AERODYNAMIC LIFT AND DRAG COEFFICIENTS

By Tom Irvine
Email: tomirvine@aol.com

March 26, 2001

A Cessna Citation leaves a deep trough in the clouds beneath it, proof that it stays aloft by pushing air down.

Photo Courtesy of Cessna Aircraft Company

Introduction

An airflow applies both static and dynamic forces to structures. Lift and drag coefficients are required to calculate the airflow forcing functions. The purpose of this tutorial is to summarize data for these empirical coefficients.

Lift

The lift coefficient C_L is defined as

$$C_L = \frac{L / A}{\frac{1}{2}\rho V_o^2} \tag{1}$$

where

> L is the lift force normal to the free stream
> A is the usually project area in the direction of the freestream
> ρ is the density of the air or fluid medium
>
> V_o is the freestream velocity

Note that A is the planform area for the case of an airfoil. The planform area is the area of the airfoil as seen normal to the chord line of the airfoil.

The lift coefficient for a circular cylinder is shown in Figure 1, as taken from Reference 1. The lift coefficient is given as a function of Reynolds number. The Reynolds number is the ratio of the inertia force to the friction force.

Note that the Reynolds number Re for flow around a cylinder is

$$Re = \frac{\rho V D}{\mu} \tag{2}$$

where μ is the absolute viscosity.

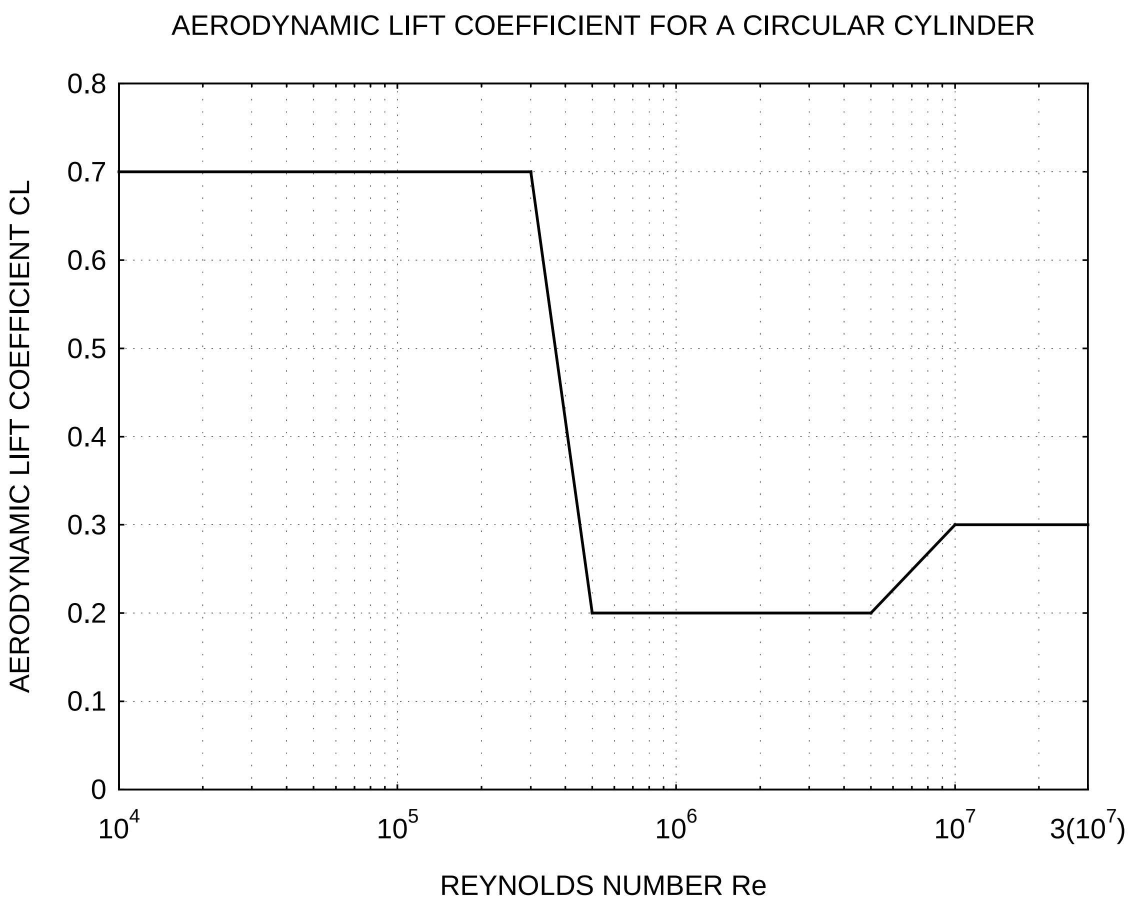

AERODYNAMIC LIFT COEFFICIENT FOR A CIRCULAR CYLINDER
AERODYNAMIC LIFT COEFFICIENT CL
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^4
10^5
10^6
10^7
$3(10^7)$
REYNOLDS NUMBER Re

Figure 1.

Drag

The drag coefficient C_D is defined as

$$C_D = \frac{D/A}{\frac{1}{2}\rho V_o^2} \qquad (1)$$

where

D is the drag force parallel to the free stream,
A is the usually project area in the direction of the freestream,
ρ is the density of the air or fluid medium,

V_o is the freestream velocity.

Note that A is the planform area for the case of an airfoil. The planform area is the area of the airfoil as seen normal to the chord line of the airfoil. Similarly, A is the actual plate area for a plate oriented parallel to the flow.

Furthermore, the drag coefficient C_D accounts for both skin and pressure drag.

The drag coefficient for a circular cylinder is given in Figure 2, as taken from Reference 2.

The drag coefficients for a flat plate oriented normal to the flow are given in Figure 3, as taken from Reference 3. The drag coefficient is given as a function of aspect ratio.

The drag coefficients for two-dimensional bodies are given in Table 1, as taken from Reference 2.

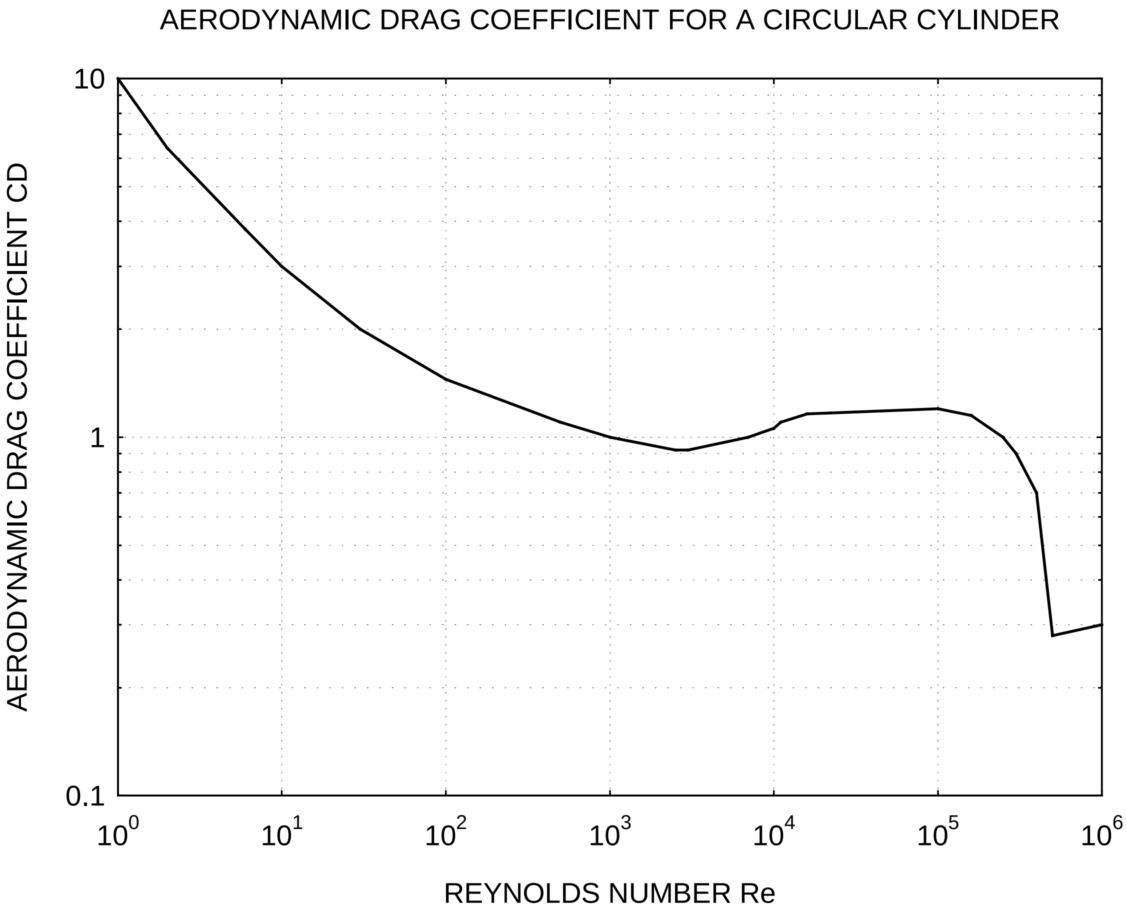
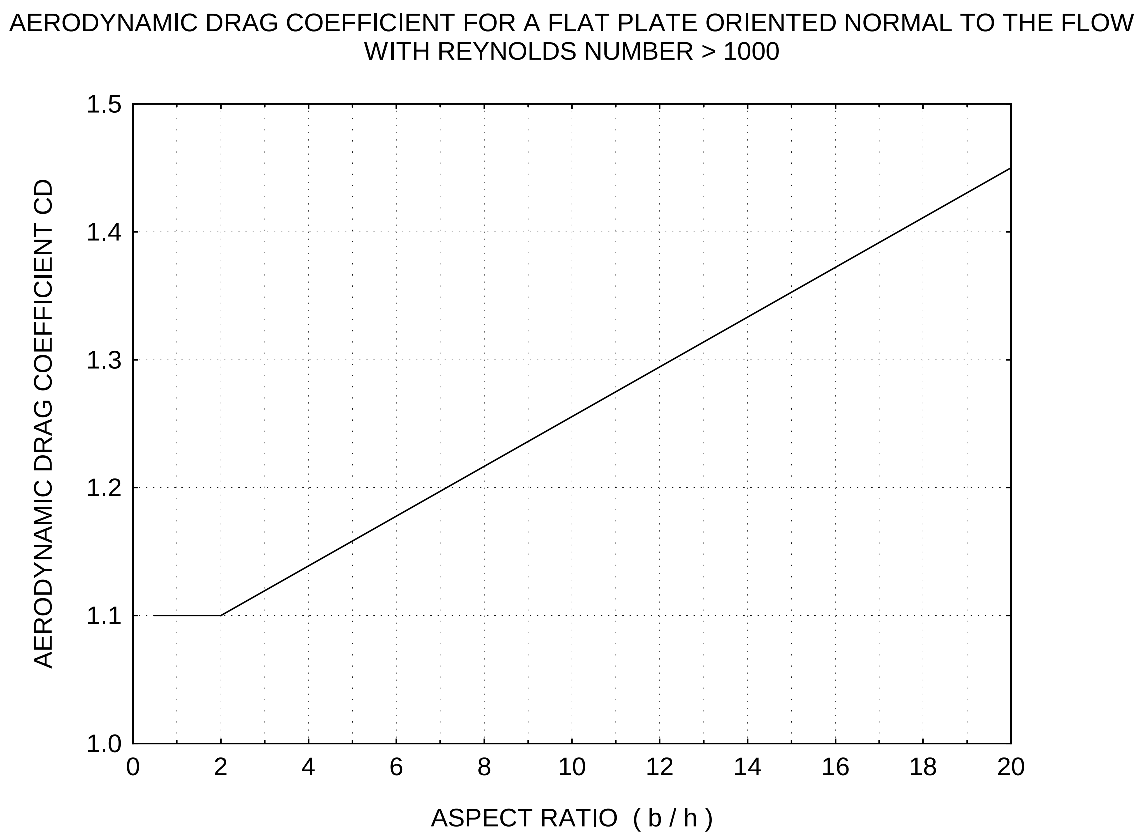

AERODYNAMIC DRAG COEFFICIENT FOR A CIRCULAR CYLINDER
10
1
0.1
AERODYNAMIC DRAG COEFFICIENT CD
10^0
10^1
10^2
10^3
10^4
10^5
10^6
REYNOLDS NUMBER Re

Figure 2.


AERODYNAMIC DRAG COEFFICIENT FOR A FLAT PLATE ORIENTED NORMAL TO THE FLOW
WITH REYNOLDS NUMBER > 1000
AERODYNAMIC DRAG COEFFICIENT CD
1.5
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8
10
12
14
16
18
20
ASPECT RATIO (b / h)

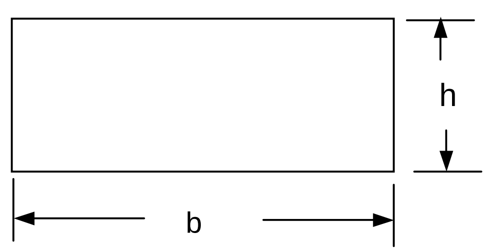
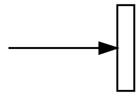
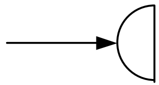
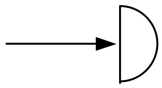
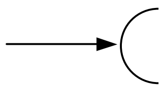
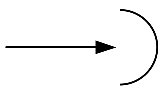
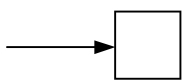
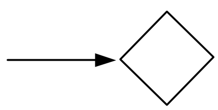
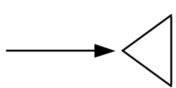
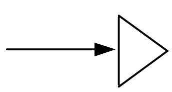

h
b

Figure 3.

Table 1. Drag Coefficients for Two-dimensional Bodies at Reynolds Number $\approx 10^5$

Shape	C_D
Plate 	2.0
Half -cylinder 	1.2
	1.7
Half-tube 	1.2
	2.3
Square Cylinder 	2.1
	1.6
Equilateral Triangle 	1.6
	2.0

Examples

Sample problems are given in Appendix A.

References

1. H. Bachmann, et al., Vibration Problems in Structures, Birkhauser Verlag, Berlin, 1995.
2. I. Shames, Mechanics of Fluids, McGraw-Hill, New York, 1982.
3. F. Fox and A. McDonald, Introduction to Fluid Mechanics, Wiley, New York, 1978.

APPENDIX A

Example

A missile is bolted to a launch stool.

The missile is exposed to a uniform wind speed of 50 km/hr (13.9 m/sec) at standard atmospheric conditions.

Model the missile as a cylinder 1 m in diameter and 25 m tall.
Assume that the stool height is negligible.
Neglect end effects and gusts.
Assume that the bending moment is due to the drag force.
Assume that the air temperature is 25 degrees C.
Estimate the bending moment at the base of the missile due to wind forces.

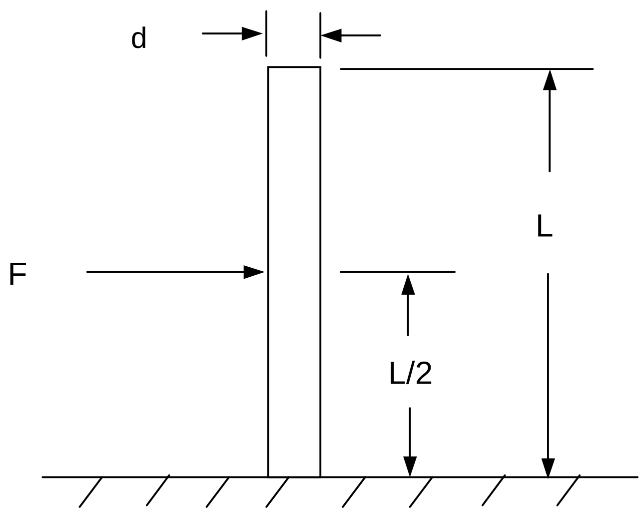


Figure A-1.

The Reynolds number is

$$\mathrm{Re} = \frac{\rho V D}{\mu} \tag{A-1}$$

$$\rho = 1.184 \ \mathrm{kg/m^3} \quad \text{for air at 25 degrees C} \tag{A-2}$$

$$\mu = 1.955\text{e-}05 \ \ \text{N sec/m}^2 \quad \text{for air at 25 degrees C} \tag{A-3}$$

$$\text{Re} = \frac{\left(1.184 \text{ kg/m}^3\right)\left(13.9 \text{ m/s}\right)\left(1 \text{ m}\right)}{\left(1.955\text{e-}05 \text{ N sec/m}^2\right)} \tag{A-4}$$

$$\text{Re} = 8.412\text{e+}05 \tag{A-5}$$

The coefficient of drag from Figure 2 is

$$C_D \approx 0.3 \tag{A-6}$$

The drag force is

$$D = \frac{1}{2}\rho {V_o}^2 A C_D \tag{A-7}$$

$$D = \frac{1}{2}\left(1.184 \text{ kg/m}^3\right)\left(13.9 \text{ m/s}\right)^2\left[\left(1 \text{ m}\right)\left(27 \text{ m}\right)\right]\left(0.3\right) \tag{A-8}$$

$$D = 857.9 \text{ N} \tag{A-9}$$

The moment M is

$$M = DL/2 \tag{A-10}$$

$$M = \left(857.9 \text{ N}\right)\left(25\text{m}\right)/2 \tag{A-11}$$

$$M = 10{,}700. \ \text{Nm} \tag{A-12}$$