\alpha \\ \sin\alpha \end{array} \right\} V_s \\ &= \frac{1}{l}[-y_i \; x_i] \left\{ \begin{array}{c} \Delta X \\ \Delta Y \end{array} \right\} V_s \end{aligned} \tag{6.68}$$

Let (X_s, Y_s) be the coordinates of the shear centre referenced from the centroid.
Then for $V_x = 1$,

$$-V_x Y_s = \sum (M_z)_i \tag{6.69}$$

gives Y_s and for $V_y = 1$,

$$V_y X_s = \sum (M_z)_i \tag{6.70}$$

gives X_s. These are the coordinates of the shear centre measured from the centroid of the cross section.
The equation (6.61), is written as,

$$q_{sJ} - q_{sI} = \Delta q \tag{6.71}$$

and in terms of q_{sI} and Δq,

$$\left\{ \begin{array}{c} q_{sI} \\ q_{sJ} \end{array} \right\} = \left\{ \begin{array}{c} q_{sI} \\ q_{sI} + \Delta q \end{array} \right\} \tag{6.72}$$

Consider the simple channel section in Figure 6.15. then using equation (6.72), the node equilibrium equations in the z direction can be written,

$$\begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{array}{c} q_{R1} \\ q_{R2} \\ q_{R3} \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ \Delta q_{R1} \\ \Delta q_{R2} \\ \Delta q_{R3} \end{array} \right\} = 0 \tag{6.73}$$

These equations can be solved for q_{R1}, q_{R2}, q_{R3} and then the equations (6.71) used to obtain q_{L1}, q_{L2}, q_{L3}. The shear flow at the mid-point of an element is obtained by substituting $s = l/2$ in equation (6.62).
The shear flow distributions for $V_x = 1$, $V_y = 1$ are viewed in perspective with the command

PLTTHN A M S A=HA,VA,ZOOM N=[5,6]

N=5 plots the V_x distribution and N=6 plots the V_y distribution. The values of (HA,VA) are the horizontal and vertical viewing angles respectively and ZOOM the scale factor. The matrix S gives both shear flow distributions, A contains the nodal coordinates and M topology and thicknesses.

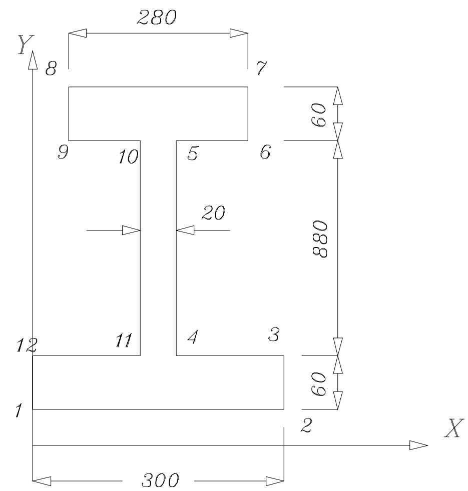

Figure 6.16: Symmetric I section.

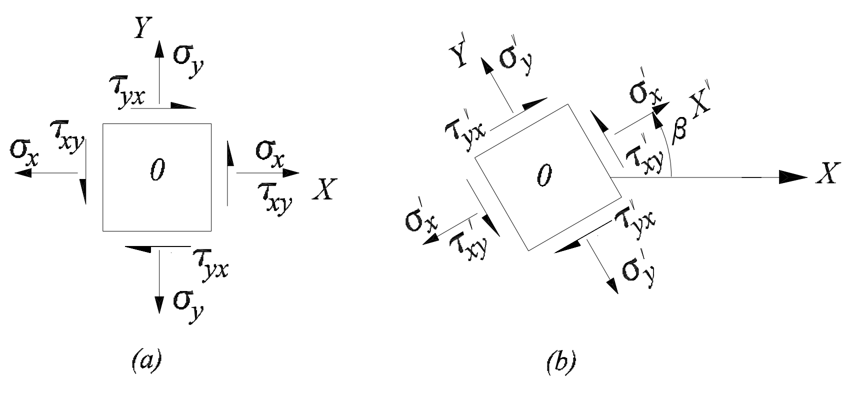

Figure 6.17: Stress tensor for rotation of axes

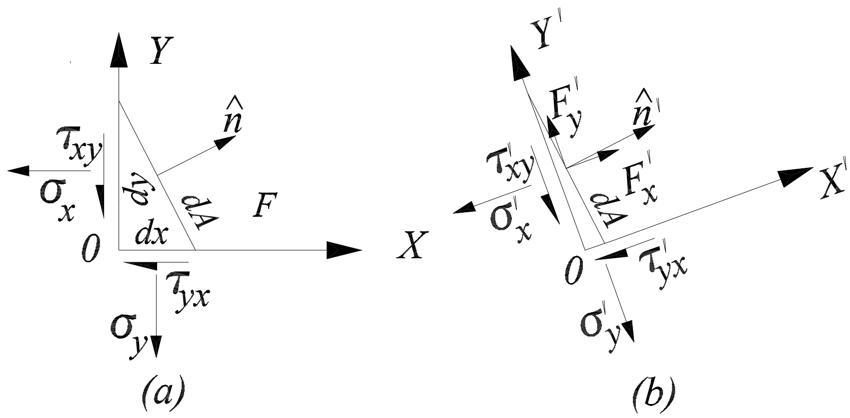

Figure 6.18: Elementary wedge of material in X, Y and X', Y' axes

6.6 Stress tensor-rotation of axes-principal axes

6.6.1 Two dimensional stress state

In two dimensions the stress components (σ_x, τ_{xy}) and (σ_y, τ_{yx}) act on the on the $X - Y$ faces of the unit cube shown in Figure 6.17. The sign convention for these stresses is positive if a face whose outwards normal is in a positive coordinate axis direction and the stress is also in a positive coordinate direction. The reverse is true for a stress on a face whose normal is is in a negative coordinate direction. In Figure 6.17 all components are shown in their positive sense. For the shear stresses τ_{xy}, τ_{yx}, the first index refers to the normal to the surface on which the shear acts and the second to its direction. Figures 6.17(a) and (b) represent the same stress state at the point O, the second set of axes being rotated from the first by an angle of β. It is required to calculate the transformation between the two stress states, and also to determine the angle for which the shear stress components are zero. It is found that this is an identical problem to that for the transformation of the second moment of area tensor, see sections 6.2.1 and 6.2.2. Consider the two sets of axes (X, Y) and (X', Y') shown in Figure 6.18(a) and (b) with the area dA and of the same orientation to OX (the original axis) in both figures. The intercepts with the two sets of coordinate axes are (dx, dy) and (dx', dy') respectively. The stress matrix $[\sigma]$ is defined in both coordinate systems,

$$[\sigma_x] = \begin{bmatrix} \sigma_x & \tau_{yx} \\ \tau_{xy} & \sigma_y \end{bmatrix} \quad \text{and} \quad [\sigma'] = \begin{bmatrix} \sigma'_x & \tau'_{yx} \\ \tau'_{xy} & \sigma'_y \end{bmatrix} \tag{6.74}$$

The sides of the wedge are related to the area dA by the projection,

$$\left\{ \begin{array}{c} dy \\ dx \end{array} \right\} = \left\{ \begin{array}{c} \cos\alpha \\ \sin\alpha \end{array} \right\} dA \tag{6.75}$$

and similarly in the rotated axes by replacing quantities by their primed values. From the Figure 6.18(a) the forces (F_x, F_y) on the face dA are expressed in terms of the stress components of the (X, Y) faces by

$$\left\{ \begin{array}{c} F_x \\ F_y \end{array} \right\} = \left[\begin{array}{cc} \sigma_x & \tau_{yx} \\ \tau_{xy} & \sigma_y \end{array} \right] \left\{ \begin{array}{c} \cos\alpha \\ \sin\alpha \end{array} \right\} dA \tag{6.76}$$

That is,

$$\{F\} = [\sigma]\hat{n}dA \tag{6.77}$$

and by the same reasoning in Figure 6.18(b) for the rotated axes,

$$\{F'\} = [\sigma']\hat{n}'dA \tag{6.78}$$

The transformation of vector components for rotation of axes is expressed,

$$\{F\} = [L]^T\{F\}' \quad \text{and} \quad \{F\}' = [L]\{F\} \tag{6.79}$$

$$\{\hat{n}\} = [L]^T\{\hat{n}\}' \quad \text{and} \quad \{\hat{n}\}' = [L]\{\hat{n}\} \tag{6.80}$$

In these equations,

$$[L] = \left[\begin{array}{cc} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{array} \right] \tag{6.81}$$

Premultiply both sides of equation (6.77) by $[L]$,

$$\{F\}' = [L]\{F\} = [L][\sigma]\hat{n}dA \tag{6.82}$$

Substituting for $\hat{n}$,

$$\{F\}' = [L]\{F\} = [L][\sigma][L]^T\hat{n}'dA \tag{6.83}$$

However the values of F' in equations (6.78) and (6.83) are equal and since $\hat{n}$ is arbitrary it follows that,

$$[\sigma'] = [L][\sigma][L]^T \tag{6.84}$$

and since $[L]$ is an orthogonal matrix, the reverse transformation is given,

$$[\sigma] = [L]^T[\sigma'][L] \tag{6.85}$$

This are the identical transformation as in equation (6.27) for the second moment of area matrix $[I]$. It follows that the principal stresses (zero shear stress state), are obtained from equation (6.27) substituting the stress values,

$$\beta = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{6.86}$$

and,

$$[\sigma_\beta] = [L][\sigma][L]^T \tag{6.87}$$

Two commands are available in STATICS-2020 for transformations of the two dimensional stress tensor. The first command is for performing the transformation, the second plots the rotated cube with its stress components.

STRTRN A B E F T=?

This command has the stress matrix A (2×2) as input with a rotation angle given by T=?. The rotated stress matrix components are in B. In addition the principal stresses are calculated in E and the angle from the positive X axis to the maximum principal stress axis is in F. The rotated cube with its stresses is plotted with the command,

PLTRTN B T=?

B being the rotated matrix and T=? the angle of rotation.

6.6.2 Three dimensional stress state

The three dimensional stress state has its components arranged in the (3×3) matrix, $[\sigma]$,

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \tag{6.88}$$

The principal stresses and principal stress directions may be obtained by using an eigen value analysis for the (3×3) matrix $[\sigma]$ which in effect finds the transformation matrix $[L]$ and performs the axes rotation in equation (6.87). If the stresses have been input into the (3×3) matrix A, the command,

JACOBI A B

returns the principal stresses in A and their principal directions with respect to the XYZ axes in B. For example if

$$A = [\sigma] = \begin{bmatrix} 10.0 & -5.0 & 5.0 \\ -5.0 & 20.0 & 5.0 \\ 5.0 & 5.0 & 30.0 \end{bmatrix} \tag{6.89}$$

then , Principal stresses and principal directions are given,

$$\mathrm{A} = \begin{bmatrix} 6.108 & 0.0 & 0.0 \\ 0.0 & 21.446 & 0.0 \\ 0.0 & 0.0 & 32.446 \end{bmatrix} \quad \mathrm{B} = \begin{bmatrix} 0.872 & -0.471 & 0.137 \\ 0.410 & 0.853 & 0.321 \\ -0.268 & -0.224 & 0.937 \end{bmatrix} \tag{6.90}$$

6.7 Example of cross section properties

Examples are given for both thick and thin walled sections to illustrate use of STATICS-2020 commands. These exampls use data input rather than that generated from the SECTEX and THSECT commands. For the nodal point numbering see Figure 6.14.

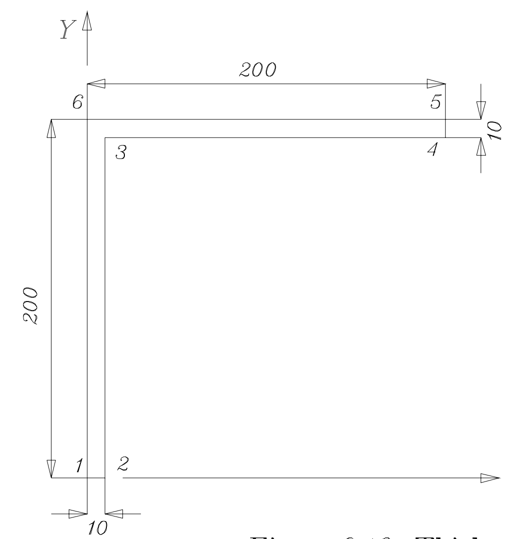

Figure 6.19: Thick angle

6.7.1 Symmetric I section

See Figure 6.1 for node numbering.

coordinates

nodes	x-y values	
1	0	0
2	300	0
3	300	60
4	160	60
5	160	940
6	280	940
7	280	1000
8	20	1000
9	20	940
10	140	940
11	140	60
12	0	60

Commands

```
LOADR   A R=12 C=2
(Coordinates entered here)
PERIM   A M=1 N=12 S=1
PROPER  B C D
PRINT   A
PRINT   B
PRINT   C
PRINT   E F
PRINT   F
```

Results:

Area (B) $=$ 47600.0

Centroid (C) $x_c = 150.00$

$y_c = 440.76$

Inertia matrix (D) $I_{cc} = \begin{bmatrix} 0.17559 & 0.0 \\ 0.0 & 7.6047 \end{bmatrix}$

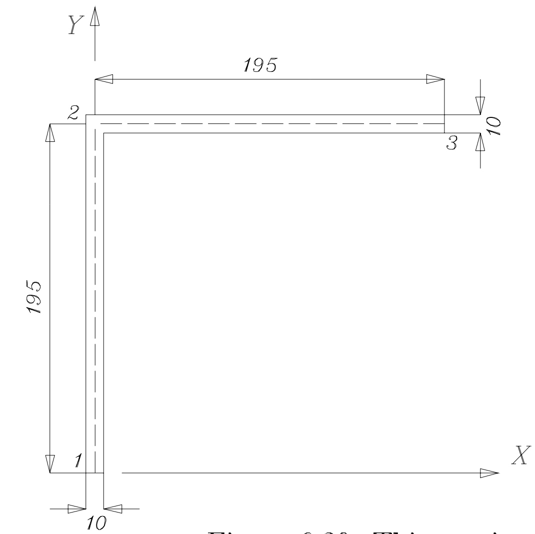

Figure 6.20: Thin section angle

6.7.2 Thick angle section

Section coordinates
Figure 6.16)

	x	y
1	0	0
2	10	0
3	10	190
4	200	190
5	200	200
6	0	200

Command sequence
Same as for the
Section
in Example 6.5.1

RESULTS:

Area (B)		=	3900.0
Centroid (C)	x_c	=	53.72
	y_c	=	146.28
Inertia matrix (D)	I_{cc}	=	$10^7 \begin{bmatrix} 1.548 & 0.926 \\ 0.926 & 1.548 \end{bmatrix}$
(E)	I_{cc}	=	$10^7 \begin{bmatrix} 0.622 & 0.0 \\ 0.0 & 2.473 \end{bmatrix}$
(F)	θ	=	-45^o

6.7.3 Thin section angle

coordinates

nodes	x-y values	
1	0.0	0.0
2	0.0	195.0
3	195.0	195.0

topology-thickness

member	node numbers		t
1	1	2	10
2	2	3	10

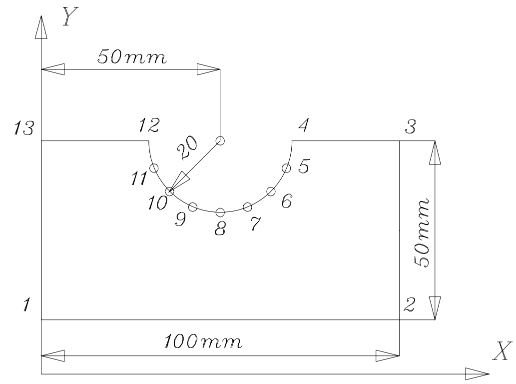

Figure 6.21: Rectangle with semi-circular cutout

Command sequence

LOADR A R=3 C=2
(x-y coordinates)
LOADR M R=2 C=3
(Topology and thickness)
THSECT A M
PROPER A B C D
(same commands as
in example 6.5.1

RESULTS:

Area (B)		=	3900.0
Centroid (C)	x_c	=	48.75
	y_c	=	146.25
Inertia matrix (D)	I_{cc}	=	$10^7 \begin{bmatrix} 1.544 & 0.927 \\ 0.927 & 1.544 \end{bmatrix}$
(E)	I_{cc}	=	$10^7 \begin{bmatrix} 0.618 & 0.0 \\ 0.0 & 2.472 \end{bmatrix}$
(F)	θ	=	-45^o

6.7.4 Example: section properties of rectangle with semi-circular cut out

The beam cross section is shown in Figure 6.21. The semi-circular cutout has a radius of 20 mm and its perimeter will be divided into 8 equal arcs as shown in Figure (6.18). A moment of $M_x = 1000$ will be applied and the stresses σ_z calculated at the node points.

Command sequence

LOADR A R=13 C=2

0 0	CIRCOR	A1	A2 R=9 C=2 L=1,1	Stresses σ_z	
100 0	STOSM	A	A2 L=4,1	1	2.73
100 50	PERIM	A	M=1 N=13 S=1	2	2.73
0 0	PROPER	A	B C D	3	-3.28
0 0	PRINT	B		4	-3.28
0 0	(area=4387.			12	-3.28
0 0	PRINT	C		13	-3.28
0 0	($\bar{x} = 50.0$ $\bar{y} = 22.68$)				
0 0	STRESS	A	G F=0 M=0.0,100000.0		
0 0	PRINT	G			
0 0					
0 0					
0 50					

6.7.5 Example of shear flow distribution in thin walled sections

The following section describes the procedure for calculating shear flow distributions in thin walled open sections and locating the shear centre. These routines use the basic commands already developed for thin walled sections, together with an additional shear flow command. To start, the basic data, coordinates and element topology is first stored in the data base with the commands:

LOADR A R=? C=2
(coordinates)
LOADR M R=? C=3
(node numbers I-J, thickness t)

The command:

THSECT A M

then calculates area, first moments about $(x-y)$ axes and the inertia matrix all referenced to the initial coordinate axes. Now the command,

PROPER A B C E

calculates coordinates C of the centroid, area of the cross section B and the inertia matrix E about the centroidal axes. Coordinates in A are now referenced to the centroidal axes. The flexibility matrix of the cross section is then calculated in S using,

INVERT E T=1

Finally the shear flow distributions for $V_x = 1$, $V_y = 1$, are calculated and printed with the commands,

SHFLOW A M C E S
PRINT S

The shear centre coordinates from the centroid are given in the array, SC, and are displayed

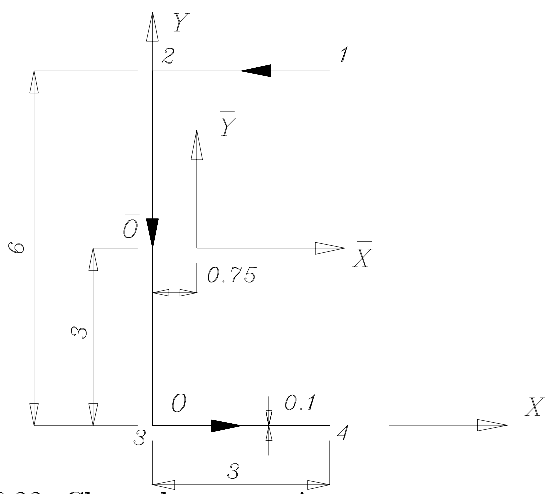

Figure 6.22: Channel cross section

by the print command,

PRINT SC.

An example is given of the calculation of the calculation of shear distributions for a channel section shown in Figure 6.22. From the figure the coordinates and topology are,

coordinates		
nodes	x-y values	
1	3.0	6.0
2	0.0	6.0
3	0.0	0.0
4	3.0	0.0

topology-thickness			
member	node numbers		t
1	1	2	0.1
2	2	3	0.1
3	3	4	0.1

Computer command sequence

```
LOADR A R=4 C=2
3 6
0 6
0 0
3 0
LOADR M R=3 C=3
1 2 0.1
2 3 0.1
3 4 0.1
THSECT A M
PROPER A B C E
INVERT E T=1
SHFLOW A M C E S
PRINT S
PRINT SC
```

The calculations give the following results, for $V_x = 1, V_y = 1$.

		$V_x = 1$	$V_y = 1$
	R	0.0	0.0
1	C	-0.2	-0.0625
	L	-0.2	-0.125
	R	-0.2	-0.125
2	C	0.0	-0.1875
	L	0.2	-0.125
	R	0.2	-0.125
3	C	0.2	-0.0625
	L	0.0	0.0

Location of the shear centre from $\bar{0}$ = -1.875in,

6.8 Section properties and stresses in beams

6.8.1 Hollow section

A typical section is shown in Figure 6.15. The command sequence and explanations are given below:

LOADR	A	R=?	C=2		load all $x - y$ coordinates of corner nodes in sequential order
PERIM	A	M=1	N=12	S=1	Traverse outside perimeter
PERIM	A	M=13	N=16	S=−1	Traverse inside void
PROPER	A	B	C	D	calculate section properties relative to the centroidal axes A=coordinates referenced to centroid B=area C=coordinates of centroid from origin D=second moment of area matrix about centroidal axes
STRESS	A	G	F=?	M=?,?	F=axial force $M = M_y, M_x$, moments positive as shown G=axial stress values at all node points
PRINC	E	H			Calculates the principal inertia marix E and angle(H) to the x axis

6.9 Section properties module

The purpose of this teaching module is to give the student familarity with bending resistance and bending stress calculation in beams with various cross section types that may be met in engineering practice. The theory for the calculation of the properties of bema cross sections bounded by straight lines(and including internal cutouts), is given in Section 6.1, and for thin walled sections in Section 6.2 Portions of circular sections may be included in the perimeter by using the CIRCOR command to generate straight line segment approximations too the circular arc. The typical cross sections, included in these two groups that may be generated by STATIC-2020 are shown in Figures 6.8 and 6.11. The cross sections are defined in terms of a number of basic dimensions. Breadth (b) and depth (d) and web and flange thicknesses (t_1, t_2) respectively. Either metric or imperial units can be used in these exercises and appropriate force and moment units chosen to give the desired stress units. The sections given are simple shapes that are either efficient in the use of material for resisting bending moments or are in the second group used to model rolled or extruded thin walled sections. Other simple shapes are shown in Figures 6.21. the theory for the calculation of bending stresses in beam cross sections for bending moments about the coordinate (nonprincipal) axes is given in Section 6.2.3, see equation(6.37). This theory applies irrespective of whether the cross section has an axis of symmetry or the axes are principal axes. Some sections in Figures 6.8 and 6.11 can have axes of symmetry depending on the chosen dimensions. the units used in the two systems of measurement are:

(a) Metric system

1. Cross section dimensions in millimetres (mm)
2. Bending moment in Newton millimetres

Then stress is calculated in mega Pascals (mPa).

(b) Imperial system

1. Cross section in inches in.
2. Bending moment in inch pounds
3. Axial force in pounds (lb)

Then stresses are lb / square inch psi. If kips are used as the measure of force (1kip = 1000 lb) the stress is calculated in ksi.

It will be found that when millimetres are used the magnitudes of the numbers giving section properties may be large whereas when metres are used these numbers may be small. There are a number of aspects of the theory that should be introduced in the section property exercises.

1. The location of the centroid of the cross section.
2. The calculation of the second moment of area matrix and the nature of this matrix for crosss sections such as in 1,2,3, and 5 in Figure 6.8 where one of the $X - Y$ axes coincide with an axis of symmetry of the cross section.
3. The angle section 4 in the Figure 6.8 has a nonzero I_{xy}.
4. For nonsymmetric sections location of principal axes.

When the cross section properties command is executed, the location of the centroid, the area of the cross section and the second moment of the area matrix are always calculated. In the case of the section (4), the section still has an axis of symmetry if $b = d$, and this can be shown to coincide wuth one of the principal axes. After section properties of a section have been calculated, the calculation of axial stress σ_x for all cross section nodes can be carried out. It will be noted that the sum of various contributions from (F_X, M_y, M_x) are automatically given for each of the points. The same theory can then be studied for thin walled cross sections, Section 6.4 for which the thickness of the elements is much smaller that the overall dimensions. The theory developed in Section 6.2 can be applied to the calculation of shear flow in open cross sections and the location of the shear centre (centre of twist), see Section 6.4.

6.10 Section property exercises

Exercises S1 to S9 are based on the sections given in Figure 6.8 their input data is generated using the command,

See Figure 6.8 for definition of b, d, t_1, t_2. The dimensions given in exercises S2 to S9 will all refer to these values.

(S1) For cross sections (1) and (5) in Figure 6.8 there are two axes of symmetry. By inspection locate the centroid and the principal axes.

(S2) For sections (2), (3) and (4), $b = 100mm$, $d = 150mm$, $t_1 = t_2 = 10mm$. Use STATICS-2020 to calculate area, locate the centroid and determone the second moment of area matrix. For which sections is I_{xy} nonzero? Explain why this is so.

(S3) The sections (1) and (5) have $b = 100mm, d = 150mm, t_1 - 5mm, t_2 = 20mm$. Calculate the section properties,

1. Using STATICS-2020.
2. Using properties of solid section bd and subtracting off the properties of the voids.

(S4) For the section (4) in exercise (**S2**), use STATIC-2020 to calculate the principal values and the location of the principal axes.

6.10.1 Stress calculations

Axial stress due to axial force F and bending moments M_x, M_y are calculated using equation (6.37), $\sigma_x = \frac{F}{A} - [\bar{x}\ \bar{y}][I_{cc}]^{-1} \begin{Bmatrix} M_y \\ M_x \end{Bmatrix}$

This equation is used by STATICS-2020 in the stress calculation command STRESS. The exercises S5 to S9 illustrate its use for thick sections.

(S5) The sections (10 and (5) in Figure 6.8 have the same dimensions as given in S3. Moments $M_x = 100kNm, M_y = 100kNm$ are applied to these sections.

1. Calculate the bending stresses in each section. Determine at which points the maximum tension and compression stresses occur and their magnitudes in each case.
2. From (1) which section has the highest stresses and from the matrix $[I_{cc}]^{-1}$ explain why this is so.
3. If M_x only is applied plot the bending stresses for the cross section.
4. If the maximum allowable stress is $400mPa$ tension and compression, calculate the allowable values of M_x and M_y.

(S6) The section (4) in Figure 6.8 has the dimensions, $b = 100mm, d = 100mm, t_1 = t_2 = 5mm$.

1. Calculate the bending stresses for $M_x = 100kNm$.
2. What effect has been produced because $I_{xy} \neq 0$?

(S7) The section (2) in Figure 6.8 has the same dimensions as those in (**S6**).

1. Calculate bending stresses for $M_x = 100kNm$
2. Compare these stresses with those of the sections in (**S6**).
3. What is the ratio of the areas of the two sections?

(S8) The section (5) in Figure 6.8 has the dimensions as in (**S3**). A moment $M_x = 100kNm$ is applied to the section.

1. Calculate the maximum tensile stress.
2. An axial force is applied at the centroid. Calculate its magnitude if the tension stress in (1) is to be made equal to zero. Note: This force will be compressive.
3. Apply a compression force F to the cross section at an excentricity y_F producing an $M_x = -Fy_F$. Using equation (6.37), find the eccentricity y_F so that the maximum tensile stress is then zero.

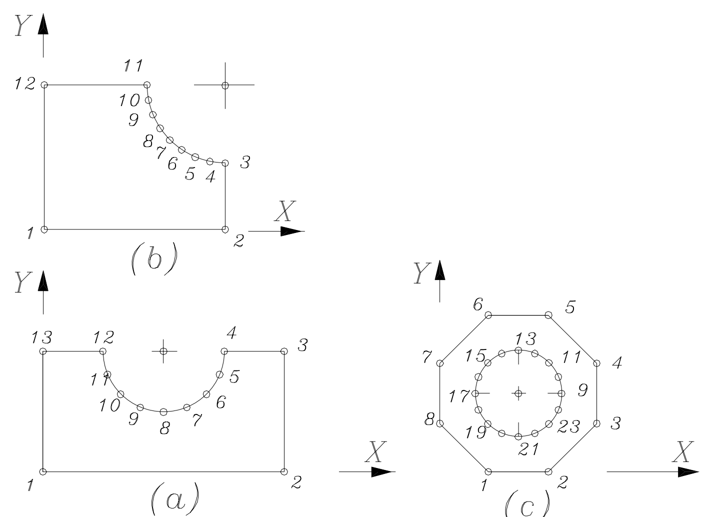

Figure 6.23: Solid sections with circular cutouts