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July 2006 Newsletter

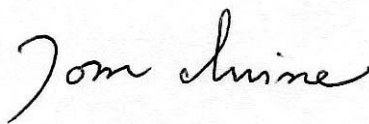
## Pax Vobiscum

Given the absence of a third choice, the Universe is either filled with some hypothetical dark matter or Isaac Newton's laws of motion must be modified for extraordinarily low acceleration rates. Such is the dilemma that astrophysicists have presented us, considering orbital data for stars about their respective galactic centers.

Most astronomers accept the dark matter theory along with that of dark energy. On the other hand, Mordehai Milgrom and his colleagues have proposed a modified Newtonian dynamics (MOND) theory. This theory provides a numerical explanation of star orbits, although it still needs a true theoretical foundation in modern physics.

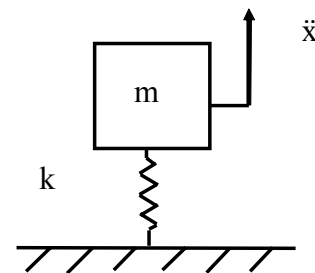
The purpose of this Newsletter is to set forth the idea that a terrestrial oscillator can be used to test the MOND theory. The results of this test could be an important step in resolving the validity of the MOND theory. I would be happy to collaborate with other researchers in performing these experiments.

Sincerely,



Tom Irvine  
Email: tomirvine@aol.com

## Feature Articles



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The material in this Newsletter may be freely used for scientific research as long as the author is acknowledged.

## **In Search of a Terrestrial Milgrom Oscillator**

By Tom Irvine

*A Milgrom Oscillator is an oscillator that demonstrates MOND at extraordinarily low acceleration levels.*

### Newton's Laws

Isaac Newton's second law of motion states that force is equal to the change in momentum per change in time. Momentum is mass times velocity.

The following equation thus applies for the force F acting on an object with constant mass:

$$F = m a \quad (1-1)$$

where

m is mass

a is acceleration

Newton's second law of motion accurately accounts for the respective orbits of the nine planets about the Sun. The law explains why Mercury only takes about 88 days to orbit the Sun, but faraway Pluto takes 90,550 days for its orbit.

This law is also readily verifiable using experiments with simple objects in a high school physics lab.

Furthermore, Newton's second law is the basis of vibration analysis. The natural frequencies of familiar objects such as a pendulum, tuning fork or wind chime can be derived from Newton's second law.

### Einstein's Relativity

Albert Einstein, however, developed the Special Theory of Relativity which showed that Newton's laws only apply if the velocity is substantially less than the speed of light, which is approximately  $3.0 \times 10^8$  km/sec in a vacuum.

*If Newton's laws are only applicable below some speed threshold, could there also be some lower limit of validity in terms of velocity or acceleration?*

As an aside, ordinary human experience occurs at speeds that are a tiny fraction of the speed of light. As a rather extreme example, astronauts traveling to the Moon must accelerate to a speed of 11.2 km/sec to escape the Earth's gravity. This is only 0.004% of the speed of light. As of July 2006, only twelve men have set foot on the Moon. Several others have flown around the Moon without landing.

A greater number of people have traveled at a supersonic speed either in a military aircraft or in the Concorde jet. Mach one at sea level, however, is only 0.0001% of the speed of light.



Figure 1-1. Spiral Galaxy NGC 1232

### Spiral Galaxies and Dark Matter

Consider stars in a spiral galaxy, orbiting about the galactic center. The orbital velocity of a given star should follow Newton's laws. Specifically, the speed should be proportional to the inverse square of the distance between the star and the center, as is the case with the nine planets orbiting about the Sun.

Surprisingly, the velocity tends to flatten out to a nearly constant value at large distances from the center. Thus a star in the middle part of the galaxy would have nearly the same speed as a star on the outer edge, as shown in Figure 1-2.

Astronomers have tried to resolve this discrepancy by assuming the existence of a halo of dark matter around every galaxy. Swiss astrophysicist Fritz Zwicky was the first to infer the existence of what came to be known as dark matter in 1933.

This hypothetical dark matter does not emit light but it does induce a gravitational pull.

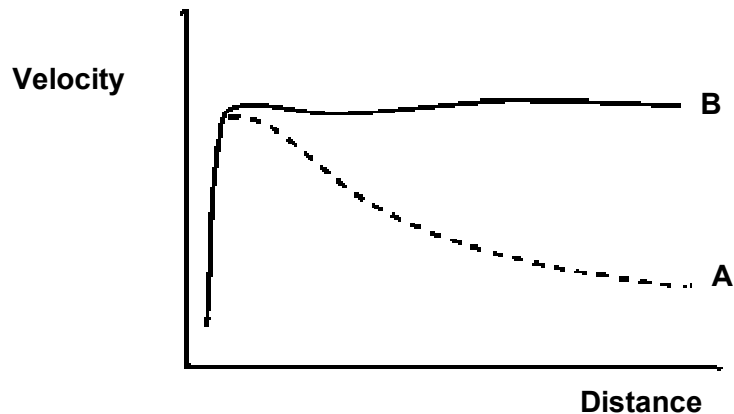


Figure 1-2. Typical Spiral Galaxy Rotation Curve  
(Image Courtesy of Wikipedia)

### Milgrom's MOND

Mordehai Milgrom is a physics professor in the department of Condensed Matter Physics at the Weizmann Institute in Rehovot, Israel.

His work was the subject of an article "Gravity's Gadfly" in Discover magazine, August 2006. He has also published numerous papers in scientific journals going back to 1983.

Milgrom has put forward the modified Newtonian dynamics (MOND) as a physical theory which attempts to explain the galaxy rotation problem by changing Newton's second law of motion. It is an alternative to the dark matter theory.

Milgrom has proposed

$$F = m\mu\left(\frac{a}{a_0}\right)a \quad (1-2)$$

where

$$\mu(x)=1 \text{ if } x \gg 1$$

$$\mu(x)=x \text{ if } x \ll 1$$

and

$$a_0 = 1.2e-10 \text{ meters/sec}^2$$

Thus,

$$F = m \frac{a^2}{a_0} \quad \text{for } a \ll a_0 \quad (1-3)$$

Again, MOND theory provides a numerical explanation of star orbits, but it still needs a true theoretical foundation in modern physics.

Furthermore, if equation (1-3) is correct, it must hold true for a mechanical oscillator on the Earth as well as for a star's orbit.

### Some Basic Equations

The following formulas are given to introduce the approach for the recommended tests in the following sections.

$$T = \frac{1}{f_n} \quad (1-4)$$

$$\omega_n = 2\pi f_n \quad (1-5)$$

$$A_{\max} = \omega_n^2 X_{\max} \quad (1-6)$$

where

- T is the period in seconds
- $f_n$  is the natural frequency in Hertz
- $\omega_n$  is natural frequency in rad/sec
- $A_{\max}$  is peak acceleration
- $X_{\max}$  is peak displacement

Equation (1-6) holds true for linear Newtonian dynamics. Nonlinear systems, whether Newtonian or otherwise, have more complicated relationships, however.

### Recommended Tests

An intermediate goal is to find and measure an acceleration amplitude much less than

$$a_0 = 1.2e-10 \text{ meters/sec}^2$$

An accelerometer with such a fine resolution may not exist.

On the other hand, displacement can be measured to very low values. The goal then is to find an oscillator with such an extraordinarily low frequency that it experiences measurable displacement but with a corresponding acceleration well below the  $a_0$  threshold. Note that an oscillator with a very low natural frequency would have a very long period.

The resulting measured displacement time history can then be differentiated twice to yield the corresponding acceleration time history. The resulting displacement and acceleration time histories can then be compared with both the Newtonian and Milgrom oscillator models, as discussed in the next article.

Three candidate test methods are discussed in the following sections. Each method uses either a man-made or a natural oscillator. In each case, the oscillator is idealized as an undamped, spring-mass system, as discussed in the next article. Each choice will have many technical hurdles, but these hurdles should be surmountable with dedication and persistence.

The final recommendation is to pursue all of these options in parallel.

### Man-made Oscillator

There are several possibilities for a man-made oscillator to test the MOND theory. The challenge is that the period should be at least one hour.

A highly modified Lehman seismometer might be the answer. This seismometer uses a horizontal pendulum in a “swinging gate” manner. Using some simple parts, I constructed a Lehman seismometer with a 14-second period as described in the November 2004 Newsletter.

Now assume that a Lehman seismometer with a 1-hour period could be devised, perhaps with a “very long” boom. The purpose of this seismometer would not be to measure earthquakes, however. Rather it would be given an initial displacement and allowed to vibrate freely.

The natural frequency would be 278 micro Hertz. A displacement of 10 microns peak-to-peak would yield an acceleration of  $1.52e-11$  meters/sec<sup>2</sup> per Newton’s law, which is less than the  $a_0$  threshold.



Figure 1-3. The Author's Lehman Seismometer

*I invite readers to submit ideas for their own "very long period" oscillators. Perhaps, NASA, USGS, Los Alamos or some university already has a suitable device.*

### Earth's Fundamental Vibration Mode

The Earth experiences seismic vibration. The Earth itself may be a suitable oscillator to test the MOND theory per the following example.

The fundamental natural frequency of the Earth is 309.286 micro Hertz. This is equivalent to a period of 3233.25 seconds, or approximately 54 minutes.

The frequency reference is:

T. Lay and T. Wallace, Modern Global Seismology, Academic Press, New York, 1995.

A typical amplitude is not immediately available.

Assume a displacement amplitude of 10 microns peak-to-peak. The corresponding acceleration per Newton's laws is  $1.885e-11$  meters/sec<sup>2</sup>, which is less than the  $a_0$  threshold.

## Seiche Method

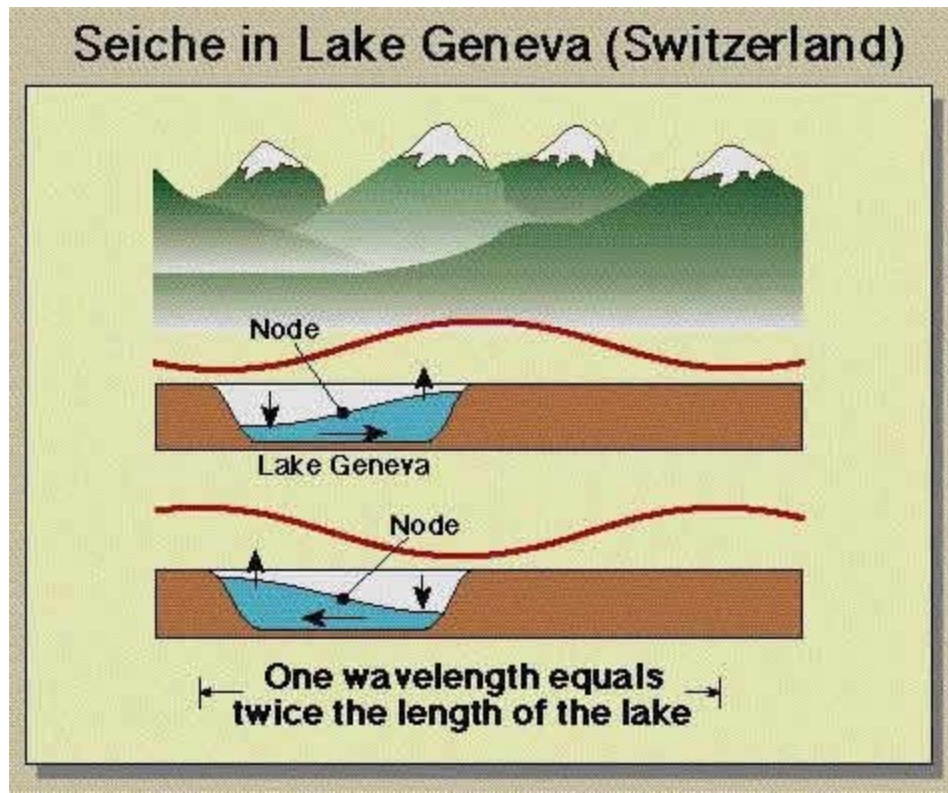


Figure 1-4. (Image courtesy of Professor Brennan, Geneseo State Univ. of New York)

A body of water with a very long seiche period may be a suitable oscillator for the MOND test. This body can rock back and forth in its basin. The water may be initially excited by wind or some other source.

The period of this oscillation depends on the basin's geometry. In a cylindrical tank, this rocking motion is called "slosh." In a natural basin, it is called a "seiche." This term is pronounced as "saysh." It is an old Swiss French word meaning "to sway."

There are a number of candidate bodies of water with very long periods, although Lake Geneva's period is too brief.



As an example, Cayuga Lake in New York has a seiche period of 60 hours per the following reference:

[http://test.utilities.cornell.edu/utl\\_lsceis\\_toc.html](http://test.utilities.cornell.edu/utl_lsceis_toc.html)

[http://test.utilities.cornell.edu/utl\\_lsceis\\_thermal.html#232132](http://test.utilities.cornell.edu/utl_lsceis_thermal.html#232132)

The corresponding natural frequency is 4.63 micro Hz. Assume that this a linear Newtonian value.

Now assume a 20 mm peak-to-peak (10 mm zero-to-peak) displacement. The corresponding acceleration per Newton's laws is  $8.46e-12$  meters/sec<sup>2</sup>, which is much less than the  $a_0$  threshold.

The water displacement measurements could be made over a period of perhaps weeks or months. The data should be lowpass filtered at a frequency of perhaps 20 micro Hz. This filtering can be performed as a post-processing step, but a suitable analog anti-aliasing filter should be used in the data collection.

*The best approach for evaluating terrestrial MOND would be a multidisciplinary effort involving seismologists, oceanographers, and mechanical engineers!*

## The Mathematics of a Milgrom Oscillator

By Tom Irvine

Consider a spring-mass system.

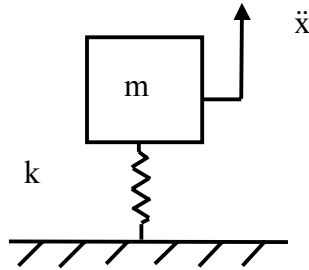


Figure 2-1.

where

- m is the mass
- k is the stiffness
- x is the displacement of the mass
- $\ddot{x}$  is the acceleration of the mass

Assume the following:

1. The system is undamped.
2. The system has linear stiffness.
3. The peak acceleration is  $\ll a_0$ .
4. The system obeys the hypothetical Milgrom equation for force at extraordinarily low acceleration levels.
5. The system is a single-degree-of-freedom system.
6. Require that displacement, velocity and acceleration must each be real in the time domain.
7. The system undergoes free vibration due to an initial displacement or velocity.
8. The acceleration must have the opposite polarity as displacement for a stable oscillation.

The free body diagram is

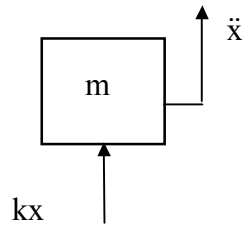


Figure 2-2.

Recall that Milgrom's equation for MOND is

$$F = m \frac{a^2}{a_0} \quad \text{for } a \ll a_0 \quad (2-1)$$

Apply MOND to the spring-mass system.

$$\sum F = \frac{m}{a_0} \ddot{x}^2 \quad (2-2)$$

$$\frac{m}{a_0} \ddot{x}^2 = -kx \quad (2-3)$$

The equation of motion is

$$\frac{m}{a_0} \ddot{x}^2 \text{sign}(\ddot{x}) + kx = 0 \quad (2-4)$$

The acceleration must be real and it must have the opposite polarity as displacement in equation (2-1). Thus the sign function is needed.

Note that for a variable  $y$

$$\text{sign}(y) = \frac{y}{|y|} \quad (2-5)$$

Divide through by mass.

$$\ddot{x}^2 \text{sign}(\ddot{x}) + \frac{k a_0}{m} x = 0 \quad (2-6)$$

Define a constant

$$\beta = \frac{k a_0}{m} \quad (2-7)$$

Note that  $\beta > 0$ .

$$\ddot{x}^2 \text{sign}(\ddot{x}) + \beta x = 0 \quad (2-8)$$

Again,

$$\text{sign}(\ddot{x}) = -\text{sign}(x) \quad (2-9)$$

Substitute equation (2-9) into (2-8).

$$-\ddot{x}^2 \text{sign}(x) + \beta x = 0 \quad (2-10)$$

$$\ddot{x}^2 \text{sign}(x) = \beta x \quad (2-11)$$

$$\ddot{x}^2 = \beta x \text{sign}(x) \quad (2-12)$$

$$\ddot{x} = \sqrt{\beta x \text{sign}(x)} \quad (2-13)$$

$$\ddot{x} = \sqrt{\beta|x|} \quad (2-14)$$

Again, the acceleration must have the opposite polarity as displacement for a stable oscillation. Thus, a  $\text{sign}(x)$  term is applied with some hand-waving.

$$\ddot{x} = -\text{sign}(x)\sqrt{\beta|x|} \quad (2-15)$$

The resulting equation is nonlinear. It lacks a closed-form solution. A numerical solution is thus required.

### State Space Model

The governing second-order ODE can be reduced to a pair of first-order ODEs.

$$\dot{v} = -\text{sign}(x)\sqrt{\beta|x|} \quad (2-16)$$

$$v = \dot{x} \quad (2-17)$$

The pair of equations can be solved numerically using the Runge-Kutta method via Matlab.

Note that the theoretical period  $\tau$  for this system is

$$\tau \approx 5.98 \left( \frac{u_m}{\beta} \right)^{1/4} \quad (2-18)$$

where  $u_m$  is the maximum displacement in terms of consistent units.

Equation (2-18) was derived using a method in

Weaver, Timoshenko, and Young; *Vibration Problems in Engineering*, Wiley-Interscience, New York, 1990.

### Example

Consider the Cayuga Lake seiche as a single-degree-of-freedom system. The fundamental frequency is 4.63 micro Hz, as discussed in the first article.

First solve for

$$\beta = \frac{k a_0}{m} \quad (2-19)$$

Explicit knowledge of the mass and stiffness is not required. Use the assumption from linear, Newtonian dynamics that

$$\omega_n = 2\pi f_n = \sqrt{\frac{k}{m}} \quad (2-20)$$

$$\beta = [2\pi f_n]^2 a_0 \quad (2-21)$$

The actual calculation of  $\beta$  is performed via Matlab for convenience. The result is

$$\beta = 1.016e-19 \text{ (m/sec}^4\text{)} \quad (2-22)$$

Assume a 10 mm zero-to-peak initial displacement, with zero initial velocity. The resulting displacement and acceleration are shown in Figures 2-3 and 2-4, respectively, for several cycles. The phase portrait is given in Figure 2-5.

Note that the Milgrom natural frequency is 9.0 micro Hz, as calculated by Matlab. The theoretical frequency per equation (2-18) is about 3% lower.

The calculated 9.0 micro Hz frequency is nearly double that from the linear Newtonian model, which was 4.63 micro Hz.

This shift has the undesired consequence of increasing the peak absolute acceleration to  $3.7e-11 \text{ m/sec}^2$  which is still  $< a_0$ , but perhaps not  $\ll a_0$ .

Detection of the frequency increase might be the best metric for evaluating MOND.

## Matlab Scripts

The Matlab scripts are;

Milgrom\_free\_function.m

Milgrom\_oscillator\_ode45.m

The scripts are posted at:

<http://www.vibrationdata.com/matlab.htm>

*Comments including suggestions for additions and corrections are welcome.*

*Further research is absolutely needed!*

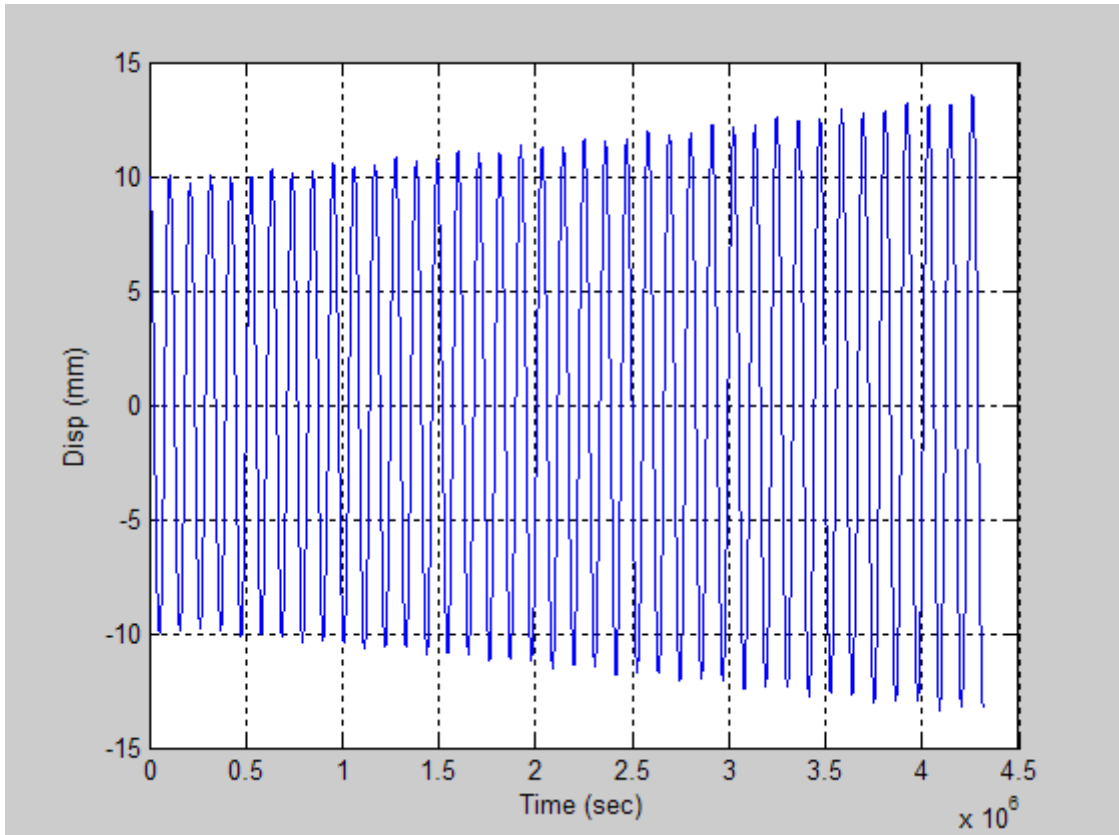


Figure 2-3. Displacement Time History



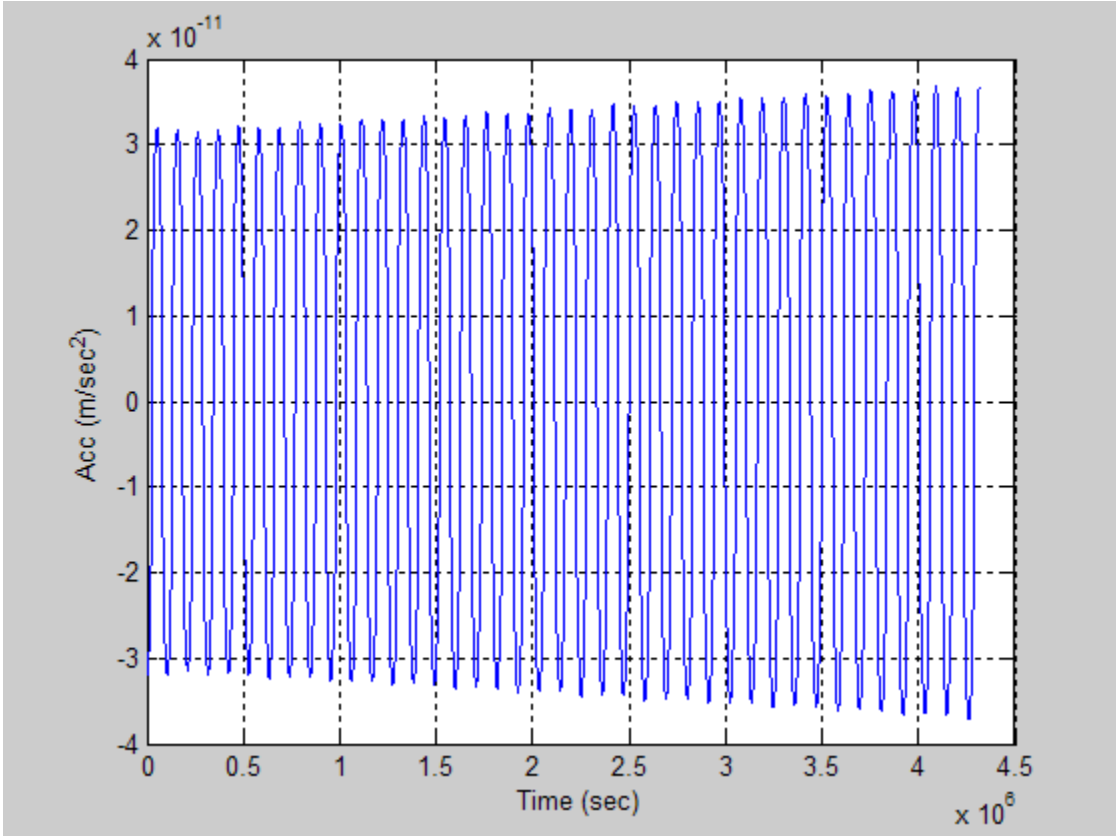


Figure 2-4. Acceleration Time History

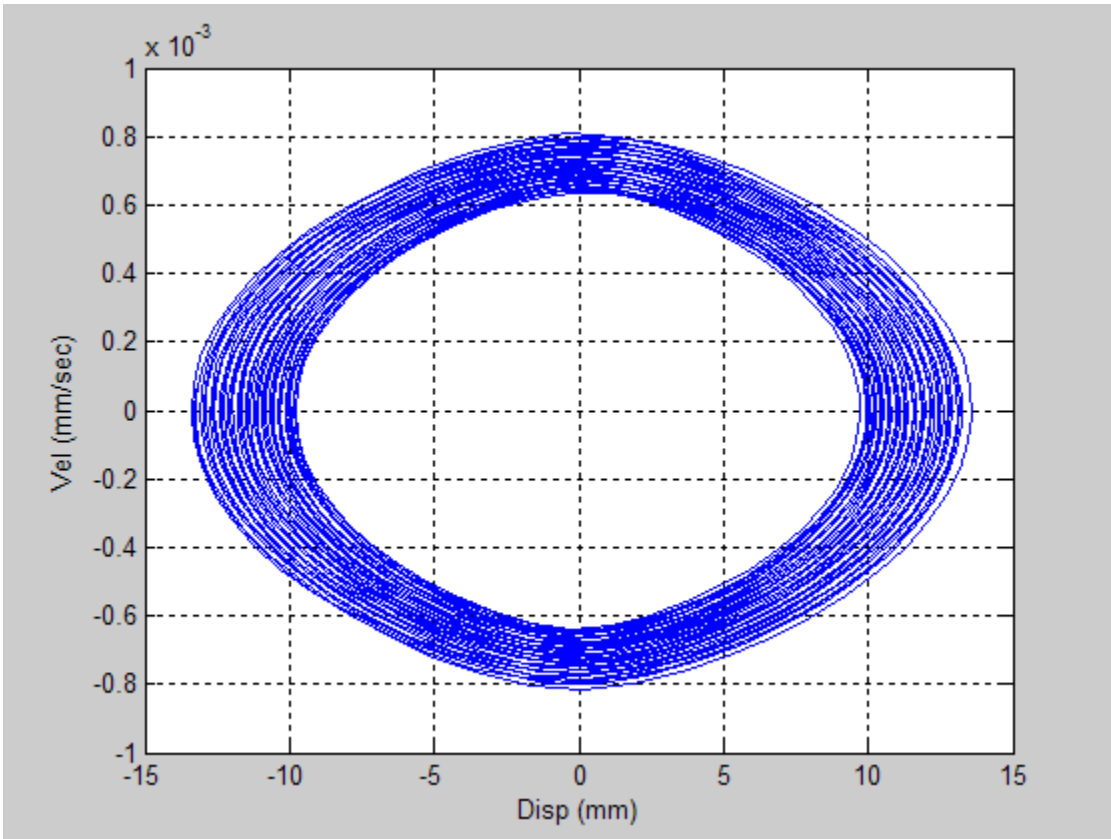


Figure 2-5. Phase Portrait

## Postscript

By Tom Irvine

### Milgrom, Abramovici, and Vager

Since initially releasing this Newsletter, I have corresponded with Professor Milgrom who directed me to several papers.

Professor Milgrom's colleagues published a paper with the following paragraph:

This modified dynamics seems to be consistent with many observed features of galaxies and galaxy clusters. Observational aspects of open clusters in the solar neighborhood indicate that Newtonian behavior is reestablished once the system moves in an external acceleration field larger than  $a_0$ , even if the system's internal movements proceed at accelerations smaller than  $a_0$ , which amounts to a violation of the strong equivalence principle (SEP). As a consequence, no deviation from Newton's second law would be detectable in earthbound laboratory experiments, since the apparatus is immersed in the strong gravitational field of the Earth. If, on the other hand, the partial restoration of Newtonian behavior in the neighboring open clusters is due to some, as yet, unsuspected mechanism, peculiar to these clusters, while the strong equivalence principle is preserved, laboratory experiments involving accelerations small than  $a_0$  should display deviations from  $F=ma$ .

Reference:

Abramovici and Vager, Test of Newton's Second Law at Small Accelerations, Physical Review D, Volume 34, Number 10, 15 November 1986.

These researchers then described a laboratory experiment which verified Newton's second law down to  $3e-09$  cm/sec<sup>2</sup>. This is equivalent to  $3e-11$  meters/sec<sup>2</sup>.

### Strong Equivalence Principle

There are several "equivalence principles" regarding gravity and motion.

An article in the Wikipedia encyclopedia describing these principles notes:

They are related to the Copernican idea that the laws of physics should be the same everywhere in the universe, to the equivalence of gravitational and inertial mass, and also to Albert Einstein's assertion that the gravitational "force" as experienced locally while standing on a massive body (such as the Earth) is actually the same as the *pseudo-force* experienced by an observer in a non-inertial (accelerated) frame of reference.

Furthermore, this reference describes the “strong equivalence principle” as

The outcome of any local experiment, whether gravitational or not, in a laboratory moving in an inertial frame of reference is independent of the velocity of the laboratory, or its location in spacetime.

The strong equivalence principle is related to Einstein's theory of general relativity.

Circumventing the strong equivalence principle may be a greater challenge than modifying Newtonian dynamics for Milgrom and his colleagues.

### Final Comments

I have profound respect for the work of physicists involved in all aspects of research, whether they are proponents of dark matter or MOND. Yet, there is simply no direct satisfactory explanation for the observed behavior of stars in spiral galaxies.

My suggested experiment with Cayuga Lake may or may not be an appropriate test of MOND, depending on the disposition of the strong equivalence principle.

I still consider that Cayuga Lake is potentially an excellent platform for extraordinarily low-amplitude, extraordinarily low-frequency vibration research. Given its 60 hour period, it may be Earth's ultimate natural seismometer, when equipped with proper instrumentation.

This lake may yet reveal serendipitous discoveries whether related to astrophysics or to other branches of physics, Earth sciences and engineering.