Solution of the Terminal Velocity Ordinary Differential Equation

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You bail out of a helicopter and pull the ripcord of your parachute such that your downward velocity satisfies the initial value problem:

$$dv/dt = 32 - 1.6v \quad \text{where } v(0) = 0.$$

Construct a slope field and sketch the solution curve. How long will it take you to reach 19 ft/second.

$$\frac{dv}{dt} = 32 - 1.6v \tag{1}$$

$$\frac{dv}{32 - 1.6v} = dt \tag{2}$$

$$\int_{v_1}^{v_2} \left[\frac{dv}{32 - 1.6v} \right] = \int_{t_1}^{t_2} dt \tag{3}$$

$$\int_{v_1}^{v_2} \left[\frac{dv}{32 - 1.6v} \right] = t_2 - t_1 \tag{4}$$

Let

$$u = 32 - 1.6v \tag{5}$$

$$du = -1.6\, dv \tag{6}$$

$$\frac{-1}{1.6} du = dv \tag{7}$$

$$\frac{-1}{1.6} \int_{u_1}^{u_2} \left[\frac{du}{u} \right] = t_2 - t_1 \tag{8}$$

$$\frac{-1}{1.6}\ln[u]\Big|_{u_1}^{u_2} = t_2 - t_1 \tag{9}$$

$$\frac{-1}{1.6}\ln[32-1.6v]\Big|_{v_1}^{v_2} = t_2 - t_1 \tag{10}$$

$$\frac{-1}{1.6}\{\ln[32-1.6v_2]-\ln[32-1.6v_1]\} = t_2 - t_1 \tag{11}$$

$$v_1 = 0 \tag{12}$$

$$t_1 = 0 \tag{13}$$

Let

$$t = t_2 \tag{14}$$

$$v = v_2 \tag{15}$$

$$\frac{-1}{1.6}\{\ln[32-1.6v]-\ln[32]\} = t \tag{16}$$

$$\frac{-1}{1.6}\left\{\ln\left[\frac{32-1.6v}{32}\right]\right\} = t \tag{17}$$

The problem statement can be solved using equation (17). Now solve for the velocity as a function of time.

$$\left\{\ln\left[\frac{32-1.6v}{32}\right]\right\} = -1.6t \tag{18}$$

$$\left[\frac{32-1.6v}{32}\right] = \exp[-1.6t] \tag{19}$$

$$32 - 1.6\mathrm{v} = 32\exp[-1.6\mathrm{t}] \tag{20}$$

$$-1.6\mathrm{v} = 32\{\exp[-1.6\mathrm{t}] - 1\} \tag{21}$$

$$\mathrm{v}(\mathrm{t}) = 20\{1 - \exp[-1.6\mathrm{t}]\} \tag{22}$$