Modal Transient Analysis of a Multi-degree-of-freedom System with Enforced Motion via a Ramp Invariant Digital Recursive Filtering Relationship
Revision A

By Tom Irvine
Email: tomirvine@aol.com

February 8, 2012

Variables

M	Mass matrix
K	Stiffness matrix
F	Applied forces
F_d	Forces at driven nodes
F_f	Forces at free nodes
I	Identity matrix
Π	Transformation matrix
u	Displacement vector
u_d	Displacements at driven nodes
u_f	Displacements at free nodes

The equation of motion for a multi-degree-of-freedom system is

$$[M][\ddot{u}]+[K][u]=F \tag{1}$$

$$[u]=\begin{bmatrix} u_d \\ u_f \end{bmatrix} \tag{2}$$

Partition the matrices and vectors as follows

$$\begin{bmatrix} M_{dd} & M_{df} \\ M_{fd} & M_{ff} \end{bmatrix} \begin{bmatrix} \ddot{u}_d \\ \ddot{u}_f \end{bmatrix} + \begin{bmatrix} K_{dd} & K_{df} \\ K_{fd} & K_{ff} \end{bmatrix} \begin{bmatrix} u_d \\ u_f \end{bmatrix} = \begin{bmatrix} F_d \\ F_f \end{bmatrix} \tag{3}$$

The equations of motions for enforced displacement and acceleration are given in Appendices A and B, respectively.

Create a transformation matrix such that

$$\begin{bmatrix} u_d \\ u_f \end{bmatrix} = \Pi \begin{bmatrix} u_d \\ u_w \end{bmatrix} \tag{4}$$

$$\Pi = \begin{bmatrix} I & 0 \\ T_1 & T_2 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} M_{dd} & M_{df} \\ M_{fd} & M_{ff} \end{bmatrix} \Pi \begin{bmatrix} \ddot{u}_d \\ \ddot{u}_w \end{bmatrix} + \begin{bmatrix} K_{dd} & K_{df} \\ K_{fd} & K_{ff} \end{bmatrix} \Pi \begin{bmatrix} u_d \\ u_w \end{bmatrix} = \begin{bmatrix} F_d \\ F_f \end{bmatrix} \tag{6}$$

Premultiply by Π^T,

$$\Pi^T \begin{bmatrix} M_{dd} & M_{df} \\ M_{fd} & M_{ff} \end{bmatrix} \Pi \begin{bmatrix} \ddot{u}_d \\ \ddot{u}_w \end{bmatrix} + \Pi^T \begin{bmatrix} K_{dd} & K_{df} \\ K_{fd} & K_{ff} \end{bmatrix} \Pi \begin{bmatrix} u_d \\ u_w \end{bmatrix} = \Pi^T \begin{bmatrix} F_d \\ F_f \end{bmatrix} \tag{7}$$

Equation (7) can then be uncoupled using the normal modes per the method in Reference 1.

The modal transient analysis can be then be performed on the modal coordinates using the ramp invariant digital recursive filtering relationship in Reference 2.

The resulting modal responses are then transformed into physical responses per the method in Reference 1.

Next, the transformation in equation (4) is performed.

Finally, the physical responses are reassembled in the correct order.

References

1. T. Irvine, The Generalized Coordinate Method for Discrete Systems, Revision F, Vibrationdata, 2012.

2. T. Irvine, Modal Transient Analysis of a System Subjected to an Applied Force via Ramp Invariant Digital Recursive Filtering Relationship, Revision B, Vibrationdata, 2012.

APPENDIX A

Enforced Displacement

Again, the partitioned equation of motion is

$$\Pi^T \begin{bmatrix} M_{dd} & M_{df} \\ M_{fd} & M_{ff} \end{bmatrix} \Pi \begin{bmatrix} \ddot{u}_d \\ \ddot{u}_w \end{bmatrix} + \Pi^T \begin{bmatrix} K_{dd} & K_{df} \\ K_{fd} & K_{ff} \end{bmatrix} \Pi \begin{bmatrix} u_d \\ u_w \end{bmatrix} = \Pi^T \begin{bmatrix} F_d \\ F_f \end{bmatrix} \tag{A-1}$$

Transform the equation of motion to uncouple the mass matrix so that the resulting mass matrix is

$$\begin{bmatrix} \hat{M}_{dd} & 0 \\ 0 & \hat{M}_{ww} \end{bmatrix} \tag{A-2}$$

Apply the transformation to the mass matrix

$$\Pi^T \, M \, \Pi = \begin{bmatrix} I & T_1{}^T \\ 0 & T_2{}^T \end{bmatrix} \begin{bmatrix} M_{dd} & M_{df} \\ M_{fd} & M_{ff} \end{bmatrix} \begin{bmatrix} I & 0 \\ T_1 & T_2 \end{bmatrix} \tag{A-3}$$

$$\Pi^T M \Pi = \begin{bmatrix} I & T_1{}^T \\ 0 & T_2{}^T \end{bmatrix} \begin{bmatrix} M_{dd} + M_{df} T_1 & M_{df} T_2 \\ M_{fd} + M_{ff} T_1 & M_{ff} T_2 \end{bmatrix} \tag{A-4}$$

$$\Pi^T M \Pi = \begin{bmatrix} M_{dd} + M_{df} T_1 + T_1{}^T (M_{fd} + M_{ff} T) & M_{df} T_2 + T_1{}^T M_{ff} T_2 \\ T_2{}^T (M_{fd} + M_{ff} T_1) & T_2{}^T (M_{ff} T_2) \end{bmatrix} \tag{A-5}$$

$$\Pi^T M \Pi = \begin{bmatrix} M_{dd} + M_{df} T_1 + T_1{}^T (M_{fd} + M_{ff} T_1) & \left(M_{df} + T_1{}^T M_{ff}\right) T_2 \\ T_2{}^T (M_{fd} + M_{ff} T_1) & T_2{}^T (M_{ff} T_2) \end{bmatrix} \tag{A-6}$$

$$\Pi^T M \Pi = \begin{bmatrix} M_{dd} + T_1{}^T M_{fd} + \left(M_{df} + T_1{}^T M_{ff}\right) T_1 & \left(M_{df} + T_1{}^T M_{ff}\right) T_2 \\ T_2{}^T (M_{fd} + M_{ff} T_1) & T_2{}^T (M_{ff} T_2) \end{bmatrix} \tag{A-7}$$

Let

$$T_2 = I \tag{A-8}$$

$$\Pi^T M \Pi = \begin{bmatrix} M_{dd} + T_1{}^T M_{fd} + \left(M_{df} + T_1{}^T M_{ff}\right) T_1 & \left(M_{df} + T_1{}^T M_{ff}\right) \\ (M_{fd} + M_{ff} T_1) & M_{ff} \end{bmatrix} = \begin{bmatrix} \hat{M}_{dd} & 0 \\ 0 & \hat{M}_{ww} \end{bmatrix} \tag{A-9}$$

$$M_{df} + T_1{}^T M_{ff} = 0 \tag{A-10}$$

$$T_1{}^T = - M_{df} M_{ff}{}^{-1} \tag{A-11}$$

$$T_1 = -M_{ff}^{-1} M_{fd} \tag{A-12}$$

The transformation matrix is

$$\Pi = \begin{bmatrix} I_{dd} & 0 \\ T_1 & I_{ff} \end{bmatrix} \tag{A-13}$$

$$\hat{M}_{dd} = M_{dd} + T_1^T M_{fd} + \left(M_{df} + T_1^T M_{ff}\right) T_1 \tag{A-14}$$

$$\hat{M}_{ww} = M_{ff} \tag{A-15}$$

$$\Pi^T K \Pi = \begin{bmatrix} I_{dd} & T_1^T \\ 0 & I_{ff} \end{bmatrix} \begin{bmatrix} K_{dd} & K_{df} \\ K_{fd} & K_{ff} \end{bmatrix} \begin{bmatrix} I_{dd} & 0 \\ T_1 & I_{ff} \end{bmatrix} \tag{A-16}$$

By similarity, the transformed stiffness matrix is

$$\begin{bmatrix} \hat{k}_{dd} & \hat{k}_{dw} \\ \hat{k}_{wd} & \hat{k}_{ww} \end{bmatrix} = \begin{bmatrix} K_{dd} + T_1^T K_{fd} + \left(K_{df} + T_1^T K_{ff}\right) T_1 & \left(K_{df} + T_1^T K_{ff}\right) \\ \left(K_{fd} + K_{ff} T_1\right) & K_{ff} \end{bmatrix} \tag{A-17}$$

$$\begin{bmatrix} \hat{F}_d \\ \hat{F}_w \end{bmatrix} = \begin{bmatrix} I_{dd} & T_1 \\ 0 & I_{ff} \end{bmatrix} \begin{bmatrix} F_d \\ F_f \end{bmatrix} \tag{A-18}$$

$$\begin{bmatrix} \hat{F}_d \\ \hat{F}_w \end{bmatrix} = \begin{bmatrix} I_{dd} F_d + T_1 F_f \\ I_{ff} F_f \end{bmatrix} \tag{A-19}$$

$$\begin{bmatrix} \hat{F}_d \\ \hat{F}_w \end{bmatrix} = \begin{bmatrix} F_d + T_1 F_f \\ F_f \end{bmatrix} \tag{A-20}$$

$$\begin{bmatrix} \hat{M}_{dd} & 0 \\ 0 & \hat{M}_{ww} \end{bmatrix} \begin{bmatrix} \ddot{u}_d \\ \ddot{u}_w \end{bmatrix} + \begin{bmatrix} \hat{k}_{dd} & \hat{k}_{dw} \\ \hat{k}_{wd} & \hat{k}_{ww} \end{bmatrix} \begin{bmatrix} u_d \\ u_w \end{bmatrix} = \begin{bmatrix} \widehat{F}_d \\ \hat{F}_w \end{bmatrix} \tag{A-21}$$

$$\hat{M}_{ww} \ddot{u}_w + \hat{k}_{wd} u_d + \hat{k}_{ww} u_w = \hat{F}_w \tag{A-22}$$

The equation of motion is thus

$$\hat{M}_{ww} \ddot{u}_w + \hat{k}_{ww} u_w = \hat{F}_w - \hat{k}_{wd} u_d \tag{A-23}$$

The final displacement are found via

$$\begin{bmatrix} u_d \\ u_f \end{bmatrix} = \Pi \begin{bmatrix} u_d \\ u_w \end{bmatrix} \tag{A-24}$$

$$\Pi = \begin{bmatrix} I_{dd} & 0 \\ -M_{ff}{}^{-1} M_{fd} & I_{ff} \end{bmatrix} \tag{A-25}$$

APPENDIX B

Enforced Acceleration

Again, the partitioned equation of motion is

$$\Pi^T \begin{bmatrix} M_{dd} & M_{df} \\ M_{fd} & M_{ff} \end{bmatrix} \Pi \begin{bmatrix} \ddot{u}_d \\ \ddot{u}_w \end{bmatrix} + \Pi^T \begin{bmatrix} K_{dd} & K_{df} \\ K_{fd} & K_{ff} \end{bmatrix} \Pi \begin{bmatrix} u_d \\ u_w \end{bmatrix} = \Pi^T \begin{bmatrix} F_d \\ F_f \end{bmatrix} \tag{B-1}$$

Transform the equation of motion to uncouple the stiffness matrix so that the resulting stiffness matrix is

$$\begin{bmatrix} \hat{K}_{dd} & 0 \\ 0 & \hat{K}_{ww} \end{bmatrix} \tag{B-2}$$

$$\Pi^T K \Pi = \begin{bmatrix} I & T_1{}^T \\ 0 & T_2{}^T \end{bmatrix} \begin{bmatrix} K_{dd} & K_{df} \\ K_{fd} & K_{ff} \end{bmatrix} \begin{bmatrix} I & 0 \\ T_1 & T_2 \end{bmatrix} \tag{B-3}$$

$$\Pi^T K \Pi = \begin{bmatrix} I & T_1{}^T \\ 0 & T_2{}^T \end{bmatrix} \begin{bmatrix} K_{dd} + K_{df} T_1 & K_{df} T_2 \\ K_{fd} + K_{ff} T_1 & K_{ff} T_2 \end{bmatrix} \tag{B-4}$$

$$\Pi^T K \Pi = \begin{bmatrix} K_{dd} + K_{df} T_1 + T_1{}^T \left(K_{fd} + K_{ff} T \right) & K_{df} T_2 + T_1{}^T K_{ff} T_2 \\ T_2{}^T \left(K_{fd} + K_{ff} T_1 \right) & T_2{}^T \left(K_{ff} T_2 \right) \end{bmatrix} \tag{B-5}$$

$$\Pi^T K \Pi = \begin{bmatrix} K_{dd} + K_{df} T_1 + T_1^T (K_{fd} + K_{ff} T_1) & (K_{df} + T_1^T K_{ff}) T_2 \\ T_2^T (K_{fd} + K_{ff} T_1) & T_2^T (K_{ff} T_2) \end{bmatrix} \tag{B-6}$$

$$\Pi^T K \Pi = \begin{bmatrix} K_{dd} + T_1^T K_{fd} + (K_{df} + T_1^T K_{ff}) T_1 & (K_{df} + T_1^T K_{ff}) T_2 \\ T_2^T (K_{fd} + K_{ff} T_1) & T_2^T (K_{ff} T_2) \end{bmatrix} \tag{B-7}$$

Let

$$T_2 = I \tag{B-8}$$

$$\Pi^T K \Pi = \begin{bmatrix} K_{dd} + T_1^T K_{fd} + (K_{df} + T_1^T K_{ff}) T_1 & (K_{df} + T_1^T K_{ff}) \\ (K_{fd} + K_{ff} T_1) & K_{ff} \end{bmatrix} = \begin{bmatrix} \hat{K}_{dd} & 0 \\ 0 & \hat{K}_{ww} \end{bmatrix} \tag{B-9}$$

$$K_{df} + T_1^T K_{ff} = 0 \tag{B-10}$$

$$T_1^T = -K_{df} K_{ff}^{-1} \tag{B-11}$$

$$T_1 = -K_{ff}^{-1} K_{fd} \tag{B-12}$$

$$\Pi = \begin{bmatrix} I_{dd} & 0 \\ T_1 & I_{ff} \end{bmatrix} \tag{B-13}$$

$$\hat{K}_{dd} = K_{dd} + T_1{}^T K_{fd} + \left(K_{df} + T_1{}^T K_{ff}\right)T_1 \tag{B-14}$$

$$\hat{K}_{ww} = K_{ff} \tag{B-15}$$

$$\Pi^T M \Pi = \begin{bmatrix} I_{dd} & T_1{}^T \\ 0 & I_{ff} \end{bmatrix} \begin{bmatrix} M_{dd} & M_{df} \\ M_{fd} & M_{ff} \end{bmatrix} \begin{bmatrix} I_{dd} & 0 \\ T_1 & I_{ff} \end{bmatrix} \tag{B-16}$$

By similarity, the transformed mass matrix is

$$\begin{bmatrix} \hat{m}_{dd} & \hat{m}_{dw} \\ \hat{m}_{wd} & \hat{m}_{ww} \end{bmatrix} = \begin{bmatrix} M_{dd} + T_1{}^T M_{fd} + \left(M_{df} + T_1{}^T M_{ff}\right)T_1 & \left(M_{df} + T_1{}^T M_{ff}\right) \\ \left(M_{fd} + M_{ff} T_1\right) & M_{ff} \end{bmatrix} \tag{B-17}$$

$$\begin{bmatrix} \hat{F}_d \\ \hat{F}_w \end{bmatrix} = \begin{bmatrix} I_{dd} & T_1 \\ 0 & I_{ff} \end{bmatrix} \begin{bmatrix} F_d \\ F_f \end{bmatrix} \tag{B-18}$$

$$\begin{bmatrix} \hat{F}_d \\ \hat{F}_w \end{bmatrix} = \begin{bmatrix} I_{dd} F_d + T_1 F_f \\ I_{ff} F_f \end{bmatrix} \tag{B-19}$$

$$\begin{bmatrix} \hat{F}_d \\ \hat{F}_w \end{bmatrix} = \begin{bmatrix} F_d + T_1 F_f \\ F_f \end{bmatrix} \tag{B-20}$$

$$\begin{bmatrix} \hat{m}_{dd} & \hat{m}_{dw} \\ \hat{m}_{wd} & \hat{m}_{ww} \end{bmatrix} \begin{bmatrix} \ddot{u}_d \\ \ddot{u}_w \end{bmatrix} + \begin{bmatrix} \hat{K}_{dd} & 0 \\ 0 & \hat{K}_{ww} \end{bmatrix} \begin{bmatrix} u_d \\ u_w \end{bmatrix} = \begin{bmatrix} \hat{F}_d \\ \hat{F}_w \end{bmatrix} \tag{B-21}$$

$$\hat{m}_{wd}\ddot{u}_d + \hat{m}_{ww}\ddot{u}_w + \hat{K}_{ww}u_w = \hat{F}_w \tag{B-22}$$

The equation of motion is thus

$$\hat{m}_{ww}\ddot{u}_w + \hat{K}_{ww}u_w = \hat{F}_w - \hat{m}_{wd}\ddot{u}_d \tag{B-23}$$

The final displacement are found via

$$\begin{bmatrix} u_d \\ u_f \end{bmatrix} = \Pi \begin{bmatrix} u_d \\ u_w \end{bmatrix} \tag{B-24}$$

$$\Pi = \begin{bmatrix} I_{dd} & 0 \\ -K_{ff}{}^{-1}K_{fd} & I_{ff} \end{bmatrix} \tag{B-25}$$

APPENDIX C

Enforced Acceleration Example

The diagram of a sample system is shown in Figure 1.

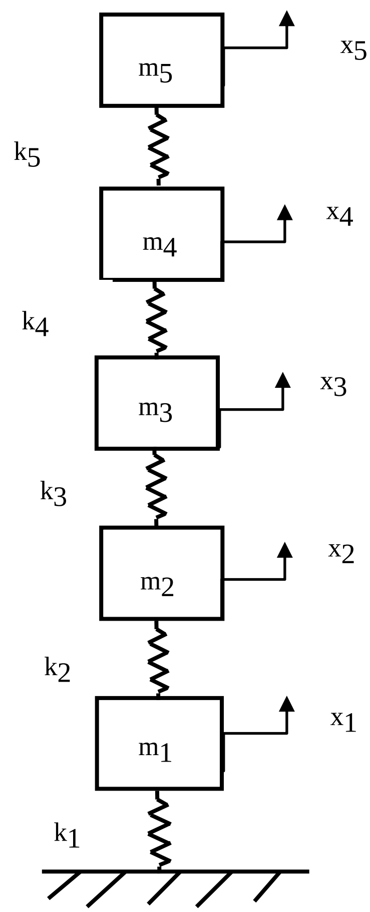

Figure C-1.

k1	10e+07
k2	10e+07
k3	8e+07
k4	8e+07
k5	6e+07

m1	65,000
m2	65,000
m3	65,000
m4	60,000
m5	45,000

English units:
stiffness (lbf/in), mass(lbf sec^2/in), force(lbf)

Assume modal damping of 5% for all modes.

The equation of motion is

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \\ \ddot{x}_5 \end{bmatrix}$$

$$+ \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix} \tag{C-1}$$

Drive mass 4 with acceleration as follows:

$$a_4(t) = (386 \text{ in/sec}^2) \sin [\, 2\pi \, (4 \text{ Hz}) \, t \,] , \quad 0 \le t \le 3 \text{ sec} \tag{C-2}$$

Set the sample rate at 200 samples/sec.

The results are shown in Figure C-2, as calculated by Matlab script:
mdof_modal_enforced_acceleration_ri.m.

The Matlab scripts implements the method given in the main text.

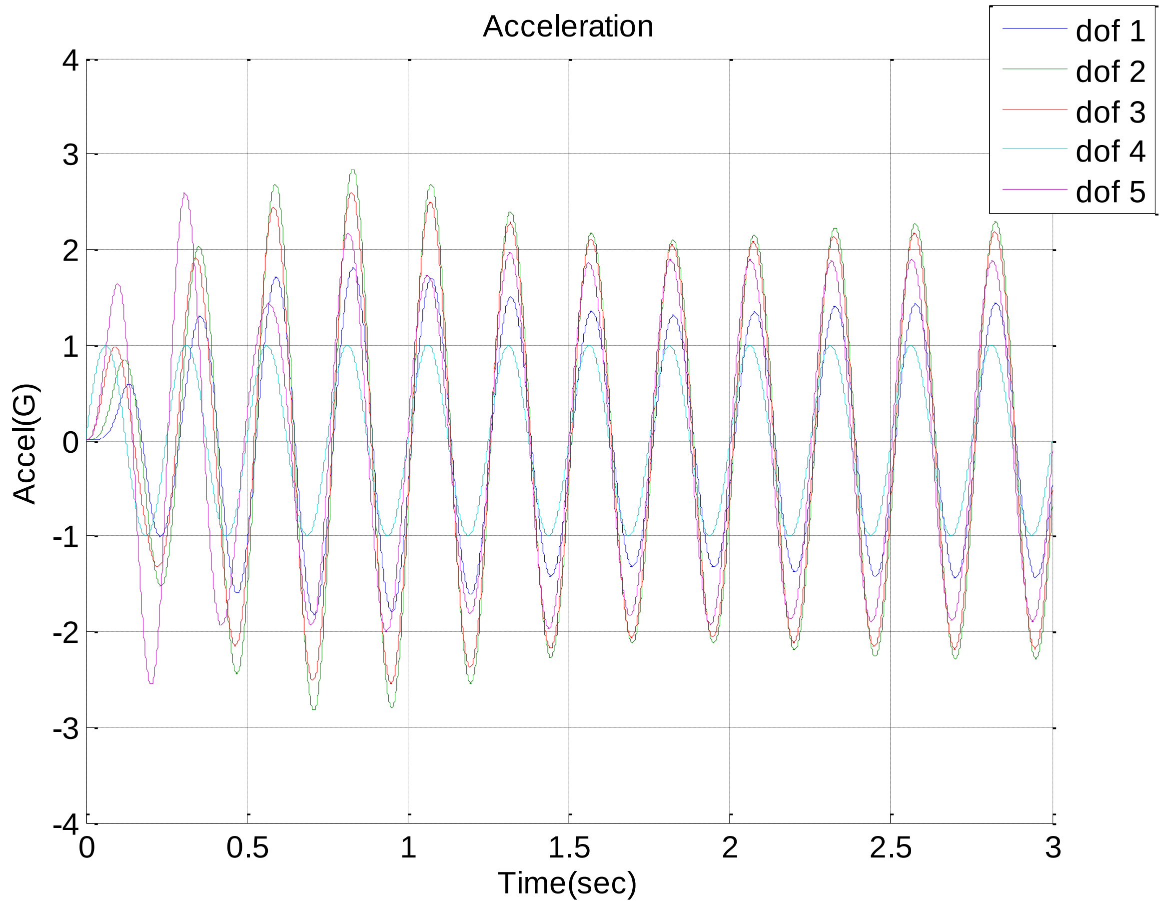

Figure C-2.

```
>> mdof_modal_enforced_acceleration_ri

  mdof_modal_enforced_acceleration_ri.m  ver 1.0  February 7, 2012
  by Tom Irvine

  Response of a multi-degree-of-freedom system to enforced
  acceleration at selected point mass nodes via a ramp invariant
  digital recursive filtering relationship.


 Enter the units system
 1=English  2=metric
 1
 Assume symmetric mass and stiffness matrices.
```

```
 Select input mass unit
  1=lbm  2=lbf sec^2/in
 2
 stiffness unit = lbf/in

 Select file input method
   1=file preloaded into Matlab
   2=Excel file
1

 Mass Matrix
 Enter the matrix name:  mass_5dof

 Stiffness Matrix
 Enter the matrix name:  stiff_5dof
 Input Matrices

mass =

       65000           0           0           0           0
           0       65000           0           0           0
           0           0       65000           0           0
           0           0           0       60000           0
           0           0           0           0       45000


stiff =

   200000000  -100000000           0           0           0
  -100000000   180000000   -80000000           0           0
           0   -80000000   160000000   -80000000           0
           0           0   -80000000   140000000   -60000000
           0           0           0   -60000000    60000000

 Natural Frequencies
 No.       f(Hz)
1.         1.8283
2.         4.9465
3.         7.4613
4.         9.7491
5.         11.171

  Modes Shapes (column format)

ModeShapes =

    0.0007    0.0017    0.0020    0.0019    0.0021
    0.0013    0.0023    0.0011   -0.0008   -0.0025
    0.0019    0.0013   -0.0020   -0.0017    0.0018
    0.0023   -0.0008   -0.0016    0.0027   -0.0011
    0.0026   -0.0027    0.0024   -0.0015    0.0004
```

```
 Select modal damping input method
  1=uniform damping for all modes
  2=damping vector
 1

 Enter damping ratio
 0.05

 number of dofs =5


 Enter the duration (sec)
 3

 Enter the sample rate (samples/sec)
 (suggest >    223.4 )
 1000
 Each acceleration file must have two columns: time(sec) & accel(G)

 Enter the number of acceleration files
 1

 Note: the first dof is 1

 Enter acceleration file 1
 Enter the matrix name:  sine_4Hz

 Enter the number of dofs at which this acceleration is applied
 1

 Enter the dof number for this acceleration
 4
 begin interpolation
 end interpolation


enforced_string =

accel



MT =

  1.0e+005 *

    1.5495    0.1444    0.2889    0.4694    0.4500
    0.1444    0.6500         0         0         0
    0.2889         0    0.6500         0         0
    0.4694         0         0    0.6500         0
    0.4500         0         0         0    0.4500
```

```
KT =

  1.0e+008 *

    0.2222   -0.0000    0.0000   -0.0000         0
   -0.0000    2.0000   -1.0000         0         0
    0.0000   -1.0000    1.8000   -0.8000         0
   -0.0000         0   -0.8000    1.6000         0
         0         0         0         0    0.6000

 Natural Frequencies
 No.      f(Hz)
1.        1.8283
2.        4.9465
3.        7.4613
4.        9.7491
5.        11.171

  Modes Shapes (column format)

ModeShapes =

    0.0023   -0.0008   -0.0016   -0.0027   -0.0011
    0.0002    0.0018    0.0023   -0.0013    0.0024
    0.0002    0.0026    0.0018    0.0020   -0.0021
    0.0002    0.0018   -0.0008    0.0036    0.0026
    0.0003   -0.0020    0.0040    0.0041    0.0015



Mwd =

  1.0e+004 *

    1.4444
    2.8889
    4.6944
    4.5000


Kwd =

  1.0e-007 *

   -0.2235
    0.0745
   -0.1490
         0


Mww =
```

```
       65000           0           0           0
           0       65000           0           0
           0           0       65000           0
           0           0           0       45000


Kww =

   200000000  -100000000           0           0
  -100000000   180000000   -80000000           0
           0   -80000000   160000000           0
           0           0           0    60000000

 Natural Frequencies
 No.      f(Hz)
1.        4.5268
2.        5.8115
3.        8.2749
4.        11.021

  Modes Shapes (column format)

ModeShapes =

    0.0019         0    0.0024    0.0024
    0.0028         0    0.0006   -0.0027
    0.0021         0   -0.0030    0.0014
         0    0.0047         0         0

 Participation Factors

part =

  435.2223
  212.1320
    0.9398
   74.7038
```

APPENDIX D

Enforced Displacement Example

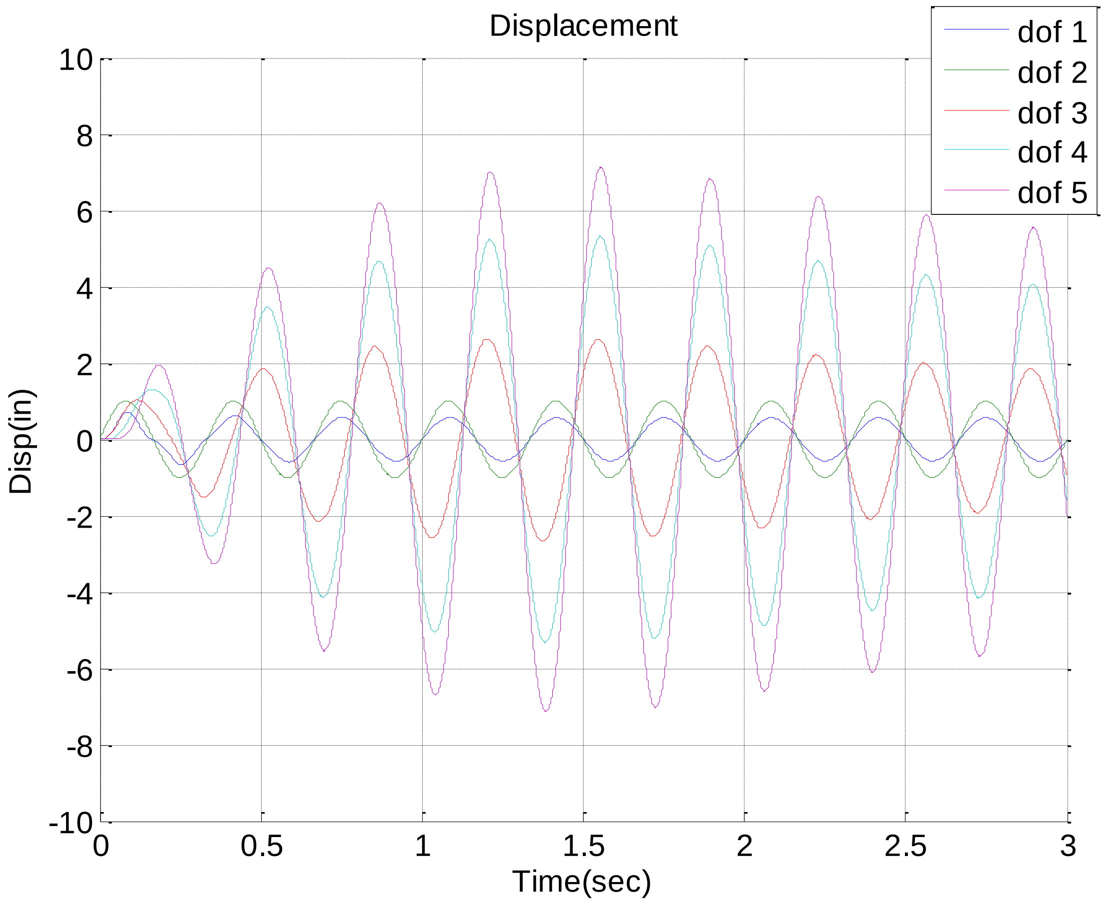

Figure D-1.

Consider the spring-mass system from Appendix C.

Drive mass 2 with displacement as follows:

$$d_2(t) = (1 \text{ inch}) \sin [\, 2\pi \,(3 \text{ Hz})\, t \,] , \quad 0 \leq t \leq 3 \text{ sec} \tag{D-1}$$

The results are shown in Figure D-1, as calculated by Matlab script:
mdof_modal_enforced_displacement_ri.m.

```
>> mdof_modal_enforced_displacement_ri

  mdof_modal_enforced_displacement_ri.m  ver 1.1  February 8, 2012
  by Tom Irvine

  Response of a multi-degree-of-freedom system to enforced
  displacement at selected point mass nodes via a ramp invariant
  digital recursive filtering relationship.


 Enter the units system
 1=English  2=metric
 1
 Assume symmetric mass and stiffness matrices.
 Select input mass unit
  1=lbm  2=lbf sec^2/in
 2
 stiffness unit = lbf/in

 Select file input method
   1=file preloaded into Matlab
   2=Excel file
1

 Mass Matrix
 Enter the matrix name:  mass_5dof

 Stiffness Matrix
 Enter the matrix name:  stiff_5dof
 Input Matrices

mass =

       65000           0           0           0           0
           0       65000           0           0           0
           0           0       65000           0           0
           0           0           0       60000           0
           0           0           0           0       45000


stiff =

   200000000  -100000000           0           0           0
  -100000000   180000000   -80000000           0           0
           0   -80000000   160000000   -80000000           0
           0           0   -80000000   140000000   -60000000
           0           0           0   -60000000    60000000

 Natural Frequencies
 No.      f(Hz)
```

```
1.        1.8283
2.        4.9465
3.        7.4613
4.        9.7491
5.        11.171

  Modes Shapes (column format)

ModeShapes =

    0.0007    0.0017    0.0020    0.0019    0.0021
    0.0013    0.0023    0.0011   -0.0008   -0.0025
    0.0019    0.0013   -0.0020   -0.0017    0.0018
    0.0023   -0.0008   -0.0016    0.0027   -0.0011
    0.0026   -0.0027    0.0024   -0.0015    0.0004

 Select modal damping input method
  1=uniform damping for all modes
  2=damping vector
 1

 Enter damping ratio
 0.05

 number of dofs =5


 Enter the duration (sec)
 3

 Enter the sample rate (samples/sec)
 (suggest >    223.4 )
 1000
 Each displacement file must have two columns: time(sec) & disp(in)

 Enter the number of displacement files
 1

 Note: the first dof is 1

 Enter displacement file 1
 Enter the matrix name:  sine_3Hz

 Enter the number of dofs at which this displacement is applied
 1

 Enter the dof number for this displacement
 2
 begin interpolation
 end interpolation
```

```
enforced_string =

disp



MT =

       65000           0           0           0           0
           0       65000           0           0           0
           0           0       65000           0           0
           0           0           0       60000           0
           0           0           0           0       45000


KT =

   180000000  -100000000   -80000000           0           0
  -100000000   200000000           0           0           0
   -80000000           0   160000000   -80000000           0
           0           0   -80000000   140000000   -60000000
           0           0           0   -60000000    60000000

 Natural Frequencies
 No.      f(Hz)
1.        1.8283
2.        4.9465
3.        7.4613
4.        9.7491
5.        11.171

  Modes Shapes (column format)

ModeShapes =

    0.0013    0.0023    0.0011   -0.0008   -0.0025
    0.0007    0.0017    0.0020    0.0019    0.0021
    0.0019    0.0013   -0.0020   -0.0017    0.0018
    0.0023   -0.0008   -0.0016    0.0027   -0.0011
    0.0026   -0.0027    0.0024   -0.0015    0.0004



Mwd =

     0
     0
     0
     0


Kwd =
```

```
  -100000000
   -80000000
           0
           0


Mww =

       65000           0           0           0
           0       65000           0           0
           0           0       60000           0
           0           0           0       45000


Kww =

   200000000           0           0           0
           0   160000000   -80000000           0
           0   -80000000   140000000   -60000000
           0           0   -60000000    60000000

 Natural Frequencies
 No.      f(Hz)
1.        2.7278
2.        6.9133
3.        8.8283
4.        9.9997

  Modes Shapes (column format)

ModeShapes =

         0         0    0.0039         0
    0.0014    0.0027         0    0.0024
    0.0025    0.0013         0   -0.0029
    0.0033   -0.0031         0    0.0015

 Participation Factors

part =

  392.7638
  115.3874
  254.9510
   49.2174
```