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COMPARISON OF MILES' RELATIONSHIP TO THE TRUE MEAN SQUARE VALUE OF RESPONSE FOR A SINGLE DEGREE OF FREEDOM SYSTEM

BY

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I. Introduction

During ascent of a space vehicle system from its launching pad, the noise generated by its propulsion system creates considerable vibratory motion in the vehicle structure. Consequently, any component and/or payload attached to the structure will experience a dynamic loading condition. Predicting these acoustically induced "random" loads is difficult because of the complexity of the acoustic environment and the vehicle structure. Conventional finite element analysis can be used to predict the response, i.e., loads, for relatively low frequency vibration, but the number of degrees of freedom becomes prohibitive for higher frequencies.

In 1954, John W. Miles performed an analysis[1] that led to a simplified method for calculating random loads. This analysis dealt with the response of a single degree of freedom (SDOF) system to a white noise input. This analysis, later to become known as Miles' relationship, provides estimates of the MSV (mean square value) of the response of a SDOF system to a white noise excitation. Miles' relationship (MSV) as used in industry today is:

$$\text{Miles} \cong (\pi/2) Q\, F_n\, G_o \tag{1}$$

where:

Q = system amplification factor = $1/2\zeta$

ζ = system damping

F_n = system natural frequency

G_o = power spectral density of input signal (a constant)

The form of the Miles' relationship as shown in equation (1) was obtaind by integrating the SDOF expression for all positive frequencies, i.e., MSV = $G_o \int_0^\infty |H(f)|^2 df$.

II. Analytical Procedure

Miles' relationship gives a simple method of estimating the mean square response of a component excited by white noise. However, Miles' relationship is commonly used to predict the response of a component to a more complex input excitation with the assumption that the complex environment can be approximated by white noise, i.e., to any arbitrary shaped input spectra. Actually, a component will respond at many different frequencies with different amplification factors, but the greatest response, in general, is at the fundamental frequency and the responses at all other modes are assumed to be insignificant. By estimating a system gain (Q) and natural frequency (F_n) and using equation (1), the mean square value of response for a component or payload can be generated very quickly and easily if the above assumption of a white noise spectrum is valid.

Miles' relationship has proved very popular when a random load must be generated quickly, precluding complex computer analysis. Now that small, inexpensive desk-top micro computers are available a more exact but equally rapid method of estimating mean square responses for any type of input spectra is being sought. This more accurate method would in effect reduce conservatism in many cases by reducing inaccuracies. A comparison of one such method with Miles' relationship is shown herein. Consider the following single input constant parameter linear system:

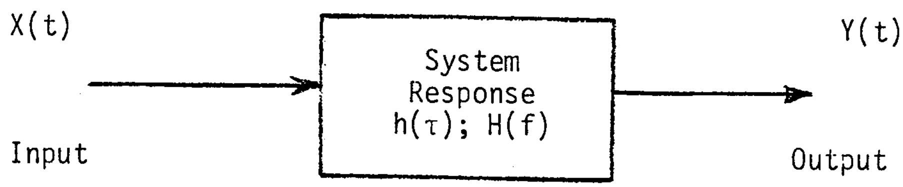


Figure 1

where: H(f) is the frequency response function and h(τ) is the impulse response function . For this system, the input and output can be related as follows:

$$G_y(f) = |H(f)|^2 G_x(f) \tag{2}$$

Where: $G_x(f)$ = power spectral density (PSD) of the input (excitation)
$G_y(f)$ = power spectral density of the output (response)

The mean square value of the response is simply

$$\psi^2 y = \int_0^\infty G_y(f)df = \int_0^\infty |H(f)|^2 G_x(f)df \tag{3}$$

which is the area under the response curve.

For a single-degree-of-freedom system $|H(f)|^2$ can be written as

$$|H(f)|^2 = \frac{1}{[1 - (f/f_n)^2]^2 + 4\zeta^2(f/f_n)^2} \tag{4}$$

where f_n = natural frequency.

The above described SDOF system is illustrated in figure 2a.

For anything other than a white noise input spectrum, a closed form solution of equation (3) is very cumbersome and tedious and can only be obtained for very idealized types of sloped input spectra. However, the mean square value can be approximated by replacing the integral form of equation (3) with a summation:

$$\psi_y^2 = \sum_{i=0}^{N} G_i(f_i)|H_i(f_i)|^2\Delta f \qquad (5)$$

If N is large enough, equation (5) converges to the exact solution. The real advantage to equation (5), however, is that it can be solved with a computer very easily.

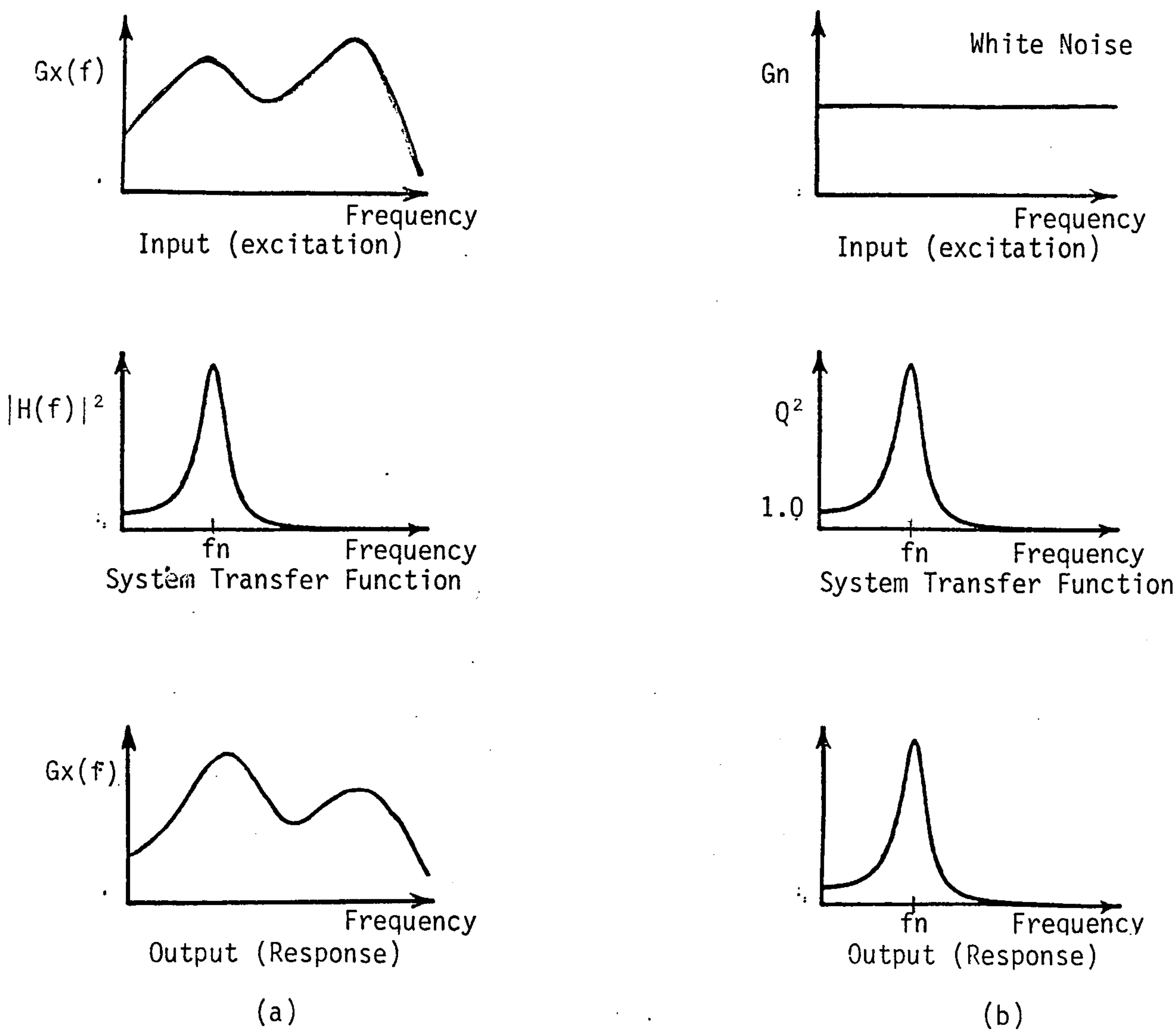


Figure 2. Typical SDOF Systems

A computer program that solves equation (5) has been developed and was used to compare an essentially exact solution to Miles' relationship. Three parameters were varied in the comparison: the input spectrum shape, the amplification factor, and the system natural frequency. The results of equation (5) were then ratioed to the mean square value estimate provided by equation (1) and plotted versus Q for each case. This ratio, Miles/ψ_y^2, then provides an immediate comparison between the idealized estimate of the MSV of the response obtained by using the Miles' relationship and the true MSV.

III. Comparison of Results

The first case that was examined was a flat (white noise) input spectrum to validate Miles' relationship (see figure 2b). Figure 3 presents the ratio of the Miles estimate to the true mean square value as determined by equation (5) plotted versus Q. The natural frequency of the SDOF system in this case was 100 Hz and the limit of the summation was 0 - 2000 Hz (dashed line) and 20 - 2000 Hz (solid line). The results indicate very good agreement with Miles' relationship (<6%) at low Q values. Very little error associated with truncating the lower end of the spectrum is evident and, further, the truncation at the high frequencies (2000 Hz) produced essentially no error.

Figure 4 shows a Miles/ψ_y comparison with a shaped input spectrum. The input spectrum has an increasing slope from 20 to 300 Hz that varies from 2 dB per octave to 20 dB per octave. This figure shows a very significant result: for a natural frequency at the break frequency (300 Hz), Miles' relationship is considerably conservative, but at a natural frequency on the slope (100 Hz) Miles' relationship is unconservative. In the range of damping (Q = 4 to 12) and spectral slopes (2 to 12 dB per octave) Miles' relationship varied from the true mean square value by as much as ±50%. When the natural frequency of the system is moved to the flat portion of the input spectrum above the break frequency (400 Hz) the input begins to approach that of the white noise case and Miles' relationship approaches the true mean square value (figure 5).

Next, a typical vibration criteria was used as an input to the SDOF system. Figure 6 shows the spectral shape of the criteria and figure 7 shows the comparison. There is a rather large variation between Miles' relationship and the true mean square value at the higher damping, depending upon the natural frequency. The most deviation occurred when the natural frequency was at the transition from a slope to a flat portion of the input spectrum (200 and 650 Hz). This case illustrates the transition from conservatism to unconservatism by simply moving the natural frequency.

The final case shown is that of an actual Space Shuttle criteria (figure 8). The comparison is shown in figure 9. This input spectrum is relatively flat (low slopes), i.e., approaching a white noise input, and Miles' relationship gives a fairly good estimate of the true mean square value (within 20%).

The evidence shown above indicates that using Miles' relationship for other than that for which it was intended, i.e., white noise input, is an unreliable estimator of the true mean square value of a SDOF system. Miles' relationship is usually assumed to give a conservative estimate, but in many cases that is not true and, in addition, it is very difficult to predict in advance whether the

Miles estimate will be conservative or not. By using a computer algorithm based on equation (5), a much more accurate mean square value can be calculated. This would reduce uncertainty and, consequently, conservatism or unconservatism in random loads since there is more confidence in the answer.

IV. Further Consideration for Obtaining Design Loads

Two more areas in predicting space vehicle design loads must be examined more thoroughly in the future. First, the statistical crest factor for a random response should be investigated. Currently, random loads are calculated by multiplying a crest factor by the RMS value of the predicted response. This gives a statistical estimate of the peak response value, which would be the maximum expected load. Generally this crest factor is based on a 99.87% confidence level, which is the so-called "3σ" value. This 3σ value may in fact be too conservative and should be reviewed.

Second, the method of combining low and high frequency (random) loads should be examined. The current method is to combine the loads so that the absolute worst case for both types of loads is used in the structural analysis. A statistical survey should be conducted of the phasing as well as direction of the low and high frequency loads so that all loading conditions are not ultra-conservative. Finally, these load prediction methods must be backed up with flight instrumentation to verify the procedures. This study of random loads prediction procedures is part of a long-term study of loads combination technology.

Flight instrumentation will become increasingly important for verification of the combined loads prediction procedure. The only way to have a reasonable amount of confidence in the loads is to measure the response of several payloads on the Space Shuttle and correlate them with various prediction methods. To reduce the conservatism in dynamic loads, more work remains to be done both in reverifying old methods and defining new methods.

V. Conclusion

The results presented herein show very clearly the limitations of the use of Miles' relationship for estimating the mean square value (MSV) of the response of a single-degree-of-freedom system. Considerable uncertainty in the predicted response results when applying Miles to any type of realistic shaped input spectra (criteria). These predicted responses can be either conservative or unconservative and depend critically upon where the SDOF system's natural frequency falls with respect to the input criteria.

It is also shown that with the aid of small, desk-top microprocessors, software can be very quickly and efficiently written which will provide a very accurate and reliable value for the "true MSV" of the response of a SDOF system to any arbitrary input spectrum shape (criteria).

References

1. Miles, John W., "On the Structural Fatigue Under Random Loading," Journal of the Aeronautical Sciences, November 1954

2. Bendat, Julius and Piersol, Allan, "Random Data: Analysis and Measurement Procedures," Wiley-Interscience, 1971

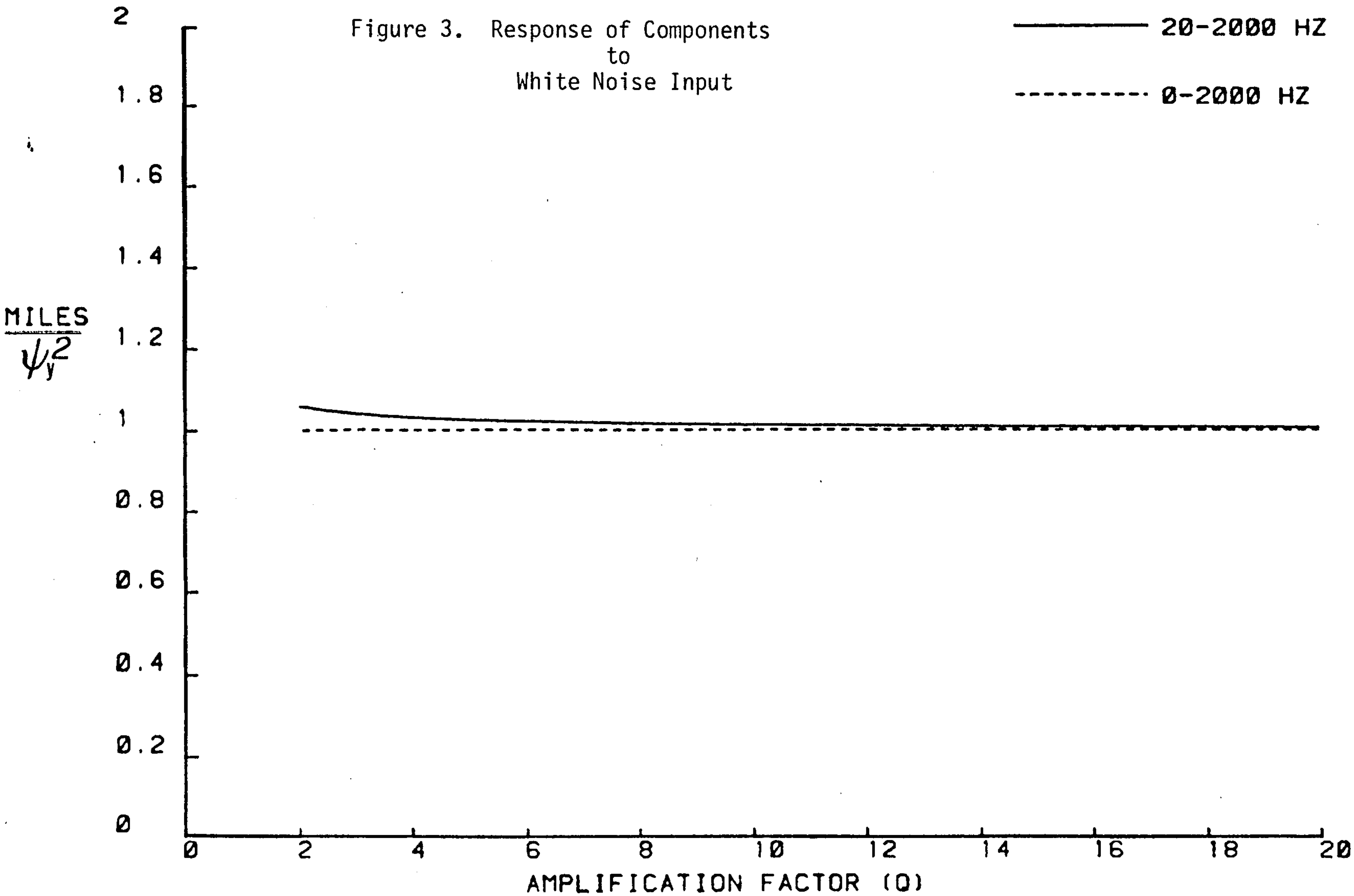


Figure 3. Response of Components to White Noise Input

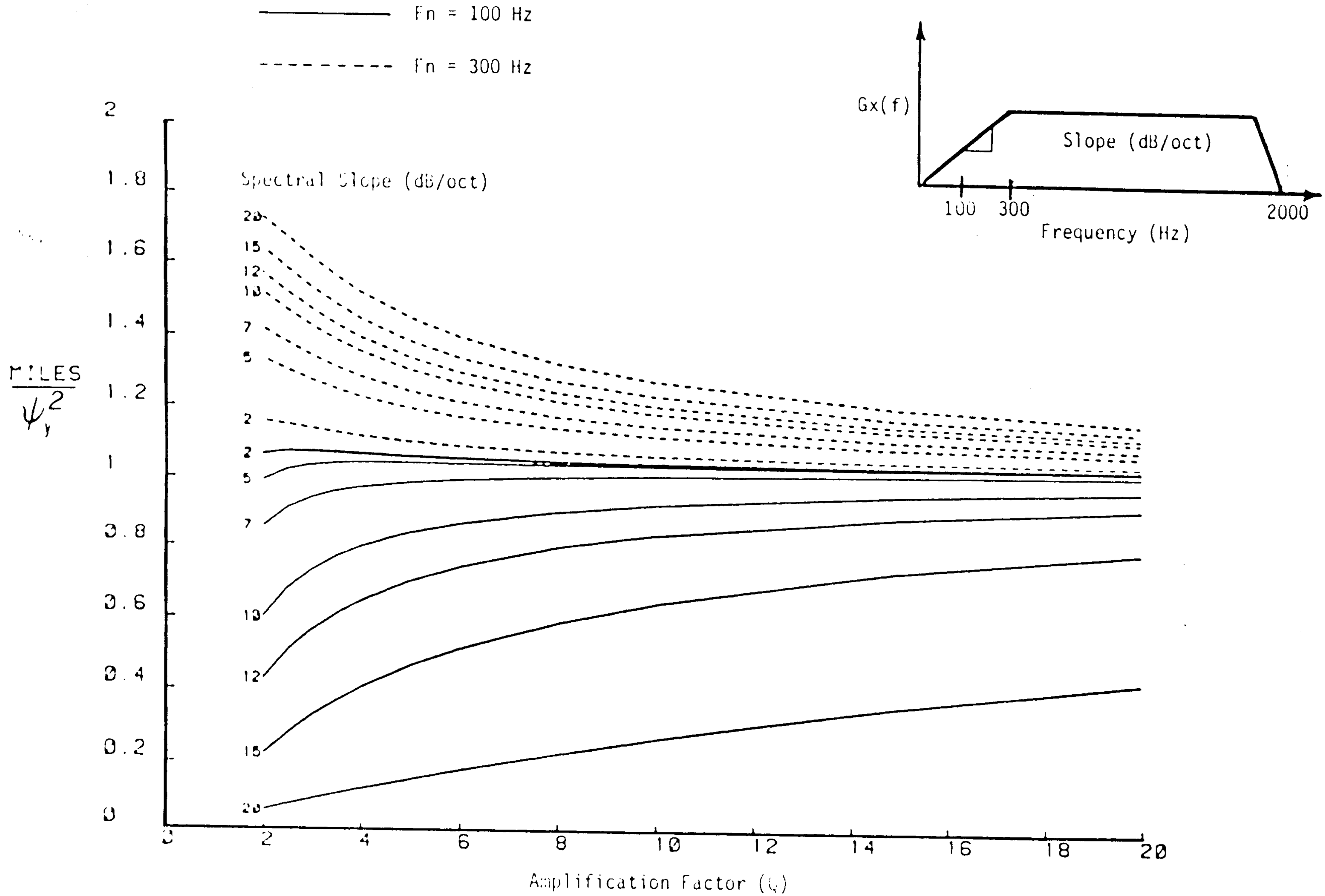
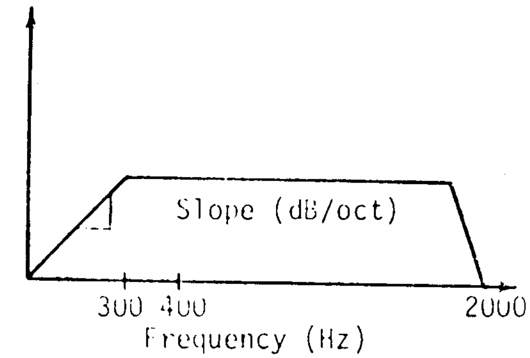


Figure 4. Response of Components to Shaped Input Spectra

Figure 5. Response of Component to Shaped Input Spectra



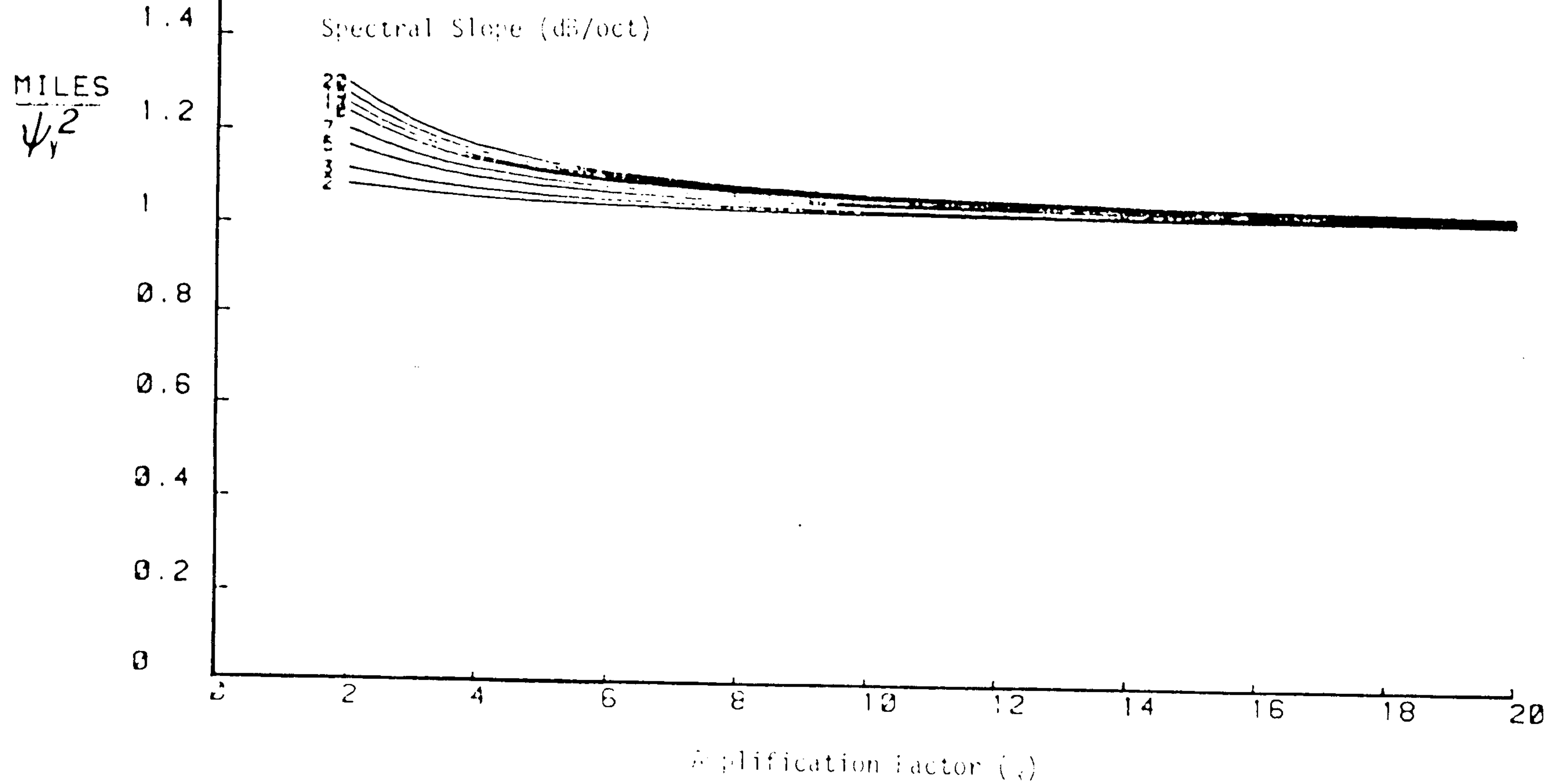


Figure 6. Typical Vibration Criteria.

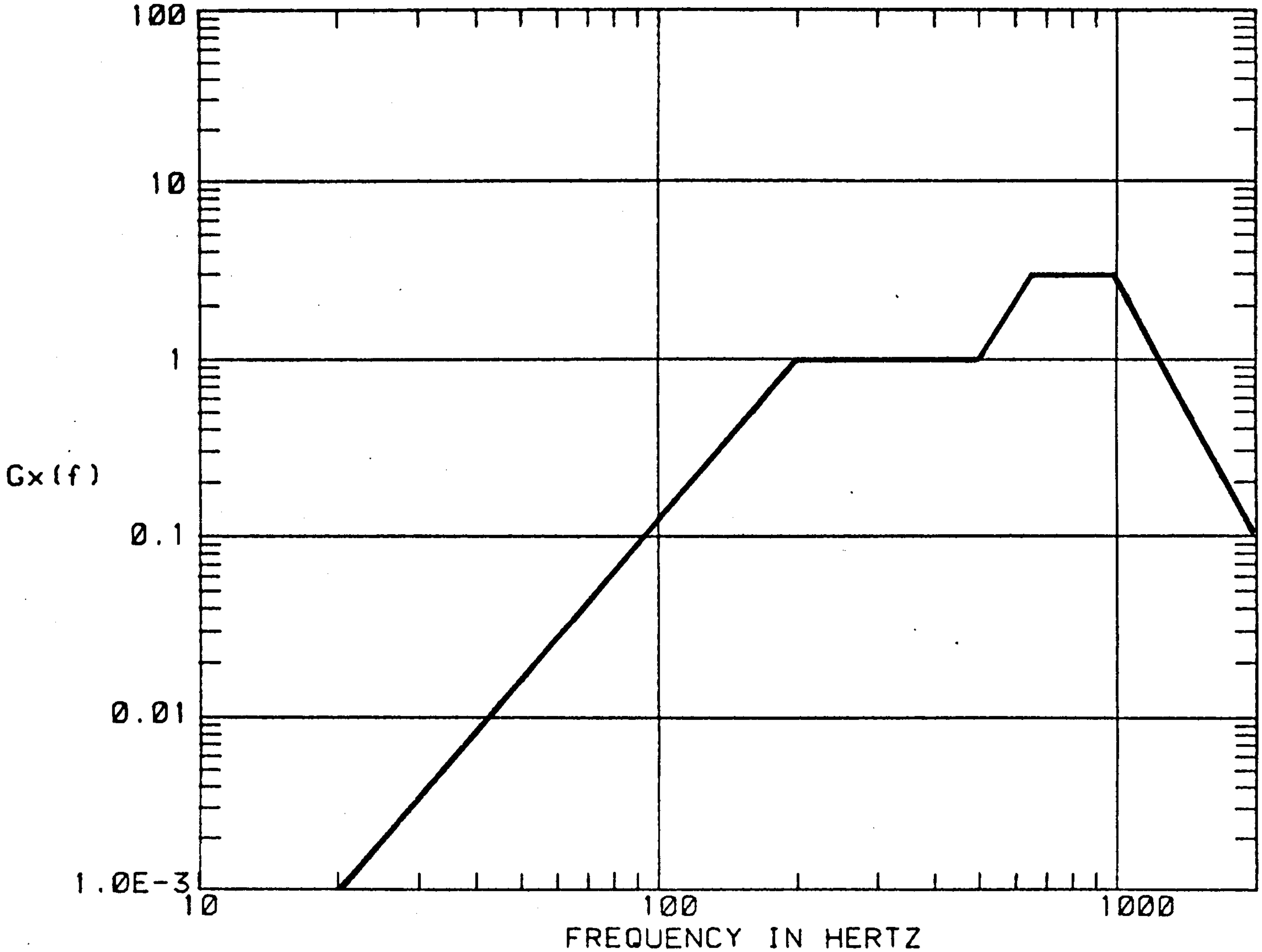


Figure 7. Response of Component to a Typical Vibration Criteria

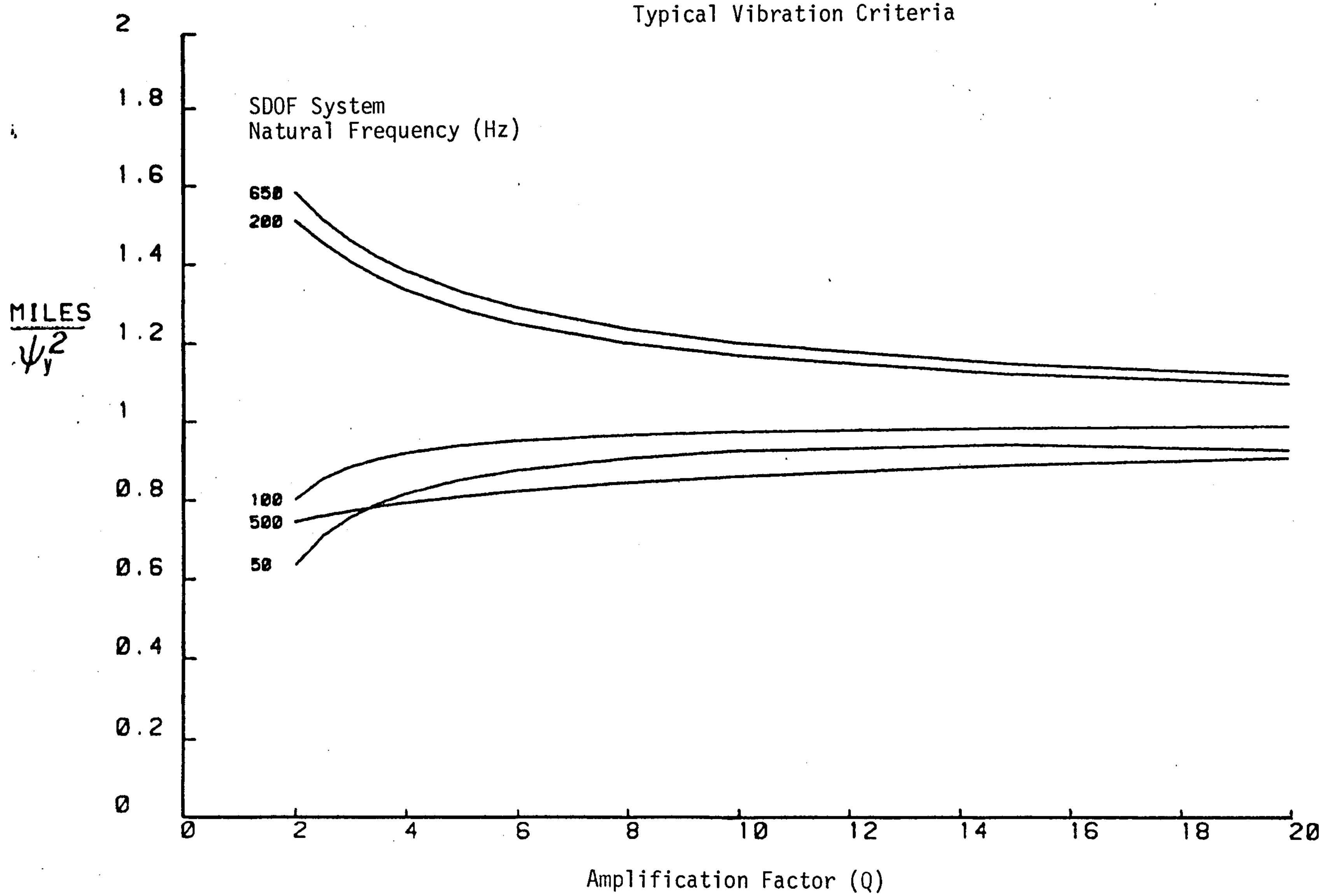


Figure 8. Space Shuttle Vibration Criteria.

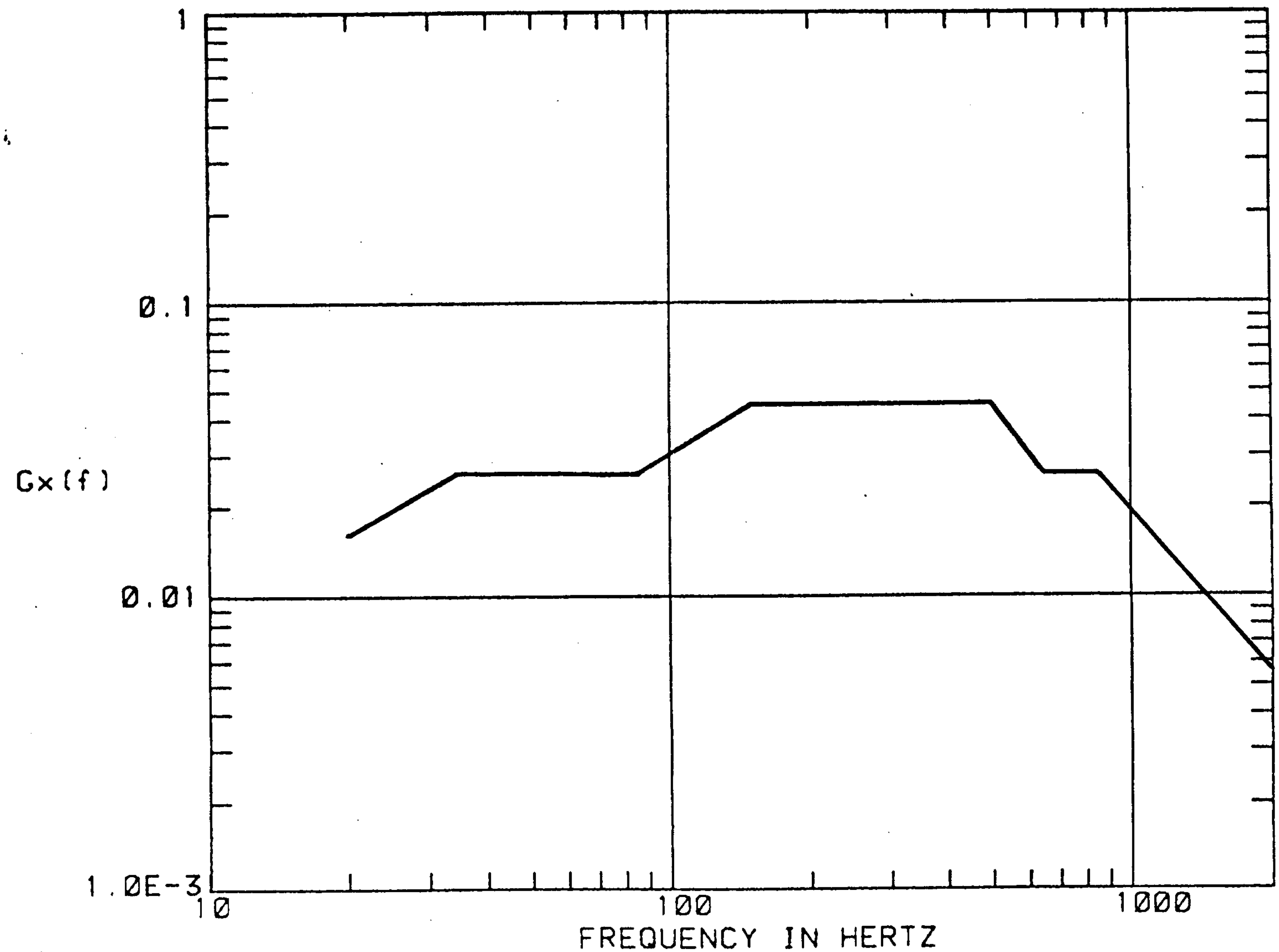


Figure 9. Response of Component to a Space Shuttle Criteria