STRUCTURAL LOADS PREDICTION IN FORCE-LIMITED VIBRATION TESTING

Kurng Y. Chang
Jet Propulsion Laboratory, California Institute of Technology
4800 Oak Grove Dr., MS 157-410, Pasadena, CA 91109-8099, USA
Phone: 01 818 354 8565, fax: 01 818 393 1156

Abstract

Flight equipment is exposed to random vibration excitations during launch and is functionally designed to survive a shaker random vibration test. In the test, the random vibration design levels are applied at the equipment-mounting interface and are often force limited to reduce over-testing at shaker hardmount resonance frequencies. As is commonly practiced for heavier, lower resonant frequency, spacecraft equipment, the equipment housing is also frequently designed to a structural flight limit load. The purpose of the work presented herein is to discuss the results of force limit notching during vibration testing with respect to the traditional limit load design criteria. By using a single-degree-of-freedom (SDOF) system approach, this work shows that the structural test load is a function of the force limit factors, the equipment predominant resonant frequency, and random input excitation levels, but is independent of the damping values of the equipment. With an appropriate force specification the notched response due to force limiting will result in loads comparable with the structural design limit criteria. A simplified formula is presented to predict the equipment vibration test loads for SDOF system approximation. The work is currently being expanded to include multi-degree-of-freedom vibratory systems.

INTRODUCTION

Flight equipment is exposed to random vibration excitations during launch and is functionally designed to survive shaker random vibration testing. In the testing, the random vibration design levels are applied at the equipment-mounting interface. For lightweight aerospace structures, the mechanical impedance of equipment and of the mounting structure are typically comparable, so that the vibration of the combined structure and load involves modest interface forces and responses. Most of the high amplification resonances and resultant mechanical failures in conventional vibration tests are test artifacts associated with the essentially infinite mechanical impedance and unlimited force capability of the shaker. An improved vibration testing technique has been recently developed [Ref. 1,2] and applied to eliminate overtesting caused by the infinite mechanical impedance of the shaker in conventional vibration tests. With the newly developed technique, the acceleration input is automatically notched at the resonance frequencies of the test item by specifying a force limit in order to limit the test loads to that predicted for flight.

FORCE LIMIT SPECIFICATION IN RANDOM VIBRATION TESTING

Implementation of force-limited vibration testing requires derivation of a force limit specification. The flight force at the equipment interface, which may be derived from Norton and Thevenin's equivalent electrical circuit theorems, can be written as follows [Ref. 3]:

$$S_{FF}(f) = \left| \frac{Z_{EE}(f)\, Z_{SS}(f)}{Z_{EE}(f) + Z_{SS}(f)} \right|^2 S_{AA}(f) \tag{1}$$

where

$S_{FF}(f)$ = Interface force spectrum
$Z_{SS}(f)$ = Input impedance of support structures
$Z_{EE}(f)$ = Input Impedance of component or equipment
$S_{AA}(f)$ = Acceleration spectrum at component mounting points of unloaded structures

Input impedances in the above equation are specified in terms of the "force/acceleration" format. For a rigid body system, the impedance is equal to its mass only. For complex structural systems the force limit values must be calculated from measurements or analyses of the flight mounting structure and the test items mechanical impedances. An alternate approach to compute the driving force spectrum has been achieved by the replacement of the impedance term by a general dynamic mass.

$$S_{FF}(f) = |M_D(f)|^2\ S_{AA}(f) \tag{2}$$

In this expression, $S_{AA}(f)$ is re-defined as the loaded interface acceleration spectrum and the load dynamic mass, $M_D(f)$, is a frequency response function (FRF) that includes mass, damping, and stiffness effects. The frequency dependence is shown explicitly to emphasize the relationship between force and acceleration applied at each frequency. Since little flight vibratory force data are available and due to structural complexities of space vehicles, precise analytical approaches to obtain the parameters defined in the above equation are not practical. In order to validate Equation (2), several approximate methods [Ref. 4,5] along with measured data have been used to predict the force spectra quantitatively. The semi-empirical method [Ref. 6] has also been developed to simplify the development of shaker force limited vibration testing criteria. In this simplified method, the input force spectrum at the fundamental resonance frequency is properly enveloped by multiplying the total mass, M, of the test item by the input acceleration spectral density specification and by a constant, C. The required force spectrum value is then related to the shaker control acceleration spectrum as

$$\begin{aligned} S_{FF}(f) &= C^2\ M^2\, S_{AA}(f_o)\,, && f < f_o \\ S_{FF}(f) &= C^2\ M^2\, S_{AA}(f_o) \,/\, (f \,/\, f_o)^n, && f \geq f_o \end{aligned} \tag{3}$$

where f_o is the fundamental resonance of the test item on the shaker. Some judgment and reference to previous test data for similar configurations must be considered to choose the constant value of C and the roll-off ratio, n, in the above equation.

Semi-empirical force specifications require only the acceleration specification and data from a low-level vibration pretest and are, therefore, much simpler to determine than previously described force limits based on analytical models and measurements of the mounting structure mechanical impedance. The force limit conservatism is dependent on the chosen constant or so-called fudge factor, C^2. In normal conditions, this factor may be as high as $C^2 = 5$ for directly mounted lightweight loads, whereas for heavier strut mounted equipment a factor as low as $C^2 = 2$ may be considered. Comparison of in-flight measurements shows agreement with these specifications [Ref. 7]. The simplified method to derive the test force specification has been successfully used in many JPL spacecraft and component tests [Ref. 8,9 & 10].

QUASI-STATIC ACCELERATION FOR EQUIPMENT STRUCTURAL DESIGN

The design limit loads for aerospace equipment are usually given in terms of the "quasi-static" acceleration of the center-of-gravity (C.G.) of the hardware. The preliminary Limit Load Factor (LLF) or so-called the Mass Acceleration Curve (MAC) has been adapted over many years for use in the preliminary structural design of spacecraft structures and equipment [Ref. 11]. The accelerations shown on the MAC, as illustrated in Figure 1 for a typical spacecraft, are applied at the C.G. of the equipment in the low frequency range (i.e., usually up to 80 or 100 Hz). In practice, the MAC frequently bounds the equipment launch vibration loads for all frequency ranges, except for very lightweight and high stiffness payload equipment.

During the equipment vibration qualification tests, loads induced in structural elements are normally not allowed to exceed the specified limit loads. In cases where the shaker test induced loads would otherwise exceed the limit loads of the structure, input notching or other response-limiting measures must be taken to protect the flight hardware being tested. The purpose of this paper is to study whether the force limiting has accomplished the notching requirement to limit the equipment structural response in low frequency vibration tests to something less than the design load.

ACCELERATION RESPONSES FOR SDOF SYSTEM

To illustrate the concept of the force limited vibration excitation, consider first a single-degree-of-freedom (SDOF) system as illustrated in Figure 2, where a single mass M is suspended from a moving support by means of a linear spring in parallel with a linear dashpot. The system is subjected to a constant wide-band random excitation of its base. The narrow band response of the SDOF due to random excitation is very similar to sinusoidal motion with randomly varying amplitude. The response acceleration spectral density ASD_x of the mass is given by

$$ASD_x = S_{AA}(f) \, |T(f)|^2 \tag{4}$$

where $|T(f)|$ is the sinusoidal transmissibility for such a system. The area under the transmissibility curve is finite so that for a constant input acceleration spectrum, the root mean square (RMS) acceleration response of the mass can be given in closed form [Ref. 12].

$$\sigma_x = [\, 0.5\, \pi \; f_o \; Q \; S_{AA}(f_o) \,]^{1/2} \tag{5}$$

where Q is the dynamic amplification factor and f_o is the resonant frequency of the system. The procedure to determine the RMS acceleration for one single resonant mode can be applied to more practical cases, which usually exhibit many resonant peaks. As shown in Figure 3, the response acceleration spectral density curve, in the vicinity of resonance of a single resonant mode, can be replaced by an equivalent narrow band with an effective bandwidth equal to

$$\Delta f_{eff} = 0.5\,\pi\ f_o/Q \tag{6}$$

and a constant acceleration spectral density, ASD_{max}, equals to the maximum value at resonance. The effective bandwidth is equal to the so-called half-power bandwidth f_o /Q of the resonant system times a correction factor $\pi/2$ which accounts for response outside this bandwidth. The mean square acceleration within this equivalent band is then equal the product of the above two quantities. Since $ASD_{max} = S_{AA}(f_o)\ Q^2$, the RMS acceleration is

$$\sigma_x \cong [\,\Delta f_{eff}\ \mathrm{x}\ ASD_{max}\,]^{1/2} = [\,0.5\,\pi\ f_o\ Q\ S_{AA}(f_o)\ ]^{1/2} \tag{7}$$

where $S_{AA}(f_o)$ is the acceleration spectral density of the input excitation at frequency f_o. This RMS acceleration response includes the effect of response at all frequencies above and below resonance. The expression in Equation (7) is exactly identical to Equation (5) for a SDOF base excitation system. However, if the actual acceleration spectral density curve in Figure 3 is integrated graphically, it is found that this RMS acceleration corresponds approximately to the area under the curve up to a frequency of approximately $2f_o$. Thus, the RMS acceleration as calculated will be in error if there are any additional resonant peaks, i.e., two or multi-degree-of-freedom vibratory systems, at frequencies less than $2f_o$. This upper frequency limit will decrease for higher values of Q.

ACCELERATION RESPONSES UNDER FORCE-LIMITED EXCITATION

For a SDOF system with force limited excitation, and if the force limit value as defined from Equation (3) is less than the maximum reaction force, the response force spectral density FSD_x will be reduced or notched to the controlled force limiting value as shown in Figure 4. The depth of the force notching is

$$A^2 = Q^2 / C^2 \tag{8}$$

The notched RMS response, $\underline{\sigma}$, of the force-limited excitation will be equal to the total integration of the remaining shaded area. No closed form solution is available to represent the integration result. The approximate values can be calculated by utilizing the results previously described in the random vibration textbook by Crandall and Mark [Ref. 12] and also later reproduced in Hendrickson's approximate formula [Ref. 13]. The results obtained from Hendrickson's formula are shown in Figure 5, where the fraction of mean square notched response, f_r, of a SDOF system is plotted against the notching value. Based on this approximation, the notched RMS response can be expressed as follows:

$$\underline{\sigma}_{C.G.} \cong \sqrt{f_r}\ \sigma_x \tag{9}$$

and hence

$$f_r = 1 - \frac{2}{\pi} \left[\tan^{-1}\sqrt{A^2-1} - \frac{\sqrt{A^2-1}}{A^2} \right] \tag{10}$$

Substituting all these values back into Equation (9) results in

$$\underline{\sigma}_{C.G.} = \left[Q \left(\pi/2 - \tan^{-1}\sqrt{Q^2-C^2}/C \right) + C\sqrt{Q^2-C^2}/Q \right]^{1/2} \sqrt{f_o\, S_{AA}(f_o)} \tag{11}$$

As expected, the mathematical expression of the notched RMS response, $\underline{\sigma}$, is a function of several variable factors like: system damping values, force limit factors, as well as equipment resonant frequencies, and random input excitation levels. For aerospace flight equipment, the dynamic amplification, Q, at resonance is in the range from 5 up to much higher values; and the force limit factor, C^2, normally is between 2 to 5 as indicated earlier. Since $Q >> C$ in general flight hardware cases, the above expression can be simplified to become

$$\underline{\sigma}_{C.G.} = \left[Q \left(\pi/2 - \tan^{-1} Q/C \right) + C \right]^{1/2} \sqrt{f_o\, S_{AA}(f_o)} \tag{12}$$

By relating to trigonometry, the first term ($\pi/2 - \tan^{-1} Q/C$) of the above equation represents the acute angle of a right triangular where C and Q represent two sides of the right angle. This angle is also approximately equal to C/Q for small angles. Substituting this value into Equation (12) in which the Q factor drops out, simplification yields an approximate formula to predict the notched RMS response value for a single mode system.

$$\underline{\sigma}_{C.G.} \cong \left[2\ C\ f_o\ S_{AA}(f_o) \right]^{1/2} \tag{13}$$

To demonstrate the above prediction process, consider a flight equipment system with Q=10, and f_o = 80 Hz, and a force limit factor of $C^2 = 5$. The notch depth due to force limiting can be computed and is equal to $10^2/5 = 20$, which corresponds to a 13 dB notch. Assume that the weight of the equipment is 20 Kg and is subjected to a 0.2 g^2/Hz input acceleration spectral density. The RMS C.G. acceleration response for the force limited excitation would be equal to

$$\underline{\sigma}_{C.G.} \cong \left[2\ C\ f_o\ S_{AA}(f_o) \right]^{1/2} = \left[2 \times 2.236 \times 80 \times 0.2 \right]^{1/2} = 8.46\ g$$

For this example, this corresponds to a 3-sigma peak acceleration of 3 x 8.46 = 25.4 g, which is exactly the acceleration value given by the typical MAC curve as shown in Figure 1.

CONCLUDING REMARKS

- A simplified formula for predicting the equipment structural test loads for force-limited vibration testing has been derived. By properly choosing the force limit factor along with the equipment predominant resonant frequency, the force-limited vibration test will result in test loads comparable with the structural design limit criteria derived from the MAC. This prediction formula is an extremely useful tool that allows the test engineer to estimate equipment structural responses and test induced loads before testing is conducted.

- The structure test loads prediction method presented herein is so far considered to be an approximation. A parametric investigation of the notched prediction has also confirmed the dependence of the response on typical dynamic parameters like: force limit factors, component resonant frequencies, and random vibration excitation levels, but the response is independent from the test system damping values [Ref. 14].

- As noted herein the RMS acceleration computation is correct for a single resonant mode system, but will be in error for two or multi-degree-of-freedom systems. Nevertheless, the results obtained from SDOF systems to random excitations will be the foundation for analyzing response of more complex structural systems subjected to random excitations. In order to complement the development and to assess the validity of the simplified formula for more general usage, the work is currently being extending to study the additional contribution due to other resonant frequency responses in the vicinity of the predominate resonant mode.

ACKNOWLEDGMENTS

The work described in this paper was carried out by the Jet Propulsion Laboratory (JPL), California Institute of Technology under a contract with the National Aeronautics and Space Administration (NASA). The many contributions of the personnel in the JPL Dynamics Environments Group and its supervisor, Dennis Kern, are appreciated. Moreover, the author would like to acknowledge the contributions of Dr. Terry Scharton for his development of force limiting technology and many helpful suggestions in preparing this paper.

REFERENCES

1. Smallwood, D. O., " An Analytical Study of a Vibration Test Method Using Extremal Control of Acceleration and Force," Proc. of Institute of Environmental Sciences 35th Annual Technical Meeting, 1989, pp. 263-271.

2. Scharton, T. D., Boatman, D. J., and Kern, D. L., " Dual Control Vibration Testing," Proc. of 60th Shock and Vibration Symposium, vol., 1989, pp. 199-217.

3. Chang, K. Y., and Kao, G. C., "Simplified Techniques for Predicting Vibro-Acoustic Environments," Shock and Vibration Bulletin, No. 45, Part 3, June 1975, pp. 167-181.

4. Scharton, T. D., " Force Specifications for Extremal Dual Controlled Vibration Tests," Proc. of Institute of Environmental Sciences 36th Annual Technical Meeting, 1990, pp. 140-146.

5. Scharton, T. D., "Vibration-Test Force Limits Derived From Frequency-Shift Method," AIAA Journal of Spacecraft and Rockets 32(2), 1995, pp. 312–316.

6. Chang, K. Y., and Scharton, T. D., "Cassini Spacecraft and Instrument Force Limited Vibration Testing," Proc. of the 3rd International Symposium on Environmental Testing for Space Programmes, ESTEC, June 1997.

7. Scharton, T. D., " In-flight Measurements of Dynamic Force and Comparison with the Specifications used for Limiting the Forces in Ground Vibration Testing," European Conference on Spacecraft Structures, Materials and Mechanical Testing, Braunschweig, Germany, November 1998.

8. Scharton, T. D., " Force Limited Vibration Testing Monograph," NASA Reference Publication RP-1403, May 1997.

9. McNelis, M. E., and Scharton, T. D., "Benefits of Force Limiting Vibration Testing," NASA/TM-1999-209382, August 1999.

10. Chang, K. Y., "DS1 Spacecraft Vibration Qualification Testing," Proc. of Institute of Environmental Sciences Annual Technical Meeting, May 2000, pp.351-356.

11. Trubert, M. R., "Mass Acceleration Curve for Spacecraft Structural Design," JPL D-5882, November 1989.

12. Crandall, S. H., and Mark, W. D., RANDOM VIBRATION IN MECHANICAL SYSTEM, Academic Press, New York, 1973.

13. Scharton, T. D., Bamford, R. M., and Hendrickson, J., " Force Limiting R&D at JPL," Spacecraft and Launch Vehicle Dynamics Technical Interchange Meeting, June 1995.

14. Chang, K. Y., " Force Limit Specifications Vs. Design Limit Loads in Vibration Testing," Proc. of the European Conference on Spacecraft Structures, Materials and Mechanical Testing, Noordwijk, The Netherlands, November 2000, pp.295-300.

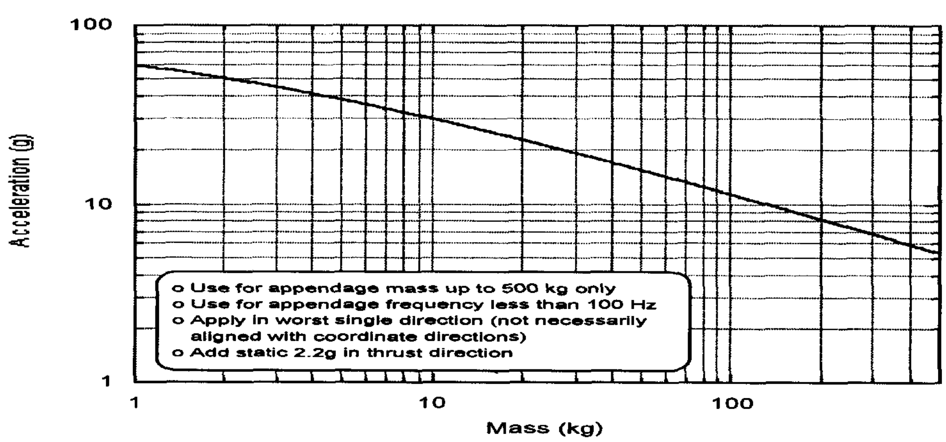

Figure 1. Typical Physical Mass Acceleration Curve for Flight Equipment

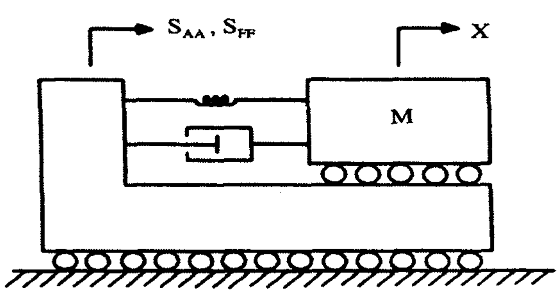

Figure 2. Single-Degree-of-Freedom System Driven by Base Random Excitation

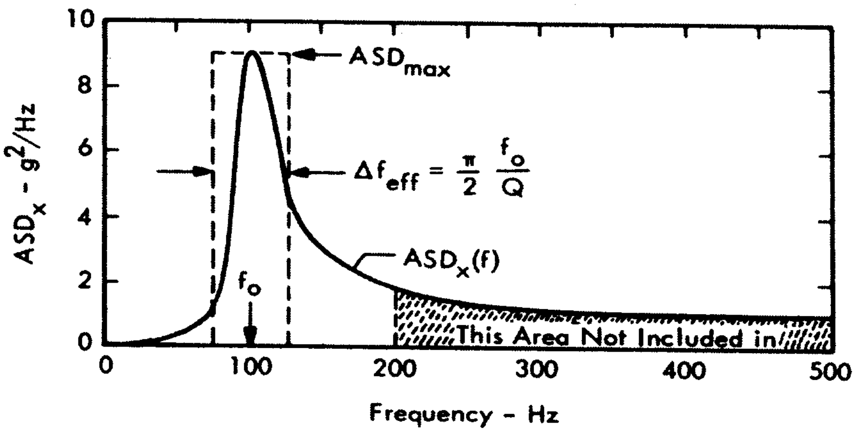

Figure 3. Acceleration Spectral Density Curve for Response of Single Resonant Mode Systems

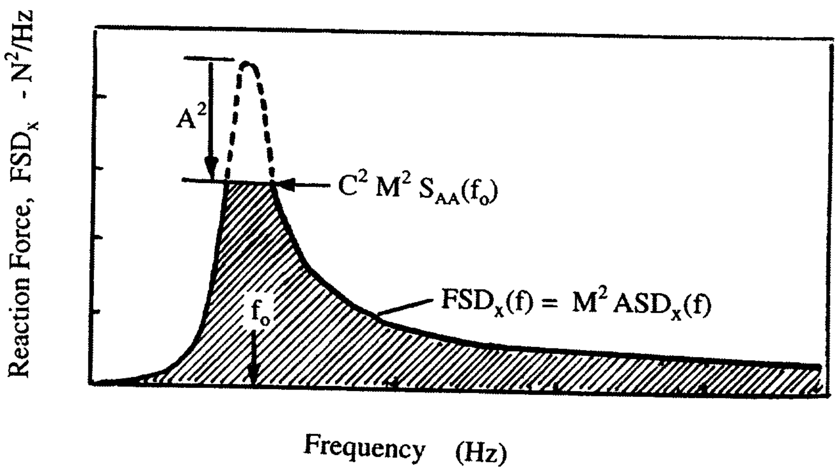

Figure 4. Force-Limited Responses of Single Resonant Mode Systems

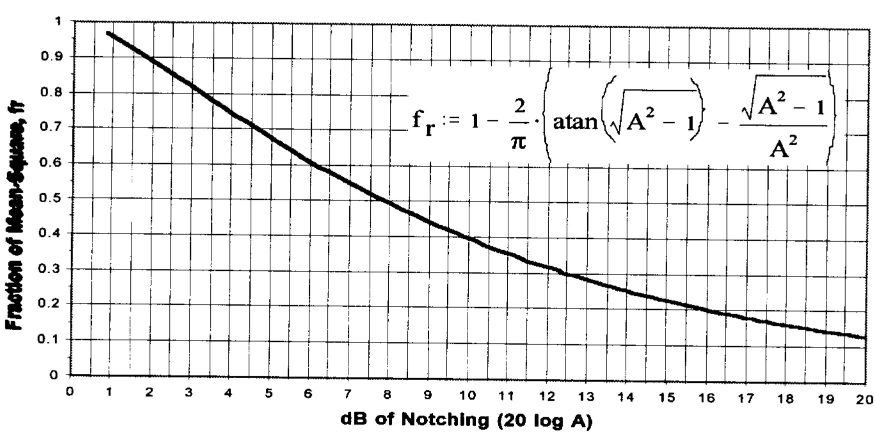

Figure 5. Notched Mean-Square Responses of SDOF Systems Subjected to Base Random Excitation