NAVSEA 0908-LP-000-3010
Rev.1

SHOCK DESIGN CRITERIA FOR SURFACE SHIPS

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SHOCK DESIGN CRITERIA FOR SURFACE SHIPS

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LIST OF SYMBOLS AND ABBREVIATIONS

A_o	Acceleration limited shock design value (equivalent acceleration in g's)
A	Area, in^2
a	Mode number
ASM	Algebraic Summation Method
c	Distance from the neutral axis of a beam cross section, inches
[C]	Damping matrix
c_{ij}	Damping matrix element
CP	Controllable Pitch
D	Shock design value, g's
D_{va}, D_{aa}, D_{fa}	Specified spectrum design values (for mode a) along ship coordinate axes, g's
D_a	Shock design value for mode a
[D]	Diagonal matrix of $[L][D][L]^T$, decomposition of [K]
d	Displacement, inches
DDAM	Dynamic Design Analysis Method
DDS	Design Data Sheet
DOF	Degrees of Freedom
E	Modulus of elasticity, lbs/in^2
f	Shape factor
F	Force
{F(t)}	Force vector as a function of time
f	Frequency, Hertz
G	Shear modulus of elasticity
GRP	Glass Reinforced Plastic
g	Acceleration due to gravity, 386 in/sec^2
g_{ij}	Intermediate variable of [K] decomposition
HVAC	Heating Ventilation and Air Conditioning
Hz	Hertz
I	Moment of inertia, in^4
i	Mode counter
J	Polar moment of inertia, in^4
j	Mode counter
K	Spring constant (stiffness), lbs/in
$[\bar{K}]$	Generalized stiffness matrix
[K]	Stiffness matrix
k_{ij}	Stiffness matrix element
ksi	Kips per square inch
L	Length, inches
[L]	Lower triangular matrix of $[L][D][L]^T$, decomposition of [K]
l_{ij}	Element of [L] matrix

LIST OF SYMBOLS AND ABBREVIATIONS

$[\bar{M}]$	Generalized mass matrix
$[M]$	Mass matrix
m_{ij}	Mass matrix element
M_c	Modal force per given mode
M_c^t	Member force at time t
MDR	Multi-Directional Response
N	Total number of degrees of freedom in math model
n	Mass number counter
N^*	Master degrees of freedom
NOM	Reduced number of modes
NDOF	Number of degrees of freedom
NRL	Naval Research Laboratory
P(t)	External force as a function of time
P_a	Participation Factor for mode a
{Q}	Load vector
{q}	Decomposed load vector
$\bar{R}_i$	Response at point i
{r}	Influence coefficient vector
RPM	Revolutions per minute
T	Torque, in-lbs
t	Time, seconds
t_{max}	Maximum time (seconds)
t_{inc}	Incremental time step
u	Displacement
V_o	Velocity limited shock design value (pseudo-velocity), in/sec
V_c	Shear force on plane c
W	Weight, kips
X,Y,Z	Coordinate directions
$\bar{X}$	Displacement, inches
X_k	Intermediate eigenvector for iteration k
X_k	Normalized eigenvector for iteration k
$\dot{X}$	Notation for velocity
$\ddot{X}$	Notation for acceleration
Z	Section modulus, in^3

LIST OF SYMBOLS AND ABBREVIATIONS

Δ	Flexibility matrix
δ_{ij}	Flexibility matrix element
θ	Angle between direction of attack and vertical axis
ω_n	Natural frequency, radians
ξ	Fraction of critical damping
Σ	Summation of
σ	Direct or bending stress
τ	Shear stress
Φ	Angle between direction of attack and a transverse plane through the ship
$[\Phi]$	Mode shape matrix
$\{\Phi\}_a$	Mode shape for the a^{th} mode
Φ_{ia}	Mode shape for the i^{th} degree of freedom in mode a
λ	Eigenvalue
μ	Shift parameter for vector iteration
η	Shifted eigenvalues $(\lambda - \mu)$
ρ	Eigenvalue at iteration k
π	3.14159
$[\ ]$	Matrix notation
$\{\ \}$	Vector notation
$\{\ \}^T$	Transpose of a vector
$[\ ]^{-1}$	Inverse of a matrix
*	Notation for reduced set of characteristics

Chapter 1. INTRODUCTION

The primary purposes of this report are as follows:

a. Provide technically oriented shock design criteria for Navy review and approval of shock design calculations.

b. Provide a limited amount of general background/educational material concerning application of the Dynamic Design Analysis Method (DDAM).

This report is intended to convey Navy dynamic shock analysis requirements to engineers who possess an educational or experience background in the fields of vibration analysis, structural dynamics and stress analysis. If the user finds that this report does not provide information sufficient to permit full and efficient satisfaction of all specified dynamic shock analysis requirements, the cognizant contracting officer should be contacted for additional information.

The requirements indicated by this report are subject to modification by applicable specifications. Users of this report should carefully review applicable specifications to determine whether any of the provisions of this report have been modified.

The contents of this report are founded upon dynamic analysis procedures originally developed by the Naval Research Laboratory, Washington, D.C. These procedures were originally reported in the following reports:

NAVSHIPS Report "Shock Design of Shipboard Equipment, Dynamic Design Analysis Method", R.O. Belsheim and G. O'Hara 250-423-30 dated May 1, 1961,

BUSHIPS Report "Shock Design of Shipboard Equipment, Interim design Inputs for Submarines and Surface Ship Equipment"(U), 250-423-31, dated January 1, 1961 (Confidential), and

Naval Research Laboratory Report, "Interim Design Values for Shock Design of Shipboard Equipment", G.J. O'Hara and R.O. Belsheim NRL 1396, dated February 1, 1963

This report is a revision of NAVSEA 0908-LP-000-3010 which was prepared by the Supervisor of Shipbuilding, Conversion and Repair, USN, Brooklyn under the direction of the Naval Sea Systems Command. Portions of this report are directly derived from the reports referred to above. Where the requirements of this document are in conflict with previous DDAM guidance, this document shall take precedence.

Sections 3, 4 and 5 of this report contain example engineering calculations that illustrate

DDAM. These calculations were performed in the US customary system of units (inch - pound - seconds). Equivalent metric, System International (SI) units, appear in parentheses after or alongside of the US customary values.

1.1 Summary of Revisions to NAVSEA 0908-LP-000-3010

The Dynamic Design Analysis Method (DDAM) is the Navy's specified analytical method of qualifying non-testable equipment and supporting structures to withstand the effects of shock. The technique is based on experimental investigations conducted in the 1960s. The original design guidance document was issued in May 1976. This document was developed when the primary shock analysis capabilities available to industry and the Navy were hand calculations, and limited use of computers. Availability of powerful computer codes, advances in computer technology and analytical correlation studies conducted during recent shock trials has not changed the basic credibility of the DDAM (modal analysis) design method. These developments however, have indicated that the rules of application of the DDAM method require adjustment to account for this enhanced computer usage and capability.

The following discussion summarizes the major revisions to NAVSEA 908-LP-000-3010 developed in order to accommodate the advance over the past 25 years.

The primary areas addressed in the revision to NAVSEA 0908-LP-000-3010 are:

A) Closely Spaced Modes Phenomenon,

B) Multi-Directional Response Analysis,

C) Dynamic Reduction Techniques

D) Mode Selection Criteria

d) Other miscellaneous technical and administrative modifications which will clarify the requirements and improve/expedite the analysis process.

1.1.1 Closely Spaced Modes Phenomenon.

Recent post shock trial evaluations have indicated that response predictions based on DDAM analyses may, in some cases, be conservative due to the mathematical consequences of a phenomenon termed "Closely Space Modes". Modes whose frequencies are nearly equal are defined as closely spaced modes. This phenomenon, which occurs most often in large finite element models, has been determined to be directly related to the method of combining responses across the modes. Phasing, which is not considered in DDAM, can be an important factor when combining normal modes which have nearly the same frequency. Early in the shock induced motion, responses of closely spaced modes can be 180 degrees out of phase and cancel each other whereas later in the shock induced motion, the modal responses may come into phase and add to each other. For most shipboard equipment however, late time responses are not characteristically associated with shock damage. The revised version of NAVSEA 0908-LP-000-3010 suggests that the existence of closely spaced modes is a result of modelling error or an apparent resonant condition and should be resolved by redesign or remodelling. The report also introduces an alternate method of combining responses across modes which reduces the conservative aspect of the closely spaced modes phenomenon.

1.1.2 Multi-Directional Response Analysis (Coupled Modes Analysis)

The current version of NAVSEA 908-LP-000-3010 discourages the use of multi-directional response analysis and suggests that uni-directional analyses are preferred. This original recommendation was partially associated with the limited computer capability available at the time of introduction of DDAM. Current capabilities make the use of multi-directional analyses routine and often development of unit-directional analysis requires many simplifying assumptions, increased effort on the part of the analyst and reduced reliability of the results. The revised version of NAVSEA 908-LP-000-3010 clearly indicates that multi-directional analyses should be used where appropriate and provides corresponding application criteria.

1.1.3 Dynamic Reduction Techniques

The proliferation of the use large finite element mathematical models brings with it the increased reliance on some form of reduction technique to economically conduct the analysis and distill the huge amount of information that is frequently generated during the analysis process. Unfortunately, there are currently no clear guidelines for the application of various reduction techniques. This lack of guidance hampers both the Navy reviewing activity and the shock analyst. The revision to NAVSEA 908-LP-000-3010 presents some basic guidelines for application of dynamic reduction procedures.

1.1.4 Mode Selection Criteria

The current version of NAVSEA 908-LP-000-3010 requires that DDAM consider half the number of modes of response in the stress analysis. This requirement is no longer relevant with respect to the current use of large finite element models. Clearly new mode selection criteria is needed that considers the vast population of modes of response in a typical shock analysis of a contemporary finite element model. It is important that this new selection criteria ensure that critical modes of response are not omitted and also help to avoid unnecessary expense in the analysis process.
Under present guidelines the mode selection in a DDAM is random and uncontrolled because of the difficulty in reviewing extremely large mathematical models. The proposed revision to NAVSEA 908-LP-000-3010 presents basic parameters to consider in a selecting the important modes of response to consider in the shock design.

1.2 Change Summary

The following summary highlights and locates areas associated with significant changes to NAVSEA 0908-LP-000-3010:

Chapter 2. DEFINITIONS

Algebraic Summation Method (ASM) - ASM is a time domain method of assessing the shock response of a system from the results of a modal analysis of the system. In the DDAM, ASM is an alternate method of summing responses across the modes. This method combines the responses algebraically, thereby considering the effect of modal phase relationship on the total response and is used to evaluate the effects of closely spaced modes in the DDAM. (See Section 3.5.7 for detailed discussion).

Closely Spaced Modes - Response modes of a DDAM calculation whose frequencies are nearly equal (within $\pm 10\%$ of a common mean frequency). (Also see "split modes" and "uncombined modes", and detailed discussion in Section 3.5.)

Closely Spaced Modes (CSM) Method - The Closely Spaced Modes Method is an analysis procedure which provides a technique for combining responses from two closely spaced modes. Once this combination is determined, it is used in the NRL sum of responses in place of those two modes.

Cut-Off Frequency - As used in DDAM, the cut-off frequency is the upper bound of the frequencies of interest. The cut-off frequency reflects the level of refinement in a mathematical model used to represent a system.

Dynamic Degrees of Freedom - The number of displacement components which must be considered in order to represent the effects of all significant inertia forces on a structure.

Fixed Base - The primary ship structure such as decks, bulkheads, longitudinals and transverse frame members. For dynamic analysis purposes, a fixed base may be considered to act as a rigid, stationary (relative to the item mounted upon it) boundary in the direction of shock motion through which the shock motion is transmitted to the mounted equipment or structure.

Fixed Base Frequencies - The natural frequencies of a structure or system assuming that the mounting base of the analyzed structure or system is infinitely rigid in the direction of shock motion.

Mathematical Model - A mass-elastic system which is devised to possess a computed shock response simulating that of an actual physical system. All modeled structural elements are assumed to possess linear elastic properties.

Modal Effective Weight (Mass) - A weight (mass) that can be determined from normal mode theory which, when used in a single degree of freedom model with a similarly determined spring, results in a natural frequency which is identical to that of a given mode in a multi-degree of freedom system. The modal effective weight (mass) is also that portion of the item weight (mass) which is effectively accelerated in a given mode.

Modal Mass Force - The force accelerating a given mass in a given mode of system shock response.

Mode Shape - The relative amplitudes of displacement of the system masses in a normal mode of vibration.

Multi-Directional Response (MDR) Analysis - Shock analysis which evaluates system responses (translation and rotation) in the direction of shock input as well as other directions of response.

Node - In a finite element model, a node represents an interface joint between two separate finite elements of the model. A finite element node can include inertial properties (lumped mass) or function only as a structural connection between elements. Also, a node is a point on a structure which does not deflect during vibration in a given mode. An anti-node is a point on a structure where deflection is maximum during vibration in a given mode.

Normal Mode - A natural vibrating configuration of a linear mass-elastic system.

NRL Summation Method - The primary method, within DDAM, of determining the shock response of a system from the results of a modal analysis of the system. This method combines the responses across the modes by adding the absolute value of the largest response to the square-root of the sum of the squares of the other responses. This method takes a statistical approach to modal phasing.

Participation Factor - A value which is computed for each mode of shock response considered and indicates the relative importance of the system mode of shock response. Higher participation factors, regardless of sign, are associated with the more important system modes of shock response.

Quasi-Fixed Base - A modeling technique that eliminates certain mathematical anomalies inherent in the DDAM and permits evaluation of relative displacements between two items mounted to the same fixed base. The Quasi-Fixed Base is a fictitious mass/spring arrangement that is inserted between a mathematical model (or models) and the conventional fixed base (ship structure). The Quasi-Fixed Base mass in the equipment mathematical model(s) is to be no larger than 1% of the total model mass. The Quasi-Fixed Base mass is connected to the actual fixed base by a very stiff spring which is selected to assure that the frequency of the lowest dominant mode of the system is not changed by more than 10%.

Resilient Mount - An isolation device that acts to reduce the unwanted effects of shock, noise or vibration disturbances on a mechanical system. The term "Resilient Mount" is a generic term which includes shock, noise and vibration mounts.

Shock Design Value (D) - Numerical representation of shock response (acceleration or velocity) used for each mode in a dynamic analysis. The values depend on the mounting location of the equipment, structure, or foundation, the direction of shock response (vertical, athwartship, or fore and aft) and the item's design requirements (elastic or elastic-plastic). Formulas for the computation of shock design values are contained in "Shock Design Values", Design Data Sheet DDS 072-1 (CONFIDENTIAL).

Shock Grade - Classification category of required system or equipment performance (operability) levels in a combat environment. Items classified as Grade A are systems or equipment which are essential to the safety and continued mission capability of the ship. Accordingly they must remain operable and not create a hazard when exposed to combat environment corresponding to full shock design levels. Grade B items are items whose operation is not essential to the safety or mission capability of the ship but could become a hazard to personnel, Grade A items or the ship as a whole as a result of exposure to design level shock loading.

Shock Input - Refers to the shock design values as an input to the DDAM or to the physical shock loading due to an underwater explosion.

Shock Response - The dynamic behavior of an equipment, structure or foundation due to shock loading. Shock response generally refers to the displacement, velocity, acceleration, force, stress or strain experienced by an item.

Split Modes - A closely spaced modes phenomenon where, for example, a normal mode of the mathematical model is divided into two modes, close in frequency with each mode containing approximately equal portions of modal effective mass. The sum of the modal effective mass of the two split modes is approximately equal to that of the original single normal mode. Since the shock design values are inversely related to the modal effective mass, this artificial splitting of a mode results in a potentially erroneous increase in shock loading to the system.

Uncombined Modes - A closely spaced modes phenomenon where similar portions of the system are prevented from combining into a single mode.

Uni-Directional Response Analysis - Shock analysis which evaluates system response in only the direction of shock input. The model may be linear, planar or three dimensional.

Chapter 3. DYNAMIC DESIGN ANALYSIS METHOD

A shipboard equipment or structure, when subjected to a specified shock motion, will experience stresses and deflections in excess of those present under static conditions. The Dynamic Design Analysis Method (DDAM), developed to supersede the static G design method, is used to evaluate the shock capability of various shipboard equipment and structures. A static G analysis does not constitute an alternative to a dynamic response analysis. The first step in the evaluation process involves representing the item in question by a mathematical model. DDAM models essentially reduce an equipment or structure to an equivalent mass-elastic system which is used to design the system to sustain dynamic stresses induced by shock response motions. The desired strength levels are specified in terms of spectral values which are frequency and mass dependent. By setting up and solving the equations of motion of a mass-elastic system, forces and displacements associated with each mass and structural element in the system are determined. These forces and displacements are used to determine the stresses and/or deflections of various components of the equipment, the foundation and the hold-down means. These forces, stresses, or deflections are then compared with specified allowable values to determine the acceptability of the analyzed items from a shock standpoint.

As part of Total Quality Management (TQM) a graphical description of the total shock hardening design process is provided (as Figures 3-1, 3-2 and 3-3) to aid in understanding the material contained in this report. The process description covers the analysis methodology, evaluation considerations, applicable resources and interaction between the Navy approval agency and the analyst's organization.

Figure 3-1 is an overview of the process showing the relation of testing and analysis for Grade A and Grade B equipment and their foundations. Figure 3-2 describes the process of equipment shock qualification by DDAM and Figure 3-3 shows the procedural steps associated with foundation shock qualification. The details of the flow charts are presented throughout the text of this report.

In order to simplify discussion of the shock analysis procedure mentioned above, it will be divided into five distinct, yet interrelated, phases. These five phases will be called:

(1) Problem formulation phase
(2) Mathematical modeling phase
(3) Coefficient computation phase
(4) Dynamic computation phase
(5) Evaluation phase

Each of these phases is discussed on the following pages. The analysis criteria presented are applicable to all dynamic analyses, unless otherwise stated herein. Special considerations which apply to design of foundations and Grade B items are described in Chapters 4 and 5 of this report, respectively.

When the DDAM was first implemented in the 1960's, only manual calculation methods or simplistic computer codes were available. The calculations were performed strictly in the five-phase approach described above. With the advent of the powerful finite element computer programs, the distinctions between the various phases have become less clear. For example, current finite element programs generally permit the user to perform the coefficient computations and dynamic computations (phases (3) and (4)) in one step. The DDAM, in conjunction with finite element analysis is described throughout this report.

The limitations of the DDAM must be clearly recognized by the users of the method so that, if necessary, they can initiate a request for approval of an alternate approach or approval of special modeling considerations. First of all, the procedure is based on the presumption that the equipment being analyzed can be represented as a linear, elastic system with discrete modes. Second, except as inherent in the shock design values, damping is neglected in the DDAM which, for most shipboard equipment, is a reasonably valid assumption since shock-induced motions persist for only a few cycles of vibratory motion. For very low frequency systems (less than 5 Hertz) the DDAM may not be appropriate. Finally, where closely spaced modes exist in an analysis, DDAM may produce excessive responses. For these cases, as well as cases of non-linear or non-elastic systems, appropriate modeling assumptions must be developed or a NAVSEA approved alternate analysis method should be used to overcome the limitation. Similarly, analyses of foundations for very light weight equipment and analyses of equipment external to the hull will require appropriate modeling assumptions or alternate analysis/qualification methods. The specified shock spectrum design acceleration and velocity values are general in nature. While they have been derived from large scale model tests as well as data recorded in past full ship shock tests, they make no distinction between sizes and types of ships (e.g. cruisers, frigates, and aircraft carriers). Although DDAM defines shock design values for various mounting locations, for a given model, DDAM assumes that the shock design value is the same at every point where that mathematical model is attached to the fixed base. This may not be strictly applicable for widely distributed systems. The shock design values also do not distinguish between the motion differences expected at various decks within a given type of ship. Where such distinctions are expected to be critical in the evaluation of the equipment under consideration, inclusion of portions of the ship's structure in the analysis may be essential. Alternate methods of analysis, if specified by the Navy, employing motion inputs measured in a test of an identical or similar ship may be used when the general DDAM inputs are judged by the cognizant Navy approval authority to be inapplicable to the analysis of a particularly critical item of equipment.

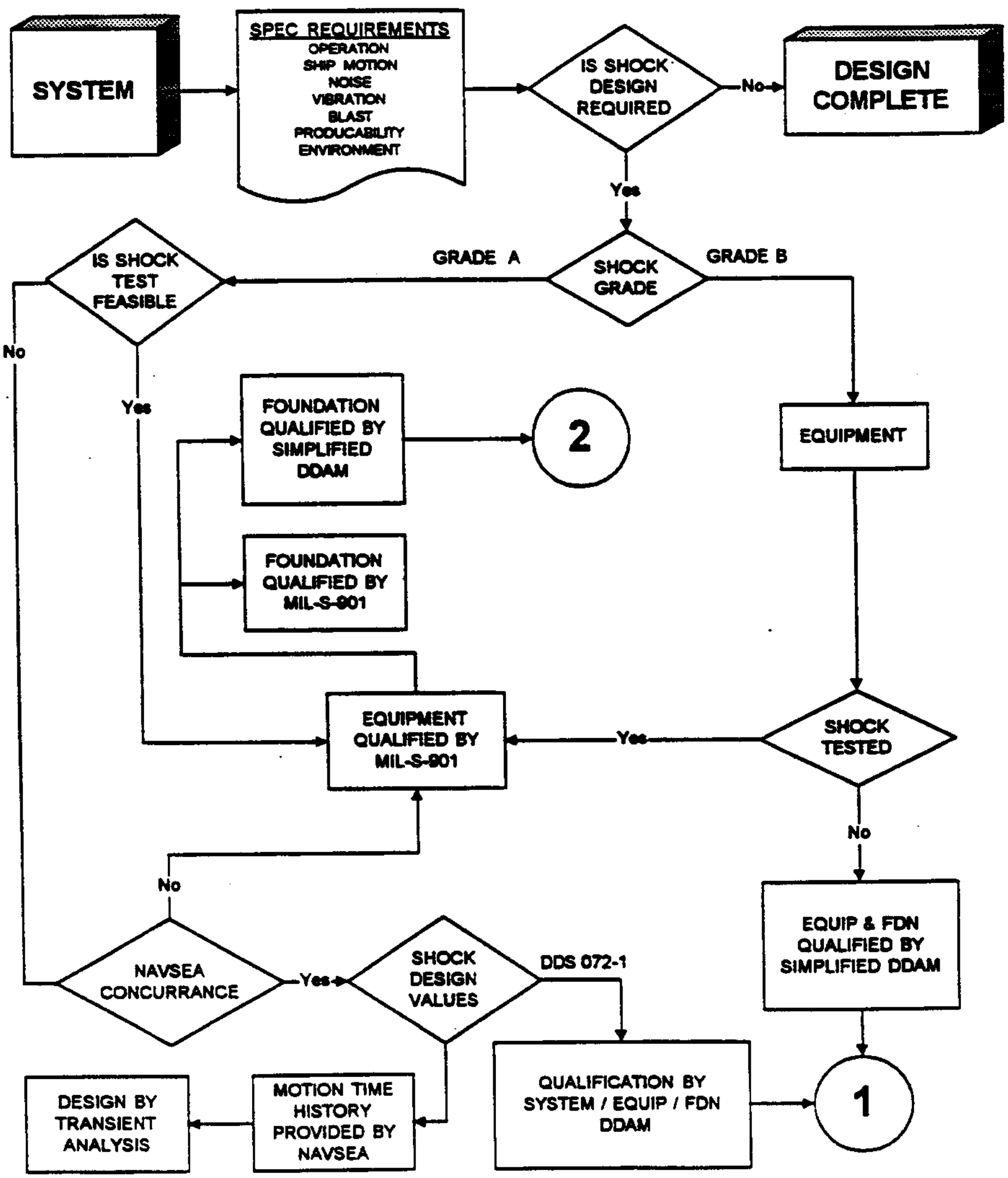


Figure 3-1 Shock Qualification Process - Overview

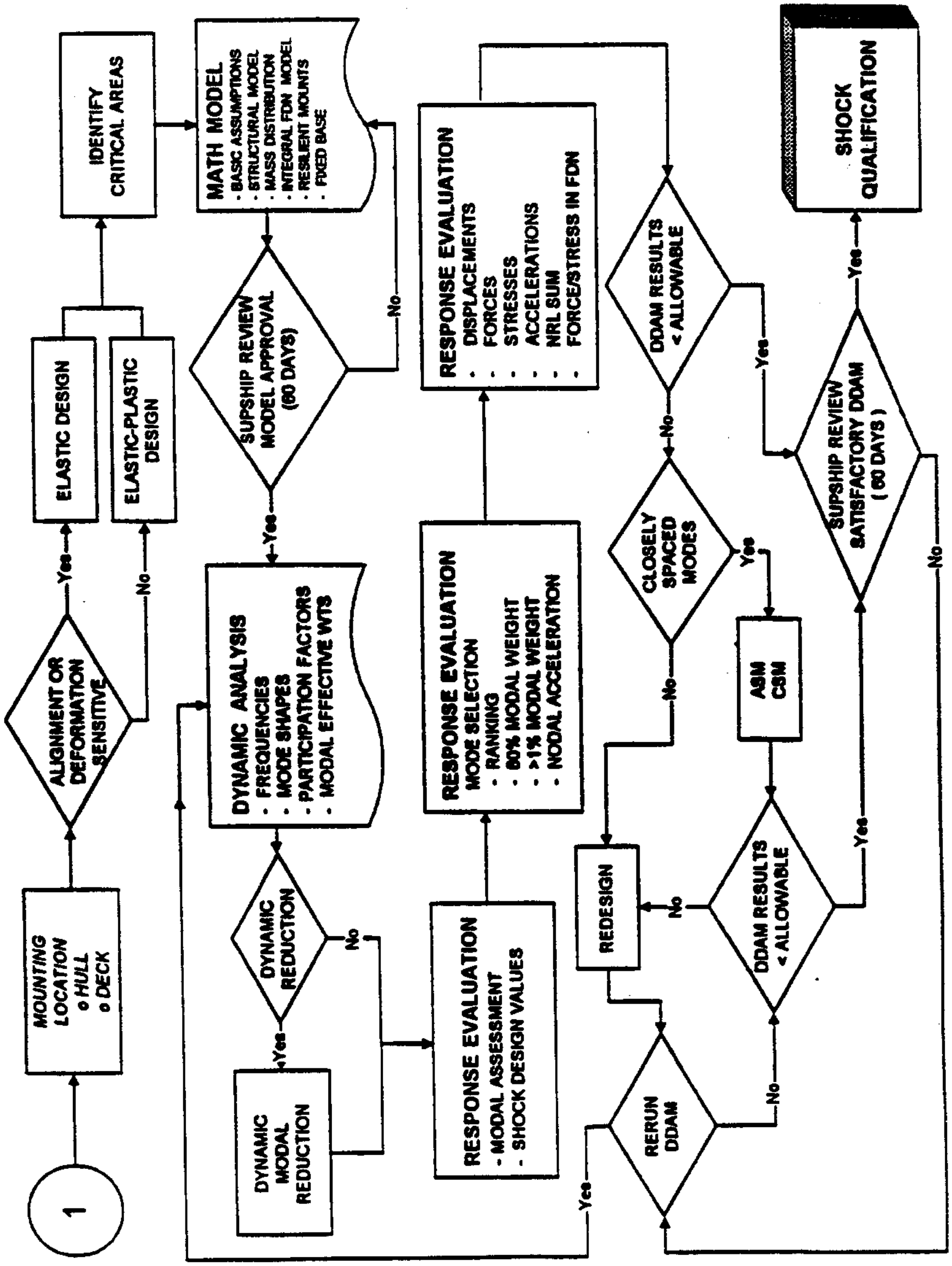


Figure 3-2 Equipment Shock Qualification By DDAM

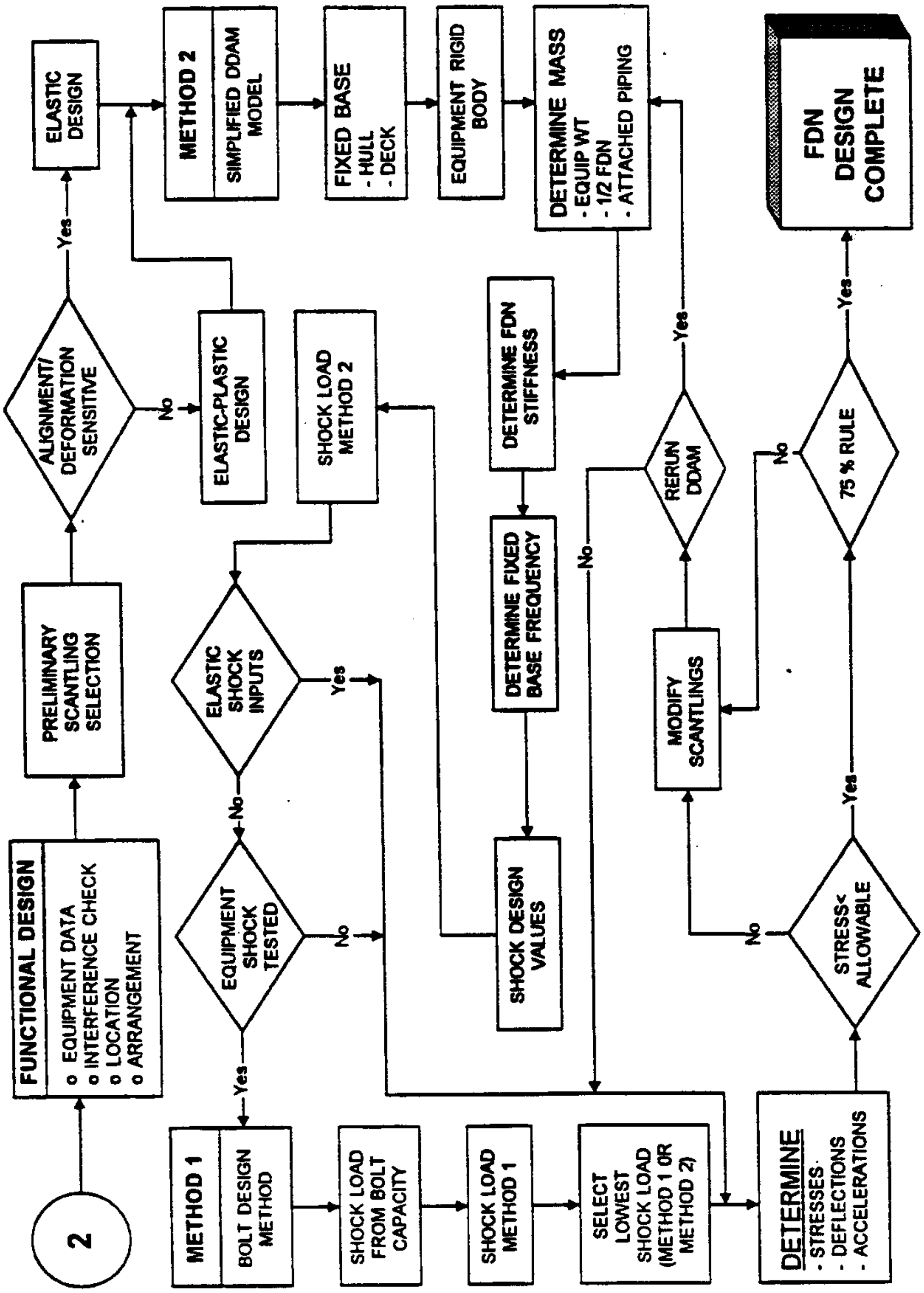


Figure 3-3 Foundation Shock Design By DDAM

3.1 Problem Formulation Phase

This phase involves a detailed study of the equipment or structure under consideration by the analyst. The analyst must determine the shock grade of the equipment or structure, the mounting location of the foundation, the shock design value to be used and the critical areas of the system which may require specific modeling considerations. For these determinations, the following requirements apply:

3.1.1 Shock Grades

The shock grades (A and B) are defined by the contract specifications. Criteria for determining shock grade requirements for an item are provided below. Grade A items are identified as such by the ship contract specifications. The specifications also designate certain Grade B items and provide general criteria for determining the shock grade of items which are other than Grade A.

Grade A shock criteria, as defined in Chapter 2, are applicable to the items which are required for the performance or direct and vital support of mission-essential functions aboard shock hardened ships. The following are often specified as mission-essential functions:

(1) Ship control and propulsion
(2) Command and control
(3) Navigation
(4) Communications
(5) Surface, air and underwater surveillance
(6) Countermeasures
(7) Launching, retrieving, fueling, defueling, rearming, and handling of aircraft and small surface craft
(8) Essential checkout and maintenance of aircraft and ordnance
(9) Fire control, firing or launching and guidance of missiles and other weapons
(10) Stowage, handling and reloading of weapons
(11) Replenishment-at-sea (stowed configuration)
(12) Mine-hunting and sweeping
(13) Transporting and landing troops and combat payload (assault ships)
(14) Casualty and damage control
(15) Collective protection system

Grade B shock criteria, as defined in Chapter 2, are applicable to items whose operation is not essential to the safety of the ship or to the direct and vital support of mission-essential functions identified above but which, due to either location or function, could become a hazard to personnel, to Grade A items or to the ship as a whole as a result of exposure to shock.

3.1.2 Mounting Locations

All shipboard equipment and structures are, for purposes of DDAM analysis, considered to be either hull mounted, deck mounted or shell mounted through their foundations. Shock inputs for each of these types of mounting locations are defined in DDS 072-1. Figure 3-4 describes various mounting locations with respect to the level of shock design input that should be applied. Proper identification of the mounting location (See Section 4.4) is important as this will determine the proper shock design value to use for dynamic analysis (See Section 3.1.3 below). This is particularly important in the case of major items of equipment mounted on decks or on bulkheads above the main deck. In the context of the following discussion "main deck" is used to indicate the "bulkhead deck" or the uppermost deck up to which the transverse or longitudinal watertight bulkheads and shell are carried. Major equipment items are often directly connected to the keel through structural bulkheads or stanchions and may thus be subjected to hull-mounted, rather than deck-mounted shock design values. The influence of the particular ship's structure supporting such items must therefore be carefully considered prior to initiating the analysis. The symmetry of the ship's structure supporting an item of equipment must also be considered. Severe asymmetry may cause undesirable rocking motions and uneven structural loading. Since the shock design values are predicated on uniform translational motion of the fixed base and rotation of the fixed base is not considered, sufficient ship structure must be considered in the development of the mathematical model such that the location of the fixed base conforms to that DDAM assumption.

The following definitions, used in the context of DDAM, are provided for the purpose of determining the category of shock inputs to apply:

"Hull Mounted" shock design values are used for equipment mounted on basic hull framing, tank tops, inner bottom, shell plating above the water-line and structural bulkheads below the main deck (bulkhead deck). Where a structural bulkhead (grounded on the inner bottom) ends at the Main Deck, or a deck below, an item attached to the deck at that location shall be considered hull mounted.

"Deck Mounted" shock design values are used for equipment mounted on decks, platforms, non-structural bulkheads and structural bulkheads above the main deck (bulkhead deck).

"Shell Mounted" shock design values are used for equipment mounted directly to the shell plating below the water line.

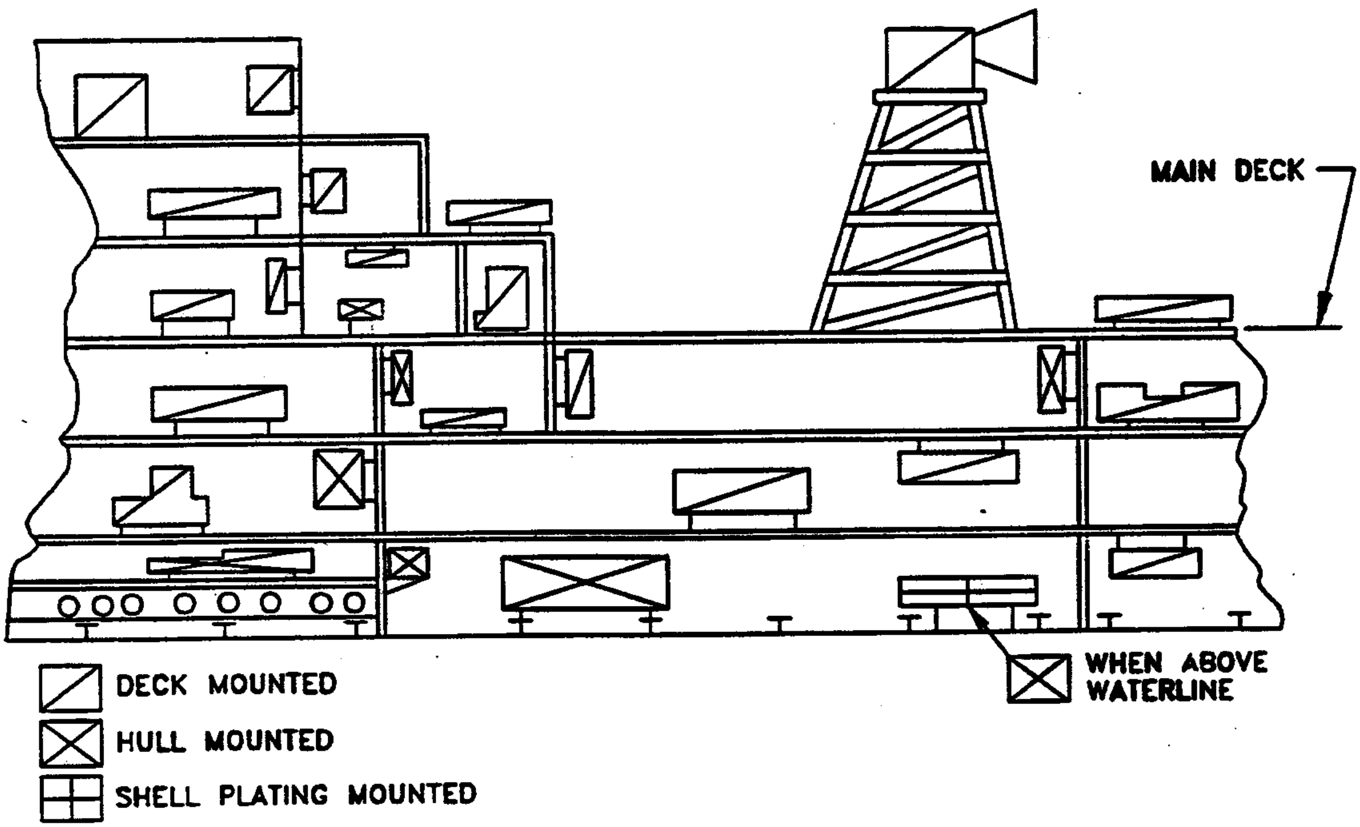


Figure 3-4 Mounting Locations for Surface Ships

In the event that an item is mounted to two different parts of the ship, for which different shock design values are specified, the larger shock design value shall be used for the analysis of the item.

Where it is necessary to evaluate specific characteristics associated with the deck structure such as load path within the ship structure or relative deflections of independent items mounted on the deck, the deck structure shall be included as part of the mathematical model. Where this is done, the fixed base of the mathematical model should extend to the structural bulkheads, stanchions, or hull framing. In these cases hull level shock inputs shall be used for design.

The following considerations shall apply for items not mounted directly on a ship's deck or on the basic hull structure:

a. <u>Shock Design Values for Items Mounted on Structural Bulkheads</u> - As indicated by DDS 072-1, hull mounted shock design values are to be used in the design of foundations mounted on structural bulkheads below the bulkhead deck. For this purpose, structural bulkheads are defined as any main transverse or longitudinal bulkhead that carries ship's loading and other bulkheads which, if removed, would require the addition of a stanchion to carry these loads. These are:

   (1) Main subdivision bulkheads.
   (2) Main longitudinal bulkheads.
   (3) Bulkheads that replace stanchions, web frames, or any other load-carrying members.
   (4) Bulkheads located or constructed such that they must be considered capable of transmitting shock loads, regardless of their function. These would include any bulkhead below the bulkhead deck which is thicker than 1/8 inch (31.75 mm) and which attaches directly to the shell or inner-bottom, or which is aligned with bulkheads, floors, or stanchions which are attached to the inner-bottom.

   For the design of foundations mounted on all other bulkheads below the bulkhead deck, and structural bulkheads above the bulkhead deck, deck inputs shall be used.

b. <u>Criteria for Lightweight Items Mounted on Machinery Space Upper Levels</u> - In analyzing lightweight items such as HVAC duct or piping systems which are supported from upper levels, the levels may be treated as decks and deck-mounted inputs applied. These criteria do not apply to analysis of the upper levels themselves. See Section 4.4.2 applicable criteria.

3.1.3 Shock Design Values

Elastic and elastic-plastic shock design values are contained in DDS 072-1. Criteria for selection of elastic versus elastic-plastic shock design values are as follows:

a. Elastic Shock Design Values - Elastic shock design values shall be used in cases where it is necessary to preserve the original physical dimensions after exposure to shock. All foundations which support rotating elements in the propulsion train (turbines, reduction gear and propeller shafting), and foundations for other alignment-critical components shall be designed to perform elastically. Foundations for rotating auxiliary equipment shall be designed elastically unless it canbe shown that plastic deformation or tilting of the equipment mounting surface will not occur or will not result in impaired equipment performance. (Note that standoff chocks may often be used to eliminate prying effects resulting from distortion of equipment mounting surfaces). Shipboard items which are known to be alignment sensitive (for purpose of shock design) are listed below. Omission of alignment sensitive items from this list does not relieve the contractor of his responsibility to assure proper selection of shock design values for all applicable items.

Main Propulsion Machinery	Auxiliary Propulsion Machinery
Ship Service Generators	Propulsion Shafting
Propulsion Shaft Bearings	Main Propulsion Reduction Gear
Propulsion Clutches	Propulsion Couplings
Turbine Brake	Main Thrust Bearing
Main CP Servo Pump	Gyroscopic Compass
Radar Antenna	Radio Antenna
Missile Directors	Gun Directors
Steering Gear (Ram Unit)	Steering Rudder System
Ammunition Hoists	Elevators and Elevator Machinery
Sonar Transducers	Catapult Machinery
Arresting Gear	Missile Launchers
Guns	Torpedo Tubes
Controllable Pitch Propeller	

b. Elastic-Plastic Shock Design Values - If elastic design is not required for the reasons stated above, elastic-plastic shock design values shall be used in cases where design by dynamic analysis is required.

c. Special Criteria for Displacement-Critical Items - In cases where deflections (rather than stresses) are critical from a shock standpoint, deflection calculations shall be based upon elastic design values.

d. Special Criteria for Hold-down/Locating Devices - In cases where equipment and/or foundations are designed to suit elastic-plastic, velocity limited shock design values, shock loadings shall be redeveloped on the basis of elastic shock design values for purposes of analysis of bolting, dowels, and similar hold-down or locating devices if shock qualification of these items by dynamic analysis is intended. Applicability of this criterion shall be limited, however, to hold-down or locating devices which are directly attached to the shipboard foundation. Hold-down or locating devices which are not at the equipment/foundation interface shall be designed to suit the same criteria that apply to other structural elements of the equipment in question.

3.1.4 Critical Areas

The critical areas of an equipment or structure are defined as those areas or components which are most likely to exceed failure criteria under shock loading. For purposes of these requirements, "failures" in a Grade A system are those which could cause functional impairment of the system. "Failures" in a Grade B system are those which will constitute a hazard as defined for Grade B items in the applicable contract specifications. The analyst shall construct the model so that necessary information (stresses, deflections) can be obtained for these critical areas. Typical critical areas of investigation for major systems normally required by the shipbuilding specifications to be designed by DDAM are contained in the SUPSHIP Brooklyn guidance manuals referred to in Section 3.6.1. The systems include: the rudder and rudder stock, main propulsion shafting system (excluding propeller), masts, and main reduction gear.

The intent of these SUPSHIP Brooklyn manuals is to provide the analyst with guidance in modeling and dynamically analyzing a specific system or equipment. Besides critical areas of investigation, the aforementioned manuals also contain information on basic assumptions used in modeling, frequency calculations for modeling purposes, sample mathematical models, and mass lumping procedures. For features of components not specifically treated by the aforementioned guidance manuals, the analyst should rely on the following means to determine which areas of an equipment or structure shall be considered critical:

(a) Frequency calculations
(b) Previous analyses
(c) Damage history
(d) Shock test information for similar equipment

In relation to the four factors listed above, engineering judgement must be used. For example, under vertical shock loading high stresses would be expected in an equipment's foundation. High stress would also be expected in bolting between an upper and lower housing.

Fixed base natural frequency calculations of individual system components are useful in determining regions which should be explicitly modeled. It is known that relatively low frequency items are likely to undergo relatively large displacements under shock. Therefore, low frequency structural components should be included in the mathematical model.

Previous analyses of similar equipment, damage history, and shock test information for similar equipment may provide useful information concerning critical areas.

3.2 Mathematical Modeling Phase

The mathematical modeling phase consists of constructing a system of masses and structural elements (beams, springs, plates, etc.) to represent the significant dynamic characteristics of the system under consideration. In the case of a reduction gear, for example, the system under consideration will include the reduction gear, its foundation, a portion of the line shafting, connections to the turbines and any other piece of attached equipment which will affect the response of the gear under shock loading. A separate dynamic analysis shall be performed for each principal direction of shock loading (e.g. vertical, athwartship, and fore and aft), and the shock resistance of the item to each direction of loading shall be evaluated separately. For uni-directional response analyses a separate mathematical model is required for each of the three directions of shock input. If a Multi-Directional Response (MDR) analysis is performed, a single mathematical model may suffice for analysis in each of the three directions of input. An MDR analysis is required where the structure or equipment is such that an input motion in a specified direction produces significant responses in other directions. Examples of such structures are:

(a) Flexible structure subject to whipping (e.g. masts)
(b) Structures oriented in directions oblique to the ships axes (e.g. radar arrays)
(c) Structures with large unbalanced masses (e.g. air conditioning plants)

To simplify discussion of the mathematical modeling phase, the following major steps will be considered separately:

(1) Basic modeling assumptions
(2) Frequency calculations
(3) Mass lumping
(4) Mass locations
(5) Designation of structural model
(6) Special modeling criteria

3.2.1 Basic Modeling Assumptions

Basic modeling assumptions must be formulated to permit reduction of a real structure to a simplified linear system of lumped masses and elastic structural elements. For certain major items required by contract specifications to be dynamically analyzed, the SUPSHIP Brooklyn guidance manuals referenced in Section 3.6.1 of this chapter describe typical basic assumptions for specific items.

a. Selection of the Fixed Base -A fundamental assumption necessary in the application of DDAM is the selection of the fixed base. A fixed base acts as a rigid stationary boundary in the direction of shock motion through which the shock motion is transmitted to the mounted equipment or structure. Inherent in the selection process is the determination of important characteristics of the fixed base. The fixed base is assumed to be at the interface of the system foundation and the basic ship structure. Section 3.1.2 describes the character of the fixed base at different shipboard mounting locations. Proper selection of the fixed base for a system, whether hull or deck mounted, also defines the proper choice of shock design values to be applied. It is necessary for the mathematical model to reflect local flexibilities of the interface which can affect the system response. For example, if rocking of the supporting ship structure is a dominant response characteristic for the system, the mathematical model should include this feature of the interface.

3.2.2 Frequency Calculations

As stated in Section 3.1.4, fixed base natural frequency calculations are used to determine those components which may be critical. These components may require a separate mass or masses to properly model them. The cut-off frequency is defined as the frequency of the highest mode of vibration to be considered in the dynamic analysis corresponding to conditions specified in Section 3.5.3. Those components whose frequency (which may be approximated by the fixed base frequency) falls below the cut-off frequency of the system shall be modeled.

3.2.3 Mass Lumping

Having determined critical areas and frequency values, the analyst can now proceed to model the equipment or structure. To aid the analyst in this task, the following guidelines are given:

a. The model should be as simple as possible. The analyst should strive for the simplest model which yields all the information required for a complete analysis of the equipment or structure.

b. High frequency components should be lumped together. The analyst is justified in combining adjacent high frequency (frequencies above the cut-off frequency) components into one mass. This justification is based upon the fact that adjacent high frequency components tend to move as a single rigid mass under shock loading, and so may be analyzed as a unit. Some high frequency components, however, may require separate modeling. This may be the case where it is required to know the relative deflection between two components of the system.

c. Low frequency components shall be represented as separate masses. A critical component whose frequency is below cut-off frequency shall be represented by one or more masses in the mathematical model. Non-critical low frequency components shall be represented by one or more masses if the weight of the component is such that it will significantly influence the shock response of a critical part of the system. To illustrate this situation the analyst is referred to the main reduction gear guidance manual referred to in Section 3.6.1. In this manual it can be seen that in modeling the main reduction gear for vertical and athwartship shock loading, the relatively low frequency line shafting adjacent to the gear is represented. Even though the shafting is not required to be stress analyzed with the reduction gear, its effect on the critical bull gear bearing requires that it be included in the gear model. The number of masses needed to model a component depends on fixed base natural frequency and the distribution of the component mass. For example, if the second mode fixed base frequency of a component is below the system cut-off frequency, then at least two masses are required to adequately model it.

To illustrate this point, assume that the simply supported shaft shown in Figure 3-5 is a part of an equipment which has an estimated cut-off frequency of 200 Hertz. Assume the shaft weight between supports is W = 19,776 lbs (87.97 kN) and that the length between supports is L = 192 inches (4.88 m).

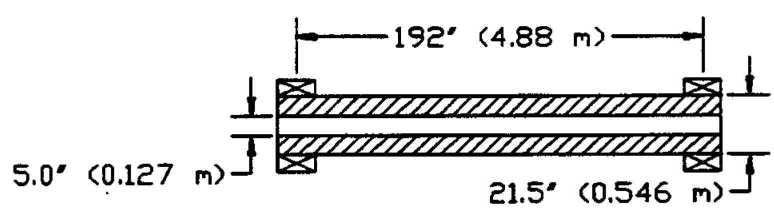
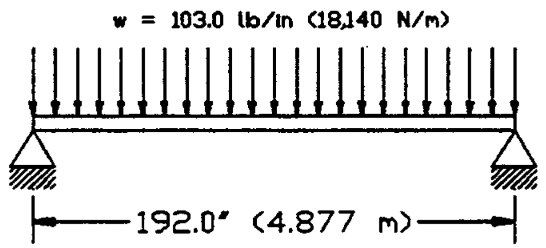


Figure 3-5 Simply Supported Shaft

The shaft shown in Figure 3-5 may be represented schematically as shown in Figure 3-6.



where:
E = 29×10^6 psi (200 GPa)
I = 10,485 in^4 (4.35×10^{-3} m^4)
g = 386 in/sec^2 (9.81 m/s^2)
w = 103 lbs/in (18,040 N/m)

Figure 3-6 Schematic Representation of Simply Supported Shaft

Using the natural frequency equation (which reflects the consistent mass nature of the beam) for a simply supported beam with a uniformly distributed weight to determine the fixed base frequency of this component:

$$f = B\sqrt{\frac{EIg}{wL^4}} = 28.96\ B\ (\mathrm{Hz})$$

Where B = 1.571 for the first mode, B = 6.283 for the second mode and B = 14.137 for the third mode frequency, the following results are obtained:

f_1 = 45.49 cps

f_2 = 181.96 cps

f_3 = 409.4 cps

Therefore, the shaft shown above is required to be modeled with two or more masses so that the effect of two significant modes of response on the equipment can be adequately evaluated.

d. Shock tested items shall be included in the model. Small shock testable items that are a part of a larger equipment shall be shock tested rather than being separately analyzed as part of the equipment DDAM. This applies to such items as tachometers, gauges and motors. The NAVSEA policy to test such items avoids the uncertainties involved in analyzing small mechanical components. Whether they are represented by a single mass or lumped into adjacent masses, shock tested items must be included if they are a part of the system under consideration. However, items which have been shock qualified should be stress analyzed only to the extent of determining the adequacy of their hold-down means unless these fasteners have also been previously qualified by shock testing of the items. For example, a turning gear motor which has been shock tested and found acceptable requires analysis to determine the adequacy of its hold-down means but does not require analysis of its internal parts (e.g. armature, brushes, etc.). Analysis should be limited to the structural portions of the equipment under consideration. For completeness, the mathematical model report shall contain information on the status of the MIL-S-901 testing of any components. If testing has been completed, references to the test report and approval documentation shall be provided. If testing is to be done in the future, planned test schedules shall be indicated.

3.2.4 Mass Locations

The concentrated masses having been determined, the analyst must then proceed to properly place them in relation to a fixed origin (the analyst should choose any origin convenient to the system under consideration).

The masses of high frequency components are added and considered to be a single mass located at a node. Center of gravity calculations locate each lumped mass relative to the origin of the coordinate system. This is done by determining the mass center of gravity of each component making up the mass point and locating the component's position along a principal axis relative to the origin. For example, to locate the X-coordinate of a mass point relative to the origin, the following formula is used:

$$\bar{X} = \frac{\sum Wx}{\sum W}$$

where,

$\bar{X}$ = distance between mass point and origin as measured along the X-axis

W = weight of individual component contained in the lumped mass.

x = distance between the origin and the center of gravity of the individual component as measured along the X-axis.

The same procedure is used to determine the Y- and Z-coordinates of a mass point. The overall center of gravity of the model should match the center of gravity of the actual item.

3.2.5 Designation of Structural Model

A structural model (linear, elastic, mathematical description) of an item can be a finite element description of the item or a mass-spring, lumped parameter representation. The structural model describes the item in terms of physical characteristics which when combined with the concentrated masses will produce dynamic characteristics representative of the equipment or system under investigation. All material properties used in generating the mathematical model shall be values at the expected operating temperature of the item.

3.2.6. Special Modeling Criteria

During the modeling, coefficient computation and dynamic computation phases, resilient mounts shall be assumed to be rigid in translation (in the direction of application of shock motion only) unless it can be shown that the mounts will remain linear and elastic during shock excursions. The effects of overturning characteristics of a resiliently mounted system shall be considered in determining the degree and extent to which the mount flexibility will be included in the mathematical model. Regardless of the representation of the mounts in the mathematical model, the actual mount physical characteristics shall be considered during the evaluation phase. It is noted that shock isolation or protection devices shall not be used in foundation systems without approval of NAVSEA.

For equipment with attached external piping which is not separately modeled, the analyst shall include the weight of five feet of this piping (including fluid) as mass when modeling the equipment.

Where an item is modeled as a lumped mass with rigid links, the equipment model should not provide constraint to the support structure.

Where foundations are grounded on deep frames, inner bottom structure, built-in tanks, or similar structure above the shell plating, this local structural flexibility may be included (but is not required) in the mathematical model. Incorporation of this structure in the model may serve to reduce the calculated shock response.

3.3 Coefficient Computation Phase

Having developed a mass-spring or finite element representation (structural model) of the equipment or structure under consideration, the analyst must then determine how this model reacts to a pre-determined shock design value (DDS 072-1). In order to determine this reaction, evaluation of the dynamic equations of motion are required.

$$[\,M\,]\{\ddot{X}\} + [\,C\,]\{\dot{X}\} + [\,K\,]\{X\} = \{\,P(t)\,\}$$

Solution of the equations of motion requires the formulation of the associated coefficient matrices. Damping is not considered in the DDAM and therefore the damping coefficient matrix, [C], is assumed to be null. The mass coefficient matrix [M] (called the mass matrix) is the matrix of elements m_{ij} where:

m_{ij} = Force corresponding to coordinate i due to a unit acceleration at coordinate j only.

The stiffness coefficient matrix, [K] is the matrix of elements k_{ij} where:

k_{ij} = Force corresponding to coordinate i due to a unit displacement of coordinate j (and no other coordinate displacements are permitted)

X, $\dot{X}$ and $\ddot{X}$ = Displacement, velocity and acceleration respectively of a nodal degree of freedom.

P(t) = Externally applied forcing function

The inverse relation of the stiffness matrix is called the flexibility matrix $[\Delta]$ and is a matrix of elements δ_{ij} where:

δ_{ij} = deflection of coordinate i due to a unit load applied to coordinate j.

Methods of determining these matrices can be found in standard structural dynamics textbooks.

3.3.1 Mass Matrices

The mass coefficient matrix can be determined by either the lumped mass or the consistent mass formulation. In the lumped mass method the mass properties of a component or model element are typically associated only with the translational degrees of freedom at the nodes of that element. However, this does not preclude the use of rotational inertia where desired. The simplest procedure for defining the lumped mass properties of any structure is to assume that the nearby distributed mass is concentrated at the nodes where translational displacements are defined. The usual procedure for defining the magnitude of mass to be located at each node is to assume that the structure is divided into regions or elements with nodes serving as connection points. The mass of each element is assumed to be concentrated as point masses at its node points. The distribution of the element mass to the node points is determined by geometric relations. The total mass concentrated at any node point is the sum of all the nodal contributions of the elements attached to that node. For the lumped parameter system the mass matrix has a diagonal form.

A consistent mass matrix is defined using a consistent shape function for both the potential and kinetic energies. Unlike the lumped mass matrix, the consistent mass matrix includes off-diagonal coefficients that couple related degrees of freedom.

The dynamic analysis of a consistent mass system generally requires considerably more computational effort than a lumped mass system does, for the following reasons:

(1) The lumped mass matrix is diagonal while the consistent mass matrix has many off-diagonal terms (leading to what is called mass coupling).

(2) Unmassed degrees of freedom can be eliminated from a lumped mass analysis by static condensation, whereas all rotational and translational degrees of freedom must be included in a consistent mass analysis.

As the lumped mass model is refined, the influence of the missing off-diagonal terms will diminish and the calculated response will converge to that of the consistent mass model.

3.4 Dynamic Computation Phase

The dynamic computation phase usually involves placing the pertinent data developed in the previous phases into a suitable computer program in order to obtain the modal characteristics present in the system. Many computer programs which perform the computations associated with the Dynamic Design Analysis Method are available or are developed external to commercially available general purpose structures programs. A sample computation for extracting characteristic values (frequencies and mode shapes) is shown for a three degree of freedom system in Appendix A.

3.4.1 Modal Analysis

The dynamic analysis of a mathematical model representation of a system or structure initially involves the definition of the modal (frequency) equations of motion for that system. The undamped free-vibration, modal equations of motion for a multi-degree of freedom system in matrix notation become:

$$-\omega^2 \, [M]\{\Phi\}_a + [K]\{\Phi\}_a = \{0\}$$

Solution of the equations (the eigenvalue problem) produces natural frequencies ω_a and mode shapes $\{\Phi\}_a$.

N	=	Number of degrees of freedom within the mathematical model
[M]	=	Mass matrix of the system
$\{\Phi\}_a$	=	Mode Shape for the a[th] mode

For the purpose of the following discussions an influence coefficient vector {r} is defined to represent displacements of all degrees of freedom resulting from a unit support translation. The influence coefficient vector {r} has the following characteristics:

(a) For a uni-directional response analysis, {r} is a column of ones.

(b) For a multi-directional response analysis in which the orientation of ship input motion coincides with the orthogonal axis of the model, {r} is a column of ones and zeros.

(c) For a multi-directional response analysis in which the orientation of the input motion is arbitrary with respect to an orthogonal axis of the model, {r} is a column of direction cosines and zeros.

Given the above characteristics (i.e. N, [M], ω_a and $\{\Phi\}_a$) the following quantities are determined for each mode and each direction of motion:

$$\bar{M}_a = \{\Phi\}_a^T [M] \{\Phi\}_a$$

Generalized mass of the a[th] mode.

$$\bar{M}_a = \sum_{i=1}^{N} \Phi_{ia}^2 M_i$$

Where Φ_{ia} is the a[th] mode shape for a lumped mass system represented by a diagonal mass matrix

$$P_a = \frac{\{\Phi\}_a^T [M] \{r\}}{\bar{M}_a}$$

Participation factor for the a[th] mode

$$P_a = \sum_{i=1}^{N} \frac{\Phi_{ia} M_i r_i}{\bar{M}_a}$$

Participation factor for a lumped mass system represented by a diagonal mass matrix

$M_a = P_a^2 \bar{M}_a$ Modal effective mass for the a[th] mode

$\{F\}_a = D_a P_a [M] \{\Phi\}_a$ Nodal forces for the a[th] mode

$\{A\}_a = D_a P_a \{\Phi\}_a$ Nodal accelerations for the a[th] mode

D_a is the design acceleration of the a[th] mode and is equal to the lesser of $V\omega_a$ or Ag as obtained from DDS 072-1 (See Section 3.5.2)

3.4.2 Dynamic Reduction Techniques

The number of dynamic degrees of freedoms used in DDAM mathematical models has increased dramatically over the years since DDAM was first introduced. As a consequence of this increase in model complexity, reliance on matrix reduction techniques has also increased. Matrix reduction techniques allow the use of a large number of static degrees of freedom while reducing the number of dynamic degrees of freedom to a fraction of the static.

There is an inherent risk in using dynamic reduction techniques as a means of simplifying complicated models. Reduction techniques attempt to convert extremely detailed models into smaller models for computational efficiency. However, these reduced models are difficult to review in detail and they may not satisfy all the requirements of Chapter 3. It is preferable to rely on engineering judgement rather than an automatic selection process available in various dynamic reduction techniques as a means of creating simplified structural models. Certain criteria must be met where dynamic reduction is used. Consider the following procedure as a minimum verification of the adequacy of any reduction technique considered within the DDAM:

Assume that the original dynamic system, with N degrees of freedom, has mass matrix [M] and stiffness matrix [K]. By any reduction method this system is reduced to a system with mass matrix [M*] and stiffness matrix [K*] with N* master degrees of freedom. This reduced dynamic system is then solved for:

NOM = Reduced number of modes

$\{\Phi^*\}$ = Mode shapes of reduced set

ω^* = Natural frequencies of the reduced set

Transform back to the original system and obtain each mode shape $\{\Phi\}$ in the original degrees of freedom.

Determine whether these mode shapes, obtained by the back transformation process, are orthogonal with respect to the original mass and stiffness matrices.

$$[\Phi]^T\,[M]\,[\Phi] \;=\; [\bar{M}]$$

$$[\Phi]^T\,[K]\,\;[\Phi] \;=\; [\bar{K}]$$

$$[\bar{K}]\,[\bar{M}]^{-1} \;=\; [\omega^2]$$

where $[\Phi]$ = A mode shape matrix with the number of columns equal to the number of degrees of freedom and the number of rows equal to the number of modes

$[\bar{M}]$ = Generalized mass matrix

$[\bar{K}]$ = Generalized stiffness matrix

$[\omega^2]$ = A diagonal matrix with the diagonal equal to the squared natural frequencies of the original system

As a check, $[\bar{K}]$ and $[\bar{M}]$ should be diagonal matrices and hence the mode shapes are orthogonal with respect to the mass and stiffness matrix. ω^* should be the same as ω and the modal masses should add up to the total modal effective weight of the system. For lumped parameter systems:

$$\sum_{a=1}^{NOM} \frac{\left(\sum_{i=1}^{N} \Phi_{ia} M_i\right)^2}{\sum_{i=1}^{N} \Phi_{ia}^2\, M_i} = \text{Total Modal Effective Weight}$$

At least three general approaches have been used effectively to reduce the number of dynamic degrees of freedom:

(1) Kinematic Condensation (Guyan Reduction)
(2) Generalized Dynamic Reduction (Rayleigh-Ritz)
(3) Component Mode Synthesis (Sub-structuring)

Kinematic Condensation is based on the assumption that inertia forces are associated with only certain selected degrees of freedom of the original idealization. The remaining degrees of freedom are not explicitly involved in the dynamic analysis and can be condensed from the dynamic matrix. In the Generalized Dynamic Reduction approach, the number of dynamic degrees of freedom are limited by assuming that the displacements of the structure are combined in selected patterns, the amplitudes of which become generalized coordinates of the dynamic analysis. Component Mode Synthesis reduces the problem by dividing the solution into a series of substructures, solving the reduced substructure and combining the substructure analyses into a single reduced analysis.

When considering the number of master degrees of freedom, the following should be used as guidance:

(a) The model should be kept as simple as possible.

(b) High frequency components should be considered as acting together.

(c) Low frequency critical components shall be represented as separate degrees of freedom.

(d) The number of master degrees of freedom selected should be at least two to three times the number of modes of interest.

(e) Include master degrees of freedom at locations having relatively large mass and/or rotary inertia.

(f) Master degrees of freedom should not be defined where the structure has an insignificant mass.

(g) Retain a uniform spatial distribution, such that the center of gravity of the master degrees of freedom closely represents that of the system modeled.

(h) Retain critical items as master degrees of freedom.

3.5 Evaluation Phase

The evaluation phase of DDAM is essentially one of determining the stresses and deflections in the equipment, structure and/or foundation and comparing them to specified failure criteria established by material and operational considerations. Having obtained the deflections of and forces on the masses of the mathematical model, the analyst may then proceed with the analysis of the equipment. The analysis at this point becomes a static analysis, i.e. within each mode the system is in equilibrium. Presented below are requirements for:

(1) Modal assessment
(2) Shock Design Values to apply
(3) The number of modes to use
(4) Combining stresses within each mode
(5) Summing stresses across the modes
(6) Combining operating and shock stresses
(7) Response assessment

3.5.1 Modal Assessment

The mathematical model used to define the equipment, system or structure is the fundamental tool by which satisfactory shock performance can be demonstrated by analysis. A modal analysis of the system generates dynamic response characteristics (frequencies and mode shapes). The results of this analysis should be examined for credibility before proceeding with subsequent steps in the design process. The results of the analysis should demonstrate that the basic requirements of DDAM are satisfied and that the model does not produce conditions that exceed the limitations of DDAM. The following are potential conditions wherein the requirements or limitations of DDAM may be exceeded:

(a) very low frequency systems (less than 5 Hz)
(b) closely spaced modes

The analyst should not continue with the analysis until the conditions which do not agree with the basic DDAM assumptions are resolved or specifically approved by the cognizant Navy acceptance authority.

One of the critical areas where the results of an analysis could exceed the limitations of the basic DDAM assumptions is the existence of closely spaced modes. Closely spaced modes are defined as two modes whose frequencies are within 10% of the common mean frequency. Closely spaced modes can become a problem when their modal effective masses are significant and are approximately of the same order of magnitude. Closely spaced modes will frequently occur in a dynamic analysis without resulting in any notable amplification of the component responses. These cases are generally associated with modes which have relatively low modal effective mass.

When closely spaced modes involve modes with large modal masses, they can produce significant responses which indicate a shock hazard to the equipment. Therefore, some preliminary assessment must be conducted to determine whether closely spaced modes that have been identified will have any significant effect on the design loading.

The following outline describes the basic approach for the treatment of closely spaced modes:

(1) Prepare a bar graph of modal effective mass versus modal frequency. This graph provides an overview of the system dynamic response and permits early identification of closely spaced modes.

(2) Identify closely spaced modes which are defined as modes which are separated by less than 10% of the common mean frequency. Potentially hazardous closely spaced modes are usually two or more modes close in frequency, each with significant modal mass of relatively significant magnitude. Selection criteria of Section 3.5.3 can be used to identify modes that are likely to be significant.

(3) Compare the mode shape (shape function times the participation factor) of the closely spaced modes suspected of being potentially damaging. The comparison should be conducted for each node point. An indication of a potentially hazardous closely spaced mode condition exists where the maximum response of similar magnitude and opposite sign occurs for the two closely spaced modes. This is indicative of a split modes phenomenon. Under these conditions it is concluded that either the model is incorrect or the design of a local component will result in an apparent resonance and should be detuned. Another indication of potentially hazardous closely spaced modes condition exists when the modal masses of each of the apparent closely spaced modes is contained in distinctly different sets of degrees of freedom. This is indicative of an uncombined mode phenomenon. Uncombined modes may occur for either of two reasons: each portion may have been modeled with independently fixed bases and are too lightly coupled, or, one of the portions may have been modeled so as to become a split mode. Under these conditions it is concluded that the fixed base may be inappropriately selected. An acceptable change would be to extend the boundries of the mathematical model so that it includes more of the supporting ship structure.

(4) Show the extent of detuning necessary to eliminate the split mode condition. Similarly, where uncombined modes exist, the analysis should show what quasi fixed base is needed to eliminate this condition.

(5) Determine if damaging effects of closely spaced modes cannot be eliminated by remodeling or redesigning (detuning). If this cannot be done, the analyst should request Navy approval of application of an alternate techniques such as the methods described in the remainder of this section. Section 3.5.7 discusses the ASM and the CSM techniques used to evaluate closely spaced modes. Sections 3.5.7 and 7.2.2.8 discuss the ASM analysis submittal and approval requirements with regard to supplementary information to be supplied in the corrective action report.

3.5.2 Shock Design Values to Apply

As noted in Chapter 2, the shock design values to apply when performing a DDAM analysis are contained in Design Data Sheet DDS 072-1 (Confidential). The shock design values are given in the form of frequency-dependent and modal weight-dependent equations of pseudo-velocity or acceleration.

The shock design values were derived from data recorded in full ship shock tests. The data were first converted into conventional response spectra and discrete points were extracted from the spectra at the known fixed-base natural frequencies of equipment (for which the modal masses had been calculated) mounted aboard the ships. In this way, a series of tests were used to generate the design shock spectra contained in DDS 072-1. At the fixed based natural frequencies, the various items of equipment tend to act as vibration absorbers and suppress to some degree the motions of the basic ship structure. It is these fixed-base natural frequencies which give rise to the major equipment and foundation responses to shock. Because the test shock spectra tend to show minima at these fixed-base frequencies rather than peaks, the phrase "spectrum dip effect" is often used to describe the derivation of the design shock spectra. For more information on the derivations of the design values, see the reports cited in Section 3.6.2.

Although the DDAM shock design values are to be applied in each of the three translational directions (vertical, athwartship, and fore/aft) separately, responses may be calculated in all three directions (multi-directional response analysis). For cases in which the equipment or foundation's principal axes do not coincide, even approximately, with the directions of shock design values defined in DDS 072-1, special combinations of the shock design values may be appropriate. See Appendix D for discussion of oblique shock design values.

While the DDS 072-1 shock design values have been derived from test data on steel hulls, the inputs are also considered to be the best available data for analyses of equipment on wood and glass reinforced plastic (GRP) hulls. However, special design criteria must be applied in these cases. While it is usually not considered necessary to check the stresses in basic ship structure for steel hulls, such calculations are necessary for wood and GRP hulls. Special attention must be paid to the strength of interface connections, such as bolted connections between steel foundations and non-metallic hull structure. The wood frames must be checked for continuity to ensure that local failure of the ship structure under the loads transmitted by the bolts will not occur.

3.5.3 Number of Modes to Use

The number of modes to be calculated prior to the selection process shall be sufficient to satisfy the modal weight requirement listed below and the additional modes likely to contribute to the localized high responses. See Figure 3-7 for an overview description of the mode selection process.

A cut-off frequency may be selected in the mode calculation phase of the analysis which is sufficiently high to guarantee the selection requirements are complete. This cut-off frequency is to be consistent with the frequency of the system, and the level of refinement of the mathematical model used to represent it. Nominally, 250 Hz may be taken as an upper bound on the frequencies of interest. Frequencies beyond this level are, for most equipment items aboard ship, of lesser importance in a shock environment in which the ship structure filters the input motions. Alternatively, a number of modes may be selected in the mode calculation phase of the analysis which is sufficiently high to guarantee the selection requirements are satisfied. Iterations may be required if the number of modes to be extracted is specified too low to guarantee compliance with the selection requirements.

The calculated modes shall be sorted by modal effective weight, in descending order, prior to the mode selection process. The number of modes considered shall be sufficient so that their total modal effective weights shall not be less than 80% of the total weight of the system.

In this sorting process it is useful to construct a graph of the modal effective weight versus frequency. The graph will provide an overview of the system modal responses and will provide early identification of the existence of closely spaced modes. See Section 3.5.7 for further discussion of closely spaced modes.

All calculated modes contributing a modal effective weight in excess of a minimum percent of the total weight of the system analyzed shall be included in the selection. The value of the minimum percent of the total weight of the system shall be the greater of one percent or twenty divided by the number of dynamic degrees of freedom (NDOF) in the model expressed as a percent. The value 20/NDOF, expressed as a percent of the total weight, is intended to exclude the least massive modes of small dynamic systems. However, for a two degree of freedom model both modes are to be considered regardless of this minimum percent of total weight criteria. Similarly, for a three degree of freedom model, at least two modes must be used. When a system consists of a series of repeated cells or modules, the minimum percent of total system weight criterion shall be based on the weight of a single cell or module, not the total weight of the system. This will reduce the chance of omitting a mode which is primarily responsible for the movement of a given cell.

All additional modes of systems with modal effective weights less than the minimum percent of the total weight of the system which are deemed likely to produce critical stresses within the model are to be included in the selection. Specifically, relatively light weight sub-components may derive a significant portion of their localized response to shock from a seemingly insignificant mode. Examples of such critical areas include antennas on yardarms,

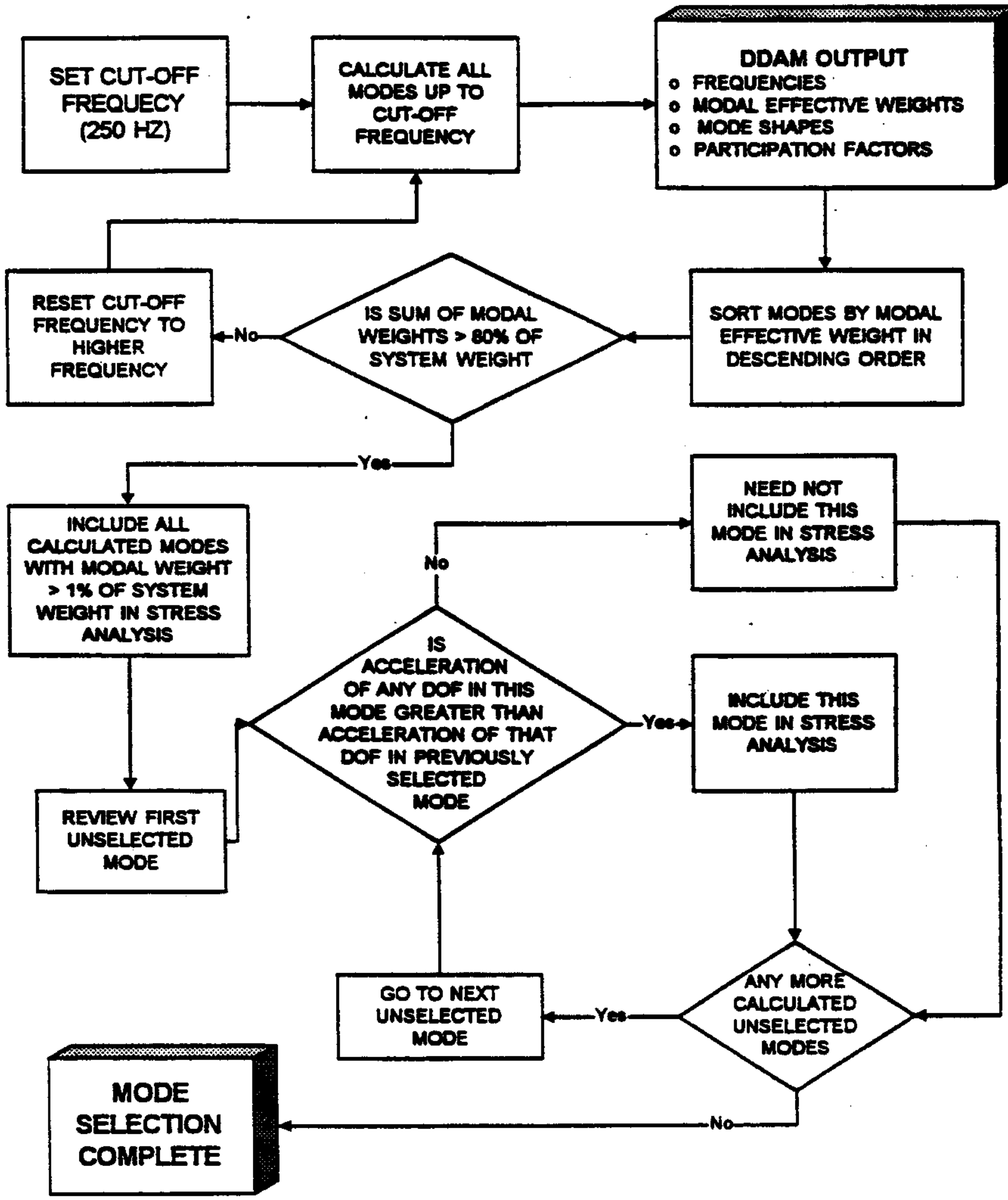


Figure 3-7 Mode Selection Process

control panels and gages. The additional modes to be included shall be those in which the nodal acceleration exceeds 10% of the maximum nodal acceleration (of a corresponding node) from any previously selected mode. Only the responses of those nodes representing critical areas or components need be considered. Alternative mode selection criteria may be used if approved by NAVSEA.

3.5.4 Calculating Stresses Within Each Mode

The following stress formula shall be used in each mode to determine the maximum modal stress. The NRL summation procedure outlined in Section 3.5.5 is then applied to obtain a total shock stress summed across the modes.

The Von Mises Theory of Failure is used to determine the modal stress σ_a in a structural member subjected to both normal and shear stresses. Modal stresses may require modification before summing across the modes. See Sections 6.3.2 and 6.4. The formulas are as follows:

For the uni-directional case the modal stress σ_a for the ath mode is given by

$$\sigma_a = \sqrt{\sigma_{norm}^2 + 3\,\tau_{shear}^2}$$

where σ_{norm} is the total normal stress produced by axial and bending loads and τ_{shear} is the total shear stress produced by either shearing or torsional loads.

For two-dimensional analysis;

$$\sigma_a = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\,\tau_{xy}^2}$$

where σ_x is the normal stress in the x direction of the element coordinate system, σ_y is the normal stress in the y direction of the element coordinate system and τ_{xy} is the shear stress.

For the three-dimensional case,

$$\sigma_a = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x \sigma_y - \sigma_y \sigma_z - \sigma_x \sigma_z + 3\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right)}$$

where σ_x, σ_y and σ_z are the normal stresses in the x, y and z directions and τ_{xy}, τ_{yz} and τ_{xz} are the shear stresses.

Consider an element of a mathematical model of a multi-degree of freedom system that has the following stresses in a particular mode of response:

$$\sigma_x = 20. \;\; \text{ksi} \; (137.9 \times 10^6 \; \text{N/m}^2)$$

$$\sigma_y = -15. \;\; \text{ksi} \; (-103.4 \times 10^6 \; \text{N/m}^2)$$

$$\tau_{xy} = 10.4 \; \text{ksi} \; (71.7 \times 10^6 \; \text{N/m}^2)$$

The combined shock stress for this element is:

$$\begin{aligned} \sigma_a &= \sqrt{20^2 - (-15)(20) + (-15)^2 + 3(10.4)^2} \\ &= 35.3 \; \text{ksi} \end{aligned}$$

$$\left(\begin{aligned} \sigma_a &= 10^6 \sqrt{137.9^2 - (-103.4)(137.9) + (-103.4)^2 + 3(71.7)^2} \\ &= 243.7 \; x \; 10^6 \; \text{N/m}^2 \end{aligned} \right)$$

3.5.5 Summing of Stresses and Deflections Across the Modes By NRL Method

The following NRL Sum formula developed by the Naval Research Laboratory shall be used when calculating the total shock stress or total relative deflection at point i:

$$R_i = |R_{ia}| + \sqrt{\left(\sum_{b=1}^{N} R_{ib}^2\right) - R_{ia}^2}$$

where R_{ia} is the value of the largest modal stress or deflection (for all the modes selected) at the point i and R_{ib} represents each member of the complete set of stress or deflection contributions at the same point under consideration. Unless the stresses or deflections under consideration are directly proportional to the forces, this formula is never to be used to combine modal forces on a mass(es) where these resultant forces are then used to calculate stresses or deflections.

Example: Suppose the following modal stresses were calculated for a point on an element of a two dimensional model (Note: The combined stress in each mode is determined as described in paragraph 3.5.4.):

Mode Number a	σ_x ksi (x10^6 N/m^2)	σ_y ksi (x10^6 N/m^2)	τ_{xy} ksi (x10^6 N/m^2)	σ_a ksi (x10^6 N/m^2)
1	10. (68.95)	3.59 (24.75)	1.32 (9.1)	9.1 (62.74)
2	20. (137.89)	-15.0 (-103.42)	10.4 (71.7)	35.3 (243.7)
3	3.0 (20.38)	2.0 (13.79)	1.63 (11.24)	3.9 (26.89)
4	1.2 (8.27)	- 0.2 (-1.38)	2.03 (13.99)	3.8 (26.2)
5	8.2 (56.54)	1.0 (6.89)	1.92 (13.24)	8.4 (57.92)

Then R_{ia} = 35.3 ksi (243.7 x 10^6 N/m^2) and the formula is applied as follows:

$$R_i = |35.3| + \sqrt{9.1^2 + 35.3^2 + 3.9^2 + 3.8^2 + 8.4^2 - 35.3^2}$$
$$= 35.3 + 13.5 = 48.8 \text{ ksi}$$

$$R_i = |243.7(10^6)| + 10^6\sqrt{62.7^2 + 243.7^2 + 26.89^2 + 26.2^2 + 57.9^2 - 243.7^2}$$
$$= 10^6(243.7 + 93.27) = 336.97 x 10^6 \text{ N/m}^2$$

Therefore the value of the total shock stress summed across the five modes is σ_{shock} = 48.8 ksi (336.97 x 10^6 N/m^2).

3.5.6 Combining Operating and Shock Stresses (Total Stresses)

In order to compare the stresses produced by shock loading to a specified failure criterion, the analyst shall combine the Von Mises stresses derived by dynamic analysis with the continuous Von Mises operating stresses present in the area under consideration. Continuous operating stresses are defined as those stresses, present in the system due to the system's operating characteristics (e.g. rotating elements, steam pressure, etc), which will not be relieved by minor yielding. An example of a continuous operating stress is that which is produced by the torsional effect of a rotating element. Non-continuous operating stresses, such as thermal stresses, shall be ignored. Gravity loads need not be considered. Bolt pre-load tensile stress shall not be added to shock stress.

For dynamic analysis purposes, the total stress shall be the combination of the shock stress summed across the modes by the NRL method described in Section 3.5.5 and the continuous operating stress. The total stress at a point shall be calculated by the following formula:

$$\sigma_{total} = |\sigma_{shock}| + |\sigma_{oper}|$$

The total stress, σ_{total}, is compared to the allowable stress of the material to determine whether failure will occur. Allowable shock stress criteria are contained in Chapter 6 of this report.

As an example of the method used to combine operating stresses at right angles to each other, assume a 20,000 HP (14.91 MW) shaft in an equipment is rotating at 2,000 RPM (209.3 rad/s) (continuous operating load). This rotation yields a continuous operating torque of:

$$\begin{aligned} T &= \frac{33,000\ (HP)\ 12}{2\pi(RPM)} \\ &= \frac{33,000\ (20,000)\ 12}{2(3.14)(2,000)} \\ &= 630.57 \text{ inch -Kips} \end{aligned}$$

The maximum torsional stress on the shaft surface is

$$\tau_{tor} = \frac{Td}{2J}$$

Assume the shaft diameter d = 7 inches (177.8 mm) and the shaft cross section polar moment of inertia J = 236 in^4 ($9.82 \times 10^{-5}\ m^4$). Then

$$\tau_{tor} = \frac{630.57(7)}{2(236)} = 9.35 \text{ ksi} \quad (64.4 \times 10^6 \text{ MPa})$$

This operating torsional stress is now added to the operating axial stress at the point of maximum stress. Assume the axial operating stress to be 34,700 psi (239.25 x 10^6 MPa) in compression. The total operating stress is:

$$\sigma_{oper} = \sqrt{\sigma_{axial}^2 + 3\tau_{tor}^2}$$

Therefore the total operating stress is:

$$\sigma_{oper} = \sqrt{34.7^2 + 3(9.34)^2} = 38.3 \text{ ksi}$$

$$\left(\sigma_{oper} = 10^6 \sqrt{239.25^2 + 3(64.4)^2} = 264 \ x \ 10^6 \text{ MPa} \right)$$

The total stress is a combination of the Von Mises shock stress and the Von Mises operational stress. With the total operating stress of 38.3 ksi (264 x 10^6 MPa) and the result shown previously (Section 3.5.5) for the shock stress, the total stress becomes;

$$\sigma_{total} = 48.8 + 38.3 = 87.1 \text{ ksi}$$

$$(\sigma_{total} = (336.97 + 264.) \ x \ 10^6 = 600.97 \ x \ 10^6 \text{ MPa})$$

3.5.7 Response Assessment

The basic method of determining the acceptability of a design is by DDAM using the NRL method of combining the responses over the modes. Where the NRL method produces responses that are within the allowable limits, the requirements of this section do not apply. Where the NRL method produces results significantly greater than the allowable failure criteria, the analyst shall conduct further analysis of the equipment to determine if the responses can be reduced to levels within the allowable limits. In these cases, the following three options are available to the analyst.

a. Redesign or Remodel - If the high responses are not caused by closely spaced modes, the item shall be redesigned to reduce the responses to acceptable limits. If the overstress results from a closely spaced modes condition, the analysis should show the extent of detuning necessary to eliminate the overstress condition. If damaging effects of closely spaced modes cannot be eliminated by remodeling or redesigning (detuning), the analyst should request NAVSEA approval of application of an alternate assessment in accordance with the Closely Spaced Modes Method (CSM) or by using the Algebraic Summation Method (ASM). Both methods consider the effect of modal phasing. These methods can only be presented as a supplemental calculation to the NRL summation method of Section 3.5.5, and should only be used as a cost effective alternative to redesigning the foundation or equipment.

b. The Closely Spaced Modes Method - The CSM is a method for combining two or more closely spaced modes into one mode. This method is restricted to mode pairs which have frequencies within 10 percent of the common mean frequency, and have amplitudes which are opposite in sign. The contributions of these closely spaced modes are then included in the NRL sum as a single effective mode. The method can be easily applied by using Figure 3-10 to account for the combined effect of two modes. Refer to Section 3.5.7.1 for the details of CSM and for an example calculation.

c. The Algebraic Summation Method - The ASM is an alternate method of combining modal responses that preserves the phase relationships among the modes. The set of modes required to be used in the ASM calculation is the same as those selected in accordance with Section 3.5.3 for the NRL summation. Refer to Section 3.5.7.2 for the details of ASM and for an example calculation.

Application of CSM or ASM will produce more credible results if closely spaced modes are the primary cause of the high shock responses. If closely spaced mode phenomena are not the cause of the high calculated response, then application of CSM or ASM will not have a significant effect on the results. The phenomenon known as closely spaced modes is an artificial amplification of the response of a system. It occurs when the phase relationship between individual modes with very close natural frequencies is ignored in the NRL method of summing modal shock responses.

When the responses calculated by the ASM or CSM are significantly less than the responses calculated by the NRL method, the ASM or CSM responses provide a technical basis for determining the acceptability of a design. However, ASM or CSM should only be used in cases when the NRL method cannot produce a cost effective design.

NAVSEA will determine the extent to which the results of the ASM or CSM supplementary analysis will influence the final decision to accept the lower stress values as the shock response levels in the item. NAVSEA will decide whether or not to modify the structure to withstand the loads associated with the NRL summation results. This decision will depend on, among other things, the criticality of the item, the reliability of the mathematical model and the relative impact of implementing design modifications.

Sections 3.5.7.1 and 3.5.7.2 describe the CSM and ASM, respectively, and provide example calculations. It must be pointed out that these examples represent the peak response at only one location and serve only to illustrate a sample calculation procedure for CSM and ASM. In practice, the calculations must be performed at all points that are being assessed for closely spaced modes. When performing calculations for beam elements, multiple points of the cross-section must be checked to ensure that the most critical location is evaluated.

3.5.7.1 Closely Spaced Modes Method

The analysis method described below provides a method for combining responses from two closely spaced modes. The method does not eliminate the need to calculate a response which includes all significant participating modes, but it does provide a method for calculating the combined effect of closely spaced modes. Once this combination is determined, it may be used in the NRL sum of responses as a single effective mode.

In a DDAM shock analysis, the normal practice is to combine the responses from individual modes using the NRL sum. This practice does not explicitly treat either the relative phasing of the individual modes or the effects of damping.

For finite element models which have significant responses in modes which are close in frequency and for which the modal responses are nearly equal in amplitude and are opposite in sign (180 degrees out of phase), damping becomes very important in determining the combined response. Since they are initially out of phase, these modal responses tend to cancel each other during early portions of the response. As time passes, the frequency difference causes the responses to shift in phase so that the magnitudes eventually add. For close frequencies, this time will be large enough so that the combined amplitude can be significantly reduced by the effects of damping. See Figures 3-8 and 3-9 for examples of the superposition of two modes with and without damping.

The associated amplitude reductions are most significant where the responses of the closely spaced pair of modes are about equal in amplitude. Section 3.5.7.1.1 provides an explicit, closed-form method for determining the reduction that can be achieved, as well as alternate numerical and graphical methods for determining the amplitude reduction.

The treatment given below and associated derivations assume that the phasing is that associated with a velocity step input. The justification for the method, however, is based on comparison of analysis to full scale ship shock test data. Therefore, no restriction relative to step velocity is included in the method. However, the method is limited to closely spaced modes, which are defined here as having frequencies within 10 percent of the common mean frequency of the modes considered.

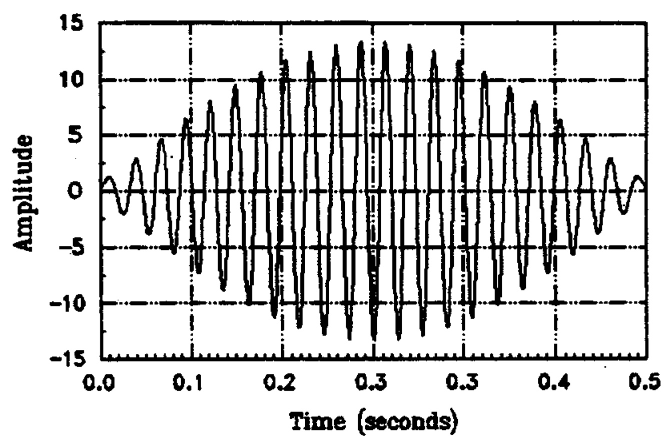


Figure 3-8 Combined Response of Two Undamped, Closely Spaced Modes

Damping has been set at 2 percent of critical as a lower bound estimate of the damping normally associated with the shock response of welded structures.

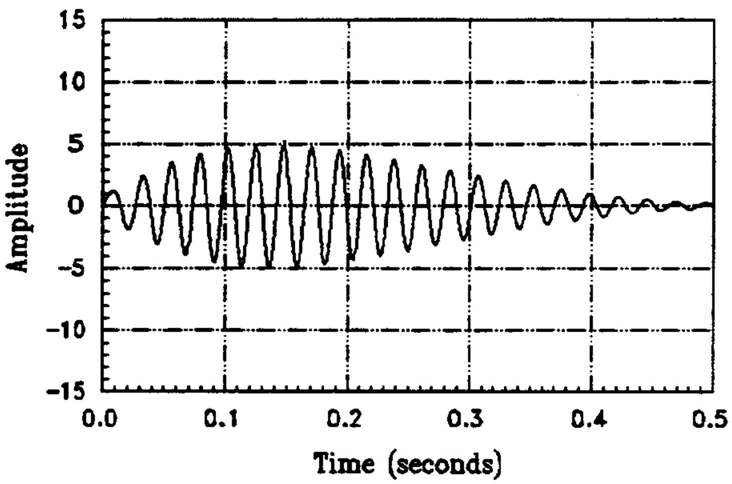


Figure 3-9 Combined Response of Two Damped (2%), Closely Spaced Modes

3.5.7.1.1 Analysis Methods - This section presents the closed form, numerical and graphical procedures for evaluating closely spaced modes using CSM. The closed form may be slightly more conservative because it is based on determining the peak of the envelope rather than the peak of the superposed values. The numerical procedure, while more tedious, provides an alternative method which might be (in the future) extended to a cluster of several modes. The graphical approach is the simplest to apply, but the graph in Figure 3-10 is strictly limited to 2 percent damping.

Damping is not associated with a particular mode because application of the procedure to date has included only cases with uniform damping.

All three procedures require an amplitude correction from the DDAM-determined modal values in order to account for the effect of damping during the first quarter cycle. Omission of this correction will result in lower modal amplitudes (about a 3 percent error for 2 percent damping).

Modal Amplitude Correction

The DDAM response spectra do not explicitly include damping. However, the values for relative amplitude or acceleration implicitly include any damping forces which act during the time from shock arrival to the maximum component response. For a step velocity model of the input, this would imply damping had been acting for one quarter of a cycle when the peak acceleration or displacement is reached. C_j accounts for damping during this time.

$$C_j = A_j \, e^{\xi \frac{\pi}{2}} \tag{1}$$

where:

A_j = mode algebraic amplitude from DDAM for the j^{th} mode.

C_j = mode algebraic amplitude for the j^{th} mode with quarter cycle correction.

ξ = damping as fraction of critical = 0.02.

The effect of the correction is not large. For ξ =2 percent, C_j/A_j=1.032. For larger damping values, the correction would be larger.

3.5.7.1.1.a Closed Form Treatment

The envelope of the sum of two decaying sinusoids (modes j and k) may be written as a function of the algebraic amplitude and damping for each sinusoid.

$$E(t) = e^{-at} \sqrt{(C_j + C_k)^2 - 4 C_j C_k \sin^2(dt)} \tag{2}$$

where:

E(t) = combined effect of two modes

C_j, C_k = mode algebraic amplitudes with quarter cycle correction.

a = $\xi \, \Omega_m$

d = $0.5 \sqrt{1-\xi^2} \left| \Omega_j - \Omega_k \right|$

Ω_m = average undamped natural frequency in radians per second.

ξ = damping in fraction of critical damping

t = time in seconds

The times at which this function is an extreme (a minimum or a maximum) are t = 0 and the times given by

$$t_n = \frac{\sin^{-1}\left[\dfrac{-a\left(C_j^2 + C_k^2\right)}{2C_j C_k \sqrt{a^2 + d^2}}\right] - \theta}{2d} \tag{3}$$

where:

$$\theta = \tan^{-1}(a/d) \qquad \text{and} \qquad 0 \le \theta \le \pi/2$$

Equation 3 has multiple solutions only if

$$S = \frac{-a\left(C_j^2 + C_k^2\right)}{2C_j C_k \sqrt{a^2 + d^2}} \le 1 \tag{4}$$

If S is greater than one, CSM cannot be used to reduce the NRL sum. If S is equal to one, there is one solution, t_1, to Equation 3 and E_{max} is the greater of E(0) and E(t_1). If S is less than one, E(t) must be calculated at t=0 and at the first two positive values of t_n from Equation 3. E_{max} is then the greatest of the three values.

Once E_{max} is determined, the modified NRL (or CSM) sum may be written:

$$\sum_{CSM} = \max|A_m, E_{\max}|_{m \neq j,k} + \sqrt{E_{\max}^2 + \sum_{m \neq j,k} A_m^2 - \left(\max|A_m, E_{\max}|_{m \neq j,k}\right)^2} \tag{5}$$

where the index m ranges from 1 to the highest mode considered, excluding the closely spaced modes, and A_m is the unsigned amplitude.

3.5.7.1.1.b Numerical Treatment

The individual modal contribution may also be combined using the numerical procedure described below. If the corrected mode algebraic amplitude for an individual component is C_j at a natural frequency, Ω_j, in radians per second with damping, ξ, as a fraction of the critical damping, the amplitude at any time may be written

$$D_j(t) = C_j\, e^{-\xi \Omega_j t} \sin\left(\sqrt{1-\xi^2}\, \Omega_j\, t\right) \qquad (6)$$

Thus, for two modes;

$$D(t) = D_j(t) + D_k(t) \qquad (7)$$

may be calculated to identify the maximum amplitude, $E_{max} = D(t)_{max}$.

Equation 5 may then be used to determine the CSM sum.

The accuracy of the above procedure is dependent upon the time step used in the numerical procedure. If the time step is too large, an unconservative sampling error will result. The time step shall be, as a minimum, 1/32 of the shorter period of the two frequencies to keep the error in any mode due to time resolution below 2 percent.

As a minimum, D(t) should be calculated for one half the "beat cycle" of the combined frequencies. That is, for

$$0 < t < \frac{0.5}{|f_k - f_j|}$$

3.5.7.1.1.c Graphical Treatment

As another alternate to evaluation of the equations of Section 3.5.7.1.1.a, Figure 3-10 provides a graphical representation for the combined effect of two modes. This figure allows determination of the combined effect of two modes without direct calculation. The ratio of the envelope magnitude to the sum of the unsigned magnitudes of the original modes may be read from the figure given a magnitude ratio (smaller divided by the larger) and a non-dimensional frequency difference $2\,(f_k - f_j)\,/\,(f_k + f_j)$.

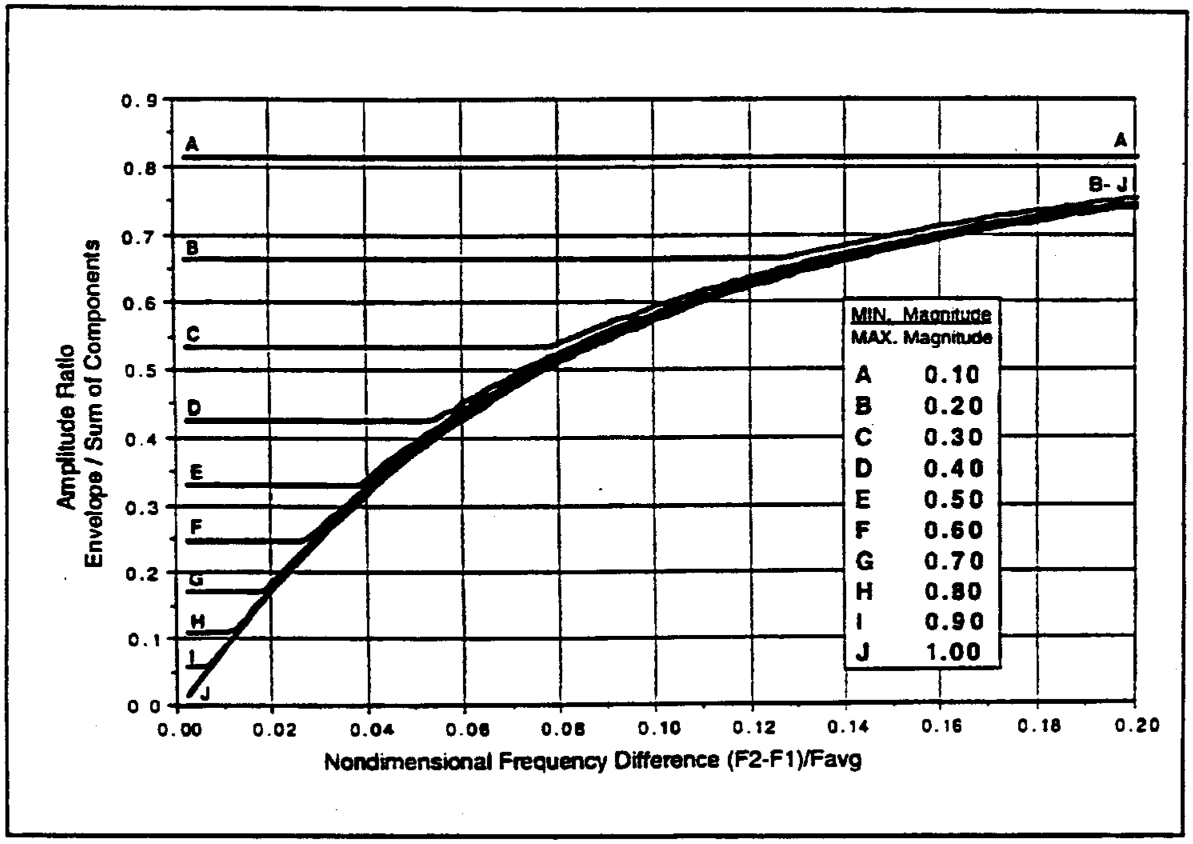


Figure 3.10 Envelope/Sum of Components for use with Closely Spaced Modes Sum (This figure was generated for a damping of 2%)

3.5.7.1.2 Example Problem - Assume that a DDAM analysis has resulted in the following fixed base frequencies and modal responses of some point, P, on the structure or equipment being analyzed.

Mode	Frequency (Hz)	Acceleration of P (g's)
1	27 Hz	5.0 g's
2	43	7.0
3	45	-6.0
4	87	-3.0
5	91	-2.0

$$\sum_{NRL} = 7.0 + \sqrt{5.0^2 + 6.0^2 + 3.0^2 + 2.0^2} = 15.6 \text{ g's}$$

Modes 2 and 3 are close in frequency and the acceleration responses at P have the opposite sign. The relative difference in frequency for modes 2 and 3 is calculated

$$\frac{2(f_3 - f_2)}{f_3 + f_2} = 0.045 = 4.5\ \%$$

Since the difference is less than 10 percent, one may proceed.

Note that the frequencies of modes 4 and 5 are also within 10 percent of their common mean frequency of 89 Hz. A reduction can not be achieved by combining these modes, however, since the modal accelerations have the same sign.

The modal amplitude correction is then applied to both modes 2 and 3. From Equation 1.

$$C_j = A_j\ e^{\pi \xi / 2}$$

$$C_2 = 7.0\ e^{\pi\, 0.02 / 2} = 7.22 \text{ g's}$$

$$C_3 = -6.0\ e^{\pi\, 0.02 / 2} = -6.19 \text{ g's}$$

Approach 1. Closed Form Treatment (Example Problem)

For an analytical solution, the equations from Section 3.5.7.1.1.a may be evaluated directly. First the preliminary calculations:

$$a = \xi\, \Omega_m = 0.02\ (2\pi)\ 44 = 5.5292 \text{ sec}^{-1}$$

$$d = \sqrt{1-\xi^2}\ \frac{(\Omega_3 - \Omega_2)}{2} = \sqrt{1-\xi^2}\ \pi\,(f_3 - f_2)$$

$$= \sqrt{1-(0.02)^2}\ \pi\,(45 - 43) = 6.2819 \text{ sec}^{-1}$$

Use Equation 4 to check that a solution exists:

$$S = \frac{-a\left(C_2^2 + C_3^2\right)}{2 C_2 C_3 \sqrt{a^2 + d^2}}$$

$$= \frac{-5.5292 \ (7.22^2 + (-6.19)^2)}{2\,(7.22)\,(-6.19)\sqrt{5.5292^2 + 6.2819^2}} = 0.6685 \quad \leq 1$$

Because S is less than one, multiple solutions to Equation 3 exist. Only the first two solutions are of interest as they are potential absolute maximums of the envelope.

Equation 3 gives the times at which the envelope of the sum of the damped sinusoids is at a relative extreme (minimum or maximum). The first two solutions are given by the following expressions:

$$t_1 = \frac{\sin^{-1}(S) - \tan^{-1}\left(\frac{a}{d}\right)}{2d}$$

$$= \frac{\sin^{-1}(0.6685) - \tan^{-1}\left(\frac{5.5292}{6.2819}\right)}{2\,(6.2819)} = 0.00083 \quad \text{sec}$$

and

$$t_2 = \frac{\pi - \sin^{-1}(S) - \tan^{-1}\left(\frac{a}{d}\right)}{2d}$$

$$= \frac{\pi - \sin^{-1}(0.6685) - \tan^{-1}\left(\frac{5.5292}{6.2819}\right)}{2\,(6.2819)} = 0.1343 \quad \text{sec}$$

The inverse trigonometric functions in the above expressions were evaluated to yield results in radians. Substituting into Equation 2 with $t = t_2$ gives

$$E(t_2) = e^{-at_2}\sqrt{(C_2 + C_3)^2 - 4C_2C_3\sin^2(dt_2)}$$

$$= e^{-5.5292\,(0.1343)}\sqrt{(C_2 + C_3)^2 - 4C_2C_3\sin^2 dt}$$

$$= 0.4759\,\sqrt{(7.22 + (-6.19))^2 - 4(7.22)(-6.19)\sin^2(6.2819\,(0.1343))}$$

$$= 4.8\ g\text{'s}$$

The height of the envelope at the other times, t = 0 and t = t1, must also be calculated. The results of those calculations are

$$E(0) = 1.0\ g\text{'s and}$$
$$E(t_1) = 1.0\ g\text{'s}$$

Therefore $E_{max} = E(t_2) = 4.8$. The CSM sum may now be calculated from the following modal contributions:

Mode	Acceleration
1	5.0 g's
2 & 3	4.8
4	-3.0
5	-2.0

$$\sum_{CSM} = 5.0 + \sqrt{4.8^2 + 3.0^2 + 2.0^2} = 11.0\ g\text{'s}$$

Comparing the closely spaced modes sum with the NRL sum for point P in this example, a reduction of (15.6-11.0)/15.6 or 29 percent is obtained. This is slightly more reduction than the graphical solution.

Approach 2. Numerical Treatment (Example Problem)

This treatment (described in Section 3.5.7.1.1.b) requires calculation of Equation 6 at many times for each mode. The time step must be less than 1/32 of the shorter period.

$$\left(\frac{1}{32}\right)\left(\frac{1}{45}\right) = 0.0006944 \quad \text{seconds}$$

For convenience, choose Δt = 0.000667 seconds. The total time considered must be for

$$0 < t < \frac{0.5}{f_3 - f_2} = \frac{0.5}{45 - 43} = 0.25 \ \text{seconds}$$

Thus 0.25/0.000667 or 375 solutions of Equation 6 are required for each mode. This obviously requires a computer even for this simple example.

For modes 2 and 3 of the sample, Equation 6 becomes

$$D_2(t) = 7.22\, e^{-5.4025\, t} \sin\,(270.123\, t)$$

$$D_3(t) = -6.19\, e^{-5.6537\, t} \sin\,(282.687\, t)$$

$$D(t) = D_2(t) + D_3(t)$$

The calculation is not reproduced here. Figure 3-9 shows a typical plot of D(t) as a function of time. The maximum value determined at 136 msec is:

$$D\,(t)_{max} = |\,D(0.136068)\,| = 4.8 \text{ g's}$$

The CSM sum may now be calculated from the "modal contributions"

Mode	Acceleration
1	5.0 g's
2 & 3	4.8
4	-3.0
5	-2.0

$$\sum_{CSM} = 5.0 + \sqrt{4.8^2 + (-3.0)^2 + (-2.0)^2} = 11.0 \text{ g's}$$

Comparing this closely spaced modes sum with the NRL sum of 15.6, a reduction of (15.6-11.0)/15.6 or 29 percent is achieved.

Approach 3. Graphical Treatment (Example Problem)

The nondimensional frequency ratio calculated above is 0.045. The amplitude ratio is 6.19/7.22 = 0.857. Examination of Figure 3-10 gives

$$E/(\text{sum of magnitudes}) = 0.37$$

or

$$E = 0.37\ (6.19 + 7.22) = 5.0 \text{ g's}$$

The closely spaced modes sum is then calculated from the following contributions:

Mode	Acceleration
1	5.0 g's
2 & 3	5.0
4	-3.0
5	-2.0

$$\sum_{CSM} = 5.0 + \sqrt{5.0^2 + (-3.0)^2 + (-2.0)^2} = 11.2 \text{ G's}$$

Comparing the closely spaced modes sum with the NRL sum for this example shows a reduction of (15.6-11.2)/15.6 = 28%.

3.5.7.2 The Algebraic Summation Method

ASM uses repetitive calculations that are not practical for manual calculation but can be easily programmed for any computer. ASM can be applied to any response characteristic, for example stress, member force, acceleration, velocity, displacement or relative displacement. As an example, the ASM is applied to a beam element from a mathematical model in the following manner:

Step 1. A set of discrete times at which to calculate the stress time history is selected. The calculations should be made over a time interval beginning at time zero and continuing until the lowest natural frequency mode of the summation (first mode) has been damped by 50% or until the envelope of any closely spaced pairs reaches a maximum, whichever is greater. The fraction of critical damping should be 2%. The discrete times should be evenly distributed over the interval at a spacing of one tenth of the period of the highest mode in the summation. Larger time steps are not allowed.

$$e^{-2\pi\xi f_1 T_{max}} = 0.50$$

$$T_{max} = \frac{\ln(2)}{2\pi\xi f_1}$$

$$= \frac{5.516}{f_1}$$

and

$$t_{inc} = 1000 \text{ msec/sec} \times 1/10 \times 1/f_n$$
$$= 100/f_n$$

where

T_{max}	=	duration of time interval. in seconds
f_1	=	natural frequency of the first mode, in Hz
f_n	=	natural frequency of highest mode in summation
t_{inc}	=	time step increment, in milliseconds

Step 2. A set of points of interest on the periphery of the cross-section of the beam is selected. These are the points of possible maximum stress at which the NRL stresses were determined. It should be noted that the maximum ASM stress may not occur at the same point on the cross-section as did the maximum NRL stress. Therefore, all potential locations on the cross-section must be evaluated. For each of the points of interest steps 3 through 6 are performed:

Step 3. At each discrete time the equivalent static force vector and/or moment vector in each mode at the end of each beam element under consideration is multiplied by the damping factor and the wave amplitude of the corresponding mode to give the ASM modal force at time (t). The wave amplitude of each mode at time t is equal to the sine of the product of the natural frequency (in radians/sec) of the mode and the time (in seconds).

$$M_c^t = M_c e^{-2\pi\xi f t} \sin\left(2\pi f \sqrt{1-\xi^2}\ t\right)$$

where:

M_c^t	=	member force at time t for a given mode
M_c	=	maximum member force for a given mode
$\exp(-2\pi\xi f t)$	=	damping factor
$\sin(2\pi f (1-\xi^2)^{1/2} t)$	=	wave amplitude
c	=	subscript which indicates the plane in which the member force acts
f	=	frequency of the mode
t	=	the discrete time

Step 4. At each discrete time, an algebraic (vector) summation of the ASM modal forces is performed over all the modes (n) considered to be acting at each point of interest.

$$M_c^T = \sum_n M_c^t$$

Step 5. The sum of the forces at each discrete point and time is used to calculate the resultant normal and shear stresses acting at the point by the conventional methods of strength of materials.

Step 6. At each discrete point and time the Von Mises stress is calculated from the resultant normal and shear stresses. For each point of interest the maximum combined stress is the maximum response calculated at all of the discrete times at that point. The ASM stress for the beam element is the greatest Von Mises stress of all the points at a cross section of the element.

When the calculated ASM stress is less than the NRL stress, it may be compared to allowable values given in Chapter 6 to determine the adequacy of a design for shock. If the ASM combined stress for any member exceeds the allowable values, the design should be modified to eliminate the over-stress determined by the NRL method. If the ASM combined stress is less than the allowable value, the design may be accepted by the Navy as adequate for shock.

In the event that parameters other than stresses are used to determine the shock adequacy of a design, the above calculation procedure may still be applied. Likely alternatives to the stresses are forces, relative displacements and accelerations, etc. For modal forces, the above procedure should be amended by omitting the stress calculations (steps 5 and 6) and substituting the vector quantity of the desired response characteristic (modal forces) in step 3 above. The vector sum determined in step 4 will be the value of the response characteristic time history at the particular point and time.

The ASM value of the response characteristic would be the greatest magnitude achieved by the response-time history during the time interval considered. When stresses are used to determine acceptability, the algebraic sum of the forces (and moments) is used to determine the stresses rather than calculating a stress contribution for each mode and summing them as is done in the NRL method.

As an example of DDAM-ASM, consider a hypothetical beam element with an arbitrary cross-section in bending and shear only (See Figure 3-11). Suppose the mathematical model contains the following cross-sectional properties in some consistent system of units (the subscripts 'c' and 'd' refer to the two transverse directions about which the member bends):

Sectional modulus for bending in two directions:
$Z_c = 1.0 \qquad Z_d = 2.0$

Shear areas for transverse shear in two directions:
$A_c = 0.1 \qquad A_d = 0.2$

Assume an allowable stress of 100 (in consistent units) and assume that DDAM has resulted in the following modal forces and frequencies:

MODE	BENDING MOMENTS		SHEAR FORCES		FREQ
	PLANE c M_c	PLANE d M_d	PLANE c V_c	PLANE d V_d	Hz
1	10	-20	3	2	30
2	-12	18	-4	-2	31
3	5	5	1	1	45

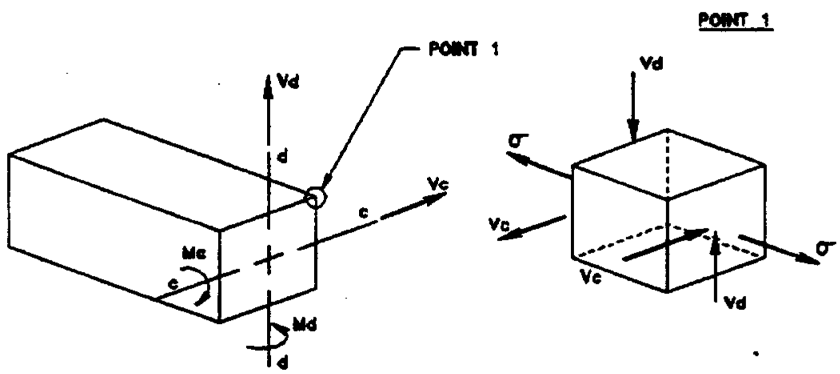


Figure 3-11. Bending and Shear Forces on Beam Element

The NRL method of Section 3.5.5 and the method of combining stresses of Section 3.5.4 would result in the following stresses:

Mode 1:

$$\sigma_{bend} = M_c / Z_c + M_d / Z_d$$

$$\sigma_{bend} = \frac{10}{1} + \left(-\frac{20}{2}\right) = 0$$

$$\tau_c = V_c / A_c = 3 \ / \ .1 \ = 30$$

$$\tau_d = V_d / A_d = 2 \ / \ .2 \ = 10$$

$$\sigma = \sqrt{\sigma_{bend}^2 + 3\left(\tau_c^2 + \tau_d^2\right)}$$

$$\sigma = \sqrt{0 + 3\left(30^2 + 10^2\right)} = \sqrt{3000} = 54.8 \text{ (consistent units assumed)}$$

Mode 2:

$$\sigma_{bend} = -12/1 + 18/2 = -3$$

$$\tau_c = -4/.1 = -40$$

$$\tau_d = -2/.2 = -10$$

$$\sigma = \sqrt{3^2 + 3(40^2 + 10^2)} = 71.5$$

Mode 3:

$$\sigma_{bend} = 5/1 + 5/2 = 7.5$$

$$\tau_c = 1/.1 = 10$$

$$\tau_d = 1/.2 = 5$$

$$\sigma = \sqrt{7.5^2 + 3(10^2 + 5^2)} = 20.8$$

The NRL sum of the Von Mises stresses is

$$\sum_{NRL} = 71.5 + \sqrt{54.8^2 + 20.8^2}$$

$$= 71.5 + 58.6$$

$$= 130.1 \quad > \quad 100 \quad \text{(greater than the assumed allowable)}$$

Note: This NRL summed stress is for only one point on the member, i.e. "point 1" shown on Figure 3-10. By the usual methods of strengths of materials, the same calculations would be repeated for all points of interest on the periphery of the cross-section.

From the frequencies above (shown in the previous table) it is seen that the first two modes are closely spaced and the NRL summed stress may be too conservative. Since the calculated NRL result exceeds the allowable, we will examine the ASM results as a basis for further technical evaluation.

Step 1:

A. Find the total time interval from the lowest frequency:

$$T_{max} = 5.516 / 30 \text{ Hz} = 0.1848 \text{ sec}$$

B. Find the time step spacing from the highest frequency:

$$T_{inc} = 100 / 45 = 2.2 \text{ msec.}$$

Step 2: For this example procedure only one point, the same one considered in NRL summation above, will be used.

Step 3: The following calculations (steps 3 through 6) would be repeated for each of the 84 discrete times in the set { 2.2, 4.4, 6.6, ... 184.8 }. In this example calculations for all times were conducted, but only the details for $t = 116.6$ milliseconds, which was the worst case, are shown here.

A. Find the product of the damping factor and the wave amplitudes in each mode at the discrete time:

$$\sqrt{1-\xi^2} = \sqrt{1-0.02^2} = 0.99980$$

$$e^{-2\pi\xi f t}\sin\left(2\pi\sqrt{1-\xi^2}\,f t\right) = e^{-2\pi(0.02)f(0.1166)}\sin\left[2\pi(0.9998)f(0.1166)\right] = e^{-0.01465 f}\sin(0.73247\,f)$$

$$e^{-0.01465(30)}\sin[0.73247(30)] = 0.6444(0.01705) = 0.0110$$

$$e^{-0.01465(31)}\sin[0.73247(31)] = 0.6340(-0.65594) = -0.4159$$

$$e^{-0.01465(45)}\sin[0.73247(45)] = 0.5172(0.99967) = 0.5170$$

Note: The frequency, f, is in Hertz, the time (t) is in seconds so that the argument of the sine function is in radians. The products of the damping factor and the wave amplitude are dimensionless.

B. Multiply the modal member force components by the corresponding wave amplitude for that mode at the selected time (the superscript 't' is used to denote "at time t"). For example, the bending moment in plane c in mode 1 is calculated as follows:

1. $$M_c^t = M_c\, e^{-2\pi\xi f t}\sin\left(2\pi f\sqrt{1-\xi^2}\; t\right) = (10)(0.0110) = 0.11$$

2. Repeating the calculation for each force component in each mode gives:

MODE	BENDING MOMENTS		SHEAR FORCES	
	PLANE c	PLANE d	PLANE c	PLANE d
	M_c^t	M_d^t	V_c^t	V_d^t
1	0.110	-0.220	0.033	0.022
2	4.991	-7.486	1.664	0.832
3	2.585	2.585	0.517	0.517

Step 4: Calculate the algebraic (signed) sum over the modes of the force components at the selected time (the superscript 'T' is used to denote "total over all the modes at time t"):

A.
$$M_c^T = \sum_N M_c^t$$
$$= 0.110 + 4.991 + 2.585$$
$$= 7.686$$

B. Repeating the summation for each of the force components gives:

SUM OF THE BENDING MOMENTS		SUM OF THE SHEAR FORCES	
PLANE c	PLANE d	PLANE c	PLANE d
M_c^t	M_d^t	V_c^t	V_d^t
7.686	-5.121	2.214	1.371

Step 5. Based on the algebraic sum of the force components (the vector sum of the modal member forces), calculate the normal and shear stresses at the selected time:

$$\sigma_{bend} = \frac{7.686}{1} + \frac{-5.121}{2} = 5.126$$

$$\tau_c = \frac{2.214}{.1} = 22.14$$

$$\tau_d = \frac{1.371}{.2} = 6.855$$

Step 6. Based on the normal and shear stresses, calculate the Von Misses stress at the selected time:

$$\begin{aligned}\sigma &= \sqrt{\sigma_{bend}^2 + 3\left(\tau_c^2 + \tau_d^2\right)} \\ &= \sqrt{5.126^2 + 3\,(22.14^2 + 6.855^2)} \\ &= 40.5\end{aligned}$$

Note: The calculation indicates the Von Mises stress at time t = 0.1166 seconds only. Steps 3 through 6 must be repeated at each point of interest at each of the 84 discrete times. The greatest value of the Von Mises stress so obtained is the ASM stress.

The above results represent the peak response at one location and serve only to illustrate a sample calculation procedure for DDAM-ASM. Although the NRL summed stress above was evaluated at only one point on the periphery of the cross-section of the beam element, it may be larger at another point on the beam cross-section.

The ASM summed stress in step 6 is for only one point and at only one time. However, for this one point, a complete ASM stress-time history was calculated and the largest stress did occur at 116.6 milliseconds. Therefore it is appropriate to compare the NRL summed stress to the DDAM-ASM stress.

Assuming the example above resulted in a final NRL summed stress of 130.1 for the member and an ASM stress of 40.5 for the member, the member should be designed for a shock induced stress of at least 40.5. The relative responses reflected in this example indicate that the closely spaced modes phenomenon acts to artificially amplify the stress results when using the NRL summation method.

3.6 Sources of Additional Guidance

The following is a list of documents currently available to aid in the development of a dynamic shock analysis. Where the guidance provided by the following SUPSHIP Manuals is in conflict with the provisions of this document, this document takes precedence.

3.6.1 Guidance Manuals

a. "Mathematical Model and Dynamic Shock Analysis Guide for Main Propulsion Shafting" - Report No. SUPSHIP 280-1.

b. "Mathematical Model and Dynamic Shock Analysis Guide for Rudders, Rudder Stock and Bearings" - Report No. SUPSHIP 280-2.

c. "Mathematical Model and Dynamic Shock Analysis Guide for Main Reduction Gear" - Report No. SUPSHIP 280-3.

d. "Mathematical Model and Dynamic Shock Analysis Guide for Masts" - Report No. SUPSHIP 280-6.

The above listed guidance manuals may be obtained from Supervisor of Shipbuilding, Conversion and Repair, USN, Code 280, Portsmouth Detachment, Colts Neck, 201 South State Route 34, Colts Neck, NJ 07722

3.6.2 DDAM Background

a. O'Hara, G.J. and Cunniff, P.F., "Elements of Normal Mode Theory", NRL Report 6002, November 1963.

b. Cunniff, P.F. and O'Hara, G.J., "Normal Mode Theory for Three Dimensional Motion", NRL Report 6170, January 1965.

c. Remmers, G., "The Evolution of Spectral Techniques in Navy Shock Design", Shock and Vibration Bulletin 53, Part 1, May 1983.

d. O'Hara, G.J., "Background for Mechanical Shock Design of Ships Systems", NRL Report 6267, March 12, 1965

e. O'Hara, G.J. and Petak, L.P., "Effect of a Second Mode and Nearby Structures on Shock Design Values", NRL Report 6676, April 1968.

f. Clements, E.W., "Shipboard Shock and Navy Devices for its Simulation", NRL Report 7396, July 14, 1972.

g. Cunniff, P.F. and O'Hara, G.J., "A Procedure for Generating Shock Design Values", Journal of Sound and Vibration, Volume 134, No. 1, pp 155-164, 1989.

h. Belsheim, R. and Dick, R., "Shock Design of Shipboard Equipment Part III - Experimental Evaluation of the Dynamic Design Analysis Method", NRL Report 6478, January 23, 1967.

Chapter 4. FOUNDATION SHOCK DESIGN

4.1 General

All foundations which support Grade A or B equipment shall be assigned the same shock grade as the supported equipment. For foundations which require shock qualification, shock testing as described in the contract specifications or the design methods described herein shall be employed to demonstrate that the foundation is adequate from a shock standpoint. In general, possible shock damage shall be minimized. If misalignment would not interfere with operation of equipment, energy dissipation through permanent deformation of the foundation is preferable to damage to the equipment or the hull. In any case, deformation should take the form of buckling and bending of local structure, rather then permitting the equipment to tear loose from its attachment. Accordingly, joints shall develop the ultimate strength of the weakest member of the connection. Foundation deformation shall not act to compromise or invalidate the grade of shock for which the supported equipment was qualified. Foundation structures shall be proportioned to give approximately uniform stress distribution, permitting maximum absorption of energy through elastic deformation. Structural attachments or connections which minimize stress concentrations shall be used where possible. In general, brittle materials, with low ductility, as defined in Section 6.10, shall not be used. Where practical, under vertical shock, bolts should be loaded in tension rather than in shear.

The designer should not assume that a heavier/stiffer foundation is required when developing the design of shock resistant foundations. Foundations which are initially designed without regard for shock loadings will generally satisfy shock requirements specified for any ship with little or no modification required. The procedure to follow in meeting shock requirements for foundations is to first design the foundation to meet normal operating requirements (e.g. ship motion, vibration, air blast, wave slap, etc.) and then check the foundation to determine its adequacy from a shock standpoint. When the analysis indicates local over-stresses in the foundation, it is usually a simple matter to redesign the over-stressed area to meet shock stress requirements. To achieve an efficient design in cases where shock governs the design of a foundation, total stresses (shock plus operating) in at least the primary members shall exceed 75% (but not 100%) of the allowable stress (see Section 6.8).

See the shipbuilding or contract specifications for permissible bolt hole clearances. Applicable shock criteria for equipment hold-down bolts are cited in Section 3.1.3.d of this report and are illustrated in Example 1 of Chapter 5 of this report.

For systems suitable for modeling with a single degree of freedom, two alternate methods of designing shock resistant foundations, Method 1 and Method 2, are presented herein. For cases in which Method 1 applies, analysis shall be conducted using both methods and the lesser shock design loading shall be used. Method 1 or Method 2 may be used independently for each direction of shock.

Method 1 may only be used in cases where both of the following apply:

(1) The mounted equipment has been qualified on the basis of shock testing. It is essential in such cases that the foundation designer not compromise the shock qualification of the equipment by his foundation design. The designer shall consider the type of support used in the shock testing of the equipment. For example, if a support of uniform stiffness at each mounting point was used in the testing, the foundation being designed should also have uniform stiffness,

(2) The design of the foundation based on a single mass model to suit elastic-plastic shock criteria would be acceptable (see Chapter 3 for criteria pertaining to applicability of elastic-plastic shock design values).

In the procedures outlined below, the term "hold-down means" refers to hold-down bolts, dowels, keys, and any other devices which serve to locate or secure equipment to its foundation.

4.2 Method 1

Method 1 procedures for design of foundations for a specific shock direction are as follows:

(1) Determine the magnitude of the maximum shock loads which can be transmitted to the foundation by the equipment hold-down means by assuming the shock loading is applied at the center of gravity of the mounted equipment (or at the centers of gravity of each separately mounted equipment, if appropriate) and that the maximum load is developed when stress in one or more of the hold-down means equals 90% of ultimate strength in either shear or tension. For those cases in which the hold-down means are loaded for only one condition of a shock direction (e.g. bolts loaded in vertical downward direction but not in vertical upward direction), the analyst shall perform the Method 1 calculations for that condition in which the hold-down means are under loading.

(2) Check all critical areas of the foundation except the connection to ship's structure to assure that the foundation can resist the loads determined by 1, above.

(3) Increase the magnitude of the shock loadings obtained in step 1, above, by a factor equal to the ratio of foundation weight to equipment weight,

$$\text{Shock Load at base of Foundation} = \left[\frac{Weight\ \ of\ \ Foundation}{Weight\ \ of\ \ Supported\ \ Equipment} + 1\right] Shock\ \ Load_{step1}$$

Use these increased loadings for purposes of checking the connection between the foundation and the ship's structure.

(4) Repeat the above three steps for the other two principal directions of shock loadings.

(5) Calculate stresses in foundation members separately for each direction of shock loading. Allowable stresses are the same as for dynamically analyzed foundations which are designed to elastic-plastic shock design values. See Chapter 6 for allowable stresses.

(6) If necessary, stiffen the foundation to achieve acceptable stress levels. Whenever practical, employ local stiffening only (such as by gussets) to reduce stresses to acceptable levels.

4.3 Method 2

Method 2 is the conventional dynamic analysis method of foundation design, and is acceptable for all foundations. For purposes of foundation dynamic analysis, the item supported may generally be considered a single rigid mass and the foundation may be designed in accordance with procedures outlined in Sections 4.6 and 4.7. Where components which must be kept in alignment are not mounted on a rigid sub-base, each component must be considered a separate mass for foundation design purposes. If shock will induce significant rocking (rotation) of the foundation in addition to translation in the shock input direction, a simplified multi-degree of freedom mathematical model should be used to represent the equipment, as illustrated in Section 4.8. In the model, that portion of the foundation weight consistent with its dynamic response characteristics shall be lumped with the equipment weight. The remainder of the foundation weight shall be ignored (assumed part of the fixed base). See Sections 4.6 and 4.7.

Three or more masses may be required to adequately represent complicated foundation/equipment arrangements. In general, any major mass whose deflection under shock can be expected to differ significantly from the deflection of other portions of the structure must be separately represented by a mass point in the dynamic model.

Foundations for which multi-mass equipment representation is known to be required are listed below. Omission of equipment from this list does not relieve the Contractor from his responsibility to properly model other equipment for purposes of foundation dynamic analysis.

(1) Main propulsion gas turbine
(2) Main propulsion reduction gear
(3) Ship service diesel generator

(4) Air conditioning compressor
(5) Air conditioning chiller, condenser and receiver
(6) Ship service diesel engine heat exchanger
(7) Lube oil cooler
(8) Weapon systems (missile launchers, gun systems, torpedo tubes, etc.)

4.4 Extent of Foundation

For shock design purposes, foundations shall generally be considered to end at the point where primary ship structure begins (decks, longitudinals, web frames, structural bulkheads, etc.). The primary ship structure is considered to act as a fixed base (See Section 3.2.1.a). Shock design values shall be applied at the assumed fixed base (the interface of the foundation and the primary ship structure) in accordance with Section 3.1.2. Since basic ship structure is not required to be designed for shock, a clear definition of the interface between ship structure and the foundation is required. The design requirements for that interface (structural continuity) must be specified. Care must be taken to avoid any sudden structural discontinuity between foundations and ship structure. Chocks, brackets, or local strengthening of ship structure shall be used to provide structural continuity where necessary and checked for strength, but this added structure need not be included in the foundation mathematical model.

4.4.1 Equipment Mounted on Shell Framing - Shell framing is not normally considered as part of the foundation, although local strengthening may be required to insure structural continuity.

4.4.2 Equipment Mounted on Upper Levels of Machinery Spaces - Machinery space upper levels which are provided solely as a support for auxiliary machinery shall be considered as foundations, grounded on ship's structural web frames, transverse structural bulkheads, and bottom framing (or inner bottom) through stanchion connections. The shock response and design of these levels shall consider all equipment and piping or other distributed weights supported thereon. The upper levels shall be analyzed using DDAM multi-mass techniques with hull inputs.

4.4.3 Equipment Mounted on Decks - Deck mounted equipment fall into two categories distinguished by the alignment sensitivity of the equipment. For non-alignment sensitive installations, only the structure between the deck and equipment mounting surfaces need be considered in the foundation analysis. If necessary, to ensure structural continuity or adequacy, local headers or pads shall be added to stiffen the plating or framing in way of the equipment. Beams added in the plane of the deck to suit the arrangement of foundations and to provide points for attachment of foundations, shall be designed to transmit shear forces (associated with shock loadings) to primary ship structure (longitudinals and transverse web frames).

For alignment sensitive installations, all structure expressly added for support of the equipment (including additional headers, pads, and "normal" structural members whose size has been locally increased specifically to suit the installation) shall also be demonstrated suitable from a shock standpoint. This is accomplished by imposing foundation reaction loads upon the ship structure to determine whether additional stiffening of the added structure is required. Structural continuity shall be provided between this added structure and "normal" ship structure.

4.4.4 Equipment Mounted on Structural Bulkheads - Local stiffening should be used, where necessary, to insure structural continuity between the foundation and the supporting structural bulkhead. No general strengthening of the bulkhead should be considered solely for shock purposes.

4.4.5 Equipment Supported by Stanchions - Stanchions which are provided primarily to support heavy equipment shall be treated as an extension of the foundation and designed accordingly. Local stiffening of the interface between the stanchion and the structure upon which the stanchion falls must be provided to ensure structural continuity. Stanchions which are part of the basic ship structure are designed primarily as compression members for dead, live and sea loads. Stanchions that are part of a foundation must be capable of supporting tensile as well as compressive shock loads.

4.4.6 Equipment Supported by Pallets - Pallet type structures utilized for support of electronic equipment or other Grade A or B equipment shall be considered as foundations and shall be designed accordingly. Structural continuity between the pallet and the ship structure must be checked as part of the foundation shock design.

4.4.7 Equipment Mounted on Nonstructural Bulkheads - Nonstructural bulkheads include joiner, non-load bearing and non-tight, lightweight bulkheads. Where shock Grade A and B equipment are mounted to non-structural bulkheads, it is required that the bulkhead panels be considered as foundations and designed to withstand design shock loads. Bulkhead foundation systems for Grade A and B equipment should have top, bottom and inter-panel connections designed to support design shock loads. Deflection connections and/or additional reinforcements shall be provided as required. For equipment mechanically fastened to nonstructural bulkheads considered as foundations for Grade A and B equipment the designer should ensure that shock loads at local attachment points can be sustained by the fastener/bulkhead configuration.

4.4.8 Mechanical Attachments for Non-Metallic Hulls - The mechanical attachment of foundations to nonmetallic structure requires the designer/engineer to consider potential foundation instabilities which could occur if the design of bolted foundation attachments cannot sustain shock design loads. These attachments are typically provided by through-bolted connections attaching foundation structure or bearing brackets to ship structure. Consideration should be given to the effects of local crushing of ship structure in way of bolt attachments due to significant bolt bearing loads under shock conditions. This localized distortion of bolt openings may account for loss of equipment alignment. For alignment sensitive equipment, the local effect of bolt bearing loads should be considered in the foundation design.

Under dynamic shock load conditions the bearing strength of wood or composite structure in way of local attachments shall be considered in order to minimize the number and size of bolts required to attach foundations to ship structure.

4.5 Requirements for Supporting Ship Structure

Shock tests of ships, in which bulkheads, decks, etc., were not specifically designed for shock, have shown that structure designed for normal ship dynamic loads is generally adequate for shock loading. Nonetheless, attention shall be given to shock considerations when planning installations of certain weapon system components and any other items which are known to be alignment-critical sensitive. Structure (below foundations) supporting such items should possess the following characteristics:

4.5.1. Supporting ship structure should be "balanced" from the standpoint of resistance to deflection in the vertical direction to minimize tilting (angular misalignment) due to vertical shock. For instance, alignment-sensitive deck mounted items should be mounted squarely over bulkheads or squarely between framing members, other factors permitting. It is usually advantageous to have uniform stiffness at each mounting point of the equipment to avoid load concentrations at any one point during shock. Numerous equipment failures during ship shock testing have been traced to a disregard for this principle.

4.5.2 Plating or web frames should not be depended upon to resist angular deflections. Ensure that full structural continuity exists between alignment-critical equipment foundations and adjacent structural bulkheads or structural framing.

4.5.3 In order to avoid high lateral shock loading of stanchions and to avoid eccentric loading of stanchions (due to vertical shock), equipment having a cumulative weight of more than 1000 pounds shall not be attached directly to structural stanchions.

4.6 Dynamic Analysis of a Foundation - Single Degree of Freedom System

The simplest model of a foundation structure is a single degree of freedom system in which the foundation forms the spring and the equipment itself is the major portion of the mass. A schematic model of this type of system is shown in Figure 4-1. The shock loads, the total stress and the displacements of such a system can be determined by Method 2 using the following steps:

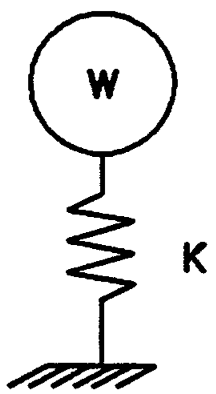


Figure 4-1. Schematic Representation of a Single Degree of Freedom System

Step 1 - Compute the spring constant K for a direction of shock loading. The spring constant is a measure of the stiffness of the structure and is equal to the load causing unit deflection. For the vertical shock model, the spring constant in lbs/in is numerically equal to the amount of force (lbs) acting down through the center of gravity of the equipment foundation system required to deflect the center of gravity down one inch. Simultaneous deflections of the center of gravity in other directions are ignored. Generally there will be a different spring constant in each shock direction.

Step 2 - Determine the modal effective weight W. For an item of equipment mounted on a foundation which is to be represented as a single mass, W may be assumed equal to the equipment weight plus one-half of the weight of the foundation.

Step 3 - Calculate the angular frequency, ω, by the following equation:

$$\omega = \sqrt{K g / W}$$

where: g is the gravitational constant in consistent terms

Step 4 - Using the shock design value formulas contained in DDS 072-1 (CONFIDENTIAL), determine the design velocity value (V) and the design acceleration value (A) based on mounting location, direction of shock loading and type of design category (elastic or elastic-plastic).

Step 5 - Calculate the design acceleration of the system, D (in gravity units), in accordance with DDS 072-1 by using,

$$D = (V)\ \omega / g \qquad \text{or}$$

$$D = A \qquad \text{whichever is less.}$$

Step 6 - Determine the effective static force F applied to the equipment at its center of gravity by use of the formula,

$$F = W\ D.$$

Step 6a (Optional) - Where appropriate, forces resulting from application of Method 1 (See Section 4.2) may be compared with those derived from Method 2 (See step 6 above.) Assessment of the foundation design would then be based on the shock loads which result in the least foundation weight.

Step 7 - Apply the shock load calculated in Step 6 or 6a, plus any continuous operation loads (as defined in Chapter 3). Analyze the structure using conventional static analysis procedures to determine the total stresses. If the equipment hold-down bolts are to be shock qualified by dynamic analysis, repeat Step 6 with D derived from elastic shock design values and with W in Step 6 equal to equipment weight only.

Step 8 - If required for displacement-sensitive items, the maximum relative displacement of the center of gravity of the equipment with respect to the fixed base may be determined by the formula:

$$X = F/K$$

F is determined on the basis of elastic shock design values in all cases.

Step 9 - Repeat the above steps for the other principal directions of shock loading.

4.6.1 Example - Single Degree of Freedom System

To illustrate the aforementioned procedure for determining the shock load on a single degree of freedom system, consider the equipment-foundation system shown in Figure 4-2. Assume that this shock tested equipment is rigid and symmetrical and that a single mass is sufficient to represent it. The shock adequacy will be determined for the vertical shock direction for upward motion of the ship (i.e. web in compression). The equipment shown in this example is not considered to be alignment sensitive; therefore, the foundation is not required to remain within the elastic range and the use of elastic-plastic shock design values is considered acceptable.

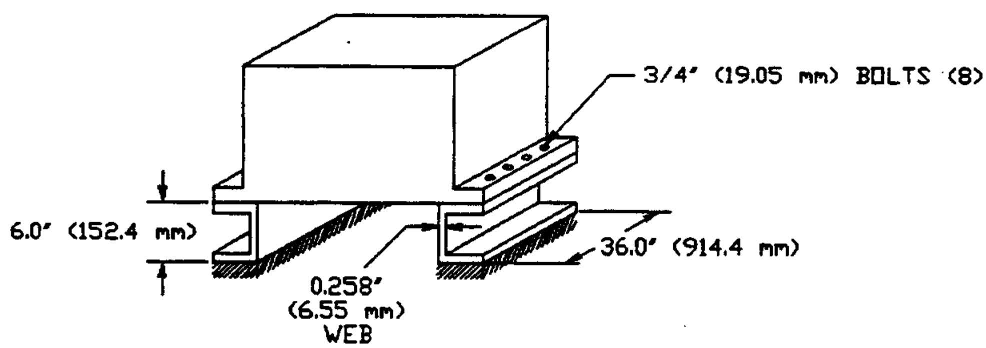


Figure 4-2. Single Degree of Freedom Foundation Model

For the system shown in Figure 4-2, assume the following characteristics:

Equipment Weight	- 5000 lbs (22.241 kN)
Foundation Weight	- 720 lbs (3.202 kN) each beam
Equipment Location	- Deck
Category of Shock Design Value	- Elastic-Plastic
Foundation Material	- Steel, $E = 30 \times 10^6$ PSI (210×10^9 Pa)

For the system shown in Figure- 4-2, the center of gravity of the equipment is equidistant from the supports. The supports land on the fixed base (rigid frame of reference) throughout their length.

Step 1 - Spring Constant K

For shock in the upward direction (web in compression).

$$K_1 = \frac{AE}{L} \quad (\text{for one channel})$$

$$= \frac{36\,(0.258)\,30 \times 10^6}{6}$$

$$= 46.44 \times 10^6 \text{ psi}$$

$$\left(\begin{aligned} K_1 &= \frac{0.9114\,(6.55 \times 10^{-3})\,2.068 \times 10^{11}}{0.1524} \\ &= 8.127 \times 10^9 \text{ N/m} \end{aligned} \right)$$

$$K_2 = K_1$$

$$K = K_1 + K_2 \quad (\text{springs in parallel})$$

$$= 2\,(46.44 \times 10^6) = 92.88 \times 10^6 \text{ psi}$$

$$\left(K = 2\,(8.127 \times 10^9) = 1.625 \times 10^{10} \text{ N/m} \right)$$

Step 2 - Weight W

$$W = \text{weight of equipment} + 1/2 \text{ weight of foundation}$$

$$= 5,000 + \frac{720 + 720}{2}$$

$$= 5,720 \text{ lbs}$$

Using the values obtained in Steps 1 and 2 above, the system shown in Figure 4-2 is schematically represented in Figure 4-3.

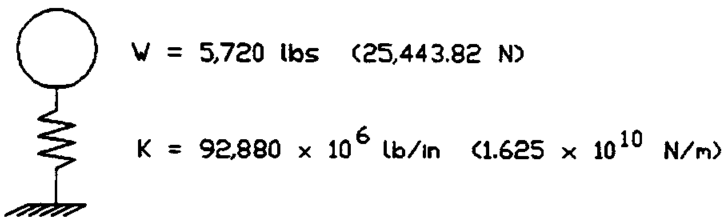


Figure 4-3 Schematic Representation of a Single Degree of Freedom System

Step 3 - Angular Frequency

$$\omega = \sqrt{\frac{Kg}{W}}$$

$$= \sqrt{\frac{92.88 \text{ x } 10^{6}(386)}{5,720}} \qquad \left(\sqrt{\frac{1.625 \text{ x } 10^{10}(9.81)}{25,443.82}}\right)$$

$$= 2,504 \ \frac{\text{rad}}{\text{sec}}$$

Step 4 - Design Velocity (V) and Design Acceleration (A)

DDS 072-1 contains formulas that give the shock design values as a function of the modal effective weight in Kips. From those formulas, for a system with a modal effective weight of 5,720 lbs (or 5.72 Kips), vertical shock loading, deck mounting, and elastic-plastic design, the shock design values are:

$$V = 22.68 \text{ in/sec} \qquad (0.5765 \text{ m/sec})$$

$$A = 40.7 \text{ g's}$$

Step 5 - Absolute Acceleration D

Based on velocity:

$$D = \frac{V\omega}{g}$$

$$= \frac{22.68\ (2504)}{386} \qquad \left(\frac{0.5765\ (2504)}{9.81}\right)$$

$$= 147.12 \text{ g's}$$

Based on acceleration, D = A = 40.7 g's.

The shock design value to use is the lesser of the these values. Therefore, use D = 40.7 g's.

Step 6 - Effective Static Force F

$$F = WD$$

$$= 5,720\ (40.7) \qquad \left(25,443.82\ (40.7)\right)$$

$$= 232,804 \text{ lbs} \qquad \left(1.036 \times 10^6 \text{ N/m}\right)$$

Step 6a (Optional) - Computation of Effective Static Force F by Method 1

For the system shown in Figure 4-2, it is assumed that a load applied at the center of gravity of the equipment in the downward shock direction (bolts in loaded condition) will load the 8 hold-down bolts equally. Therefore:

Area/bolt = .3340 in.^{2} (2.155×10^{-4} m^2)

Area (8 bolts) = 8(.3340) = 2.672 in.^{2}
($8(2.155 \times 10^{-4}) = 1.724 \times 10^{-3}$ m^2)

Ultimate strength (Grade 5) = 120,000 psi (MIL-S-1222)
(827.37×10^6 N/m^2)

90% Ultimate Strength = 108,000 psi (744.64×10^6 N/m^2)

Force F = 108,000 (2.672) = 288,576 lbs
($744.64 \times 10^6 (1.724 \times 10^{-3}) = 1.284 \times 10^6$ N)

Step 7 - Structural Analysis (Stresses)

Use the force F calculated in step 6 above since that value is less than the corresponding force determined by Method 1 in step 6a.

Due to the symmetry of the system, each support will experience a loading of 232,804/2 or 116,402 pounds ($(1.036 \times 10^6)/2$ or 5.18×10^5 N). This is schematically represented in Figure 4-4. Note that these loadings would be increased by continuous operating loads (defined in Chapter 3), if any are present.

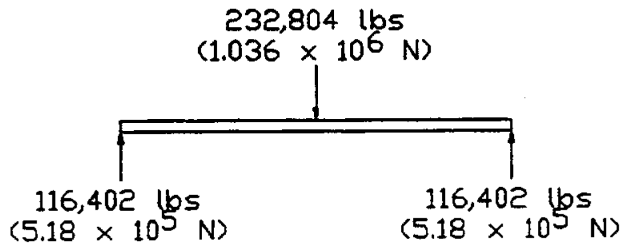


Figure 4-4. Schematic Representation of a Simply Supported Beam Loaded at the Center

Using standard stress formulas, the compressive stress in each web of the foundation is equal to,

$$\sigma = \frac{P}{A}$$
$$= \frac{116,402}{36(0.258)}$$
$$= 12,532 \text{ psi}$$

$$\left(\begin{aligned} \sigma &= \frac{5.18 \times 10^{5}}{0.9144\ (6.55 \times 10^{3})} \\ &= 86.49 \times 10^{6} \text{ N/m}^2 \end{aligned} \right)$$

Step 8 - <u>Structural Analysis (Deflection)</u>

$$X = \frac{F}{K}$$
$$= \frac{232,804}{92.88 \times 10^{6}}$$
$$= 0.0025 \text{ inches}$$

$$\left(\begin{aligned} X &= \frac{1.036 \times 10^{6}}{1.625 \times 10^{10}} \\ &= 0.06375 \text{ mm} \end{aligned} \right)$$

The values calculated in Steps 7 and 8 above shall be compared to the allowable criteria cited in Chapter 6 of this report to determine the shock adequacy of the foundation in the upward shock direction.

Step 9 - Shock Loading in Other Directions

Step 1 through 8 shall be repeated for the athwartship, fore-and-aft, and vertical downward (web in tension) directions of shock loading, if required, using the appropriate spring constant values for those particular directions. For the downward shock direction (ship moving down), the foundation flanges will be in bending and the equipment hold-down bolts and webs will be in tension.

4.7 Example - Uni-Directional Response Analysis of a Foundation - Multi-Mass System

Foundations for Grade A, alignment-sensitive equipment such as those listed in Section 3.1.3.a, have, as a rule, been modeled as multi-degree of freedom systems. Analysis of multi-degree of freedom foundation systems generally require the use of computer solutions. Multi-degree of freedom models used to analyze foundations have the following characteristics:

(a) The model is three-dimensional and represents the equipment and foundation.

(b) The model should minimize the complexity of the analysis i.e. sound engineering judgement should be used in the preparation of the model. It is not necessary to model the supported equipment with the same degree of refinement as is used in an equipment analysis. However, it is necessary to model the equipment such that the overall mass distribution of the equipment and its flexibility are properly represented.

The basic steps necessary to analyze a multi-mass system are as follows:

Step 1 - Divide the system into N regions that adequately describe the system and calculate the mass of each; i.e. M_1, M_2..........M_N, where

$$\sum_{i=1}^{N} M_i = \text{Total Mass}$$

These masses represent the dynamic degrees of freedom of the system and are located at nodes in accordance with Section 3.3.

Step 2 - Calculate the influence (or stiffness) coefficients for these nodes and form the influence (or stiffness) coefficient matrix.

Step 3 - Using the method shown in Appendix A, or other suitable methods, find a number of mode shapes and natural frequencies necessary to satisfy the mode selection criteria of Section 3.5.3. The frequency of the highest mode calculated need not exceed 250 Hertz unless it is determined that the cumulative modal effective weight requirement of 80%, noted in Section 3.5.3, will not be satisfied at that frequency.

Step 4 - For the first mode, mode "a", complete the following table:

MODAL COMPUTATION TABLE (MODE "a")

Mass Number, i	Mass, M_i	Mode Shape, Φ_{ia}	$M_i\ \Phi_{ia}$	$M_i\ \Phi^2_{ia}$
1	M_1	Φ_{1a}	$M_1\ \Phi_{1a}$	$M_1\ \Phi^2_{1a}$
2	M_2	Φ_{2a}	$M_2\ \Phi_{2a}$	$M_2\ \Phi^2_{2a}$
-	-	-	-	-
n	M_n	Φ_{na}	$M_n\ \Phi_{na}$	$M_n\ \Phi^2_{na}$
		Σ	$M_i\ \Phi_{ia}$	$M_i\ \Phi^2_{ia}$

Step 5 - Calculate the participation factor*, P_a

$$P_a = \frac{\sum_{i=1}^{N} M_i \Phi_{ia}}{\sum_{i=1}^{N} M_i \Phi_{ia}^2}$$

Step 6 - Calculate the modal effective mass*, M_a

$$M_a = P_a\left(\sum_{i=1}^{N} M_i \Phi_{ia}\right) = \frac{\left(\sum_{i=1}^{N} M_i \Phi_{ia}\right)^2}{\sum_{i=1}^{N} M_i \Phi_{ia}^2}$$

Step 7 - Multiply M_a by g to get the modal effective weight and divide this value by the total model weight to obtain the percent modal effective weight.

Step 8 - Using the shock design value formulas in DDS 072-1, with the modal effective weight as W (in Kips), determine the design velocity value (V) and the design acceleration value (A).

* The definition of participation factor and modal effective mass shown herein apply only to the uni-directional models. See Section 4.8 for general definition of these parameters.

Step 9 - Calculate the values of V ω_a and A g. Determine the modal shock design value D_a to be the lesser of the two:

$$D_a = V\,\omega_a$$

or

$$D_a = A\,g$$

Step 10 = Calculate the effective static force applied at each mass:

$$F_{ia} = M_i\,\Phi_{ia}\,P_a\,D_a$$

Step 11 - Apply the effective external static forces calculated in Step 10 to their respective nodes and calculate the desired response (e.g. stresses, reaction forces, bending moments, deflections, etc) by the usual methods of structural analysis of static structures.

Step 12 - Repeat Steps 4 through 11 for modes "b", "c", etc., as necessary (see Section 3.5.3). The values obtained in step 11 for all calculated modes shall be summed across the modes by the NRL summing method described in Section 3.5.5. The resultant value (combined with continuous operating stresses, if present) shall be compared to the failure criteria given in Chapter 6 of this report.

If required, the following quantities may be determined from the information obtained above:

1. Relative displacement between any two nodes, within a mode,

$$X_{ia} - X_{ka} = (\Phi_{ia} - \Phi_{ka})\,P_a\,(D_a/\omega_a^2)$$

2. Relative displacement between any node and the fixed base, within a mode,

$$X_{ia} = \Phi_{ia}\,P_a\,(D_a)/\omega_a^2$$

Relative displacements can also be summed across the modes using the NRL summing method described in Section 3.5.5. The NRL summing method shall not be used to sum absolute deflections across the modes unless total displacement of a point on the structure with respect to the fixed base is required.

4.8 Dynamic Analysis of a Foundation - Multi-Directional Response Analysis

The analytical technique for a Multi-Directional Response (MDR) analysis is analogous to that for uni-directional analysis. The basic principles are derived from normal mode theory and are valid for a maximum of six directions of response motion at each node. The full theory, for rotations as well as translations, is considered too involved for presentation here. Most three-dimensional systems can be adequately described by translational motions alone. Therefore, the analysis procedure for three directional response motions, as given below, is applicable in most cases. Some of the basic concepts of modal analysis for multi-direction response are:

(a) Stiffness Matrix:

k_{ij} = the reaction force at the i^{th} degree of freedom due to a unit deflection at the j^{th} degree of freedom, with all other degrees of freedom restrained

$k_{ij} = k_{ji}$ for linear elastic structures,

where i and j are arbitrary degree of freedom indicators.

(b) Influence Coefficient vector: The influence coefficient vector {r} is a vector of direction cosines between the direction of shock input and the direction of response for each degree of freedom.

(c) Participation factor:

P_a = participation factor for mode a

$$P_a = \frac{\sum_{i=1}^{N} m_i \Phi_{ia} r_i}{\sum_{i=1}^{N} m_i \Phi_{ia}^2}$$

where

m_i = mass associated with the i^{th} degree of freedom
Φ_{ia} = mode shape for i^{th} degree of freedom in mode a
r_i = direction cosine for the i^{th} degree of freedom

(d) Modal effective mass:

$$m_a = \frac{\left(\sum_{i=1}^{N} M_i \Phi_{ia} r_i\right)^2}{\sum_{i=1}^{N} M_i \Phi_{ia}^2}$$

the modal effective mass (acting in the direction of shock input) for the a^{th} mode.

For shock input in one selected direction (vertical, athwartship or fore/aft), the basic steps for evaluating the dynamic response for a particular mode, mode a, are given below. (Steps 1, 2 and 3 are generally done once and apply to the system for all three directions of shock input. Steps 4 through 7 are repeated for each mode and for the other two directions of shock input.) The steps outlined are illustrative of the DDAM procedure, however, numerically equivalent steps may be substituted for calculational efficiency.

(1) Determine the stiffness and mass matrices for the mathematical model.

(2) Calculate the modal characteristics Φ_{ia} and ω_a.

(3) Determine vector $\{r_i\}$, the direction cosines for each degree of freedom with respect to the direction of shock input considered.

(4) Calculate the participation factor and modal effective mass as shown above.

(5) Determine the design velocity value (V) and the design acceleration value (A) from DDS 072-1 using the calculated modal effective weight, W_a (in Kips). Calculate the values of V ω_a and Ag. Determine the modal shock design value D_a to be the lesser of the two:

$$D_a = V\,\omega_a$$
$$D_a = Ag$$

(6) Calculate the effective static force applied for each degree of freedom:

F_{ia} = force at node i in mode a

$$F_{ia} = m_i \Phi_{ia} P_a D_a$$

(7) Apply the effective static forces calculated above at their respective nodes. Since these forces F_{ia} occur simultaneously, the ensuing stress analysis will properly consider the concurrent effects of the forces in all directions. The modal displacements may be calculated directly:

$$X_{ia} = \phi_{ia} P_a D_a / \omega_a^2$$

4.8.1 Example - Multi-Directional Response Analysis

This example is provided to demonstrate the application of DDAM for Multi-Directional Response (MDR) analysis. Consider a simply supported structure as shown in the figure below. This model may represent a mast yardarm with mounted antennas (masses M_1, M_2 and M_3). The vertical members below the masses represent the antenna foundations. In the context of this report an MDR analysis is defined as an analysis that uses a model which allows response degrees of freedom in all directions including directions other than the direction of input motion. Thus, under vertical shock, masses M_1, M_2 and M_3 will have lateral as well as vertical shock responses. It is obvious that under vertical shock (shock input motion at the supports in the Y direction) bending of the vertical members cannot be evaluated unless an MDR DDAM analysis is conducted. Omission of lateral degrees of freedom for each mass in the vertical mathematical model will significantly alter the results and conclusions of the analysis.

The shock inputs for an MDR model are applied independently as they are for a uni-directional model. A separate analysis is conducted for each direction of shock input.

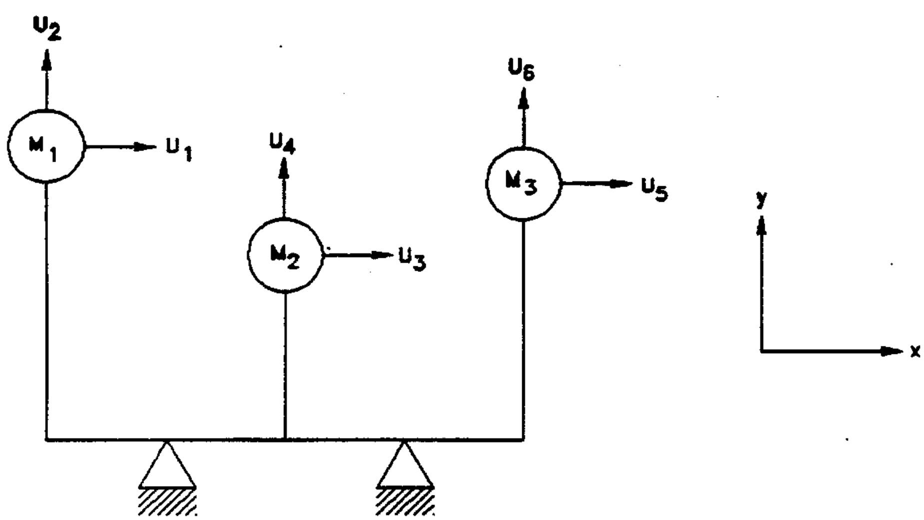


Figure 4-5 Schematic Representation of a Mathematical Model for an MDR Analysis

(1) Mass and Stiffness Matrices:

$$[K] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \qquad [M] = \begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{66} \end{bmatrix}$$

Using quantities from the mass matrix above;

$$M_1 = m_{11} = m_{22}$$

$$M_2 = m_{33} = m_{44}$$

$$M_3 = m_{55} = m_{66}$$

(2) Frequency Response:

DOF	MODE	1	2	3	4	5	6
	FREQ	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
u_1		Φ_{11}	Φ_{12}	Φ_{13}	Φ_{14}	Φ_{15}	Φ_{16}
u_2		Φ_{21}	Φ_{22}	Φ_{23}	Φ_{24}	Φ_{25}	Φ_{26}
u_3		Φ_{31}	Φ_{32}	Φ_{33}	Φ_{34}	Φ_{35}	Φ_{36}
u_4		Φ_{41}	Φ_{42}	Φ_{43}	Φ_{44}	Φ_{45}	Φ_{46}
u_5		Φ_{51}	Φ_{52}	Φ_{53}	Φ_{54}	Φ_{55}	Φ_{56}
u_6		Φ_{61}	Φ_{62}	Φ_{63}	Φ_{64}	Φ_{65}	Φ_{66}

(3) The influence coefficient vector {r} for shock in the Y direction is:

$$r = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{Bmatrix}$$

(4) Modal Composition (shown for mode 1)

Degree of Freedom	Mass m_{ii}	Mode Shape Φ_{ia}	r	$\{\Phi\}_a^T[M]\{r\}$	$\{\Phi\}_a^T[M]\{\Phi\}_a$
u_1	M_1	Φ_{11}	0	$M_1\ \Phi_{11} = 0$	$M_1\ (\Phi_{11})^2$
u_2	M_1	Φ_{21}	1	$M_1\ \Phi_{21}$	$M_1\ (\Phi_{21})^2$
u_3	M_2	Φ_{31}	0	$M_2\ \Phi_{31} = 0$	$M_2\ (\Phi_{31})^2$
u_4	M_2	Φ_{41}	1	$M_2\ \Phi_{41}$	$M_2\ (\Phi_{41})^2$
u_5	M_3	Φ_{51}	0	$M_3\ \Phi_{51} = 0$	$M_3\ (\Phi_{51})^2$
u_6	M_3	Φ_{61}	1	$M_3\ \Phi_{61}$	$M_3\ (\Phi_{61})^2$

For shock input in the "Y" direction, the participation factor for mode a is:

$$P_a = \frac{M_1\Phi_{2a} + M_2\Phi_{4a} + M_3\Phi_{6a}}{M_1(\Phi_{1a})^2 + M_1(\Phi_{2a})^2 + M_2(\Phi_{3a})^2 + M_2(\Phi_{4a})^2 + M_3(\Phi_{5a})^2 + M_3(\Phi_{6a})^2}$$

The modal effective mass for mode a in the direction of shock input is:

$$m_a = \frac{(M_1\Phi_{2a} + M_2\Phi_{4a} + M_3\Phi_{6a})^2}{M_1(\Phi_{1a})^2 + M_1(\Phi_{2a})^2 + M_2(\Phi_{3a})^2 + M_2(\Phi_{4a})^2 + M_3(\Phi_{5a})^2 + M_3(\Phi_{6a})^2}$$

(5) Shock design value:

The shock design values to be applied in each mode are obtained from DDS 072-1. These values are a function of modal effective weight (in kips) and the modal frequency in radians.

(6) Effective static forces:

The effective static forces in mode a for each degree of freedom are:

Mass 1:

$$F_{1a} = M_1 \Phi_{1a} P_a V_a \omega_a \quad or \quad M_1 \Phi_{1a} P_a A_a g$$

$$F_{2a} = M_1 \Phi_{2a} P_a V_a \omega_a \quad or \quad M_1 \Phi_{2a} P_a A_a g$$

Mass 2:

$$F_{3a} = M_2 \Phi_{3a} P_a V_a \omega_a \quad or \quad M_2 \Phi_{3a} P_a A_a g$$

$$F_{4a} = M_2 \Phi_{4a} P_a V_a \omega_a \quad or \quad M_2 \Phi_{4a} P_a A_a g$$

Mass 3:

$$F_{5a} = M_3 \Phi_{5a} P_a V_a \omega_a \quad or \quad M_3 \Phi_{5a} P_a A_a g$$

$$F_{6a} = M_3 \Phi_{6a} P_a V_a \omega_a \quad or \quad M_3 \Phi_{6a} P_a A_a g$$

(7) Stress Analysis:

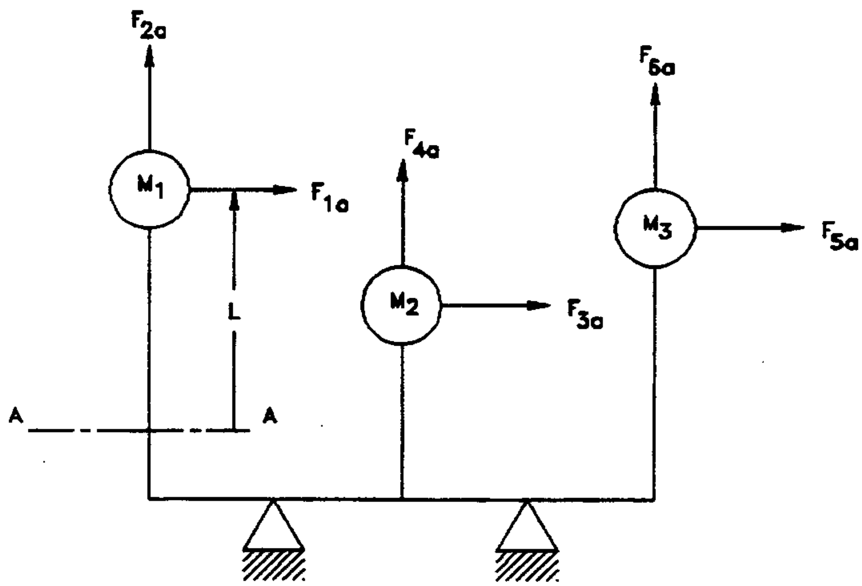


Figure 4-6 Force Schematic for an MDR Analysis

Stress at section A-A:

$$\sigma_a = \frac{Mc}{I} + \frac{F}{A} = \frac{(F_{1a} L)\ c}{I} + \frac{F_{2a}}{A}$$

where I, A and c are the member section properties

I	=	Section Moment of Inertia
A	=	Section Area
c	=	Distance from the neutral axis to the fiber carrying the greatest stress

4.9 Finite Element Application of DDAM

To illustrate the finite element modeling of a complicated structure, consider the example shown in Figure 4-7 below. Appendix E provides details for the format and content of a finite element mathematical model and dynamic analysis. The model used for this example is a typical finite element representation for a rack type foundation. Each equipment mounted in the rack is modeled with its weight concentrated at its center of gravity. The weight of the rack structure, associated cooling water piping, cabling, mounting hardware and other distributed weight is included in the model. The flexibility of the equipment should be included if known. Otherwise, the equipment can be considered rigid bodies.

This model is used to design the foundation structure and can be used to check the shock loading in the equipment hold-down bolting. The foundation model and analysis is not used to evaluate the equipment itself since the equipment is normally qualified for shock by testing in accordance with MIL-S-901. If the equipment is a Grade B item, its shock adequacy can be demonstrated by analysis in lieu of testing. The results of application to the equipment and equipment appendages of acceleration values derived from the DDAM analysis of this model can be evaluated in accordance with Section 6.4 to determine whether the item meets Grade B shock requirements.

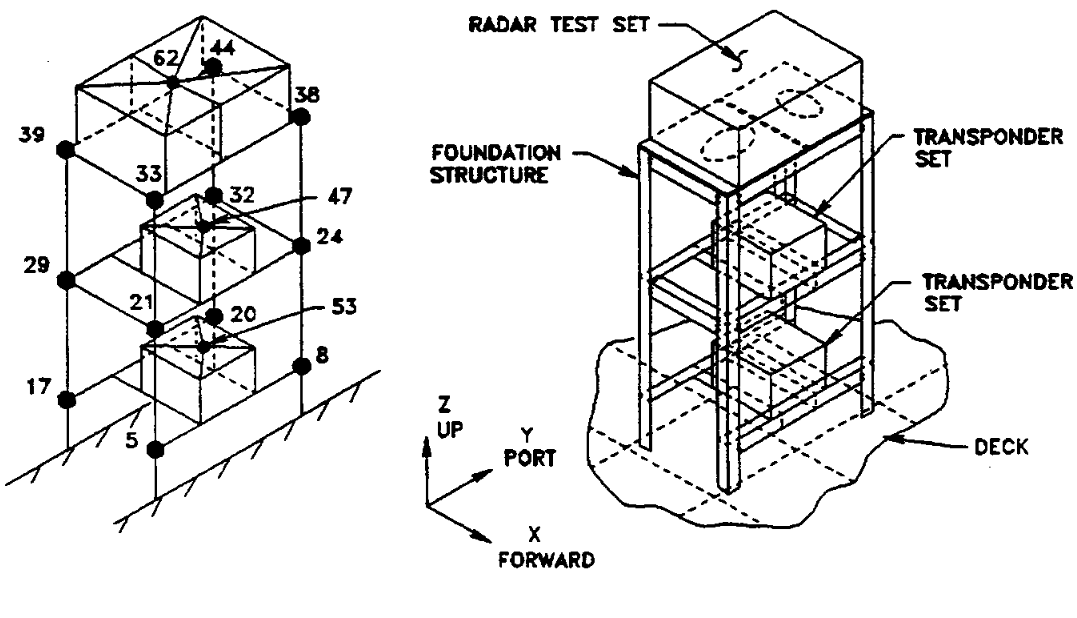


Figure 4-7 Schematic Representation of a Multi-Mass Finite Element Foundation Mathematical Model

Chapter 5. DDAM OF GRADE B ITEMS

Shock qualification of a Grade B item by dynamic analysis (in lieu of shock testing) is permitted in cases where the item has been assigned Grade B status solely because the item or portions thereof could possibly cause a hazard by coming adrift due to shock.

In cases where the dynamic model of a Grade B item would be relatively complicated or where the Grade B item does not lend itself well to dynamic analysis (due to non-linearities or doubt concerning possible failure modes), it is recommended that the item be shock qualified by shock testing instead of by dynamic analysis.

Dynamic analysis criteria contained in Chapter 3 apply to analysis of Grade B items, with the exception that low frequency components need not be modeled as separate masses unless they are items which can cause a hazard. For example, if an item could cause a shock hazard by coming adrift external to the equipment, it should be considered as a separate mass.

Generally elastic-plastic shock design values apply to dynamic analyses of Grade B items. However, hold-down means must be designed based on elastic inputs. Elastic shock design values shall also be used for Grade B equipment where a hazard can arise by overstressing a component which releases a toxic material from a bolted joint or where a hazard can arise as a result of excessive deformation or fracture of a brittle container. Allowable stress criteria are contained in Chapter 6.

The following two examples illustrate procedures for dynamic analysis of Grade B items.

Example 1. Consider the deck mounted, Grade B, equipment shown in Figure 5-1. The analysis for this item is required to show that it will not come adrift under shock. This is accomplished by ensuring that failure will not occur in the equipment legs or the hold-down bolts under shock loading. (Only vertical shock is shown in this example)

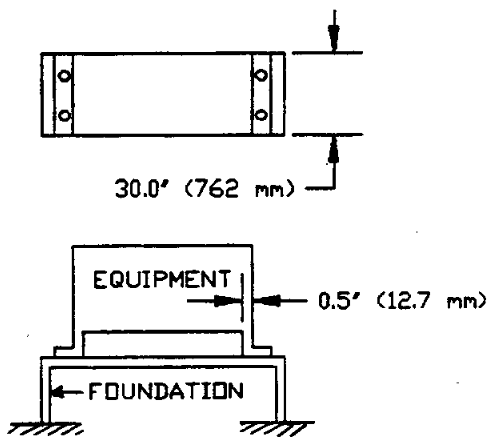


Figure 5-1 Single Degree of Freedom Foundation Model for Dynamic Analysis of Grade B Item

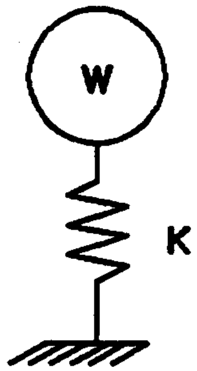


Figure 5-2 Schematic Representation of a Single Degree of Freedom Foundation

The system can be analyzed as a single degree of freedom system as shown in Figure

$$W = \text{equipment weight} + \frac{\text{foundation weight}}{2}$$

$$= 3,000 + \frac{450}{2} \qquad \left(13,345 + \frac{2001.7}{2} \right)$$

$$= 3,225 \text{ lbs} \qquad \left(14.346 \times 10^6 \text{ N} \right)$$

5-2. Assume K = 1.92×10^6 lbs/in (336.24×10^6 N/m) and assume

The angular frequency of the system is derived as follows:

$$\omega = \sqrt{\frac{Kg}{W}}$$

$$= \sqrt{\frac{1.92 \times 10^6 (386)}{3,225}} \qquad \left(\sqrt{\frac{336.2 \times 10^6 (9.81)}{14,346}} \right)$$

$$= 479 \frac{\text{rad}}{\text{sec}}$$

From DDS 072-1, for a system with a modal effective weight of 3.225 kips vertical shock loading, deck mounting, and elastic-plastic design, the shock design values are,

$$V = 24.8 \text{ in/sec} \qquad (0.63 \text{ m/sec})$$
$$A = 49.0 \text{ g's}$$

Therefore,

$$D = \frac{V\omega}{g}$$

$$= \frac{24.8\,(479)}{386} \qquad \left(\frac{0.63\,(479)}{9.81}\right)$$

$$= 30.8 \text{ g's}$$

$$D = A = 49.0 \text{ g's}$$

$$\text{Use } D = 30.8 \text{ g's}$$

To analyze the stress in each foot, a force of

$$F = (3{,}000)(30.8) = 92{,}400 \text{ lbs.} \qquad (13{,}346\,(30.8) = 411.1 \text{ kN})$$

would be divided between the two legs. It will be noted that 3,000 lbs. (13,345 N) was used instead of 3,225 lbs. (14,346 N) to calculate the force. This was done because only the weight of the equipment effectively acts on the legs (and bolts).

$$\sigma_{foot} = \frac{F}{A_{web}}$$

$$= \frac{92{,}400}{2(.5)(30)} \qquad \left(\frac{4.111 \times 10^5}{2(0.0127)(0.762)}\right)$$

$$= 3{,}080 \text{ psi (axial)} \qquad \left(21.24 \times 10^6 \text{ N/m}^2\right)$$

In the interest of expediency for this problem, bending stresses in the legs will not be examined. To stress analyze the four hold-down bolts, the force of 92,400 lbs (4.097×10^5 N) is not appropriate because all bolts, dowels, pins and similar hold-down means must be designed for shock on the basis of elastic shock inputs.

Thus, for elastic inputs, the shock design values for this system would be,

$$V = 49.6 \text{ in/sec} \qquad (1.26 \text{ m/sec})$$

$$A = 49.0 \text{ g's}$$

and $$D = \frac{V\omega}{g}$$

$$= \frac{49.6\,(479)}{386} \qquad \left(\frac{1.26\,(479)}{9.81}\right)$$

$$= 61.6 \text{ g's}$$

$$D = A = 49.0 \text{ g's}$$

$$\text{Use } D = 49.0 \text{ g's}$$

To determine bolt stresses, the shock force is

$$F = 3{,}000\,(49) = 147{,}000 \text{ lbs}$$
$$(13{,}346(49) = 654.0 \text{ kN})$$

and $$\sigma_{bolt} = \frac{147{,}000 \text{ lbs}}{4A_{bolt}} \qquad \left(\frac{654.0 \text{ kN}}{4A_{bolt}}\right)$$

The stress values determined above for the legs and the bolts shall be compared to the allowable stress criteria in Chapter 6 of this report to determine if the design criteria is met.

Example 2. Consider that the equipment shown in Figure 5-1 has a 200 lb. (889.6 N) motor attached to it as shown in Figure 5-3.

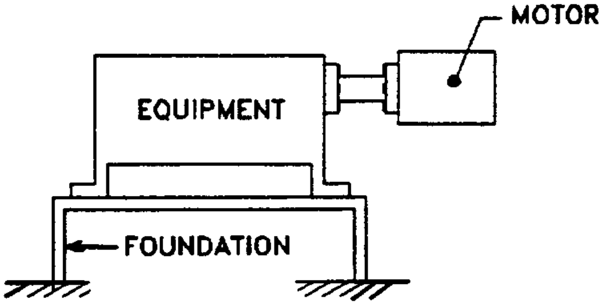
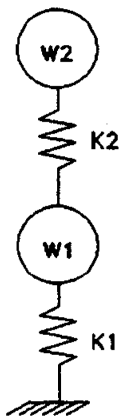


Figure 5-3. Equipment/Foundation Configuration with a Cantilevered Motor

To determine whether this Grade B system will create a hazard by coming adrift under shock loading the analyst must check that neither the legs, hold-down bolts nor motor attachment will fail under shock loading because any one of them would cause the equipment or motor to come adrift. To analyze this system, a two mass model such as the one shown in Figure 5-4 is required.



where:

W1	=	weight of equipment + 1/2 foundation weight
W2	=	weight of motor, portion of motor shaft and attachment
K1	=	foundation spring constant
K2	=	motor attachment spring constant

Figure 5-4. Schematic Representation of a Two Degree of Freedom System

The method used for the dynamic analysis of a two mass system has been discussed in Sections 4.7 and 4.8. The analyst shall determine whether failure of the motor mount bolting will occur by using the forces developed in Spring 2 (K2). The feet and bolt stresses are determined from the forces developed in Spring 1 (K1). In keeping with criteria presented in Chapter 3, elastic shock design criteria would apply to the design of the hold-down means which secure the equipment to the foundation, but not to the bolting which secures the motor to the equipment.

Chapter 6. ALLOWABLE STRESS CRITERIA

6.1 General Criteria

Each principal direction of shock loading (vertical, athwartship, and fore-and-aft) shall be considered separately. Continuous operating stresses (as defined in Chapter 3) shall be added to calculated shock stresses. Allowable stress criteria presented herein should be compared to calculated stresses based on the Von Mises Failure Theory. Comparison of combined shock and operating stresses to allowable stresses will generally determine design acceptability. The allowable stress described in sections 6.2 through 6.5 apply to Navy standard metal materials (e.g. steel, aluminum, K-Monel, etc.). Allowable stresses for other non-standard materials (e.g. GRP, composite, epoxy chock, titanium, wood, etc.) shall be provided by the contract specifications. If the shipbuilding specifications do not address the material design properties of these materials, the contractor shall propose material properties for Navy approval. Other failure criteria, as discussed in Section 3.1.4 and specified in the approved model report shall also be considered. In addition, it shall also be assured that column buckling for those items designed to elastic shock design values will not occur and the deflection of foundations must not lead to overloading of flexible couplings or other displacement-critical components. Figure 6-1 is a summary table for the allowable stress criteria reflected in this report for Grade A and B systems.

Design stresses are categorized as general or local, and as membrane or membrane plus bending. Definitions of these categories are provided below with examples for their application provided in Appendix F. (Note: stresses derived from one-dimensional beam elements are limited solely by the general stress categories). In finite element analyses, local high stresses, analogous to stress concentrations, may be reported. Examples of regions of local high stress include inadequate mesh refinement in areas of complex stress gradients, loading and geometry, or modeling distributed connections where the results are in terms of a point load rather than the true distributed load. In these cases, engineering judgement must be applied to the results to properly determine the allowable stress requirements.

6.1.1 General Stress - General stress is the average (normal and/or shear) stress resulting from global deformation of the structure under consideration.

6.1.2 Local Stress - Local stress (normal and/or shear) occurs in regions of load application or structural discontinuity. Stresses which exceed the general stress allowables may be considered local if the area over which the stress exceeds the general stress allowable does not exceed 10% of the effective area. Definitions of the effective area are shown in Table F.1 of Appendix F. The 10% limit can be waived if it can be demonstrated that the load carrying capacity of the structure is adequate.

6.1.3 General Membrane Stress - General membrane stress is calculated from the average normal and/or shear stress across the thickness or depth of a section under evaluation. For one-dimensional beam elements, this includes mean axial, shear and torsional shear stresses. The mean axial stress is the normal stress averaged over the effective cross-section under evaluation. It should be noted that for Grade A, elastic, case 2; Grade A, elastic-plastic and Grade B elastic, the membrane stresses (normal, shear), defined as the average stress components through the load carrying section, must be separated from the total stress prior to a Von Mises stress combination (see step 9 of 6.1.8.1).

6.1.4 General Membrane Plus Bending Stress - General membrane plus bending stress is calculated from stresses at the outermost fibers of the subject section. The bending stress is the variable component of the stress (normal and/or shear) across the thickness or depth of a section, but excludes peak stresses caused by geometric discontinuities. The variation may or may not be linear across the thickness or depth of a section. The depth of a section may be that of a composite section made up of effective plate elements of a finite element model or the thickness of a single plate element. General membrane plus bending stress includes membrane stress categorized as local in the evaluation of the adequacy of the cross section. The consideration of local membrane stresses may result in lower magnitudes of general bending stresses being considered acceptable.

6.1.5 Local Membrane Stress - Local membrane stress is calculated from the total membrane stress produced by mechanical loads, including the effects of constraint of adjacent material or self-constraint of the structure. It can occur in regions of gross or local structural discontinuities and at locations of intersecting structural members. Peak stresses are not limited.

6.1.6 Local Membrane Plus Bending Stress - Local membrane plus bending stress is calculated from the total stress evaluated at the outermost fibers of the subject section produced by mechanical loads including self-limiting stresses developed by the constraint of adjacent material or self-constraint of the structure. It can occur in regions of gross or local structural discontinuities and at locations of intersecting structural members. Peak stresses are not limited.

6.1.7 Adjacent Local Stressed Regions - Table F.2 of Appendix F provides examples of adjacent local stressed regions. Adjacent areas of local stress due to the introduction of concentrated loads may not overlap. The centers of adjacent local stressed regions cannot be closer than 2.5 times the average of the dimensions of the two locally stressed areas. The length of each locally stressed region shall be based on the limit of local stress exceeding general stress limits and shall be measured along a line of action between the center of each pair of adjacent locally stressed areas.

Figure 6-1. ALLOWABLE STRESS CRITERIA AND APPLICABLE DESIGN LEVELS

GRADE	ALLOWABLE DEFORMATION	SHOCK INPUT	TYPE OF STRESS	ALLOWABLE STRESS GENERAL	ALLOWABLE STRESS LOCAL	SECT
GRADE A	CASE 1. ALIGNMENT SENSITIVE, NO PERMANENT DEFORMATION ALLOWED	ELASTIC	$\sigma_{MEMBRANE}$	1.0 σ_{YIELD}	1.0 σ_{YIELD}	6.2.1
			$\sigma_{MEMBRANE} + \sigma_{BENDING}$	1.0 σ_{YIELD}	1.0 σ_{YIELD}	
			σ_{COLUMN}	BUCKLING	BUCKLING	
			$\sigma_{BEARING}$	1.6 σ_{YIELD}	1.6 σ_{YIELD}	
	CASE 2. ALIGNMENT SENSITIVE, SLIGHT PERMANENT DEFORMATION ALLOWED	ELASTIC	$\sigma_{MEMBRANE}$	1.0 σ_{YIELD}	1.5 σ_{YIELD}	6.2.2
			$\sigma_{MEMBRANE} + \sigma_{BENDING}$	1.0 σ_{EFF}	2.0 σ_{EFF}	
			σ_{COLUMN}	BUCKLING	BUCKLING	
			$\sigma_{BEARING}$	1.6 σ_{YIELD}	1.6 σ_{YIELD}	
	NON-ALIGNMENT SENSITIVE EQUIPMENT AND FOUNDATIONS, SLIGHT PERMANENT DEFORMATION ALLOWED	ELASTIC - PLASTIC	$\sigma_{MEMBRANE}$	1.0 σ_{YIELD}	1.0 σ_{YIELD}	6.3.1
			$\sigma_{MEMBRANE} + \sigma_{BENDING}$	1.0 σ_{YIELD}	1.0 σ_{YIELD}	
			σ_{COLUMN}	NO LIMIT	NO LIMIT	
			$\sigma_{BEARING}$	NO LIMIT	NO LIMIT	
	NO PERM DEFORM ALLOWED; BOLTS	ELASTIC	$\sigma_{MEMBRANE}$	1.0 σ_{YIELD}	1.0 σ_{YIELD}	6.5
	NON-ALIGNMENT SENSITIVE EQUIPMENT AND FOUNDATIONS, CONSIDERABLE PERMANENT DEFORMATION ALLOWED	ELASTIC - PLASTIC	$\sigma_{MEMBRANE}$	1.0 σ_{YIELD}	1.0 σ_{YIELD}	6.3.2
			$\sigma_{MEMBRANE} + \sigma_{BENDING}$	2.0 σ_{YIELD}	2.0 σ_{YIELD}	
			σ_{COLUMN}	NO LIMIT	NO LIMIT	
			$\sigma_{BEARING}$	NO LIMIT	NO LIMIT	
GRADE B	NO PERMANENT DEFORMATION ALLOWED	ELASTIC	$\sigma_{MEMBRANE}$	1.0 σ_{YIELD}	1.5 σ_{YIELD}	6.2.2
			$\sigma_{MEMBRANE} + \sigma_{BENDING}$	1.0 σ_{EFF}	2.0 σ_{EFF}	
			σ_{COLUMN}	BUCKLING	BUCKLING	
			$\sigma_{BEARING}$	1.6 σ_{YIELD}	1.6 σ_{YIELD}	
	NON-ALIGNMENT SENSITIVE, PERMANENT DEFORMATION ALLOWED (EQUIPMENT AND FOUNDATIONS)	ELASTIC - PLASTIC	$\sigma_{MEMBRANE}$	1.0 σ_{YIELD}	NO LIMIT	6.4
			$\sigma_{MEMBRANE} + \sigma_{BENDING}$	NO LIMIT	NO LIMIT	
			σ_{COLUMN}	NO LIMIT	NO LIMIT	
			$\sigma_{BEARING}$	NO LIMIT	NO LIMIT	
	NO PERM DEFORM ALLOWED; BOLTS	ELASTIC	$\sigma_{MEMBRANE}$	1.0 σ_{YIELD}	1.0 σ_{YIELD}	6.5

6.1.8 Stress Evaluation and Classification - A procedural outline is presented for finite element stress evaluation and classification to help ensure consistent application of the criteria to structural evaluations. The outline is specific to thin plate/shell stress evaluations, which represent a significant portion of the structural evaluations. Considerations for one-dimensional beam elements are also presented.

6.1.8.1 Stress Evaluation Procedure for Thin Plate/Shell Elements

Compute finite element stress components at the bottom, middle, and top surfaces of the plate element. Stresses may be evaluated at integration points of the element or extrapolated to nodal points for joint averaging. Significant differences in unaveraged nodal stresses from adjacent elements indicate a stress concentration or an inadequate mesh size. Joint averaging should not be performed at thickness discontinuities, at material modulus changes, or at geometric discontinuities such as the intersection of two plates. Stress components should be oriented such that predominant stress states (e.g., beam bending, axial stress, hoop stress, radial stress, etc.) can be evaluated.

1. Compute the stress within each mode based on the Von Mises Failure Theory as defined in Section 3.5.4. Membrane stresses are computed from mid-surface stress components. Membrane plus bending stresses are computed at extreme fibers of the plate.

2. Compute NRL summed total stress as defined in Section 3.5.5.

3. Review stresses computed in step 2 (i.e., through use of fringe/contour stress plots) noting which intensities exceed the established stress allowable (general membrane) for the particular shock grade of the structure. General membrane stress limits apply to mean axial and shear stress states over the member cross-sectional area.

4. In cases where the general membrane allowable is exceeded, further investigation of component level stresses in each mode will be required to classify the stress component as general membrane, general membrane plus bending, local membrane, or local membrane plus bending. A deformed plot for each mode of the subject structure can aid in classifying stresses.

5. To classify an outer fiber stress as general membrane plus bending, a variable component of stress through the thickness or depth of the section must be present. If general bending of the structural member is present, use of the general membrane plus bending limit is permitted.

6. To classify an outer fiber stress as local membrane plus bending, the stress must exist at a location of load introduction or structural discontinuity. The bending stress variation is predominantly through the thickness of a plate and limited to 10% of the effective area. Average shear stresses derived from plate punch-through and plate tear-out calculations shall be limited to the general membrane allowables.

7. The stress at a load introduction or structural discontinuity identified in step 6 may exceed the local membrane plus bending stress limit if it is confined to less than 5% of an effective area. No limits are established within this area. Average shear stresses derived from plate punch-through and plate tear-out calculations shall be limited to the general membrane allowables.

8. Classification of local membrane stresses is similar to classification of local membrane plus bending stresses except that local membrane stresses are evaluated at the plate mid-surface.

9. For those elements classified with general membrane plus bending or local stress, re-evaluate the Von Mises stress as follows. Re-compute the combined stress within each mode using component level stresses adjusted by the factor of the general membrane stress allowable over the stress allowable applicable to each respective component stress. Re-compute the NRL summation of stresses. Compare the NRL summation of combined stresses to the membrane stress allowables.

10. Failure to meet the specified allowables is cause for structural modification and re-analysis in accordance with Section 3.5.7 or in cases of local stresses further demonstration that the load carrying capacity of the structure is adequate.

    Note: General bending of a cross section may result in membrane stresses at the element level (i.e., for I-beam in strong axis bending, flanges will be predominantly membrane). It is not the intent of these criteria to limit element level membrane stresses to membrane allowables. However, such limitation would be conservative.

6.1.8.2 Stress Evaluation Procedure for Beam Elements

Stress evaluation for one-dimensional beam element models is limited to the general membrane and general membrane plus bending stress categories. Member mean axial and shear stresses are limited to the membrane stress allowables. Stresses evaluated at the extreme fibers of a beam cross section that includes bending stresses are limited to general membrane plus bending stress category. Transverse shear distributions may be averaged for use in Von Mises stress calculations.

6.2 Allowable Design Stresses for Grade A and Grade B Items Designed to Suit Elastic Shock Design Values

6.2.1 CASE 1 - Where deflection is critical, combined operating and shock stresses shall not exceed the material static yield strength (0.2% offset).

6.2.2 CASE 2 - Where slight permanent deformation of a cross-section can be tolerated, general membrane stresses (average normal and/or shear stress) are limited to the material static yield stress. The criterion of failure for general membrane plus bending stresses is the effective yield strength of the material. This effective yield strength is defined by

$$\sigma_{EFF} = \sigma_y + f\ (\sigma_u - \sigma_y)$$

In this equation σ_y is the 0.2% offset yield strength, elastic limit, or other accepted definition of material yield strength. σ_u is the conventional definition of material ultimate strength. All strengths are the values at the expected operating temperature.

The symbol f represents a factor which takes account of the efficiency with which the material in the member being analyzed is utilized. Examples of f are given below. The efficiency is computed by comparing the load required to just initiate yielding of the member with the load required to have the member completely yielded. In this computation it is assumed that the stress-strain curve of the material is bi-linear, with no strain hardening. The factor f (the efficiency minus one) is thus dependent on the kind of loading, i.e. tension, bending, etc., and on the cross-section of the member. For example, the factor is zero for any member in pure tension and 0.5 for a rectangular section in pure bending.

In general, brittle materials, as defined in Section 6.10, may not be used. However, where exceptions are granted the following applies: for any brittle material (one which has less than ten percent elongation before fracture in a tension test) the factor f is always zero. This is often true for ultra high strength steels and cast material (steel or aluminum). The factor f must be taken as zero for any application where a slight plastic set cannot be permitted.

The value of the factor f is taken from limit design theory, in which the existence of a "plastic hinge" is postulated. The plastic hinge occurs when the member's cross-section is fully yielded, as described above, in bending. Limit design theory may be used to define allowable component stresses under shock loading provided that the operability of any Grade A equipment is not compromised by the permanent distortion associated with yielding. For example, limit design theory permits the use of multiple plastic hinges under certain limited conditions.

Sample Factors f

Consider a rectangular bar subject to pure bending. The ratio of the fully plastic moment obtained by limit analysis to the bending moment at yield is well known to be 1.50.

So, $f = 1.50 - 1 = 0.5$ and the allowable stress is,

$$\sigma = \sigma_y + 0.5 (\sigma_u - \sigma_y)$$

For a typical I section,

$$f = A/(6 + 2A)$$

$$\text{where } A = \frac{\text{(web width) (depth of section)}}{2 \text{ (flange width) (flange thickness)}}$$

For a solid shaft in bending, $f = 0.7$

For a hollow shaft in bending, $f = 0.913 - 0.638(R^1/R)$ where R^1/R is the ratio of the inner to the outer radius and R^1/R is equal to or greater than 0.6.

If bending is combined with torsion, shear, tension or compression, then the analyst should compute the ratio of the maximum load to the yield load, and subtract one, to obtain the factor f.

6.2.3 For CASE 2, local stresses have higher limits than general stresses. The local membrane stress limits are 1.5 times the general membrane stress allowables. The local membrane plus bending stress limits are twice the general membrane plus bending stress allowables.

6.2.4 For CASE 1 and CASE 2, combined continuous operating and shock loads shall not exceed allowable column loads. Allowable bearing stresses are 160 percent of the material static yield strength.

6.2.5 Special design criteria must be considered in the case of equipment or foundation structures fabricated from aluminum or incorporating bimetallic (steel to aluminum) elements. Tabulated nominal yield stresses from contract specifications for welded aluminum alloys should be used to determine allowable design stresses. Manufacturers' specified yield strengths should be used as the basis for shock design evaluations of bimetallic elements. Consideration must be given in such evaluations to the increased width of the elements in comparison to the thicknesses of the steel or aluminum structural members adjacent to the bimetallic elements. In general, the design of the bimetallic elements should be such that their strength in shock is greater than that of adjacent structural members.

6.3 Allowable Design Stresses for Grade A Items Designed to Suit Elastic-Plastic Shock Design Values

6.3.1 In cases where it is necessary to limit permanent deflection to approximately twice the maximum elastic deflection at yield, the calculated stresses (from elastic-plastic analysis) shall not exceed the material static yield strength (0.2% offset). The limiting elastic-plastic deflection used for evaluation is twice the deflection that occurs at yield. Where deflections are critical, elastic-plastic analysis cannot be used.

6.3.2 In cases where considerable plastic bending can be tolerated (as is usually the case with foundations designed to suit elastic-plastic shock design values), membrane plus bending stresses not exceeding 200% of the material static yield strength will be considered acceptable. Membrane stresses shall not exceed the material static yield strength.

a) Where 200% allowable stress criteria apply, continuous operating stresses (if present) shall be doubled before combining same with shock stresses.

b) Combined stresses, calculated as described in Sections 3.5.4 and 3.5.6, shall not exceed the material static yield strength. (Calculated bending stresses subject to 200% allowable stress criteria shall be halved before inserting into the combined stress formula.)

6.3.3 Allowable stress criteria for areas of foundations or equipments in way of holddown bolts are the same as for other areas of the foundation and equipment. However, average shear stresses derived from plate punch-through and plate tear-out calculations shall be limited to general membrane stress allowables. Higher loadings resulting (in some cases) from special criteria applied for purposes of holddown bolt design are applicable solely to holddown bolting and shall not be transferred for design purposes to foundations or equipment.

6.3.4 Column buckling and bearing stresses need not be considered.

6.4 Allowable Design Stresses for Grade B Items Designed to Suit Elastic-Plastic Shock Design Values - Allowable design stresses for Grade B items are the same as those which apply to Grade A items, except that bending stresses need not be considered in cases where it is evident that plastic bending of the members in question will not lead to violation of Grade B criteria. There are no limits placed on local stresses. In cases where the above cannot be assured, the allowable stress criteria described previously for Grade A items shall apply.

6.5 Allowable Bolt Stresses - For bolts, where MIL-S-1222 applies, the elastic proof stress may be considered as the yield stress. For bolts fabricated from materials other than the materials included in MIL-S-1222, the material static yield strength is the allowable stress. See Section 3.1.3.d of this report for related criteria. If not shock qualified with the equipment, fasteners used as holddown devices under shock loading shall be designed for axial and shear loads so that the stress measure does not exceed the static yield strength of the material. The Von Mises Failure Theory shall be used to combine the normal and shear stresses. Typically, fastener bending stresses are not considered under shock loading. However, where consideration for fastener bending is required, the maximum value of stress measure at the periphery of the fastener resulting from direct tension, shear, and bending, but excluding stress concentration, shall not exceed the static yield strength.

6.6 Allowable Stresses for Wire Rope

6.6.1 For Grade A systems in which no permanent deformation can be tolerated, 60% of the specified nominal breaking strength used in conjunction with elastic inputs shall be the basis for shock design of wire rope.

6.6.2 For Grade A and B systems in which permanent deformation can be tolerated, 75% of the breaking strength in conjunction with elastic inputs shall be the basis for the design of wire rope. Elastic-plastic inputs will not be used in conjunction with the design of wire rope.

6.6.3 Reduction of effective breaking strength due to wear, abrasion, lubrication, corrosion, etc. are included in the determination of the preceding values. The fact that wire rope does not possess the same degree of energy absorption (beyond the elastic limit) as a solid steel bar is also included in the 60% and 75% values noted above.

6.7 Allowable Stress for Non-Metallic Material - For material where the creep strength is low in relation to the yield strength and where pre-load is an important factor in shock design, the allowable stress for joint design shall be creep strength rather than yield.

6.8 Special Stress Criteria for Foundations - In order to minimize weight, maximum shock stresses on foundation members whose size is governed by shock shall exceed 75% (but not 100%) of allowable tensile, compressive or shear stresses in at least one primary member for all foundations supporting Grade A and B machinery and equipment systems weighing more than 125 pounds. A primary member is any main structural supporting member. Foundations for machinery and equipment systems that weigh less than 125 pounds are not covered by this requirement. Shock design values to be used for foundation dynamic analysis are specified in design data sheet DDS 072-1. Allowable stresses for foundations designed by method 1 (see Chapter 4) are the same as apply to foundations which are designed to suit elastic-plastic shock design values.

6.9 Special Criteria for Piping Connections - When determining the stress in nozzles due to restraint of attached piping, maximum shock motion of mounts shall be considered or the nozzles shall be designed to withstand the fully plastic moment of the attached piping.

6.10 Ductility - In developing the allowable stress criteria presented in this chapter it was assumed that the material under consideration has adequate ductility (expressed, for example, as % elongation measured in a tensile test). Adequate ductility means that the material is not subject to a brittle fracture failure, but will yield plastically before fracturing. Many types of cast materials do not exhibit adequate ductility and thus cannot be analyzed with the criteria contained herein. Elements with less than 10% ductility shall not be used in structural applications which are intended to withstand shock loading.

6.11 Special Criteria for Design of Hold-Down Bolts - When a bolted joint is loaded in tension (pre-load), shock loads do not directly increase the stress in the bolt, but decrease the clamping force between the bolt flange and the foundation. If the bolt load exceeds the clamping force the flanges will separate and the bolts will begin to stretch. Acceptability criteria are exceeded when the load exceeds the yield strength or proof load of the bolt. The adequacy of the joint in a quasi-static condition (when the load is gradually applied) depends more on the bolt material strength than the tightness of the joint. Under dynamic loads, however, the stiffness of the joint decreases radically when the flanges separate and the system goes through a part of its cycle at a reduced frequency, with correspondingly increased deflection, until the gap re-closes with associated hammering and chatter. The initial tightness of the bolted joint therefore, is of vital importance for system shock resistance since this hammering may be a more significant damage mechanism for the equipment than direct acceleration associated with the shock motion. In shock design calculations a bolted connection may not be adequate if the pre-stress is exceeded regardless of the strength of the bolt. In order not to waste their strength, bolts subjected to shock loading should be tightened to near their yield strength. Generally, achieving bolt loads of 80% to 90% of yield are considered practicable. To prevent separation of the equipment flange and its foundation, shipbuilding specifications require that threaded fasteners, which are used to hold down machinery and equipment to sub-bases and foundations, shall be of the self-locking type. In connection with this requirement, the pre-load torque necessary to achieve the desired clamping force for hold-down bolts of Grade A machinery and equipment must be determined in the associated foundation shock design calculations and specified on the applicable installation drawings.

Chapter 7 - DYNAMIC SHOCK ANALYSIS REVIEW AND APPROVAL PROCEDURES

7.1 Background Because of the specialized nature of shock design requirements and in particular the extreme importance of consistent and qualified determination of compliance, the need for a responsible centralized review activity was recognized by the Navy. Shipbuilding specifications generally indicate that review and approval of the mathematical model and the dynamic analysis will be made by NAVSEA. To meet this need, a special group was established and trained within the office of the Supervisor of Shipbuilding, Brooklyn, N.Y. In 1965 the Dynamic Shock Analysis Division, Code 280, Supervisor of Shipbuilding, Brooklyn, New York was assigned responsibilities to provide centralized technical support in review/approval of dynamic analysis. Currently the responsibility resides with the Supervvisor of Shipbuilding, Conversion & Repair, USN, Portsmouth Detachment, Colts Neck, NJ

In June 1966 NAVSHIPS INSTRUCTION 9400.13 was issued. That document outlined the Navy's mathematical model report and dynamic shock analysis report review and approval procedures. NAVSHIPS INSTRUCTION 9400.13, with modifications, forms the basis of this chapter. The mathematical model report and dynamic shock analysis review and approval requirements described in this chapter shall be considered to apply unless specifically modified by applicable contract specifications.

7.2 Report Format and Content The format and content required by the Navy for mathematical model reports and dynamic shock analysis reports are as follows:

7.2.1 Mathematical Model Report Format and Content The mathematical model report describes the structural and functional characteristics and the mathematical model of a shipboard equipment or structure, with its foundation, for purposes of dynamic shock analysis. The report is used to provide assurance that the equipment or structure will be properly modeled prior to submittal of the dynamic shock analysis report. The mathematical model report shall contain the following information as a minimum:

7.2.1.1 An introductory description of the equipment or structure being analyzed and its normal function or operation.

7.2.1.2 The planned location and orientation of the equipment or structure with respect to the ship's axes.

7.2.1.3 The shock Grade (A or B) to which the equipment is to be qualified.

7.2.1.4 Mounting location (hull, deck or shell) of the equipment.

7.2.1.5 Type of shock design value (elastic or elastic-plastic) to be used in the analysis.

7.2.1.6 Procurement specification(s) under which the equipment is procured.

7.2.1.7 Description of proposed method of analysis.

7.2.1.8 A list of specific areas of concern of the equipment or structure which might be subject to high stresses or deflections under shock loading. Particular attention should be given to the proposed failure criteria for each area. Yield stress or effective yield stress criteria (at normal equipment operating temperatures) shall be described. The consequences of failure in each critical area shall be considered. The effects of a postulated failure on equipment operability or on potential personnel hazards must be included.

7.2.1.9 Assumptions which have been made in the preparation of the model and justification for such assumptions.

7.2.1.10 An estimate of the weight and location of center of gravity of the equipment or structure. A listing of weights of components which are used to arrive at the equipment weight shall also be included.

7.2.1.11 A description of the proposed breakdown of the equipment or structure for analysis. The description must indicate how the proposed mass breakdown permits determination of stresses or deflections in the previously defined areas of concern.

7.2.1.12 A separate list of all lumped masses considered in the mathematical model shall be provided. This list shall specify the location with respect to a specified coordinate system and the composition, magnitude and direction of associated degrees of freedom for each lumped mass. The model report shall discuss the extent and magnitude of computer generated distributed mass used in the problem.

7.2.1.13 If dynamic reduction techniques are to be used in the shock analysis, the mathematical model report shall fully describe the controls that will be applied to ensure that important response characteristics will not be overlooked. The center of gravity of the mathematical model masses for the original and reduced model shall be determined and identified in the model report. See Section 7.2.1.10. The model report shall also provide a list of the master degrees of freedom. The planned dynamic reduction process and associated criteria for reducing the problem size must be specifically approved by the Navy.

7.2.1.14 A description of the extent and structural characteristics of the foundation. Sketches or drawings are required as part of the model report to indicate the arrangement of the equipment and its foundation.

7.2.1.15 Properly labeled figures and text to describe the model for each direction of shock shall be provided. The text shall discuss:

(1) Formulation of the model.

(2) Representative element properties.
(3) Details associated with combining shock stresses with continuing operating stresses.

When the model is prepared for finite element computer analysis, the following information shall also be included:

(4) A description of the applicable portion of the computer program and the characteristics of the elements to be used.
(5) A complete printout and description of the input data used.
(6) The node and element numbering system and plots of the model to help the reviewer correlate specific nodes, elements and lumped mass locations with the input data.
(7) Boundary conditions used in the model.

Where special modeling techniques are used such as mesh generation routines, sub-structuring, etc., additional information shall be furnished to clearly describe the process including objectives and limitations.

7.2.1.16 A map of the finite element model (figures or sketches) shall be provided showing grid point (or node) numbers, element numbers and lumped mass locations (this information can be provided by separate figures). Computer generated mathematical model figures (graphics) are often difficult to read. Care should be taken so that the material is legible and clear.

7.2.1.17 Fixed-base natural frequency calculations of suspected low frequency system components, (e.g., shafts, cantilevered equipment, yardarm) should be provided. A comparison of these frequency values to the cut-off frequency of the system shall be made and the components modelled accordingly.

7.2.1.18 References to the source of analysis method, formulas, constants, curves and all other sources used. Shock tested items which are a part of the equipment or structure to be analyzed must be included in the model but need not be modelled in detail. Wherever qualification of components is to be through MIL-S-901 testing, rather than through analysis, the mathematical model report shall contain information on the status of the testing. If testing has been completed, references shall be given to the test report and applicable approvals by NAVSEA or its representatives. If testing is to be done in the future, schedules and planned test facilities should be described.

7.2.1.19 Equipment outline and assembly drawings, support, sub-base and foundation plans. The report shall include preliminary drawings when final drawings are not available. If no drawings are available, sketches shall be provided. These drawings or sketches shall disclose a level of design detail commensurate with the analysis. Detailed working drawings are not required.

7.2.1.20 A simplified bench-mark model, including all input and output, shall be provided separately or with the model report if requested by the Navy. The purpose of this bench-mark problem is to ensure that the DDAM criteria are correctly applied. The characteristics and parameters of the bench-mark model shall be as specified by the Navy (or a simple three degree of freedom model that can easily be verified by hand calculations). Stress calculations in the bench-mark problem should be limited to beam-type stresses. The bench-mark problem shall also demonstrate pre- and post-processing routines and any special modeling procedures or capabilities that are planned for the shock design analysis.

7.2.2 Dynamic Analysis Report Format and Content The dynamic analysis report demonstrates the ability of equipment, structures and systems to resist shock as defined by the Dynamic Design Analysis Method (DDAM). The report is used in conjunction with the mathematical model report when an item's shock resistance cannot be determined by shock testing or extension from a previously qualified item. The dynamic analysis report shall contain the following information as a minimum:

7.2.2.1 A printout of the input data used in the analysis. This data shall include all nodal point locations, element connectivity, material properties, element properties and mass distribution. The DDAM report shall include a full description of the mathematical model used. The approved mathematical model report may be submitted as an appendix to the final DDAM report. Any differences between the approved mathematical model and the model presented in the DDAM shall be noted, fully explained and justified. When computer output on large finite element analyses is too voluminous for inclusion in the dynamic analysis report, Supervisor of Shipbuilding, Portsmouth Detachment, Colts Neck, NJ. should be consulted to obtain a precise definition of the data which may be excluded.

7.2.2.2 A list of all calculated modal frequencies, modal effective weights, participation factors and modal design inputs for all modes of the system including those not considered in the stress analysis process. This list shall also identify the modes which are used in the stress or deflection calculations. Mode shapes and associated forces and deflections for all modes considered in the stress or deflection calculations shall be included in the report. If computer output is used directly, adequate references and sufficient explanatory detail must be provided to facilitate review.

7.2.2.3 A graph showing modal effective weight versus modal frequency. Closely spaced modes occurring in a DDAM analysis can produce misleading results. The existence of closely spaced modes can best be determined by a graph showing the modal effective weight versus modal frequency for all the modes chosen for analysis. This representation will show potential closely spaced modes. All DDAM analyses reports must contain this graph in order to show that the assumptions of the DDAM with respect to closely spaced modes have not been violated. Where closely spaced modes exist an additional graphic representation is required to evaluate the effect of the closely spaced modes on the system design. This second graph shows the modal response versus node point for the modes which are considered to be closely spaced.

7.2.2.4 Calculations of stresses and deflections at those specific areas of concern on the equipment or structure under shock loading, as defined in the mathematical model report. References to the source of data used in these calculations shall be provided. Drawings which aid in an independent review of the calculations shall be provided. If no drawings are available, sketches shall be provided.

7.2.2.5 Tabulated summaries of calculated and allowable stresses and deflections. These summaries shall include the sources of the tabulated stresses and deflections (for example, tensile, shear and operating loads). NRL sum of stresses for all elements in the mathematical model shall be provided in the DDAM report. Where the element is an equivalent elastic member, such as a spring or a uniform beam rather than a comprehensive finite element description, the effective forces or stresses on the actual structural element shall be derived and presented in separate calculations.

7.2.2.6 A list of any elements with a negative margin of safety. Where an over-stress is indicated, a proposed remedy for the condition is required. The effect of any such changes on the overall analysis shall be provided. A re-analysis may be required by the Navy. If re-analysis is required a formal plan of action and milestones (POAM) must be submitted which defines the dates by which necessary NAVSEA approvals for the design change must be obtained, as well as dates for completion of detail design and installation of the change.

7.2.2.7 A comprehensive analysis of the foundation, when such foundation is supplied by the equipment vendor. When the foundation is provided by the shipbuilder, the vendor shall provide a summary of the shock forces into the foundation for use by the shipbuilder in his analysis.

7.2.2.8 A full description of the application of ASM shall be submitted if ASM is used to evaluate responses as part of a corrective action recommendation report. This discussion shall provide the following information as a minimum:

- o description of the response characteristics under investigation
- o time step used
- o period of duration of the ASM
- o lowest modal frequency
- o highest modal frequency considered in the analysis
- o the suspected closely spaced modes for each member evaluated

7.2.2.9 A list of modal accelerations for sub-component appendages (such as antennas on mast yard arms). This list shall include all modes of response and shall be sorted in decreasing order by magnitude of the acceleration. The DDAM analysis shall include, in addition to the normal mode selection, the modal stresses or deflections for at least the two most severe responses associated with each appendage.

7.2.2.10 Where plate finite elements are used in the mathematical model, for which forces and stresses are calculated at each node point in the plate element, the values at high stress areas may not be averaged between elements unless it can be demonstrated that the variations in unaveraged stresses in the region of interest are within acceptable limits. A hard copy printout of the unaveraged node stresses in the region of interest can be used to supplement contour plots with averaged stresses. The evaluation of adequacy of mesh discretization will be based on the relative magnitudes of stress among adjacent elements. Typically, in an adequately refined mesh, the contour plots of Von Mises effective stresses will reveal "Stress Bands" which are slightly discontinuous across element boundaries. Large discontinuities indicate a mesh which is too large.

7.3 Review and Approval Authority - Mathematical model reports, dynamic shock analyses, and shock extension requests based upon dynamic analyses shall be forwarded to the appropriate Navy agency as indicated by this document or ship contracts and/or specifications. All dynamic analysis submittals not covered by this section shall be forwarded to NAVSEA for review and approval.

7.3.1 Equipment, Weapons and Systems Analyses - Mathematical models, dynamic shock analyses, and extension requests based upon approved dynamic analyses developed to satisfy contractual requirements shall be forwarded to the Supervisor of Shipbuilding, Portsmouth Detachment, Colts Neck NJ. for approval action.

7.3.2 Foundation Analyses - Where required, foundation dynamic analyses shall be subject to review and approval by the local design approval agency unless otherwise stated by applicable specifications. (Review of foundation analyses performed by Government activities will be by the Supervisor of Shipbuilding, Portsmouth Detachment, Colts Neck NJ., upon request, or as required by applicable directives.)

7.3.3 Ten Sample Foundations - Where the ship's specifications/contracts require the shipbuilder to submit sample foundation calculations to the Navy for review, these calculations shall be forwarded to NAVSEA or its designated approval authority. The shipbuilder shall prepare sample shock calculations for a series of at least ten foundations covering all elements noted below. This selected set of calculations will constitute a diverse and representative sample describing the application of shock design requirements by the shipbuilder. The math model and analysis may be submitted together. Calculations for additional foundations shall be provided if requested.

All the following categories shall be included in the sampling. One foundation model may be used to address more than one of the categories listed below. Foundations associated with equipment DDAM analysis shall be prepared with the equipment DDAM analysis and shall not be included in the list of sample foundations.

a. Foundation for hull mounted equipment
b. Foundation for deck mounted equipment
c. Equipment foundation including a sway brace configuration
d. Foundation for resiliently mounted equipment
e. Foundation for overhead mounted equipment
f. Foundation for bulkhead mounted equipment (structural bulkheads)
g. Foundation for bed-plate, raft, or pallet mounted equipment (items with two or more mounted components)
h. Foundation for a typical electrical power distribution switchboard
i. Foundation for bulkhead mounted equipment (joiner bulkhead)
j. Foundation with an upper support in addition to a base mount
k. Foundation for Grade A alinement sensitive equipment
l. Foundation for typical Grade B equipment
m. Foundation for a fire pump
n. Foundation for equipment with critical clearance requirement
o. Typical deck-to-deck foundation
p. Foundation for free standing tank

7.4 Navy Review and Approval/Disapproval Cycle

7.4.1 Unless modified by the shipbuilding or contract specification, the Navy will complete action on math modal reports within 60 days of receipt of same. Provisional approvals may be granted to permit proceeding with the analyses in cases where only minor corrections and/or additional reference material are required. In such cases the cognizant design approval agency will ensure that supplemental material is forwarded promptly.

7.4.2 For mathematical models which are disapproved, the forwarding letter will indicate the basis for disapproval. The cognizant design approval agency is expected to follow-up the rejection to ensure that the shipbuilder or contractor is aware of the need for timely response.

7.4.3 Unless modified by the shipbuilding or contract specification, the Navy will complete action on dynamic shock analysis reports within 60 days of receipt of same.

7.4.4 For dynamic analysis reports which are not approved, the forwarding letter will indicate the basis for disapproval. The cognizant design approval agency is expected to follow-up the rejection to ensure that the shipbuilder or contractor is aware of the need for timely response.

7.4.5 Re-submittals of model reports and dynamic analyses which involve the review of extensive modifications shall be treated as new submittals and subject to the applicable Navy review times stated above.

7.4.6 The allotted time for Navy review and approval/disapproval of all other dynamic analysis submittals shall be determined by NAVSEA on a case basis.

7.5 Guidelines and Requirements

7.5.1 A list of all equipment requiring dynamic shock analysis shall be prepared by the shipbuilder or contractor and forwarded within 60 days of the signing of the contract, unless otherwise indicated by appropriate specification or contract.

7.5.2 A planned schedule of submittals of mathematical models and dynamic shock analysis shall be prepared by the shipbuilder or contractor and forwarded within 30 days of item 7.5.1 above. The schedule shall be updated at 30 day intervals unless otherwise indicated in the appropriate specification or contract. This schedule shall be based on realistic vendor information and shall reflect the shipbuilder's or contractor's requirements for orderly plan development and production/delivery schedules.

7.5.3 Each mathematical model report and dynamic analysis report for an equipment being analyzed must provide sufficient information and detail to permit timely review. Items indicated in Sections 7.2.1 and 7.2.2 of this chapter are needed to establish the suitability of these reports. The cognizant design approval agency will screen all mathematical model reports and dynamic analysis reports for conformance with guidelines of this chapter, prior to submittal to Supervisor of Shipbuilding, Portsmouth Detachment, Colts Neck NJ. In order to expedite review, the local design approval agency may authorize direct liaison between SUPSHIP and the shipbuilder or contractor.

7.5.4 Since it is the responsibility of the cognizant design approval agency to ensure that characteristics of the equipment are in conformity with the applicable ship or equipment specification, modifications to equipment or foundations which are indicated by the analysis shall be monitored by the design approval agency to ensure that the equipment installation complies with the analyzed system. Responsibilities for approval of plans and installations are not transferred to Supervisor of Shipbuilding, Portsmouth Detachment, Colts Neck NJ. by this document.

7.5.5 The shipbuilder (or his design agent or the prime contractor for Government furnished material) shall ensure that all model reports and analyses are acceptable and shall indicate in the forwarding letter that such documentation satisfies all of the requirements of the applicable specifications.

7.5.6 In order to provide for timely submittals and reviews, all local design approval agencies shall incorporate the reporting and review actions of this document in all contracts involving dynamic shock design requirements and on outstanding contracts where applicable and permissable under existing provisions.

APPENDICES

APPENDIX A
SAMPLE COMPUTATION OF NORMAL MODES OF A STRUCTURE

It is the purpose of this appendix to illustrate by a simple numerical example, the computation of required normal modes of a structure. Consider the following system:

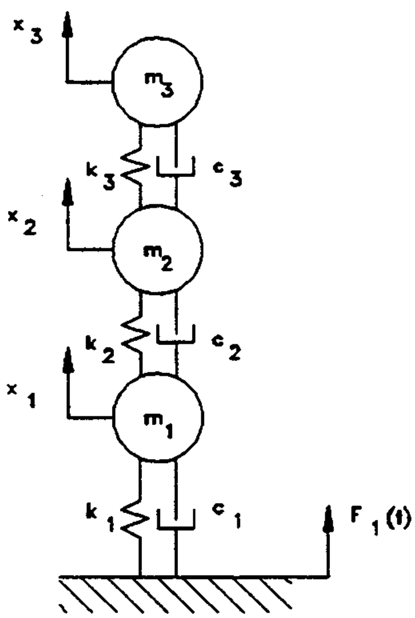


Figure A-1 Mathematical Model - 3 Degree of Freedom System

m_i = Mass Value

k_i = Stiffness Coefficient

c_i = Damping Coefficient

x_i = Displacement Coordinate

The equations of motion for the system, which are obtained by considering the dynamic equilibrium of each mass are:

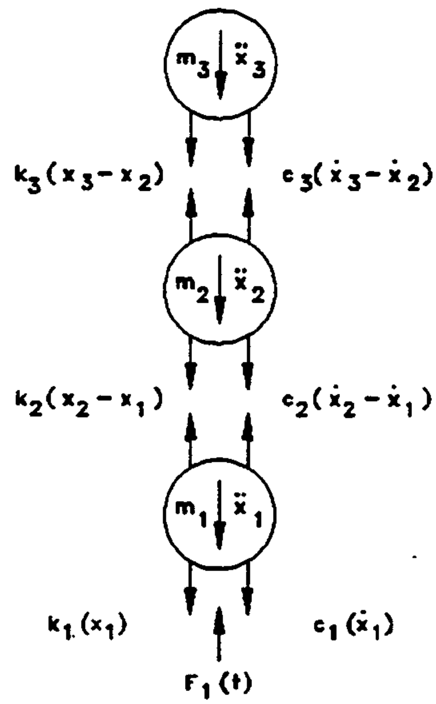


Figure A-2
Free Body Diagram
3 Degree of Freedom System

$\dot{x}_i$ = Velocity

$\ddot{x}_i$ = Acceleration

$$m_1\ddot{x}_1 + k_1x_1 - k_2(x_2 - x_1) + c_1\dot{x}_1 - c_2(\dot{x}_2 - \dot{x}_1) = 0$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) - k_3(x_3 - x_2) + c_2(\dot{x}_2 - \dot{x}_1) - c_3(\dot{x}_3 - \dot{x}_2) = 0$$

$$m_3\ddot{x}_3 + k_3(x_3 - x_2) + c_3(\dot{x}_3 - \dot{x}_2) = 0$$

These equations may be conveniently written in matrix form as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \qquad (1)$$

where:

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \quad [C] = \begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \quad [K] = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \quad \{F(t)\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

For undamped free vibration, the damping matrix [C] and the forcing vector {F(t)} are equal to zero and equation (1) reduces to:

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \qquad (2)$$

These equations are solved by substituting:

$$x_1 = a_1 \sin(\omega t)$$

$$x_2 = a_2 \sin(\omega t)$$

$$x_3 = a_3 \sin(\omega t)$$

$$\ddot{x}_1 = -a_1 \omega^2 \sin(\omega t)$$

$$\ddot{x}_2 = -a_2 \omega^2 \sin(\omega t)$$

$$\ddot{x}_3 = -a_3 \omega^2 \sin(\omega t)$$

into equation (2), and canceling the factor $\sin(\omega t)$ to obtain:

$$-m_1 a_1 \omega^2 + k_1 a_1 - k_2(a_2 - a_1) = 0$$

$$-m_2 a_2 \omega^2 + k_2(a_2 - a_1) - k_3(a_3 - a_2) = 0$$

$$-m_3 a_3 \omega^2 + k_3(a_3 - a_2) = 0$$

In matrix form:

$$\begin{bmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 - m_2\omega^2 & -k_3 \\ 0 & -k_3 & k_3 - m_3\omega^2 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (3)$$

For a non-trivial solution, we require that the determinant of the coefficient matrix be equal to zero (eigenvalue problem), that is:

$$\begin{vmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 - m_2\omega^2 & -k_3 \\ 0 & -k_3 & k_3 - m_3\omega^2 \end{vmatrix} = 0 \qquad (4)$$

The expansion of the determinant gives a cubic equation in ω^2 namely:

$$m_1m_2m_3\omega^6 - [k_3m_1m_2 + (k_1+k_2)m_2m_3 + (k_2+k_3)m_1m_3]\omega^4 +$$

$$[k_2k_3m_1 + (k_2k_3 + k_1k_3)m_2 + (k_1k_2 + k_2k_3 + k_1k_3)m_3]\omega^2 - k_1k_2k_3 = 0$$

Substituting the values for m_1, m_2, m_3, k_1, k_2 and k_3

$m_1 = 7.764$ lb-sec²/in.

$m_2 = 5.176$ lb-sec²/in.

$m_3 = 2.588$ lb-sec²/in.

$k_1 = 8.4804 \times 10^6$ lb/in.

$k_2 = 5.6536 \times 10^6$ lb/in.

$k_3 = 2.8268 \times 10^6$ lb/in.

the cubic equation becomes:

$$y^3 - 4.551133 \times 10^6 y^2 + 5.36876 \times 10^{12} y - 1.303144 \times 10^6 = 0$$

where: $y = \omega^2$

The roots of this cubic are:

$$\omega_1^2 = 326,722$$

$$\omega_2^2 = 1,424,591$$

$$\omega_3^2 = 2,799,875$$

Therefore, the natural frequencies of the system are:

$$\omega_1 = 571.60 \ \frac{\text{rad}}{\text{sec}}$$

$$\omega_2 = 1193.56 \ \frac{\text{rad}}{\text{sec}}$$

$$\omega_3 = 1673.28 \ \frac{\text{rad}}{\text{sec}}$$

or in cycles per second:

$$f_1 = 90.97 \text{ Hz}$$

$$f_2 = 189.96 \text{ Hz}$$

$$f_3 = 266.31 \text{ Hz}$$

The modal shapes are then determined by substituting each of the values for the natural frequencies into equation (3), deleting one of the equations, and solving the remaining two equations for two of the unknowns in terms of the third unknown. The first parameter a_1 is set to 1.00. Performing these operations, we obtain the following values for the modal shapes:

$a_{11} = 1.00 \quad a_{12} = 1.00 \quad a_{13} = 1.00$

$a_{21} = 2.05 \quad a_{22} = 0.54 \quad a_{23} = -1.34$

$a_{31} = 2.93 \quad a_{32} = -1.79 \quad a_{33} = 0.86$

or in graphical form:

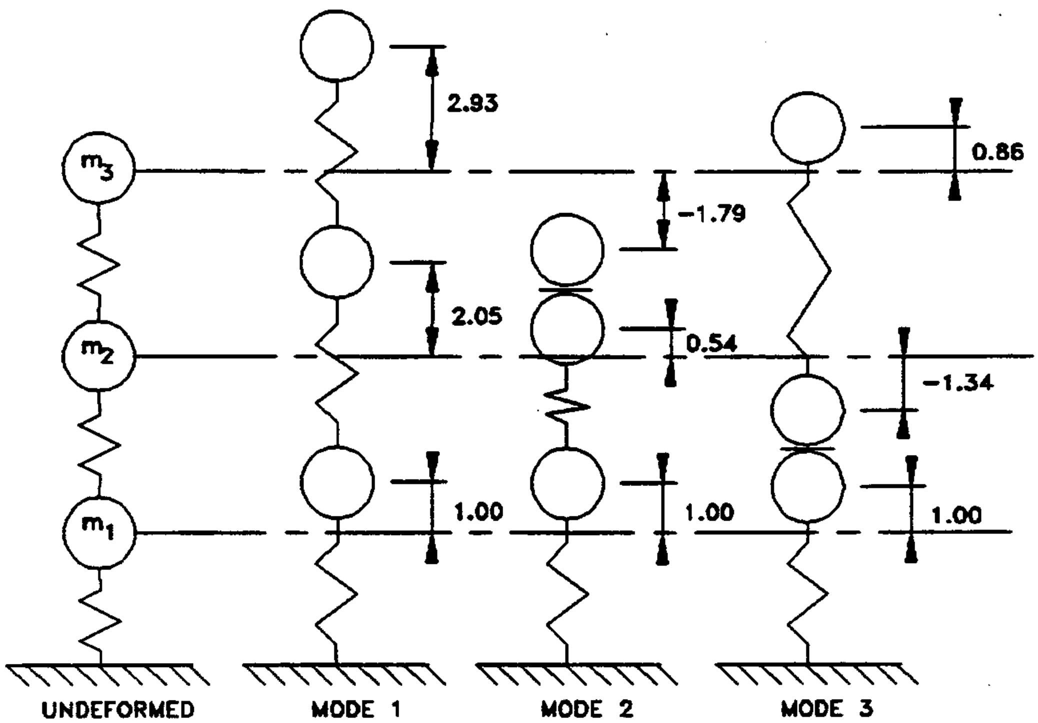


Figure A-3 - Mode Shapes, Three Degree of Freedom System

The eigenvalue problem may also be solved by numerical techniques. There are many methods which can be used to solve the eigenvalue problem. However, the inverse iteration technique is demonstrated here, since it is employed in various important iteration procedures including the determinant search and subspace iteration. The following discussion is presented to illustrate a typical computer analysis method rather than provide the reader with a manual computation approach which would rarely be used. The method presented below converges to the lowest eigenpair, however, shifts may be applied to obtain the higher order eigenpairs.

In the solution, a starting iteration vector $\{X_1\}$ is assumed and then equation (4) is evaluated in each iteration step k=1, 2.....

$$[K]\{X_{k+1}\} = [M]\{X_k\} \qquad (4)$$

after convergence, equations (5) and (6) are evaluated.

$$X_{k+1} = \frac{X_{k+1}}{\sqrt{X_{k+1}^T[M]X_{k+1}}} \qquad (5)$$

$$\rho(X_{k+1}) = \frac{X_{k+1}^T[K]X_{k+1}}{X_{k+1}^T[M]X_{k+1}} \qquad (6)$$

as k goes to infinity, X_{k+1} goes to ϕ_1 (eigenvector) and $\rho\{x_{k+1}\}$ goes to ω_1 (eigenvalue).

The solution for the first eigenpair using this technique will be demonstrated for the sample problem. The higher order pairs may be obtained by imposing a shift on the original matrices and proceeding in the same fashion.

$$[K] = \begin{bmatrix} 14.134 & -5.6536 & 0 \\ -5.6536 & 8.4804 & -2.8265 \\ 0 & -2.8265 & 2.8265 \end{bmatrix} \times 10^6$$

$$[M] = \begin{bmatrix} 7.76 & 0 & 0 \\ 0 & 5.176 & 0 \\ 0 & 0 & 2.588 \end{bmatrix}$$

To solve the equations in (4), it is first necessary to decompose the stiffness matrix [K] into its triangular factors [D] and $[L]^T$. The general equations for the decomposition are as follows:

$$g_{i,j} = k_{i,j}$$

$$g_{ij} = k_{ij} - \sum_{r=1}^{i-1} l_{ri} g_{rj} \qquad i = 2, \ldots, j-1$$

$$l_{ij} = \frac{g_{ij}}{d_{ii}} \qquad i = 1, \ldots, j-1$$

$$d_{jj} = k_{jj} - \sum_{r=1}^{j-1} l_{rj} g_{rj}$$

The particular solution is:

$$d_{11} = k_{11} = 14.134 \times 10^6$$

$$g_{12} = k_{12} = -5.6536 \times 10^6$$

$$l_{12} = g_{12}/d_{11} = (-5.6536 \times 10^6) / (14.134 \times 10^6) = -0.4$$

$$d_{22} = k_{22} - l_{12}g_{12} = (8.4804 \times 10^6) - (-0.4)(-5.6536 \times 10^6) = 6.226 \times 10^6$$

$$g_{23} = k_{23} = -2.8268 \times 10^6$$

$$l_{23} = g_{23}/d_{22} = (-2.8268 \times 10^6) / (6.226 \times 10^6) = -.454$$

$$d_{33} = k_{33} - l_{23}g_{23} = (2.8268 \times 10^6) - (-0.454)(-2.8268 \times 10^6) = 1.543 \times 10^6$$

The resulting decomposed matrices are:

$$[D] = \begin{bmatrix} 14.134 & 0 & 0 \\ 0 & 6.226 & 0 \\ 0 & 0 & 1.543 \end{bmatrix} \times 10^6$$

$$[L]^T = \begin{bmatrix} 1 & -0.4 & 0 \\ 0 & 1 & -0.454 \\ 0 & 0 & 1 \end{bmatrix}$$

Equation (4) may now be written as:

$$10^{6}\begin{bmatrix}14.134 & 0 & 0\\ 0 & 6.226 & 0\\ 0 & 0 & 1.543\end{bmatrix}\begin{bmatrix}1 & -0.4 & 0\\ 0 & 1 & -0.454\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}X_{12}\\ X_{22}\\ X_{32}\end{Bmatrix} = \begin{bmatrix}7.76 & 0 & 0\\ 0 & 5.176 & 0\\ 0 & 0 & 2.588\end{bmatrix}\begin{Bmatrix}1\\ 1\\ 1\end{Bmatrix}$$

Multiplying through:

$$10^{6}\begin{bmatrix}14.134 & -5.6536 & 0\\ 0 & 6.226 & -2.827\\ 0 & 0 & 1.543\end{bmatrix}\begin{Bmatrix}X_{12}\\ X_{22}\\ X_{32}\end{Bmatrix} = \begin{Bmatrix}7.76\\ 5.176\\ 2.588\end{Bmatrix}$$

Reducing the right side vector:

$$q_i = Q_i - \sum_{r=m_i}^{i-1} l_{ri}\, q_r$$

$$q_1 = 7.764$$

$$q_2 = 5.176 - l_{12}(V_1) = 5.176 - (-0.4)(7.76) = 8.282$$

$$q_3 = 2.588 - l_{23}(V_2) = 2.588 - (-0.454)(8.28) = 6.340$$

solving for $\{\bar{X}_2\}$

$$10^{6}\begin{bmatrix}14.134 & -5.6536 & 0\\ 0 & 6.226 & -2.827\\ 0 & 0 & 1.543\end{bmatrix}\begin{Bmatrix}X_{12}\\ X_{22}\\ X_{32}\end{Bmatrix} = \begin{Bmatrix}7.764\\ 8.282\\ 6.340\end{Bmatrix}$$

$$\bar{X}_{32} = 6.340/1.543 \times 10^{6} = 4.10894 \times 10^{6}$$

$$\bar{X}_{22} = (8.282 + (2.827 \times 10^{6})(4.1089 \times 10^{-6}))\ /\ (6.226 \times 10^{6}) = 3.1956 \times 10^{-6}$$

$$\bar{X}_{12} = (7.764 + (5.6536 \times 10^{6})(3.1956 \times 10^{-6}))\ /\ (14.134 \times 10^{6}) = 1.828 \times 10^{-6}$$

Dividing $\{\bar{X}_2\}$ by the first component $\bar{X}_{12}$ gives the first iteration $\{X_2\}$ approximation to the lowest eigenvector.

Therefore $X_2 =$

$$\begin{matrix} 1.0 \\ 1.75 \\ 2.25 \end{matrix}$$

Continuing the iteration process using the resulting vector $\{X_i\}$ from the previous iteration, as the starting vector in equation (4), the resulting iterations are:

Vector X_3	Vector X_4	Vector X_5	Vector X_6
1.0	1.0	1.0	1.0
1.99	2.04	2.05	2.05
2.76	2.89	2.92	2.93

evaluating equation (5), to generate a normalized mode shape:

$$\sqrt{[X_{k+1}]^T\,[M]\,[X_{k+1}]}$$

$$[1.0 \quad 2.05 \quad 2.93] \begin{bmatrix} 7.76 & 0 & 0 \\ 0 & 5.176 & 0 \\ 0 & 0 & 2.588 \end{bmatrix} \begin{bmatrix} 1.0 \\ 2.05 \\ 2.93 \end{bmatrix}$$

Multiplying through and taking the square root:

$$= 7.1923$$

The normalized mode shape is:

$$\begin{matrix} 0.1391 \\ 0.2853 \\ 0.4071 \end{matrix}$$

Now evaluating the eigenvalue from equation (6):

$$= \frac{[.1391 \quad .2853 \quad .4071] \begin{bmatrix} 14.134 & -5.6536 & 0 \\ -5.6536 & 8.4804 & -2.8268 \\ 0 & -2.8268 & 2.8268 \end{bmatrix} \times 10^6 \begin{bmatrix} .1391 \\ .2853 \\ .4071 \end{bmatrix}}{[.1391 \quad .2853 \quad .4071] \begin{bmatrix} 7.764 & 0 & 0 \\ 0 & 5.176 & 0 \\ 0 & 0 & 2.588 \end{bmatrix} \begin{bmatrix} .1391 \\ .2853 \\ .4071 \end{bmatrix}}$$

$$\rho = \omega^2 = 326,719.83 \left(\frac{\text{rad}}{\text{sec}}\right)^2$$

$$\omega = 571.59 \frac{\text{rad}}{\text{sec}}, \qquad f = 2\pi\omega = 90.97 \text{ Hz}$$

Now imposing a shift, the eigenvalue problem becomes:

$$[K - \mu M]\phi = \eta M\phi$$

where:

$$\eta_i = \lambda_i - \mu$$

assumming a shift of 1×10^6, the [K-μM] matrix becomes:

$$[K] = \begin{bmatrix} 6.374 & -5.6536 & 0 \\ -5.6536 & 3.304 & -2.8265 \\ 0 & -2.8265 & 0.2385 \end{bmatrix} \times 10^6$$

The decomposition and iteration may now proceed as before.

APPENDIX B
FINITE ELEMENT METHOD FOR DDAM ANALYSIS

When performing a dynamic analysis for any system with two or more degrees of freedom, it is necessary to create a flexibility or stiffness coefficient matrix (see Section 3.3). For complicated mass-spring systems hand calculations are impractical. The finite element method is currently being used by many analysts to perform this analysis. The method is described below. For dynamic analyses required by the shipbuilding specifications, the finite element method is acceptable.

Finite element codes provide the user with a library of element types which represent distinct patterns of structural response reflected by rods, beams, plates, continuum, etc. These finite elements are derived from the principal of Minimum Potential Energy based on assumed shape functions and are therefore approximate. However, sufficiently refined assemblages of finite elements can be constructed to represent the behavior of structural systems. At element intersections, displacements and rotational compatibility may be enforced or released by the user. The finite element method is a systemized method for assembling sophisticated mass elastic systems and therefore must conform to the guidelines provided within this document.

The following is a list of the type of information that the analyst must assemble for a discrete element type model:

a. Type of material - steel, aluminum, etc.
b. Type of structure - frame or truss
c. Type of loading
d. Degrees of freedom - description of all releases and constraints
e. Description of each finite element
f. Mass distribution

The above type of information, when entered into an appropriate computer program, will produce the stiffness matrices necessary for the performance of the dynamic analysis.

State-of-the-art finite element programs are capable not only of producing the stiffness matrices, but also of calculating natural frequencies and mode shapes in one step. Such programs tend to eliminate the distinctions made in Chapter 3 between the coefficient computation phase and the dynamic computation phase. Certain proprietary versions of finite element programs even calculate the DDAM motion inputs, modal stresses in beam or plate elements, NRL stress and margins of safety relative to allowable design stresses. The evaluation phase described in Chapter 3 can therefore largely be done in conjunction with the coefficient computation phase with such programs.

It must be emphasized that the use of large finite element models for DDAM analyses does not relieve the analyst from the obligations to exercise judgement and to properly interpret the analytical results. For example, shock stresses calculated directly by finite element models are often only gross approximations. In many instances, complicated geometrical parts are represented by simple constant-section beam elements for purposes of generating system flexibility or stiffness properties. The program-calculated stresses in such elements must be checked by means of manual calculations which account for the true geometry of the parts being evaluated. Alternatively, secondary finite-element analyses with more modelling detail in the areas in question may be conducted. These secondary analyses may be static ones, with the applied loads being the DDAM-calculated inertia loadings.

The capabilities available in modern finite element programs tend to encourage the use of large mathematical models for DDAM analysis. Figures B-1 and B-2 illustrate a finite element mathematical model of moderate complexity. Included in the model are both beam and plate elements. The tendency to use models of ever-increasing complexity should be discouraged. Overly complicated models have the following disadvantages:

A. Difficulty in performing review and check. Extremely voluminous input/output data sets make checking of the analytical results difficult for both the Contractor and the Navy and thus reduces the overall level of confidence in the shock hardness of the design.

B. Misleading accuracy of results. Since the dynamic analysis by DDAM of most large complicated models generally requires the use of reduction techniques the accuracy of the results may not be as reliable as expected. Since the solution of the dynamic problem has been obtained from a reduced mathematical model the accuracy has not been increased by excessive refinement of the model. In fact, if the reduction process is improperly applied, a lower level of accuracy will be achieved for the more complicated model.

C. The larger the model the higher the probability of producing closely spaced modes.

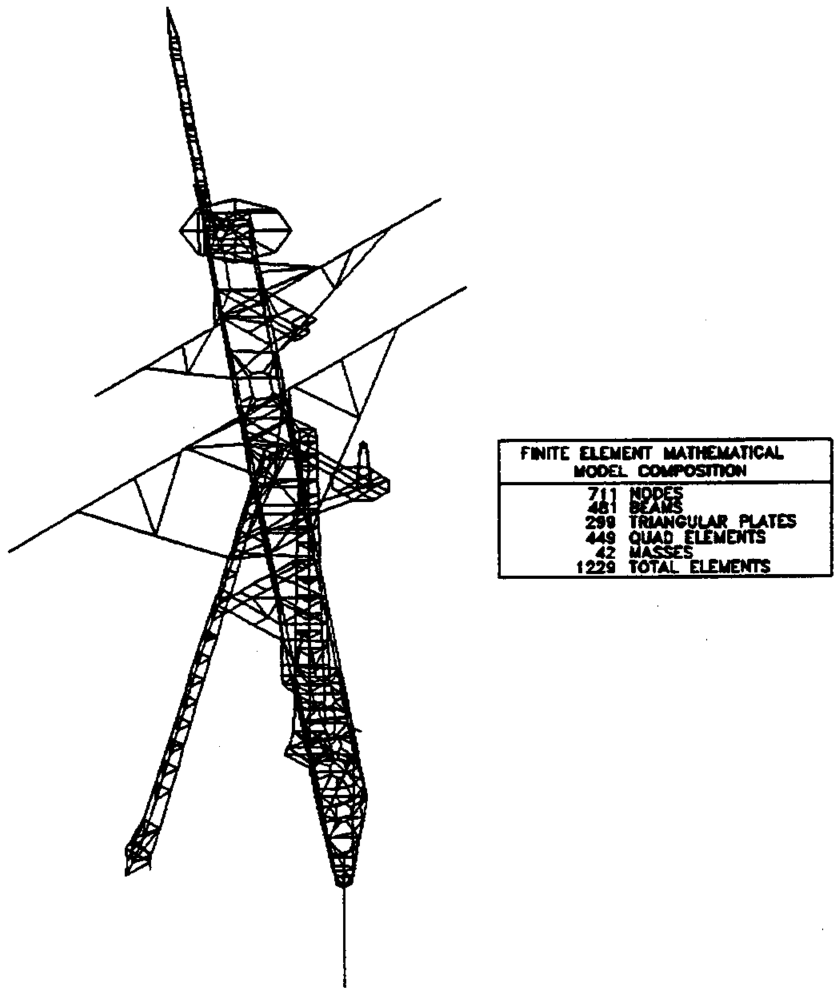


Figure B-1 - Mathematical Model Representation of a Mast - Isometric View

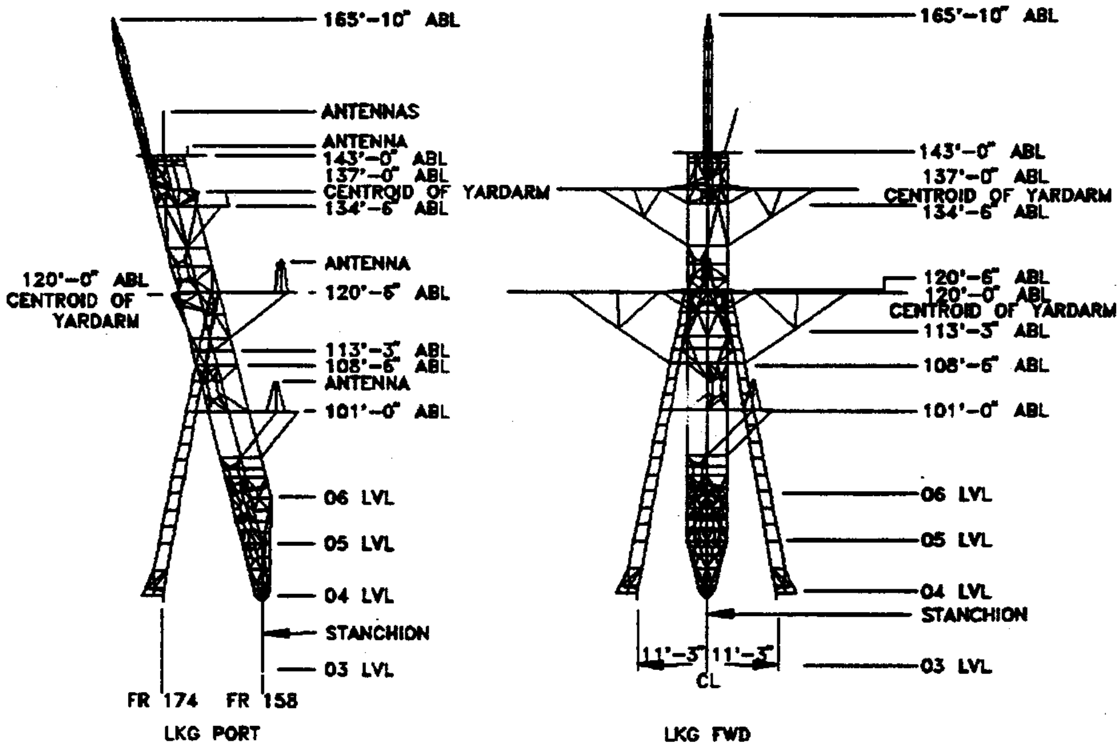


Figure B-2 - Mathematical Model Representation of a Mast - View Looking Forward and View Looking Port

APPENDIX C
TRANSIENT ANALYSIS METHODS AND ENERGY METHODS

As noted throughout this guide, the DDAM is considered appropriate for use on linear, elastic shipboard systems for which the DDS 072-1 shock design values are considered applicable and appropriate. Other procedures, such as a transient analysis method or an energy method, may be substituted for DDAM if approved by NAVSEA.

The transient analysis method, similar to the modal summation technique of DDAM, requires a mathematical model to be developed which represents both the elastic and inertial properties of the system. Whereas the DDAM assumes an undamped steady state solution by combining the maximum responses of each modal contribution regardless of the times at which these modal maxima occur, transient analyses determine the phased responses within a finite response interval. It is not likely, in the presence of structural damping, that the peak modal contributions will constructively combine as assumed in the DDAM. The high frequency responses will likely diminish very rapidly and many of the analytical difficulties within DDAM associated with closely spaced modes will not be present in a transient analysis. Furthermore, lightweight equipment mounted on low frequency structures in tune with the ship's hull girder frequencies will be subjected to vibratory motion rather than a shock loading, sometimes referred to as shock induced resonance.

Differential base motions can be supplied for larger equipment items with multiple supports to reflect variation in support structure. Non-linear effects and the ability of redundant structures to redistribute forces can also be accounted for in sophisticated transient analyses. However, unlike the modal methods in which enveloped spectral response motions can be supplied to the analyst, transient analyses require time history forcing functions or base motions as inputs. These transient inputs depend on the characteristics of the UNDEX parameters and the ship structure. As yet no set of general inputs has been determined by the Navy which constitute a transient design environment.

Transient response calculations can be conducted on three distinct levels of analytical rigor:

a) Full ship responses in which a three dimensional hull model of the ship structure is loaded via a fluid structure interaction algorithm. The pressures and motions within the fluid, resulting from a postulated attack geometry and charge weight, load the ship structure and the response of the internal equipment is calculated interactively with the full ship response. This methodology allows the analyst to consider the effects of shock, cavitation and bubble pulsations on the full ship, thereby providing the most complete representation of the three dimensional ship structure response. Transient analysis techniques may also be applied to the analysis of external appendages. The transient analysis approach however, has several obstacles to overcome before it can be implemented in a shipbuilding program. The input parameters have not been defined, in fact, multiple analyses may be required to determine the most severe response to various attack configurations which all correspond to the same shock environment level. The full ship transient approach is potentially an expensive method to apply and is not a practical substitute for DDAM in a production mode for most if not all equipment foundation design.

b) Beam model responses in which the ship structure is reduced to an equivalent beam loaded by a more simplistic momentum transfer algorithm. The equipment is driven by the beam motions projected to the equipment locations. In these beam analyses, simplistic characterizations of the fluid loadings may be prescribed. Beam models, however, globally constrain entire components of motion, exclude significant coupling which may be important to equipment response and filter the frequency content of the motion delivered to the equipment.

c) Local equipment responses can be determined by subjecting the equipment to measured shock test data. These analyses can only be used following ship shock tests as an evaluation tool for equipment response not equipment design. Care must be exercised in selecting boundaries for the equipment model and the application of the input motions. Gage records must be chosen prudently to best represent the characteristics of the equipment structure interaction.

The limitations inherent in any of the transient analysis approaches discussed above must be clearly understood. The transient analysis approach requires a very accurate definition of the base input motion. As explained in Chapter 3, test data have shown the great importance of the spectrum dip, or equipment feedback, effect on ship base motions. Determination of this effect requires that an accurate model of the equipment under consideration be included in the hull model being used to derive the input motions. Errors in the determination of the spectrum dip effect will cause the transient analysis to over-predict equipment and foundation responses to shock. Similarly, responses caused by multiple resonant conditions within the hull model used to generate input motions will generally lead to over-predictions. In general, it is considered prudent to do a shock spectrum analysis of proposed transient analysis inputs and to compare them for reasonableness with the DDS 072-1 inputs before proceeding with a complete transient analysis.

APPENDIX D
OBLIQUE DIRECTIONAL SHOCK INPUTS

Components of the design spectrum levels can be used to solve for equipment response to an oblique shock or for redefining the shock design values into equipment oriented axes. Consider that the three specified design spectrum values, D_v, D_a and D_f form an ellipsoid (not of revolution). The octant of space occupied by this ellipsoid intersects the X, Y and Z axes at values which correspond to the maximum (or principal) ship oriented design shock spectrum inputs. Figure D-1 shows the relationship between the three axes of a hypothetical damage surface. If we let the Y axis correspond to the ship's vertical direction and its principle design spectrum value is D_v, the Z axis correspond to the athwartship direction with its principal design spectrum value as D_a and the X axis correspond to the fore/aft ship direction with its principal design spectrum value of D_f we can develop parametric equations for any angle of attack. The point P on the surface of this ellipsoid represents the components of the design shock spectrum values to be used for oblique angles of attack or to determine responses along axes other than the principal ship axes.

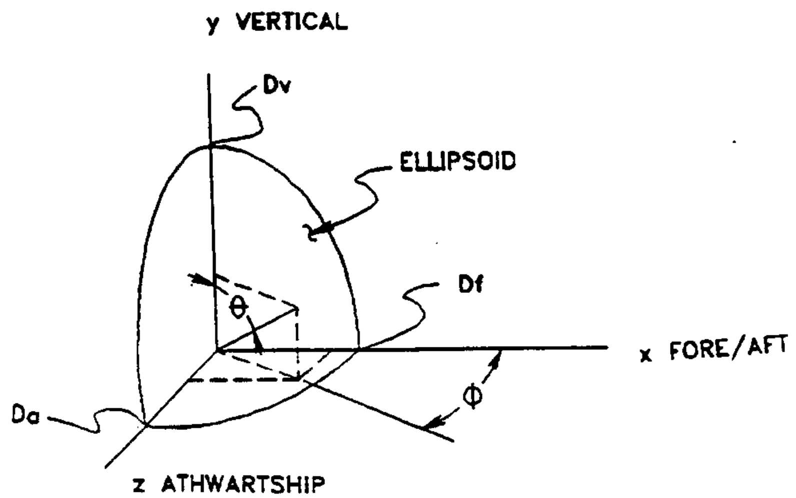


Figure D-1
Hypothetical Damage Surface

Oblique Equipment Orientation

In a similar manner if we rotate the response axis to correspond with the equipment axis rather than principal ship axes the analysis method requires determination of three coordinate input values for each individual direction of design input. That is, components of the specified vertical design shock input are required to be determined along each of the three equipment axes. These component inputs are to be applied simultaneously and the solutions combined on a mode by mode basis.

The design produced from shock inputs that have been re-oriented to coincide with equipment axes is the same as the design produced by inputs along the ship axes and these alternate inputs can be used if desired for ease of calculation and design.

Oblique Equipment Orientation - Illustrative Example

Consider a mass-elastic model of the equipment oriented in the fore/aft - vertical plane of the ship whose local axes, x and y are rotated an angle θ with respect to the global axes of the ship.

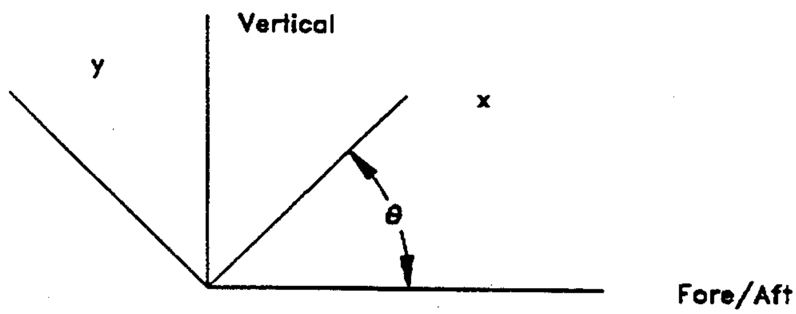


Figure D-2 Orientation of Equipment Axis with Respect to Ship Axis

For each mode "a" of the equipment analysis a frequency, ω_a and a mode shape, $\{\Phi_a\}$ are defined in the local x-y coordinate system. Correspondingly, for this multi-directional response analysis, Participation Factors are calculated for each mode and direction of motion as:

$$P_a = \frac{\{\Phi_a\}^T \ [M] \ \{r\}}{\{\Phi_a\}^T \ [M] \ \{\Phi_a\}}$$

where the vector {r} relates the orientation of the motion of the ship to the local coordinates of the equipment. For a simple two degree of freedom system with one degree of freedom in the local x axis and the other in the local y axis, the {r} vector will be:

$$\{r\} = \begin{Bmatrix} \cos\theta \\ -\sin\theta \end{Bmatrix} \quad \text{for fore/aft motion}$$

$$\{r\} = \begin{Bmatrix} \sin\theta \\ \cos\theta \end{Bmatrix} \quad \text{for vertical motion}$$

Modal masses are calculated for each mode "a" and assumed direction of ship motion and the spectral response values are obtained from DDS 072-1. For fore/aft motion the spectral value is D_{fa} and for vertical motions the spectral response value is D_{va}.

The equipment response displacements for each mode and each direction of ship motion is calculated from normal mode theory as:

$$\{d_{fa}\} = \{\Phi_a\} \; P_{fa} \; D_{fa} \quad \text{for fore/aft ship motion}$$

$$\{d_{va}\} = \{\Phi_a\} \; P_{va} \; D_{va} \quad \text{for vertical ship motion}$$

For the two degree of freedom example previously described, the two components of equipment response, X and Y, for a particular mode "a" will be, for fore/aft ship motion:

$$\{d_{fa}\} = \begin{Bmatrix} X_{fa} \\ Y_{fa} \end{Bmatrix}$$

$$X_{fa} = \frac{\Phi_{a1}\left(\Phi_{a1} M_1 \cos\theta - \Phi_{a2} M_2 \sin\theta\right)}{M_1 \Phi_{a1}^{\;2} + M_2 \Phi_{a2}^{\;2}} D_{fa}$$

$$Y_{fa} = \frac{\Phi_{a2}\left(\Phi_{a1} M_1 \cos\theta - \Phi_{a2} M_2 \sin\theta\right)}{M_1 \Phi_{a1}^{\;2} + M_2 \Phi_{a2}^{\;2}} D_{fa}$$

and for vertical ship motion

$$\{d_{va}\} = \begin{Bmatrix} X_{va} \\ Y_{va} \end{Bmatrix}$$

$$X_{va} = \frac{\Phi_{a1}\left(\Phi_{a1}\, M_1 \sin\theta + \Phi_{a2}\, M_2 \cos\theta\right)}{M_1\, \Phi_{a1}^{\;2} + M_2\, \Phi_{a2}^{\;2}} D_{va}$$

$$Y_{va} = \frac{\Phi_{a2}\left(\Phi_{a1}\, M_1 \sin\theta + \Phi_{a2}\, M_2 \cos\theta\right)}{M_1\, \Phi_{a1}^{\;2} + M_2\, \Phi_{a2}^{\;2}} D_{va}$$

Alternatively, spectral response values can be prescribed in the orientation of the local coordinates, N_x and N_y.

$$\begin{Bmatrix} N_{xa} \\ N_{ya} \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} D_{fa} \\ D_{va} \end{Bmatrix}$$

Participation factors P_{xa} and P_{ya} are determined as before, however, the {r} vector will now relate the new orientation of the ship motion to the local coordinates of the equipment. For the two degree of freedom example the vector {r} will now be:

$$\{r\} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \text{for x motion and}$$

$$\{r\} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \quad \text{for y motion}$$

The equipment response displacements for each mode and each direction of ship motion is calculated from normal model theory as before

$$\{d_{xa}\} = \{\Phi_a\}\ P_{xa}\ N_{xa} \quad \text{for x direction ship motion}$$

$$\{d_{ya}\} = \{\Phi_a\}\ P_{ya}\ N_{ya} \quad \text{for y direction ship motion}$$

For the example of the two degree of freedom system, the two components of equipment response, X and Y for a particular mode will be, for x direction ship motion;

$$\{d_{xa}\} = \begin{Bmatrix} X_{xa} \\ Y_{xa} \end{Bmatrix}$$

$$X_{xa} = \frac{\Phi_{a1}^2\, M_1 \left(D_{fa} \cos\theta + D_{va} \sin\theta\right)}{\Phi_{a1}^2\, M_1 + \Phi_{a2}^2\, M_2}$$

$$Y_{xa} = \frac{\Phi_{a1}\ \Phi_{a2}\, M_1 \left(D_{fa} \cos\theta + D_{va} \sin\theta\right)}{\Phi_{a1}^2\, M_1 + \Phi_{a2}^2\, M_2}$$

Similarly for the y direction ship motion:

$$\{d_{ya}\} = \begin{Bmatrix} X_{ya} \\ Y_{ya} \end{Bmatrix}$$

$$X_{ya} = \frac{\Phi_{a1}\ \Phi_{a2}\, M_2 \left(-D_{fa} \sin\theta + D_{va} \cos\theta\right)}{\Phi_{a1}^2\, M_1 + \Phi_{a2}^2\, M_2}$$

$$Y_{ya} = \frac{\Phi_{a2}^2\, M_2 \left(-D_{fa} \sin\theta + D_{va} \cos\theta\right)}{\Phi_{a1}^2\, M_1 + \Phi_{a2}^2\, M_2}$$

These modal response motions can be related by recognizing the relationship between the input motions prescribed in the two coordinate systems. For a pure fore/aft ship motion the response value $D_{va} = 0$, and there will be two components of spectral response values to be applied simultaneously:

$$N_{xa} = D_{fa} \cos\theta$$

$$N_{ya} = -D_{fa} \sin\theta$$

Correspondingly, the equipment response in the x direction will be the sum of the x direction response resulting from N_{xa} and N_{ya}.

$$\{\bar{d}_{fa}\} = \begin{Bmatrix} \bar{X}_{fa} \\ \bar{Y}_{fa} \end{Bmatrix}$$

$$\bar{X}_{fa} = \frac{\Phi_{a1}^2 M_1 (D_{fa} \cos\theta)}{\Phi_{a1}^2 M_1 + \Phi_{a2}^2 M_2} - \frac{\Phi_{a2}\Phi_{a1} M_2 (D_{fa} \sin\theta)}{\Phi_{a1}^2 M_1 + \Phi_{a2}^2 M_2}$$

$$\bar{Y}_{fa} = \frac{\Phi_{a1}\Phi_{a2} M_1 (D_{fa} \cos\theta)}{\Phi_{a1}^2 M_1 + \Phi_{a2}^2 M_2} - \frac{\Phi_{a2}^2 M_2 (D_{fa} \sin\theta)}{\Phi_{a1}^2 M_1 + \Phi_{a2}^2 M_2}$$

In this fashion, equivalent equipment responses can be calculated to motions in either the global ship axes or the local equipment coordinate system.

APPENDIX E
SAMPLE FINITE ELEMENT DDAM ANALYSIS - FORMAT AND CONTENT

1.0 Introduction

This appendix is provided as an example of the format and content of a dynamic analysis report for a typical finite element model. It is not the intent of this example to provide technical guidance in the performance of DDAM.

This appendix presents the mathematical model and the vertical dynamic analysis of the foundation for a radar test set, two transponder sets and an electronic controller (Section 5 of this appendix, Figure E-1). The appendix verifies that the foundation structure shown in Section 7 of this appendix, Figures E-4 through E-10, is adequate for Grade A vertical shock. Since some permanent deformation would not invalidate the design for its intended grade of shock, elastic-plastic inputs were used. The appendix also identifies and eliminates closely spaced modes from the modal analysis. It is noted that the finite element model and shock analysis for athwartship and longitudinal shock, although not presented here, will have the same format as the vertical shock analysis.

The material presented in the example problem is representative of the material that would be provided in a finite element DDAM submittal to the Navy for final approval. The following data is provided:

Note: For this sample problem, modal output and internal member forces are provided for a typical mode only to limit the size of Appendix E.

2.0 Mathematical Model

The rack type foundation, shown in Section 5, Figure E-1, supports a radar test set, two transponder sets and an electronic controller. The electronic controller is attached to the center transponder set. The equipment sketches for the radar test set are shown in Section 6, Figure E-2. The equipment sketches for the transponder set are shown in Section 6, Figure E-3. The electronic controller is a small rectangular box and the equipment sketches are not provided. Scantling drawings for the rack foundation are shown in Section 7, Figures E-4 through E-10. The foundation was modeled using prismatic beam elements for the entire model. The plates shown in Figure E-8 were represented as flanges of beams using effective plate widths. The radar test set and transponder sets are represented with a rigid frame configuration. The electronic controller is modeled as a linear spring and mass. At the rigid frame(equipment)/foundation interface, the moments about each of the three local axes were released to simulate the effects of the bolted connections. The foundation frame is fixed for all translations and rotations at the deck. Computer generated plots for the full structural model are shown in Section 8, Figures E-11 and Section 9, Figure E-12.

The mass distribution for the mathematical model is shown in Section 10, Figure E-13. The three equipment masses are given dynamic degrees of freedom in the three global directions (fully coupled). Because of its size, the electronic controller is given only a vertical dynamic degree of freedom. Due to the symmetry of the structural masses, and resulting small coupled motions in the horizontal plane for vertical inputs, these masses were given only vertical dynamic degrees of freedom for the vertical shock analysis.

3.0 Computer Analysis

A particular computer program and dynamic solution technique has been chosen for this example. There are numerous other programs available to perform a DDAM analysis. It is not the intent of this example to restrict the finite element analysis to any one computer code. A copy of the computer input data used for this shock analysis is shown in Section 11.

A system with three phases to the analysis was used for the shock analysis of the example foundation. The first phase (a general structures program) calculates the stiffness matrix, member loads, support reactions and joint deflections. The second phase performs the dynamic analysis and determines the natural frequencies and effective static forces associated with each mode. The last phase used in conjunction with the output of the general structures program determines the forces, stresses in each member and all joint displacements associated with the shock loading. This final phase also combines (NRL sum) the member stresses developed in the modes analyzed.

4.0 Results

The results of the foundation analysis are provided on the following pages. To demonstrate the identification and elimination of closely spaced modes, an iteration prior to the final iteration is shown for demonstration purposes only. This iteration would not normally be submitted with the final analysis report.

12.d	Internal member force calculations for each mode.	E-41 - E-45
12.e	NRL summation of stresses for each member.	E-46 - E-50

Allowable bending stresses are twice yield for an elastic-plastic analysis.
Allowable stress is 2 x 33,000 psi = 66,000 psi.
Allowable shear stresses are 60% of twice yield.
Allowable shear stress is 0.6 x 66,000 psi = 39,600 psi.

Reviewing the modal results shown in Figure E-16 of Section 12.b, it can be seen that modes 6, 7 and 8 are closely spaced (within 10% of the lower mode). Further review of the eigenvectors of the three modes, Figure E-17 of Section 12.b reveals that the 1 lb. electronic controller (node 66) is out of phase and dominates in modes 6 and 7. The force that the 1 lb. electronic controller is anticipated to be excessive. When these modes are summed in the NRL procedure, the cancelling effect of the small mass is lost and erroneous results occur.

The problem is eliminated by stiffening the interface so that the mass of the electronic controller may be combined with that of the transponder set. Section 12.b, Figure E-18 shows the modified modal data. It can be seen that modes 6 and 7 have been combined into a single mode having the same modal weight as the two previous modes.

After elimination of the closely spaced modes, the critical normal stress (NRL) in member 18, at joint 21 is:

$$\sigma_{max} = 59{,}574 \text{ psi} < 66{,}000 \text{ psi}$$

After elimination of the closely spaced modes, the critical shear stress (NRL) in member 7 at joints 40/39 is:

$$\tau_{max} = 8{,}233 \text{ psi} < 39{,}600 \text{ psi}$$

All other stresses are also below allowable limits.

Section 5.0

Sketch or Arrangement of Item

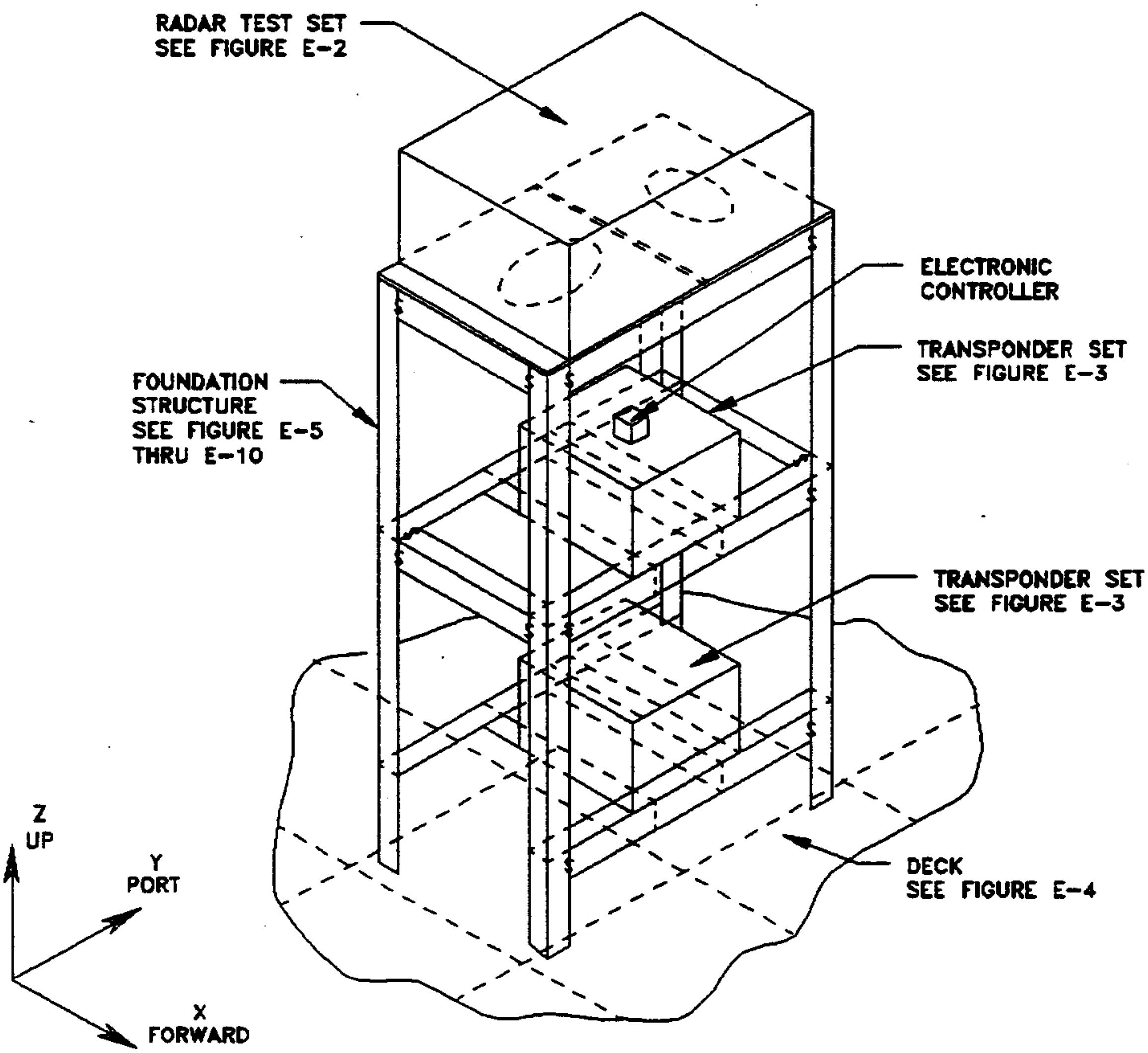
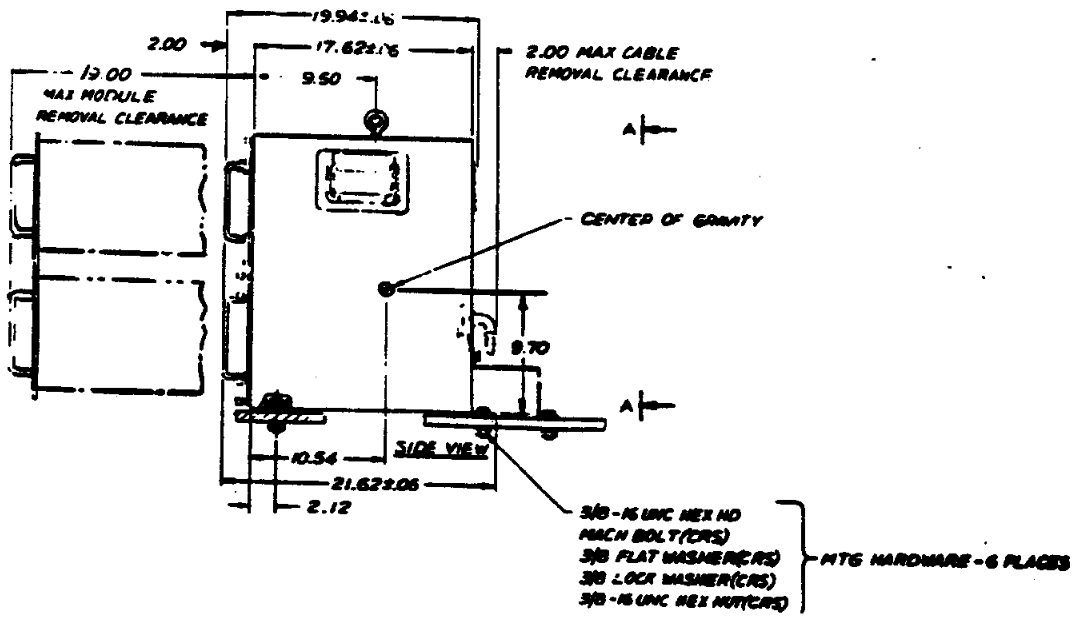


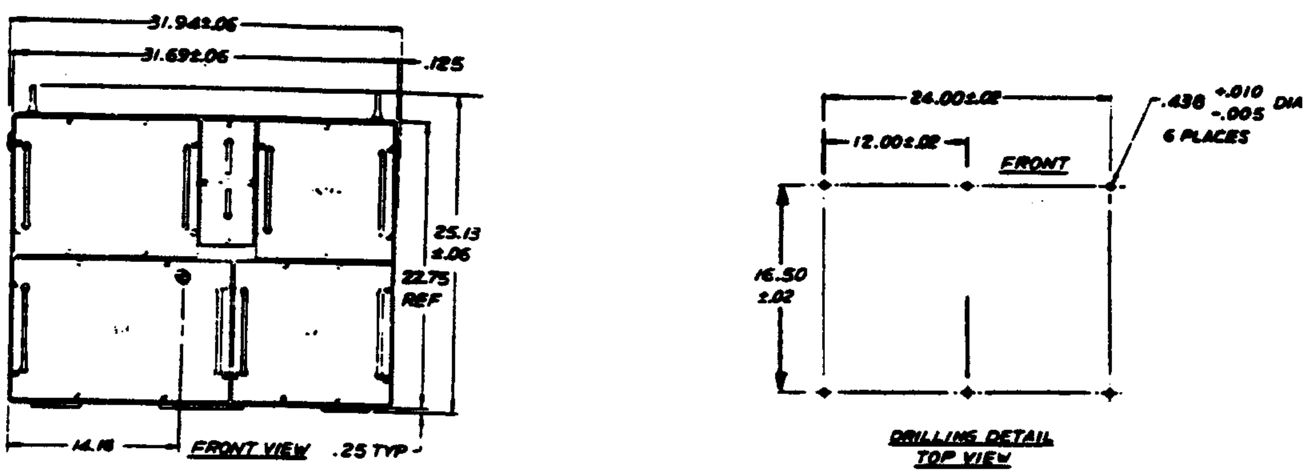
Figure E-1 General Arrangement of Foundation

Section 6.0

Sketch of Equipment



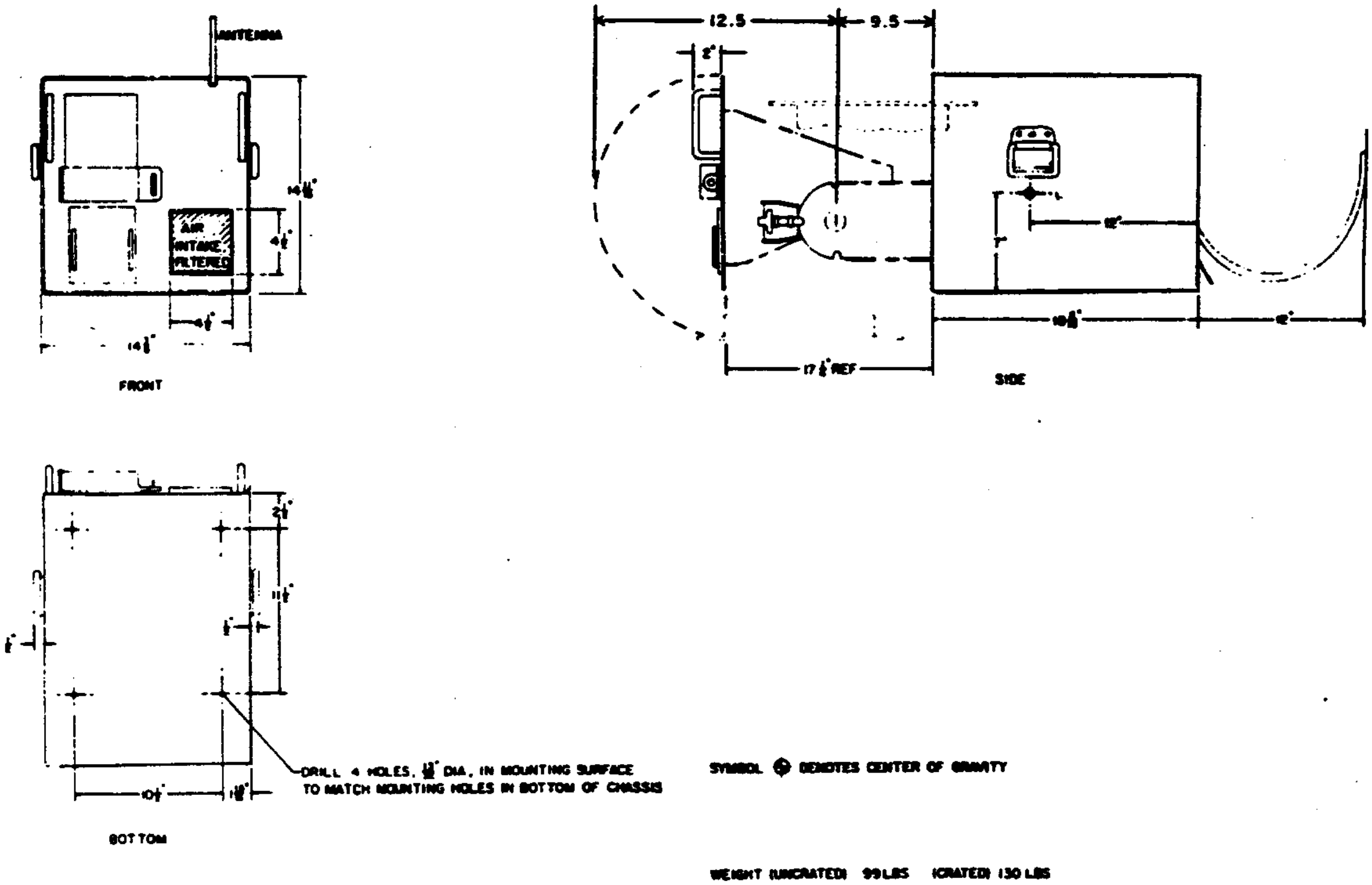
WEIGHT = 190 lbs.



RADAR TEST SET

FIGURE E-2

Transponder Set
AN/UPX-28(V)

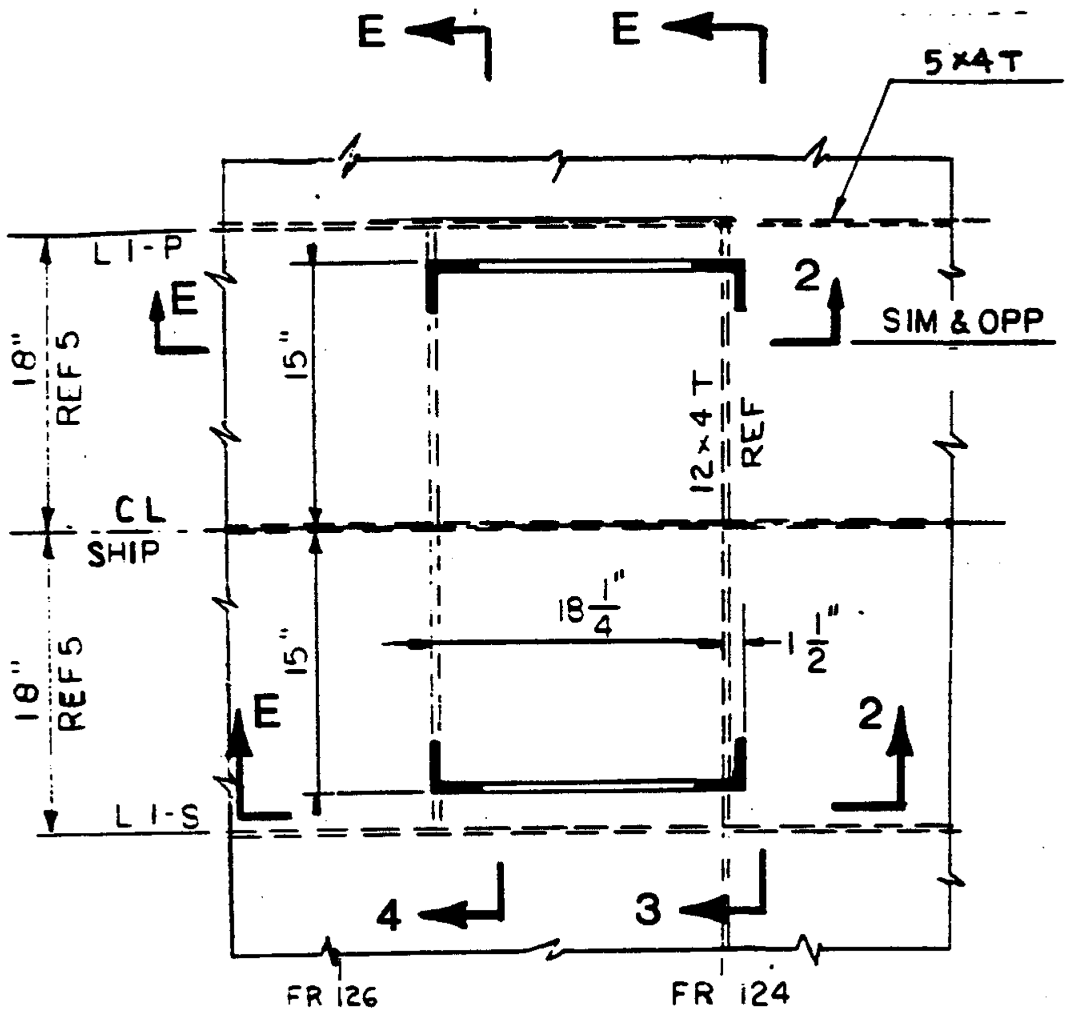


TRANSPONDER SET

FIGURE E-3

Section 7.0

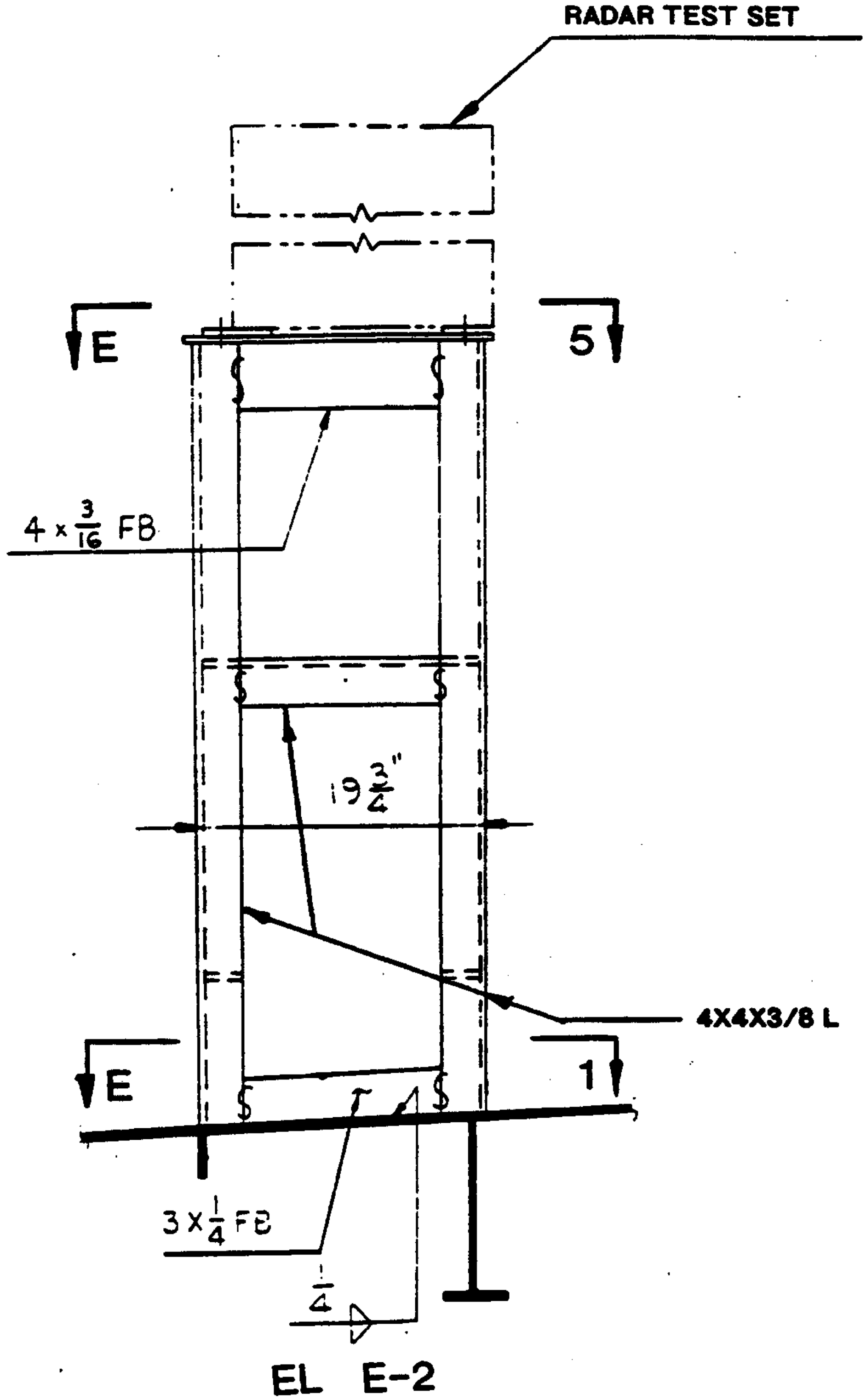
Sketches of Foundation



PLAN **E-1**
AT MAIN DECK

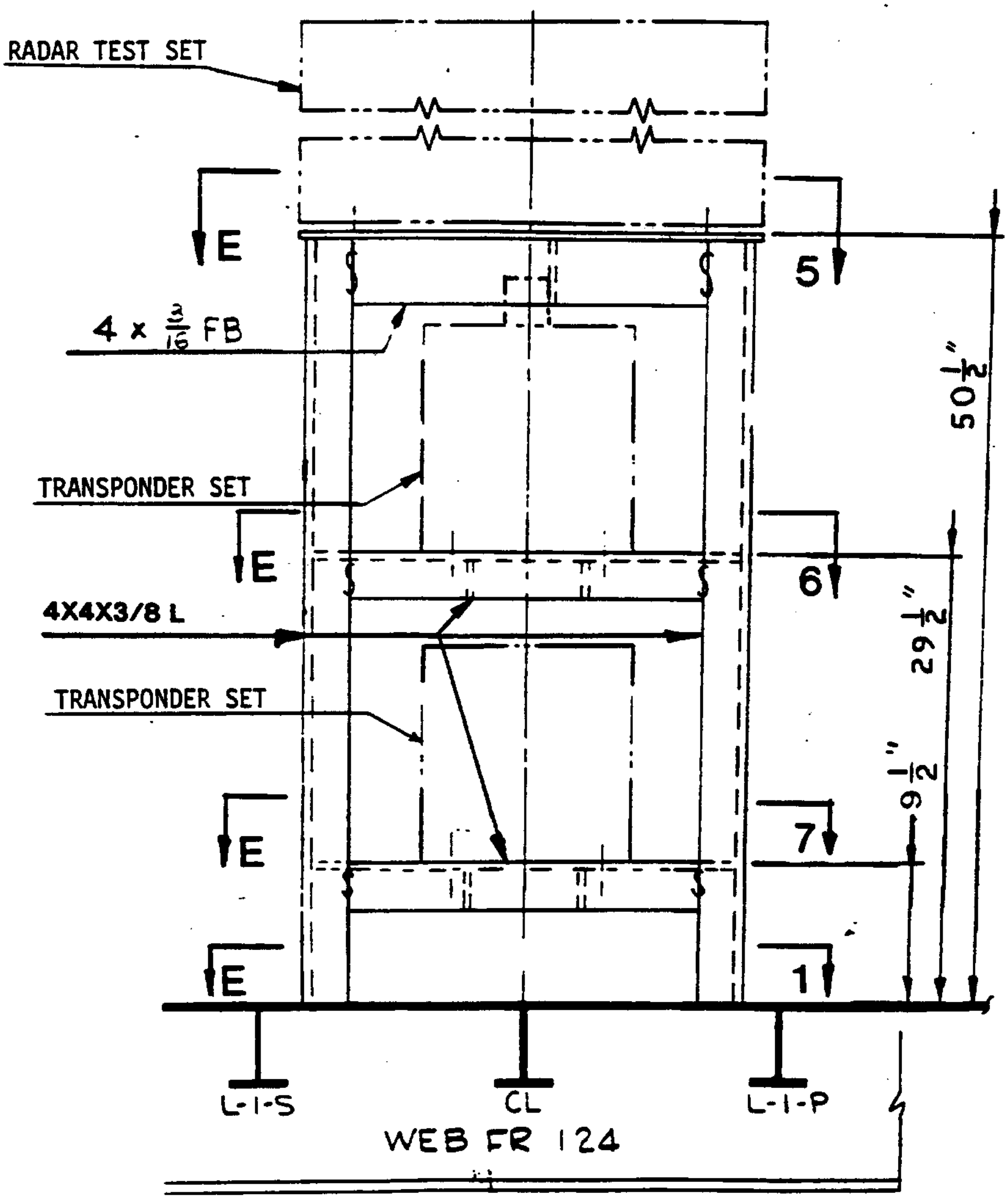
SCANTLING DRAWING FOR THE FOUNDATION

FIGURE E-4



SCANTLING DRAWING FOR THE FOUNDATION (CONTD)

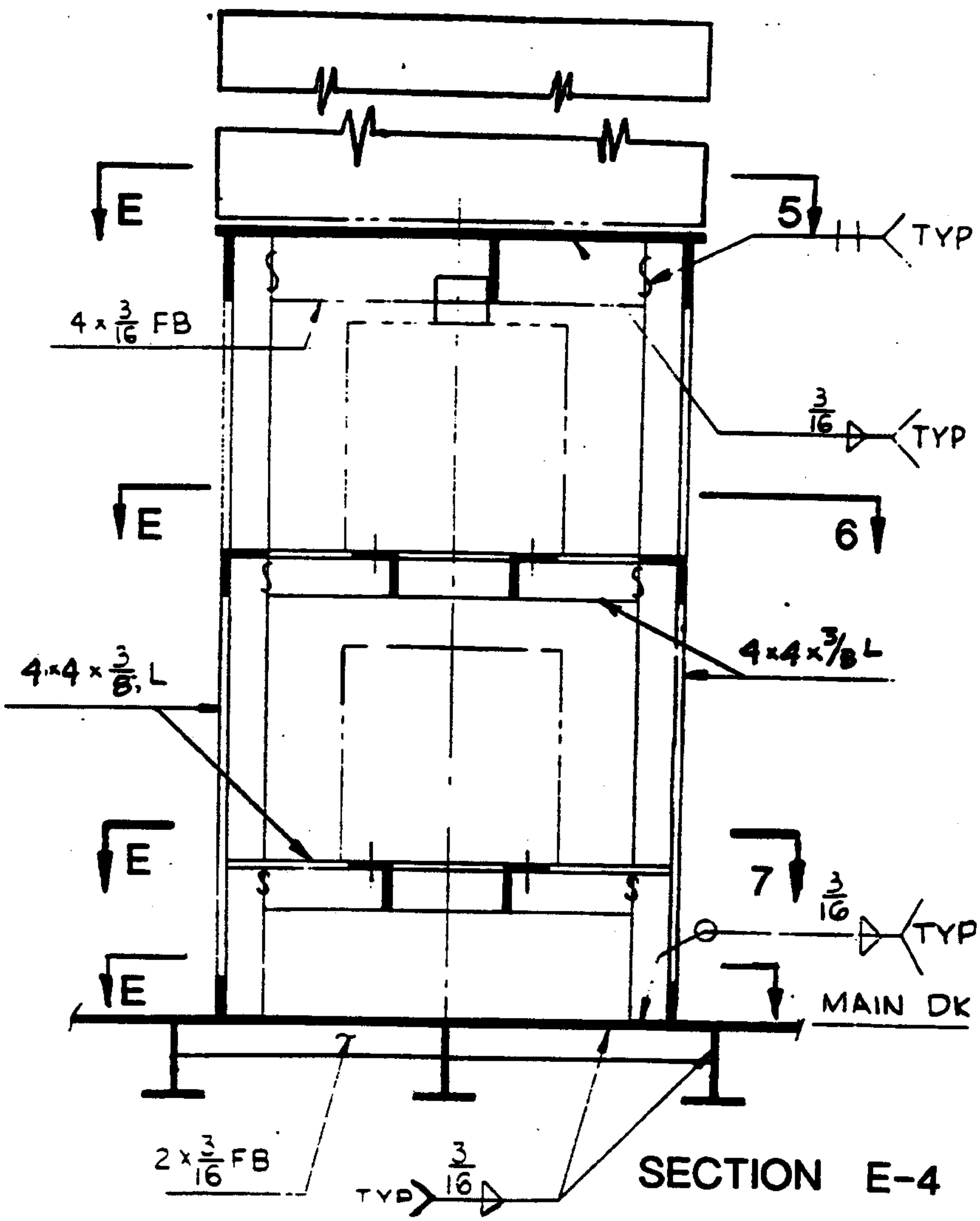
FIGURE E-5



SECTION E-3

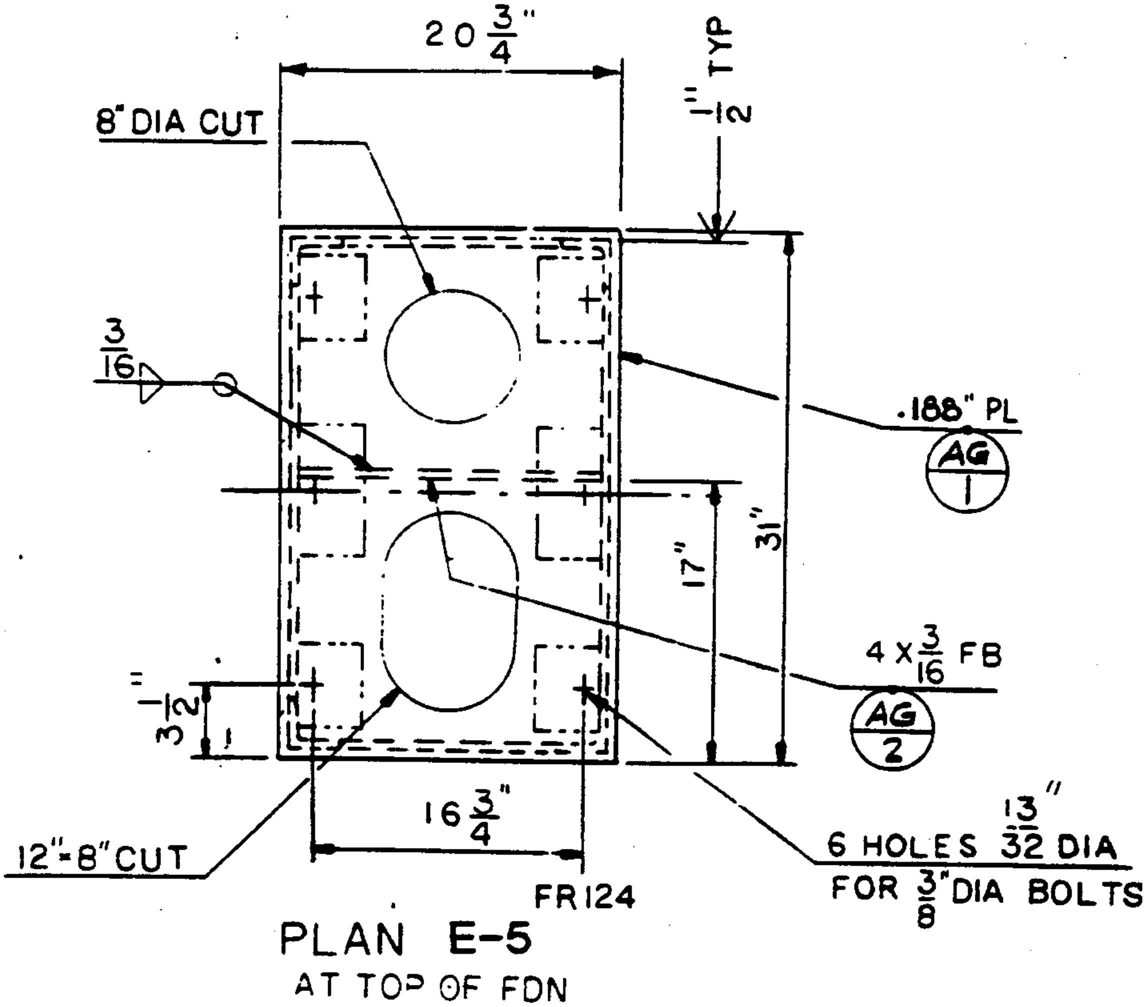
SCANTLING DRAWING FOR THE FOUNDATION (CONTD)

FIGURE E-6



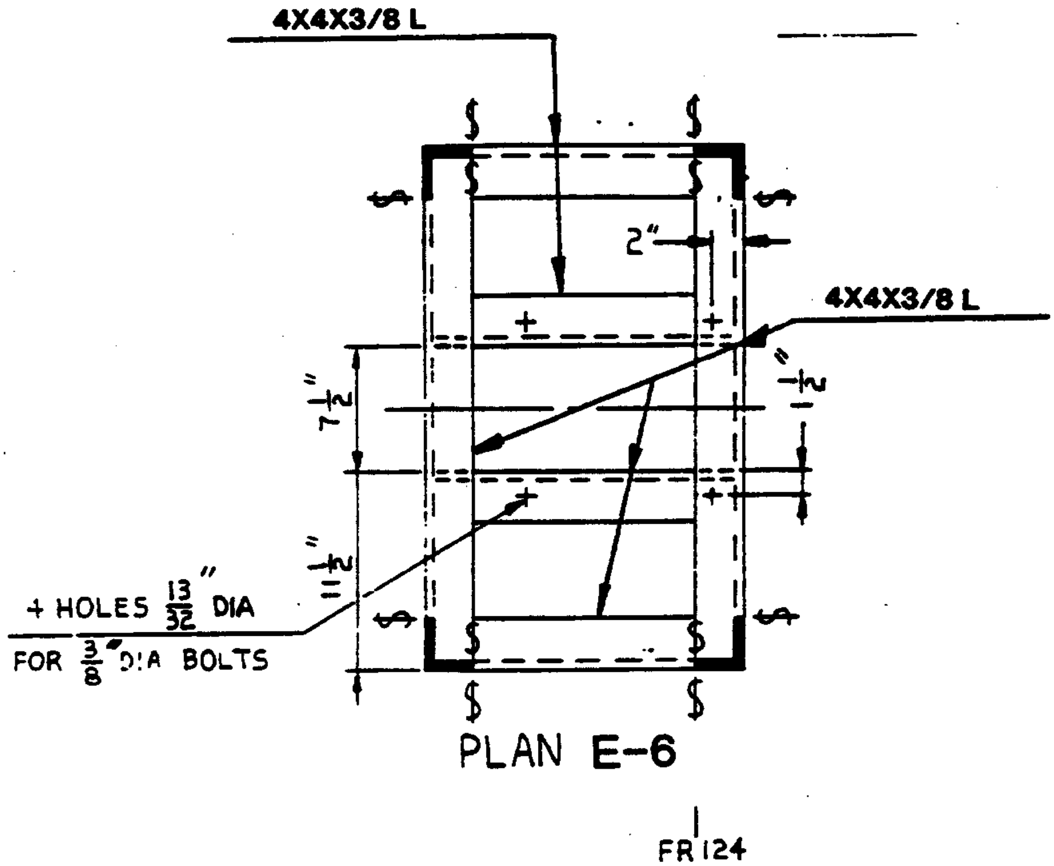
SCANTLING DRAWING FOR THE FOUNDATION (CONTD)

FIGURE E-7



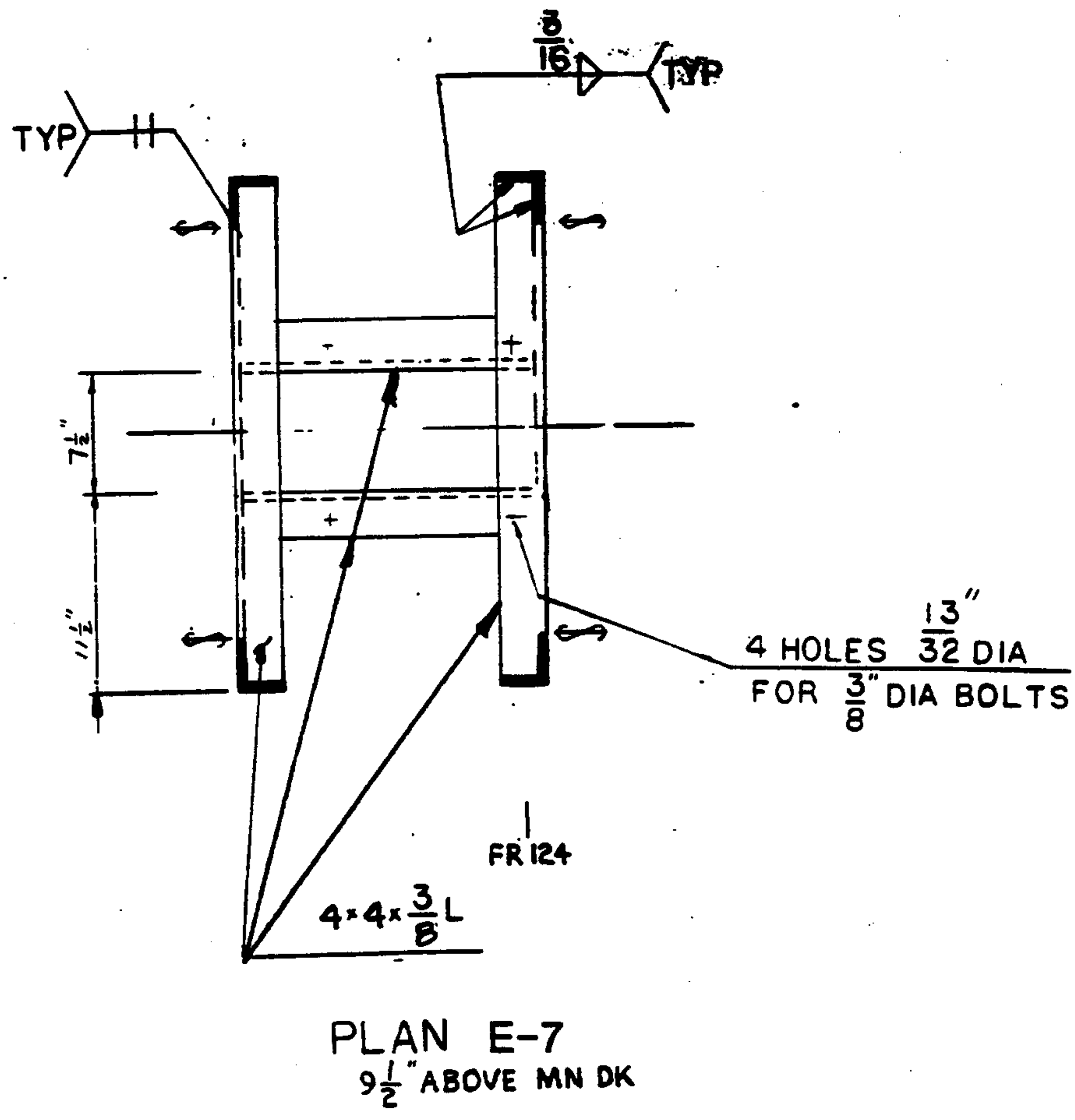
SCANTLING DRAWING FOR THE FOUNDATION (CONTD)

FIGURE E-8



SCANTLING DRAWING FOR THE FOUNDATION (CONTD)

FIGURE E-9

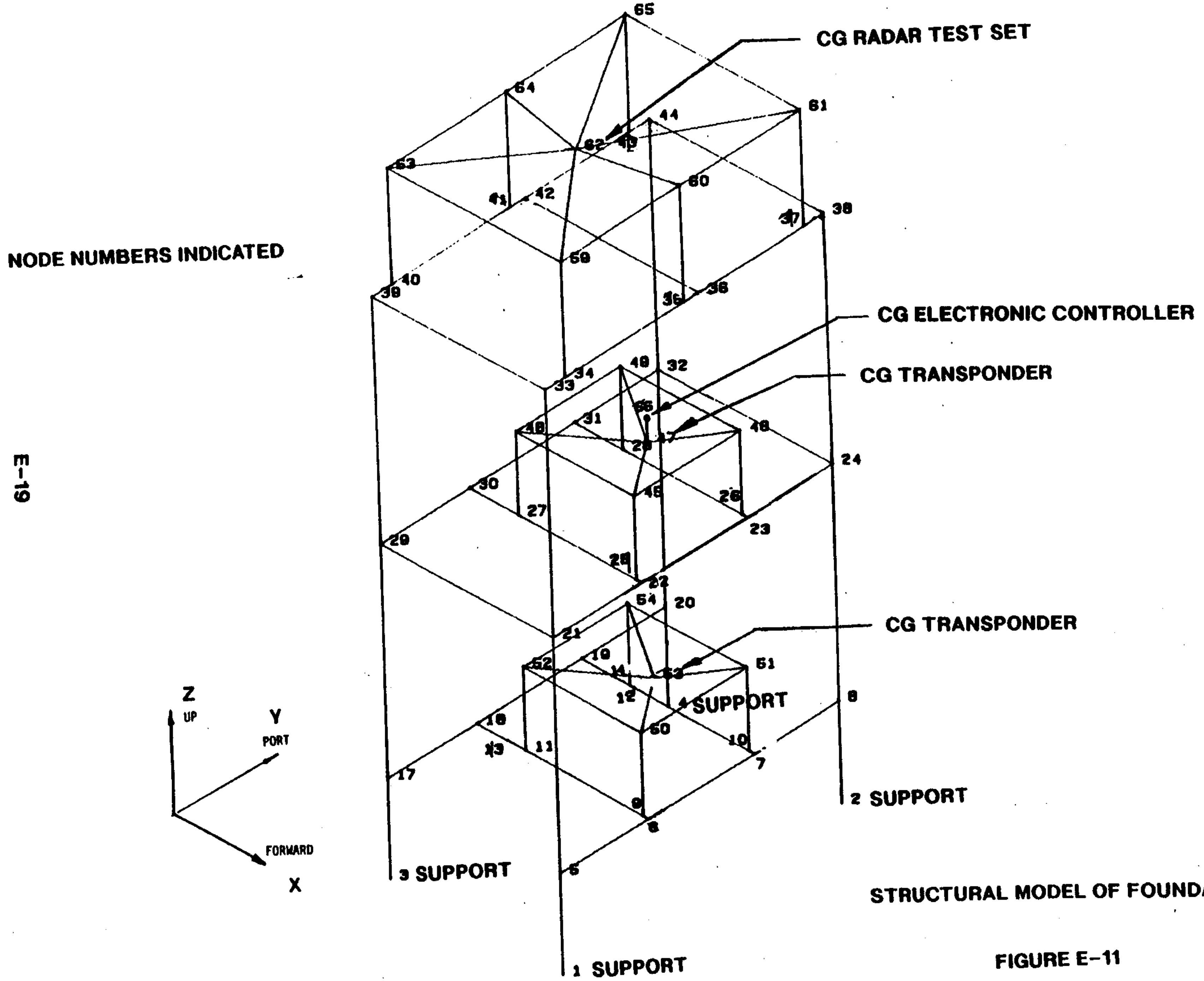


SCANTLING DRAWING FOR THE FOUNDATION (CONTD)

FIGURE E-10

Section 8.0

Mathematical Model Sketch (node numbers)

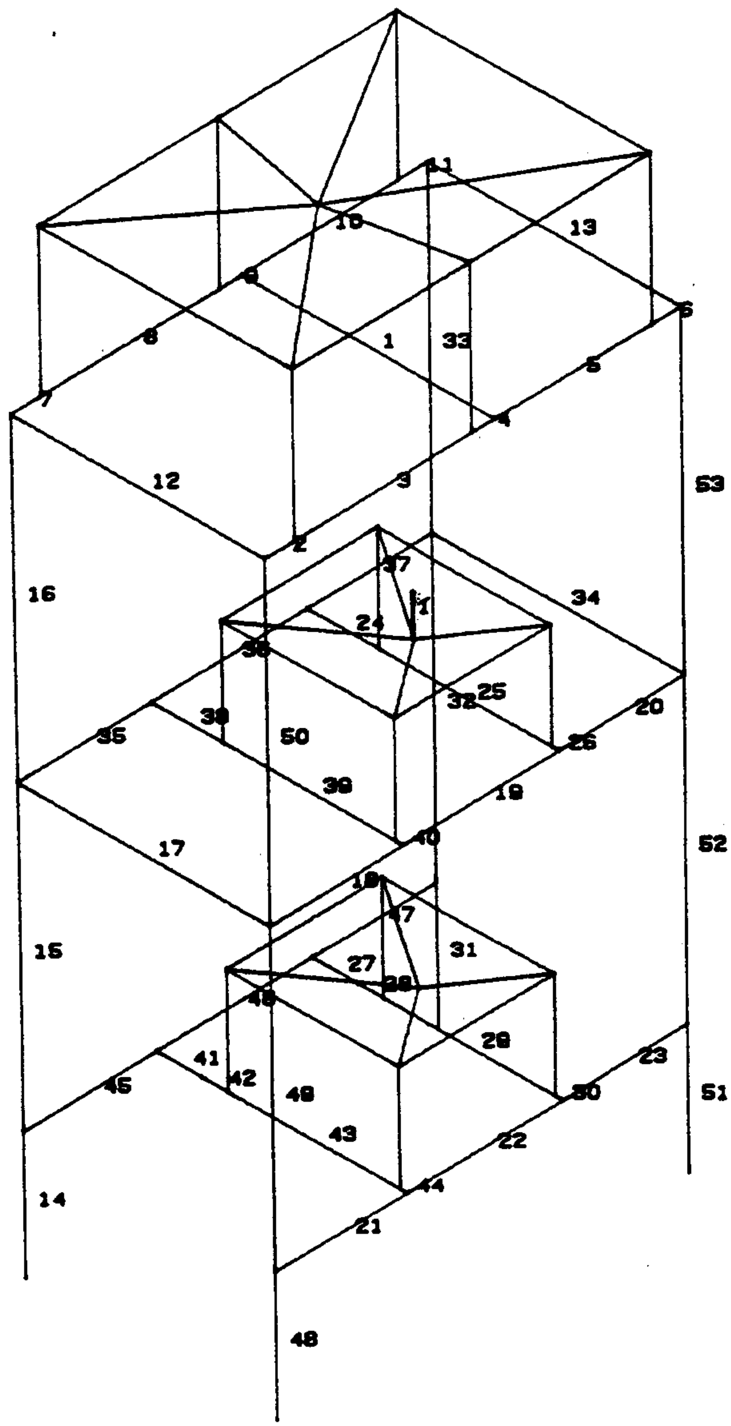


STRUCTURAL MODEL OF FOUNDATION

FIGURE E-11

Section 9.0

Mathematical Model Sketch (element numbers)



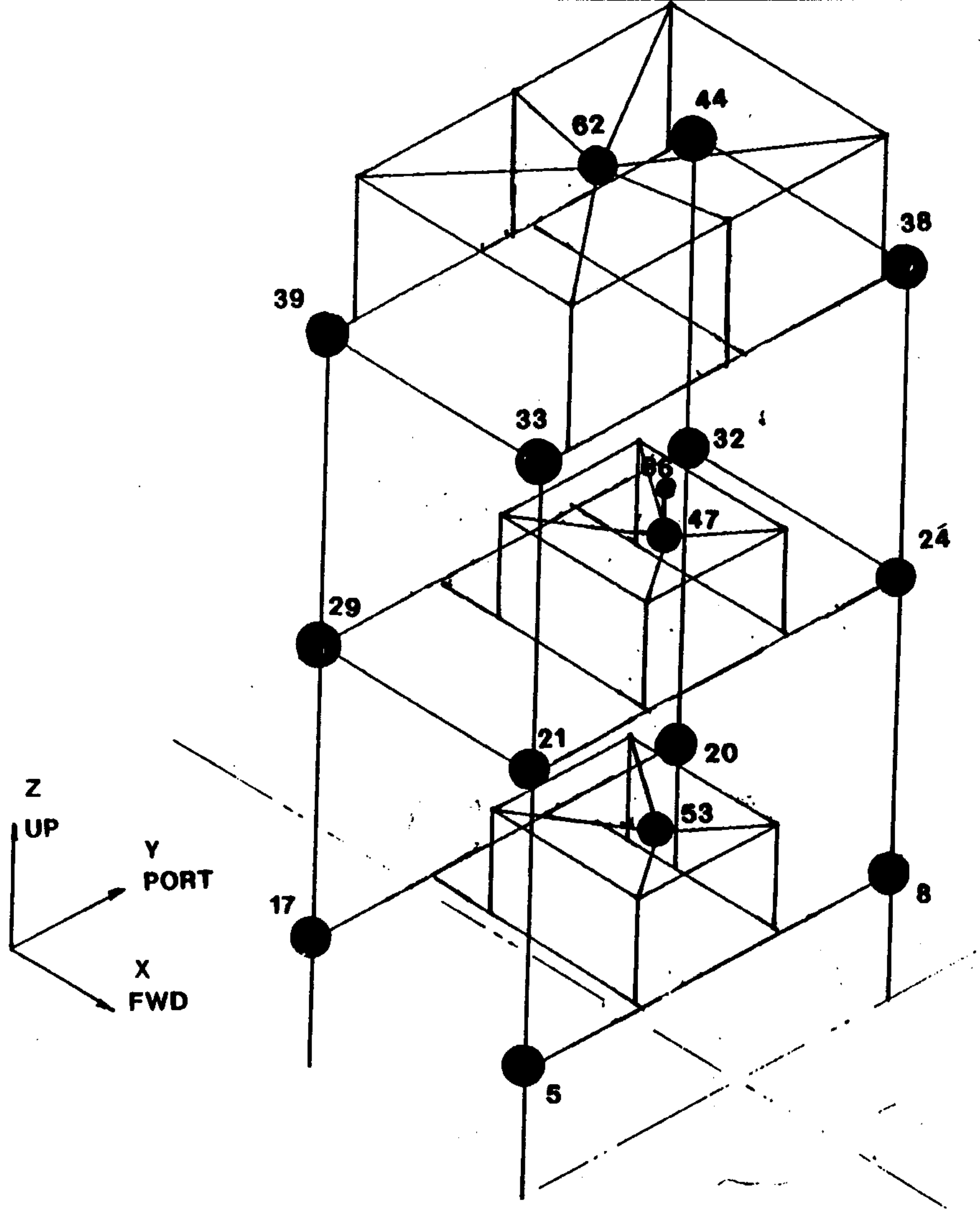
ELEMENT NUMBERS INDICATED

STRUCTURAL MODEL OF FOUNDATION (CONTD)

FIGURE E-12

Section 10

Mathematical Model Sketch (mass locations)



EQUIPMENT MASSES

NODE	WEIGHT(LBS.)
47	99.0
53	99.0
62	190.0
66	1.0

STRUCTURAL MASSES

NODE	WEIGHT(LBS.)
5	32.56
8	32.56
17	32.56
20	32.56
21	36.44
24	36.44
29	36.44
32	36.44
33	23.61
38	22.54
39	23.61
44	22.54

MASS DISTRIBUTION

FIGURE E-13

Section 11

Computer Input

```
TYPE SPACE FRAME

$
JOINT COORDINATES

$
$ Global Cartesian Coordinate System Origin:
$  +X = 4-3/4 in. fwd of Frame 126
$  +Y = 15 in. off ship's CL (stbd)
$  +Z = Main Deck
$
$ Joint Coordinate Syntax is defined as follows:
$   first value indentifies the joint number.
$   second value defines the X coord., where +X is forward.
$   third value defines the Y coord., where +Y is port.
$   fourth value defines the Z coord., where +Z is vertical up
$   last character "S" specifies a support location (fully fixed).
$
1 16.75 0.0 0.0 S

2 16.75 28.0 0.0 S

3 0.0 0.0 0.0 S

4 0.0 28.0 0.0 S

5 16.75 0.0 8.5

6 16.75 9.0 8.5

7 16.75 19.5 8.5

8 16.75 28.0 8.5

9 16.25 9.0 8.5

10 16.25 19.5 8.5

11 4.75 9.0 8.5

12 4.75 19.5 8.5

13 3.0 9.0 8.5

14 3.0 19.5 8.5

17 0.0 0.0 8.5

18 0.0 9.0 8.5

19 0.0 19.5 8.5

20 0.0 28.0 8.5

21 16.75 0.0 28.5

22 16.75 9.0 28.5

23 16.75 19.5 28.5

24 16.75 28.0 28.5

25 16.25 9.0 28.5

26 16.25 19.5 28.5

27 4.75 9.0 28.5

28 4.75 19.5 28.5

29 0.0 0.0 28.5

30 0.0 9.0 28.5

31 0.0 19.5 28.5

32 0.0 28.0 28.5

33 16.75 0.0 49.5

34 16.75 2.0 49.5

35 16.75 14.0 49.5

36 16.75 15.5 49.5

37 16.75 26.0 49.5

38 16.75 28.0 49.5

39 0.0 0.0 49.5

40 0.0 2.0 49.5

41 0.0 14.0 49.5

42 0.0 15.5 49.5

43 0.0 26.0 49.5

44 0.0 28.0 49.5

45 16.25 9.0 35.5

46 16.25 19.5 35.5

47 12.438 14.25 35.5

48 4.75 9.0 35.5

49 4.75 19.5 35.5

50 16.25 9.0 15.5

51 16.25 19.5 15.5
```

```
52 4.75 9.0 15.5

53 12.438 14.25 15.5

54 4.75 19.5 15.5

59 16.75 2.0 59.2

60 16.75 14.0 59.2

61 16.75 26.0 59.2

62 8.33 12.287 59.2

63 0.0 2.0 59.2

64 0.0 14.0 59.2

65 0.0 26.0 59.2

66 12.438, 14.25, 37.5

$
MEMBER INCIDENCES

$
$ Member Incidence Syntax is defined as follows:
$   first integer identifies the member number
$   second integer identifies the "start joint" of the member
$   third integer identifies the "end joint" of the member
$
1 42 36

2 33 34

3 34 35

4 35 36

5 36 37

6 37 38

7 40 39

8 41 40

9 42 41

10 43 42

11 44 43

12 39 33

13 38 44

14 3 17

15 17 29

16 29 39

17 29 21

18 21 22

19 22 23

20 23 24

21 5 6

22 6 7

23 7 8

24 31 28

25 28 26

26 26 23

27 19 14

28 14 12

29 12 10

30 10 7

31 4 20

32 20 32

33 32 44

34 32 24

35 29 30

36 30 31

37 31 32

38 30 27

39 27 25

40 25 22

41 10 13

42 13 11
```

```
43 11 9
44 9 6
45 17 18
46 18 19
47 19 20
48 1 5
49 5 21
50 21 33
51 2 8
52 8 24
53 24 38
54 34 59
55 35 60
56 37 61
57 40 63
58 41 64
59 43 65
60 59 62
61 60 62
62 61 62
63 63 62
64 64 62
65 65 62
66 9 50
67 10 51
68 11 52
69 12 54
70 50 51
71 50 52
72 51 54
73 52 53
74 53 54
75 54 53
76 50 53
77 51 53
78 52 54
79 25 45
80 26 46
81 27 48
82 28 49
83 45 46
84 45 48
85 46 49
86 48 49
87 45 47
88 48 47
89 49 47
90 46 47
91 59 63
92 63 64
93 64 65
94 65 61
95 61 60
96 60 59
$
ELEMENT INCIDENCES
97 47 66
```

```
$
CONSTANTS

$
$ Material Constants Syntax is defined as follows:
$   first character, or word, identifies the material property
$   second real number defines the value of the material property
$   third word, "ALL", indicates that the property is to be applied to all
$     elements in the model
$
E 29600000.0 ALL

G 12000000.0 ALL

POI 0.3 ALL

DENSITY 0.0 ALL


$
MEMBER PROPERTIES PRISMATIC

$
$ Member Property Syntax is defined as follows:
$   first integer, or set of integers, specifies element(s) which are defined
$       by the property
$   AX      = cross-sectional area
$   AY, AZ = areas effective in resisting shear in the local
$       y and z directions
$   IX      = torsional constant
$   IY, IZ = moments of inertia about the local y and z axes
$   SXF     = value of the pseudo torsional section modulus (Ix/t)
$
$ Member Property No. 1 - 4 in. x 3/16 in. FB on 5 in. of 3/16 in. pl
1 AX 1.688 AY 1000.0 AZ 0.7852 IX 0.02 IY 2.829 IZ 10000.0 SXF 0.107

$
$ Member Property No. 2 - 4 in. x 3/16 in. FB on 6 in. of 3/16 in. pl
2 TO 11 AX 1.875 AY 1000.0 AZ 0.7852 IX 0.02 IY 2.976 IZ 100000.0 -
SXF 0.117

$
$ Member Property No. 3 - 4 in. x 3/16 in. FB on 3 in. of 3/16 in. pl
12 AX 1.313 AY 1000.0 AZ 0.7852 IX 0.015 IY 2.411 IZ 100000.0 SXF 0.08

$
$ Member Property No. 4 - 4 in. x 3/16 in. FB on 4 in. of 3/16 in. pl
13 AX 1.5 AY 1000.0 AZ 0.7852 IX 0.018 IY 2.646 IZ 100000.0 SXF 0.096

$
$ Member Property No. 5 - 4 in. x 4 in. x 3/8 in. L
14 TO 53 AX 2.86 AY 1.5000 AZ 1.5000 IX 0.1406 IY 4.360 IZ 4.360 -
SXF 0.375

$
$ Member Property No. 6 - Rigid Elements
54 TO 96 AX 1000.0 AV 1000.0 AZ 1000.0 IX 10000.0 IY 100000.0 -
IZ 100000.0 SXF 100000.0

$
$
SPRING ELEMENT PROPERTIES DOF DX

1 K 11615.7899

MEMBER RELEASES

$
$ Member Release Syntax is defined as follows:
$   set of integers specifies elements which will have DOF(s) r
$   START defines which joint number of a member is to be relea
$   MOMENT X Y Z defines rotational DOFs are to be released
$
54 55 56 57 58 59 66 67.68 69 82 83 84 85 START MOMENT X Y Z

$
$ Flexibility Matrix Load Syntax is defined as follows:
$   first line defines "INFLUENCE COEFFICIENT LOAD" identifier
$   second line defines the joint number, direction of force,
$     gravitational multiplier, and value of load in pounds
$
INFL COE LOAD 'P1'

JOINT '5' FORCE Z 1.0 MASS 32.56

INFL COE LOAD 'P2'

JOINT '8' FORCE Z 1.0 MASS 32.56

INFL COE LOAD 'P3'

JOINT '17' FORCE Z 1.0 MASS 32.56

INFL COE LOAD 'P4'

JOINT '20' FORCE Z 1.0 MASS 32.56

INFL COE LOAD 'P5'

JOINT '39' FORCE Z 1.0 MASS 23.610

INFL COE LOAD 'P6'

JOINT '44' FORCE Z 1.0 MASS 22.54

INFL COE LOAD 'P7'

JOINT '33' FORCE Z 1.0 MASS 23.610

INFL COE LOAD 'P8'

JOINT '38' FORCE Z 1.0 MASS 22.54

INFL COE LOAD 'P9'
```

```
JOINT '21' FORCE Z 1.0 MASS 36.44
INFL COE LOAD 'P10'
JOINT '24' FORCE Z 1.0 MASS 36.44
INFL COE LOAD 'P11'
JOINT '29' FORCE Z 1.0 MASS 36.44
INFL COE LOAD 'P12'
JOINT '32' FORCE Z 1.0 MASS 36.44
INFL COE LOAD 'P13'
JOINT '47' FORCE Z 1.0 MASS 99.0
INFL COE LOAD 'P14'
JOINT '53' FORCE Z 1.0 MASS 99.0
INFL COE LOAD 'P15'
JOINT '62' FORCE Z 1.0 MASS 190.0
INFL COE LOAD 'P16'
JOINT '47' FORCE X 1.0 MASS 99.0
INFL COE LOAD 'P17'
JOINT '53' FORCE X 1.0 MASS 99.0
INFL COE LOAD 'P18'
JOINT '62' FORCE X 1.0 MASS 190.0
INFL COE LOAD 'P19'
JOINT '47' FORCE Y 1.0 MASS 99.00
INFL COE LOAD 'P20'
JOINT '53' FORCE Y 1.0 MASS 99.00
INFL COE LOAD 'P21'
JOINT '62' FORCE Y 1.0 MASS 190.0
INFL COE LOAD 'P22'
JOINT '66' FORCE Z 1.0 MASS 1.0
PRINT DATA
```

Section 12

DDAM Output

Section 12.a

Frequency, participation factors and modal weights for each mode used in the NRL sum

MODAL ANALYSIS RESULTS WITH CLOSELY SPACED MODES

MODE NO.	FREQUENCY (HZ)	VERTICAL PARTICIPATION FACTOR	MODAL WEIGHT LONGITUDINAL (LBS.)	MODAL WEIGHT TRANSVERSE (LBS.)	MODAL WEIGHT VERTICAL (LBS.)
1	56.62	-0.0100	295.82	0.01918	0.0224
2	64.68	0.0026	0.0148	275.70	0.0014
3	198.80	-0.0870	42.68	1.056	1.0609
4	227.43	-0.0187	2.178	36.164	0.0415
5	253.22	-0.0512	45.54	0.171	0.3077
6	323.32	10.5355	0.0217	0.0011	206.37
7	350.79	-11.0740	0.1375	0.0218	275.29
8	380.54	0.8610	1.237	3.6897	63.28
9	401.14	-0.2271	0.0518	74.43	5.5472
10	441.45	0.6362	0.1365	0.0012	92.06
11	676.50	0.1602	0.0020	0.5714	3.349

FIGURE E-14

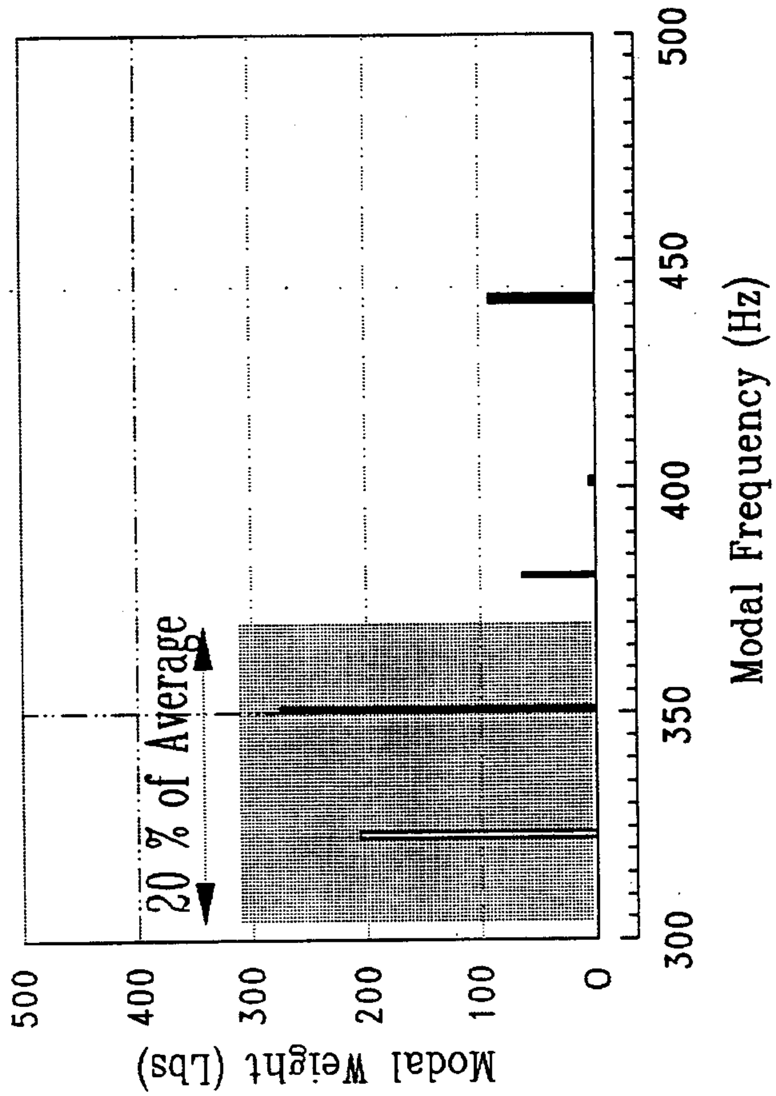
MODAL ANALYSIS RESULTS WITHOUT CLOSELY SPACED MODES

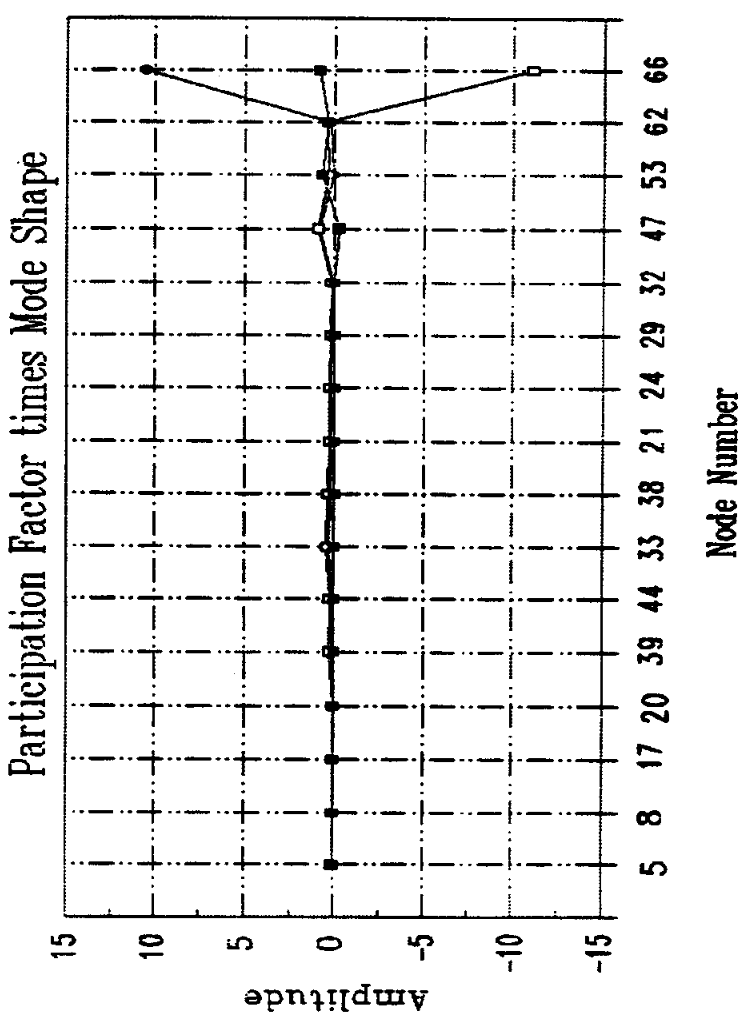
MODE NO.	FREQUENCY (HZ)	VERTICAL PARTICIPATION FACTOR	MODAL WEIGHT LONGITUDINAL (LBS.)	MODAL WEIGHT TRANSVERSE (LBS.)	MODAL WEIGHT VERTICAL (LBS.)
1	56.616	-0.0099	295.82	0.0192	0.0220
2	64.680	0.0026	0.0148	275.6973	0.0014
3	198.804	-0.0854	42.7008	1.0570	1.022393
4	227.429	-0.0183	2.1749	36.1630	0.0395
5	253.237	-0.0564	45.5134	0.1714	0.3731
6	337.352	1.7753	0.1159	0.0071	459.8850
7	379.979	0.8230	1.2811	3.5690	80.3386
8	401.124	-0.2347	0.0500	70.5675	5.8884
9	440.776	0.5239	0.1397	0.0008	92.4231
10	676.498	0.1602	0.0020	0.5714	3.3495
11	736.586	0.0705	0.10558	0.0007	0.8406

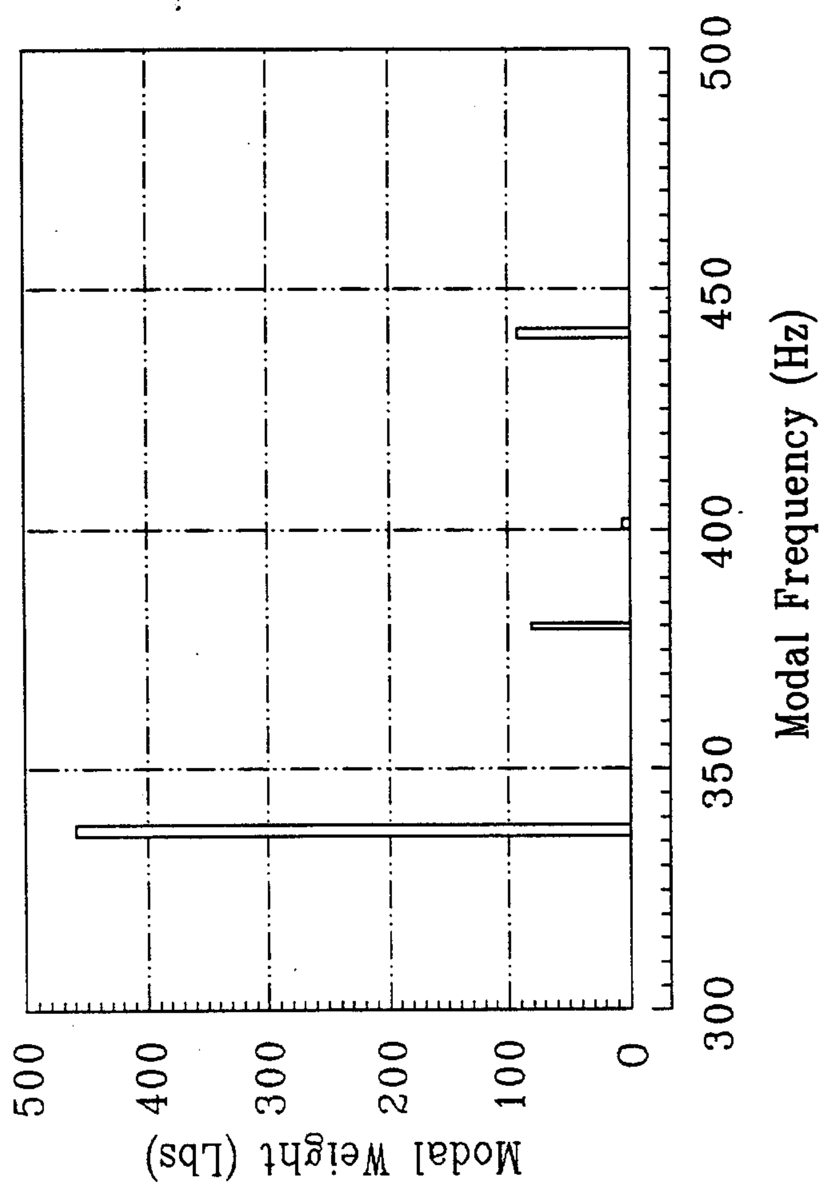
FIGURE E-15

Section 12.b

Modal mass vs. Frequency and Eigenvector vs. Node Number Charts


20 % of Average
Modal Frequency (Hz)
300
350
400
450
500
Modal Weight (Lbs)
0
100
200
300
400
500


Participation Factor times Mode Shape
Amplitude
15
10
5
0
-5
-10
-15
Node Number
5
8
17
20
39
44
33
38
21
24
29
32
47
53
62
66


Modal Weight (Lbs)
0
100
200
300
400
500
Modal Frequency (Hz)
300
350
400
450
500

Section 12.c

Modal output (mode shape, forces, deflections) for each mode

MODE 6 PAGE

FREQUENCY = 0.21196482E 04 RAD/SEC
= 0.33735229E 03 HERTZ
FREQUENCY SQUARED = 0.44929080E 07 (RAD/SEC) SQUARED

X-DIRECTION MODEL

PART. FACTOR = 0.2818041E-01
EFF. WEIGHT = 0.1158785E 00
EFF. MASS = 0.3002035E-03
CUMULATIVE MASS SUM = 0.1000808E 01
PERC. OF TOTAL MASS = 99.59

Y-DIRECTION MODEL

PART. FACTOR = 0.6995454E-02
EFF. WEIGHT = 0.7140689E-02
EFF. MASS = 0.1849919E-04
CUMULATIVE MASS SUM = 0.8111787E 00
PERC. OF TOTAL MASS = 80.71

Z-DIRECTION MODEL

PART. FACTOR = 0.1775294E 01
EFF. WEIGHT = 0.4598850E 03
EFF. MASS = 0.1191412E 01
CUMULATIVE MASS SUM = 0.1195189E 01
PERC. OF TOTAL MASS = 61.01

* * * TRANSLATIONAL * * *

MASS NO.	MASS	MODE SHAPE
41	0.98999939E 02	0.87237895E-01
53	0.98999939E 02	-0.20963188E-01
62	0.18999989E 03	-0.12886442E-01

MASS NO.	MASS	MODE SHAPE
47	0.98999939E 02	0.30670548E-04
53	0.98999939E 02	0.13716683E-01
62	0.18999989E 03	-0.17896874E-02

MASS NO.	MASS	MODE SHAPE
5	0.32559967E 02	0.96940637E-01
8	0.32559967E 02	0.93950629E-01
17	0.32559967E 02	0.59981514E-01
20	0.32559967E 02	0.51879659E-01
39	0.23609970E 02	0.25540978E 00
44	0.22539978E 02	0.21816552E 00
33	0.23609970E 02	0.37619847E 00
38	0.22539978E 02	0.34938973E 00
21	0.36439957E 02	0.29236048E 00
24	0.36439957E 02	0.28312588E 00
29	0.36439957E 02	0.18876404E 00
32	0.36439957E 02	0.16276109E 00
47	0.98999939E 02	0.10000000E 01
58	0.98999939E 02	0.22408730E 00
62	0.18999989E 03	0.35035175E 00

MODE 6 PAGE

SHOCK RESPONSES

	X-DIRECTION INPUT	Y-DIRECTION INPUT	Z-DIRECTION INPUT
G FACTORS:			
ACC. =	0.0	0.0	65.86
VEL. =	0.0	0.0	150.99

X-DIRE. TRANS. WEIGHTS

MASS NO.		X FORCE	X DEFLECTION	Y FORCE	Y DEFLECTION	Z FORCE	Z DEFLECTION
47	0.25643015E 00					0.1009580E 04	0.8762821E-03
53	0.25643015E 00					-0.2426009E 03	-0.2105698E-03
62	0.49213868E 00					-0.2862112E 03	-0.1294410E-03

FORCE SUM: 0.48076782E 03

Y-DIRE. TRANS. WEIGHTS

MASS NO.		X FORCE	X DEFLECTION	Y FORCE	Y DEFLECTION	Z FORCE	Z DEFLECTION
47	0.25643015E 00					0.3549418E 00	0.3080777E-06
53	0.25643015E 00					0.1587394E 03	0.1377805E-03
62	0.49213868E 00					-0.3974945E 02	-0.1797694E-04

FORCE SUM: 0.11934488E 03

Z-DIRE. TRANS. WEIGHTS

MASS NO.		X FORCE	X DEFLECTION	Y FORCE	Y DEFLECTION	Z FORCE	Z DEFLECTION
5	0.84336996E-01					0.8689692E 03	0.9737441E-03
8	0.84336996E-01					0.3575889E 03	0.9437101E-03
17	0.84336996E-01					0.2282978E 03	0.6024991E-03
20	0.84336996E-01					0.1974611E 03	0.5211178E-03
39	0.61154675E-01					0.7049099E 03	0.2565525E-02
44	0.58383178E-01					0.5748311E 03	0.2191417E-02
33	0.61154675E-01					0.1038277E 04	0.3778818E-02
38	0.58383178E-01					0.9205859E 03	0.3509530E-02
21	0.94386995E-01					0.1245366E 04	0.2936686E-02
24	0.94386995E-01					0.1206030E 04	0.2843927E-02
29	0.94386995E-01					0.8040774E 03	0.1896085E-02
32	0.94386995E-01					0.6933127E 03	0.1634894E-02
47	0.25643015E 00					0.1157272E 05	0.1004475E-01
53	0.25643015E 00					0.2593299E 04	0.2250898E-02
62	0.49213868E 00					0.7781406E 04	0.3519193E-02

FORCE SUM: 0.30287121E 05

Section 12.d

Internal member force calculations for each mode

INTERNAL MEMBER FORCES (UNITS: INCH LBF) FOR MODE 6

SHOCK INPUT IN Z-DIRECTION

MEMBER	JOINT	FORCES AXIAL	FORCES SHEAR Y	FORCES SHEAR Z	MOMENTS TORSIONAL	MOMENTS BENDING Y	MOMENTS BENDING Z
1	42	-0.1954712E 00	0.1383920E 01	-0.4322690E 00	-0.9483546E-01	0.3626894E 01	0.1849832E 03
1	36	0.1954712E 00	-0.1383920E 01	0.4322690E 00	0.9483546E-01	0.3613611E 01	-0.1618025E 03
2	33	-0.1998118E 04	0.1983928E 03	-0.3157361E 04	0.2404441E 01	0.2510977E 05	0.1331114E 04
2	34	0.1998118E 04	-0.1983928E 03	0.3157361E 04	-0.2404441E 01	-0.1879505E 05	-0.9343289E 03
3	34	-0.1032040E 02	0.1606599E 03	-0.1566009E 04	0.2404477E 01	0.1879505E 05	0.9343289E 03
3	35	0.1032040E 02	-0.1606599E 03	0.1566009E 04	-0.2404477E 01	-0.2934170E 01	0.9935889E 03
4	35	0.8511022E 01	0.2624963E 02	-0.1568310E 04	0.2404605E 01	0.2934186E 01	-0.9935889E 03
4	36	-0.8511022E 01	-0.2624963E 02	0.1568310E 04	-0.2404605E 01	0.2349531E 04	0.1032964E 04
5	36	0.9894941E 01	0.2644508E 02	-0.1568742E 04	-0.1209004E 01	-0.2349436E 04	-0.8711614E 03
5	37	-0.9894941E 01	-0.2644508E 02	0.1568742E 04	0.1209004E 01	0.1882123E 05	0.1148835E 04
6	37	0.1889729E 04	-0.1478954E 03	-0.2939957E 04	-0.1208838E 01	-0.1882122E 05	-0.1148835E 04
6	38	-0.1889729E 04	0.1478954E 03	0.2939957E 04	0.1208838E 01	0.2470114E 05	0.8530444E 03
7	40	-0.2002704E 04	0.2023477E 03	0.3783514E 04	0.2411650E 01	0.1783334E 05	0.1062189E 04
7	39	0.2002704E 04	-0.2023477E 03	-0.3783514E 04	-0.2411650E 01	-0.2540038E 05	-0.6574934E 03
8	41	-0.1005858E 02	-0.4372334E 02	0.1478254E 04	0.2411884E 01	0.9429599E 02	0.5375078E 03
8	40	0.1005858E 02	0.4372334E 02	-0.1478254E 04	-0.2411884E 01	-0.1783335E 05	-0.1062189E 04
9	42	0.1123364E 02	-0.1107252E 03	0.1556068E 04	0.2411948E 01	-0.2239806E 04	0.3714202E 03
9	41	-0.1123364E 02	0.1107252E 03	-0.1556068E 04	-0.2411948E 01	-0.9429601E 02	-0.5375078E 03
10	43	0.9849720E 01	-0.1109207E 03	0.1555635E 04	-0.1214944E 01	-0.1857407E 05	-0.6082632E 03
10	42	-0.9849720E 01	0.1109207E 03	-0.1555635E 04	0.1214944E 01	0.2239900E 04	-0.5564031E 03
11	44	0.1890854E 04	-0.1446765E 03	0.2755554E 04	-0.1214911E 01	-0.2408518E 05	-0.8976160E 03
11	43	-0.1890854E 04	0.1446765E 03	-0.2755554E 04	0.1214911E 01	0.1857407E 05	0.6082692E 03
12	39	0.6849659E 00	-0.4198747E 02	-0.3912288E 03	0.6293908E 00	0.3278578E 04	0.6426099E 03
12	33	-0.6849659E 00	0.4198747E 02	0.3912288E 03	-0.6293908E 00	0.3274507E 04	-0.1345900E 04
13	38	-0.7540998E 00	-0.1585578E 01	-0.1546824E 03	0.3151386E 00	0.1294008E 04	0.8618423E 03
13	44	0.7540998E 00	0.1585578E 01	0.1546824E 03	-0.3151386E 00	0.1296923E 04	-0.8884014E 03
14	3	-0.1467388E 05	-0.4794039E 04	0.1260691E 04	-0.1316880E 02	-0.1786548E 05	-0.4611179E 05
14	17	0.1467388E 05	0.4794039E 04	-0.1260691E 04	0.1316880E 02	0.7149609E 04	0.5362453E 04
15	17	-0.1172732E 05	-0.4400478E 04	0.9917087E 03	-0.5938419E 02	-0.7312898E 04	-0.4773234E 05
15	29	0.1172732E 05	0.4400478E 04	-0.9917087E 03	0.5938419E 02	-0.1252127E 05	-0.4027711E 05
16	29	-0.4333480E 04	-0.2044692E 04	0.2018628E 03	-0.1488377E 02	-0.9539277E 03	-0.1753878E 05
16	39	0.4333480E 04	0.2044692E 04	-0.2018628E 03	0.1488377E 02	-0.3280990E 04	-0.2539974E 05
17	29	0.5658542E 02	-0.3534546E 03	-0.1599562E 04	0.1639836E 02	0.1337109E 05	-0.2969490E 04
17	21	-0.5658542E 02	0.3534546E 03	0.1599562E 04	-0.1639836E 02	0.1342156E 05	-0.2950881E 04
18	21	-0.3829771E 04	0.7472783E 03	-0.7052738E 04	0.6650551E 02	0.6769962E 05	0.2905752E 04
18	22	0.3829771E 04	-0.7472783E 03	0.7052738E 04	-0.6650551E 02	-0.4224918E 04	0.3819757E 04
19	22	-0.9762149E 02	-0.6404709E 02	-0.1083455E 04	0.2025224E 01	0.4090574E 04	-0.5155652E 03
19	23	0.9762149E 02	0.6404709E 02	0.1083455E 04	-0.2025224E 01	0.7285703E 04	-0.1569289E 03
20	23	0.3516438E 04	0.4584172E 02	-0.6792656E 04	-0.8200980E 01	-0.7860945E 04	-0.2314704E 03
20	24	-0.3516438E 04	-0.4584172E 02	0.6792656E 04	0.8200980E 01	0.6559850E 05	0.6211213E 03
21	5	-0.9120051E 03	0.1974756E 03	-0.4489633E 04	0.1348679E 03	0.5294835E 05	0.4775768E 02
21	6	0.9120051E 03	-0.1974756E 03	0.4489633E 04	-0.1348679E 03	-0.1254164E 05	0.1729522E 04
22	6	-0.1629585E 03	-0.7290945E 02	-0.2678604E 04	0.1133904E 02	0.1254185E 05	-0.1139487E 04
22	7	0.1629585E 03	0.7290945E 02	0.2678604E 04	-0.1133904E 02	0.1558348E 05	0.3739377E 03
23	7	0.9000303E 03	-0.1982595E 03	-0.4160270E 04	-0.8793755E 00	-0.1558169E 05	-0.1689473E 04
23	8	-0.9000303E 03	0.1982595E 03	0.4160270E 04	0.8793755E 00	0.5094398E 05	0.4267736E 01
24	31	-0.2715076E 01	-0.1658241E 04	0.1554135E 04	-0.1528995E 02	-0.1526698E 03	-0.4732172E 04

24	28	0.2715076E 01	0.1658241E 04	-0.1554135E 04	0.1528995E 02	-0.7229461E 04	-0.3144471E 04
25	28	-0.9032055E 01	0.3524636E 03	-0.8110161E 03	-0.1528995E 02	0.7229461E 04	0.3144471E 04
25	26	0.9032055E 01	-0.3524636E 03	0.8110161E 03	0.1528995E 02	0.2097223E 04	0.9088662E 03
26	26	-0.1098885E 03	0.3614059E 04	-0.5709203E 04	0.5752419E 03	0.2844376E 04	0.1418631E 04
26	23	0.1098885E 03	-0.3614059E 04	0.5709203E 04	-0.5752419E 03	0.1022620E 02	0.3883992E 03
27	19	-0.2515519E 03	-0.2184111E 03	-0.2130930E 03	-0.1784533E 01	-0.1709988E 01	-0.7109152E 02
27	14	0.2515519E 03	0.2184111E 03	0.2130930E 03	0.1784533E 01	0.6409893E 03	-0.5841423E 03
28	14	-0.2515519E 03	-0.2184111E 03	-0.2130930E 03	-0.1784533E 01	-0.6409893E 03	0.5841423E 03
28	12	0.2515519E 03	0.2184111E 03	0.2130930E 03	0.1784533E 01	0.1013902E 04	-0.9663618E 03
29	12	-0.1374724E 01	0.1522090E 03	0.1515232E 03	-0.1784533E 01	-0.1013902E 04	0.9663618E 03
29	10	0.1374724E 01	-0.1522090E 03	-0.1515232E 03	0.1784533E 01	-0.7286143E 03	0.7840393E 03
30	10	0.1253500E 03	0.1062989E 04	-0.1481666E 04	-0.1784533E 01	0.7286143E 03	-0.7840393E 03
30	7	-0.1253500E 03	-0.1062989E 04	0.1481666E 04	0.1784533E 01	0.1221837E 02	0.1315534E 04
31	4	0.8912230E 04	-0.4585164E 04	0.9582115E 02	-0.4787227E 02	-0.8414573E 03	-0.4580775E 05
31	20	-0.8912230E 04	0.4585164E 04	-0.9582115E 02	0.4787227E 02	0.2697954E 02	0.6833855E 04
32	20	0.6271027E 04	-0.4197469E 04	-0.4359738E 02	-0.5058746E 02	-0.3277174E 02	-0.4598804E 05
32	32	-0.6271027E 04	0.4197469E 04	0.4359738E 02	0.5058746E 02	0.9035173E 03	-0.3796136E 05
33	32	0.2640620E 04	-0.1889268E 04	0.1454307E 03	-0.9215221E 01	-0.1755906E 04	-0.1558976E 05
33	44	-0.2640620E 04	0.1889268E 04	-0.1454307E 03	0.9215221E 01	-0.1298138E 04	-0.2408487E 05
34	32	-0.1530719E 03	-0.8401668E 02	-0.1059339E 03	0.1221570E 02	0.9157793E 03	-0.7435977E 03
34	24	0.1530719E 03	0.8401668E 02	0.1059339E 03	-0.1221570E 02	0.8586147E 03	-0.6636799E 03
35	29	-0.2002326E 04	0.8466306E 03	-0.5089363E 04	0.1041021E 03	0.5783229E 05	0.2924989E 04
35	30	0.2002326E 04	-0.8466306E 03	0.5089363E 04	-0.1041021E 03	-0.1202800E 05	0.4694687E 04
36	30	0.5659434E 03	0.3447549E 03	-0.2757042E 04	-0.8927852E 02	0.1204589E 05	0.1092867E 04
36	31	-0.5659434E 03	-0.3447549E 03	0.2757042E 04	0.8927852E 02	0.1690305E 05	0.2527063E 04
37	31	0.2224185E 04	0.3420398E 03	-0.4311176E 04	0.6339131E 02	-0.1691833E 05	0.2205115E 04
7	32	-0.2224185E 04	-0.3420398E 03	0.4311176E 04	-0.6339131E 02	0.5356333E 05	0.7022253E 03
9	30	-0.5018752E 03	-0.2568270E 04	-0.2332323E 04	0.1788275E 02	0.1933806E 03	-0.5787555E 04
38	27	0.5018752E 03	0.2568270E 04	0.2332323E 04	-0.1788275E 02	0.1088515E 05	-0.6411715E 04
39	27	0.5602028E 02	0.3375187E 01	-0.2396113E 01	0.9743214E-01	0.2157974E 01	0.2756471E 02
39	25	-0.5602028E 02	-0.3375187E 01	0.2396113E 01	-0.9743214E-01	0.2539781E 02	0.1124996E 02
40	25	0.8113252E 03	0.3732148E 04	0.5969285E 04	0.1343389E 03	-0.3049127E 04	0.5170262E 04
40	22	-0.8113252E 03	-0.3732148E 04	-0.5969285E 04	-0.1343389E 03	0.6448027E 02	-0.3304192E 04
41	18	-0.1567902E 03	-0.5628503E 03	-0.3781262E 03	-0.2157167E 00	0.1707930E 03	-0.2294308E 04
41	13	0.1567902E 03	0.5628503E 03	0.3781262E 03	0.2157167E 00	0.9635869E 03	0.6057563E 03
42	13	-0.1567902E 03	-0.5628503E 03	-0.3781262E 03	-0.2157167E 00	-0.9635869E 03	-0.6057563E 03
42	11	0.1567902E 03	0.5628503E 03	0.3781262E 03	0.2157167E 00	0.1625310E 04	-0.3792319E 03
43	11	0.1486597E 01	-0.5089801E 02	0.5184911E 02	-0.2157167E 00	-0.1625310E 04	0.3792319E 03
43	9	-0.1486597E 01	0.5089801E 02	-0.5184911E 02	0.2157167E 00	0.1029044E 04	-0.9645583E 03
44	9	0.2703850E 03	0.7490471E 03	0.1811031E 04	-0.2157167E 00	-0.1029044E 04	0.9645583E 03
44	6	-0.2703850E 03	-0.7490471E 03	-0.1811031E 04	0.2157167E 00	0.1235289E 03	-0.5900352E 03
45	17	-0.3935669E 03	0.2689832E 03	-0.3232777E 04	0.1632917E 03	0.4236988E 05	0.4621541E 02
45	18	0.3935669E 03	-0.2689832E 03	0.3232777E 04	-0.1632917E 03	-0.1327489E 05	0.2374635E 04
46	18	0.1692836E 03	0.1121932E 03	-0.2854650E 04	-0.7501294E 01	0.1327467E 05	-0.8032578E 02
46	19	-0.1692836E 03	-0.1121932E 03	0.2854650E 04	0.7501294E 01	0.1669915E 05	0.1258354E 04
47	19	0.3876946E 03	-0.1393586E 03	-0.2641557E 04	-0.5791351E 01	-0.1670094E 05	-0.1187263E 04
47	20	-0.3876946E 03	0.1393586E 03	0.2641557E 04	0.5791351E 01	0.3915418E 05	0.2715211E 01
48	1	-0.1431579E 05	-0.6344449E 04	0.1200417E 04	-0.1215800E 02	-0.1754316E 05	-0.5788070E 05
48	5	0.1431579E 05	0.6344449E 04	-0.1200417E 04	0.1215800E 02	0.7339617E 04	0.3952865E 04
49	5	-0.8816578E 04	-0.5432445E 04	0.1002942E 04	-0.5991566E 02	-0.7474484E 04	-0.5690121E 05
49	21	0.8816578E 04	0.5432445E 04	-0.1002942E 04	0.5991566E 02	-0.1258435E 05	-0.5174770E 05
50	21	-0.3135100E 04	-0.1956130E 04	0.1990778E 03	-0.1478629E 02	-0.9037219E 03	-0.1596834E 05
50	33	0.3135100E 04	0.1956130E 04	-0.1990778E 03	0.1478629E 02	-0.3276912E 04	-0.2511040E 05
51	2	0.1384105E 05	-0.6223762E 04	0.1464871E 03	-0.4708981E 02	-0.1136753E 04	-0.5783809E 05
51	8	-0.1384105E 05	0.6223762E 04	-0.1464871E 03	0.4708981E 02	-0.1083817E 03	0.4936086E 04
52	8	0.9438176E 04	-0.5323734E 04	-0.5177223E 02	-0.5135751E 02	0.1075019E 03	-0.5588007E 05
52	24	-0.9438176E 04	0.5323734E 04	0.5177223E 02	0.5135751E 02	0.9279382E 03	-0.5059463E 05

53	24	0.2737052E 04	-0.1891314E 04	0.1471413E 03	-0.8798150E 01	-0.1794751E 04	-0.1501614E 05
53	38	-0.2737052E 04	0.1891314E 04	-0.1471413E 03	0.8798150E 01	-0.1295217E 04	-0.2470146E 05
54	34	-0.1591351E 04	-0.1987798E 04	0.3773293E 02	0.0	0.3096214E-09	-0.2938999E-08
54	59	0.1591351E 04	0.1987798E 04	-0.3773293E 02	0.0	-0.3660095E 03	-0.1928162E 05
55	35	0.2300529E 01	-0.1883141E 02	0.1944103E 03	0.0	-0.2746479E-11	-0.2845974E-08
55	60	-0.2300529E 01	0.1883141E 02	-0.1944103E 03	0.0	-0.1303779E 04	-0.1826647E 03
56	37	0.1371214E 04	-0.1879834E 04	0.1743405E 03	0.0	-0.5257186E-09	0.1787782E-08
56	61	-0.1371214E 04	0.1879834E 04	-0.1743405E 03	0.0	-0.1691101E 04	-0.1823438E 05
57	40	-0.2305259E 04	-0.1992646E 04	0.2460711E 03	0.0	-0.6355620E-09	-0.4853238E-08
57	63	0.2305259E 04	0.1992646E 04	-0.2460711E 03	0.0	-0.2386888E 04	-0.1932865E 05
58	41	0.7781329E 02	-0.2129221E 02	0.6700186E 02	0.0	0.6934001E-10	0.4707805E-09
58	64	-0.7781329E 02	0.2129221E 02	-0.6700186E 02	0.0	-0.6499180E 03	-0.2065345E 03
59	43	0.1199918E 04	-0.1881003E 04	0.3375586E 02	0.0	0.6185419E-09	-0.2132127E-08
59	65	-0.1199918E 04	0.1881003E 04	-0.3375586E 02	0.0	-0.3274316E 03	-0.1824573E 05
60	59	-0.1386711E 04	-0.5968223E 03	0.8078774E 03	0.3014741E 03	-0.9916523E 04	-0.2003240E 04
60	62	0.1386711E 04	0.5968223E 03	-0.8078774E 03	-0.3014741E 03	-0.8230486E 03	-0.5930652E 04
61	60	0.4891914E 03	-0.9464917E 03	0.1233561E 03	0.1865116E 03	0.1401603E 04	-0.9373899E 03
61	62	-0.4891914E 03	0.9464917E 03	-0.1233561E 03	-0.1865116E 03	-0.2461537E 04	-0.7195320E 04
62	61	0.1452816E 04	-0.3214263E 03	-0.4714680E 03	0.1664707E 03	0.9076301E 04	-0.1658278E 04
62	62	-0.1452816E 04	0.3214263E 03	0.4714680E 03	-0.1664707E 03	-0.1489574E 04	-0.3514016E 04
63	63	-0.1601426E 04	-0.4769824E 03	-0.1013065E 04	-0.2694167E 03	[illegible]50750E 04	-0.1788185E 04
63	62	0.1601426E 04	0.4769824E 03	0.1013065E 04	0.2694167E 03	[illegible]550928E 04	-0.4525512E 04
64	64	0.1704340E 03	-0.9861804E 03	-0.2088116E 03	-0.1316889E 03	0.6207979E 03	-0.1017114E 04
64	62	-0.1704340E 03	0.9861804E 03	0.2088116E 03	0.1316889E 03	0.1137990E 04	-0.7369668E 04
65	65	0.1331655E 04	-0.4158311E 03	0.4342769E 03	-0.2371902E 03	-0.7679977E 04	-0.1927110E 04
65	62	-0.1331655E 04	0.4158311E 03	-0.4342769E 03	0.2371902E 03	0.7121057E 03	-0.4744809E 04
66	9	-0.1759182E 04	-0.7999451E 03	0.2688984E 03	0.0	0.2469234E-09	-0.2963065E-09
66	50	0.1759182E 04	0.7999451E 03	-0.2688984E 03	0.0	-0.1882289E 04	-0.5599617E 04
67	10	0.1633189E 04	-0.9107800E 03	0.1267247E 03	0.0	-0.8969535E-10	0.2855383E-08
67	51	-0.1633189E 04	0.9107800E 03	-0.1267247E 03	0.0	-0.8870725E 03	-0.6375453E 04
68	11	-0.4299758E 03	-0.5119521E 03	0.1582768E 03	0.0	0.1398242E-09	0.1181675E-08
68	52	0.4299758E 03	0.5119521E 03	-0.1582768E 03	0.0	-0.1107937E 04	-0.3583668E 04
69	12	-0.3646157E 03	-0.3706201E 03	0.2501771E 03	0.0	0.1918700E-11	0.1067477E-08
69	54	0.3646157E 03	0.3706201E 03	-0.2501771E 03	0.0	-0.1751240E 04	-0.2594343E 04
70	50	-0.3288037E 02	0.9126788E 02	-0.8603914E 03	0.2856372E 03	0.4587906E 04	0.3107297E 03
70	51	0.3288037E 02	-0.9126788E 02	0.8603914E 03	-0.2856372E 03	0.4446195E 04	0.6475828E 03
71	50	0.1715005E 03	-0.2737230E 02	0.9452751E 02	0.6830519E 01	0.1902156E 04	0.6211382E 03
71	52	-0.1715005E 03	0.2737230E 02	-0.9452751E 02	-0.6830519E 01	-0.2909221E 04	-0.9359202E 03
72	51	-0.1574488E 03	-0.5617030E 02	-0.3156130E 03	-0.2602640E 01	0.2058589E 02	0.5106462E 03
72	54	0.1574488E 03	0.5617030E 02	0.3156130E 03	0.2602640E 01	0.3608964E 04	-0.1156605E 04
73	52	-0.5359790E 03	-0.1764902E 03	-0.3312029E 03	-0.2335971E 02	0.4059363E 04	-0.6372478E 03
73	53	0.5359790E 03	0.1764902E 03	0.3312029E 03	0.2335971E 02	-0.1776013E 04	-0.1005798E 04
74	53	0.1132996E 03	0.2363238E 03	-0.1211519E 03	-0.7878723E 02	0.2247490E 04	0.1542333E 04
74	54	-0.1132996E 03	-0.2363238E 03	0.1211519E 03	0.7878723E 02	-0.1119621E 04	0.6577363E 03
75	54	0.1132996E 03	-0.2363237E 03	-0.1211519E 03	-0.7878723E 02	-0.1119621E 04	-0.6577363E 03
75	53	-0.1132996E 03	0.2363237E 03	0.1211519E 03	0.7878723E 02	0.2247490E 04	-0.1542333E 04
76	50	-0.5949490E 03	-0.4395654E 03	0.8042625E 03	0.3432048E 03	-0.9926350E 03	-0.3104082E 03
76	53	0.5949490E 03	0.4395654E 03	-0.8042625E 03	-0.3432048E 03	-0.4225402E 04	-0.2541482E 04
77	51	0.9387371E 03	-0.2176403E 03	-0.4571846E 03	0.2027748E 03	0.2240168E 04	0.1369365E 03
77	53	-0.9387371E 03	0.2176403E 03	0.4571846E 03	-0.2027748E 03	0.7260344E 03	-0.1548982E 04
78	52	-0.9131798E 02	-0.1331468E 02	-0.1933004E 03	0.9738043E 02	0.8308352E 03	-0.2988721E 03
78	54	0.9131798E 02	0.1331468E 02	0.1933004E 03	-0.9738043E 02	0.1198821E 04	0.1588678E 03
79	25	-0.5971880E 04	-0.3728774E 04	0.7553049E 03	-0.5181512E 04	0.3023730E 04	0.1342415E 03
79	45	0.5971880E 04	0.3728774E 04	-0.7553049E 03	0.5181512E 04	-0.8310867E 04	-0.2629564E 05
80	26	0.4898184E 04	-0.3261595E 04	-0.1008566E 03	-0.2827498E 04	-0.4941594E 04	0.5905322E 03
80	46	-0.4898184E 04	0.3261595E 04	0.1008566E 03	0.2827498E 04	0.5647596E 04	-0.2842159E 05
81	27	-0.2329927E 04	-0.2571645E 04	0.5578955E 03	0.6884148E 04	-0.1088730E 05	-0.1778532E 02

ELEMENT	NODE	ELEMENT RESULTS					
81	48	0.2329927E 04	0.2571645E 04	-0.5578955E 03	-0.6384148E 04	0.6982035E 04	-0.1798373E 05
82	28	0.2365152E 04	-0.2010706E 04	-0.6316901E 01	0.0	0.2254271E-08	0.1392358E-08
82	49	-0.2365152E 04	0.2010706E 04	0.6316901E 01	0.0	0.4421609E 02	-0.1407493E 05
83	45	-0.2212878E 03	0.3054250E 03	-0.2666609E 04	0.0	-0.2616961E-11	-0.1014457E-12
83	46	0.2212878E 03	-0.3054250E 03	0.2666609E 04	0.0	0.2799939E 05	0.3206968E 04
84	45	0.9418457E 03	-0.6004147E 02	0.8546851E 03	0.0	-0.6233659E-11	-0.2284772E-12
84	48	-0.9418457E 03	0.6004147E 02	-0.8546851E 03	0.0	-0.9828879E 04	-0.6904768E 03
85	46	-0.8616960E 03	0.1210643E 02	-0.5670300E 03	0.0	0.6202509E-12	-0.3054292E-12
85	49	0.8616960E 03	-0.1210643E 02	0.5670300E 03	0.0	0.6520848E 04	0.1392242E 03
86	48	-0.1748403E 03	0.1108430E 03	-0.2162155E 04	-0.6631707E 03	0.1430213E 05	0.2175845E 04
86	49	0.1748403E 03	-0.1108430E 03	0.2162155E 04	0.6631707E 03	0.8320484E 04	-0.1011993E 04
87	45	-0.3078688E 04	-0.1627446E 04	0.2450394E 04	0.2213977E 05	-0.1634656E 05	0.5181512E 04
87	47	0.3078688E 04	0.1627446E 04	-0.2450394E 04	-0.2213977E 05	0.4484639E 03	-0.1574034E 05
88	48	-0.2532484E 04	-0.1245656E 04	-0.1022458E 04	-0.9948279E 03	0.4929695E 04	0.3517835E 04
88	47	0.2532484E 04	0.1245656E 04	0.1022458E 04	0.9948279E 03	0.4588922E 04	-0.1511433E 05
89	49	0.1850968E 04	-0.1367869E 04	0.7700266E 03	-0.6758662E 03	-0.9214355E 04	0.1151219E 04
89	47	-0.1850968E 04	0.1367869E 04	-0.7700266E 03	0.6758662E 03	0.2045753E 04	-0.1388547E 05
90	46	0.3454590E 04	-0.1190665E 04	-0.1664550E 04	0.1880354E 04	-0.7022445E 04	0.5534461E 04
90	47	-0.3454590E 04	0.1190665E 04	0.1664550E 04	-0.1880354E 04	0.1782201E 05	-0.1325946E 05
91	59	0.3657605E 03	-0.4896302E 01	-0.1749101E 03	0.5492976E 02	0.6312957E 04	0.2599073E 04
91	63	-0.3657605E 03	0.4896302E 01	0.1749101E 03	-0.5492976E 02	-0.3383210E 04	-0.2681086E 04
92	63	-0.4528162E 03	-0.2527003E 02	-0.1117284E 04	-0.1567612E 03	0.1163618E 05	-0.8929011E 03
92	64	0.4528162E 03	0.2527003E 02	0.1117284E 04	0.1567612E 03	0.1771216E 04	0.5896606E 03
93	64	0.5261887E 03	0.1002890E 02	-0.8326584E 03	-0.1414432E 03	-0.1568625E 04	0.4274553E 03
93	65	-0.5261887E 03	-0.1002890E 02	0.8326584E 03	0.1414432E 03	0.1156052E 05	-0.3071079E 03
94	65	-0.3797437E 03	-0.8009520E 00	-0.6701828E 02	0.1798983E 01	0.3970502E 04	0.2234227E 04
94	61	0.3797437E 03	0.8009520E 00	0.6701828E 02	-0.1798983E 01	-0.2847948E 04	-0.2247643E 04
95	61	0.4743889E 03	-0.6780923E 02	0.8327271E 03	0.6827701E 02	-0.1041084E 05	0.5893608E 03
95	60	-0.4743889E 03	0.6780923E 02	-0.8327271E 03	-0.6827701E 02	0.4181165E 03	-0.1403072E 04
96	60	-0.5317969E 03	0.8845938E 02	0.9583831E 03	0.1007819E 03	-0.1305896E 03	0.4656799E 03
96	59	0.5317969E 03	-0.8845938E 02	-0.9583831E 03	-0.1007819E 03	-0.1138201E 05	0.5958323E 03

Section 12.e

NRL sum of stresses for each member

NONE GIVEN

NRL STATISTICAL SUM OF STRESSES

NON-SYMMETRICAL SECTIONS

MEMBER	JOINT	Y SHEAR	Z SHEAR	TORSION	POINT 1 NORMAL	POINT 2 NORMAL	POINT 3 NORMAL	POINT 4 NORMAL	NRL SUM OF AXIAL STRESSES
1	42	0.061	0.958	0.215	6.840	2.530			0.261
1	36	0.061	0.958	0.215	7.297	2.046			0.261
2	33	0.376	7318.672	30.787	49434.684	11575.777			2001.674
2	34	0.376	7318.672	30.787	36920.020	7995.219			2001.674
3	34	0.285	3389.730	30.788	34929.828	9985.000			10.396
3	35	0.285	3389.730	30.788	55.168	22.770			10.396
4	35	0.087	3365.822	30.790	64.564	15.180			11.226
4	36	0.087	3365.822	30.790	4372.172	1238.231			11.226
5	36	0.087	3366.710	23.381	4371.574	1238.742			9.810
5	37	0.087	3366.710	23.381	34740.070	9931.449			9.810
6	37	0.296	7142.465	23.378	36651.152	8020.680			1921.864
6	38	0.296	7142.465	23.378	40871.273	11517.195			1921.864
7	40	0.380	8233.039	30.836	35721.434	7649.566			2002.500
7	39	0.380	8233.039	30.836	49033.809	11689.484			2002.500
8	41	0.131	3255.055	30.840	173.788	36.901			10.081
8	40	0.131	3255.055	30.840	33729.207	9641.746			10.081
9	42	0.224	3411.727	30.841	4242.797	1202.237			10.441
9	41	0.224	3411.727	30.841	154.476	56.460			10.441
10	43	0.225	3410.835	23.469	35010.207	10008.281			10.188
10	42	0.225	3410.835	23.469	4243.527	1201.563			10.188
11	44	0.292	6284.555	23.468	47671.242	11176.672			1920.045
11	43	0.292	6284.555	23.468	36919.891	8098.658			1920.045
12	39	0.122	742.142	12.219	5864.980	2614.855			0.959
12	33	0.122	742.142	12.219	5863.305	2611.660			0.959
13	38	0.106	485.734	8.881	3675.468	1377.611			0.991
13	44	0.106	485.734	8.881	3681.156	1377.192			0.991
14	3	3996.187	1002.607	100.374	9766.816	27833.652	25259.566		7535.449
14	17	3996.187	1002.607	100.374	4843.488	11753.941	6120.191		7535.449

NONE GIVEN

NRL STATISTICAL SUM OF STRESSES

NON-SYMMETRICAL SECTIONS

MEMBER	JOINT	Y SHEAR	Z SHEAR	TORSION	POINT 1 NORMAL	POINT 2 NORMAL	POINT 3 NORMAL	POINT 4 NORMAL	NRL SUM OF AXIAL STRESSES
15	17	3685.800	772.545	104.054	17475.281	24613.410	33395.695		6146.840
15	29	3685.800	772.545	104.054	3155.954	10944.840	33743.484		6146.840
16	29	2525.181	239.037	46.464	10416.879	12591.266	20347.668		2725.211
16	39	2525.181	239.037	46.464	6069.980	10139.566	30057.301		2725.211
17	29	271.728	1812.547	57.443	11572.617	3553.590	6319.426		30.527
17	21	271.728	1812.547	57.443	11619.[illegible]	3539.415	6290.793		30.527
18	21	568.329	5496.340	211.586	59574.027	23286.094	20911.934		1724.010
18	22	568.329	5496.340	211.586	13075.258	5147.852	5865.691		1724.010
19	22	55.280	2356.159	6.884	10763.551	4113.480	4457.629		42.277
19	23	55.280	2356.159	6.884	13746.406	5517.758	5541.176		42.277
20	23	180.856	5130.961	95.923	15467.121	5610.047	4708.176		1631.153
20	24	180.856	5130.961	95.923	57925.723	21855.355	21856.492		1631.153
21	5	248.942	3976.554	442.301	41191.789	15778.719	15722.520		1053.725
21	6	248.942	3976.554	442.301	14969.658	5742.402	7721.621		1053.725
22	6	184.456	3028.595	113.537	14932.180	5670.977	6202.543		100.989
22	7	184.456	3028.595	113.537	16093.449	7118.000	5636.934		100.989
23	7	262.641	3766.994	206.269	15568.055	7639.406	5406.988		1029.707
23	8	262.641	3766.994	206.269	39930.602	15269.934	15291.746		1029.707
24	31	1477.373	1459.536	149.907	1505.515	1720.003	4253.859		68.866
24	28	1477.373	1459.536	149.907	5575.492	3750.919	435.518		68.866
25	28	300.536	760.180	149.907	5622.973	3708.758	381.289		4.769
25	26	300.536	760.180	149.907	1625.611	1078.792	108.797		4.769
26	26	3171.674	4998.551	4739.883	1896.646	1570.573	1044.614		115.175
26	23	3171.674	4988.551	4739.883	251.532	280.827	865.723		115.175
27	19	907.617	799.282	25.339	1042.059	1197.248	2947.539		154.228
27	14	907.617	799.282	25.339	2387.289	765.610	1542.199		154.228
28	14	907.617	799.282	25.339	2387.289	765.610	1542.199		154.228
28	12	907.617	799.282	25.339	3263.015	1866.136	1357.171		154.228

NONE GIVEN

NRL STATISTICAL SUM OF STRESSES

NON-SYMMETRICAL SECTIONS

MEMBER	JOINT	Y SHEAR	Z SHEAR	TORSION	POINT 1 NORMAL	POINT 2 NORMAL	POINT 3 NORMAL	POINT 4 NORMAL	NRL SUM OF AXIAL STRESSES
29	12	189.589	248.739	25.339	3343.966	1820.680	1204.406		1.787
29	10	189.589	248.739	25.339	1590.898	759.160	1222.547		1.787
30	10	2043.350	3135.112	25.339	1630.387	607.705	1316.258		168.132
30	7	2043.350	3135.112	25.339	546.275	610.793	1589.741		168.132
31	4	3769.005	455.107	150.462	24086.543	17287.453	35390.469		4169.176
31	20	3769.005	455.107	150.462	8570.129	6855.375	4780.523		4169.176
32	20	3502.703	324.166	190.866	20354.641	18351.195	37294.695		3071.831
32	32	3502.703	324.166	190.866	10758.660	8648.398	32966.770		3071.831
33	32	2385.790	195.856	32.123	12181.629	9619.145	21004.855		1635.637
33	44	2385.790	195.856	32.123	11273.113	8553.629	29816.984		1635.637
34	32	113.874	596.501	42.013	5116.086	1796.744	2607.063		64.212
34	24	113.874	596.501	42.013	5101.172	1824.155	2548.401		64.212
35	29	659.678	4525.824	309.939	55651.973	20541.832	22920.301		925.108
35	30	659.678	4525.824	309.939	13886.402	6307.875	4160.590		925.108
36	30	298.974	3199.660	315.251	14340.371	5759.591	6281.051		230.845
36	31	298.974	3199.660	315.251	19397.949	6734.284	8624.715		230.845
37	31	385.231	3854.158	310.747	19029.176	7153.641	6304.043		997.265
37	32	385.231	3854.158	310.747	51503.703	19690.125	19764.090		997.265
38	30	2187.917	1835.479	371.898	1533.625	1738.046	4810.648		199.442
38	27	2187.917	1835.479	371.898	6058.758	5751.238	2037.385		199.442
39	27	2.923	1.918	0.119	13.428	15.472	47.825		24.021
39	25	2.923	1.918	0.119	48.074	19.933	6.283		24.021
40	25	3224.362	4962.238	3097.401	1269.866	2224.312	3336.247		325.000
40	22	3224.362	4962.238	3097.401	617.823	681.062	2769.893		325.000
41	18	1005.824	808.689	21.805	1155.393	1406.502	3449.509		125.688
41	13	1005.824	808.689	21.805	2371.941	726.516	1551.280		125.688
42	13	1005.824	808.689	21.805	2371.941	726.516	1551.280		125.688
42	11	1005.824	808.689	21.805	3091.634	1964.227	805.906		125.688

NONE GIVEN

NRL STATISTICAL SUM OF STRESSES

NON-SYMMETRICAL SECTIONS

MEMBER	JOINT	Y SHEAR	Z SHEAR	TORSION	POINT 1 NORMAL	POINT 2 NORMAL	POINT 3 NORMAL	POINT 4 NORMAL	NRL SUM OF AXIAL STRESSES
43	11	151.695	191.100	21.805	3153.403	1930.838	683.500		1.715
43	9	151.695	191.100	21.805	1407.944	998.227	985.090		1.715
44	9	1962.292	3337.210	21.805	1609.376	816.607	1098.514		188.946
44	6	1962.292	3337.210	21.805	436.315	504.111	1276.471		188.946
45	17	341.802	2839.373	559.800	85984.129	14149.605	14205.039		479.069
45	18	341.802	2839.373	559.800	108[illegible].559	5353.281	3906.460		479.069
46	18	241.774	2539.979	222.754	11065.590	5083.230	3971.883		99.024
46	19	241.774	2539.979	222.754	15180.512	5549.570	6714.492		99.024
47	19	307.151	2214.687	275.048	15603.070	5038.187	8342.937		489.296
47	20	307.151	2214.687	275.048	32456.496	12841.656	12819.648		489.296
48	1	5327.391	948.874	99.313	43078.035	19018.277	36790.297		6388.500
48	5	5327.391	948.874	99.313	7886.387	7095.832	14227.078		6388.500
49	5	4206.684	773.984	185.435	43562.254	20490.977	27789.820		4375.508
49	21	4206.684	773.984	185.435	44870.016	8134.156	20560.910		4375.508
50	21	2480.309	237.764	45.971	21475.840	10483.168	12648.426		1998.709
50	33	2480.309	237.764	45.971	31590.543	8189.211	12370.715		1998.709
51	2	5237.914	454.268	151.643	36818.918	25881.898	21054.066		6391.992
51	8	5237.914	454.268	151.643	7100.184	9851.270	8727.207		6391.992
52	8	4115.477	321.141	193.498	39893.770	28217.863	21381.809		4723.512
52	24	4115.477	321.141	193.498	40344.855	10617.441	14188.469		4723.512
53	24	2416.974	196.489	30.871	19151.211	11384.680	8812.695		1947.162
53	30	2416.974	196.489	30.871	29323.117	10051.078	7428.590		1947.162

APPENDIX F
EXAMPLE APPLICATIONS OF THE ALLOWABLE STRESS CRITERIA

Allowable stress criteria used to evaluate the adequacy of structures under shock loading are detailed in Section 6. Examples for their applications are provided in Tables F.1 and F.2.

Table F.1 <u>Example Applications of the Allowable Stress Criteria</u>

Note: Sa = Allowable Design Stress
▒ Stress region

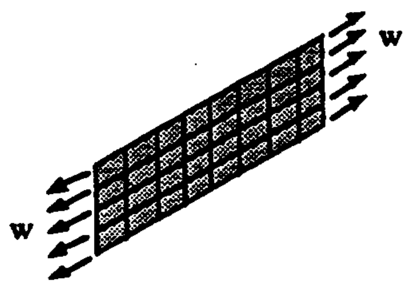
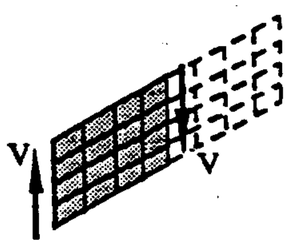
Description of Load Types and Stress Regions	Remarks
1 	General Membrane.
2 	General Membrane. For structural cross sections, stress in segments typically thought of as shear load carrying members (i.e. segments parallel to the direction of the shear load) must remain less than Sa.

Table F.1 Example Applications of the Allowable Stress Criteria (Cont'd)

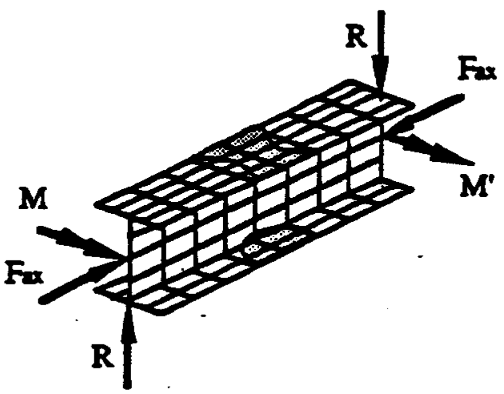
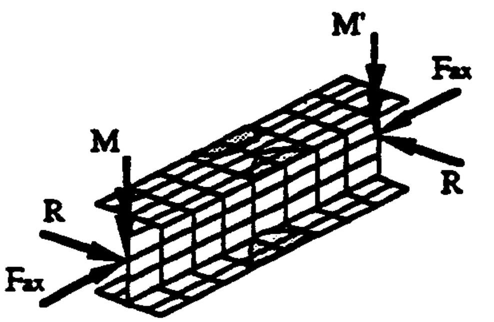
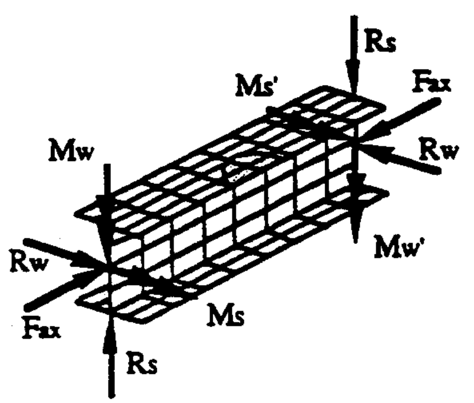
Description of Load Types and Stress Regions	Remarks
3 	General Membrane plus Bending. Bending stress is varying across the cross-section. Stresses evaluated at the outermost fibers must remain less than Sa. The web of the cross-section is the shear load carrying segment.
4 	General Membrane plus Bending. Bending stress is varying across the cross-section. Stresses evaluated at the outermost fibers must remain less than Sa. The flanges of the cross-section are the shear load carrying segments.
5 	General Membrane plus Bending. Bending stress is varying across the cross-section. Stresses evaluated at the outermost fibers must remain less than Sa. The web and flanges of the cross-section are the shear load carrying segments.

Table F.1 Example Applications of the Allowable Stress Criteria (Cont'd)

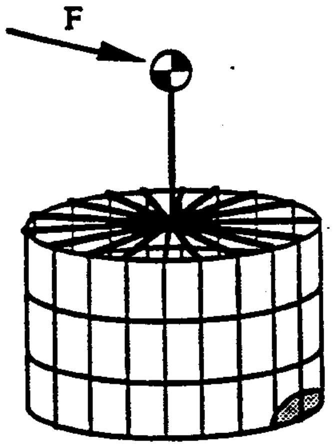
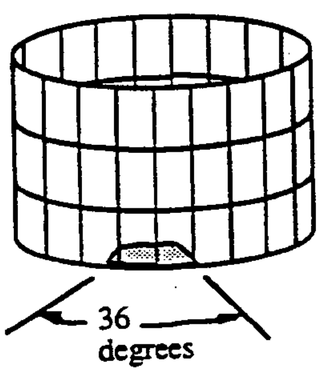
Description of Load Types and Stress Regions	Remarks
6 	General Membrane plus Bending. The bending stress distribution is through the entire cross-section. The stress in the section must remain less than Sa.
7 	Local Membrane plus Bending. The local bending stress distribution is varying through the plate thickness due to a structural discontinuity. The stress in any 36 degree section (10% of effective area) must remain less than Sa.

Table F.1 Example Applications of the Allowable Stress Criteria (Cont'd)

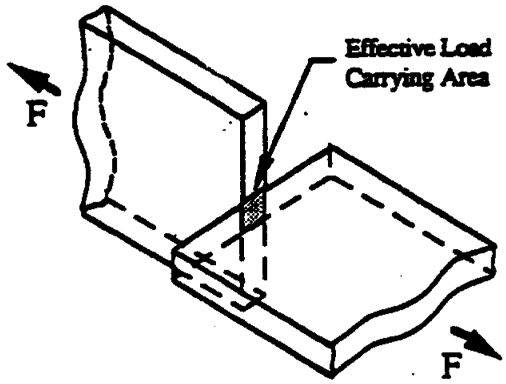
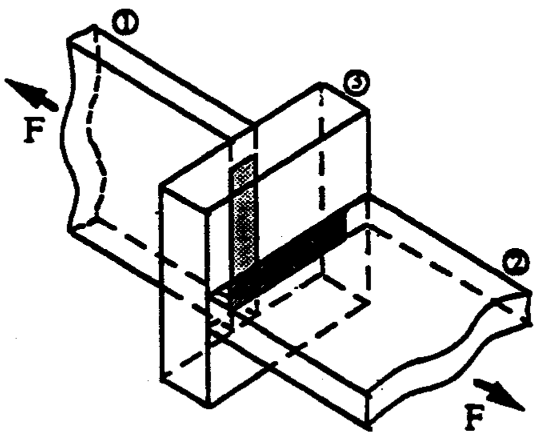
Description of Load Types and Stress Regions	Remarks
8 	**General Membrane.** **Stress levels in the effective load carrying area must remain less than Sa.**
9  Effective Load Carrying Area 1 $= t_1(t_2 + 2\,t_3)$ Effective Load Carrying Area 2 $= t_2(t_1 + 2\,t_3)$ Effective load carrying areas based on 45° load flare through thickness of plate 3.	**General Membrane.** **Stress levels in the effective load carrying area must remain less than Sa.**

Table F.1 Example Applications of the Allowable Stress Criteria (Cont'd)

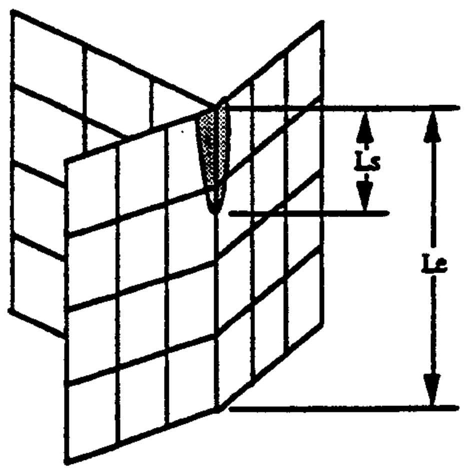
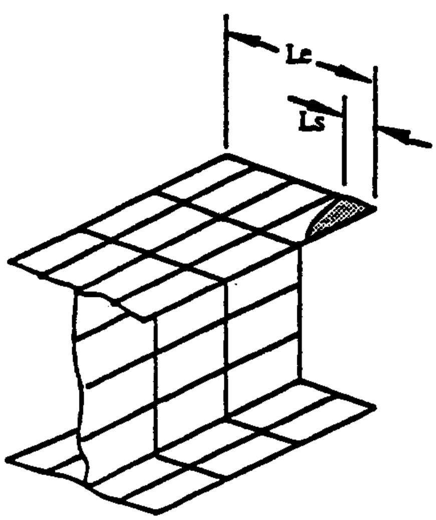
Description of Load Types and Stress Regions	Remarks
10  Le = Total Length of Intersection Ls = Length of Intersection Over Which Stresses Exceed General Allowables	Structural Discontinuity-Web Intersections Local Membrane plus Bending. No greater than 10% of the length of the line formed by the intersection of the plates may experience stress greater than the general allowables. $\frac{L_s}{L_e} < 0.10$ (10%)
11  Le = Total Length of Intersection Ls = Length of Intersection Over Which Stresses Exceed General Allowables	Local Membrane plus Bending. The local bending stress distribution is varying through the flange thickness due to a structural discontinuity. No greater than 10% of the length of the boundary of the flange may experience stress greater than the general allowables. $\frac{L_s}{L_e} < 0.10$ (10%)

Table F.1 Example Applications of the Allowable Stress Criteria (Cont'd)

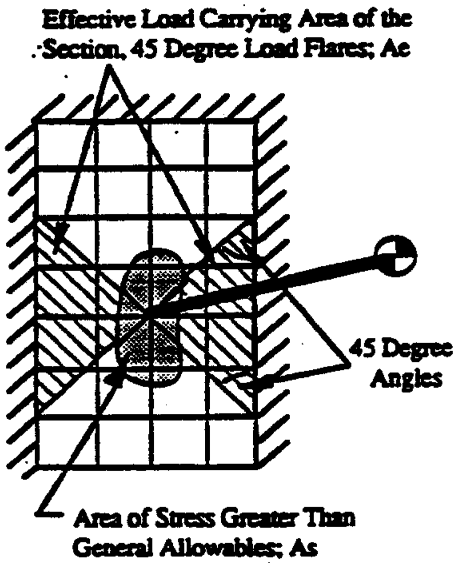
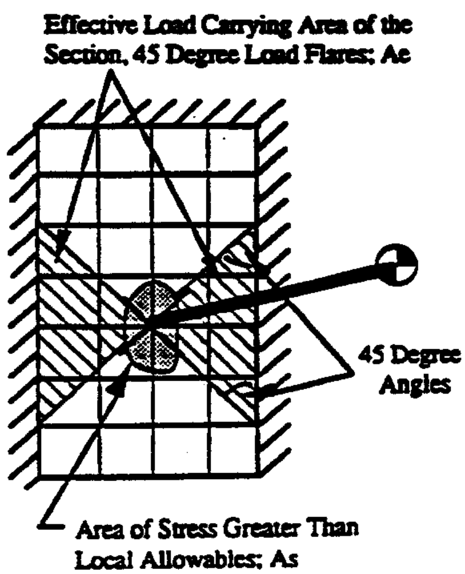
Description of Load Types and Stress Regions	Remarks
12  	Local Membrane plus Bending. The area of stress, A_s, in which general allowables are exceeded must not exceed 10% of the effective load carrying area of the section, A_e. $\frac{A_s}{A_e} < 0.10$ (10%) In order for a load path to be effective, at least one of the two angles created by the flare boundaries and the edge of the plate must be 45 degrees. Average shear stresses from tear-out and punch-through calculations are limited to general membrane allowables.
13  	Local Membrane plus Bending. In cases where local allowables are exceeded, the area of stress, A_s, greater than the local allowables must not exceed 5% of the effective load carrying area of the section, A_e. $\frac{A_s}{A_e} < 0.05$ (5%) In order for a load path to be effective, at least one of the two angles created by the flare boundaries and the edge of the plate must be 45 degrees. Average shear stresses from tear-out and punch-through calculations are limited to general membrane allowables.

Table F.1 Example Applications of the Allowable Stress Criteria (Cont'd)

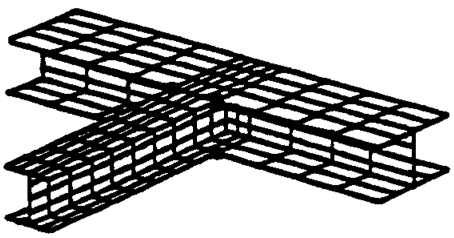
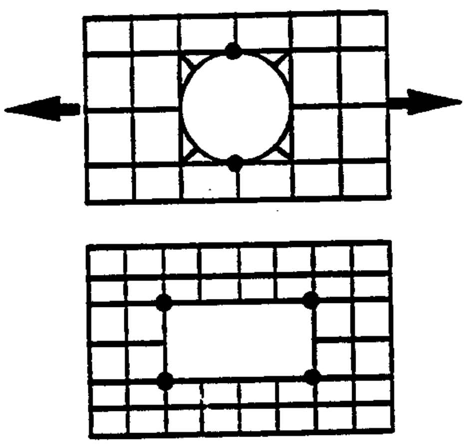
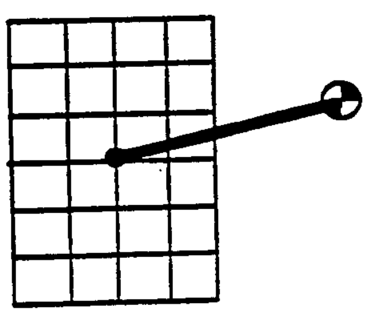
Description of Load Types and Stress Regions	Remarks
14  FLANGE INTERSECTION  CUTOUTS & PENETRATIONS  POINT OF LOAD INTRODUCTION	**Stress Concentrations** Concentrated stresses are ordinarily computed for determining fatigue adequacy of a structure. Because adequacy for fatigue is not a requirement for shock induced loads, nodal stresses occurring at points of stress concentration (i.e. corners, cutouts, points of load introduction) are unlimited. General stress requirements for the gross section must still be satisfied. In the case of local load introduction, shear, tear-out and punch-through requirements must still be satisfied.

Table .2 Special Applications of the Allowable Stress Criteria

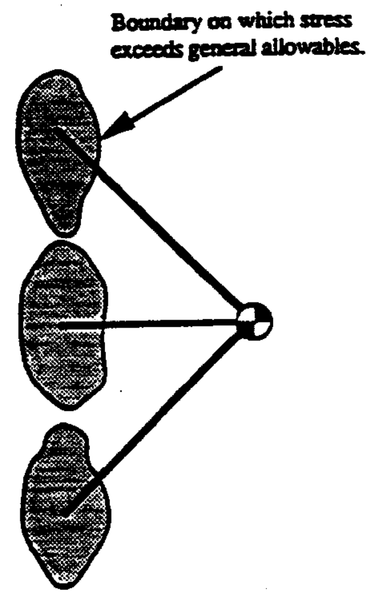
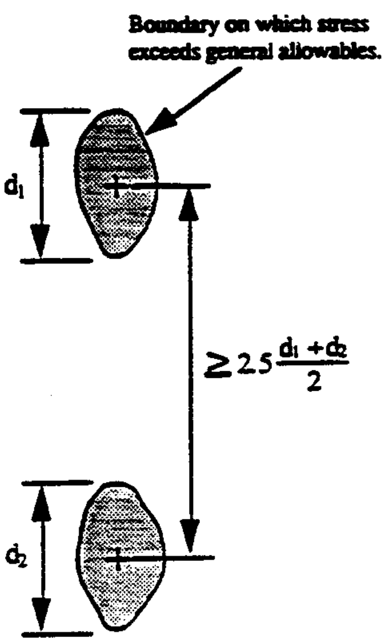
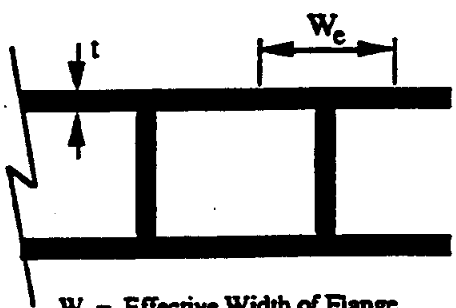
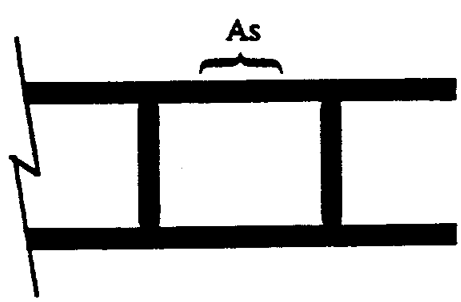
Description of Load Types and Stress Regions	Remarks
1 	**Adjacent Local Stress Regions** Membrane plus Bending. Discrete regions of local stress resulting from concentrated loads shall not overlap.
2 	**... Local Stress Regions** The centers of adjacent stressed regions classified as local cannot be closer than 2.5 times the average dimension of each locally stressed area unless the sum of the areas is less than 10% of the effective load carrying area. The length of each locally stressed area (d_1,d_2) shall be taken as the distance over which the stresses exceed the general stress limits along a line of action between the center of each pair of adjacent locally stressed areas. Not applicable to concentrated loads.

Table F.2 <u>Special Applications of the Allowable Stress Criteria</u> (Cont'd)

Description of Load Types and Stress Regions	Remarks
3  W_e = Effective Width of Flange σy = Allowable Yield Stress of Flange E = Modulus of Elasticity t = Plate Thickness	<u>Stiffened and Sandwiched Plate Structures</u> Calculate the effective flange width of the continuous plate from DDS 100-4. $W_e = 2t\sqrt{\frac{E}{\sigma_y}}$ Use as an aid in determining effective area for local stress evaluations.
4 	<u>Stiffened and Sandwiched Plate Structures</u> The region of stress, A_s, located between the effective flange widths is evaluated as a typical plate.

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