VIBRATION RESPONSE OF A CYLINDRICAL SKIN TO ACOUSTIC PRESSURE VIA THE FRANKEN METHOD Revision H

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Introduction

The front end of a typical rocket vehicle contains avionics and a payload, enclosed by a cylindrical skin. Rocket vehicles are subjected to intense acoustic loading during liftoff. The external acoustic pressure causes the skin surfaces to vibrate. The skin sections are also excited by structural-borne vibration transmitted directly from the engine or motor. Nevertheless, the acoustic field is usually the dominant excitation source.

The purpose of this report to present an empirical method for calculating the vibration response of a cylindrical skin to an external acoustic pressure field. This method is based on data collected by Franken, as documented in References 1 through 4.

The external acoustic pressure field may be due either to liftoff or to aerodynamic buffeting thereafter.

Method

A vibroacoustic response function is given in Figure 1. The function was developed from studies of Jupiter and Titan 1 acoustic and radial skin vibration data collected during static firings.

The function predicts the skin vibration level in GRMS based on the input sound pressure level in dB, vehicle diameter in feet, and surface weight density in pounds per square foot. The bandwidth for the predicted vibration levels is the same as the bandwidth for the acoustic input levels. For example, the vibration levels will be in terms of one-third octave spectra if the acoustic pressure is in terms of one-third octave spectra.

Note that the function in Figure 1 predicts levels in terms of a 6 dB amplitude range. No details are available as the statistical considerations used to arrive at this range.

Assumptions

The Franken method assumes

1. All flight vehicles to which the procedure is applied have similar dynamic characteristics to the Jupiter and Titan 1 vehicles.
2. Vibration is primarily due to the acoustic noise during liftoff or other pressure fields during flight which can be estimated.

3. The vibration magnitude is directly proportional to the pressure level of the excitation and inversely proportional to the surface weight density of the structure.
4. Predominant vibration frequencies are inversely proportional of the diameter of the vehicle.
5. Spatial variations in the vibration can be considered as a random variable.

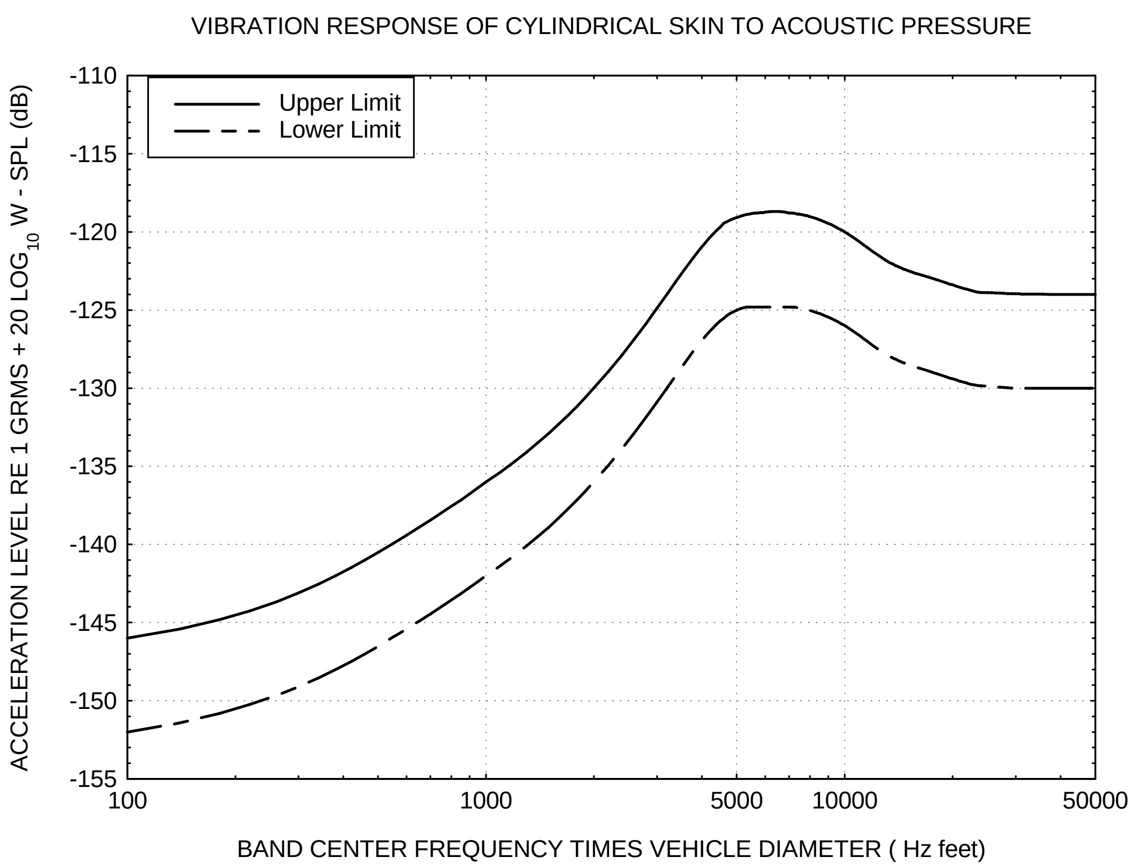

Figure 1. Vibroacoustic Frequency Response Function

Note that W is the skin surface weight density in units of psf. An equivalent graph in terms of metric units is given in Appendix A.

Ring Frequency

The ring frequency f_r is the frequency at which the longitudinal wavelength in the skin material is equal to the vehicle circumference.

$$f_r = \frac{C_L}{\pi d} \tag{1}$$

where

C_L	is the longitudinal wave speed or speed of sound in the skin material
d	is the diameter

The longitudinal wave speed in aluminum bar is approximately 16,539 feet per second. This value increases to 20,670 feet per second for aluminum bulk. The bar value is recommended for thin-walled cylinders.

Thus the ring frequency for aluminum is

$$f_r = \frac{5264 \text{ Hz feet}}{d} \tag{2}$$

Equivalent formulas are

$$f_r = \frac{63{,}172 \text{ Hz inches}}{d} \tag{3}$$

$$f_r = \frac{1604 \text{ Hz meters}}{d} \tag{4}$$

Note that the curves in Figure 1 reach their respective peaks near this ring frequency.

The corresponding mode shape is one in which all of the points move radially outward together and then radially inward together. Further information on the characteristics of the ring frequency is given in Reference 5.

Speed of Sound

Table 1. Speed of Sound				
Material	Density (lbm/in^3)	Elastic Modulus (psi)	Speed of Sound (in/sec)	Speed of Sound (ft/sec)
Aluminum	0.098	1.0E+07	198,463	16,539
Brass	0.307	1.5E+07	137,752	11,479
Copper	0.322	1.6E+07	138,492	11,541
Glass/Epoxy	0.072	3.0E+06	126,820	10,568
Graphite/Epoxy	0.058	1.0E+07	257,976	21,498
Steel	0.283	2.9E+07	197,164	16,430
Titanium	0.163	1.6E+07	191,650	15,971

The values in Table 1 are approximate.

Note that the speed of sound values are for bar rather than bulk.

Composite Sandwich Structure, Method 1

Consider a sandwich structure with a honeycomb or foam core. The sandwich design provides for greater stiffness than if the two face sheets formed a single structure with no core. The sandwich design should thus provide greater vibroacoustic attenuation than a homogeneous structure.

The Franken method can be extended to a sandwich structure by scaling in terms of mechanical impedance or equivalently by modal density.

The driving point impedance Z_{dp} for an orthotropic, thin plate is

$$Z_{dp} \approx \frac{4M}{n(f)} \tag{5}$$

where

M = mass

n(f) = modal density at frequency f

Equation (5) is taken from Reference 6.

The modal density for a cylinder can be found via the method in Reference 7.

The Franken result can be calculated for a sandwich structure by applying a reduction factor to the Franken results for the equivalent homogenous structure with no core.

The reduction factor R(f) is

$$R(f) = 20\log_{10}\left[\frac{n_2}{n_1}\right] \quad , \quad \text{dB} \tag{6}$$

where

n_2 = modal density of the composite, sandwich structure

n_1 = modal density of the equivalent homogeneous material without the core

Note that each modal density term is a function of frequency.

Composite Sandwich Structure, Method 2

Equation (6) may over-predict the attenuation of a sandwich structure in some cases. The radiation efficiency is another parameter of interest, in addition to impedance and modal density. Reference 8 gives further details, including a power transmissibility function.

Bending-to-Shear Transition

Furthermore, a honeycomb-sandwich panel may experience a bending-to-shear transition as follows:

Range	Characteristic
Low Frequencies	Bending of the entire structure as if were a thick plate
Mid Frequencies	Transverse shear strain in the honeycomb core governs the behavior
High Frequencies	The structural skins act in bending as if disconnected

See Reference 9 for further information regarding these characteristics.

The honeycomb-sandwich panel may experience an unexpectedly high vibration response in the high-frequency range where the facesheets act in bending independently from one another. In this case, the honeycomb-sandwich panel may have a higher vibration response than an equivalent homogeneous plate.

References

1. Summary of Random Vibration Prediction Procedures, NASA CR-1302, 1969.
2. Vibration, Shock, Acoustics; McDonnell Douglas Astronautics Company, Western Division, 1971.
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8. T. Irvine, Vibration Response of a Thin Cylindrical Shell to External Acoustic Pressure via Statistical Energy Analysis, Revision B, Vibrationdata, 2008.
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APPENDIX A

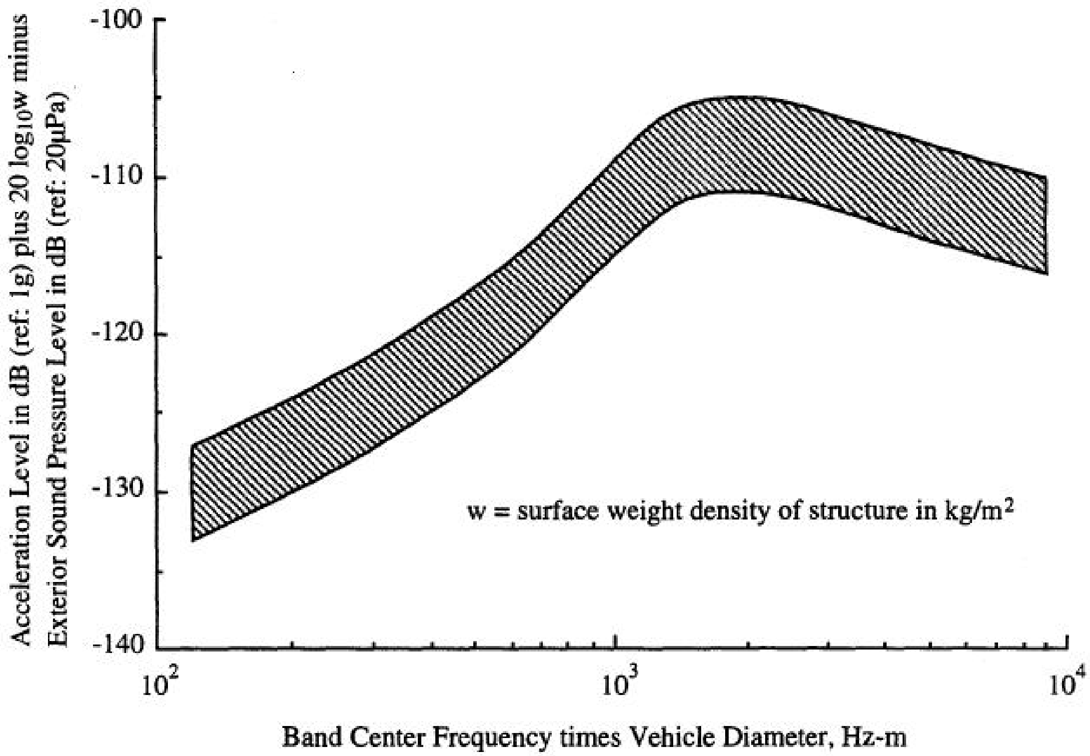

Figure A-1. Vibroacoustic Frequency Response Function, Metric Units

This graph is taken from Reference 3.

APPENDIX B

Sample Franken Method Calculation, English Units

Consider an aluminum cylinder.

The diameter is 40 inch, or 3.333 feet.

The skin thickness is 0.25 inch

The mass density of aluminum is 0.1 lbm/in^3.

The equivalent surface mass density is 0.025 lbm/in^2, or 3.6 lbm/ft^2.

Thus,

$$W = 3.6 \text{ psf} \tag{B-1}$$

Consider that the external sound pressure level is 120 dB at a center frequency of 100 Hz, in terms of a one-third octave band.

The band center frequency at 100 Hz multiplied by the vehicle diameter in feet is

$$100 \text{ Hz} \times 3.333 \text{ ft} \quad = \quad 333.3 \text{ Hz-ft} \tag{B-2}$$

The Franken upper limit curve value is

$$-142 \text{ dB at } 333.3 \text{ Hz-ft} \tag{B-3}$$

Let

F = Franken upper limit dB level

A = response acceleration level in GRMS.

The Franken formula is

$$20 \log\left(\frac{A}{1\,GRMS}\right) + 20 \log W \ - SPL\,(dB) = F \qquad (B\text{-}4)$$

$$20 \log\left(\frac{A}{1\,GRMS}\right) = F - 20 \log W \ + SPL\,(dB) \qquad (B\text{-}5)$$

Recall

F	=	142 dB	from Franken upper limit curve
W	=	3.6 psf	from problem statement
SPL	=	120 dB	from problem statement

By substitution,

$$20 \log\left(\frac{A}{1\,GRMS}\right) = -142 - 20 \log 3.6 \ + 120 \qquad (B\text{-}6)$$

$$20 \log\left(\frac{A}{1\,GRMS}\right) = -142 - 11.1 + \ 120 \qquad (B\text{-}7)$$

$$20 \log\left(\frac{A}{1\,GRMS}\right) = -33.1 \qquad (B\text{-}8)$$

$$\log\left(\frac{A}{1\,GRMS}\right) = -1.655 \qquad (B\text{-}9)$$

$$\left(\frac{A}{1\,GRMS}\right) = 0.022 \qquad (B\text{-}10)$$

$$A = 0.022 \text{ GRMS at } 100 \text{ Hz} \tag{B-11}$$

The bandwidth at 100 Hz for the one-third octave band is 23 Hz.

The PSD level at 100 Hz is

$$\frac{[0.022 \text{ GRMS}]^2}{23 \text{ Hz}} \tag{B-12}$$

$$2.13e-05 \quad \frac{\text{GRMS}^2}{\text{Hz}} \tag{B-13}$$

By convention, this is abbreviated as

$$2.13e-05 \quad \frac{G^2}{\text{Hz}} \quad \text{(at 100 Hz)} \tag{B-14}$$

The sample calculation can be repeated for other bands, given an input sound pressure level defined over these bands.

APPENDIX C

Mass Law

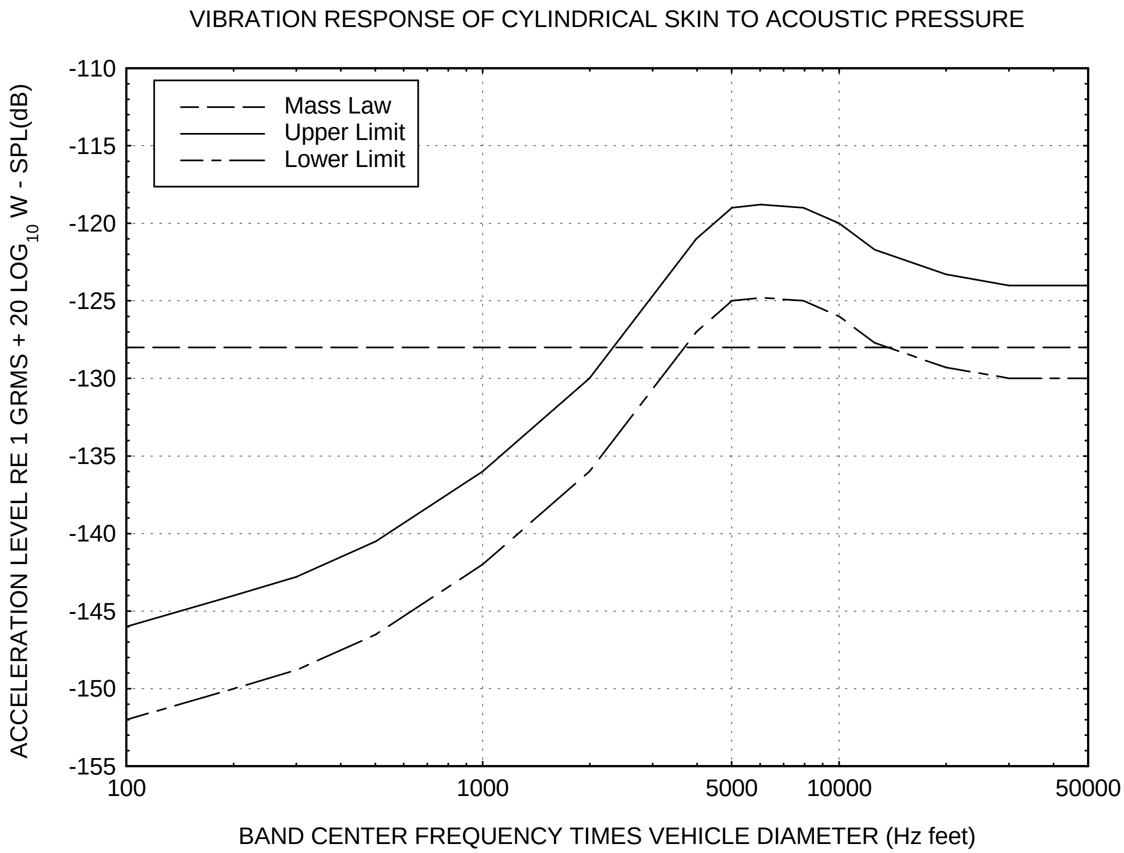

Figure C-1.

The Franken curves are shown together with the mass law curve. The mass law curve assumes that the cylinder wall has mass but neither stiffness nor damping.

The mass law ML is derived as follows.

$$\mathrm{ML\ (dB)} = 20\log\left(\frac{\mathrm{A}}{1\ \mathrm{GRMS}}\right) + 20\log(\mathrm{W}) - 20\log\left(\frac{\mathrm{P}}{\mathrm{Ref}}\right) \qquad \text{(C-1)}$$

$$\text{ML (dB)} = 20\log\left\{\left(\frac{\text{A}}{1\ \text{GRMS}}\right)(\text{W})\left(\frac{\text{Ref}}{\text{P}}\right)\right\} \qquad \text{(C-2)}$$

$$\text{ML (dB)} = 20\log\left\{\left(\frac{\text{Ref}}{1\ \text{GRMS}}\right)\left(\frac{\text{AW}}{\text{P}}\right)\right\} \qquad \text{(C-3)}$$

Note that

$$\frac{\text{AW}}{\text{P}} = \left(\frac{\text{force}}{\text{mass}}\right)\left(\frac{\text{area}}{\text{force}}\right)\left(\frac{\text{mass}}{\text{area}}\right) = 1 \qquad \text{(C-4)}$$

By substitution

$$\text{ML (dB)} = 20\log\left\{\left(\frac{\text{Ref}}{1\,\text{GRMS}}\right)\right\} \qquad \text{(C-5)}$$

The acoustic pressure reference is

$$\text{Ref} = 4.18\text{e-07 psf} \qquad \text{(C-6)}$$

$$\text{ML (dB)} = 20\log\left\{\left(\frac{\text{Ref}}{1\,\text{GRMS}}\right)\right\} \qquad \text{(C-7)}$$

$$\text{ML (dB)} = -128\ \text{dB} \qquad \text{(C-8)}$$