Bolt Calculations: Equations

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Bolt preload

$$F = c\, A_t\, S_p$$

F = preload, A_t = tensile stress area, S_p = proof strength (approximately 85% of the yield strength).
$c = 0.75$ for connections requiring reuse, $c = 0.90$ for permanent connections (Shigley).
The product $A_t\, S_p$ is the proof load.

Tensile stress area, Unified inch threads

$$A_t = \frac{\pi}{4}\left(d - \frac{0.9743}{n}\right)^2$$

d = nominal (major) diameter, n = threads per inch. The app uses tabulated A_t values for UNC and UNF sizes #2 to 1-1/2 in; all 40 agree with this formula within 0.2%.

Units, bolt preload

English: A_t in in^2, S_p in ksi, $F = 1000\, c\, A_t\, S_p$ in lbf.
Metric: ISO metric threads, A_t in mm^2, S_p in MPa, $F = c\, A_t\, S_p$ in N.
Coarse M3 to M39 (20 sizes) and fine M8×1 to M39×3 (16 sizes), with A_t from ISO 898-1 (Ref. 12):

$$A_t = \frac{\pi}{4}\left(d - 0.9382\, P\right)^2 \qquad P = \text{pitch (mm)}$$

Bolt torque-preload relation

$$T = K\, D\, F_p$$

T = tightening torque, K = nut factor, D = nominal diameter, F_p = preload force.
K accounts for thread and under-head friction, thread geometry and pitch. The default is $K = 0.2$ (a common value for dry steel fasteners); lubricated fasteners have lower values. See SAND2008-0371, Appendix A, for nut factors. The app solves for T, F_p or D from the other three quantities.

Units, torque-preload

English: T in in-lbf, D in in, F_p in lbf.
Metric: T in N-m, D in mm (converted to m in the calculation), F_p in N.

Multiaxial stress: combined tension and shear (Ref. 6, Appendices A and B)

All loads are per bolt. P_s = external shear load, P_t = external tensile load,

SF = safety factor with respect to ultimate, F_{tu} = ultimate tensile strength,

A_t = tensile stress area, A_s = thread area for shear, k_s = shear factor (0.6, Ref. 11).

$$F_{ut} = F_{tu}\,A_t \qquad F_{us} = k_s\,F_{tu}\,A_s \qquad \sigma = \frac{P_t}{A_t} \qquad \tau = \frac{P_s}{A_s}$$

F_{ut}, F_{us} = ultimate tension and shear capabilities of the bolt.

Appendix A: elliptical interaction (Refs. 6 to 8)

$$\left[SF\,\frac{P_s}{F_{us}}\right]^3 + \left[\frac{F_b}{F_{ut}}\right]^2 \le 1 \qquad P_b = \frac{K_b\,P_t}{K_b + K_m} \qquad P_0 = \frac{F_i}{1 - C}$$

$$F_b = F_i + SF\,P_b \quad (SF\,P_t < P_0) \qquad F_b = SF\,P_t \quad (SF\,P_t \ge P_0)$$

P_b = portion of P_t carried by the bolt, F_i = preload, K_b, K_m = bolt and joint stiffness, $C = K_b/(K_b + K_m)$.

When the factored load $SF\,P_t$ reaches the separation load P_0, the joint gaps and the bolt carries all of $SF\,P_t$.

With $F_i = 0$ and $K_m = 0$ this is eq. (24) of Ref. 6: $(SF\,\tau/\tau_{ult})^3 + (SF\,\sigma/\sigma_{ult})^2 \le 1$.

Maximum SF: the app solves left side = 1 for SF numerically.

Appendix B: quadratic interaction (Refs. 6, 9, 10)

$$\left(SF\,\frac{\tau}{\tau_{ult}}\right)^2 + \left(SF\,\frac{\sigma}{\sigma_{ult}}\right)^2 \le 1 \qquad \sigma_{ult} = F_{tu} \qquad \tau_{ult} = k_s\,F_{tu}$$

Equivalently $(\tau/\tau_{ult})^2 + (\sigma/\sigma_{ult})^2 \le 1/SF^2$, which is 0.64 for $SF = 1.25$.

$$SF_{max} = 1 \Big/ \sqrt{(\tau/\tau_{ult})^2 + (\sigma/\sigma_{ult})^2}$$

Note: eq. (B-2) of Ref. 6 gives a limit of 0.80 ($= 1/SF$) for $SF = 1.25$. The app applies SF to both stresses, consistent with eq. (24) and Appendix A, which gives the limit $1/SF^2$.

Units, multiaxial stress

English: loads lbf, areas in^2, F_{tu} and stresses ksi, stiffness lbf/in.

Metric: loads N, areas mm^2, F_{tu} and stresses MPa, stiffness N/mm. Only $K_b/(K_b + K_m)$ is used.

Example, Ref. 6: 10-32 cap screw

$P_s = P_t = 1333$ lbf, $A_t = 0.020$ in^2, $A_s = 0.0175$ in^2, $F_{tu} = 180$ ksi, $k_s = 0.6$, $SF = 1.25$

Appendix A with $F_i = 0$, $K_m = 0$: interaction = 0.90 (eq. 25), maximum $SF = 1.30$.

Appendix B: interaction = 0.99, maximum $SF = 1.26$.

Joint gapping (separation) (Ref. 1)

Loads are per bolt. F_i = preload, P = external tensile load, k_b, k_m = bolt and member stiffness.

$$C = \frac{k_b}{k_b + k_m} \qquad F_b = F_i + C\,P \qquad F_m = F_i - (1 - C)\,P$$

C = joint constant (fraction of P carried by the bolt), F_b = bolt load, F_m = member clamping load.

The joint separates when $F_m = 0$. The separation load and the load factor against separation are:

$$P_0 = \frac{F_i}{1 - C} \qquad SF_{sep} = \frac{P_0}{P} = \frac{F_i}{(1 - C)\,P}$$

After separation the bolt carries all of P ($F_b = P$).

Preload needed for a required factor: $F_i \geq SF_{req}\,(1 - C)\,P$.

The required SF_{sep} is set by the governing project standard (see Ref. 14). The app default is 1.25.

Thread pullout (stripping)

$$P_s = S_{us}\,A_s \qquad S_{us} = \text{ultimate shear strength of the thread material being checked}$$

Thread shear (stripping) areas for Unified threads, n = threads per inch, L_e = engagement length (Ref. 13):

$$\text{External:}\quad A_s = \pi\,n\,L_e\,K_{n,max}\left[\frac{1}{2n} + 0.57735\,(E_{s,min} - K_{n,max})\right]$$

$$\text{Internal:}\quad A_n = \pi\,n\,L_e\,D_{s,min}\left[\frac{1}{2n} + 0.57735\,(D_{s,min} - E_{n,max})\right]$$

$K_{n,max}$ = max. minor dia. of internal thread, $E_{s,min}$ = min. pitch dia. of external thread,

$D_{s,min}$ = min. major dia. of external thread, $E_{n,max}$ = max. pitch dia. of internal thread.

Diameter limits are from the ASME B1.1 tables for the thread class (Ref. 3); 0.57735 = tan 30°.

These are geometric values; effective areas are somewhat less (Ref. 13). Metric: n = 1/pitch, dimensions in mm.

Threads strip when the bolt load reaches P_s. Using F_b above, the external load at failure is

$$P_{fail} = \frac{P_s - F_i}{C} \;\text{ if this is} \leq P_0, \quad \text{otherwise}\;\; P_{fail} = P_s \qquad SF_{pull} = \frac{P_{fail}}{P}$$

The second case means the joint gaps before the threads strip. If $P_s \leq F_i$ the threads fail at preload.

Bolt pattern loads (Refs. 1, 6, 7)

Rigid mounting plate, linear elastic bolts, joint remains clamped (centroid method). Loads are per bolt.
m = mass, (G_x, G_y, G_z) = acceleration in G at the CG, (x_c, y_c) = CG plan position, h = CG height.

$$\mathbf{F} = m\,g\,(G_x,\ G_y,\ G_z) \qquad \mathbf{M} = (x_c - \bar{x},\ y_c - \bar{y},\ h) \times \mathbf{F}$$

English: m in lbm and $g = 1$ give lbf. Metric: m in kg and $g = 9.80665$ m/s^2 give N.
Bolt coordinates x_i, y_i below are measured from the pattern centroid $(\bar{x}, \bar{y})$.

$$P_{t,i} = \frac{F_z}{n} + \alpha\,x_i + \beta\,y_i \qquad \sum P_{t,i}\,y_i = M_x \qquad \sum P_{t,i}\,x_i = -M_y$$

$$\mathbf{P}_{s,i} = \frac{(F_x,\ F_y)}{n} + \frac{M_z}{J}\,(-y_i,\ x_i) \qquad J = \sum (x_i^2 + y_i^2)$$

For two rows of bolts this reduces to $R = Fh/(2a)$ per row, as in Ref. 6. Negative P_t means preload relief only.
Send passes the maximum P_t and P_s to the stress, gapping, pullout and fatigue tabs.

Joint stiffness (Ref. 1)

$$k_b = \frac{A_d\,A_t\,E_b}{A_d\,l_t + A_t\,l_d} \qquad A_d = \frac{\pi d^2}{4} \qquad l_d = l - l_t$$

l = grip; bolt and nut: $l = t_1 + t_2$; cap screw in a tapped hole: $l = t_1 + \min(t_2,\ d)/2$.
Members: two 30° frusta start at $D = 1.5\,d$ (washer face) and meet at mid-grip; each piece i has

$$k_i = \frac{\pi\,E_i\,d\,\tan 30^\circ}{\ln\left[\dfrac{(2t_i\tan 30^\circ + D_i - d)(D_i + d)}{(2t_i\tan 30^\circ + D_i + d)(D_i - d)}\right]} \qquad k_m = \left(\sum \frac{1}{k_i}\right)^{-1}$$

t_i, D_i, E_i = thickness, smaller diameter and modulus of each frustum piece; $C = k_b/(k_b + k_m)$.

Bolt fatigue (Ref. 1)

External load cycles between P_{min} and P_{max} on a preloaded bolt. Fatigue is checked at both preload limits, $F_{i,max}$ and $F_{i,min}$ (from Torque Scatter). The bolt load is

$$F_b = F_i + C\,P \quad (P < P_0) \qquad F_b = P \quad (P \geq P_0) \qquad P_0 = \frac{F_i}{1 - C}$$

After separation the bolt carries all of P. The stresses over the cycle are

$$\sigma_i = \frac{F_i}{A_t} \qquad \sigma_a = \frac{F_b(P_{max}) - F_b(P_{min})}{2\,A_t} \qquad \sigma_m = \frac{F_b(P_{max}) + F_b(P_{min})}{2\,A_t}$$

Clamped ($P_{max} < P_0$): $\sigma_a = C\,(P_{max} - P_{min})/(2\,A_t)$ and the load line starts at σ_i on the mean-stress axis. Separated ($P_{max} \geq P_0$): the preload is lost over part of the cycle and the load line starts at the origin.

$$\text{Clamped: } n_f = \frac{S_e\,(S_{ut} - \sigma_i)}{S_{ut}\,\sigma_a + S_e\,(\sigma_m - \sigma_i)} \quad \text{Separated: } n_f = \frac{1}{\sigma_a/S_e + \sigma_m/S_{ut}}$$

$$n_p = \frac{S_p\,A_t}{F_b(P_{max})} \quad \text{at } F_{i,max}$$

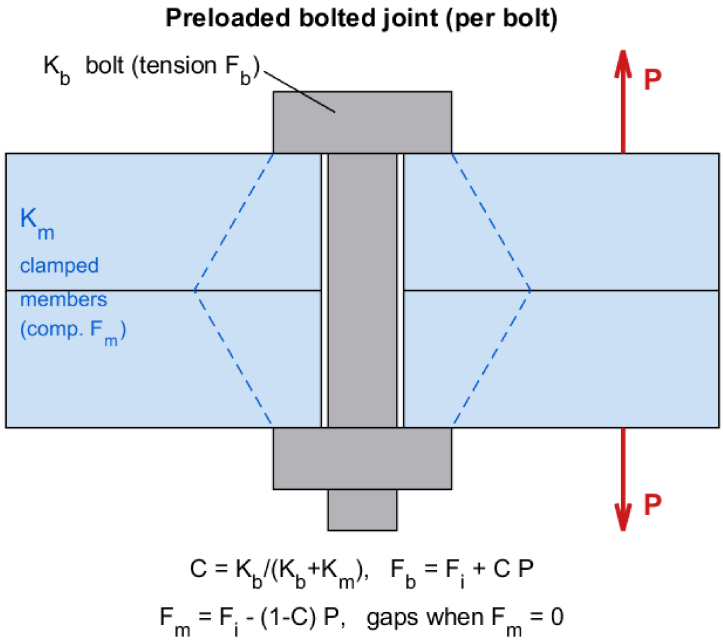
The app reports the lower n_f of the two preload limits. While the joint stays clamped, σ_a does not depend on preload, so $F_{i,max}$ governs through its higher mean stress. If the joint separates at $F_{i,min}$, σ_a can rise by up to a factor of $1/C$ and $F_{i,min}$ may govern; the app flags separation. With $F_{i,min}$ blank, one preload is checked. S_e = fully corrected endurance strength of the threaded bolt (enter the value for the bolt grade and thread manufacture, e.g. rolled threads, from Ref. 1 or test data), S_{ut} = ultimate strength, S_p = proof strength. n_p is the yielding (proof) factor of safety, at $F_{i,max}$ where the bolt load is highest. The app plots both operating points on the Goodman diagram in a separate window.

Torque scatter (Refs. 1, 4)

$$F_i = \frac{T}{K\,D} \qquad F_{i,max} = (1 + u)\,\frac{T_{max}}{K\,D} \qquad F_{i,min} = (1 - u)\,(1 - r)\,\frac{T_{min}}{K\,D}$$

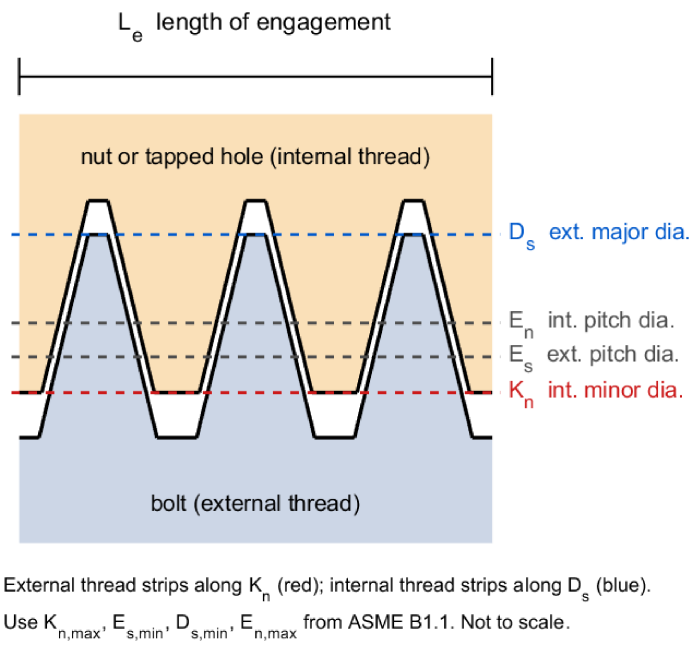
T_{max}, $T_{min} = T\,(1 \pm t_{tol})$, u = preload uncertainty of the tightening method, r = relaxation loss. Preload accuracy by method (Ref. 4, adapted from Barrett, MIL-HDBK-60 and Machinery's Handbook): by feel ±35%, torque wrench ±25%, turn-of-nut ±15%, load-indicating washer ±10%, bolt elongation ±3 to 5%, strain gages ±1%, ultrasonic sensing ±1%. Metric: T in N-m, D in mm. Send uses $F_{i,min}$ for joint gapping, $F_{i,max}$ for the multiaxial stress and pullout tabs, and both for fatigue.

Bolted joint model (Multiaxial Stress and Joint Gapping tabs; Ref. 1)



The external load P is shared between the bolt (fraction C) and the clamped members (fraction $1 - C$).

Thread nomenclature (Thread Pullout tab; Refs. 3, 13)



Diameter limits $K_{n,max}$, $E_{s,min}$, $D_{s,min}$, $E_{n,max}$ come from the ASME B1.1 tables for the thread class. The app plots the joint load diagram (Joint Gapping tab) and the interaction curves (Multiaxial Stress tab) in separate windows when Calculate is pressed.

NASA-STD-5020B fastener margins (Ref. 14)

Limit loads P_{tL} (tension) and P_{sL} (shear) on one fastener. Preload at assembly and in service:

$$P_{pi,max} = c_{max}\,(1+\Gamma)\,P_{pi,nom} \qquad P_{pi,min} = c_{min}\,(1-\Gamma)\,P_{pi,nom}$$

$$P_{p,max} = P_{pi,max} + P_{\Delta t,max} \qquad P_{p,min} = P_{pi,min} - P_{pr} - P_{pc} - P_{\Delta t,min}$$

Γ = preload variation of the tightening method, c_{max}, c_{min} = torque tolerance factors, P_{pr} = relaxation (blank: 5% of $P_{pi,min}$), P_{pc} = creep loss, $P_{\Delta t,max}$, $P_{\Delta t,min}$ = thermal gain and loss (Preload Loss tab).
Allowables, A_t = tensile stress area, A_m = minor diameter area:

$$P_{tu} = F_{tu}\,A_t \qquad P_{ty} = F_{ty}\,A_t$$

$$P_{su} = \frac{\pi}{4}\,D^2 F_{su}\ \text{(body)} \qquad P_{su} = F_{su}\,A_m\ \text{(threads)}$$

Linear joint theory, n = load introduction factor, ϕ = joint stiffness factor (joint constant):

$$P'_{sep} = \frac{P_{p,max}}{1-n\phi} \qquad P'_{tu} = \frac{P_{tu} - P_{p,max}}{n\phi} \qquad P'_{ty} = \frac{P_{ty} - P_{p,max}}{n\phi}$$

Tension capacity at ultimate: P_{tu} if the joint separates first ($P'_{sep} < P'_{tu}$), otherwise P'_{tu}. Yield: P_{ty} or P'_{ty}.
Margins of safety, FF = fitting factor, FS = factor of safety:

$$MS = \frac{\text{capacity}}{FF\,FS\,P} - 1 \qquad MS_{slip} = \frac{\mu\,P_{p,min}}{FF\,FS_s\,(P_{sL} + \mu\,P_{tL})} - 1$$

Tension ultimate and yield: capacity above on P_{tL} with FS_u and FS_y. Shear ultimate: P_{su} on P_{sL} with FS_u.
Separation: $P_{p,min}$ on P_{tL} with FS_{sep}. Joint slip: friction coefficient μ and FS_s.
Tension and shear interaction at ultimate, without bending, P^*_{tu} = tension capacity at ultimate:

$$R_t = \frac{FF\,FS_u\,P_{tL}}{P^*_{tu}} \qquad R_s = \frac{FF\,FS_u\,P_{sL}}{P_{su}} \qquad (xR_s)^a + (xR_t)^b = 1 \qquad MS = x - 1$$

Shear plane in the body: $a = 2.5$, $b = 1.5$. In the threads: $a = 1.2$, $b = 2$. The app lists all six margins, reports the lowest, and plots them in a separate window. English: lbf, ksi, in (×1000 for ksi·in^2). Metric: N, MPa, mm.
Flags: $P_{p,min} \le 0$ (no preload remains), $P_{p,max} \ge P_{ty}$ (yield at assembly).

Preload loss: embedding, creep and temperature (Refs. 14, 15)

Embedding (settling) of the thread and bearing surfaces, VDI 2230 guide values in μm (Ref. 15):

$$f_Z = f_{Z,thread} + n_b\, f_{Z,face} + N_i\, f_{Z,inner}$$

$n_b = 2$ (bolt and nut) or 1 (tapped hole), N_i = number of inner interfaces. Guide values by roughness R_z, thread / face / inner interface. Axial load: $R_z < 10\,\mu$m 3 / 2.5 / 1.5, 10 to 40 3 / 3 / 2, 40 to 160 3 / 4 / 3. Transverse load: $R_z < 10\,\mu$m 3 / 3 / 2, 10 to 40 3 / 4.5 / 2.5, 40 to 160 3 / 6.5 / 3.5.

$$k_s = \frac{K_b\, K_m}{K_b + K_m} \qquad F_Z = f_Z\, k_s \qquad F_{cr} = \frac{c_r}{100}\, F_{i,min} \qquad \Delta F = (\alpha_m - \alpha_b)\, \Delta T\, L\, k_s$$

K_b, K_m = bolt and joint stiffness, c_r = creep loss (%), α_b, α_m = expansion coefficients of bolt and joint, L = grip length, ΔT = temperature change from assembly (hot and cold). A joint that expands more than the bolt gains preload. f_Z is converted from μm to in ($\div$25400) or mm ($\div$1000).

$$P_{\Delta t,max} = \max(0,\, \Delta F_{hot},\, \Delta F_{cold}) \qquad P_{\Delta t,min} = \max(0,\, -\Delta F_{hot},\, -\Delta F_{cold})$$

$$P_{p,max} = F_{i,max} + P_{\Delta t,max} \qquad P_{p,min} = F_{i,min} - F_Z - F_{cr} - P_{\Delta t,min}$$

The app plots preload against temperature change in a separate window.

Shear joint (Refs. 8, 14, 20)

Shear load P_{sL}, tension P_{tL}, m = number of shear planes (faying surfaces), μ = friction coefficient.

$$MS_{slip} = \frac{\mu\, m\, P_{p,min}}{FF\, FS_s\, (P_{sL} + \mu\, m\, P_{tL})} - 1 \qquad P_{p,req} = \frac{FF\, FS_s\, (P_{sL} + \mu\, m\, P_{tL})}{\mu\, m}$$

$$P_{su} = m\, F_{su}\, A \qquad P_{bru} = F_{bru}\, D\, t \qquad P_{so} = 2\, t\, (e - D/2)\, F_{su,p}$$

$$MS = \frac{P_{su},\, P_{bru} \text{ or } P_{so}}{FF\, FS_u\, P_{sL}} - 1$$

$P_{p,req}$ = preload for zero slip margin. $A = \pi D^2/4$ (body in the shear plane) or A_m (threads), F_{bru} = plate bearing strength, t = thickness of the thinnest part carrying P_{sL}, e = edge distance to the hole center, $F_{su,p}$ = plate shear strength (two shear-out planes of length $e - D/2$). $e/D < 1.5$ is flagged. A check is skipped when its data are blank. If the joint slips, the bolts bear in shear: check fatigue and self-loosening.

Eccentric shear, bolt group (Refs. 7, 19)

In-plane load $P = (P_x,\, P_y)$ at $(x_p,\, y_p)$ on n bolts at $(x_i,\, y_i)$ with centroid $(\bar{x},\, \bar{y})$, $d_{x,i} = x_i - \bar{x}$, $d_{y,i} = y_i - \bar{y}$.

$$J = \sum_i \left(d_{x,i}^2 + d_{y,i}^2\right) \qquad M = (x_p - \bar{x})\, P_y - (y_p - \bar{y})\, P_x \qquad e = \frac{|M|}{|P|}$$

Elastic method: equal direct shear, plus a torsion load proportional to the distance from the centroid.

$$R_{x,i} = \frac{P_x}{n} - \frac{M}{J}\, d_{y,i} \qquad R_{y,i} = \frac{P_y}{n} + \frac{M}{J}\, d_{x,i} \qquad C_{el} = \frac{|P|}{\max_i R_i}$$

Instantaneous center (IC) method, with the Crawford-Kulak bolt load-deformation curve:

$$\frac{R_i}{R_{ult}} = \left(1 - \exp(-10\,\Delta_i)\right)^{0.55} \qquad \Delta_i = 0.34\, \frac{r_i}{r_{max}}\ \text{(in)} \qquad C = \frac{\sum_i R_i\, r_i}{R_{ult}\, \ell}$$

r_i = distance of bolt i from the IC, ℓ = moment arm of P about the IC. The IC is found from force and moment equilibrium, starting from the elastic center of rotation. C = capacity in units of R_{ult}: $P_n = C\, R_{ult}$, required $R_{ult} = |P|/C$. A concentric load gives $C = 0.98\, n$ (all bolts at Δ_{max}).

Self-loosening (Refs. 15, 16)

60° thread, pitch p, pitch diameter d_2 (blank: $D - 0.6495\, p$), bearing friction diameter D_{km} (blank: 1.3 D), thread and bearing friction coefficients μ_t, μ_b, minimum preload $F_{i,min}$.

$$\varphi = \tan^{-1} \frac{p}{\pi\, d_2} \qquad \rho' = \tan^{-1} \frac{\mu_t}{\cos 30^\circ} \qquad \text{self-locking if } \rho' > \varphi$$

$$T_{tight} = F_{i,min} \left[\frac{d_2}{2} \tan(\varphi + \rho') + \mu_b\, \frac{D_{km}}{2}\right]$$

$$T_{loose} = F_{i,min} \left[\frac{d_2}{2} \tan(\rho' - \varphi) + \mu_b\, \frac{D_{km}}{2}\right] \qquad T_p = F_{i,min}\, \frac{p}{2\pi}$$

T_p = helix (pitch) part of the torque, which acts to loosen the bolt. Torque in in-lbf, or N-m (N-mm ÷ 1000). Junker: the bolt loosens when the joint slips transversely. F_Q = transverse load, q = number of interfaces:

$$MS_{slip} = \frac{\mu_T\, q\, F_{i,min}}{FS\, F_Q} - 1 \qquad F_{i,req} = \frac{FS\, F_Q}{\mu_T\, q}$$

$F_{i,req}$ = preload for zero margin. A negative margin means a high risk of self-loosening: raise the preload, add friction or shear pins, or use a locking feature.

Random vibration fatigue (Refs. 17, 18)

Input: a stress PSD, or a load PSD carried by the bolt while the joint is clamped (C = joint constant).

$$G_S(f) = K_t^2\, G(f) \qquad G_S(f) = \left(\frac{C\, K_t}{A_t}\right)^2 G_P(f) \qquad m_k = \int f^k\, G_S(f)\, df$$

$$\sigma = \sqrt{m_0} \qquad \nu_0^+ = \sqrt{\frac{m_2}{m_0}} \qquad \nu_p = \sqrt{\frac{m_4}{m_2}} \qquad \gamma = \frac{m_2}{\sqrt{m_0\, m_4}} \quad x_m = \frac{m_1}{m_0}\sqrt{\frac{m_2}{m_4}}$$

K_t = stress concentration factor, σ = rms stress, ν_0^+ = rate of zero up-crossings, ν_p = rate of peaks, γ = irregularity factor. Moments are integrated on a log-log interpolated grid (psi^2/Hz to ksi^2/Hz).

S-N curve $N = A\, S_a^{-b}$, S_a = stress amplitude in ksi or MPa. Optional Goodman correction, $\sigma_m = F_i/A_t$:

$$g = \frac{1}{1 - \sigma_m / S_{ut}} \qquad D_{NB} = \frac{\nu_0^+\, T}{A}\, \left(\sqrt{2}\, \sigma\, g\right)^b\, \Gamma\left(1 + \frac{b}{2}\right)$$

$$D_{St} = \frac{\nu_0^+\, T}{A}\, (\sigma\, g)^b\, \left(0.683 + 0.271 \cdot 2^b + 0.0433 \cdot 3^b\right)$$

Narrowband: Rayleigh amplitudes. Steinberg three-band: 68.3%, 27.1% and 4.33% of the cycles at 1, 2 and 3 σ.

Dirlik, written in amplitude form with $Z = S_a/\sigma$:

$$D_1 = \frac{2\,(x_m - \gamma^2)}{1 + \gamma^2} \qquad R = \frac{\gamma - x_m - D_1^2}{1 - \gamma - D_1 + D_1^2} \qquad D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R}$$

$$D_3 = 1 - D_1 - D_2 \qquad Q = \frac{1.25\,(\gamma - D_3 - D_2\, R)}{D_1}$$

$$p(Z) = \frac{D_1}{Q}\, e^{-Z/Q} + \frac{D_2\, Z}{R^2}\, e^{-Z^2/2R^2} + D_3\, Z\, e^{-Z^2/2}$$

$$D_{DK} = \frac{\nu_p\, T}{A}\, (\sigma\, g)^b \left[D_1\, Q^b\, \Gamma(1 + b) + 2^{b/2}\, \Gamma\left(1 + \frac{b}{2}\right)\, \left(D_2\, |R|^b + D_3\right)\right]$$

D = Miner damage over the duration T, T/D = time to failure, $g = 1$ without the Goodman correction.

The app plots the stress PSD and the Dirlik and Rayleigh amplitude densities in separate windows.

For a nearly ideal narrowband PSD ($\gamma \to 1$) Dirlik is 0/0; the app uses the narrowband damage.

Torque table (Ref. 15)

Permissible assembly preload at a fraction ν of the yield strength S_y ($R_{p0.2}$), and the tightening torque, for each thread size. A_s = tensile stress area (ASME B1.1, ISO 898-1), P = pitch, μ_G = thread friction coefficient.

$$F_M = \frac{A_s\,\nu\,S_y}{\sqrt{1+3\left[c\,\frac{d_2}{d_0}\left(\frac{P}{\pi\,d_2}+1.155\,\mu_G\right)\right]^2}} \qquad d_2 = d - 0.6495\,P$$

$$M_A = F_M\left[0.16\,P + 0.58\,d_2\,\mu_G + \frac{D_{Km}}{2}\,\mu_K\right] \qquad d_0 = \sqrt{4A_s/\pi}$$

$c = 3/2$ in VDI 2230 (polar section modulus $W_P = \pi d_0^3/12$, partly plastic) or 2 (elastic, $W_P = \pi d_0^3/16$). $D_{Km} = r_d\,d \approx (d_W + d_h)/2$ (about 1.3 d), $\mu_K = r_k\,\mu_G$ = head friction. Units: lbf, in-lbf or kN, N-m.

Tightening stress (Ref. 15)

Thread torque and the equivalent (von Mises) stress at assembly, from the maximum preload F_i:

$$M_G = F_i\,\frac{d_2}{2}\left(\frac{P}{\pi\,d_2}+1.155\,\mu_G\right) \qquad \tau_M = \frac{M_G}{W_P} = \frac{8\,c\,M_G}{\pi\,d_0^3}$$

$$\sigma_{red,M} = \sqrt{\left(\frac{F_i}{A_s}\right)^2 + 3\,\tau_M^2} \qquad \nu = \frac{\sigma_{red,M}}{S_y}$$

$\nu \geq 1$: the bolt yields during tightening. VDI 2230 limits ν to 0.9. In service the torsion relaxes to $k_\tau = 0.5$ of the elastic value. F_{SA} = additional bolt load from the external load (e.g. $n\,\phi\,P_t$), FS = factor of safety on F_{SA}:

$$\sigma_z = \frac{F_i + FS\,F_{SA}}{A_s} \qquad \sigma_{red,B} = \sqrt{\sigma_z^2 + 3\left(k_\tau\,\frac{16\,M_G}{\pi\,d_0^3}\right)^2} \qquad MS = \frac{S}{\sigma_{red,B}} - 1$$

$S = S_y$ with FS_y, or S_u with FS_u. A_s blank: $0.7854\,(d - 0.9743\,P)^2$ (English) or $\frac{\pi}{4}(d - 0.9382\,P)^2$ (metric).

Bearing pressure under the head (Ref. 15)

Pressure on the clamped part under the head or nut, d_W = outer bearing diameter, d_h = hole diameter.

$$A_p = \frac{\pi}{4}\left(d_W^2 - d_h^2\right) \qquad p_M = \frac{F_{i,max}}{A_p} \qquad p_B = \frac{F_{i,max} + FS\,F_{SA}}{A_p} \qquad MS = \frac{p_G}{p} - 1$$

p_G = limiting surface pressure of the clamped material (values in VDI 2230). With a washer of outer and inner diameters D_o, D_i: washer on the part $A = \frac{\pi}{4}(D_o^2 - d_h^2)$ with p_G, head on the washer $A = \frac{\pi}{4}(d_W^2 - D_i^2)$ with $p_{G,w}$. This assumes a stiff (hardened) washer.

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