TECHNIQUES FOR PREDICTING LOCALIZED VIBRATORY ENVIRONMENTS OF ROCKET VEHICLES

by Robert E. Barrett
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<tr>
<td>A</td>
<td>Skin panel area</td>
</tr>
<tr>
<td>c&lt;sub&gt;a&lt;/sub&gt;</td>
<td>Velocity of sound in air</td>
</tr>
<tr>
<td>F</td>
<td>Weight attenuation factor</td>
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<tr>
<td>G</td>
<td>Acceleration due to cyclic motion divided by the acceleration of gravity</td>
</tr>
<tr>
<td>g&lt;sub&gt;o&lt;/sub&gt;</td>
<td>Acceleration due to gravity</td>
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<td>G&lt;sup&gt;2&lt;/sup&gt;/cps</td>
<td>Power spectral density of vibrating structure (in terms of acceleration)</td>
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<td>γ</td>
<td>Acoustic power-vibrational power efficiency factor</td>
</tr>
<tr>
<td>γ'</td>
<td>Mechanical power – vibration power efficiency factor</td>
</tr>
<tr>
<td>N</td>
<td>Number of rocket engines</td>
</tr>
<tr>
<td>n</td>
<td>Subscript denoting a new vehicle parameter</td>
</tr>
<tr>
<td>P&lt;sub&gt;vib&lt;/sub&gt;</td>
<td>Vibrational power</td>
</tr>
<tr>
<td>P&lt;sub&gt;ac&lt;/sub&gt;</td>
<td>Acoustic power</td>
</tr>
<tr>
<td>P&lt;sub&gt;mech&lt;/sub&gt;</td>
<td>Mechanical power</td>
</tr>
<tr>
<td>p</td>
<td>Acoustic pressure</td>
</tr>
<tr>
<td>r</td>
<td>Subscript denoting a reference parameter</td>
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<tr>
<td>ρ&lt;sub&gt;a&lt;/sub&gt;</td>
<td>Density of acoustic medium</td>
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<tr>
<td>ρ&lt;sub&gt;m&lt;/sub&gt;</td>
<td>Density of material</td>
</tr>
<tr>
<td>T</td>
<td>Thrust of rocket engine</td>
</tr>
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<td>t</td>
<td>Skin thickness</td>
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<td>V</td>
<td>Exhaust velocity of rocket engine</td>
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<td>W</td>
<td>Structural weight</td>
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<tr>
<td>SYMBOL</td>
<td>DEFINITION</td>
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<tr>
<td>$W_c$</td>
<td>Component weight</td>
</tr>
<tr>
<td>$\dot{Z}_\alpha$</td>
<td>Characteristic acoustical impedance</td>
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<td>$\dot{Z}_m$</td>
<td>Characteristic mechanical impedance</td>
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SUMMARY

It is imperative that the vibration environment of future vehicles be predicted prior to design and development so that satisfactory design and test procedures can be established. These criteria are essential to the establishment of high reliability standards necessary for man rated vehicles. The methods and techniques presented herein allow adequate predictions of future vehicle environments.

These techniques rely upon typical structural configurations which have been sufficiently defined by measured data. Subsequent statistical analyses describe the dynamic characteristics of the structure with statistical certainty. Thus, with only a knowledge of the structural geometry and mass characteristics, the anticipated dynamic environment may be established. These techniques are applicable to all rocket vehicle structure including corrugated and sandwich skin construction.

The predicted environments represent a statistical estimation since the reference spectra are established by statistical techniques. Consequently, the probability of the actual environment not exceeding the predicted environment of a future vehicle is established with a 97.5 per cent confidence. This does not infer that the predicted environment will accurately correspond to a single measured environment. Certainly, some of the measured responses of a new vehicle will be significantly lower than the predicted. This is to be expected since the criterion is such that the prediction will envelope 97.5 per cent of the situations. However, this problem is elevated somewhat by the techniques utilized of separating rocket vehicle structure into eight (8) basic categories - each possessing essentially similar dynamic characteristics. This reduces the variance about the mean so that the mode value (most likely to occur) is not greatly less than the higher confidence limits. Therefore, the 97.5 per cent criterion may be used without the concern of over conservatism in regard to a specific problem.
INTRODUCTION

The problem of dynamic analysis has been greatly complicated in recent years by the nature of the exciting forces and subsequent effects. Dynamic environments caused by rocket vehicle operation may result in failures due to over-stress and/or fatigue. Consequently, dynamic environments must be specified prior to design and development so that adequate design and testing criteria can be established. The precise prediction of these environments is a highly complex criterion which may not be reached in the near future. However, based on measured data and a few simplifying assumptions, adequate predictions may be obtained provided the necessary assumption and limitations are realized by the user.

Vibration originates primarily from four sources of excitation. These are:

1. Mechanically induced vibration from rocket engine fluctuation which is transmitted throughout the vehicle structure.
2. Acoustic pressures generated by rocket engine operation.
3. Aerodynamic pressure created by boundary layer fluctuations.
4. Self-generating machinery, etc.

Restricting notation to vibrational power quantities, the total vibration at any point on the vehicle may be expressed as:

\[ P_T = P_{\text{mech}} + P_{\text{ac}} + P_{\text{ae}} + P_{\text{mach}} \]

Where \( P \) denotes the vibrational power caused by the indicated source. The powers add arithmetically since vibrational power is proportional to the mean square cyclic response. An exact analysis of structural response would necessitate an accurate description of each individual source and the manner in which they combine. In most cases only one source is the primary forcing function. Hence, the remaining sources may be considered negligible in regard to the total dynamic response at any instant of time. However, a technique is developed on page 12 which sufficiently solves the problem of combined driving functions.

The current state of the art provides several methods for dynamic predictions. References 1 through 4 provide adequate techniques for estimating the vibrational response caused by various driving forces. There is widespread agreement that oscillatory forces are significant and should be accounted for in design and environmental testing. Conversely there is widespread disagreement as to how the dynamic environments should be established, specified, and utilized. A study of the wide differences in techniques and the amount of time required for analysis indicates the obvious necessity for standardization. The problem then is to present an adequate method, free of complex
calculations, for determining dynamic environments which may be used for design and test criteria. It is realized that the average engineer does not have available time to thoroughly understand and analyze the dynamic characteristics of each individual problem. Consequently, a generalized method must be presented which sufficiently solves the average problem with a minimum amount of calculations. Also, it should be realized that each problem is unique and requires at least some minor calculations. Hence, an optimum point must be reached which meets the requirements of both simplicity and adequacy. The method presented in the subsequent section is currently considered to satisfy these requirements. The method allows prediction of vibratory environments with only a few simple computations and excludes all complexities which might cloud the issue. The prediction methods and techniques advocated herein have proved successful. Research programs are currently being conducted in order to more precisely define the degree of accuracy obtained from these methods and direct studies for future refinements. Based upon additional information regarding the dynamic response phenomena and laboratory test results, the existing techniques may be slightly modified. However, the basic foundation upon which the methods were constructed will remain unchanged.
DESCRIPTION

Rocket vehicle structure may be separated into two dynamic categories. These are:

I. Structure susceptible to acoustic and aerodynamic pressures (i.e., skin panels).

II. Structure not susceptible to acoustic and aerodynamic pressures (i.e., structural beams).

The dynamic analysis of these two groups are obviously handled by two different techniques. Each will be explained in detail and supplemented by example problems.

Type I

First, consider Type I. This group of structure may be subdivided into three sections.

I(a) -- Skin panels. A panel is defined as a section of skin bounded by radial and longitudinal stiffeners. This type of structure is directly excited by impinging acoustic pressures. The direction normal to the panel face exhibits the most severe vibratory response; consequently, this is the direction under consideration when referring to panel vibrations.

I(b) -- Skin stiffeners (such as ring frames and stringers). This type of structure is not directly excited by acoustic forces but is driven by the motion of adjacent panels. Thus, the stiffeners are indirectly forced by impinging acoustic pressures.

I(c) -- Bulkheads. These skin segments form the upper and lower extremities of vehicle propellant tanks. The bulkheads are further subdivided into:

I(c)-1: Forward bulkheads

I(c)-2: Aft bulkheads

The added mass of liquid loading greatly reduces the vibration amplitudes experienced by the aft bulkheads; consequently, the two bulkheads are treated separately.
Although aerodynamic pressure has not been mentioned, it should be realized that structures susceptible to acoustic pressures are indeed sensitive to aerodynamic pressures. The nature of the exciting forces are essentially the same, resulting in a similar vibratory response. The maximum acoustic pressures generally occur during captive firing or the liftoff period of the vehicle. The maximum aerodynamic pressure occurs at some period during flight, generally subsequent to Mach one.

The equation for predicting the vibration environment of acoustically (or aerodynamically) susceptible structure is:

\[
G_n = G_r \left( \frac{p_n \rho_r t_r}{p_r \rho_n t_n} \right) \sqrt{F}
\]

(1)

where \(G_n\) = the vibration response of the new vehicle structure at a particular station number. The term \(G\) is the acceleration due to cyclic motion divided by the acceleration of gravity \(g_0\). Since rocket vehicle vibrations contain many frequencies the response magnitude \((G)\) is specified in spectral form.

\(G_r\) = the known vibration response of a reference vehicle structure. This value has been determined by measurements and is also presented on a spectral basis.

\(t_r\) = the thickness of the skin associated with \(G_r\)

\(\rho_r\) = the skin weight density of the reference structure

\(p_r\) = the impinging acoustic (or aerodynamic) pressure which is driving the reference structure

\(t_n\) = the skin thickness\(^1\) associated with \(G_n\)

\(\rho_n\) = the skin weight density associated with \(G_n\)

\(p_n\) = the acoustic (or aerodynamic) pressures impinging upon the new vehicle structure. This pressure must be predicted.

\(F\) = a factor which accounts for the attenuation effects produced by incorporating additional mass into the existing system.

\(^1\) For corrugated or sandwich structure this parameter is an equivalent thickness. The technique for determining this thickness is shown in Appendix I.
The above equation applies to random rms composite values or to sinusoidal values. For the mean square spectral value the total expression simply raised to the second power.

Equation (1) is applicable to localized vibratory environments and is valid for all materials. It is invalid when considering large sections of vehicle structure (i.e., entire cylindrical tank). However, the static loading of these large sections is the critical design factor and localized dynamics thereby produce only negligible effects. Further, the added mass effects of fluid loading are not considered for skin panels and stiffeners of the propellant tankage since the most severe environmental condition is the primary objective. It is assumed that all vehicle stages are captive-fired prior to launch. Thus, there will be a period of time in which the fluid level is below the point of interest resulting in extreme dynamic responses. To thoroughly understand the equation, a derivation is provided in Appendix I.

Referring again to equation (1) \( p_0 \) represents the maximum pressure impinging upon the vehicle at any time. Three conditions must be considered, one of which will result as the maximum. These are:

1. Captive-firing environments (applicable to boosters and upper stages)
2. On-pad acoustic environments
3. The period of maximum aerodynamic pressures (occurs subsequent to Mach one; therefore, the coupling of acoustic-aerodynamic pressures do not have to be considered).

A flow chart indicating the train of events necessary to determine the vibration environment of a new vehicle is shown in Figure 1. Sound pressure curves corresponding to all standard liquid rocket engines are provided in Figures 2 through 8. The acoustic environments produced as a result of rocket engine noise for the different types of engines presented herein are based on the assumption that the engine rated parameters of thrust, flow rate and exhaust velocity will be generated at sea level conditions. The acoustic environment is given in terms of overall rms (5 cps to 2000 cps) dynamic pressure, in pounds per square inch, and is plotted as a function of vehicle length. The vehicle length is measured in inches above the engine nozzle exit plane. The curves represent a single engine operation of each respective type of engine (i.e., one M-1 engine, one F-1 engine, one J-2 engine, etc.). To determine the overall acoustic pressure, at a given vehicle length, resulting from operation of two or more engines of any one particular type, the following equation can be used:

\[
p_t = \sqrt{N} p_1
\]

where: \( N \) = number of engines of one given type
\[ p_1 = \text{the pressure produced, at a given vehicle length, from the operation of a single engine. (Figures 2 through 8)} \]

\[ p_t = \text{resulting pressure when N engines are clustered.} \]

The above equation cannot be used to determine the resultant pressure from operation of two or more different types of engines, (i.e., four M-1 engines and six F-1 engines on the same stage).

The effect of rocket engine clustering is not well defined at this time; therefore, it was not explicitly treated. It is felt however, that the estimates of overall SPL obtained from Figures 2 through 8 are valid if the number of engines clustered are not great. It has been observed from measured acoustic data from clusters of one, two, four, and eight H-1 engines which propel the booster of the Saturn I vehicle, that clustering does not produce appreciable effects in peak frequency of the acoustic spectra. From model tests involving as many as 24 engines per cluster, only slight effects— if any— were exhibited in octave band frequency spectra. This model test should not be considered conclusive. The measured data should, however, be considered as indicative since they do represent test results. Therefore, it is suggested that caution be taken for use of this technique for engine clusters of more than 16 engines.

The pseudo-acoustic pressures on the vehicle surface induced by the aerodynamic boundary layer during flight may, for some locations, exceed the engine sound pressures even during the on-pad condition. Figure 9 shows the maximum in-flight rms overall sound pressures anticipated for future space vehicles utilizing liquid propellant booster stages. The words, "rms-overall" denote the total rms aerodynamic pressure in a spectral window from 5 cps to 2000 cps. Acoustic measurements from Saturn flights have shown a ratio of the overall aerodynamic oscillating pressure to the free stream dynamic pressure as high as 1.4 per cent. It is feasible to present a plot of pressure versus vehicle length for the boundary layer noise since the maximum dynamic pressure does not change appreciably when a liquid propellant booster is utilized. This is true because of the relationship between mass flow and engine thrust. Since the ratio of first stage thrust to total vehicle weight has crystallized around 1.2 to 1.5, and the flow rate-thrust relationship for liquid propellant boosters exists; there is only a small variation in the maximum dynamic pressure for any flight trajectory. Note that the distance in Figure 9 is measured from the leading edge of the vehicle. It was implicitly assumed that there would be no large projections or deviations from a smooth aerodynamic profile on the vehicle. Acoustic pressures can be magnified by a factor of 10 in the presence of an oscillating shock wave from an aerodynamic discontinuity.

With only a knowledge of the approximate component location in respect to vehicle length, the curves may be utilized to determine each of the three pressures. Obviously, the most severe pressure should be utilized in the equation. Since equation (1) depends upon reference quantities these may be lumped as a constant. Further the component weight attenuation factor was defined to be (Appendix I):
\[ F = \frac{W}{W + W_c} \]

where \( W \) = weight of structure  
\( W_c \) = component weight.

Substituting into equation (1) results in:

\[
G_n = G_r \left( \frac{p_n}{p_r} \right) \left( \frac{\rho_r t_r}{\rho_n t_n} \right) \left( \frac{W_n}{W_n + W_c} \right) = G_r K \left( \frac{p_n}{t_n} \right) \left( \frac{\rho_r}{\rho_n} \right) \left( \frac{W_n}{W_n + W_c} \right)
\]

where \( K = \frac{t_r}{p_r} \) = a constant depending upon the reference parameters

\( W_n \) = panel weight for Type I(a), stiffener weight for Type I(b), and bulkhead weight for Type I(c).

In some cases a section of vehicle skin may not incorporate longitudinal stiffeners. When determining the panel weight, the radial distance should not exceed three times the distance between rings. This is because the dynamic characteristics of thin plates remain essentially constant above aspect ratios of three. The acoustic pressures associated with the reference vehicle environments are obtained from the predicted pressure at the appropriate station numbers. Although these pressures were measured during captive tests, it is anticipated that any inherent error associated with the acoustic prediction technique will appear in both the reference and new vehicle acoustic predictions with the same magnitude. Therefore, since ratios of pressure are involved, the error is considered to cancel. Thus, with adequate reference parameters, a knowledge of the component location with respect to vehicle length and the structural mass characteristics, the vibration response of the structure may be determined. If the reference environment represents the unloaded structure (i.e., does not reflect the effect of component mass) then the environment obtained from the prediction techniques will correspond to the unloaded structure of the new vehicle. This structural response will then represent an input to a component mounted on this structure. For this reason, the attenuation factor (F) must be incorporated into the equation to account for the component mass effect. In some instances during the preliminary development period, the weight of the basic structure or component may not be known. In this case, the attenuation factor is considered as unity. This will result in an estimate of the component environment which may tend to be conservative.
Type II

Now consider the Type II structure. This structure may be subdivided into two sections. These are:

II(a) -- Structural beams such as I beams, etc. The components mounted in this section would not primarily be effected by acoustic pressures but by rocket engine vibrations.

II(b) -- Rocket engine components. These components may again be subdivided into three sections:

II(b)-1: Combustion chamber section. This section includes the components mounted on the chamber dome or side case.

II(b)-2: Turbopump section. This section includes the components located on the propellant pumps.

II(b)-3: Actuator assembly. The components located in this region are mounted on the actuator struts or actuator rods.

The equation for predicting vibrations of these structural types is:

\[ G_n = G_r \sqrt{\frac{(NTV)_n}{(NTV)_r} \frac{W_r}{W_n}} F \]  

(3)

where \( G_n \) = the vibration response of the new structure. This is considered an input to the component mounted on this structure.

\( G_r \) = the vibration response of a reference structure. This environment is determined by measured data.

\( (NTV)_n \) = number of engines, thrust and exhaust velocity of rocket engine associated with the stage under consideration in the new vehicle.

\( (NTV)_r \) = number of engines, thrust and exhaust velocity of rocket engine associated with a reference vehicle.

\( W_n \) = the structural weight corresponding to \( G_n \)

\( W_r \) = the structural weight corresponding to \( G_r \)

\( W_c \) = the weight of component to be mounted on \( W_n \)
\[ F = \frac{W_n}{W_n + W_c} \]  

is an attenuation factor which takes into account the effect of component mass loading.

Equation (3) applies to random rms composite values or to sinusoidal values. For the mean square spectral density value the total expression is simply squared.

The geometry and mass characteristics of structural Type II(a) are the most difficult to define; therefore, the problem must be approached in general terms. If the region under consideration is of beam and truss construction, the total structural weight (do not include the weight of components in the region) should be used to determine the parameter \( W_n \). Subsequent knowledge of separate component weights \( W_c \) would permit calculation of an individual attenuation factor \( F \) for each component in the region. This method is considered valid provided the reference vibration \( (G_r) \) is determined from an adequate number of measurements in this area and subsequent statistical analyses provides confidence limits. With this criterion the vibration environment of any particular component located in this region may be specified with statistical certainty.

The reference parameters \( \frac{W_r}{(NTV)_r} \) may be lumped as a constant which simplifies the expression. Incorporating this constant and the expression for the attenuation factor into equation (3) results in:

\[
\Pi(a) \quad G_n = G_r C \sqrt[2]{\frac{(NTV)_n}{W_n + W_c}}
\]

\[
C = \sqrt{\frac{W_r}{(NTV)_r}} \quad \text{for } \Pi(a)
\]

\[
C = \sqrt{\frac{W_r}{(TV)_r}} \quad \text{for } \Pi(b)
\]

\( W_n \) = weight of entire beam area of new vehicle for \( \Pi(a) \)

\( W_n \) = weight of a single rocket engine section corresponding to the stage under consideration for \( \Pi(b) \).

For Type II (b) structure, clustering does not generally affect engine vibrations; therefore, \( N_r \) and \( N_n \) are not considered. For a thorough understanding of equation (4), a derivation is provided in Appendix II. Also, for type II structure, only one case is considered the maximum. This case is the firing (either captive or inflight) of that particular stage under consideration. The vibration transmitted from other stage firings
(i.e., booster to upper stages) is negligible compared to that stage operation. Thus, with a knowledge of the necessary rocket engine parameters and structural mass characteristics, the vibratory environment of any particular component may be determined. An example problem dealing with this type of structure is provided in Appendix IV.

Adequate reference parameters have been obtained from Saturn I Block I captive firing data. Currently, the Saturn I R&D Program has provided a total of 41 captive firings representing 2,621 seconds of test time. These have resulted in 2,708 vibration measurements representing 200 different locations throughout the booster. Approximately 350 of these measurements have been utilized to define Type I structure and 725 to define Type II structure. The remainder have been used to provide design modifications for the operational Block II vehicles. Comprehensive statistical analyses were performed on these abundant data. With the results of this program, the respective environments of the eight structural types may be specified with statistical certainty. The statistical results, coupled with the necessary structural parameters, provide entirely satisfactory standards. The resultant environments and the associated parameters corresponding to the unloaded vibratory response of the structural types is given in Figures 10 through 25. These environments are presented in both sinusoidal and random form and are to serve as reference criteria. The reference spectra are shaped according to the proportion of power within a specified bandwidth. The envelopes were constructed in a manner which will cover a slight shift of modal frequencies. Specifically, large vehicles may be expected to possess a lower frequency content than smaller vehicles.

The sinusoidal and random reference environments presented for each respective type of structure are not intended to be exact equivalencies. Since the present state of the art presents no valid random-sine correlations, the respective environments are considered satisfactory interpretations based upon the limited knowledge of random-sine relationships and successful design and testing results.

Many sinusoidal testing techniques require both sweep and resonant dwell tests. The sinusoidal reference environments represent dwell criteria. For purposes of sine sweeping testing, the dwell level should be multiplied by an appropriate factor.

With the accumulation of additional data, the standards may be slightly modified. The revision will, however provide a more exact definition of that particular structural type. Statistical methods of variance analysis are being applied to the Saturn I reference structure to determine when a sufficient amount of data have been acquired. Type II structure has reached 95 per cent of the limit and Type I structure has reached 85 per cent of the limit.
CONCLUSIONS

The methods and techniques presented herein will provide satisfactory estimates of the vibratory environment associated with any particular problem. These techniques are considered indicative of the present state-of-the-art and will permit satisfactory environmental estimates with only a few simple calculations. It is not imperative that the user thoroughly understand the philosophy behind the methods. He should, however, realize the necessary requirements and limitations. A summary of the principal limitations regarding these techniques is given below:

1. The techniques do not apply to entire structure such as an entire rocket engine assembly or large structural members. They do, however, apply to components and other items of equipment mounted on these structures.

2. These methods are not applicable for prediction of combined environments such as excitation due to propellant flow and engine vibration combined. In some remote cases the type of structure may not be clearly defined or excitation may be both acoustically and mechanically induced. In these cases the two methods may be combined and a certain percentage assigned to each method depending upon degree of structural susceptibility to acoustic and mechanical excitation. Employment of the respective percentages is left to the discretion of the user. Thus:

\[ G_n = \sqrt{X\% \ G^2_n + Y\% \ G^2_n} \]

where

\[ G_n = \text{the resultant combined environment} \]
\[ G_n^{\text{I}} = \text{the environment obtained from using Type I methods} \]
\[ G_n^{\text{II}} = \text{the environment obtained from using Type II methods} \]

and

\[ X\% + Y\% = 1.0 \text{ or } 100\% \]

3. This method does not provide frequency characteristics of the structure. The method is based upon the fact that similar structure has similar response characteristics (Appendix I page 21). Precise frequency calculations tend to complicate the problem and it is doubtful that the results would justify the time and effort required. The calculation of the component natural frequency is, within itself, possibly a simple problem. In the actual system, however, this frequency is coupled with the support structure.
frequency characteristics. The determination of structure-component coupling effects is a highly complex requirement. Consequently, adequate frequency calculations and evaluation of these effects require considerable time and effort.

A summary of the prediction equations is given below in outline form. The corresponding reference parameters are lumped as a constant in each of the respective equations.

**SUMMARY OF EQUATIONS FOR PREDICTING LOCALIZED VIBRATION ENVIRONMENTS OF A NEW VEHICLE**

For Type I structure the constant represents \( \frac{t_r}{p_r} \) (in) \( \frac{1}{\text{lb/in}^2} \) (determined from Saturn I data).

For Type II structure the constant represents \( \sqrt{\frac{w_r}{[NT \ (\text{lb}) \ V \ (\text{ft/sec})]_r}} \) (determined from Saturn I data).

For all structural types, \( G_r \) is determined from Saturn I data.

**Type I (a) Skin Panels**

\[
\frac{G_n}{G_r} = 1.22 \left( \frac{p_n}{t_n} \right) \left( \frac{\rho_r}{\rho_n} \right) \sqrt{\frac{W_n}{W_n + W_c}}
\]

where

- \( W_n \) = panel weight
- \( G_r \) = Ref. Fig. 10 and 11

**Type I (b) Ring Frames and Stringers.**

\[
\frac{G_n}{G_r} = 1.22 \left( \frac{p_n}{t_n} \right) \left( \frac{\rho_r}{\rho_n} \right) \sqrt{\frac{W_n}{W_n + W_c}}
\]

where

- \( W_n \) = stiffener weight
- \( G_r \) = Ref. Fig. 12 and 13
Type I (c) Bulkheads

I (c)-1  Forward Bulkheads

\[
\frac{G_n}{G_r} = 0.95 \left( \frac{p_n}{t_n} \right) \left( \frac{\rho_r}{\rho_n} \right) \frac{\sqrt{W_n}}{W_n + W_c}
\]

I (c)-2  Aft Bulkheads

\[
\frac{G_n}{G_r} = 0.62 \left( \frac{p_n}{t_n} \right) \left( \frac{\rho_r}{\rho_n} \right) \frac{\sqrt{W_n}}{W_n + W_c}
\]

where

\[W_n = \text{bulkhead weight}\]
\[G_r = \text{Ref. Fig. 14, 15, 16, and 17.}\]

Type II (a) Beams

\[
\frac{G_n}{G_r} = 9.38 \times 10^{-4} \sqrt{\frac{(NTV)_n}{(W_n + W_c)}}
\]

where

\[W_n = \text{weight of entire beam area}\]
\[G_r = \text{Ref. Fig. 18 and 19}\]

Type II (b) Engine Components

II (b)-1 Combustion Chamber Section

\[
\frac{G_n}{G_r} = 7.6 \times 10^{-4} \sqrt{\frac{(TV)_n}{(W_n + W_c)}}
\]

where

\[W_n = \text{combustion chamber + expansion nozzle weight}\]
II (b)-2 Turbo pump Section

\[ \frac{G_n}{G_r} = 6.11 \times 10^{-4} \sqrt[3]{\frac{(TV)_n}{(W_n + W_c)}} \]

where

\[ W_n = \text{fuel + oxidizer pump weights} \]

II (b)-3 Actuator Assembly

\[ \frac{G_n}{G_r} = 4.87 \times 10^{-4} \sqrt[3]{\frac{(TV)_n}{(W_n + W_c)}} \]

where

\[ W_n = \text{weight of actuator rods and struts} \]

\[ G_r = \text{Ref. Fig. 20, 21, 22, 23, 24, and 25.} \]
APPENDIX I

DERIVATION OF EQUATION FOR PREDICTING THE VIBRATION ENVIRONMENT OF COMPONENTS MOUNTED ON SKIN STRUCTURE

Any structure responding to oscillating forces possesses a vibrational power. For the case of skin structure the primary driving function is the impinging acoustic pressure which may also be defined in terms of power. Thus, a vibrational efficiency factor may be defined as:

\[ \gamma = \frac{P_{\text{vib}}}{P_{\text{ac}}} \]  

(1)

This relationship stipulates that a certain amount of impinging acoustic power is transferred into vibrational power. Equation (1) may be written for two separate vehicles as:

\[ \gamma_r = \left( \frac{P_{\text{vib}}}{P_{\text{ac}}} \right)_r \]

and

\[ \gamma_n = \left( \frac{P_{\text{vib}}}{P_{\text{ac}}} \right)_n \]

where the subscript \( r \) denotes a reference vehicle in which the vibratory environment has been defined by many measurements and \( n \) denotes a new vehicle of unknown environments. For similar structure the above equations may be expressed as:

\[ \left( \frac{P_{\text{vib}}}{P_{\text{ac}}} \right)_n \left( \frac{P_{\text{ac}}}{P_{\text{vib}}} \right)_r = \frac{\gamma_n}{\gamma_r} \]  

(2)

Respectively, the average vibrational power and acoustic power may be expressed as:

\[ P_{\text{vib}} = \frac{1}{2\pi} \frac{\Delta f}{f} G^2/\text{cps} \ W \ g_o \]  

(3)

where \( G^2/\text{cps} = \) power spectral density of the vibrating structure

\[ W = \text{effective weight of the structure} \]

\[ g_o = \text{acceleration of gravity} \]
where $p^2$ = mean square acoustic pressure

\[ A = \text{area over which the pressure is applied} \]

\[ \rho_a = \text{mass density of the acoustic medium} \]

\[ c_a = \text{velocity of sound in the medium} \]

Substituting equations (3) and (4) into equation (2) results in:

\[
\frac{(G^2/\text{cps})_n}{(G^2/\text{cps})_r} \frac{W_n (\rho_a c_a)_n}{W_r (\rho_a c_a)_r} \frac{p^2_r A_r}{p^2_n A_n} = \frac{\gamma_n}{\gamma_r}
\]

Further, the transfer mechanism ($\gamma$) is defined as:

\[
\gamma = \frac{1}{\left[1 + \left(\frac{\dot{Z}_m}{2\dot{Z}_\alpha}\right)^2\right]^{1/2}}
\]

where

\[ \dot{Z}_m = \text{the mechanical impedance per unit area of the structure (characteristic impedance)} \]

\[ \dot{Z}_\alpha = \text{the characteristic impedance of the acoustic driving function} \]

For most practical cases $Z_\alpha << Z_m$. Thus:

\[
\gamma \approx \frac{\dot{Z}_\alpha}{\dot{Z}_m}
\]

Substituting equation (6) into equation (5) and rearranging the terms gives:

\[
\frac{(G^2/\text{cps})_n}{(G^2/\text{cps})_r} = \frac{W(A)}{A} \frac{A}{W} \frac{p^2_r (\rho_a c_a)_r}{p^2_r (\rho_a c_a)_n} \frac{Z_\alpha}{Z_m} \frac{\dot{Z}_m}{\dot{Z}_\alpha}
\]
The mechanical impedance of a structure may be expressed as:

\[
Z_m = \left[ D^2 + (M\omega - \frac{k}{\omega})^2 \right]^{\frac{1}{2}} = \frac{W}{g_o} \left[ (\eta\omega_n)^2 + \left(\omega - \frac{\omega_n^2}{\omega} \right)^2 \right]^{\frac{1}{2}}
\]

where \(D, M, \) and \(k\) are the effective damping, inertial, and elastic parameters of the system. Thus:

\[
\frac{(Z_m)_n}{(Z_m)_r} = \frac{W_n}{W_r} \left[ \frac{(\eta\omega_n)^2 + \left(\omega - \frac{\omega_n^2}{\omega} \right)^2}{(\eta\omega_n)^2 + \left(\omega - \frac{\omega_r^2}{\omega} \right)^2} \right]^{\frac{1}{2}}
\]

Since it is assumed that similar structures possess similar dynamic characteristics (i.e., similar damping and frequency content):

\[
\frac{(Z_m)_n}{(Z_m)_r} = \frac{W_n}{W_r}
\]

or

\[
\frac{(Z_m)_n}{(Z_m)_r} = \left( \frac{W}{A} \right)_n \left( \frac{A}{W} \right)_r
\]

Also the characteristic acoustic impedance may be defined as:

\[
Z_\alpha = \beta \rho_a c_a
\]

where

\[
\beta = a \text{ factor depending upon the geometry and coincident frequencies of a localized section of stiffened skin and is independent of the atmospheric impedance characteristics.}
\]

Hence for similar structure:

\[
\beta_n = \beta_r
\]

or

\[
\frac{(Z_\alpha)_n}{(Z_\alpha)_r} = \left( \frac{\beta \rho_a c_a}{\beta \rho_a c_a} \right)_n = \left( \frac{\rho_a c_a}{\rho_a c_a} \right)_n
\]

\[
\frac{(Z_\alpha)_n}{(Z_\alpha)_r} = \left( \frac{\beta \rho_a c_a}{\beta \rho_a c_a} \right)_r = \left( \frac{\rho_a c_a}{\rho_a c_a} \right)_r
\]
Substituting equations (9) and (10) into equation (7) gives:

\[(G^2/\text{cps})_n = (G^2/\text{cps})_r \left[ \frac{W}{A} \right]^2 \left( \frac{A}{W} \right) \left( \frac{p_n}{p_r} \right) \]  \hspace{1cm} (11)

\(W/A\) may be defined as weight per unit surface area. For isotropic panels:

\[W/A = \rho_m t\]  \hspace{1cm} (12)

where

\[\rho_m = \text{the weight density of the material}\]

\[t = \text{the thickness of the plate}\]

However, for corrugated or sandwich construction \(t\) is required to be an equivalent thickness. In regard to corrugation the weight over a particular surface (indicated by the sketch) is expressed as:

\[W = \rho_m t_c A = \rho_m t_c d \ell_s\]  \hspace{1cm} (13)

where

\[\ell_s = \text{surface length} = \Sigma \ell\]
Substituting equation (13) into equation (12) the expression for an equivalent "flat plate" thickness is shown to be:

\[ t_{eq} = t_c \frac{\frac{\hat{\mathbf{E}}}{\mathbf{E}}}{\frac{\hat{\mathbf{E}}}{\mathbf{E}}} \]  

(14)

This expression is only approximate since the corrugation exhibits a different set of dynamic properties than the isotropic plate. Also the geometry of the corrugation results in variable elastic properties, within itself, at the boundaries (anisotropy). However, since the resonant acceleration amplitude is primarily controlled by the system damping and mass properties, this technique is deemed sufficient.

Sandwich plate construction may be handled in a similar manner. However, the damping properties of sandwich and homogeneous plates are significantly different and must be considered. From an apparent mass (acceleration impedance) analysis, the resonant response of thin plates is:

\[ \frac{F}{X} = \frac{\rho \cdot t \cdot A \cdot \eta \cdot \alpha}{g_o} \]  

(15)

where

\[ \frac{F}{X} \] = the apparent mass
\[ \eta \] = the structural loss factor

For sandwich plates (refer to sketch for dimensions):

\[ \rho = \frac{\rho_{p1} \cdot h_{p1} + \rho_{p2} \cdot h_{p2} + \rho_c \cdot h_c}{h_{p1} + h_{p2} + h_c} \]  

(16)

where

\[ \rho_{p} \] = the density of the cover plates
\[ \rho_{c} \] = the core density

Further, experimental investigations have indicated average damping values of:

\[ \eta = .044 \] (solid panels)
\[ \eta = .108 \] (sandwich panels)
In order to establish equivalent plates, the resonant responses are required to be equal. Thus:

\[ (\rho t \eta)_{s.p.} = (\rho t \eta)_{sand}. \]  \hspace{1cm} (17)

Substituting the damping values and equation (16) into equation 17 results in:

\[ t_{eq} = 2.5 \frac{(\rho p_1 h_{p1} \rho p_2 h_{p2} \rho c h_c)}{\rho m} \]

where

\[ \rho m = \text{the density of the equivalent homogeneous plate} \]

If both the cover plates and core are of the same material then:

\[ t_{eq} = 2.5 \left( h_{p1} + h_{p2} + \frac{\rho c h_c}{\rho m} h_o \right) \]  \hspace{1cm} (18)

Further, if the cover plates are of the same thickness the above reduces to:

\[ t_{eq} = 5 h_p + 2.5 \frac{\rho c h_c}{\rho m} \]  \hspace{1cm} (19)

Generally, all rocket vehicles consist of aluminum skin which permits cancellation of the density terms. However, for the general case the densities will remain in the equation allowing calculation of the vibratory response for any type of skin material.

Applying equation (12) to equation (11) results in:

\[ (G^2/\text{cps})_n = (G^2/\text{cps})_r \left( \frac{\rho r t_r}{\rho n t_n} \right)^2 \left( \frac{p_n}{p_r} \right)^2 \]  \hspace{1cm} (20)

If \((G^2/\text{cps})_r\) corresponds to an unloaded structure (i.e., does not reflect the amplitude attenuation due to component mass), then the environment determined by equation (20) would represent the unloaded response of the new structure. For this reason an attenuation factor is required to account for the component mass effect. Equations (1) and (3) may be combined resulting in:

\[ \gamma P_{ac} = \frac{1}{2\pi} \frac{\Delta f}{f} G^2/\text{cps} \ W \ g_0 \]  \hspace{1cm} (21)

This expression indicates that for a constant driving power the acceleration response decreases as the weight increases. Now consider an item of weight, \(W_c\), to be mounted on a structure in which the vibration environment has previously been defined. The effective force (acoustic pressures) driving this structure does not change. Hence, the vibration amplitudes will be decreased by a factor \((F)\) of \(\frac{W}{W + W_c}\) or

\[ G^2/\text{cps (unloaded)} = \frac{P_{\text{vib}} 2\pi f}{\Delta f W g_0} \]
and consequently

\[
\frac{G^2/cps \text{ (loaded)}}{G^2/cps \text{ (unloaded)}} = \frac{P_{\text{ vib}} 2 \pi f}{(W + W_c) \Delta f \delta_o} = \frac{W}{W + W_c} G^2/cps \text{ (unloaded)}
\]

Further the weight of the structure is an effective weight. This may be expressed as:

\[
W_{\text{eff}} = I W_{\text{act}}
\]

where \( I = \) a constant depending upon the system stiffness and mass coupling characteristics. This relation may be extended to include the added component weight by:

\[
W_{\text{eff}} = I (W_{\text{act}} + W_c)
\]

Substituting this equation into equation (3) does not alter the results since the constant cancels out

\[
G^2/cps \text{ (loaded)} = \frac{W}{(W + W_c)} G^2/cps \text{ (unloaded)} \tag{22}
\]

A typical graph of this attenuation phenomena is shown in the sketch below:

Incorporating equation (22) into equation (20) results in:

\[
(G^2/cps)_n = (G^2/cps)_r \left( \frac{P_n}{P_r} \right)^2 \left( \frac{\rho_r}{\rho_n} \right)^2 \left( \frac{t_r}{t_n} \right)^2 \left( \frac{W_n}{W_n + W_c} \right)
\] \tag{23}

For the composite or sinusoidal case the effective bandwidths cancel resulting in:

\[
G_n = G_r \left( \frac{P_n}{P_r} \right)^2 \left( \frac{\rho_r}{\rho_n} \right)^2 \left( \frac{t_r}{t_n} \right)^2 \left( \frac{W_n}{W_n + W_c} \right)
\] \tag{24}

The entire concept is based on the assumption that similar structure exhibits similar dynamic characteristics. Once the frequency and amplitude characteristics of a typical type of structure have been adequately defined, these results may be utilized as a basis.
for predicting the vibration environment of like structure. The remaining factor to consider is the driving function. In the case discussed above the acoustic (or aerodynamic) pressures are the primary cause of vibration. These pressures must be predicted for the new vehicle in order for equation (24) to be solved.
Dense structure such as heavy "I" beams are not affected by acoustic pressures. This type of structure is dynamically excited by rocket engine vibrations. Consider an engine component subjected to a dynamic force that is random in nature. A certain amount of this force is "absorbed" by the component. The result of this absorption is exemplified as a dynamic response of the component which may be specified in terms of power. Hence, the mechanical efficiency factor is:

\[
\gamma' = \frac{P_{\text{vib}}}{P_{\text{mech}}}
\]

(1)

where

\[P_{\text{vib}} = \text{vibrational power}\]

\[P_{\text{mech}} = \text{mechanical power}\]

This relation stipulates that a certain percentage of the potential mechanical power is "absorbed" as vibrational power. To further define the relationship, consider the expressions for mechanical and vibrational power.

\[
P_{\text{vib}} = \frac{1}{2\pi} \frac{\Delta f}{f} G^2/\text{cps} W g_0
\]

(2)

where

\[W = \text{the effective structural weight}\]

\[G^2/\text{cps} = \text{power spectral density of the vibrating structure}\]

\[g_0 = \text{acceleration due to gravity}\]

Further:

\[P_{\text{mech}} = TV\]

where

\[T = \text{thrust of the rocket engine}\]

\[V = \text{exhaust velocity of the rocket engine}\]
Substituting these expressions into equation (1) results in:

$$\gamma' = \frac{\Delta f \ W \ G^2/cps \ \omega_0}{2\pi \ f \ TV}$$

Assuming similar structure possesses similar dynamic characteristics the mechanical efficiency factors may be considered equal. Thus:

$$\gamma_n' = \gamma_r'$$

(For similar structure)

Where the subscripts denote new and reference vehicle, respectively or

$$(G^2/cps)_n = (G^2/cps)_r \left( \frac{W_r}{W_n} \right) \left( \frac{T_n V_n}{T_r V_r} \right)$$

This expression may be modified further by the mass attenuation factor (Appendix I).

Thus:

$$(G^2/cps)_n = (G^2/cps)_r \left( \frac{W_r}{W_n} \right) \left( \frac{T_n V_n}{T_r V_r} \right) \left( \frac{W_n}{W_n + W_c} \right) =$$

$$(G^2/cps)_r \left( \frac{W_r}{W_n + W_c} \right) \left( \frac{T_n V_n}{T_r V_r} \right)$$

For the composite or sinusoidal case the effective bandwidths cancel, leaving:

$$G_n = G_r \sqrt{\frac{W_r}{W_n + W_c}} \ \left( \frac{T_n V_n}{T_r V_r} \right)$$

For predicting vibration environments of thrust frame areas composed of structural beams, the total mechanical power developed by that particular stage should be used. The reason for this is that the mass and geometry characteristics of thrust frame areas cannot be defined precisely enough to establish specific criteria. Therefore, the problem must be approached in general terms. This dictates use of the total mechanical power and total thrust frame area weight. The reference vibration ($G_r$) should be based upon adequate statistical analyses of measurements taken at many locations throughout the thrust frame area. Thus, $G_r$ may be specified with statistical certainty and the general environmental prediction applied. The equation for this prediction is therefore:
(beams) \[ G_n = G_r \sqrt{\frac{W_r}{W_n + W_c} \left( \frac{W_r}{W_n + W_c} \right)} \frac{(NTV)_n}{(NTV)_r} \]  \hspace{1cm} (3)

where

\( W_r \) = total weight of reference beam area

\( W_n \) = total weight of new beam area

\( W_c \) = weight of component to be located in this area

\( N \) = number of engines associated with that particular stage

In the case of engine components, the number of engines is not necessary for determining the component environment. Investigations have indicated that clustering does not appreciably affect engine component vibrations. Thus:

(engine components)

\[ G_n = G_r \sqrt{\left( \frac{W_r}{W_n + W_c} \right) \frac{(TV)_n}{(TV)_r}} \]  \hspace{1cm} (4)

\( W_r \) = basic weight of a single reference engine

\( W_n \) = basic weight of a single new engine

\( W_c \) = weight of component on the new engine

If the engine is separated into sections, then the basic weight is the weight of each individual section. These do not include component and hardware weights. It should be noted that the skeleton structure comprises approximately 80 per cent of the total engine weight. The requirements corresponding to equations (3) and (4) are that \( G_r \) represent the unloaded structure (i.e., does not reflect the effect of component mass loading) and that adequate measurements have been obtained to satisfactorily define the vibration environment.
APPENDIX III

EXAMPLE PROBLEM UTILIZING TYPE I METHODS

Statement of Problem

A component having a weight of 10 pounds is to be mounted on a 22 x 30 x .15 corrugated aluminum panel as shown in the sketch below:

The panel is located 800 inches above the engine nozzle exit plane of the second stage engines of the hypothetical vehicle configuration shown in enclosure (1).

To obtain a reliable estimate of the components' vibrational environment, the following information concerning the overall vehicle configuration should be known:

1. Total length of the vehicle configuration.

2. Distance from engine nozzle exit plane of one stage to the engine nozzle exit plane of the preceding stage for all stages of the configuration.

3. Type and number of engines for the first and second stages.

The type of structure for the given problem is in category I (a) skin structure. The equation for predicting the environment of a component on this type of structure is found on page 12. Thus:

\[
\frac{G_n}{G_r} = \frac{p_n}{p_r} \left( \frac{t_r \rho_r}{t_n \rho_n} \right) \sqrt{\frac{W_n}{W_n + W_c}} = 1.22 \frac{p_n}{t_n} \sqrt{\frac{W_n}{W_n + W_c}}
\]
where the material densities cancel because $\rho_r$ and $\rho_n$ are aluminum.

Since the structure is susceptible to dynamic pressures, the maximum acoustic or aerodynamic pressure must be determined. Calculation of three pressures is necessary of which the highest pressure is used to determine the vibration environment. First, determine the overall acoustic pressure at the component location due to the captive firing of the second stage. From Figure 1, the second stage is equipped with three M-1 engines. The equation for finding the resultant pressure produced by operation of three M-1 engines at a vehicle length of 800 inches is:

$$p_t = \sqrt[N]{p_1}$$

From Figure 8, $p_1$ is .121 psi; therefore, the resultant pressure is:

$$p_t = \sqrt[3]{.128} = .222 \text{ psi}$$

Next, determine the maximum overall acoustic pressure at the component location due to the firing of the booster stage. The distance of the component location from the booster engine nozzle exit plane is:

$$1700 + 800 = 2500 \text{ inches}$$

From Figure 7, $p_1 = .0435$ psi

Thus:

$$p_t = \sqrt[9]{.0435} = .1305 \text{ psi}$$

Next, determine the maximum aerodynamic pressure. The distance of the component location to the leading edge of the vehicle is:

$$1000 + 800 + 200 = 2000 \text{ inches}$$

The maximum aerodynamic pressure from Figure 9 is:

$$p_{ae} = .1 \text{ psi}$$

From the selection of the most extreme pressure,

$$p_n = .222 \text{ psi}$$

The equivalent thickness may be determined from Appendix I, equation (13).
Thus:

\[ t_{eq} = t_c \frac{\ell s}{\ell} = (0.15) \frac{(3 + 4 + 3 + 4)}{11} = 0.191 \text{ inches} \]

Further, the panel weight is (from equation (12), Appendix I):

\[ W = \rho m t c d \ell s = (0.1) (0.15) (30) (3 + 4 + 3 + 4) \frac{22}{11} = 12.6 \text{ lb} \]

Substituting the calculated values into equation (1) results in:

\[ \frac{G_n}{G_r} = 1.22 \left( \frac{0.222}{0.191} \right) \sqrt{\frac{12.6}{12.6 + 10}} = 1.06 \]

Now, the resultant environments may be obtained by multiplying the ratio \( \frac{G_n}{G_r} \) by the reference environments. (Figs. 10 and 11).

Hence, the vibratory environment in the given problem is:

![Graph](image1)

**(Resonant Dwell Equivalent)**

![Graph](image2)
APPENDIX IV

EXAMPLE PROBLEM UTILIZING
TYPE II METHODS

Statement of Problem

Consider an item of equipment which weighs 30 lb. This component is to be mounted in the third stage frame area of the hypothetical vehicle shown in Enclosure (1). This area is composed of a network of structural beams. A comprehensive weight breakdown indicates a total thrust structure weight of 9,000 lb. The problem is to determine the vibration environment of the component.

The first step is to determine the type of structure where the component is to be located. From the definition of structural types indicated on page 8, the problem applies to type II (a) structure. The equation for determining the vibration environment of type II (a) is shown on page 13. Thus:

\[
\frac{G_n}{G_r} = \sqrt{\frac{(NTV)_n W_r}{(NTV)_r W_n}} F = 9.38 \times 10^{-4} \sqrt{\frac{(NTV)_n}{W_n + W_c}}
\]

To solve this equation the following information is required:

(1) the number of rocket engines associated with that particular stage

(2) the thrust and exhaust velocity (ft/sec) developed by the rocket engine.

With this information:

\[
\frac{G_n}{G_r} = 9.38 \times 10^{-4} \sqrt{\frac{(6) (2 \times 10^5) (1.39 \times 10^4)}{(9,000 + 30)}} = 1.27
\]

Applying this factor to the sinusoidal and random reference environments shown in Figures 18 and 19, the resultant vibration environment of the component is:

\[
(3.2) (1.27) = 4.06 G_{pk}
\]
\[
(0.0028) (1.27) = 0.00356 \text{ In. D. A. Displ.}
\]
\[
(10.4) (1.27) = 13.2 G_{pk}
\]
and for the random:

\[(18.9) (1.27) = 24.0 \text{ Grms}\]

\[(.18) (1.27)^2 = .29 \text{ G}^2/\text{cps}\]
FIGURE 1. FLOW CHART FOR DETERMINING THE VIBRATION ENVIRONMENT OF A NEW VEHICLE STRUCTURE
Maximum overall RMS acoustic pressure, $P_t$, produced by operation of a single 165 K (H-1) engine vs. vehicle length.

To determine the resultant pressure $P_t$, from operation of two or more 165 K (H-1) engines use:

$$P_t = \sqrt{N} \cdot P_t$$

where:

$N$ = number of engines

Vehicle length (inches) vs. maximum overall RMS acoustic pressure (inches$^2$).

**Figure 2. Acoustic Pressure vs Vehicle Length 165K H-1 Engine**
MAXIMUM OVERALL RMS ACOUSTIC PRESSURE, \( p_1 \), PRODUCED BY OPERATION OF A SINGLE 188 K (H-1) ENGINE VS. VEHICLE LENGTH

TO DETERMINE THE RESULTANT PRESSURE, \( p_L \), FROM OPERATION OF TWO OR MORE 188 K (H-1) ENGINES USE -

\[
p_L = \sqrt{N} \ p_1
\]

where:

\( N = \text{NUMBER OF ENGINES} \)

Figure 3. ACOUSTIC PRESSURE VS VEHICLE LENGTH 188K H-1 ENGINE
MAXIMUM OVERALL RMS ACOUSTIC PRESSURE,
$p_t^*$, PRODUCED BY OPERATION OF A SINGLE
RL-10 ENGINE VS VEHICLE LENGTH

TO DETERMINE THE RESULTANT PRESSURE,
$p_t^*$ FROM OPERATION OF TWO OR MORE RL-10
ENGINES USE -

$$p_t = \sqrt[N]{N} \cdot p_1$$

where:

$N =$ NUMBER OF ENGINES

VEHICLE LENGTH (INCHES)

(DISTANCE ABOVE ENGINE NOZZLE EXIT PLANE)

FIGURE 4. ACOUSTIC PRESSURE VS VEHICLE LENGTH RL-10 ENGINE
MAXIMUM OVERALL RMS ACOUSTIC PRESSURE, $P_t$, PRODUCED BY OPERATION OF A SINGLE AGENA B ENGINE VS VEHICLE LENGTH

TO DETERMINE THE RESULTANT PRESSURE, $P_t^*$ FROM OPERATION OF TWO OR MORE AGENA B ENGINES USE -

$$P_t^* = \sqrt{N} \ P_t$$

where:

$N = \text{NUMBER OF ENGINES}$

10 inches or less

$$\text{MAXIMUM OVERALL RMS ACOUSTIC PRESSURE (POUNDS/INCH)}^2$$

VEHICLE LENGTH (INCHES)

(DISTANCE ABOVE ENGINE NOZZLE EXIT PLANE)

FIGURE 5. ACOUSTIC PRESSURE VS VEHICLE LENGTH AGENA B&D ENGINE
MAXIMUM OVERALL RMS ACOUSTIC PRESSURE, \( p_t \), PRODUCED BY OPERATION OF A SINGLE J-2 ENGINE VS VEHICLE LENGTH

TO DETERMINE THE RESULTANT PRESSURE, \( p_t \), FROM OPERATION OF TWO OR MORE J-2 ENGINES USE:

\[
p_t = \sqrt{N} \ p_1
\]

where:

\( N = \text{NUMBER OF ENGINES} \)

FIGURE 6. ACOUSTIC PRESSURE VS VEHICLE LENGTH J-2 ENGINE
MAXIMUM OVERALL RMS ACOUSTIC PRESSURE, $p_1$, PRODUCED BY OPERATION OF A SINGLE F-1 ENGINE VS VEHICLE LENGTH

TO DETERMINE THE RESULTANT PRESSURE, $p_r$, FROM OPERATION OF TWO OR MORE F-1 ENGINES USE -

$$p_r = \sqrt{N} \cdot p_1$$

where:

$N = \text{NUMBER OF ENGINES}$

**FIGURE 7. ACOUSTIC PRESSURE VS VEHICLE LENGTH F-1 ENGINE**
MAXIMUM OVERALL RMS ACOUSTIC PRESSURE, $p_t$, PRODUCED BY OPERATION OF A SINGLE M-1 ENGINE VS VEHICLE LENGTH

TO DETERMINE THE RESULTANT PRESSURE, $p_t'$, FROM OPERATION OF TWO OR MORE M-1 ENGINES USE:

$$p_t' = \sqrt{N} \ p_t$$

where:

$N = \text{NUMBER OF ENGINES}$

VEHICLE LENGTH (INCHES)

(DISTANCE ABOVE ENGINE NOZZLE EXIT PLANE)

FIGURE 8. ACOUSTIC PRESSURE VS VEHICLE LENGTH M-1 ENGINE
FIGURE 9. BOUNDARY LAYER INDUCED NOISE VS VEHICLE LENGTH

MAXIMUM EFFECTIVE RMS BOUNDARY LAYER INDUCED NOISE

VALID FOR LIQUID PROPELLANT BOOSTER VEHICLES

\[ 1.2 \leq \frac{T_o}{W} \leq 1.5 \]

- \( T_o \) = TOTAL FIRST STAGE THRUST (LBS.)
- \( W \) = TOTAL SPACE VEHICLE WEIGHT (LBS.)

DISTANCE FROM LEADING EDGE OF VEHICLE (INCHES)
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE TYPE I(a) SKIN PANELS
ASSOCIATED SKIN THICKNESS ($t_p$) = .19 in.
ASSOCIATED ACOUSTIC PRESSURE ($p_p$) = .156 psi (154.5 db)
MATERIAL DENSITY ($\rho$) = .1 lb/in. $^3$

VIBRATION LEVELS ARE SINUSOIDAL PEAK G'S

FIGURE 10. SINUSOIDAL REFERENCE ENVIRONMENT - TYPE I(a) SKIN PANELS
FIGURE 11. RANDOM REFERENCE ENVIRONMENT - TYPE I(a) SKIN PANELS

VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE TYPE I(a) SKIN PANELS
ASSOCIATED SKIN THICKNESS ($t_r$) = .19 in.
ASSOCIATED ACOUSTIC PRESSURE ($p_r$) = 136 psi (154.5 db)
MATERIAL DENSITY ($\rho_r$) = .1 lb/in.$^3$
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE TYPE I(b) SKIN STIFFENERS
ASSOCIATED SKIN THICKNESS \( t_{sk} \) = .19 in.
ASSOCIATED ACOUSTIC PRESSURE \( p_{ac} \) = .156 psi
\( (154.5 \text{ db}) \)
MATERIAL DENSITY \( \rho_r \) = .1 lb/in.³

VIBRATION LEVELS ARE SINUSOIDAL PEAK G's

FIGURE 12. SINUSOIDAL REFERENCE ENVIRONMENT - TYPE I(a) SKIN STIFFENERS
FIGURE 13. RANDOM REFERENCE ENVIRONMENT TYPE I(b) SKIN STIFFENERS
FIGURE 15. RANDOM REFERENCE ENVIRONMENT TYPE I(c)-1 FORWARD BULKHEADS
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE TYPE I(c)-2 AFT BULKHEADS

ASSOCIATED SKIN THICKNESS \( (t_r) \) = .095 in.

ASSOCIATED ACOUSTIC PRESSURE \( (p_r) \) = .154 psi (154.4 db)

MATERIAL DENSITY \( (p_r) \) = .1 lb/in.\(^3\)

\[ g_r \]

\[ 0065 \text{ IN. D. A. DEPL.} \]

\[ 1.2 \, G_{pk} \]

\[ 5.0 \, G_{pk} \]

VIBRATION LEVELS ARE SINUSOIDAL PEAK G's

FIGURE 16. SINUSOIDAL REFERENCE ENVIRONMENT TYPE I(c)-2 AFT BULKHEADS
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE TYPE I(c) - 2 AFT BULKHEADS
ASSOCIATED SKIN THICKNESS \( t_s = 0.095 \) in.
ASSOCIATED ACOUSTIC PRESSURE \( p_p = 0.154 \) psi (154.4 db)
MATERIAL DENSITY \( \rho_p = 1 \) lb/in.\(^3\)

FIGURE 17. RANDOM REFERENCE ENVIRONMENT TYPE I(c) - 2 AFT BULKHEADS
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE TYPE II(a) BEAMS
ASSOCIATED WEIGHT \( (W_p) = 9500 \text{ LBS.} \)
NUMBER OF ENGINE \( (N_p) = 8 \)
THRUST OF REFERENCE ENGINE \( (T_p) = 1.65 \times 10^5 \text{ LBS.} \)
EXHAUST VELOCITY OF REFERENCE ENGINE \( (V_p) = 8200 \text{ FT/SEC.} \)

VIBRATION LEVELS ARE SINUSOIDAL PEAK G's

FIGURE 18. SINUSOIDAL REFERENCE ENVIRONMENT TYPE II(a) BEAMS
VIBRATION ENVIRONMENT OF REFERENCE
STRUCTURE TYPE II(a) BEAMS
ASSOCIATED WEIGHT \( W_r \) = 9500 LBS.
NUMBER OF ENGINES \( N_r \) = 8
THRUST OF REFERENCE ENGINE \( T_r \) = 1.65 \( \times 10^6 \) LBS.
EXHAUST VELOCITY OF REFERENCE ENGINE
\( V_r \) = 8200 FT/SEC

FIGURE 19. RANDOM REFERENCE ENVIRONMENT TYPE II(a) BEAMS
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE
TYPE II(b)-1 COMBUSTION CHAMBER SECTION
ASSOCIATED WEIGHT ($W_F$) = 780 LBS.
THRUST OF REFERENCE ENGINE ($T_F$) = 165 x 10^6 LBS.
EXHAUST VELOCITY OF REFERENCE ENGINE ($V_F$) = 8200 FT/SEC

VIBRATIONAL LEVELS ARE SINUSOIDAL PEAK G's

FIGURE 20. SINUSOIDAL REFERENCE ENVIRONMENT TYPE II(b)-1 COMBUSTION CHAMBER SECTION
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE
TYPE II (b)-1 COMBUSTION CHAMBER SECTION
ASSOCIATED WEIGHT \( (W_R) = 780 \, \text{LBS.} \)
THRUST OF REFERENCE ENGINE \( (T_R) = 1.65 \times 10^6 \, \text{LBS.} \)
EXHAUST VELOCITY OF REFERENCE ENGINE
\( (V_R) = 8200 \, \text{FT/SEC} \)

FIGURE 21. RANDOM REFERENCE ENVIRONMENT TYPE II(b)-1 COMBUSTION CHAMBER SECTION
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE TYPE II(b) - 2 TURBOPUMP SECTION
ASSOCIATED WEIGHT \( (W_r) = 1.65 \times 10^6 \) LBS.
THrust OF REFERENCE ENGINE \( (T_r) = 1.65 \times 10^6 \) LBS.
EXHAUST VELOCITY OF REFERENCE ENGINE \( (V_r) = 8200 \) FT/SEC

VIBRATION LEVELS ARE SINUSOIDAL PEAK G'S

FIGURE 22: SINUSOIDAL REFERENCE ENVIRONMENT TYPE II(b) - 2 TURBOPUMP SECTION
VIBRATION ENVIRONMENT OF REFERENCE
STRUCTURE TYPE II(b)-2 TURBOPUMP SECTION
ASSOCIATED WEIGHT \( W_r \) = 505 LBS,
THRUST OF REFERENCE ENGINE \( T_r \) = 1.65 \( \times \) 10^6 LBS,
EXHAUST VELOCITY OF REFERENCE ENGINE
\( V_r \) = 8200 FT/SEC

FIGURE 23. RANDOM REFERENCE ENVIRONMENT TYPE II(b)-2 TURBOPUMP SECTION
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE TYPE II(b)-3 ACTUATOR ASSEMBLY
ASSOCIATED WEIGHT \( (W_T) = 320 \text{ LBS} \),
THURST OF REFERENCE ENGINE \( (T_T) = 1.65 \times 10^6 \text{ LBS} \),
EXHAUST VELOCITY OF REFERENCE STRUCTURE \( (V_T) = 8200 \text{ FT/SEC} \)

FIGURE 24. SINUSOIDAL REFERENCE ENVIRONMENT TYPE II(b)-3 ACTUATOR ASSEMBLY
VIBRATION ENVIRONMENT OF REFERENCE STRUCTURE TYPE II(b)-3 ACTUATOR ASSEMBLY
ASSOCIATED WEIGHT \((W_p) = 320\) LBS.
THRUST OF REFERENCE ENGINE \((T_p) = 1.65 \times 10^8\) LBS.
EXHAUST VELOCITY OF REFERENCE ENGINE \((V_p) = 8200\) FT/SEC

FIGURE 25. RANDOM REFERENCE ENVIRONMENT TYPE II(b)-3 ACTUATOR ASSEMBLY
REFERENCES


