FEASIBILITY OF FORCE-CONTROLLED SPACECRAFT VIBRATION TESTING USING NOTCHED RANDOM TEST SPECTRA

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A summary of analyses and results is presented to substantiate the feasibility of using notched random test spectra for force-controlled spacecraft vibration tests. An analytical evaluation of structural loading at the base of the spacecraft caused by application of a broadband environmental acceleration spectrum is made. By using this analytical technique, notching requirements for the broadband test spectrum are established which result in rms structural forces at the spacecraft base that are within the spacecraft's structural design load criteria. Structural loadings of other structural elements of the spacecraft and component acceleration spectra are examined on the basis of the notched spectra and are found to be within design values. The proposed method of notching is considered to be a feasible approach to force limiting when available booster driving force at the spacecraft-booster interface is not known.

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INTRODUCTION

This paper presents a summary of analyses performed on a spacecraft to show the feasibility of a force-controlled vibration test using notched random test spectra. The introduction of notches into test spectra can be accomplished with available test equipment and is presently being implemented for the environmental test of a complete spacecraft system. The requirement of qualification and verification testing of a new spacecraft to an existing booster contractor's random acceleration spectrum necessitates a force-controlled test to assure compliance with the primary spacecraft design load criteria during the test period. A broadband random test spectrum is defined at the boost vehicle-spacecraft interface in terms of a power spectral density (PSD) envelope of expected environment. Application of this environment by mechanical excitation of the base of the spacecraft is accomplished with a modified Gaussian random distribution in which g-peaks do not exceed three times the root-mean-square (rms) acceleration.

Since the test spectrum is presented as an envelope, statistically derived from previous measured flight data of other payloads, it does not reflect the constraint of having an impedance match between the booster and the new spacecraft at their interface. PSD's of measured vibration environments from which envelopes are constructed contain regions of minimum vibration energy at the major interface antiresonances and energy peaks at the combined system resonances. This is analytically shown by Kaplan and Petak [1] for a multi-spring-mass model under a constant driving force. The existence and magnitudes of the minimum energy regions will actually be dependent on the available driving force of the booster at the corresponding fixed-base frequencies of the newly installed spacecraft. The amount of driving force over the frequency spectrum is usually unknown and is, therefore, unavailable to the spacecraft test manufacturer. To limit the test spectrum in the spacecraft test, it is proposed to introduce notches into the spectrum envelope which would approximate the existence of minimum vibrational energy regions at the spacecraft-booster interface. A detailed analytical model of the spacecraft is used to define the interface antiresonance points at which the notches are located. The amount of notching is to be dependent on a compatibility between the produced rms structural forces and the primary structural load design criteria of the spacecraft.

To determine the amount of modification required for a specified spacecraft test spectrum, modal response analyses are performed which evaluate structural loading of the spacecraft when subjected to the test spectrum at the spacecraft base. Notches in the test spectrum are then determined so as to eliminate peaks in the structural force spectral densities and result in rms forces which are compatible with the structural load criteria at the interface. A limiting force spectrum envelope which determines the amount of notching and the location of notches is assumed, having the same shape as the interface acceleration spectrum envelope. The limit loads which can be transferred across the interface and those in the interface structure as defined by the load criteria are used as a basis for notching. After the notched test spectrum is determined, rms stresses and/or loads in those parts of the structure designed to minimum margins of safety are evaluated to assure that their limit loads are not exceeded during the test period. In addition, acceleration spectral densities at various equipment locations resulting from the notched test spectrum are evaluated and compared to the acceleration design environment for spacecraft components.

NOMENCLATURE

$q_n(t)$	Modal coordinate displacement in nth normal mode
g_n	Modal damping coefficient in nth normal mode
ω_n	Circular frequency of nth normal mode
m	Mass
ϕ_n	Modal deflection coefficient in nth normal mode
$\ddot{f}$	Acceleration of the fixed base of spacecraft
ω	Forcing frequency
C_n	nth normal mode modal load coefficient at base of spacecraft in the direction of excitation (force/length)
M_{eq_n}	Modal mass in nth normal mode
C_n^P	nth normal mode modal load coefficient in the Pth structural element (force/length)
F_n^P	Load in the Pth structural element in the nth normal mode
F_T^P	Total load in the Pth structural element
Φ_F^P	Transfer function, ratio of load amplitude in the Pth structural element to the amplitude of harmonic base acceleration
Φ_A^Q	Transfer function, ratio of acceleration amplitude at location Q to the amplitude of harmonic base acceleration
$A_T^{Q_j}$	Total acceleration at location Q in the jth degree of freedom
$\phi_n^{Q_j}$	nth normal mode deflection coefficient at location Q in the jth degree of freedom.
G	Gravitational constant
$W_F^P(\omega)$	Spectral density of total load in the Pth structural element (force ^{2}/cps)
$W_f(\omega)$	Spectral density of environmental base acceleration (g^2/cps)
$F_T^P rms$	Root-mean-square load
f_L	Lower frequency limit in the test spectrum (20 cps)
f_U	Upper frequency limit in the test spectrum (2000 cps)

STRUCTURAL ANALYSIS MODEL

To evaluate adequately spacecraft response to the broadband base excitation environment, a detailed structural analysis model is constructed for the prediction of a large number of normal vibration modes. The structural configuration of the spacecraft is presented in Fig. 1 and consists primarily of a glove section (skin-stringer-frame), cantilevered equipment truss, center body, aft body, fixed fins and movable flaps. The spacecraft is covered with a dense nonstructural heat protection material. Three pedestals are used to constrain the spacecraft from its aft bulkhead to the boost vehicle and are designated the support pedestals. A three-dimensional dynamic analysis model of this configuration, consisting of 1800 discrete structural elements, is derived and utilized in the matrix method of obtaining normal vibration

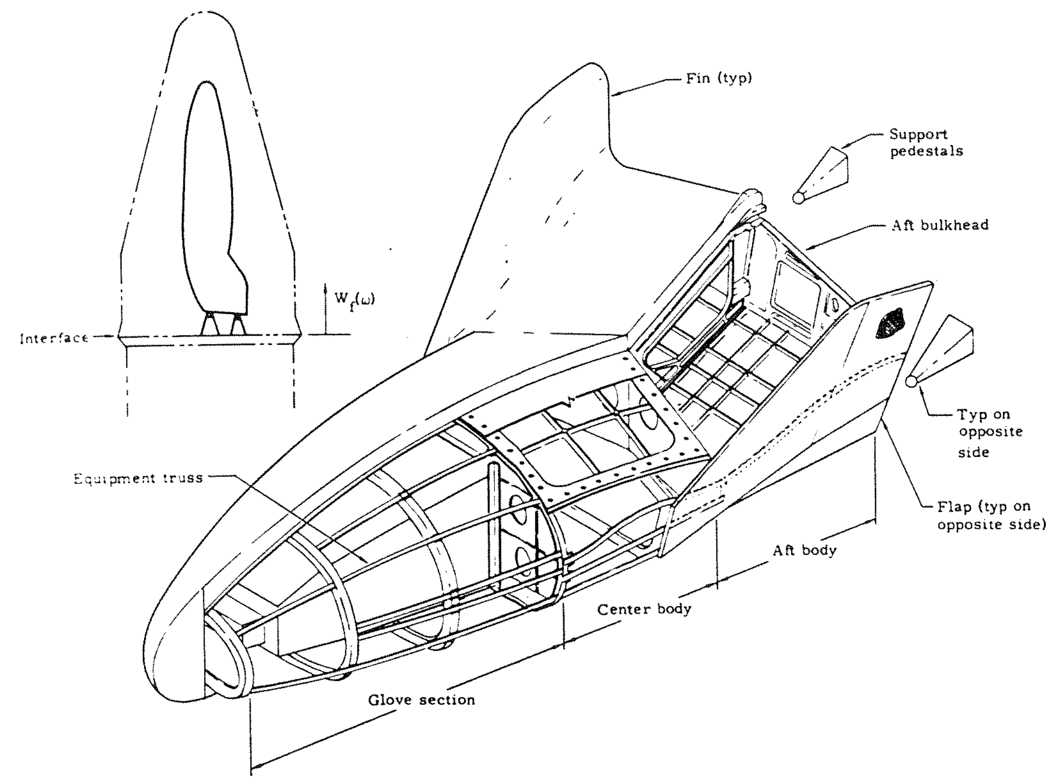


Fig. 1 - Structural configuration

modes [2]. The three-dimensional array of shear panels, tension elements, bending elements and twist elements is depicted in Fig. 2. Masses of the structure, heat shield and internal components are located at discrete points over the model. Motion of the masses is described by 300 degrees of freedom. Each of the support pedestals is represented by a set of three orthogonal reaction elements connecting the aft bulkhead to ground. Outputs from the matrix method of modal analysis consist of modal frequency, modal mass, modal deflection coefficients for each degree of freedom, and modal load and stress coefficients for each of the structural elements. A graphical description of the fundamental Z-direction bending mode is also presented in Fig. 2.

ANALYTICAL METHODS

The objective of the modal analysis is to obtain the rms structural loading at the base of the spacecraft when the structural representation is subjected to the test spectrum. These rms structural loads can then be compared to one-third the allowable base load as given by the spacecraft's design static load factors and the individual pedestal design static loads. The one-third value of allowable loads is used as a criterion to account for applied g-peaks which can be three times the rms value. Determination of loads in each of the 1800 structural elements could also be undertaken; however, the base loading is chosen as the basis for notching the test spectrum.

The modal coordinates' motion equations for a uniform beam subjected to an accelerated base motion can be obtained from the differential equation of classical beam theory and are of the form

$$\ddot{q}_n(t) + g_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) = \left(\frac{\omega_n^2 \int_0^{\ell} m(x)\, \phi_n(x)\, dx}{\int_0^{\ell} m(x)\, \phi_n^2(x)\, dx} \right) \frac{\ddot{f}(t)}{\omega_n^2} . \qquad (1)$$

This equation has been derived by Shinozuka [3]. For the discrete mass spacecraft model, the modal coordinates' motion equations become

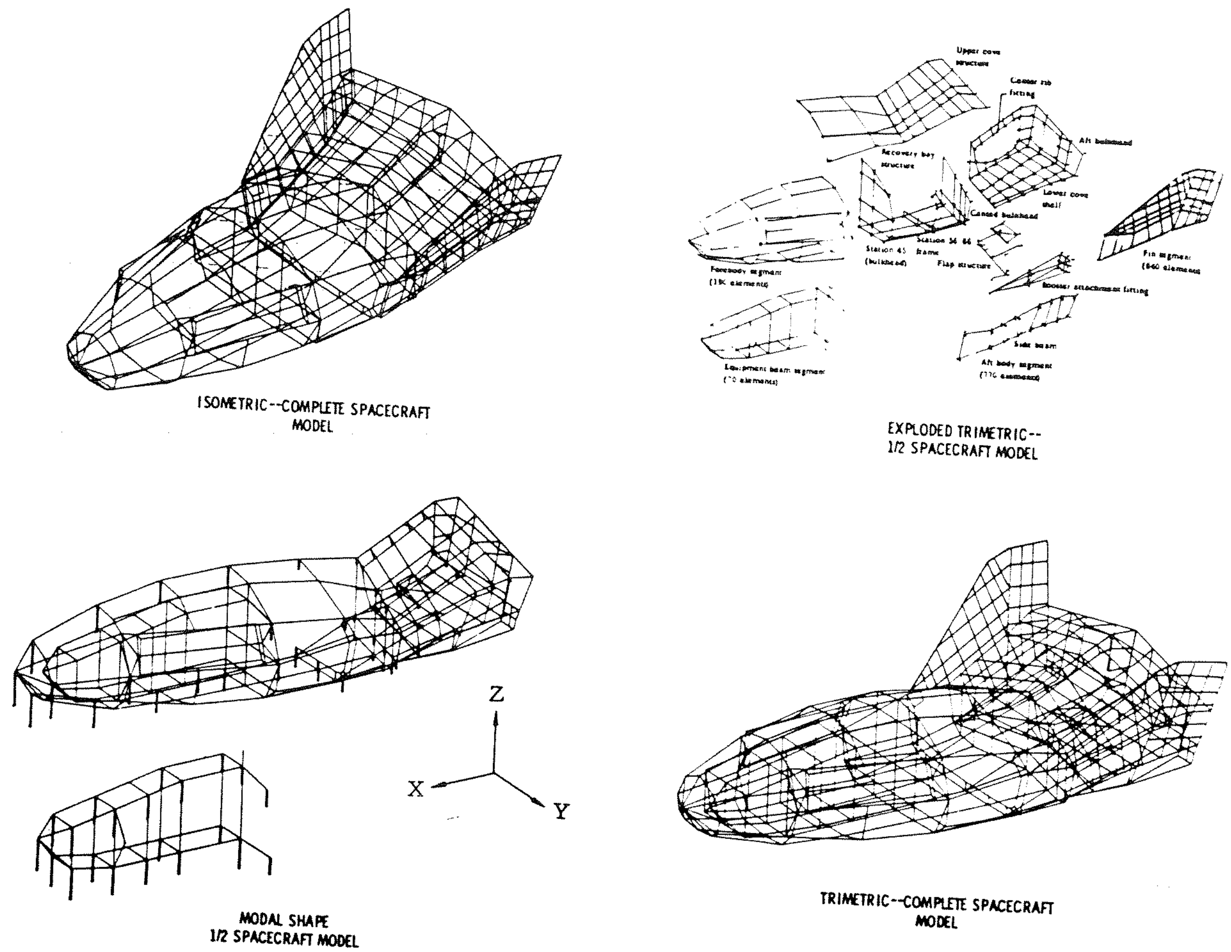


Fig. 2 - Structural analysis model of spacecraft

$$\ddot{q}_n(t) + g_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \left(\frac{\omega_n^2 \sum_j m_j \phi_{jn}}{\sum_j m_j \phi_{jn}^2}\right)\frac{\ddot{f}(t)}{\omega_n^2}, \qquad (2)$$

where the summations are made over the number of degrees of freedom in the spacecraft model. The term

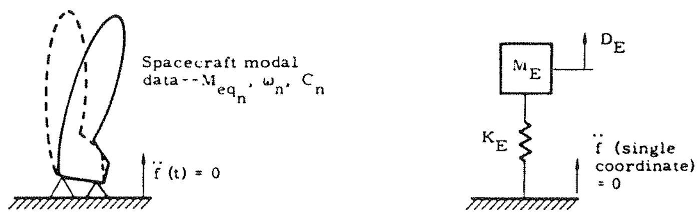
$$\omega_n^2 \sum_j m_j \phi_{jn}$$

represents the modal constraint in the nth mode at the base of the structure in the direction of excitation. This is obtained from the modal results of the spacecraft model by adding the modal load coefficients of the support pedestal elements lying in the direction of excitation. The term

$$\sum_j m_j \phi_{jn}^2$$

represents the modal mass of the nth mode. Equation (2) can also be obtained by the method used by MacNeal [4], whereby a modal model is constructed which has the proper force-displacement relationship for a single coordinate. The model consists of masses sprung from the coordinate by springs, and the values of the springs and masses can be computed from the normal modes with the coordinate constrained to zero motion. The procedure used in obtaining the motion equations by this method is summarized in Fig. 3. The sum of all the sprung masses (M_E) will equal the total mass of the spacecraft, since the system has a rigid body degree of freedom when the base constraint is removed.

Solution of Eq. (2) for harmonic accelerated base motion at the driving frequency ω gives the following relation for amplitude of the modal coordinate to the amplitude of base acceleration:



$1/2\ M_{eq}\ \dot{q}^2\ (1/2\ M_{eq}\ \omega^2 q^2)$ [kinetic energy] $1/2\ M_E \dot{D}_E^2\ (1/2\ M_E\ \omega^2\ D_E^2)$

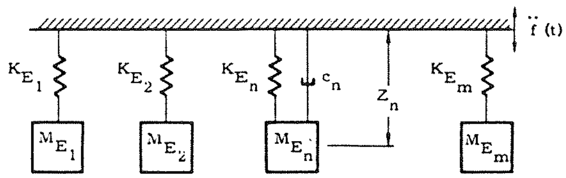
$1/2\ Kq^2\ (1/2\ M_{eq}\ \omega^2\ q^2)$ [potential energy] $1/2\ K_E\ D_E^2$

Cq [root constraint] $K_E\ D_E$

equating the kinetic, potential energies and root constraint results in

(a) $M_{E_n} = C_n^2 / M_{EQ_n}\ \omega_n^4$ (b) $K_{E_n} = \omega_n^2 M_{E_n}$

Equivalent Modal Model



Motion equations for the above system connected to the single coordinate are of the form

$$M_{E_n} \ddot{Z}_n + c_n \dot{Z}_n + K_{E_n} Z_n = - M_{E_n} \ddot{f}(t)$$

Using $C_n q_n = K_{E_n} Z_n$ (equating root constraints)

$$M_{E_n} \ddot{q}_n + c_n \dot{q}_n + K_{E_n} q_n = - \frac{M_{E_n} K_{E_n}}{C_n} \ddot{f}(t)$$

This form of the modal motion equation is equivalent to that of Equation 2

Fig. 3 - Derivation of motion equations (4)

$$q_n(\omega) = \frac{-C_n}{M_{eq_n} \omega_n^2} [A_n(\omega) - iB_n(\omega)]\ \ddot{f}(\omega), \quad (3)$$

where

$$A_n(\omega) = \frac{\omega_n^2 - \omega^2}{(\omega_n^2 - \omega^2)^2 + (g\omega_n \omega)^2}$$

and

$$B(\omega) = - \frac{g\omega_n \omega}{(\omega_n^2 - \omega^2)^2 + (g\omega_n \omega)^2} .$$

The ratio of load in a structural element in the nth mode to the amplitude of base acceleration is obtained by multiplying Eq. (3) by the element's modal force coefficient:

$$\frac{F_n^P(\omega)}{\ddot{f}(\omega)} = \frac{C_n^P q_n(\omega)}{\ddot{f}(\omega)} = - \frac{C_n^P C_n}{M_{eq_n} \omega_n^2} [A_n(\omega) + iB_n(\omega)] . \quad (4)$$

The ratio of total load in the Pth structural element, based on a modal sum approximation, to the amplitude of base acceleration becomes

$$\frac{F_T^P(\omega)}{\ddot{f}(\omega)} = \sum_{n=1}^{m} \frac{-C_n}{M_{eq_n} \omega_n^2} C_n^P [A_n(\omega) + iB_n(\omega)] = \phi_F^P . \quad (5)$$

The ratio of total structural acceleration at any of the mass locations to the amplitude of base acceleration can also be similarly obtained. Acceleration is given by

$$\frac{A_T^{Q_j}(\omega)}{\ddot{f}(\omega)} = 1.0 + \sum_{n=1}^{m} \frac{C_n}{M_{eq_n}\,\omega_n^2}\,\phi_n^{Q_j} \times [\omega^2 A_n(\omega) + i\omega^2 B_n(\omega)] = \Phi_A^Q . \quad (6)$$

The value of 1 is added to the summation if the acceleration is in the direction of excitation and is omitted if the acceleration is not in the direction of base excitation.

Equation (5) is of the form now to be utilized in obtaining PSD's of load from a base excitation environment, given terms of an acceleration spectral density. The load spectral density is given by

$$W_F^P(\omega) = (\Phi_F^P)^2\, W_f(\omega) \quad (7)$$

and

$$F_T^P\ \mathrm{rms} = \left(\int_{f_L}^{f_U} W_F^P(\omega)\, d\omega\right)^{1/2} . \quad (8)$$

ANALYTICAL RESULTS AND DISCUSSION

By using the modal data of the analytical model in Eq. (5), force transfer functions over the frequency range of interest (20 to 2000 cps) are evaluated for the total base forces and pedestal element forces in the directions of the three vehicle axes. A typical transfer function in terms of force squared to base acceleration squared is given in Fig. 4. Peaks in the transfer function occur at the fixed-base spacecraft modal frequencies for those modes having relatively large base modal force coefficients. In all of the evaluated transfer functions, the magnitudes of these peaks decreased in a broad sense with increasing frequency. The summation in the equation is made with the data of the first 35 normal modes which correspond to frequencies below 520 cps. The summation changes insignificantly when made with the data of the first 80 modes corresponding to modal frequencies below 1100 cps. This indicates that total modal forces at the spacecraft base are practically independent of the higher modes of vibration. Based on the sum of the equivalent single harmonic oscillator masses given in Eq. (a) of Fig. 3, 97.5 percent of the total spacecraft mass is represented by the first 35 modes and 98.3 percent is represented by the first 80 modes. Values of the modal damping coefficient ($2c/c_c$), which are considered to be conservative, range from 2 percent in the fundamental vibration mode to a maximum of 10 percent in the higher modes of vibration. (An experimental resonance survey of the spacecraft structure indicated a structural damping coefficient in the fundamental mode of 6 percent.) The magnitudes of the peaks in the transfer functions are expected to be significantly affected with changes in the structural damping coefficient.

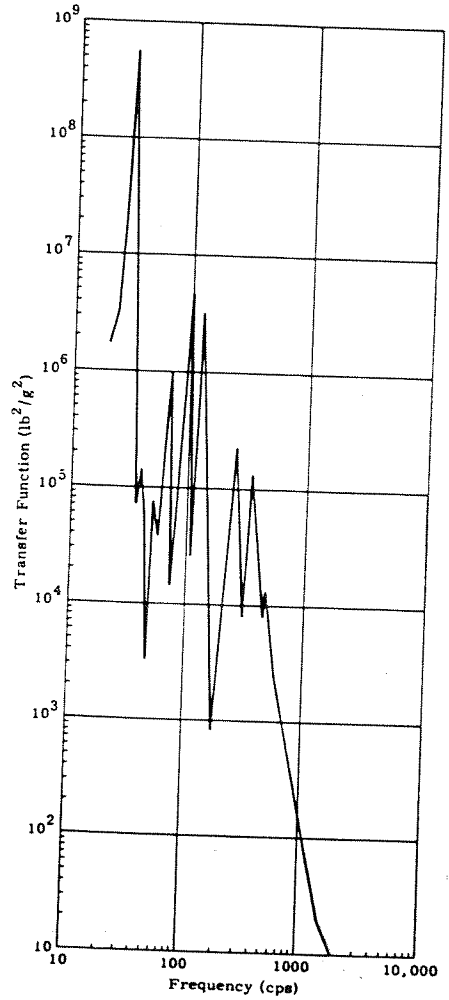


Fig. 4 - Total Y force at spacecraft base to interface acceleration transfer function

Acceleration transfer functions are also evaluated for two equipment locations by using Eq. (6). One of the locations (guidance package) is at the tip of the equipment truss and the other (actuator package) in the aft body section near the spacecraft base. Sample acceleration transfer functions relating the equipment accelerations in the same directions as base accelerations are given in Fig. 5. As is expected, the

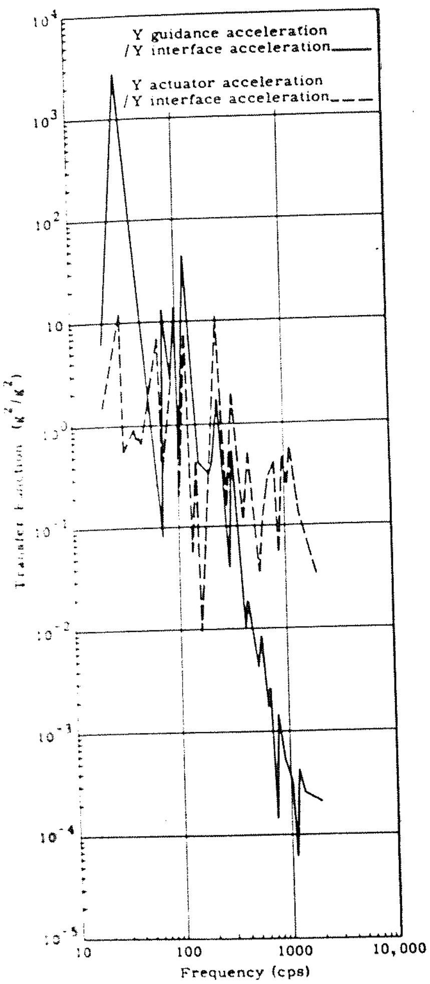


Fig. 5 - Transfer function

high-frequency portion of the acceleration transfer function is lower at the tip structural location than that near the plane of excitation. This result can be substantiated by examining the acceleration at the free end of a cantilevered beam when the driving frequency at the base approaches infinity. The expression for the ratio of free-end to fixed-end acceleration at [illegible] is given by

$$\frac{A_{free}}{A_{fixed}} \quad 1 - \sum_{n=1}^{\infty} \frac{\int_0^l m(x)\ \phi_n(x)\ dx}{\int_0^l m(x)\ \phi_n^2(x)\ dx} = 0 . \qquad (9)$$

Force spectral densities for the total base loads and pedestal loadings for the excitation applied in each of the three spacecraft axes directions were evaluated using Eq. (7). A typical force spectral density of total force across the interface is given in Fig. 6 and represents an rms force which is approximately 3.5 times greater than one-third of the peak allowable force given by the static loads criteria. To limit the force density to represent an allowable rms force, a limiting force spectrum, assumed to have the same shape as the input spectrum, is positioned with the force spectral density until the area contained within the shape represents an allowable rms force. The assumed limiting corresponds to a shape which would be obtained from a completely rigid structure. The limiting spectrum is shown in Fig. 6 with the contained shaded spectrum representing an allowable rms force. Sharp notches are then introduced into the acceleration spectral density for eliminating the spectral density peaks which exceed the limiting spectrum. Resulting notched test spectra based on each of the considered structural loadings are derived for each test excitation direction. An enveloping notched spectrum in each direction which satisfies all of the loads criteria is then proposed as the test spectrum. Figures 7 and 8 contain resulting notched spectra for two excitation axis directions and are primarily based on satisfying the total allowable loads at the base of the spacecraft. The spectrum in Fig. 7 is seen to be considerably notched between 20 and 30 cps and is caused by the fixed-base spacecraft's fundamental Y-direction bending mode frequency falling in this frequency band.

The fundamental X-direction bending mode frequency falls below 20 cps, thereby resulting in the relatively small notching requirements given in Fig. 8. The required notches are always contained in the low-frequency portions of the spectra, since the lower frequency modes produce the most significant amount of base loading. Because the notches are dependent on the sharp peaks occurring in the force spectral densities, their positioning as load limiters will be sensitive to the actual frequencies (rather than predicted frequencies) at which these peaks occur. It is, therefore, necessary to define the force transfer functions prior to the environmental tests by sinusoidal or random transmissibility testing of the spacecraft. Other structural elements, selected on a basis of minimum design margins, were also examined for rms stress levels produced by the final notched test spectra.

The modal load coefficients in Eq. (4) are replaced by the modal stress coefficients for evaluating the stress transfer functions used in obtaining the stress spectral densities. In all cases, rms stress levels produced by the notched spectra are always below one-third of

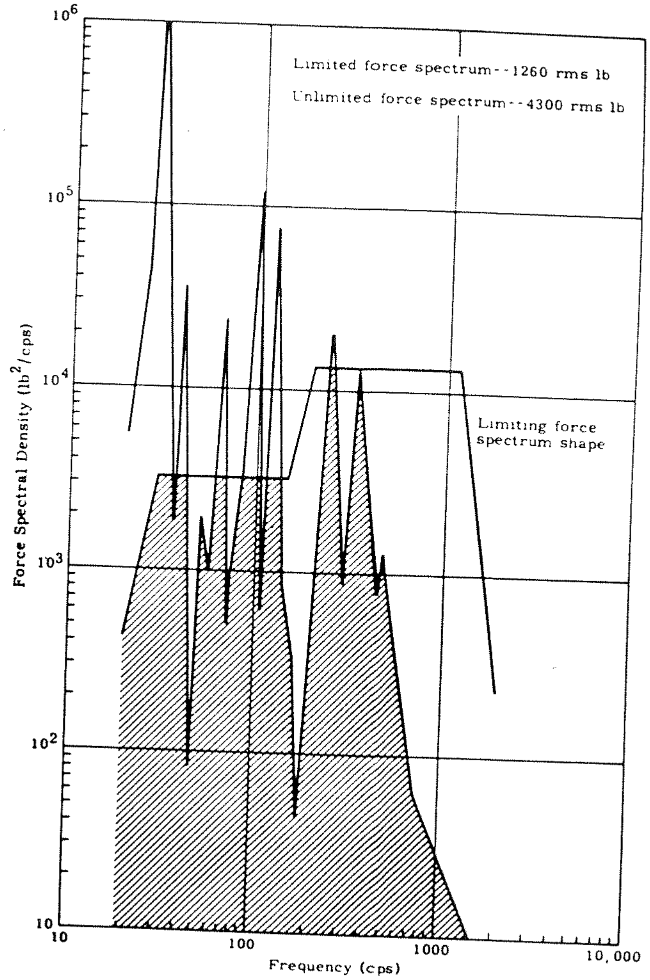
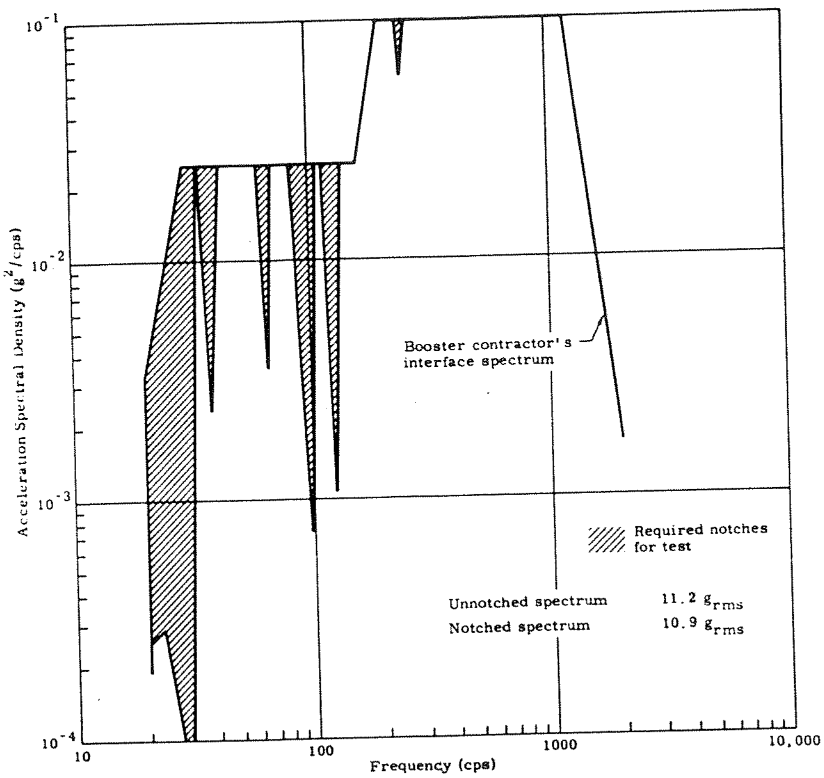


Fig. 6 - Interface force spectral density (notched and unnotched input acceleration spectra)

Fig. 7 - Y-direction test spectrum



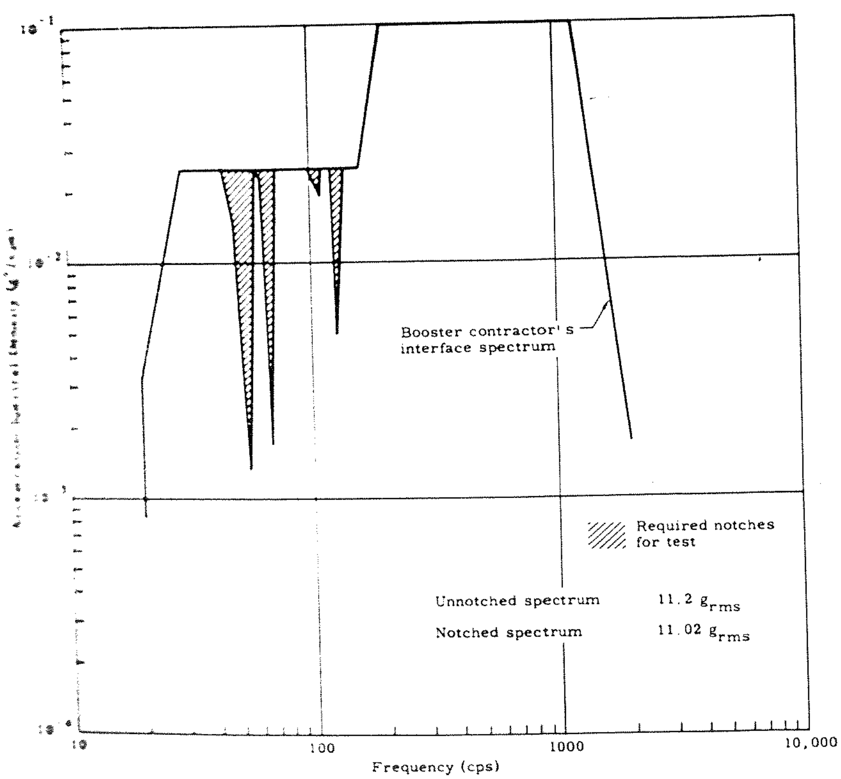


Fig. 8 - Z-direction test spectrum

the design stress levels. Acceleration spectral densities at two equipment locations (guidance package and actuator package) are also evaluated using Eq. (6) and the final notched test spectra. These densities can be compared to that used for component qualification testing to assure that the design environment will not be exceeded during the complete spacecraft testing. Sample acceleration spectral densities are given in Figs. 9 and 10 for the two locations together with the qualification test spectrum envelope.

The envelope is seen to contain almost all of the components' acceleration environment predicted from the spacecraft test. It is noted that the qualification environment envelope is based on the use of measured data from other booster vehicles in the prediction method described by McGregor et al. [5].

CONCLUSIONS

The analyses presented and analytical results show force-controlled random testing of a spacecraft is feasible by the positioning of notches in the test spectra. The notches can be considered the natural attenuations which actually occur at points of high driving impedance at the spacecraft-booster interface. When the amount of the notching is evaluated on the basis that it does not exceed the rms structural design loading at the base of the spacecraft and in the base support structure, spacecraft structural loads and component accelerations are expected to be within design values during the test period. With the assumed method of letting the position and amplitude of notches be based on a limiting force spectrum shape having the same shape as the test spectrum, notching

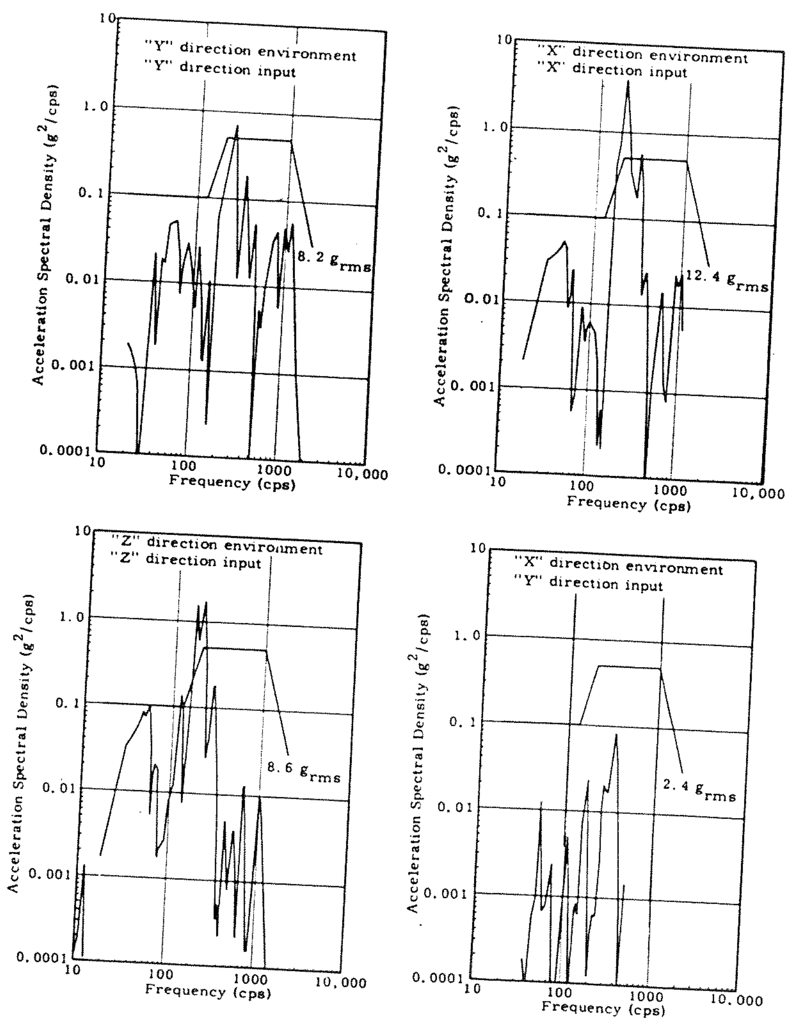


Fig. 9 - Acceleration environment at actuator

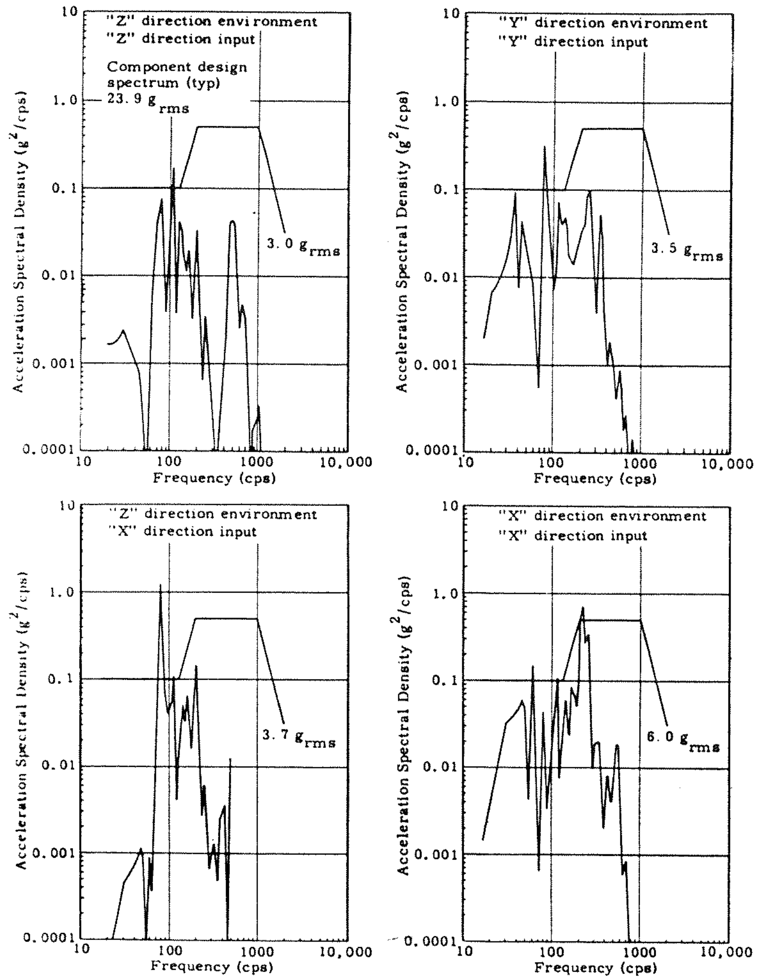


Fig. 10 - Acceleration environment at guidance package

requirements exist only in the low-frequency range of the test spectrum. A more exact method of introducing notches into the spacecraft test spectra would include the use of an available driving force spectrum of the booster; however, these data are generally not coexistent with given test spectra.

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DISCUSSION

Mr. Davis (Fairchild Hiller): Wouldn't the notched spectrum that you refer to approximate very closely the unequalized random vibration input that we usually get on a shaker before we equalize it?

Mr. Heinrichs: Usually during equalization you try to match a spectrum and envelope as closely as possible. I don't think that this type of notches occurs in the equalized spectrum shape.

Mr. Forlifer (NASA Goddard Space Flight Center): This notching procedure seems to be based on not exceeding some loads which are derived from a static load criterion, so the whole validity of it hinges on how adequate the static load criterion is. How do you check this?

Mr. Heinrichs: The spacecraft is designed to the static loads criterion and we are trying to perform the qualification test within the design load values. In other words, we do not want to test the spacecraft and then pick it up in pieces.

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