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STATISTICAL TECHNIQUES FOR DESCRIBING LOCALIZED VIBRATORY ENVIRONMENTS OF ROCKET VEHICLES

by Robert E. Barrett

George C. Marshall Space Flight Center
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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Continued)

LIST OF ILLUSTRATIONS (Concluded)

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SUMMARY

The variety of indescribable structural response characteristics such as structural dissimilarities, many-degree-of-freedom systems, non-linearities, impedance mismatches, etc. require statistical definitions to obtain a logical description of the total response within a particular section. The Saturn I Block I booster was divided into seven environmental regions and each region further subdivided into structure of essentially similar dynamic characteristics. The vibration measurements representing the response of the basic structure (data acquired from component response was not included) was statistically evaluated resulting in probability estimates on a spectral basis. Factors such as different types of structure, acoustic shadowing effects due to the one-way exhaust deflector, liquid and non-liquid loading, pressurized and unpressurized tankage are all treated. Thus statistical spectra are developed which will describe a variety of diverse cases. With these results discrete and random design and test requirements may be specified with statistical certainty for each particular location and structural type. A distinct advantage of treating each of these cases separately is that the higher confidence limits of a particular collection of data are not significantly greater than the mode value (point of maximum occurrence). If each major zone was not subzoned into similar structure, the mode and higher confidence limits would deviate significantly causing a highly conservative estimate in many cases. Thus, in many instances, over-design or overtesting would occur.

INTRODUCTION

Due to the impossibility of precise analytical descriptions of vehicle structure and response characteristics, specification of overall vehicle dynamic environments lends itself to statistical treatment. When dealing with a phenomenon that is random in nature the only logical approach is through statistical considerations. There is no statement concerning this type of process that may be made with exact accuracy. As the non-periodic process continues any statement of non-occurrence will eventually be exceeded. Further, a statistical analysis is less influenced by a single high value which may be the result of improper calibration than a maximum value approach. At the same time it does not completely ignore an extreme high or low value.

An initial approach to this problem was indicated by Reference [1]*. Further refinements to these procedures would require larger samples of data, thereby providing for more exact structural breakdowns, selective sampling, equal weighting factors in regard to each individual data location within a zone and more rigorous statistical evaluation. The concept of an adequate structural breakdown (subzoning) is important. Because of the completely different structural types within a major zone, overconservatism -- hence overdesign and testing may result if these structural characteristics are not considered. The necessary requisite of the Saturn statistical program required two separate statistical results -- one for discrete frequency presentation (used for specifying sinusoidal requirements) and one for random test requirements. These sophistications were made possible in the Saturn program because of the vast amount of vibration data collected from forty-one Saturn I Block I captive firings.

DESCRIPTION

To define the overall Saturn I vibration environment in a reasonable manner, the booster was separated into environmental zones. These major zones were further divided into subzones each of which essentially possessed similar dynamic characteristics (i.e., similar inertial, damping and elastic properties). The major zones of the cylindrical shells were subdivided into skin panels and skin stiffeners since the thin plates are significantly more elastic than the rigid frames. Also, the rocket engines were subzoned into three basic sections. These are 1) the combustion chamber section which comprises the basic engine structure, 2) the turbopump which creates a separate environment due to mounting location, impeller speeds and flow rates and 3) the actuator assembly which possesses unique dynamic characteristics because of its mounting configuration.

The vibration data which were used to define these structures had to meet the following requirements.

1. The data were required to represent a response of the basic structure which is the input to components and hardware mounted thereon. Measurements which were considered component responses were deleted from the program. In most cases the data were extracted from unloaded structure (i.e., structure which did not reflect the effects of component mass).

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* [ ] denotes reference

2. For the case of skin and stiffener structure, only the data sensing in the direction normal to the surface was used. This was done to provide a more precise definition of the most severe directional environment. For beam and engine structure no grouping by axes or direction was required. Because of the difficulty in defining the geometrical properties of this type of structure in regard to direction, separation by axes was not possible. Therefore, a bi-directional or tri-directional measurement would be considered as two or three respective data points within the population.

3. The fuel and LOX tanks were treated individually since these structures exhibited different skin thicknesses. Also the acoustic-vibration coupling effects due to dissimilar internal radiation resistance properties cause different structural responses.

4. The data were required to represent captive firing environments. Saturn flight vibratory environments would have to be evaluated independently of the captive environments due to the inherent differences in the sources of excitation.

Of the 2,800 measurements taken during Saturn captive firings approximately 1,200 satisfied the preceding criteria and were efficiently utilized for the establishment of Saturn environments.

The vibration data used for defining zonal environments were presented as rms acceleration divided by the acceleration of gravity ($\ddot{x}/g_o$). The spectral amplitudes were estimated through a 10 cps window utilizing a 5 second time slice and 2-1/2 second integrating time. Further, since all the data were acquired from static firings, the time response functions were assumed to possess self-stationary trends. Therefore, only one time slice per measurement was evaluated for each firing.

The measurements selected within a particular zone were digitized to relegate them into a useful form for analysis. This was accomplished by dividing the G_{rms} vs. frequency data into 50 cps bands. To provide a uniform method of digitizing, the peak value within each 50 cps band was used as a data point. Thus all data points taken from a particular frequency band constituted the population within a zone or sub-zone. The data were digitized from 50-2050 cps resulting in a total of 40 individual statistical populations for each subzone. A mock weighting factor was introduced so that each data location within a zone would be equally represented. Thus each point contributed an equal amount to the statistical results.

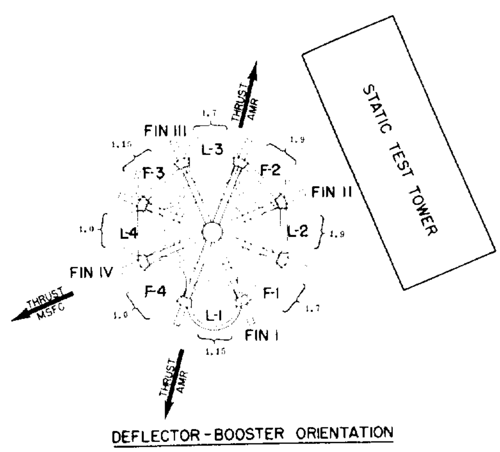

FIGURE 1

TOP VIEW OF SATURN BOOSTER SHOWING SOUND PRESSURE FACTOR USED FOR VARIOUS TANKS

Corrections were applied which acounted for the variance in sound pressures at various angles around the vehicle. These differences were created by the one-way exhaust deflector configuration as shown in Figure 1. Consequently, the skin and stiffener responses were normalized to Fin IV (exhaust direction). These corrections were accomplished by multiplying the acceleration response of tanks 1, 2, and 3 by:

$$G_4\,(\text{LOX}) = G_x \frac{p_4}{p_x} \tag{1a}$$

and

$$G_4\,(\text{fuel}) = G_x \frac{p_4}{p_x} \tag{1b}$$

where:
G_4 = the normalized acceleration response
G_x = the acceleration response of tanks 1, 2, or 3
p_4 = the measured rms acoustic pressure (20-2000 cps) at tank 4
p_x = the measured acoustic pressure corresponding to G_x.

Since the vehicle skin is principally excited by acoustic pressures and the LOX and fuel tanks are treated separately; this technique is valid assuming the sound pressure factor is uniform over the considered frequency range. The sound pressure factor is not utilized for purposes of statistically defining overall vehicle environments. However, for purposes of predicting upper stage and future vehicle environments, the factor was used in order to establish a uniform causal-effect relationship which may be utilized as reference criteria. Reference [2] provides a more thorough understanding of this line of reasoning.

The collection of data within each zonal frequency band were evaluated by statistical techniques to determine confidence limits. All frequency bands exhibited positively skewed density functions. This presented a problem since the required confidence limits could not be determined by standard methods. For large samples (one hundred and fifty or more), the cumulative percentiles of each probabilistic function may provide a satisfactory estimate of the universe. This could not be utilized as criterion, however, since the majority of subzones did not contain the required amount of data. Theoretically it is always possible to determine a function which will transform the skewed density into a normal one. Methods of transforming the skewed density functions into a known function by operating upon the sample data by logarithmic, reciprocal, or power functions were investigated. However, since each individual frequency band population exhibits unique characteristics the same transformation function did not apply to all samples. In other words, some of the skewed functions would normalize into a close approximation of a normal distribution while others would deviate significantly. This was validated by χ^2 goodness-of-fit tests which rejected approximately one-half of the transformation functions at the 5 per cent level of significance. For this reason the techniques of transformation were rejected.

A remaining alternative was to establish a criterion based upon higher order sample moments than are usually required. By operating upon some known density functions and sample functions extracted from zones with more than one hundred and fifty samples, it was possible to plot a curve of skewness versus the standard unit n_s for a 97.5 per cent confidence limit. Symbolically:

$$\bar{x} + n_s \sigma = 97.5\% \text{ C. L.} \tag{2}$$

where: $\bar{x}$ = the average value of the sample (1st moment about the origin)
σ = standard deviation of the sample (one-half power of the 2nd moment in respect to the mean).

For a normal density function n_s would be equal to 1.96. However, for a positively skewed function n_s is greater than 1.96. A second order exponential was found to fit the points with excellent accuracy. Hence, the modified unit n was found to be:

$$n_{.975} = 1.96\, e^{.2055 S - .0155 S^2} = n_s\, e^{.2055 S - .0155 S^2} \tag{3}$$

where S = the dimensionless skewness constant (3rd central moment divided by σ^3). This curve and the empirical points are shown in Figure 2. Thus each density function is modified by its degree of skewness. It was assumed that this perturbation of a normal function would extend into the lower confidence regions. Thus:

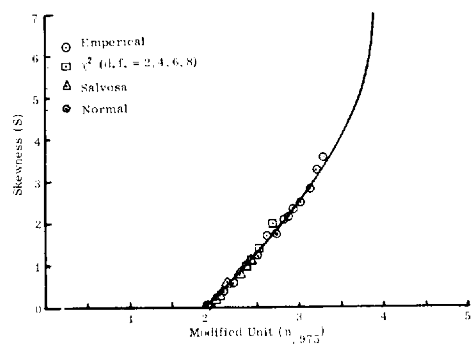

FIGURE 2.

SKEWNESS VERSUS MODIFIED UNIT FOR SEVERAL KNOWN PROBABILITY FUNCTIONS

$$\bar{x} + n_{.8413}\sigma = 84.13\% \text{ C. L.}$$

where:

$$n_{.8413} = \frac{n_{.975}}{1.96} = e^{.2055 S - .0155 S^2}.$$

This parameter was usually computed although it was of no specific use in the statistical program. In all cases the unbiased moment estimators (pg. 17) were used in computing the statistical quantities.

More accuracy may possibly have been obtained by describing the sample with an additional moment (kurtosis). Karl Pearson[1] presents a method for evaluating small sample densities utilizing the first four moments of the sample function. Investigations comparing Pearson's techniques with the above skewness concept show very small differences at the higher confidence levels [4]. This excellent comparison indicates the skewness concept to be highly satisfactory for the results required. Since this function is much easier to compute than Pearson's statistics, the modified unit, based upon the sample skewness, is always applied to the .975 percentile. This method is general enough to handle the diverse samples at hand and may be uniformly applied. However, a portion of Pearson's techniques are occasionally evaluated to obtain a more precise statistical description of a particular environmental zone. As a general point of interest the large majority of zonal probability functions correspond to a Pearson Type I freqeuncy curve. Figure 5 indicates the region in which 90 per cent of the zonal probabilities lie and the point of maximum occurrence.

Figures 6 and 7 show the Saturn I Block I booster indicating the environmental zones which were statistically evaluated. Figures 8 through 37 provide all the statistical spectra obtained from these zones. A total of 29 spectra are developed with separate statistics established for LOX and fuel tanks and liquid and non-liquid loading. Also the statistics with and without the s.p. factor (eq. 1) are indicated.

These spectra are utilized for purposes of defining resonant frequencies and enveloping these resonant peaks for discrete frequency design and testing criteria. In reality the total power under these curves are meaningless. No one individual measurement would possess the total spectrum shape and composite value that is illustrated by the confidence spectrum. For this reason additional analysis was required to obtain a realistic value for wide band composite energy which may subsequently be utilized for random spectra. For this criterion the composite value (50-2050 cps) of each measurement within a zone or sub-zone was treated as a data point within a statistical universe. Figure 3 shows a frequency histogram of rms composite values taken from Zone 3 aft fuel skirts. Since the 10 cps data are digitized in 50 cps increments, the composite value is:

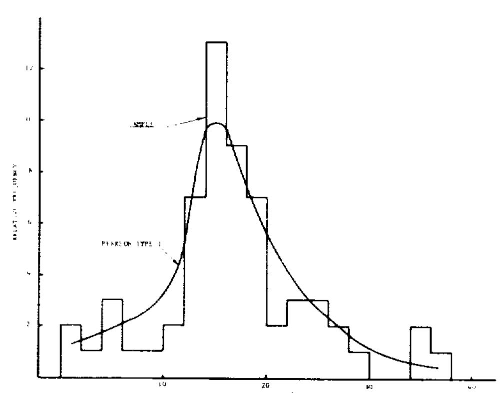

FIGURE 3.

FREQUENCY HISTOGRAM OF ZONE 3 FUEL COMPOSITE VALUES

$$G_{rms} = \sqrt{\sum_{50}^{2050} G_i^2 \left(\frac{\Delta f_i}{\Delta f_s}\right)} \qquad (4)$$

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[1] Refer to Appendix I for a description of Pearson's techniques.

where: G_i = the amplitude within a particular 50 cps band
Δf_i = the incremental bandwidth (50 cps)
Δf_s = the bandwidth of the spectral analysis (10 cps)

Thus:

$$G_{rms} = \sqrt{5 \sum_{50}^{2050} G_i^2} \qquad (5)$$

Further, the digital process selects the highest point within each Δf_i interval. Thus, the calculated composite value obtained from these points are conservatively high. For this reason the modified unit n is taken to be:

$$n_{.975}(\text{comp.}) = 1.64\, e^{.2055\, S - .0155\, S^2}. \qquad (6)$$

This value of n results in a realistic 97.5 per cent confidence composite. If the composite value could be computed by other means (such as an rms meter) then the modified unit would again be given by equation (3) for the .975 percentile. Thus it may be stated that only 2.5 per cent of the locations within a particular zone experience vibratory environments that exceed the specified limit. The resulting composite value is shaped according to the mean statistical spectrum. The mean spectrum is used as a shaping criterion based on the fact that many data points influenced the shape of the mean curve. The confidence limit spectra may have been significantly influenced by only one or two extremely high values. It would take a far greater number of high values to affect the shape of the mean spectrum. Thus it may truthfully be said that a "hump" in the mean spectrum represents a disproportionate amount of power within that bandwidth. Table I indicates the 97.5 per cent confidence composites developed from captive data. These zones and confidence composites correspond to the same zones shown in Figure 7. A derivation of a random test spectrum is provided in Appendix II as a guide to the usage of these values.

CONCLUSION

The statistical procedures provide an adequate description of the overall Saturn environments. With these results the basic structural response at any location on the vehicle may be specified with statistical certainty. Factors such as acoustical shadowing effects due to deflector geometry, liquid loading and structural dissimilarity have all been treated. Certainly, other factors remain which have not been specifically accounted for such as the measurement location in respect to the mode shape of the structure, variances in dynamic properties even within a sub-zone, etc. Treatment of these elements is highly complex. It is doubtful that the end results would justify the time and effort required even though a more precise description of the zonal environment may be established. Further, the increased number of measurements required to adequately define these additional parameters would be excessively large.

Identical statistical procedures may be performed on flight vehicles. However, since the dynamic environments are highly non-stationary a separate analysis would be required for each particular phase in flight (i.e., lift-off, mach one, etc.). The assumption of stationarity within each small time sequence is a reasonably valid statement. Thus separate statistics may be established for captive and flight phases. Comparison of the spectrum statistics between phases would yield much useful information regarding environmental diversities.

The resulting statistical environments are utilized for purposes of establishing Saturn environments and as a basis for predicting the localized environments of future rocket vehicles [2]. Methods of variance analysis are currently being applied to determine optimum requirements regarding data sampling and locations [5]. Data collected from the Saturn I Block I vehicles have been determined sufficient for satisfactorily defining the majority of zonal environments. The Saturn I Block II operational vehicles will provide an even more extensive array of vibration data since a total of twenty-six flights and approximately sixty-five captive firings are anticipated. Further, upper stage environments may be statistically determined which was not possible on the Block I dummy upper stages.

APPENDIX I. DESCRIPTION OF KARL PEARSON'S DENSITY FUNCTIONS

Karl Pearson, English biometrician, developed empirically [6] a system of generalized probability curves which satisfy most continuous distributions encountered in statistical realms. The density function p(x) of the random variable x fulfills the requirements of a differential equation given by:

$$\frac{dp(x)}{dx} = \frac{x - b_1}{b_0 + b_1 x + b_2 x^2}\, p(x) \tag{1}$$

which is the limiting case of the hypergeometric series where the origin for x is at the mean and b_0, b_1, b_2 are constants depending upon the type of density function. Only three constants appear in the differential equation because only the first four moments of a density function are considered ($\mu_1 = 0$). This equation satisfies most probabilistic functions such as the Gaussian, Beta, Chi-square, etc. If the moments in respect to the mean of a continuous density function are given by

$$\mu_r = \int_{\ell_1}^{\ell_2} x^r\, p(x)\, dx, \tag{2}$$

where ℓ_2 and ℓ_1 are the upper and lower extremities for x and $x = x_s - \overline{x}$, then the constants of equation (1) may be shown to be [7]:

$$b_1 = \frac{-\mu_3(\mu_4 + 3\mu_2^2)}{A} = \frac{-\sqrt{\mu_2\beta_1}(\beta_2 + 3)}{A'} \tag{3a}$$

$$b_0 = \frac{-\mu_2(4\mu_2\mu_4 - 3\mu^2)}{A} = \frac{-\mu_2(4\beta_2 - 3\beta_1)}{A'} \tag{3b}$$

$$b_2 = \frac{-(2\mu_2\mu_4 - 3\mu_3^2 - 6\mu_2^3)}{A} = \frac{-(2\beta_2 - 3\beta_1 - 6)}{A'} \tag{3c}$$

where:

$$A = 10\mu_4\mu_2 - 18\mu_2^3 - 12\mu_3^2$$

$$A' = 10\beta_2 - 18 - 12\beta_1$$

and

$$\beta_1 = \left(\frac{\mu_3}{\mu_2^{\frac{3}{2}}}\right)^2 \qquad \beta_2 = \frac{\mu_4}{\mu_2^2} \tag{4}$$

Further to obtain an unbiased estimate from a small sample:

$$\beta_1 = \frac{N(N-1)}{(N-2)^2}\hat{\beta}_1 \qquad \beta_2 = \frac{(N-1)}{N}\frac{\left[(N^2-2N+3)-(6N-9)\frac{1}{\hat{\beta}_2}\right]\hat{\beta}_2}{(N-2)(N-3)} \tag{5}$$

where N is the sample size and $\hat{\beta}_1$ and $\hat{\beta}_2$ are the raw moment estimators.

Equation (1) may be rewritten as:

$$\frac{d(\log p(x))}{dx} = \frac{x - b_1}{b_0 + b_1 x + b_2 x^2} \tag{6}$$

The density function p(x) may now be found by integrating the right-hand side of the above equation. The denominator will possess roots which are real of like (or opposite) signs or imaginary. Consequently, three (3) main types of density functions will evolve. The derivation of these three types are provided in References [8] and [9]. In addition, the Pearson types are broken down into transition classes which are special cases of the main types. With these transition types, a total of twelve (12) Pearson density functions are developed. These twelve types are shown in Figure 4 with y = p(x).

Pearson further developed a criterion κ which may be utilized to determine the main or transition types. This criterion is based upon the first four (4) moment estimators. Symbolically:

$$\kappa = \frac{\beta_1(\beta_2+3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} \tag{7}$$

It is possible to draw a diagram plotting β_1, β_2 to rectangular axes delineating different areas, curves or points in the plane which can be associated with particular types of Pearson probability functions. A chart of this sort is presented in Figure 5 covering all twelve Pearson types. The line $\beta_2 - \beta_1 - 1 = 0$ forms an upper boundary of the Pearson family of curves. No combination of population moments can fall above this limit. The

Pearson Type	Equation With Origin At Mean	Criterion	Remarks
MAIN TYPES			
I	$y = y_e\left(1 + \frac{x}{A_1}\right)^{m_1}\left(1 - \frac{x}{A_2}\right)^{m_2}$ where $\frac{(m_1 + 1)}{A_1} = \frac{(m_2 + 1)}{A_2}$	κ negative	Limited range; skew; usually bell-shaped, but may be U-shaped, J-shaped or twisted J-shaped.
IV	$y = y_e\left[1 + \left(\frac{x}{a} - \frac{\nu}{r}\right)^2\right]^{-m} e^{-\nu \tan^{-1}\left(\frac{x}{a} - \frac{\nu}{r}\right)}$ where $r = 2m - 2$	$0 < \kappa < 1$	Unlimited range; skew; bell-shaped.
VI	$y = y_e\left(1 + \frac{x}{A_1}\right)^{-q_1}\left(1 + \frac{x}{A_2}\right)^{q_2}$ where $\frac{(q_1 - 1)}{A_1} = \frac{(q_2 + 1)}{A_2}$	$\kappa > 1$	Unlimited range in one direction; skew; bell-shaped, but may be J-shaped.
TRANSITION TYPES			
"Normal Curve"	$y_e e^{-x^2/2\sigma^2}$	$\kappa = 0,\ \beta_1 = 0,\ \beta_2 = 3$	Unlimited range; symmetrical; bell-shaped.
II	$y = y_e\left(1 - \frac{x^2}{a^2}\right)^{m}$	$\kappa = 0,\ \beta_1 = 0,\ \beta_2 < 3$	Limited range; symmetrical; usually bell-shaped, but U-shaped when $\beta_2 < 1.8$.
VII	$y = y_e\left(1 + \frac{x^2}{a^2}\right)^{-m}$	$\kappa = 0,\ \beta_1 = 0,\ \beta_2 > 3$	Unlimited range; symmetrical; bell-shaped.

FIGURE 4a. PEARSON FAMILY OF FREQUENCY CURVES

Pearson Type	Equation With Origin At Mean	Criterion	Remarks
III	$y = y_e \left(1 + \frac{x}{A}\right)^{p} e^{-\gamma x}$ where $A = \frac{(p+1)}{\gamma}$ and $p = \gamma a$	$2\beta_2 = 6 + 3\beta_1$	Unlimited range in one direction; usually bell-shaped, but may be J-shaped.
V	$y = y_e \left(1 + \frac{x}{A}\right)^{-p} e^{\left(\frac{p-2}{1+x/A}\right)}$	$\kappa = 1$	Unlimited range in one direction; bell-shaped.
VIII	$y = y_e \left(1 + \frac{x}{A}\right)^{-m}$	κ negative; $\lambda = 0$ $5\beta_2 - 6\beta_1 - 9$ negative	Range from infinite ordinate at $-a(1 - m)/(2 - m)$ to finite ordinate at $a/(2 - m)$.
IX	$y = y_e \left(1 + \frac{x}{A}\right)^{m}$	κ negative; $\lambda = 0$ $5\beta_2 - 6\beta_1 - 9$ positive, $2\beta_2 - 3\beta_1 - 6$ negative	Range from $x = -a(m + 1)/(m + 2)$ where $y = 0$ to $x = a/(m + 2)$ where $y = y_e$.
X	$y_e e^{-\frac{x}{\sigma}}$	$\beta_1 = 4,\ \beta_2 = 9$	Exponential from finite ordinate at $-\sigma$ to infinitesimal ordinate at ∞; J-shaped.
XI	$y = y_e \left(1 + \frac{x}{A}\right)^{-m}$	$\kappa > 1,\ \lambda = 0,\ 2\beta_2 - 3\beta_1 - 6$ positive	J-shaped; starts at $x = -b/(m - 2)$ where ordinate is finite. A in this equation is opposite in sign to A for Type VIII.
XII	$y = y_e \left(1 + \frac{x}{A_1}\right)^{m_1} \left(1 - \frac{x}{A_2}\right)^{m_2}$	$5\beta_2 - 6\beta_1 - 9 = 0$	Twisted J-shaped; this type is a special case of Type I where m_1 and m_2 are equal numerically and are each <1 but of opposite sign.

The parameters given in the Criterion column are defined as follows:

$$\kappa = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)}, \quad \beta_1 = \frac{\mu_3^2}{\mu_2^3}, \quad \beta_2 = \frac{\mu_4}{\mu_2^2} \quad \lambda = \frac{(4\beta_2 - 3\beta_1)(10\beta_2 - 12\beta_1 - 18)^2 - B_1(B_2 + 3)^2(8\beta_2 - 9\beta_1 - 12)}{(3\beta_1 - 2\beta_2 + 6)[\beta_1(\beta_2 + 3)^2 + 4(4\beta_2 - 3\beta_1)(3\beta_1 - 2\beta_2 + 6)]}.$$

FIGURE 4b. PEARSON FAMILY OF FREQUENCY CURVES

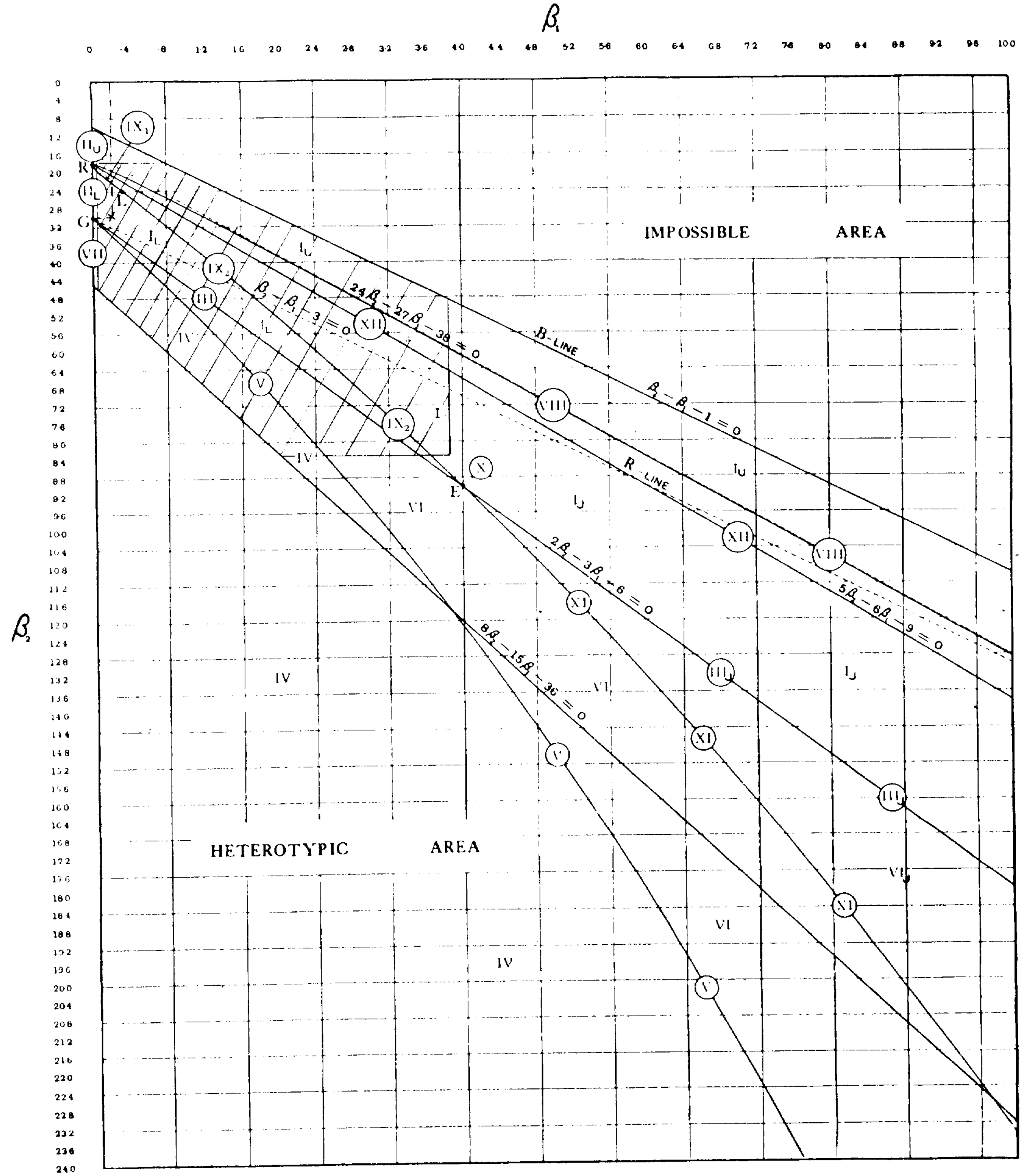


FIGURE 5. GRAPH OF PEARSON FREQUENCY CURVES-ALSO INDICATES REGION OF 90 PERCENT OCCURRENCE AND POINT OF MAXIMUM OCCURRENCE (0.20, 2.95) FOR SATURN SPECTRAL POPULATIONS

heterotypic area is where the parameters of Types IV and VI become unreliable, although a good fit might still be obtained.

The goodness-of-fit of the sample population to a particular Pearson type may be carried out by several methods. The standard Chi-square fit criterion may be applied to the number of observations in each sample class interval relative to the expected observations. However, for small samples this technique has two drawbacks. Chi-square requires at least twenty class intervals with five or more observations in each group. Thus, for small samples, the process of grouping significantly affects the resulting statistic. Robinson [10] suggests the use of a non-parametric statistic Kolmogorov-Smirnov for testing small sample fits. This function is not affected by grouping and is completely distribution-free. Symbolically:

$$\text{K - S PLUS} = \max \{S(x) - P(x)\} \tag{8a}$$

$$\text{K - S ABS} = \max \left| S(x) - P(x) \right| \tag{8b}$$

Here S(x) is the sample cumulative distribution defined as:

$$S(x) = \begin{cases} 0, & x < x_1 \\ \frac{i}{N}, & x_i \leq x < x_{(i+1)} \\ 1, & x_N \leq x \end{cases} \tag{9}$$

where x_1, x_2, ...x_N are the ordered sample elements

and P(x) is the Pearson distribution function defined as:

$$P(x) = \int_{\ell_1}^{x} p(x)\, dx. \tag{10}$$

Tables of the K - S statistics are presented in a manner similar to χ^2 tables. The degrees-of-freedom and a particular percentage point define the K - S number. This number may then be compared to a calculated K - S statistic relative to the required hypothesis.

Pearson's techniques used in conjunction with an appropriate goodness-of-fit statistic result in a satisfactory method of evaluating certain properties of statistical aggregates. It should be realized, however, that no amount of evaluation will yield information about small samples which it does not contain. Statistical models should be established with the thought in mind that the very nature of small sampling errors may provide erroneous information. The fact remains that large samples will nearly always be a closer approximation to the true universe. Inexactnesses arising from utilization of Pearson's techniques are the effect of not considering higher moments and the method of fitting probability curves by moments since the sample moments are only estimates of the population moments. In some cases a more accurate measure of the frequency curve parameters may be determined from the maximum likelihood principle [11]. In essence this function will produce the most "efficient" parameter estimate available from that particular sample. If these limitations are realized, the Pearson system may be adequately utilized. This system is general enough to treat a variety of diverse samples and may be uniformly applied to most probability functions

APPENDIX II. EXAMPLE OF A RANDOM TEST DERIVATION

Zone 3 Aft Fuel Skirts (Ring Frames and Stringers)

A list of the composite values (50 - 2050 cps) of vibration measurements taken on ring frames and stringers are given below in ascending order. These values were calculated from the digitized spectra (using eq. 5) acquired from Zone 3 aft fuel skirts.

1.46	11.86	14.26	15.76
1.75	12.15	14.53	15.76
3.70	12.18	14.56	15.93
4.38	12.31	14.82	16.09
4.76	12.99	15.01	16.09
5.61	13.60	15.02	16.11
7.67	13.68	15.24	16.19
8.13	13.79	15.41	16.89
10.58	14.02	15.43	16.94
17.18	19.28	24.28	
17.19	19.64	24.70	
17.56	20.14	26.86	
18.17	20.75	27.11	
18.30	22.10	29.98	
18.37	22.64	34.47	
18.68	22.95	34.66	
18.69	24.30	37.37	

The corresponding statistical description of this collection of data is given by the unbiased estimators:

$$\overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i = 16.67$$

$$\sigma = \sqrt{\frac{N}{N-1}}\, \hat{\sigma} = 7.36$$

$$S = \frac{\sqrt{N(N-1)}}{(N-2)} \hat{S} = +.485$$

where the biased estimates ($\wedge$) are given as:

$$\hat{\sigma} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} x_i^2 - \bar{x}^2}$$

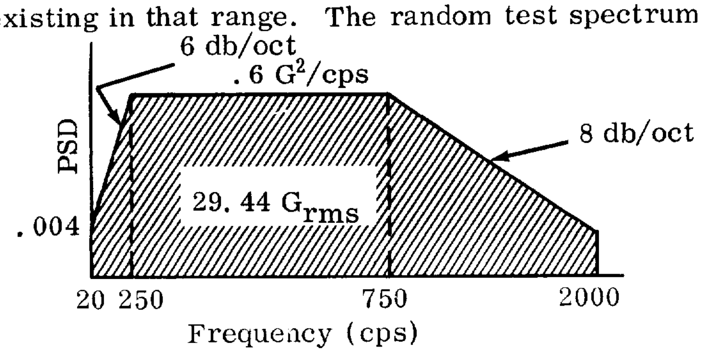
$$\hat{S} = \frac{\frac{1}{N} \sum_{i=1}^{N} x_i^3 - 3\bar{x}\left(\frac{1}{N} \sum_{i=1}^{N} x_i^2\right) + 2\bar{x}^3}{\hat{\sigma}^3} .$$

Since these composite values were computed from digitized spectra in which the highest point within a 50 cps increment was chosen, the modified unit n (comp.) is given by equation (6). Thus from equation (2):

$$97\tfrac{1}{2}\text{ \% confidence composite} = 16.67 + (7.36)(1.64)e^{.2055(.485) - .0155(.485)^2}$$

$$= 29.44 \ G_{rms}.$$

With the true 97.5 per cent confidence limit established as 29.44 G_{rms} the test spectrum is shaped according to the mean statistical spectrum (Fig. 28). Note that frequencies of 1350 - 1900 cps show a significant variation in value for the 97.5 per cent and 84.13 per cent c.ℓ. spectra, but the mean spectrum remains essentially unchanged. This implies that only one or two high values out of sixty are contained in these frequency bands. The 415 cps, 765 cps, and adjacent frequencies exhibit relatively high values on the 97.5 per cent, 84.13 per cent and mean spectra indicating a significant number of high values existing in that range. The random test spectrum is shown below:



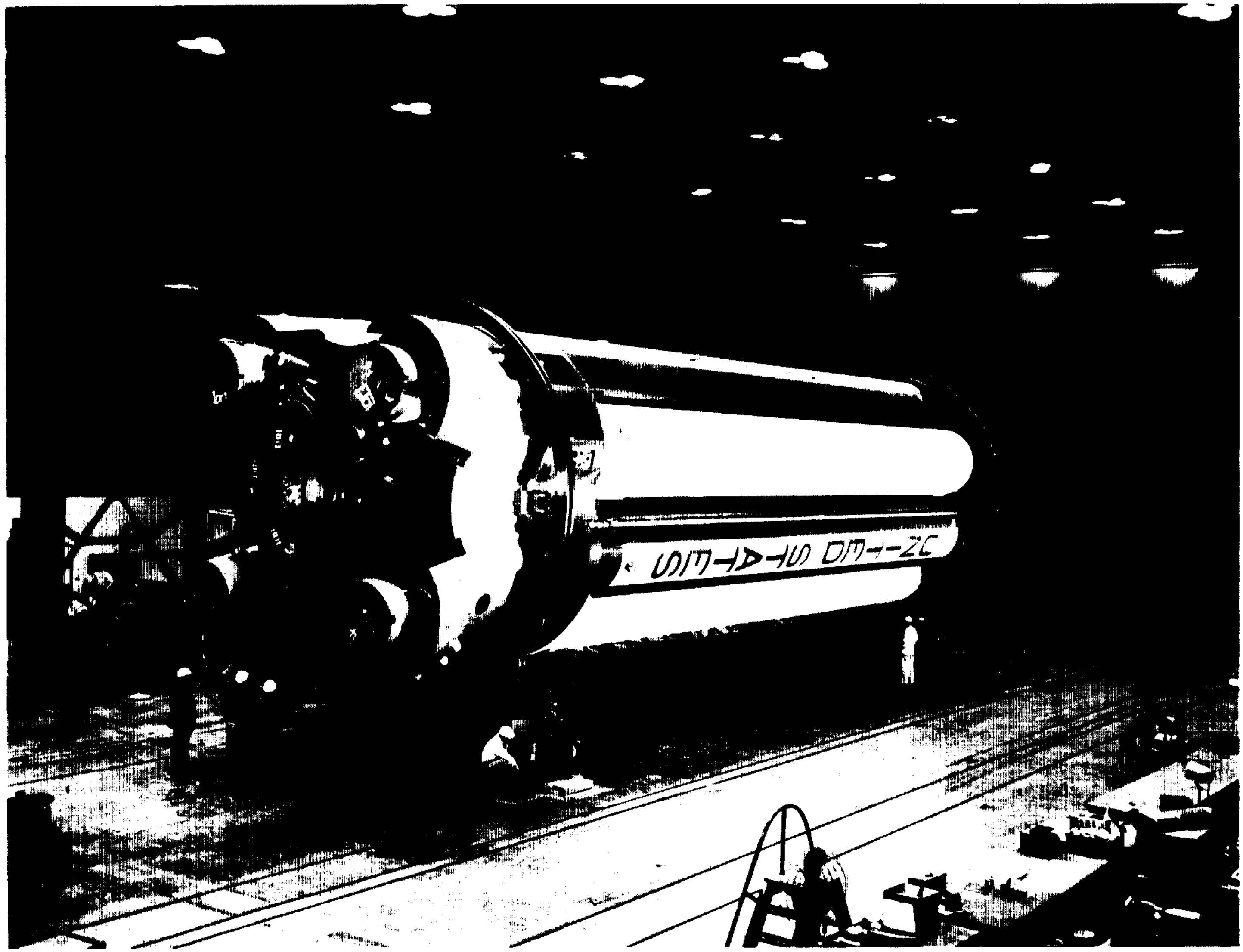


FIGURE 6. SATURN I BLOCK I BOOSTER

ZONE	DESCRIPTION		NO. OF MEAS. USED IN EVALUATION
7	SPIDER BEAM		60
6-LOX	FWD. LOX SKIRTS	(DATA OBTAINED FROM SKIN PANELS ONLY)	20
6-FUEL	FWD. FUEL SKIRTS	(DATA OBTAINED FROM SKIN PANELS ONLY)	20
6-4-FUEL	FWD. FUEL BULKHEAD		15
5-LOX	PRESSURIZED LOX TANK	(LIQUID & NON-LIQUID LOADED RESPONSES TREATED SEPARATELY)	20
5-FUEL	PRESSURIZED FUEL TANK	(LIQUID & NON-LIQUID LOADED RESPONSES TREATED SEPARATELY)	20
3-LOX	AFT LOX SKIRTS	(DATA OBTAINED FROM RING FRAMES & STRINGERS ONLY)	30
3-FUEL	AFT FUEL SKIRTS	(DATA OBTAINED FROM RING FRAMES & STRINGERS ONLY)	60
3-4-FUEL	AFT FUEL BULKHEAD		15
2	THRUST STRUCTURE		132
1-1	H-1 ENGINE TURBOPUMP (COMPONENTS)		136
1-1-0	H-1 ENGINE TURBOPUMP (GEAR CASE ONLY)		180
1-2	H-1 ENGINE COMBUSTION CHAMBER (COMPONENTS)		155
1-2-0	H-1 ENGINE CHAMBER DOME ONLY		188
1-2-C	ACTUATOR ASSEMBLY		174

FIGURE 7. SATURN I BLOCK I ZONAL BREAKDOWN (INDICATING ZONES WHICH WERE STATISTICALLY EVALUATED)

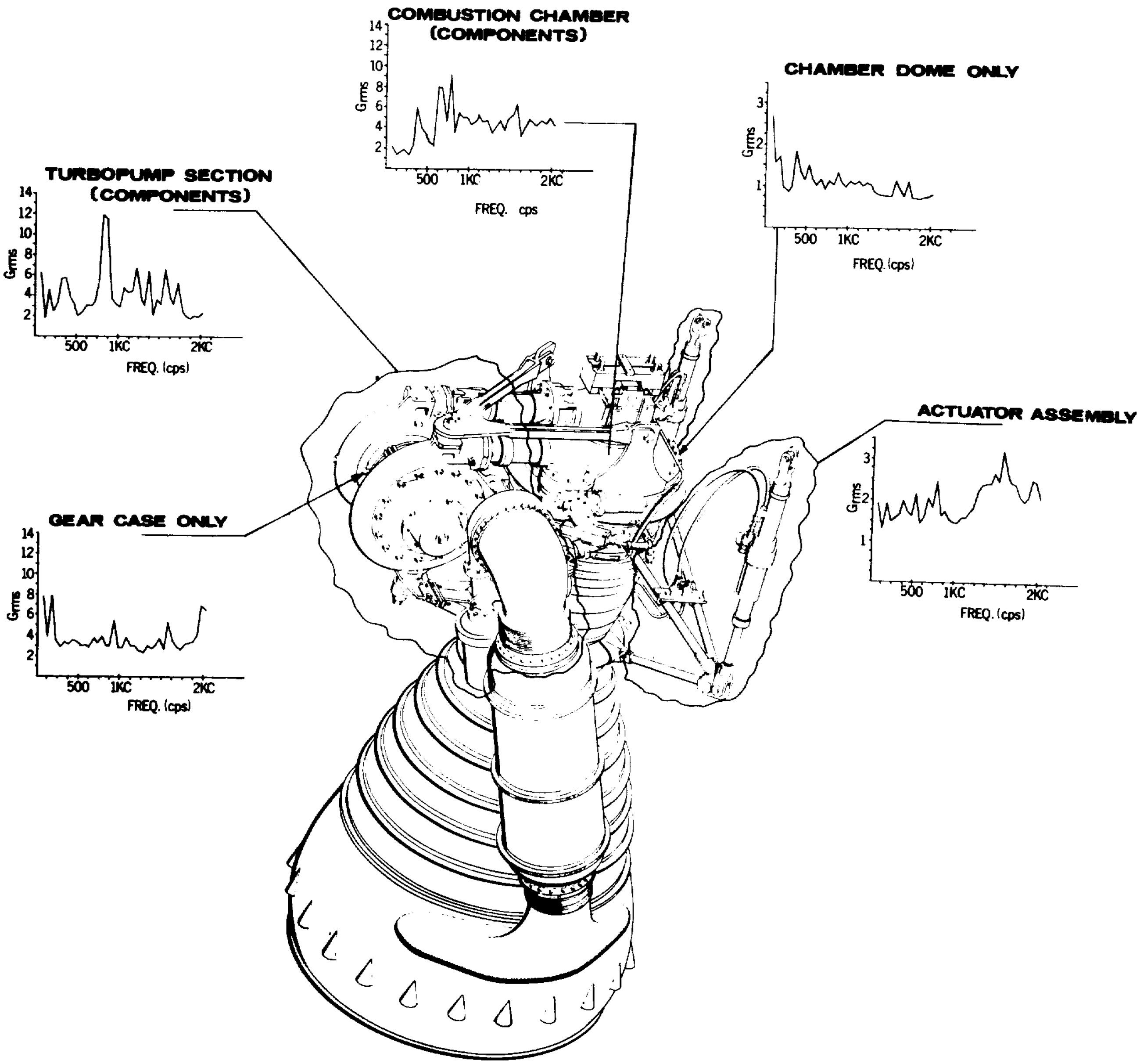


FIGURE 8. RESULTS OF H-1 ENGINE VIBRATION STATISTICAL ANALYSIS INDICATING THE 97-1/2 PERCENT CONFIDENCE SPECTRA

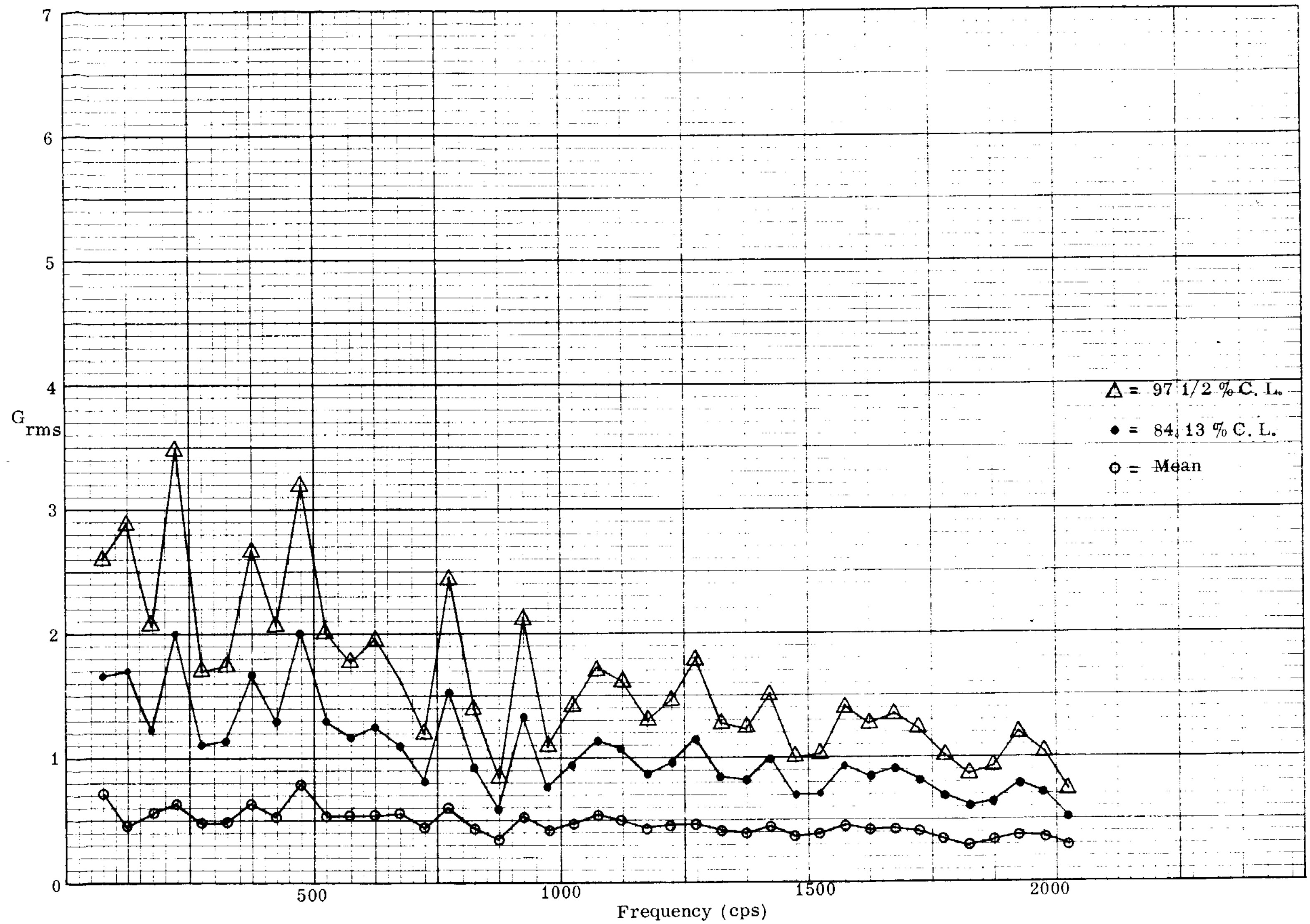


FIGURE 9. SATURN I BLOCK I (CAPTIVE) ZONE 7 SPIDER BEAM

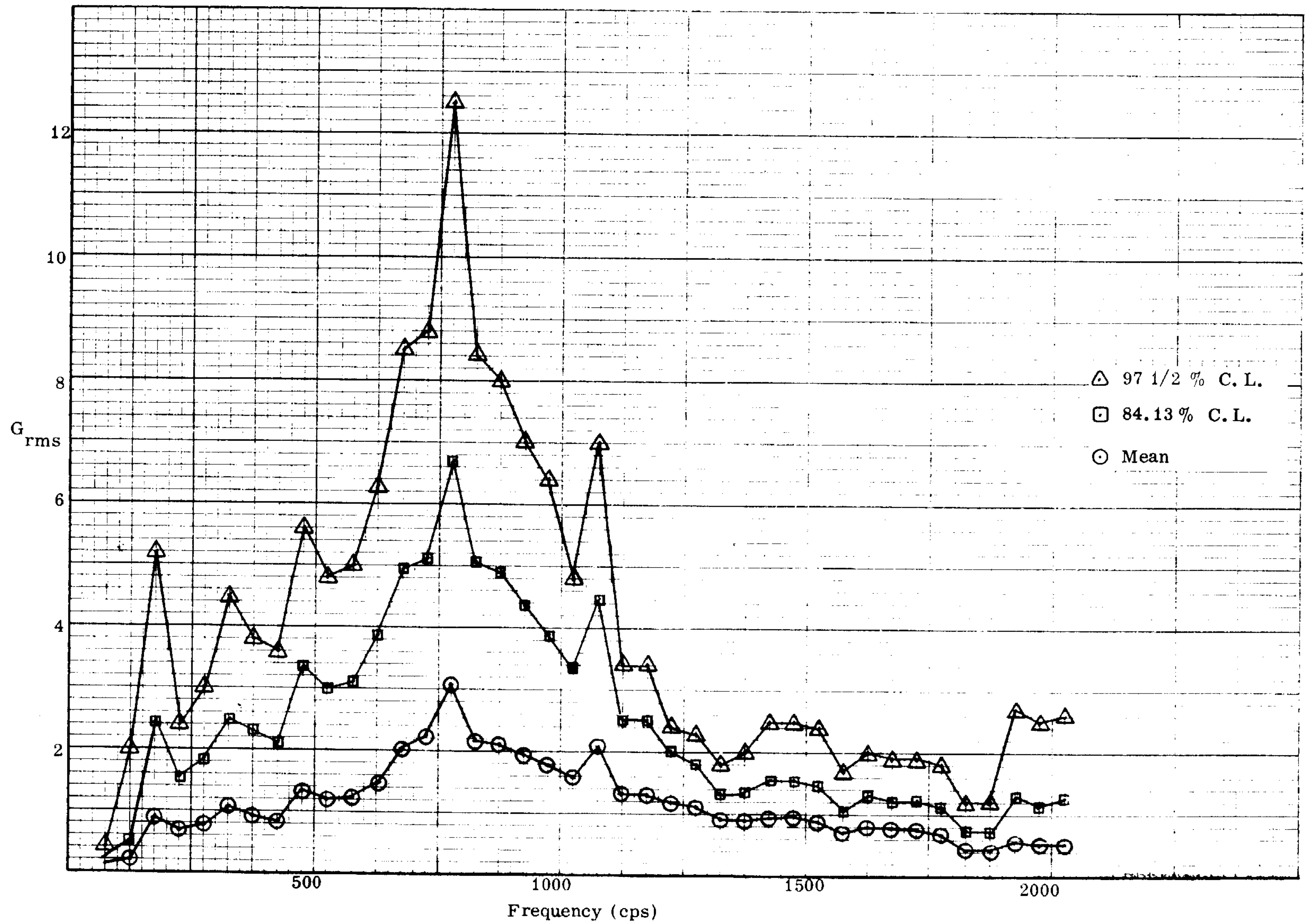


FIGURE 10. SATURN I BLOCK I (CAPTIVE) ZONE 6 LOX (WITH s.p. FACTOR) FORWARD LOX SKIRTS

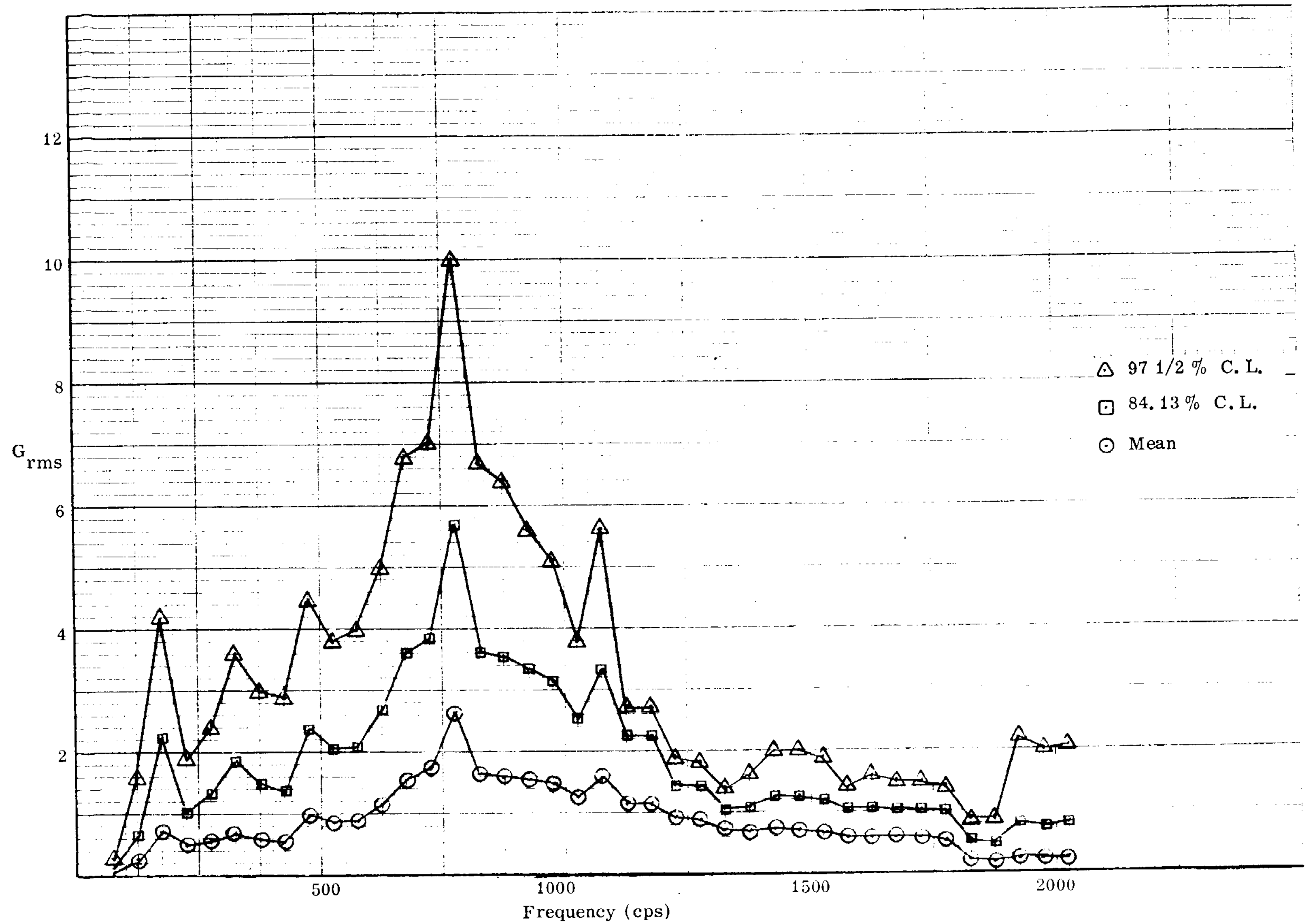


FIGURE 11. SATURN I BLOCK I (CAPTIVE) ZONE 6 LOX (WITHOUT s.p. FACTOR) FORWARD LOX SKIRTS

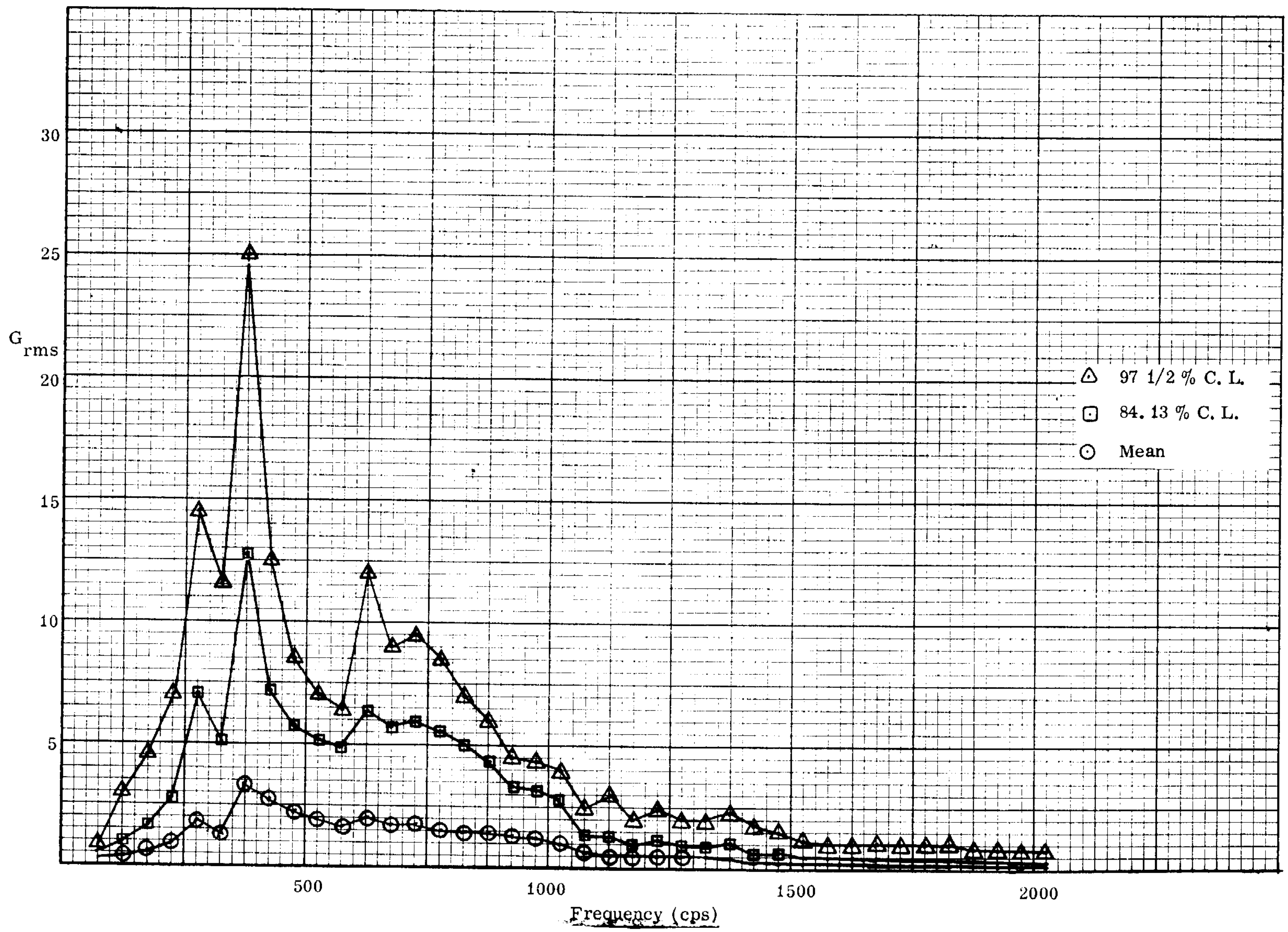


FIGURE 12. SATURN I BLOCK I (CAPTIVE) ZONE 6 FUEL (WITH s. p. FACTOR) FORWARD FUEL SKIRTS

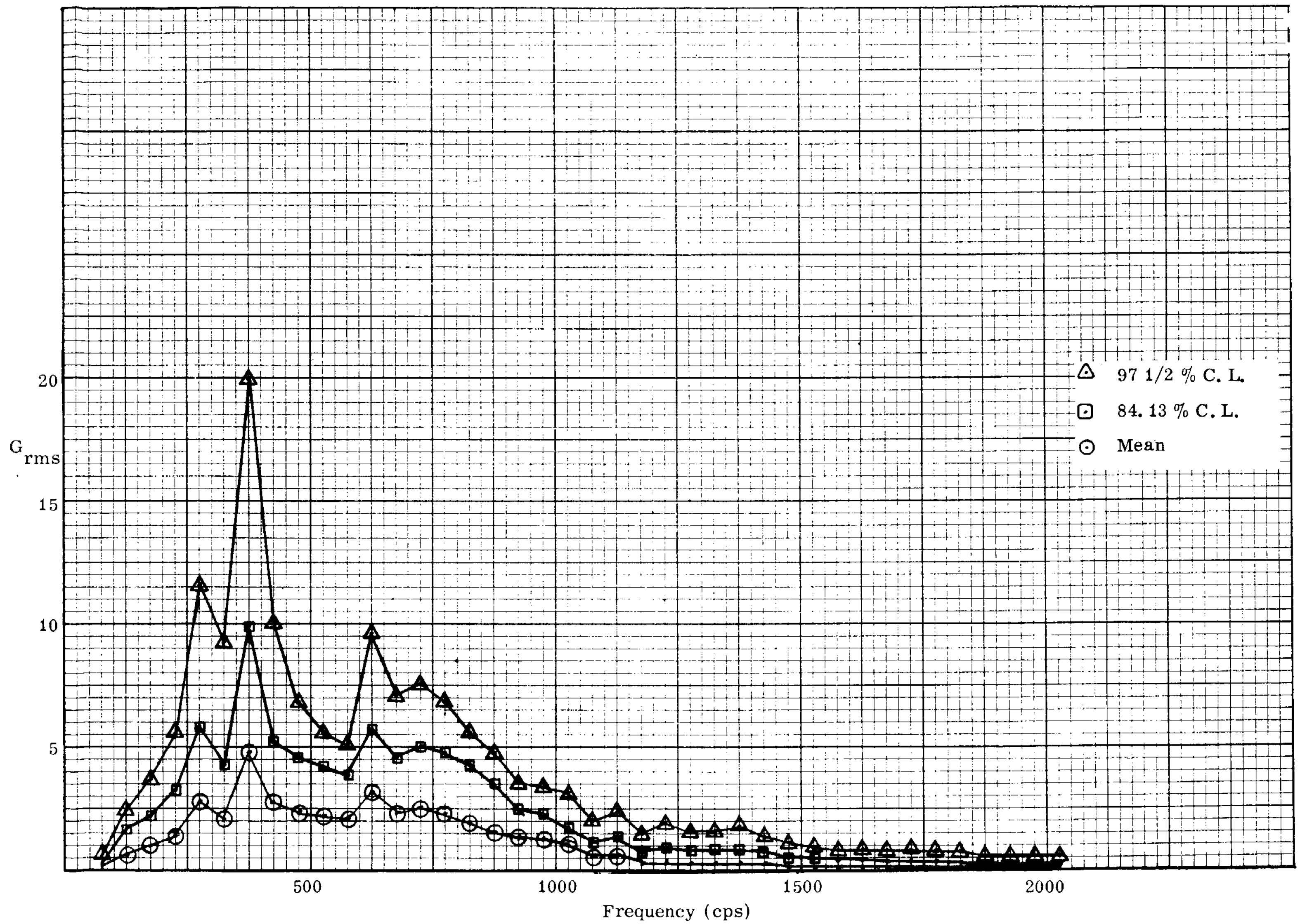


FIGURE 13. SATURN I BLOCK I (CAPTIVE) ZONE 6 FUEL (WITHOUT s. p. FACTOR) FORWARD FUEL SKIRTS

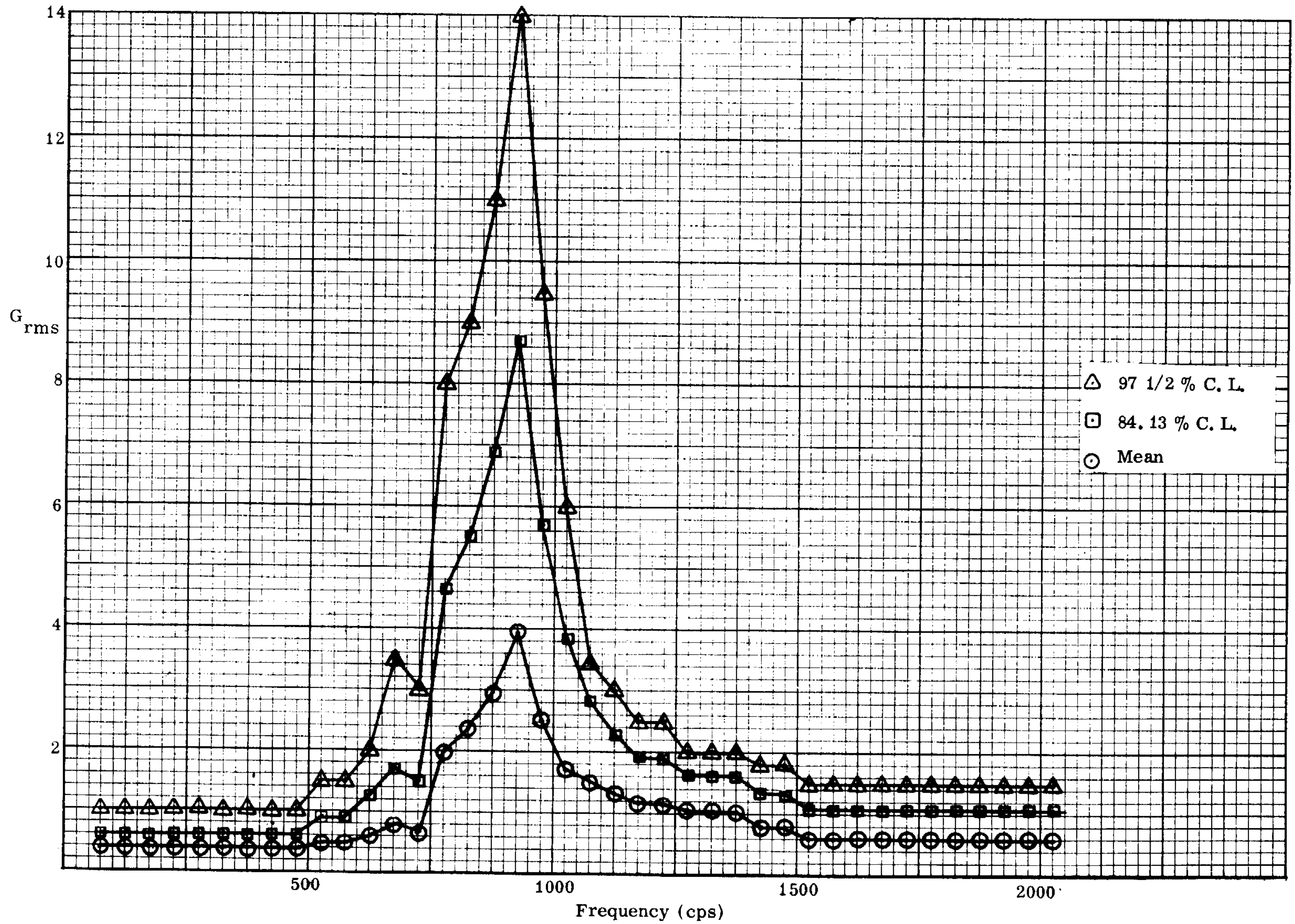


FIGURE 14. SATURN I BLOCK I (CAPTIVE) ZONE 6-4 FUEL FORWARD FUEL BULKHEAD

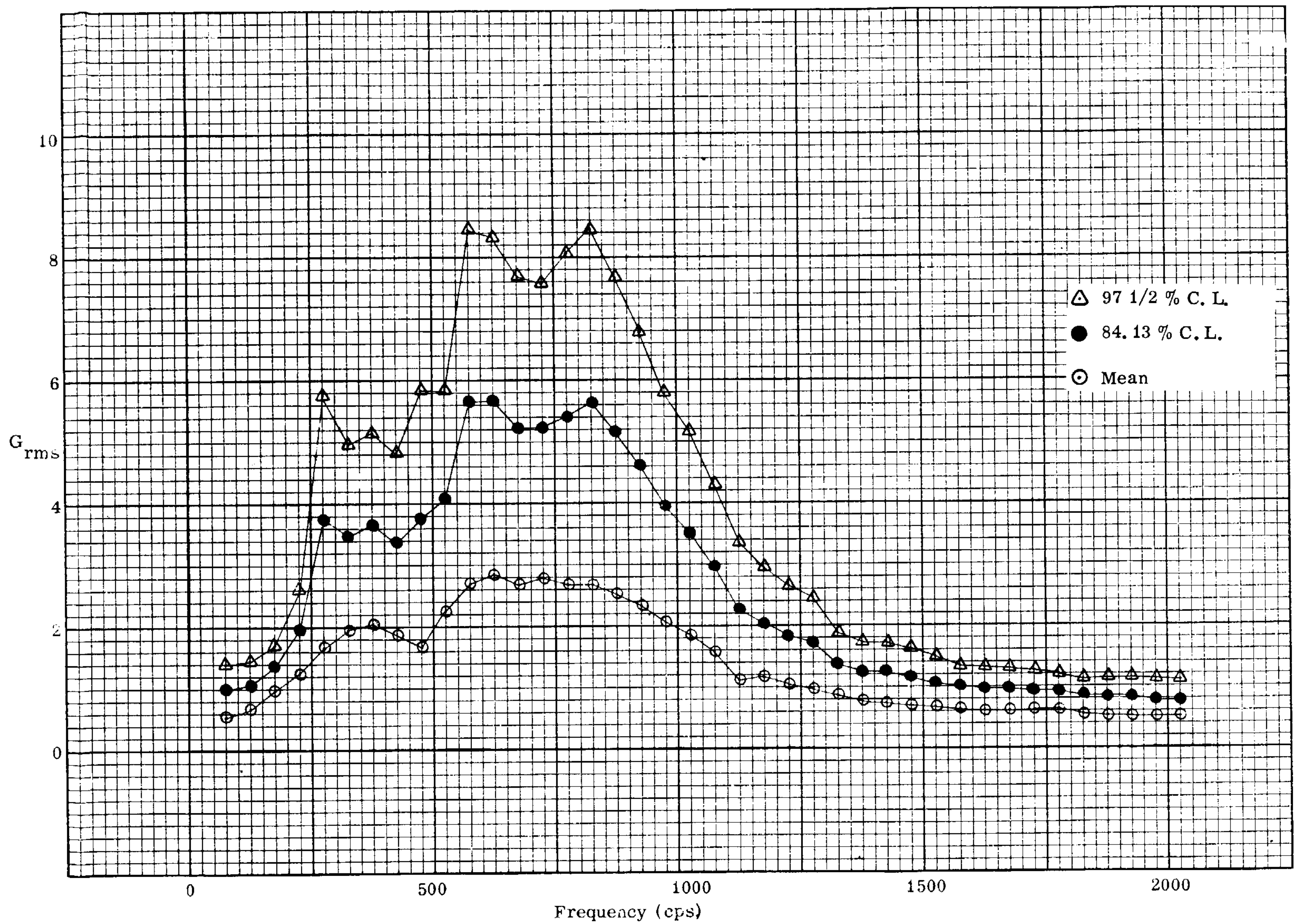


FIGURE 15. SATURN I BLOCK I (CAPTIVE) ZONE 5 LOX-LIQUID LOADING (WITH s.p. FACTOR) PRESSURIZED LOX TANKS

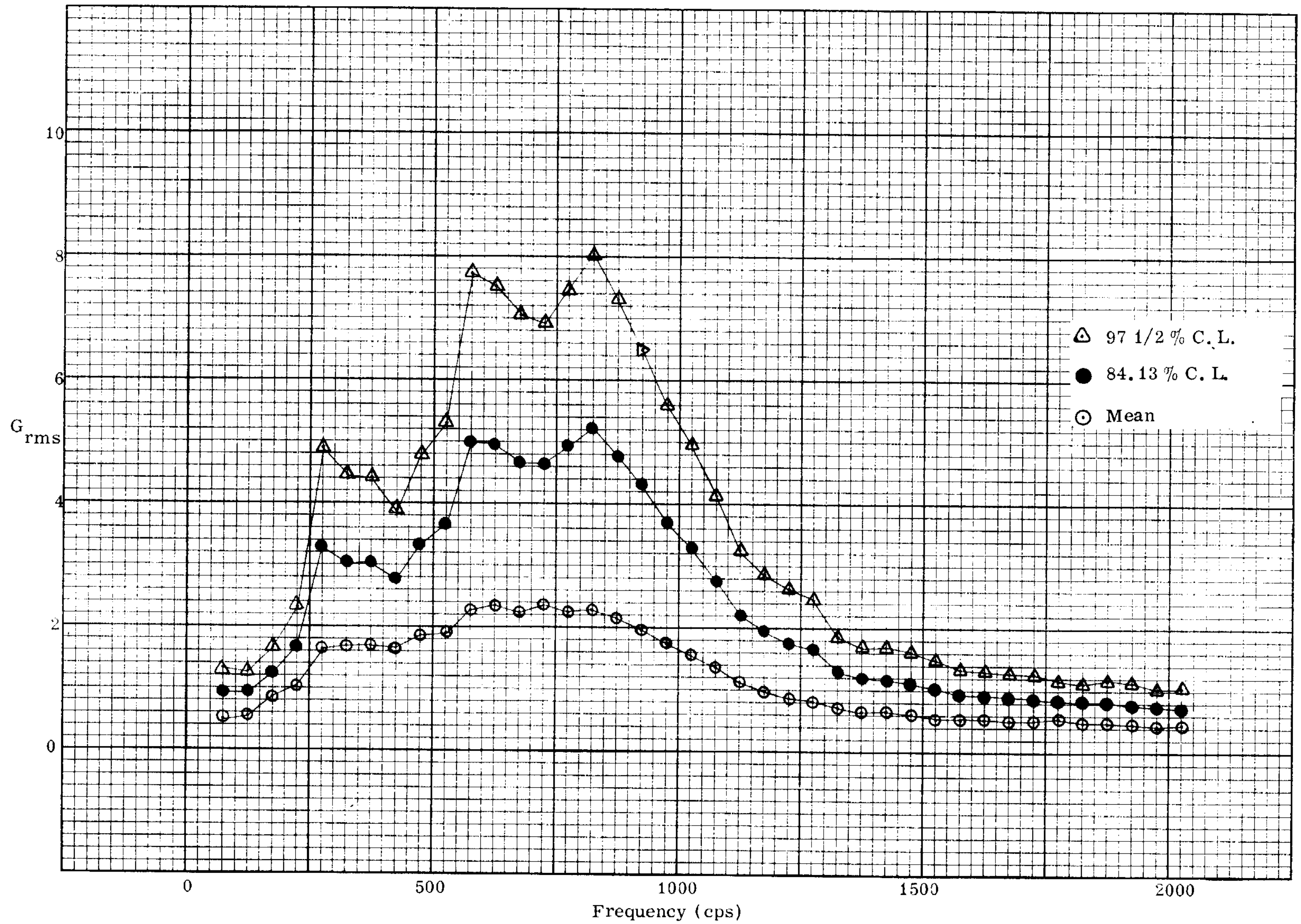


FIGURE 16. SATURN I BLOCK I (CAPTIVE) ZONE 5 LOX-LIQUID LOADING (WITHOUT s.p. FACTOR) PRESSURIZED LOX TANKS

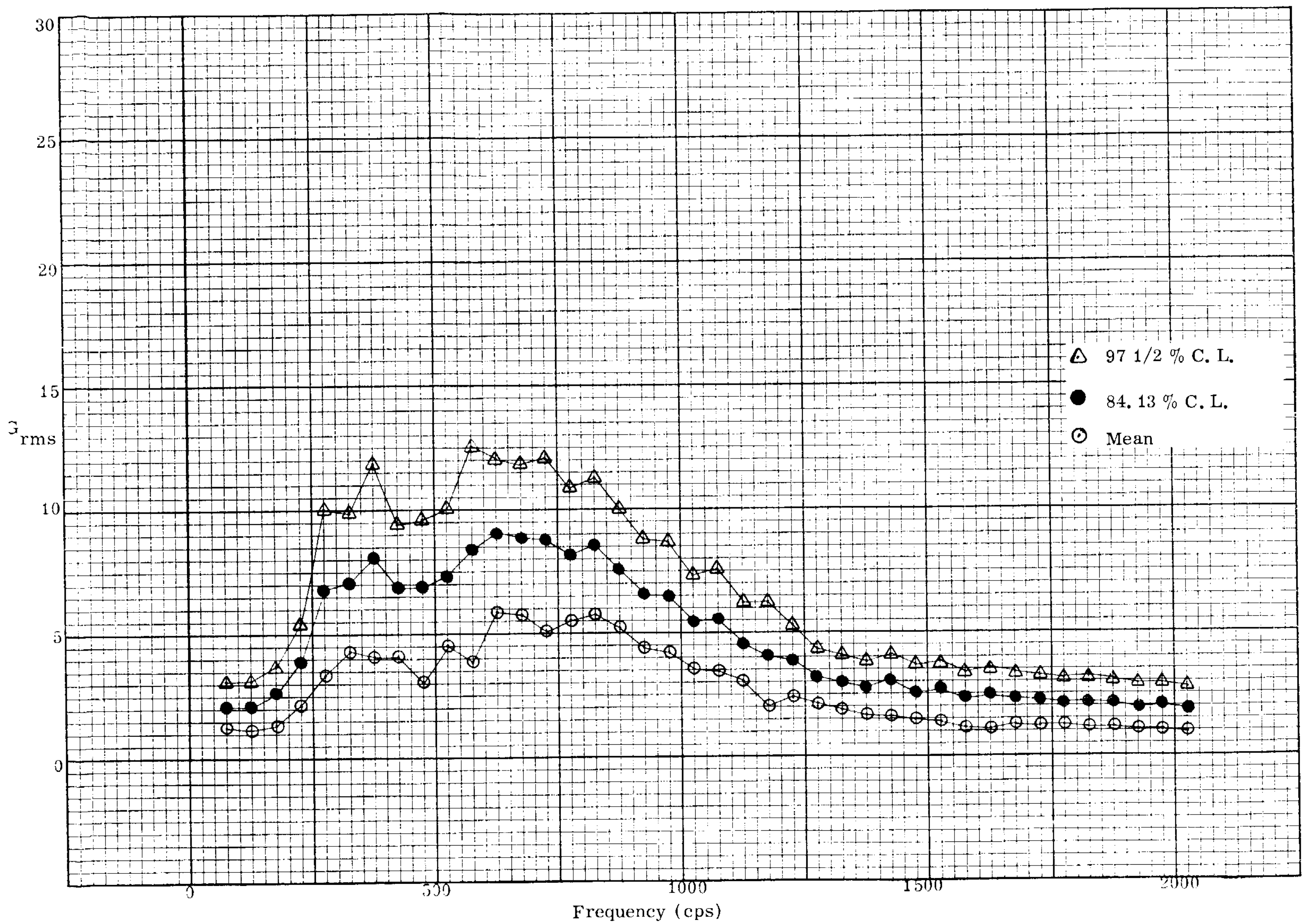


FIGURE 17. SATURN I BLOCK I (CAPTIVE) ZONE 5 LOX-NO LIQUID LOADING (WITH s.p. FACTOR) PRESSURIZED LOX TANKS

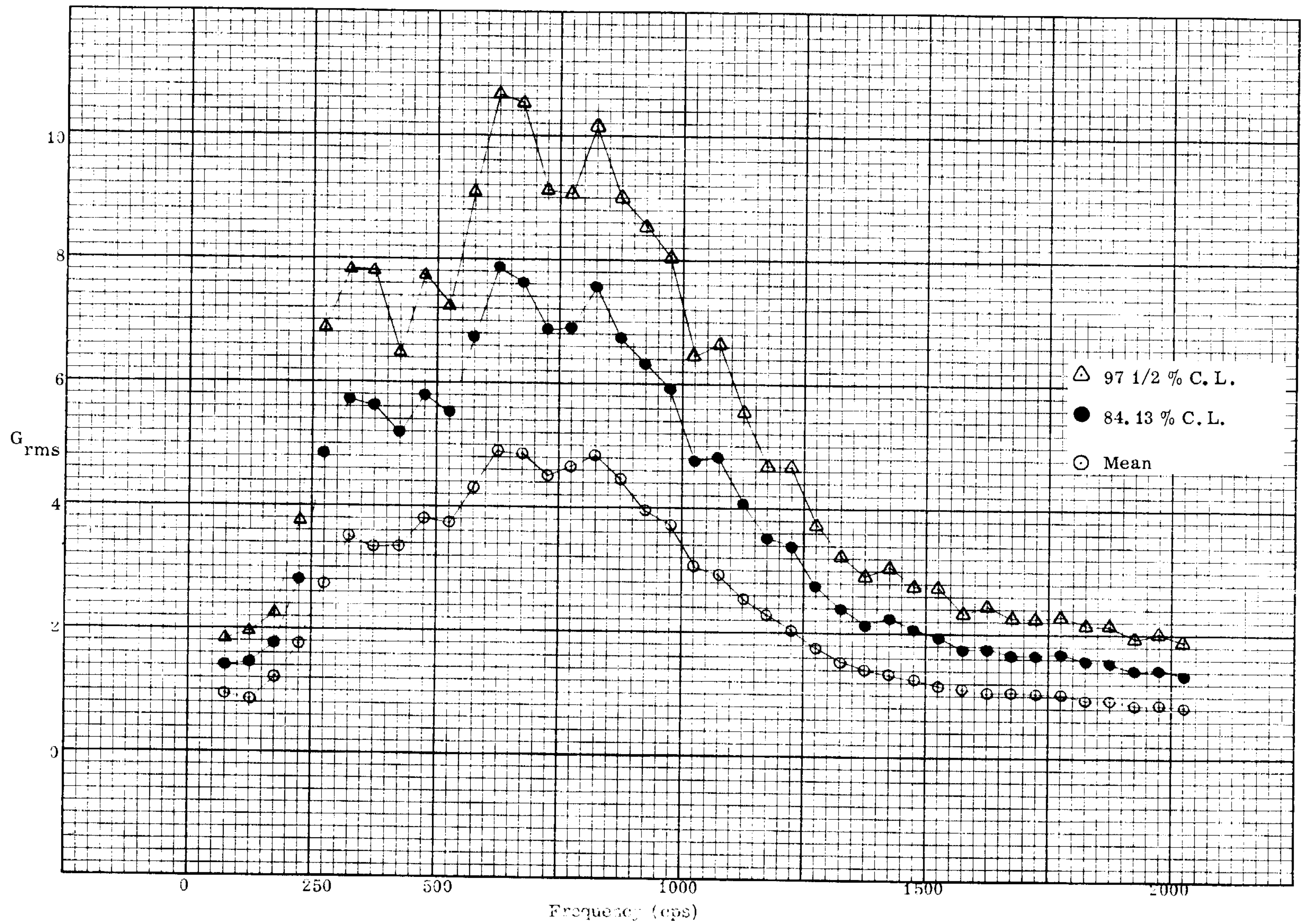


FIGURE 18. SATURN I BLOCK I (CAPTIVE) ZONE 5 LOX-NO LIQUID LOADING (WITHOUT s.p. FACTOR) PRESSURIZED LOX TANKS

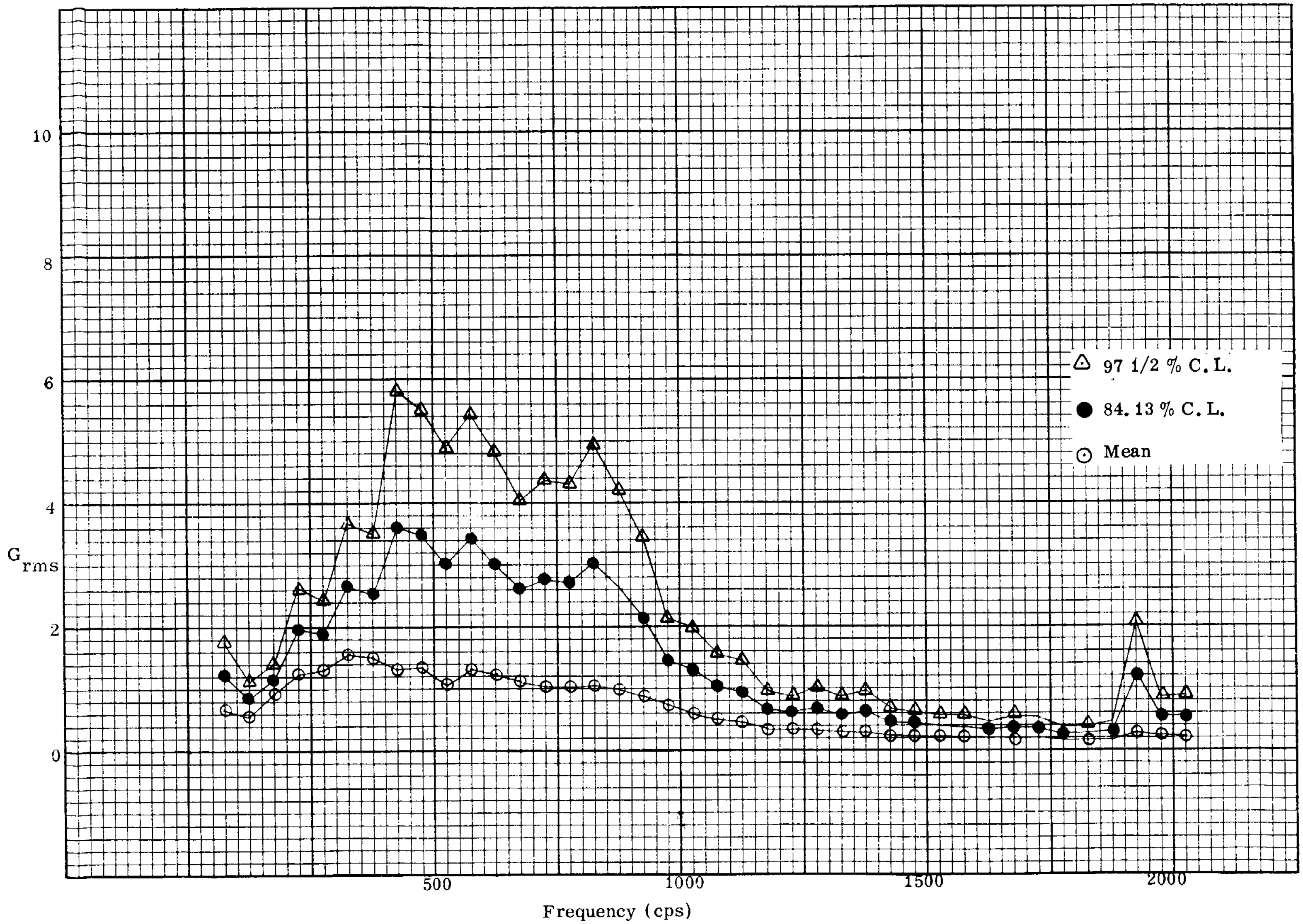


FIGURE 19. SATURN I BLOCK I (CAPTIVE) ZONE 5 FUEL-LIQUID LOADING (WITH s.p. FACTOR)

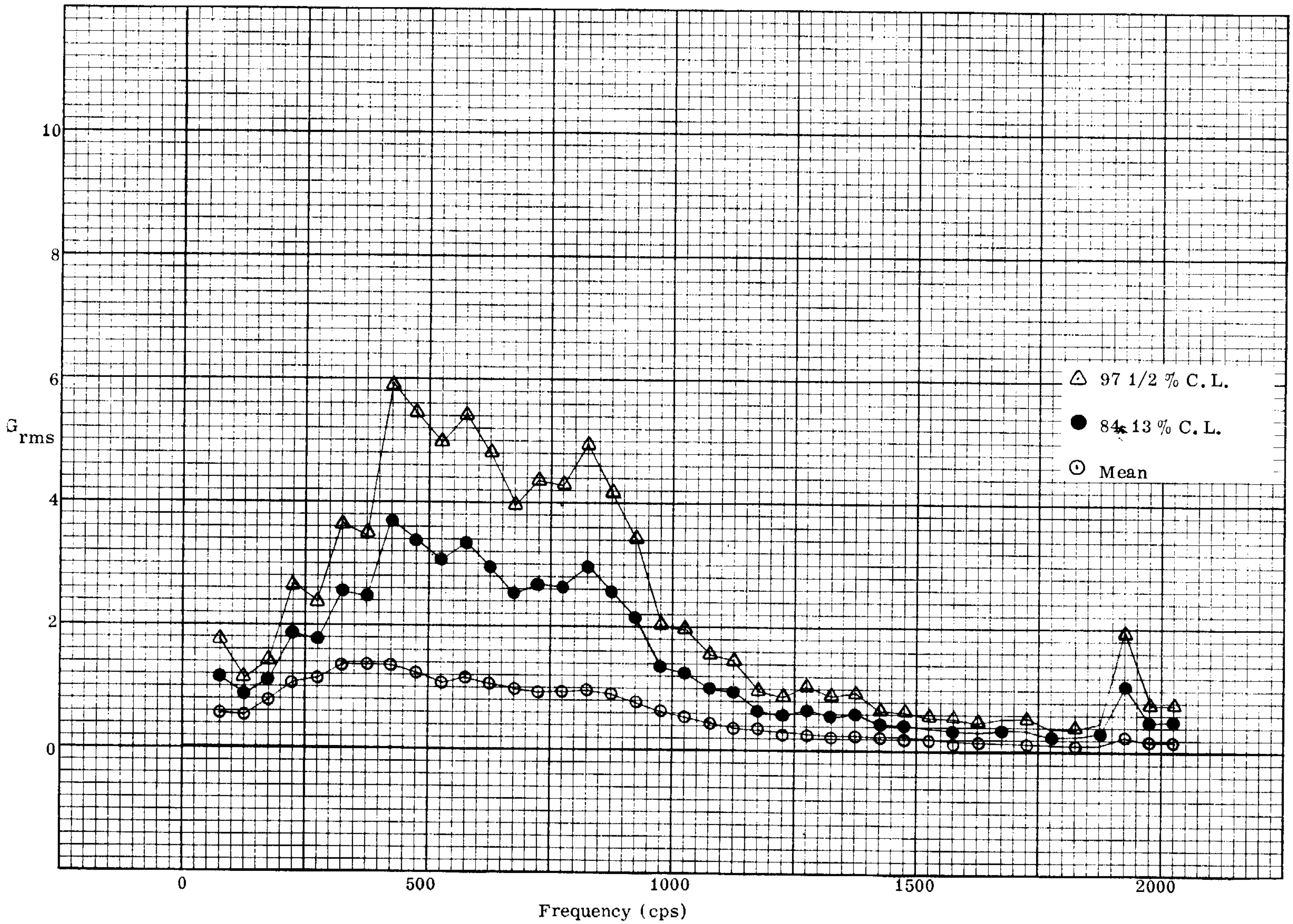


FIGURE 20. SATURN I BLOCK I (CAPTIVE) ZONE 5 FUEL-LIQUID LOADING (WITHOUT s.p. FACTOR) PRESSURIZED FUEL TANKS

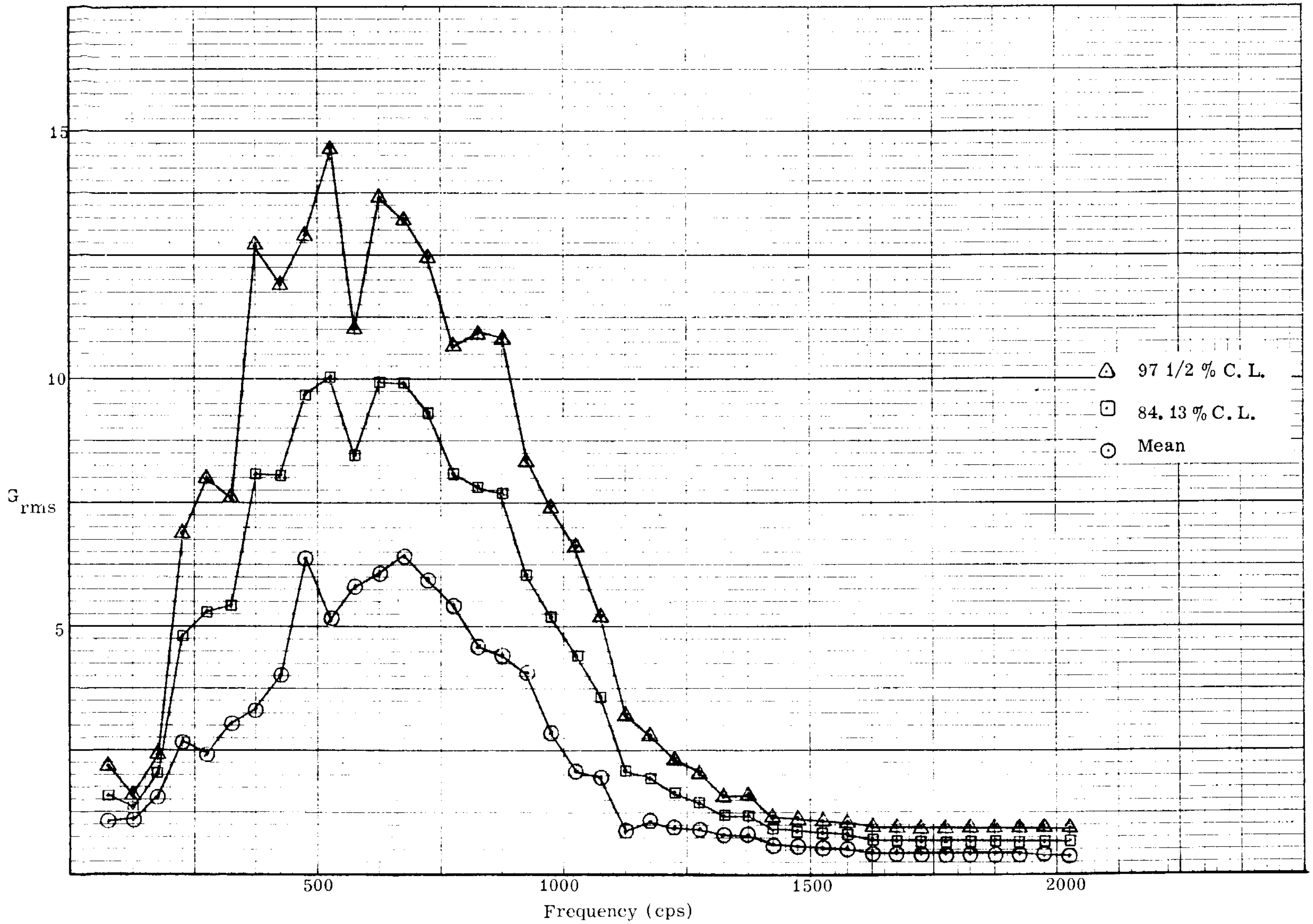


FIGURE 21. SATURN I BLOCK I (CAPTIVE) ZONE 5 FUEL-NO LIQUID LOADING (WITH s.p. FACTOR) PRESSURIZED FUEL TANKS

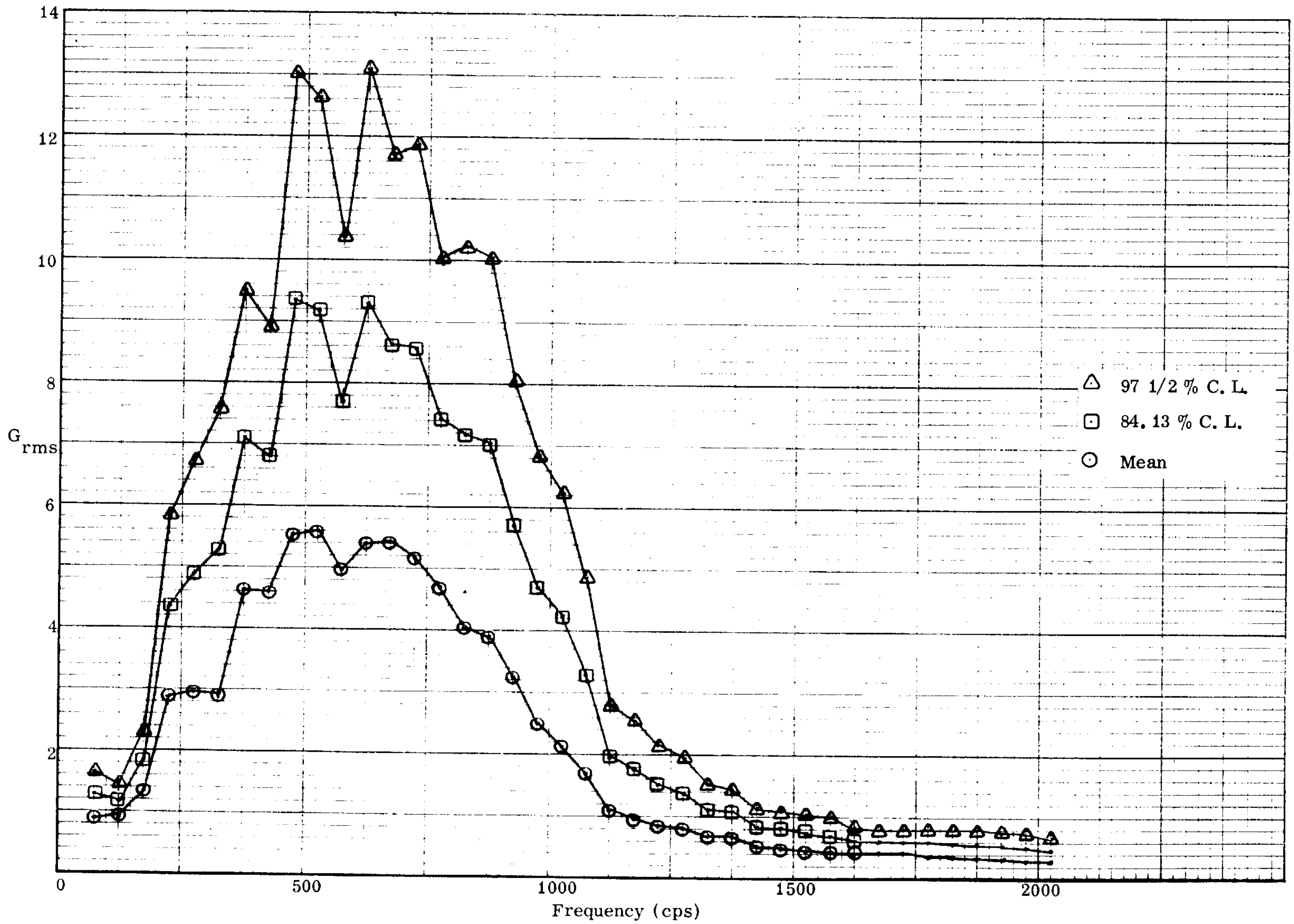


FIGURE 22. SATURN I BLOCK I (CAPTIVE) ZONE 5 FUEL-NO LIQUID LOADING (WITHOUT s.p. FACTOR) PRESSURIZED FUEL TANKS

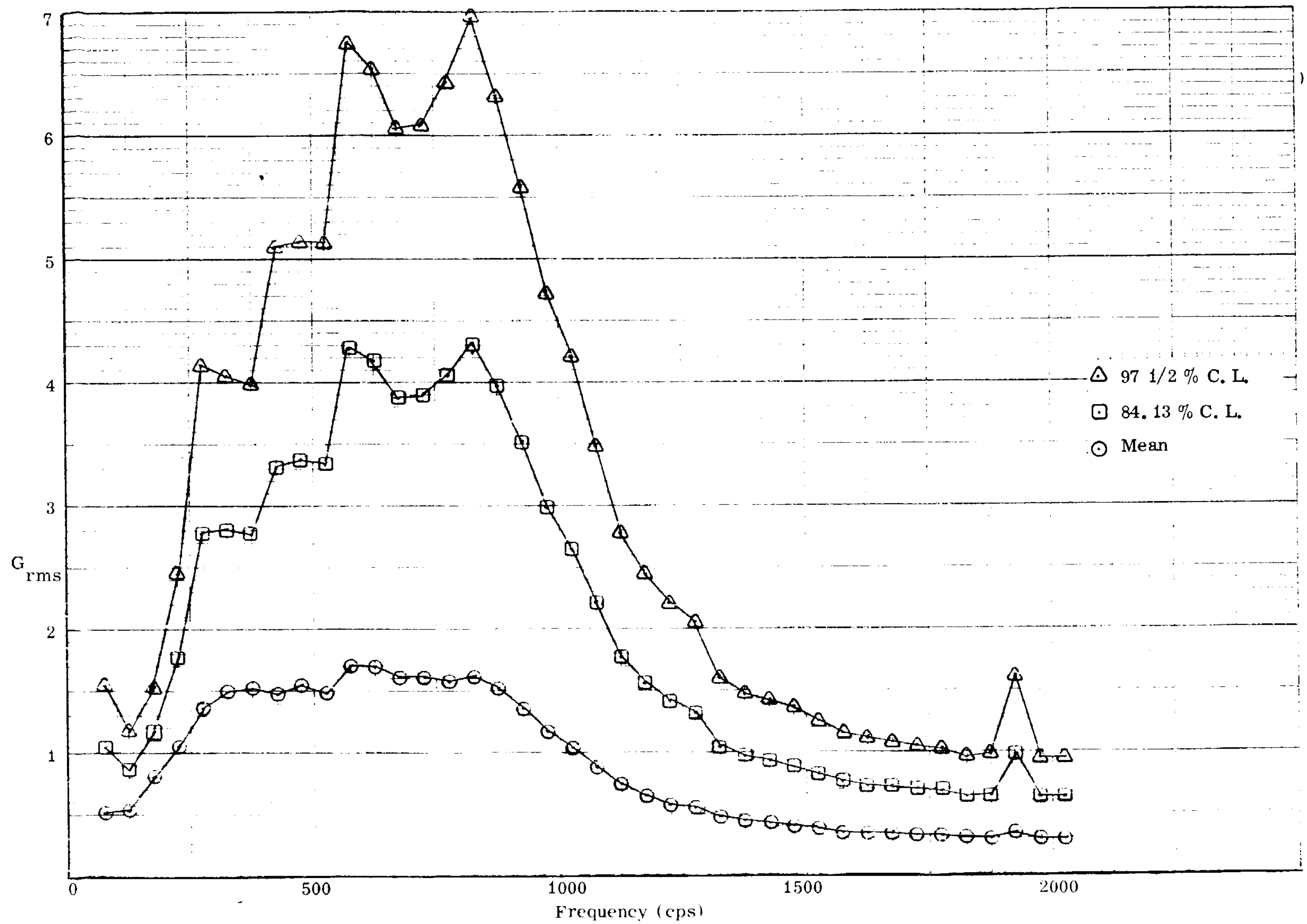


FIGURE 23. SATURN I BLOCK I (CAPTIVE) ZONE 5 FUEL AND LOX-LIQUID LOADING (WITHOUT s.p. FACTOR) PRESSURIZED LOX AND FUEL TANKS

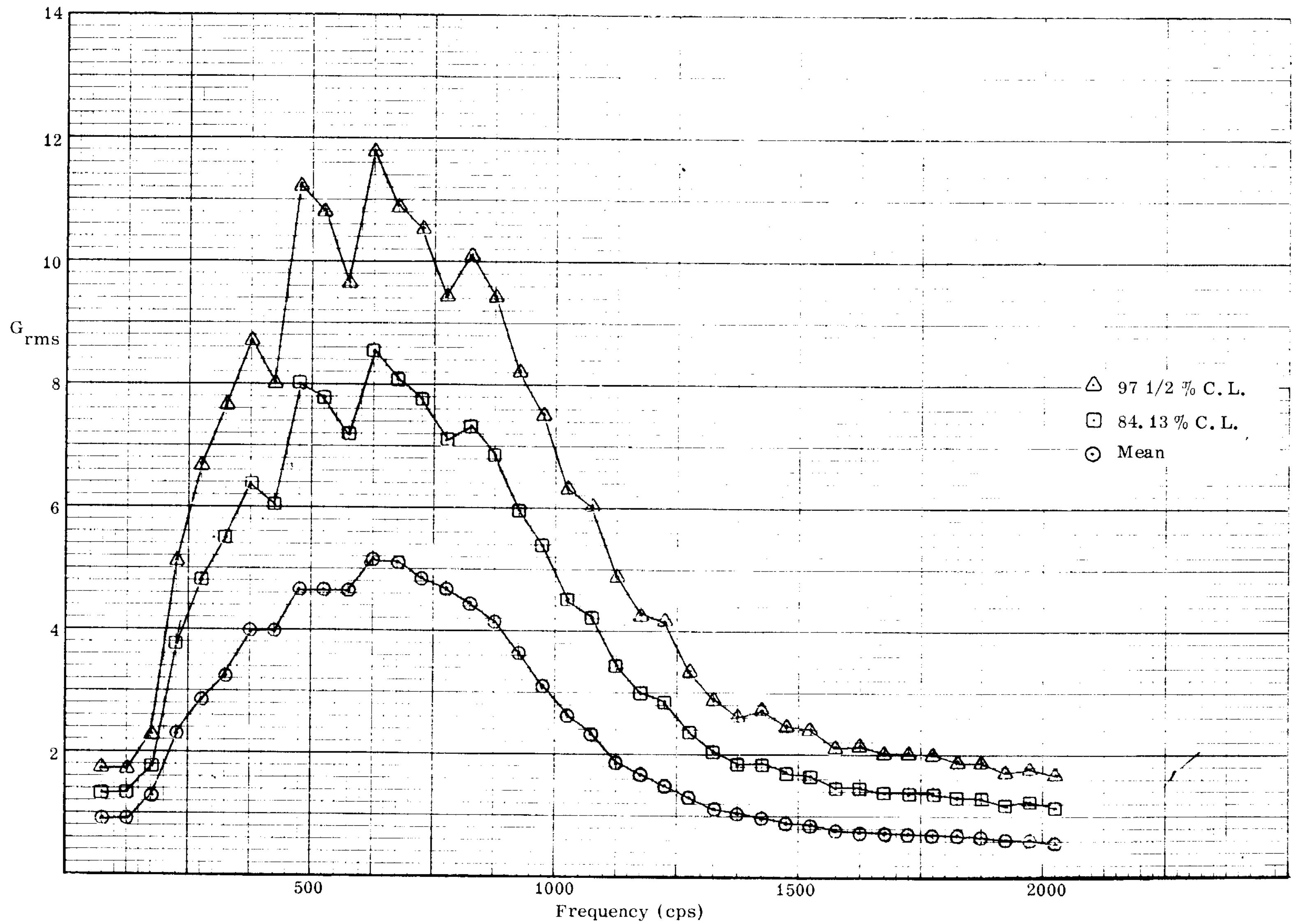


FIGURE 24. SATURN I BLOCK I (CAPTIVE) ZONE 5 FUEL AND LOX-NO LIQUID LOADING (WITHOUT s.p. FACTOR) PRESSURIZED LOX AND FUEL TANKS

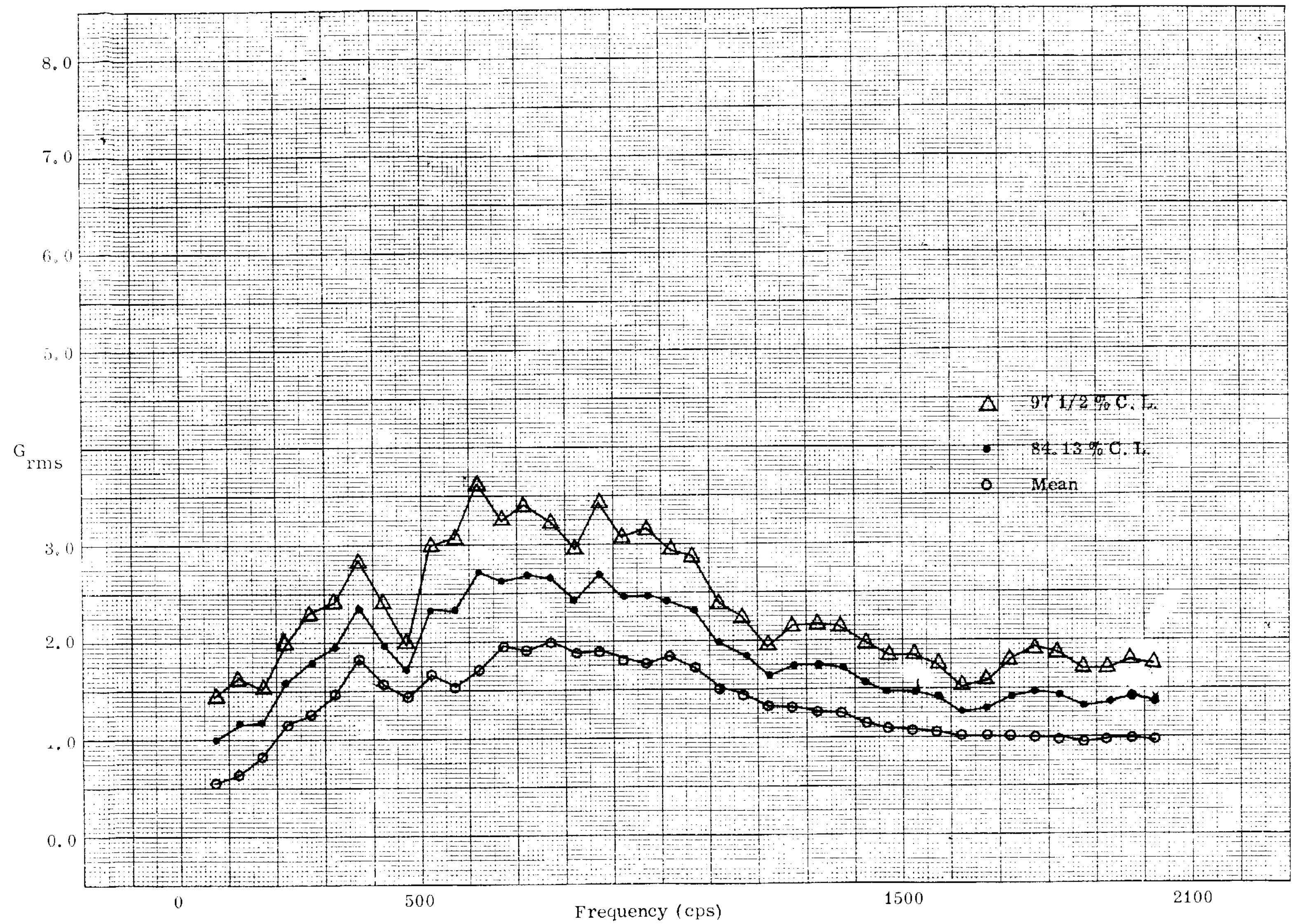


FIGURE 25. SATURN I BLOCK I (CAPTIVE) ZONE 3 LOX (WITH s.p. FACTOR) AFT LOX SKIRTS

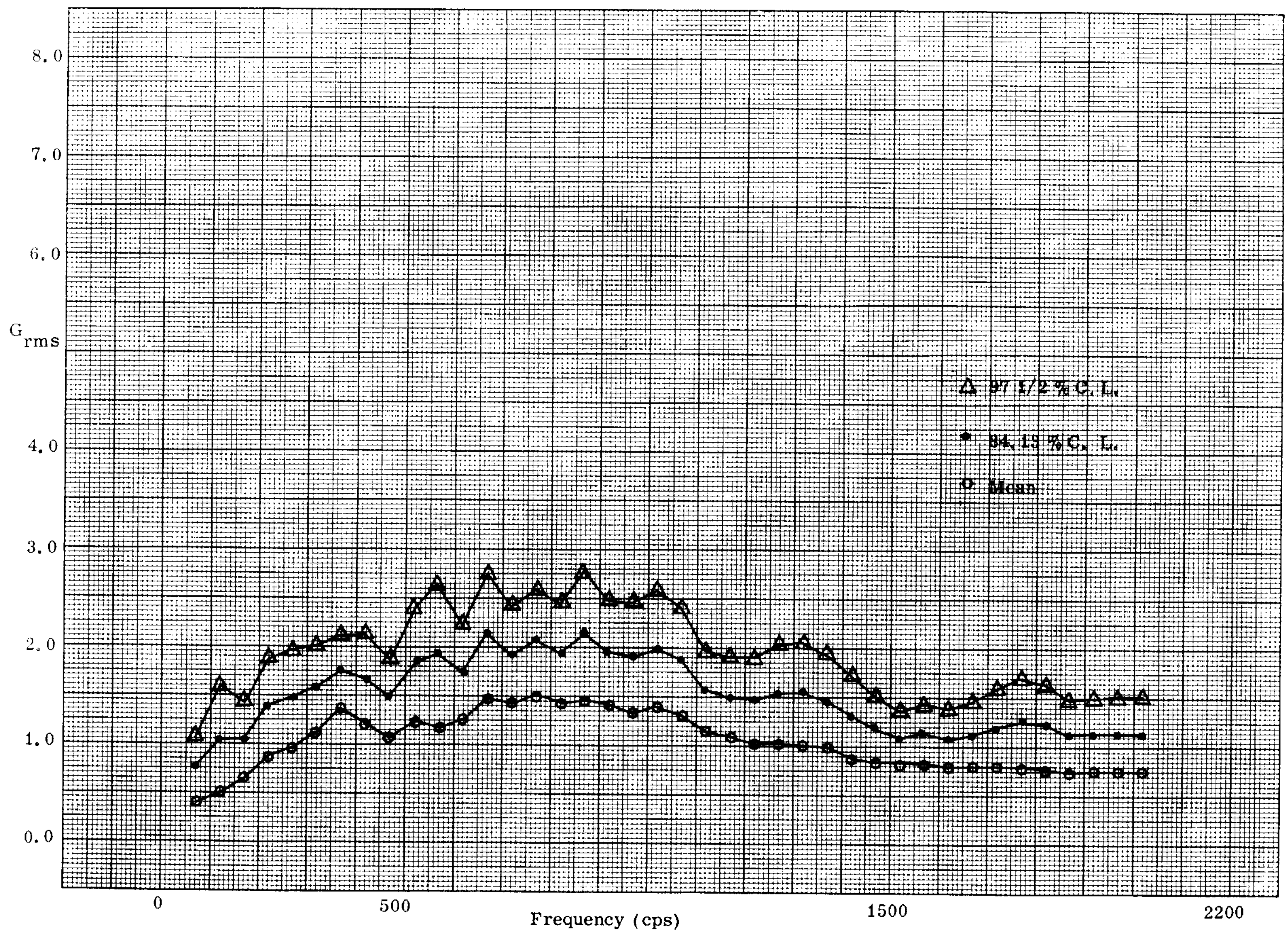


FIGURE 26. SATURN I BLOCK I (CAPTIVE) ZONE 3 LOX (WITHOUT s.p. FACTOR) AFT FUEL SKIRTS

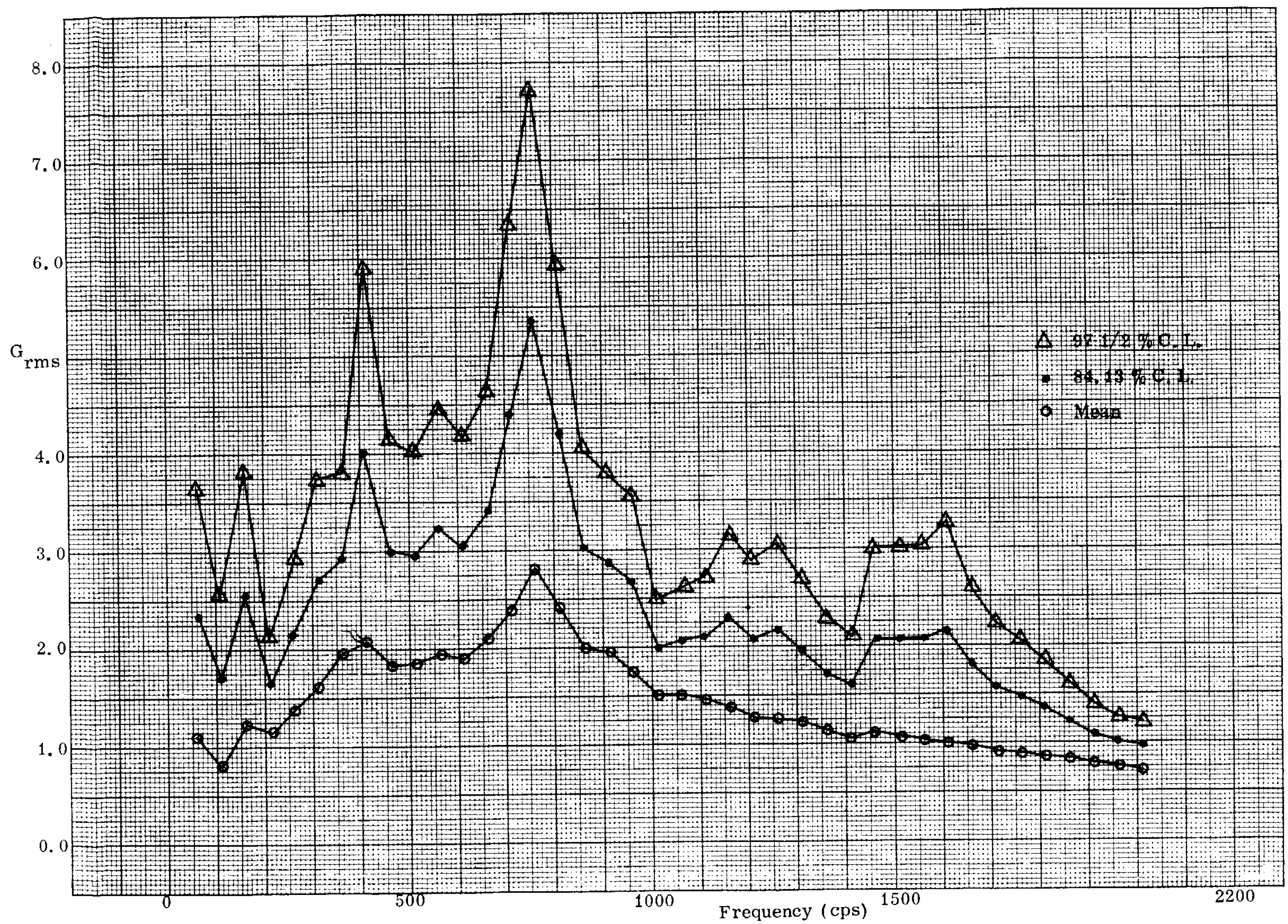


FIGURE 27. SATURN I BLOCK I (CAPTIVE) ZONE 3 FUEL (WITH s.p. FACTOR) AFT FUEL SKIRTS

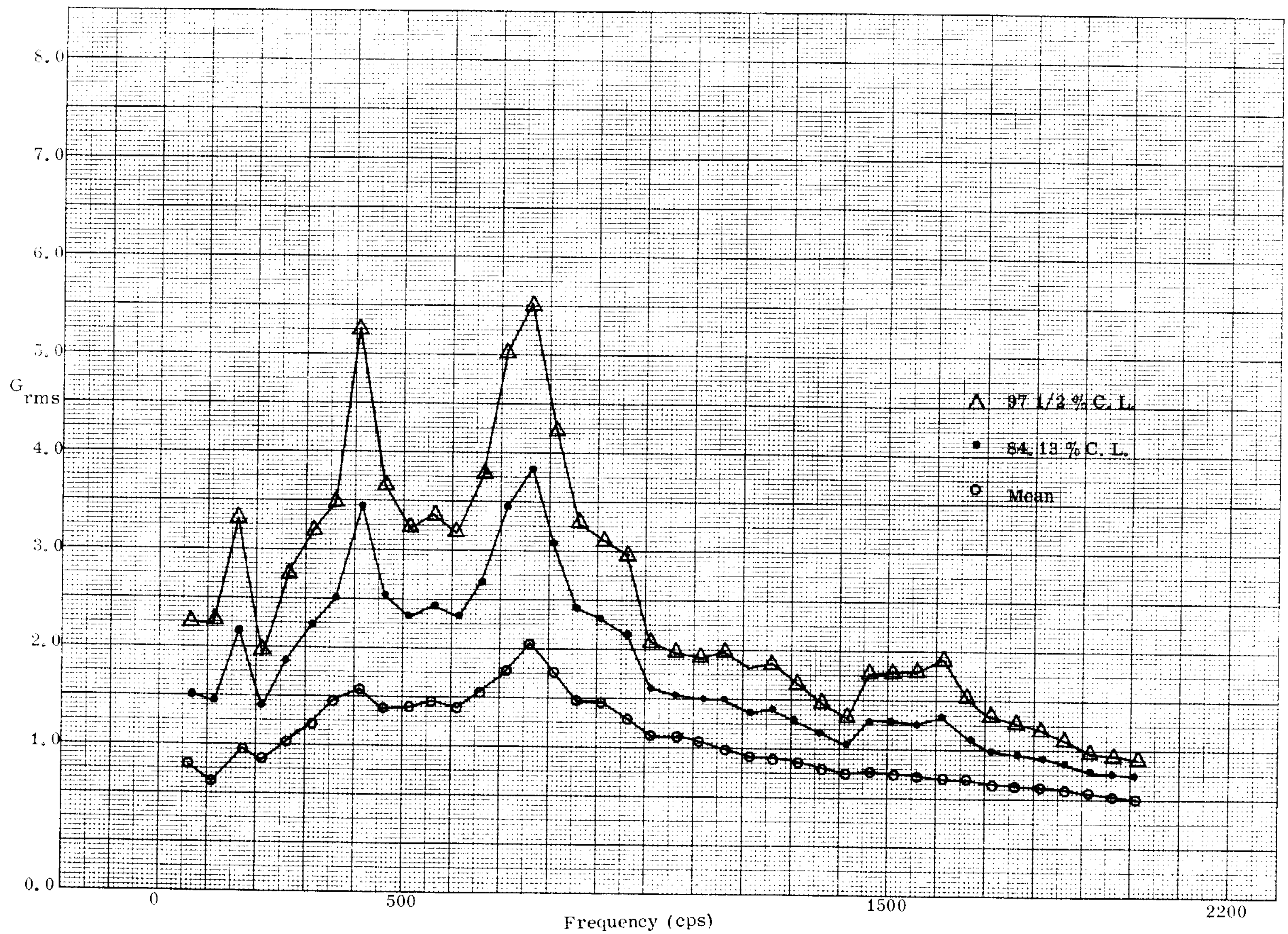


FIGURE 28. SATURN I BLOCK I (CAPTIVE) ZONE 3 FUEL (WITHOUT s.p. FACTOR) AFT FUEL SKIRTS

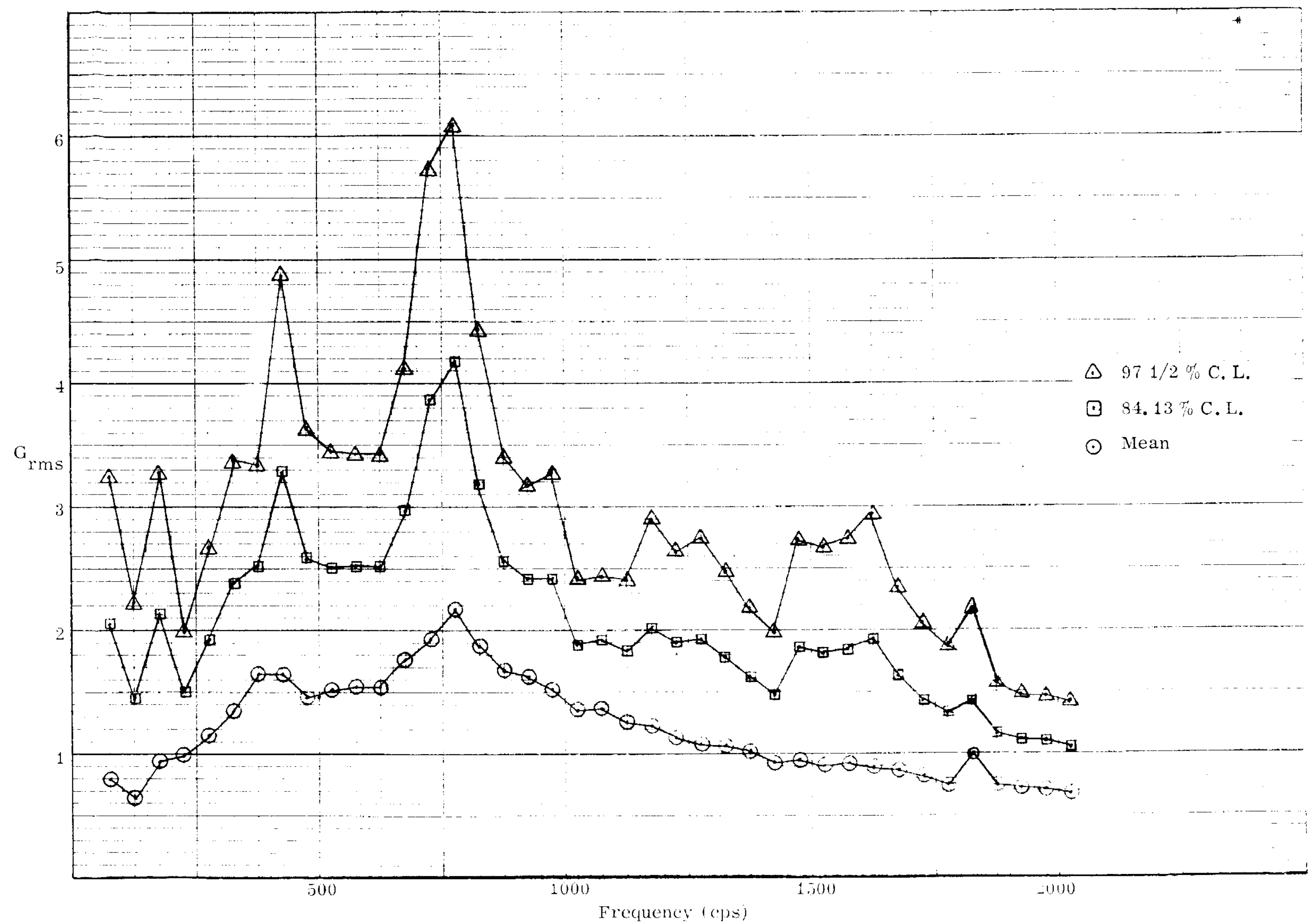


FIGURE 29. SATURN I BLOCK I (CAPTIVE) ZONE 3 LOX AND FUEL (WITH s.p. FACTOR) AFT LOX AND FUEL SKIRTS

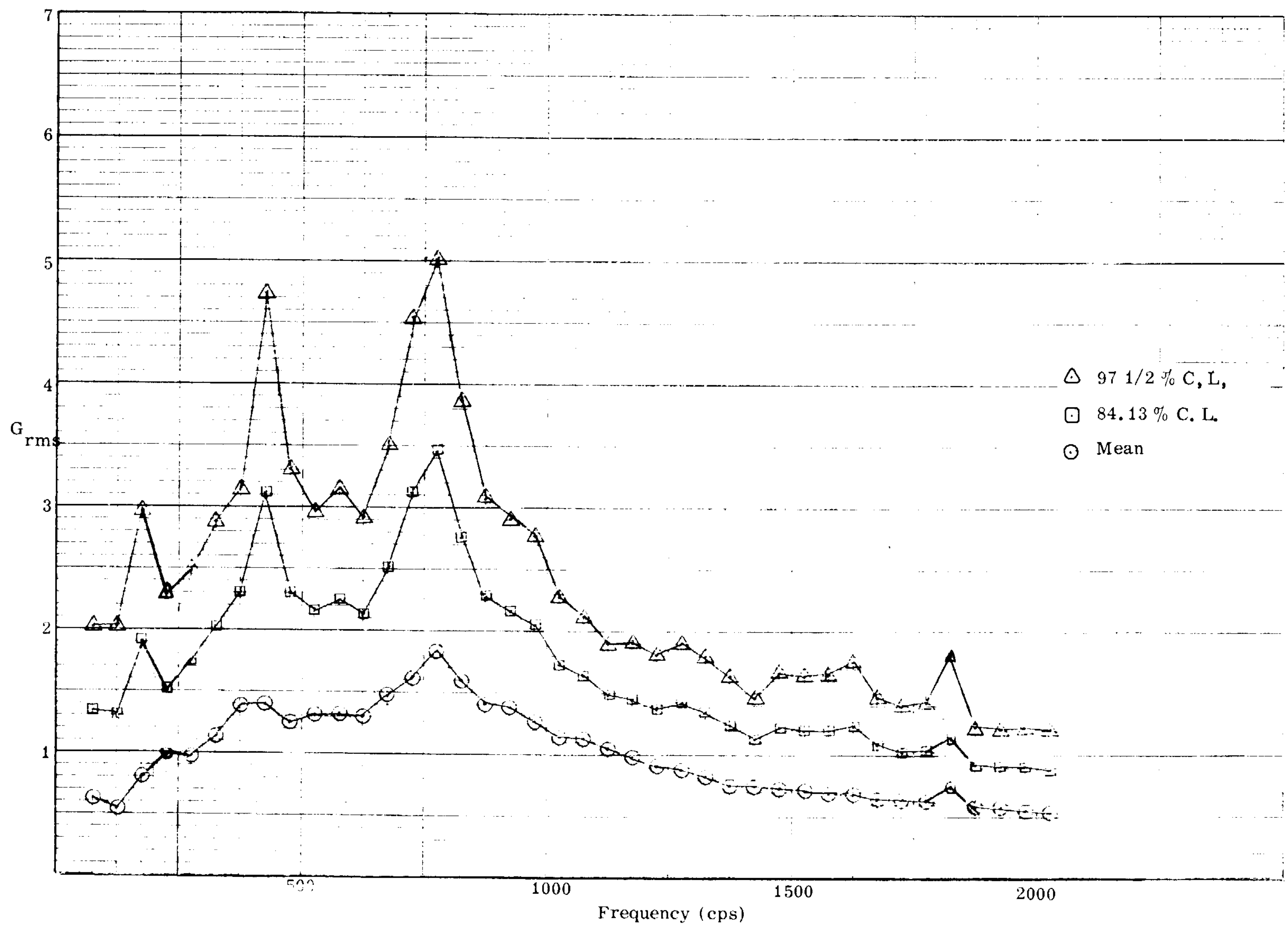


FIGURE 30. SATURN I BLOCK I (CAPTIVE) ZONE 3 LOX AND FUEL (WITHOUT s.p. FACTOR) AFT LOX AND FUEL SKIRTS

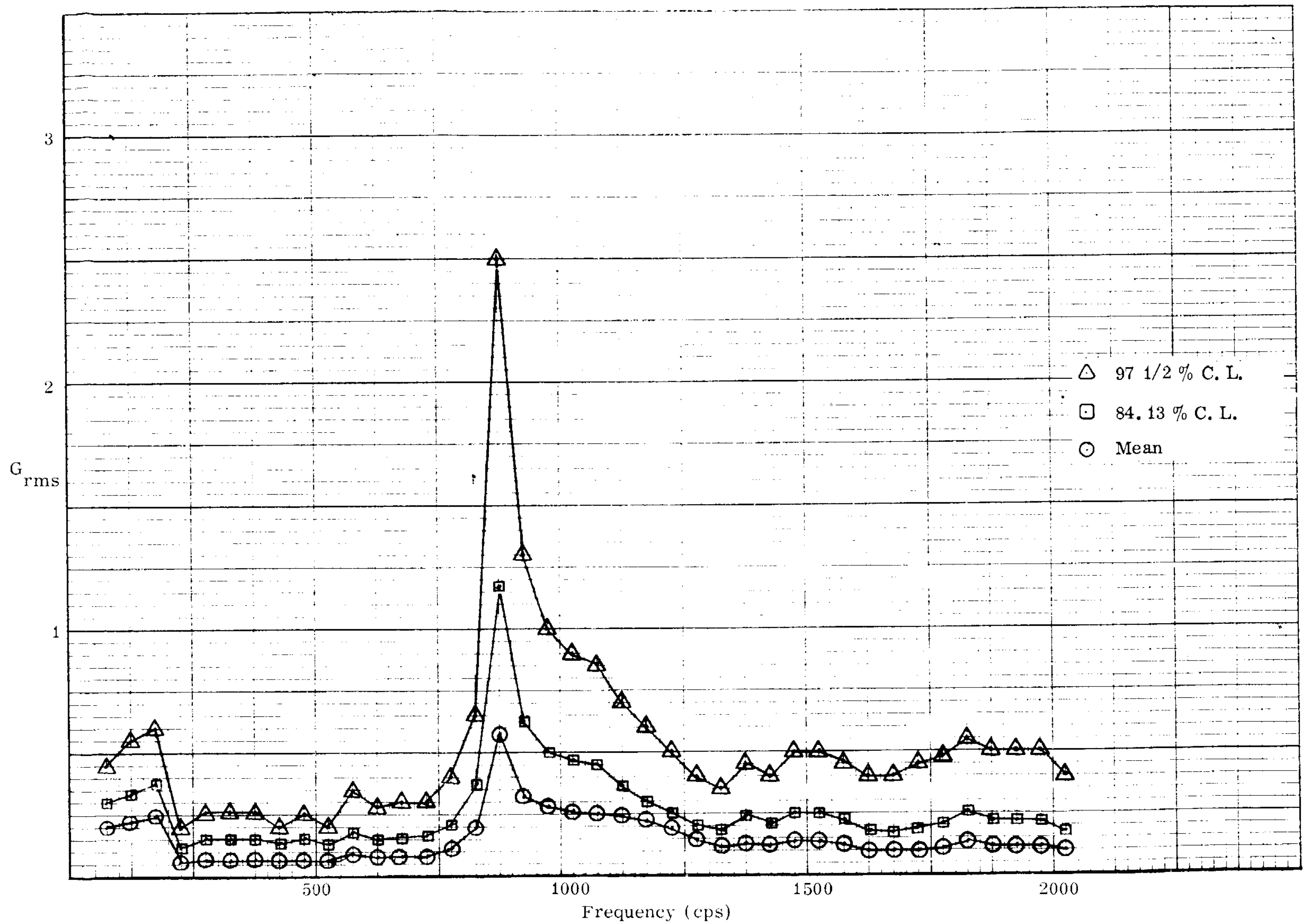


FIGURE 31. SATURN I BLOCK I (CAPTIVE) ZONE 3 - 4 FUEL AFT FUEL BULKHEAD

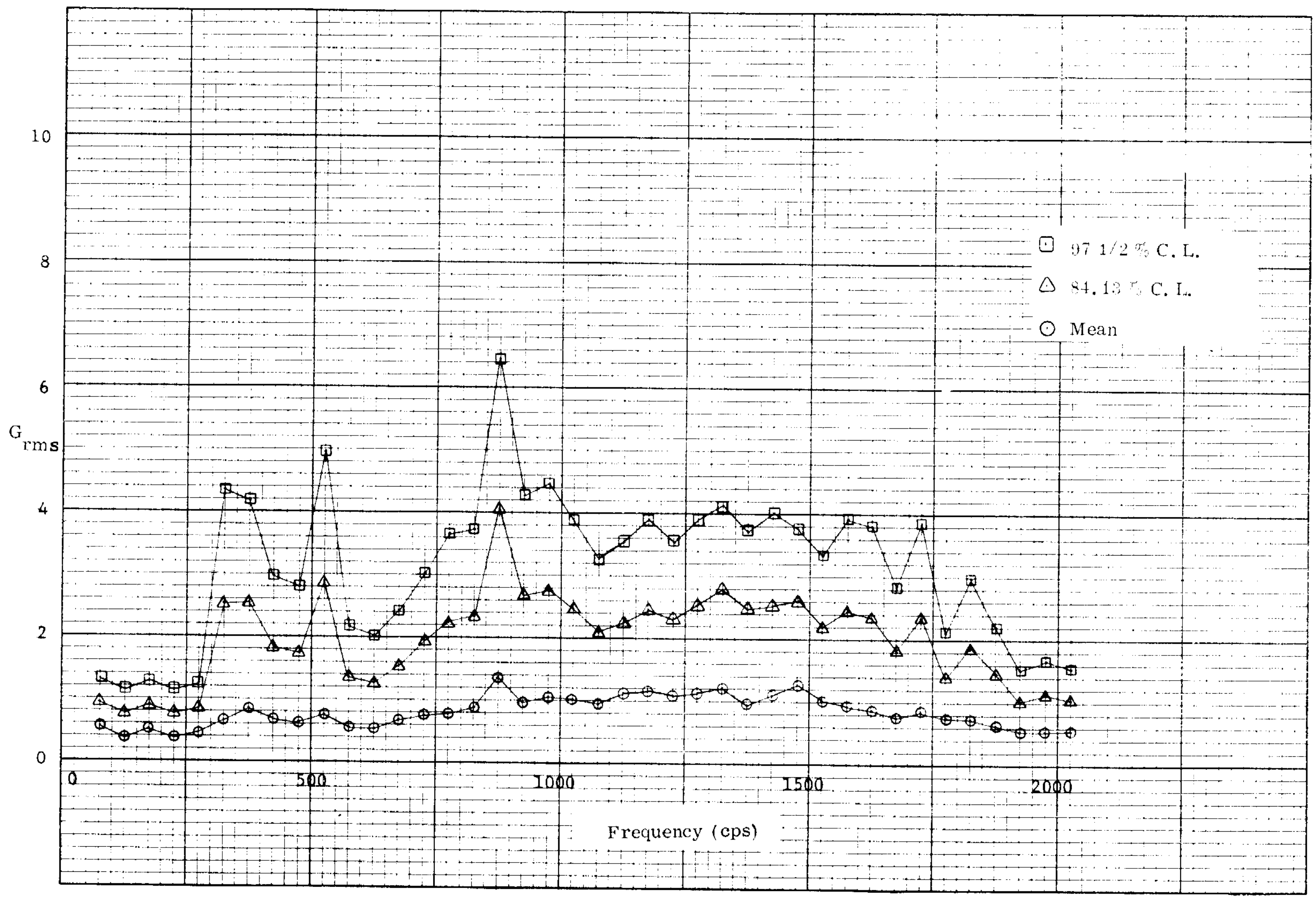


FIGURE 32. SATURN I BLOCK I (CAPTIVE) ZONE 2 THRUST STRUCTURE

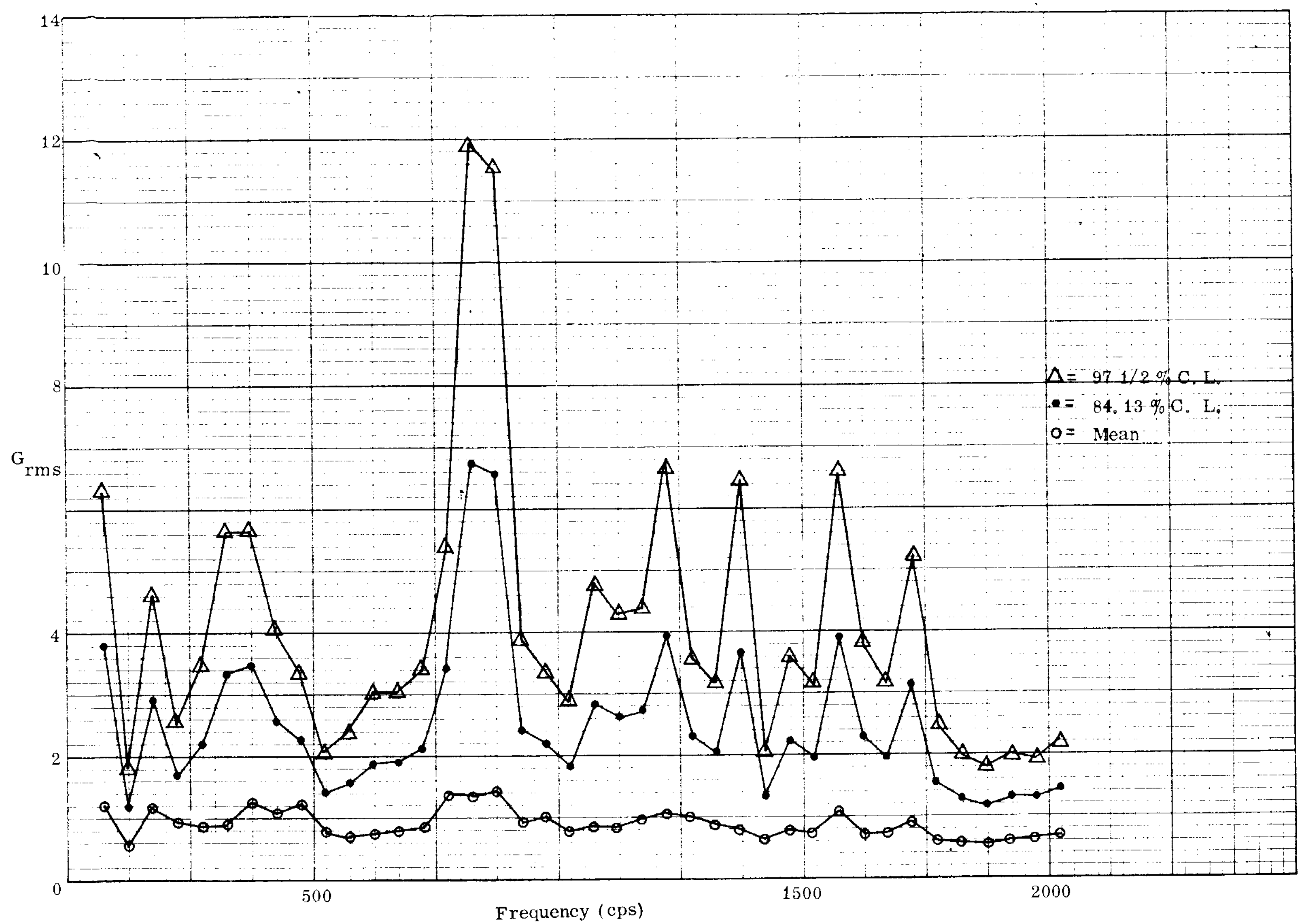


FIGURE 33. SATURN I BLOCK I (CAPTIVE) ZONE 1 - 1 H-1 ENGINE TURBOPUMP SECTION

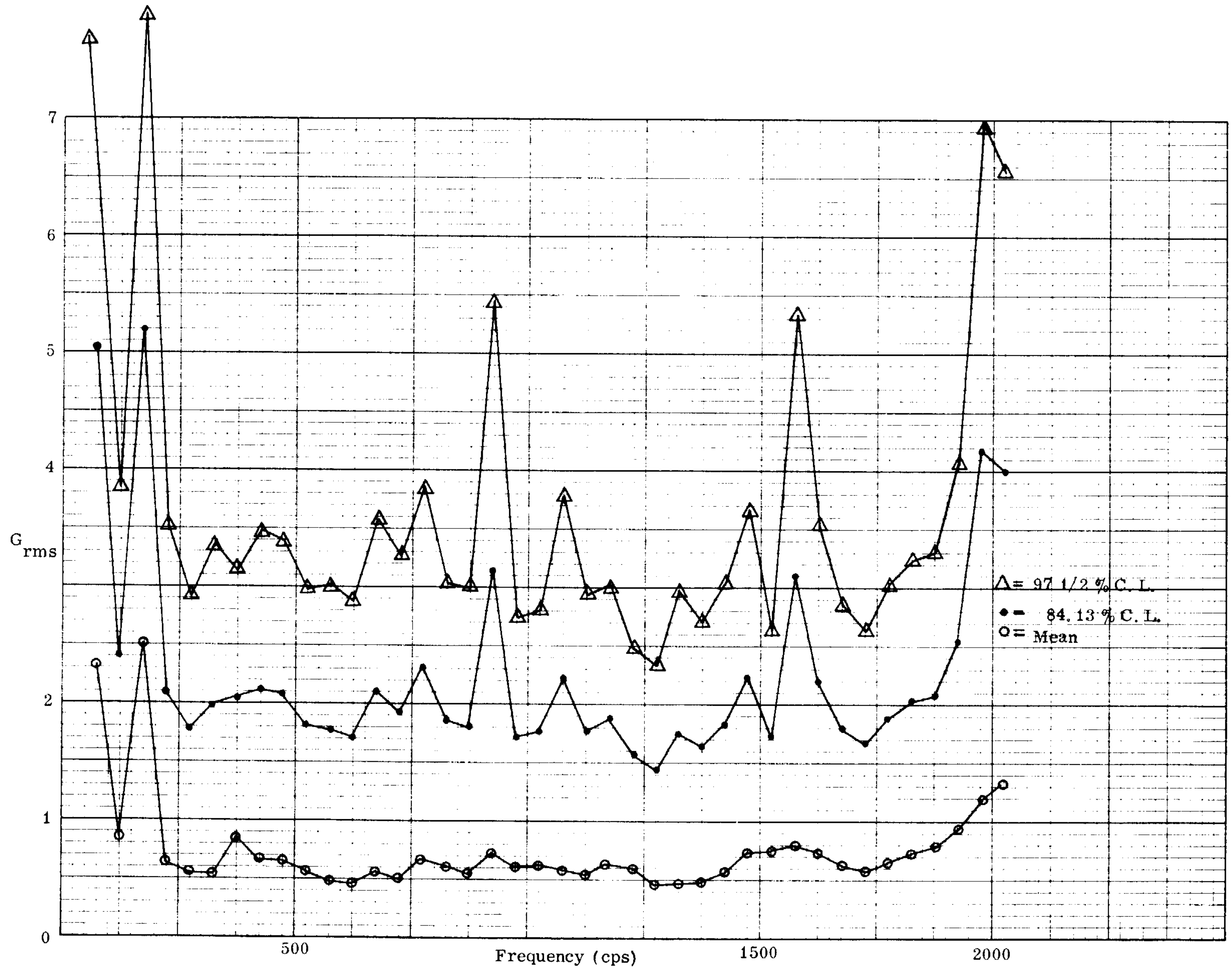


FIGURE 34. SATURN I BLOCK I (CAPTIVE) ZONE 1 - 1 H-1 ENGINE GEAR CASE ONLY (MEASUREMENT NO. 82.03)

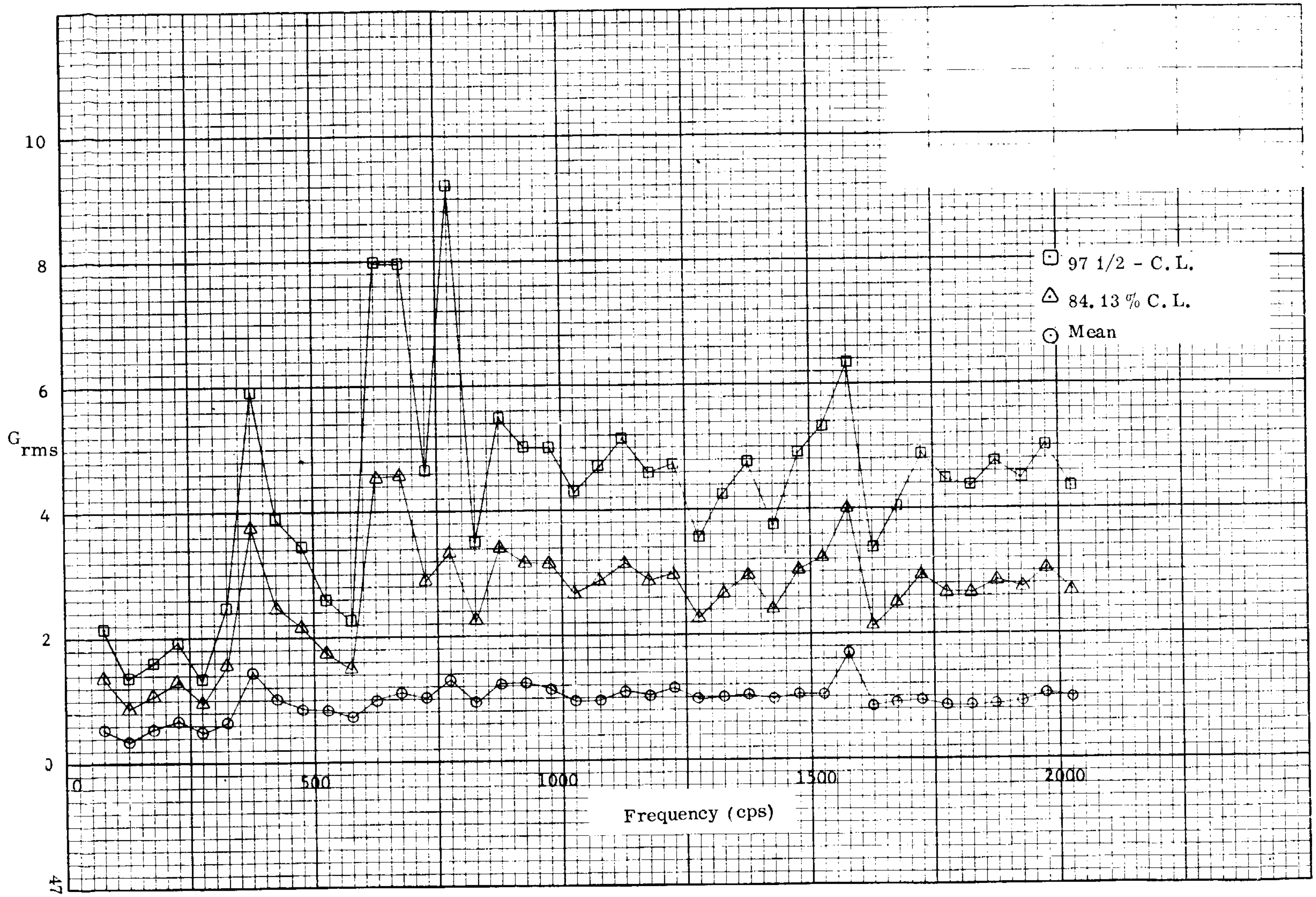


FIGURE 35. SATURN I BLOCK I (CAPTIVE) ZONE 1 - 2 H-1 ENGINE COMBUSTION CHAMBER (COMPONENTS)

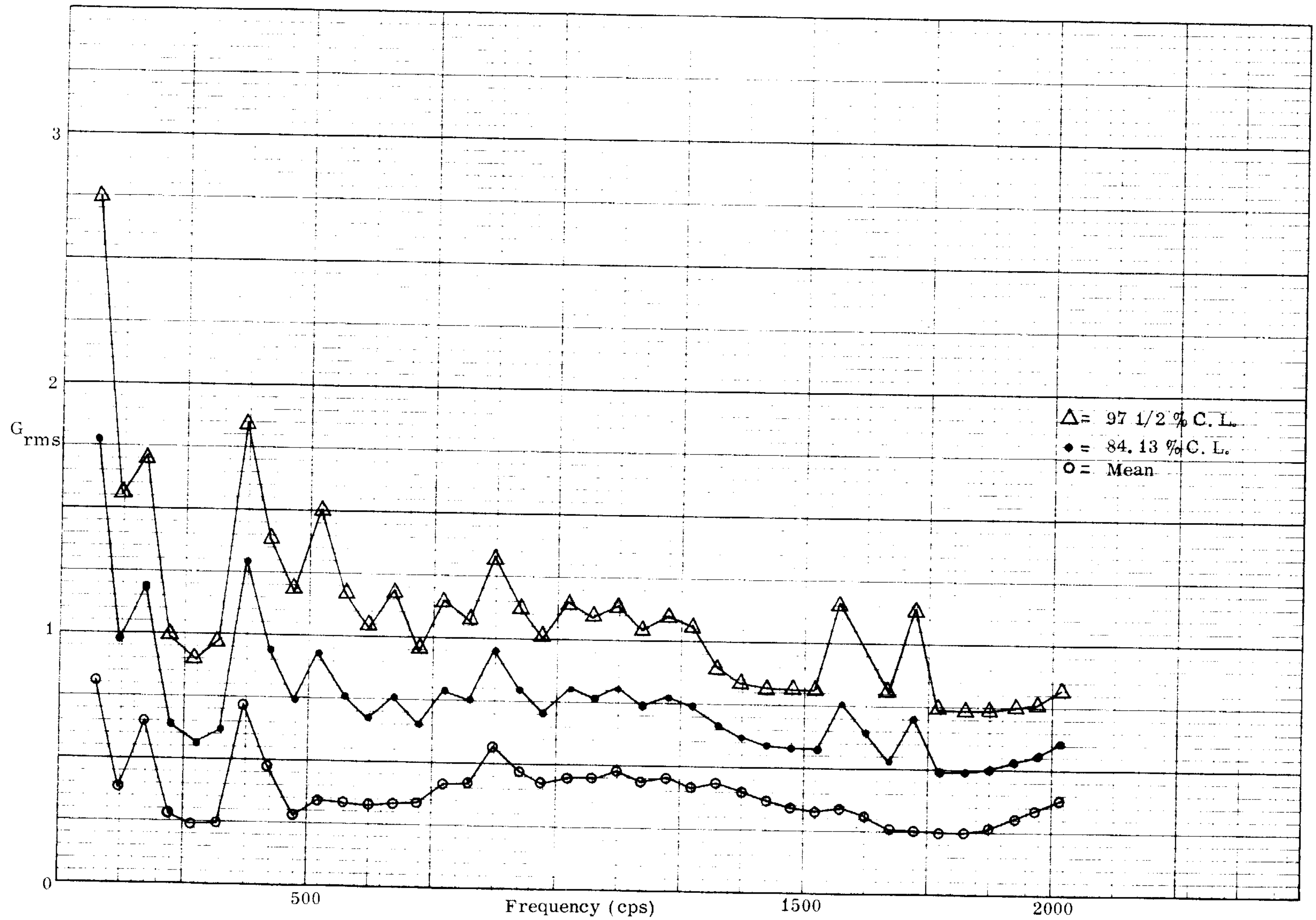


FIGURE 36. SATURN I BLOCK I (CAPTIVE) ZONE 1 - 2 H-1 ENGINE CHAMBER DOME ONLY (MEASUREMENT NO. 81.041)

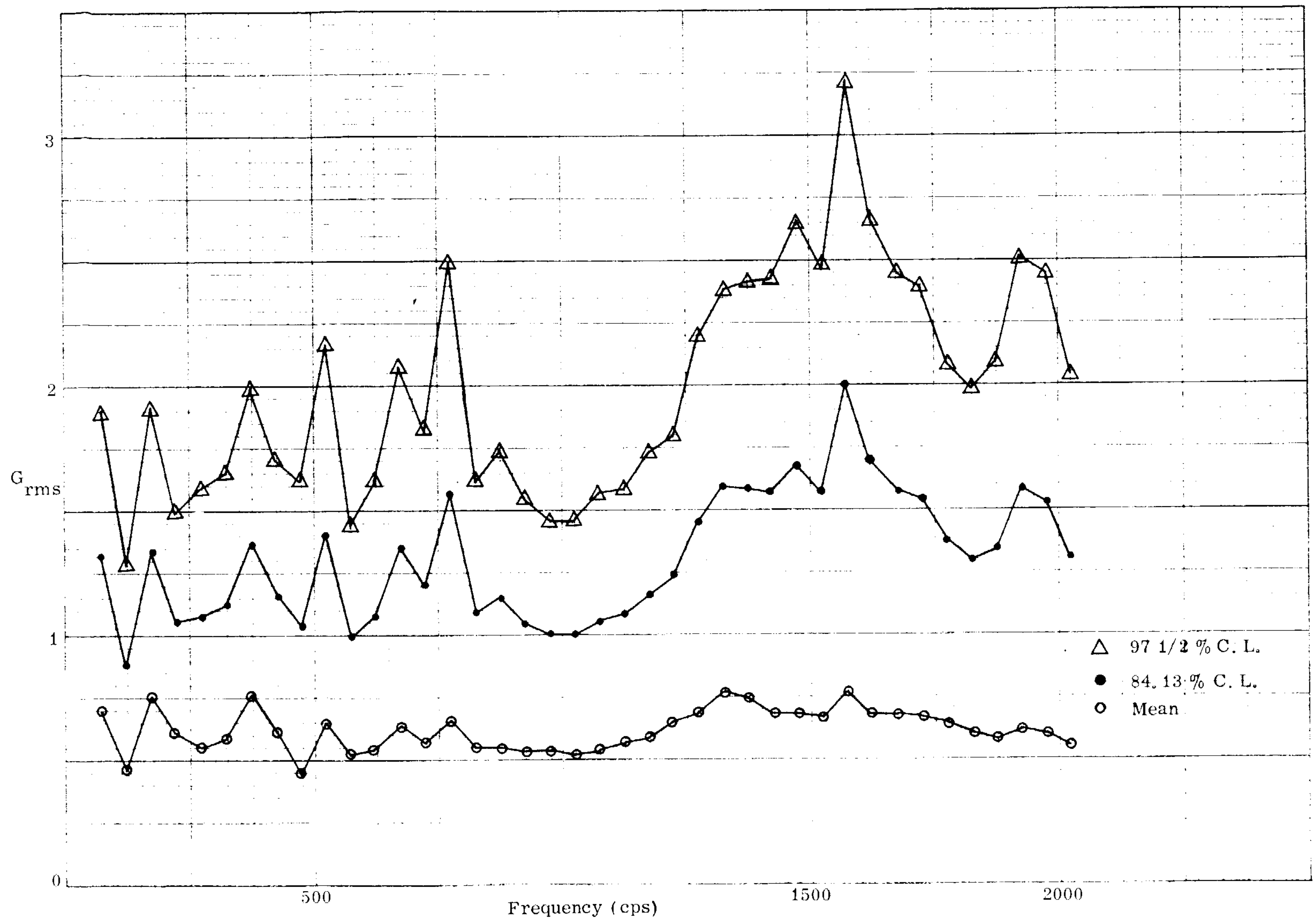


FIGURE 37. SATURN I BLOCK I (CAPTIVE) ZONE 1 - 2C H-1 ENGINE ACTUATOR ASSEMBLY

TABLE I - SUMMARY OF SATURN ZONAL COMPOSITES

ZONE	STATISTICAL DESCRIPTION OF ZONAL COMPOSITES	97-1/2 % C. L. G_{rms} (50-2050 cps)
7	$\overline{X} = 7.72$ $\sigma = 5.83$ P = I S = 1.06 K = 3.31 $P_t = 0$ V = .755	19.4
6-LOX w/s.p. FACTOR	$\overline{X} = 14.0$ $\sigma = 6.44$ P = I S = .89 K = 3.0 $P_t = 0$ V = .46	26.5
6-LOX w/out s.p. FACTOR	$\overline{X} = 11.5$ $\sigma = 6.35$ P = I S = .86 K = 2.95 $P_t = 0$ V = .552	23.8
6-FUEL w/s.p. FACTOR	$\overline{X} = 26.5$ $\sigma = 13.1$ P = I S = 1.0 K = 3.5 P_t = IX V = .495	52.5
6-FUEL w/out s.p. FACTOR	$\overline{X} = 20.8$ $\sigma = 10.7$ P = I S = 1.2 K = 3.8 $P_t = 0$ V = .515	42.3
6-4 FUEL	$\overline{X} = 31.0$ $\sigma = 10.7$ P = I S = .4 K = 3.25 P_t = III V = .345	50.0
5-LOX LIQ. LOADING w/s.p. FACTOR	$\overline{X} = 21.4$ $\sigma = 14.68$ P = I S = .90 K = 2.09 $P_t = 0$ V = .686	48.07
5-LOX LIQ. LOADING w/out s.p. FACTOR	$\overline{X} = 18.1$ $\sigma = 13.4$ P = I S = .80 L = 2.1 $P_t = 0$ V = .74	43.7

TABLE I - SUMMARY OF SATURN ZONAL COMPOSITES (Cont'd)

ZONE	STATISTICAL DESCRIPTION OF ZONAL COMPOSITES	97-1/2 % C. L. G_{rms} (50-2050 cps)
5-LOX NO LIQ. LOADING w/s.p. FACTOR	$\overline{X}$ = 33.1 σ = 21.1 P = I S = 1.06 K = 3.03 P_t = 0 V = .636	75.52
5-LOX NO. LIQ. LOADING w/out s.p. FACTOR	$\overline{X}$ = 23.1 σ = 16.7 P = I S = 1.0 K = 3.0 P_t = XII V = .723	56.17
5-FUEL LIQ. LOADING w/s.p. FACTOR	$\overline{X}$ = 12.24 σ = 8.88 P = I S = 2.78 K = 11.1 P_t = XII V = .725	35.2
5-FUEL LIQ. LOADING w/out s.p. FACTOR	$\overline{X}$ = 10.77 σ = 9.17 P = I S = 2.9 K = 11.6 P_t = 0 V = .85	34.8
5-FUEL NO. LIQ. LOADING w/s.p. FACTOR	$\overline{X}$ = 43.43 σ = 15.67 P = I S = .17 K = 1.25 P_t = II V = .361	70.12
5-FUEL NO. LIQ. LOADING w/out s.p. FACTOR	$\overline{X}$ = 36.91 σ = 16.86 P = I S = .44 K = 1.3 P_t = 0 V = .456	67.19
5-LOX & FUEL LIQ. LOADING w/out s.p. FACTOR	$\overline{X}$ = 16.13 σ = 11.32 P = I S = 1.5 K = 4.6 P_t = 0 V = .702	40.47

TABLE I - SUMMARY OF SATURN ZONAL COMPOSITES (Cont'd)

ZONE	STATISTICAL DESCRIPTION OF ZONAL COMPOSITES	97-1/2 % C. L. G_{rms} (50-2050 cps)
5-LOX & FUEL NO. LIQ. LOADING w/out s. p. FACTOR	$\overline{X} = 29.8$ $\sigma = 16.35$ $P = I$ $S = .8$ $K = 2.0$ $P_t = 0$ $V = .554$	61.04
3-LOX w/s.p. FACTOR	$\overline{X} = 19.7$ $\sigma = 5.06$ $P = IV$ $S = -.36$ $K = 3.5$ $P_t = 0$ $V = .257$ $\chi_s^2 = 9.62$ $\chi_\ell^2 = 23.68$ $(K\text{-}S+)_s = .113$ $(K\text{-}S+)_\ell = .217$ $(K\text{-}S\ ABS)_s = .113$ $(K\text{-}S\ ABS)_\ell = .241$	27.5
3-LOX w/out s. p. FACTOR	$\overline{X} = 14.7$ $\sigma = 5.35$ $P = I$ $S = -.15$ $K = 2.11$ $P_t = II$ $V = .362$ $\chi_s^2 = 15.9$ $\chi_\ell^2 = 28.87$ $(K\text{-}S+)_s = .080$ $(K\text{-}S+)_\ell = .217$ $(K\text{-}S\ ABS)_s = .090$ $(K\text{-}S\ ABS)_\ell = .241$	23.1
3-FUEL w/s.p. FACTOR	$\overline{X} = 22.8$ $\sigma = 8.8$ $P = I$ $S = -.31$ $K = 2.9$ $P_t = 0$ $V = .385$ $\chi_s^2 = 15.14$ $\chi_\ell^2 = 28.9$ $(K\text{-}S+)_s = .055$ $(K\text{-}S+)_\ell = .155$ $(K\text{-}S\ ABS)_s = .057$ $(K\text{-}S\ ABS)_\ell = .172$	36.4
3-FUEL w/out s.p. FACTOR	$\overline{X} = 16.67$ $\sigma = 7.36$ $P = IV$ $S = .48$ $K = 3.93$ $P_t = 0$ $V = .441$ $\chi_s^2 = 28.58$ $\chi_\ell^2 = 26.3$ $(K\text{-}S+)_s = .095$ $(K\text{-}S+)_\ell = .155$ $(K\text{-}S\ ABS)_s = .118$ $(K\text{-}S\ ABS)_\ell = .172$	29.44

TABLE I - SUMMARY OF SATURN ZONAL COMPOSITES (Cont'd)

ZONE	STATISTICAL DESCRIPTION OF ZONAL COMPOSITES	97-1/2 % C. L. G_{rms} (50-2050 cps)
3-LOX & FUEL w/s.p. FACTOR	$\overline{X} = 21.1$ $\sigma = 8.1$ $P = I$ $S = -.10$ $K = 2.85$ $P_t = 0$ $V = .384$ $\chi_s^2 = 6.44$ $\chi_\ell^{2t} = 21.03$ $(K\text{-}S+)_s = .041$ $(K\text{-}S+)_\ell = .127$ $(K\text{-}S\ ABS)_\ell = .041$ $(K\text{-}S\ ABS)_\ell = .141$	34.1
3-LOX & FUEL w/out s.p. FACTOR	$\overline{X} = 16.04$ $\sigma = 6.8$ $P = IV$ $S = .48$ $K = 3.97$ $P_t = 0$ $V = .424$ $\chi_s^2 = 24.64$ $\chi_\ell^{2t} = 26.3$ $(K\text{-}S+)_s = .097$ $(K\text{-}S+)_\ell = .127$ $(K\text{-}S\ ABS)_s = .097$ $(K\text{-}S\ ABS)_\ell = .141$	28.3
3-4 FUEL	$\overline{X} = 4.6$ $\sigma = 2.08$ $P = I$ $S = .2$ $K = 3.08$ $P_t = 0$ $V = .453$	8.15
2	$\overline{X} = 12.6$ $\sigma = 4.2$ $P = I$ $S = -.01$ $K = 2.96$ $P_t = 0$ $V = .333$ $\chi_s^2 = 24.4$ $\chi_\ell^{2t} = 23.68$ $(K\text{-}S+)_s = .091$ $(K\text{-}S+)_\ell = .106$ $(K\text{-}S\ ABS)_s = .091$ $(K\text{-}S\ ABS)_\ell = .118$	19.8
1-1 (COMPONENTS)	$\overline{X} = 16.6$ $\sigma = 10.6$ $P = I$ $S = .08$ $K = 3.1$ $P_t = 0$ $V = .638$	34.1
1-1 GEAR CASE ONLY	$\overline{X} = 13.9$ $\sigma = 10.65$ $P = I$ $S = 2.1$ $K = 9.3$ $P_t = 0$ $V = .766$	39.1

TABLE I- SUMMARY OF SATURN ZONAL COMPOSITES (Concluded)

ZONE	STATISTICAL DESCRIPTION OF ZONAL COMPOSITES	97-1/2 % C. L. G_{rms} (50-2050 cps)
1-2 (COMPONENTS)	$\bar{X} = 16.84$ $\sigma = 14.2$ $P = I$ $S = 1.93$ $K = 6.89$ $P_t = 0$ $V = .843$ $\chi^2_s = 78.12$ $\chi^2_\ell = 27.5$ $(K\text{-}S+)_s = .193$ $(K\text{-}S+)_\ell = .098$ $(K\text{-}S\ ABS)_s = .193$ $(K\text{-}S\ ABS)_\ell = .109$	48.4
1-2 CHAMBER DOME ONLY	$\bar{X} = 4.9$ $\sigma = 3.79$ $P = I$ $S = .9$ $K = 3.3$ $P_t = IX$ $V = .774$	12.3
1-2-C	$\bar{X} = 10.1$ $\sigma = 5.33$ $P = I$ $S = 1.27$ $K = 4.56$ $P_t = 0$ $V = .527$	21.1

DEFINITIONS OF THE STATISTICAL PARAMETERS ARE GIVEN AS FOLLOWS

$\overline{X}$	=	mean value of zonal composites
σ	=	standard deviation of the sample
S	=	skewness coefficient $\left(\frac{\mu_3}{\mu_2^{3/2}}\right)$
K	=	kurtosis coefficient $\left(\frac{\mu_4}{\mu_2^2}\right)$
μ	=	central moment
V	=	coefficient of variation $\left(\frac{\sigma}{\overline{X}}\right)$
P	=	Pearson type density function
P_t	=	Pearson transition number (sub-type)
χ^2_s	=	Chi-square number computed from the sample relative to the indicated Pearson curve
χ^2_ℓ	=	the limiting Chi-square number at the 5 per cent level of significance
$(K\text{-}S+)_s$	=	the K-S positive number computed from sample relative to the indicated Pearson curve (Kolmogorov-Smirnov nearness statistic)
$(K\text{-}S+)_\ell$	=	the limiting K-S value at the 5 per cent level of significance
$(K\text{-}S\ ABS)_s$	=	the absolute value of the relative K-S statistic
$(K\text{-}S\ ABS)_\ell$	=	the limiting absolute K-S at the 95 per cent probability limit

The criterion of acceptance for the indicated Pearson curve within the 95 per cent probability region (5 per cent level of significance) is:

$$\chi^2_s \leq \chi^2_\ell$$

$$(K\text{-}S+)_s \leq (K\text{-}S+)_\ell$$

$$(K\text{-}S\ ABS)_s \leq (K\text{-}S\ ABS)_\ell$$

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