SVB 31 (2) MAR '63

IMPEDANCE CONSIDERATIONS IN VIBRATION TESTING*

L. J. Pulgrano
Grumman Aircraft Engineering Corporation
Bethpage, New York

The assumption of negligible specimen impedance inherent in the motion input approach to vibration testing is often unsatisfied. In this paper the significance of this assumption is illustrated by considering the relationship between the operational environment and a motion input test environment for a spacecraft-launch vehicle system in which the spacecraft impedance is appreciable.

INTRODUCTION

The most widely used procedure for testing the ability of a structure or equipment to withstand a vibration environment is to subject its base or attachment points to a prescribed oscillating motion input, at test levels derived from an envelope of measured vibration data. Inherent in this testing approach is an assumption that the impedance of the test specimen is always much smaller than that of its supporting structure; or equivalently, that the measured vibration levels on which the test specification is based will not be changed appreciably by the reactions of the test specimen when it is finally installed on its supporting structure.

This motion input approach to vibration testing has been criticized in the past[1,2] on the grounds that the assumption of negligible specimen impedance is often unsatisfied when dealing with large or even moderately large structures, and the resulting tests are generally overconservative by large factors. As yet, however, a widespread appreciation of this concept is not evident, and motion input test requirements continue to appear even for very large specimens. In this paper, the significance of the small impedance assumption is illustrated by considering the relationship between the operational environment and a motion input test environment for a spacecraft-launch vehicle system in which the spacecraft impedance is appreciable.

MECHANICAL IMPEDANCE

The mechanical impedance, z_{ij}, of a structure is conventionally defined as the complex ratio of the harmonic exciting force to the resulting harmonic velocity,[3] thus

$$z_{ij} = \frac{F_j}{\dot{q}_i}, \tag{1}$$

where the subscripts i, j indicate particular coordinates on the structure. If the velocity is measured at the point of excitation (i.e., $i = j$), the resulting impedance is known as a direct or driving point impedance; otherwise, the designation transfer impedance is used.

Mechanical impedance is a convenient measure of the resistance of a structure to vibration, the impedance being high for a structure that is inherently difficult to excite, and low for a structure that is readily excited. For typical lightly damped structures the impedance varies

---

*This paper was not presented at the Symposium.
[1]Blake, R. E., "The Need to Control the Output Impedances of Shock and Vibration Machines," Shock and Vibration Bulletin No. 23 (June 1956).
[2]Blake, R. E., and Belsheim, R. O., "The Significance of Impedance in Shock and Vibration," ASME Colloquium on Mechanical Impedance Methods, ASME (1958).
[3]Crandall, S. H., "Impedance and Mobility Analysis of Lumped Parameter Systems," ASME Colloquium on Mechanical Impedance Methods, ASME (1958).

sharply as a function of frequency over a range of about two orders of magnitude. Minimum or zero values of impedance correspond to resonant frequencies, whereas maximum or infinite values of direct impedance correspond to the antiresonant frequencies associated with the driving point under consideration.

Impedance methods are especially useful in analyzing the coupling of two systems having known characteristics. For systems coupled at a single coordinate it can readily be shown[3] that the direct impedance at the interface of the coupled system, z_{TOT}, is simply the sum of the direct impedances of the two subsystems, that is,

$$z_{TOT} = z_1 + z_2 . \qquad (2)$$

For undamped systems, the resonant frequencies occur when the impedance vanishes, therefore

$$z_{TOT} = z_1 + z_2 = 0 , \qquad (3)$$

or

$$z_1 = - z_2$$

becomes the frequency equation for the coupled system.

The relative impedance requirement that must be satisfied for a motion input specification to be a realistic test is evident from these simple expressions. Assuming that the response measured at the interface of the larger of the two systems (indicated by subscript 1) is essentially the same with or without the smaller system, requires that z_{TOT} be approximately equal to z_1, or equivalently, that z_1 be much greater than z_2. If z_{TOT} is considerably different from z_1, both the resonant frequencies and the interface vibration levels will be affected.

IMPEDANCE EFFECTS FOR A TYPICAL SPACECRAFT-LAUNCH VEHICLE SYSTEM

The system used for illustration in this section consists of the Orbiting Astronomical Observatory, a 3300-pound satellite, being developed by the Grumman Aircraft Engineering Corporation for the Goddard Space Flight Center of NASA, and its Atlas-Agena B launch vehicle. As a part of the spacecraft development program, the direct impedances of both the spacecraft and the launch vehicle were calculated in the axial direction at the interface. These calculations were based on lumped parameter representations and assumed structural damping factors of 0.04. Curves of the resulting impedances are presented in Fig. 1 for frequencies up to 500 cps. Although the upper portions of this frequency band are subject to considerable inaccuracy because of the limitations of the mathematical representation, the system is only being regarded as typical, and the general discussion will not be affected by these inaccuracies.

The curves of Fig. 1 reveal that the launch vehicle impedance is much higher than the spacecraft impedance over most of the frequency range; however, there are also several ranges in which the spacecraft impedance is as high as or higher than that of the launch vehicle. Since flight vibration is primarily a resonant phenomenon, measurements made on the launch vehicle without a spacecraft or with a small spacecraft would indicate maximum vibration at the launch vehicle resonances, or frequencies of minimum impedance, and much lower levels at other frequencies. Typical motion input vibration specifications, which are based on envelopes of peak vibration levels, are therefore representative of minimum values of launch vehicle impedance; significantly, it is at these minimum values that Fig. 1 shows the spacecraft impedance to be comparable to or greater than that of the launch vehicle. The fact that the launch vehicle impedance is much larger in other ranges is of little importance since no appreciable vibration occurs in those ranges.

During a motion input vibration test, the largest amplifications of the input motion, and thus the most severe conditions, occur at the antiresonant frequencies of the spacecraft, or frequencies of maximum spacecraft impedance. This is shown in Fig. 2, for the system under consideration, where the ratio of the response at the top of the spacecraft to the input motion is plotted as a function of frequency. A maximum amplification of 27 occurs at 53 cps, the first spacecraft antiresonant frequency.

By requiring that the input motion at the spacecraft antiresonant frequencies be representative of the launch vehicle resonant response, the motion input test anticipates a condition in which a launch vehicle resonance coincides with a spacecraft antiresonance. It follows, then, that in those frequency ranges for which the motion input test is most severe, the resulting spacecraft response is realistic only if the maximum spacecraft impedance is much less than the minimum values of impedance associated with neighboring launch vehicle resonances. If reference is again made to Fig. 1, it magnitude? Impedance is complex

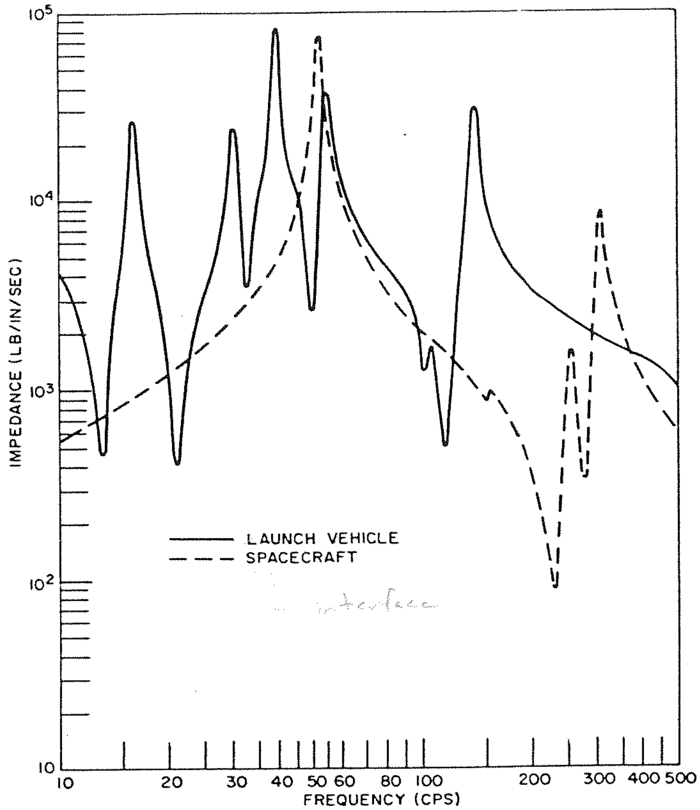

Fig. 1 - Direct impedance curves for typical spacecraft and launch vehicle

is seen that the impedance of the first spacecraft antiresonance (53 cps) is thirty times as large as the impedance of the nearest launch vehicle resonance (50 cps). Furthermore, the impedance at every one of the spacecraft antiresonances exceeds that of any neighboring launch vehicle resonance. It must therefore be concluded that for this configuration the spacecraft impedance cannot be considered small relative to that of the launch vehicle. The reaction forces generated by the vibrating spacecraft may be expected to have an appreciable effect on the vibration levels at the interface. The large margin by which the small impedance assumption is invalid in this case casts serious doubts as to its validity in many other cases, and provides an effective demonstration of the fact that it may be seriously erroneous to assume that a test specimen with relatively small mass also has relatively small impedance.

The conditions under which a motion input test is strictly valid have been discussed; next, the results of misapplying such a test by imposing it on systems having appreciable impedance must be considered. First, the changes in resonant frequencies due to the coupling of two systems will be discussed.

For systems linked at a single coordinate, a graphical solution of Eq. 3 provides a convenient method of obtaining the coupled system resonant frequencies. If the undamped impedance of the first system and the negative of the undamped impedance of the second system are plotted on the same graph, the intersections of the two impedance curves will occur at the resonant frequencies of the combined system. This plot is presented for our typical spacecraft-launch vehicle system in Fig. 3. It is seen that the addition of the spacecraft has resulted in nearly doubling the number of resonances in the frequency range below 500 cps, with the original resonances of the launch vehicle (which occur where the launch vehicle impedance curve crosses zero) being shifted in frequency to varying extents. Since flight vibration is primarily resonant, it will be at these new resonant frequencies, rather than the

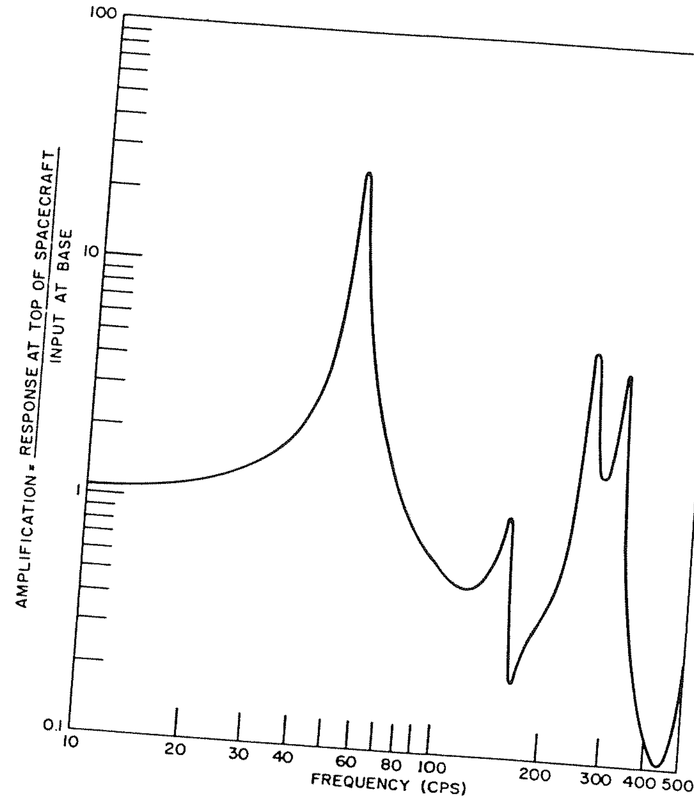

Fig. 2 - Response at top of spacecraft for a motion input at the base

original launch vehicle resonances, that the most severe vibration will occur.

Since the form shown by the direct impedance curves of Fig. 3 is characteristic of all undamped systems, some comments concerning their general behavior appear appropriate. In all mechanical systems the direct impedance alternates between resonances and antiresonances, with a change of sign occurring at each of these frequencies. When two mechanical systems are linked at a single coordinate, an antiresonance of the combined system occurs for the coupled coordinate at each antiresonance of either of the two subsystems, and again a resonance occurs between every two antiresonances. If one of the two systems has a generally small impedance compared to that of the other, the resonances of the combined system will fall near the resonances of the system with the larger impedance and near the antiresonances of the system with the smaller impedance. The validity of these comments can be deduced from the characteristic behavior of these impedance curves. For further information the reader is referred to the comprehensive discussions of Refs. 4 and 5.

The change in resonant frequencies due to the installation of a spacecraft having significant impedance has now been described; the next factor to be considered is the effect of the spacecraft reaction forces on the vibration levels at the interface. A detailed description of the actual interface flight environment for the typical launch vehicle being considered is not available for this discussion, and the actual excitation causing flight vibration is extremely complex and not known in sufficient detail to permit a calculation of these levels. The discussion will therefore be based on the vibratory response of the launch vehicle to a simple harmonic excitation. Since direct impedance curves are

[4]Bishop, R. E., and Johnson, D. C., The Mechanics of Vibration (Cambridge University Press, 1960).

[5]MacNeal, R. H., "Vibrations of Composite Systems," California Inst. of Technology Report No. 4 for ARDC (1954). (Astia AD No. 63118.)

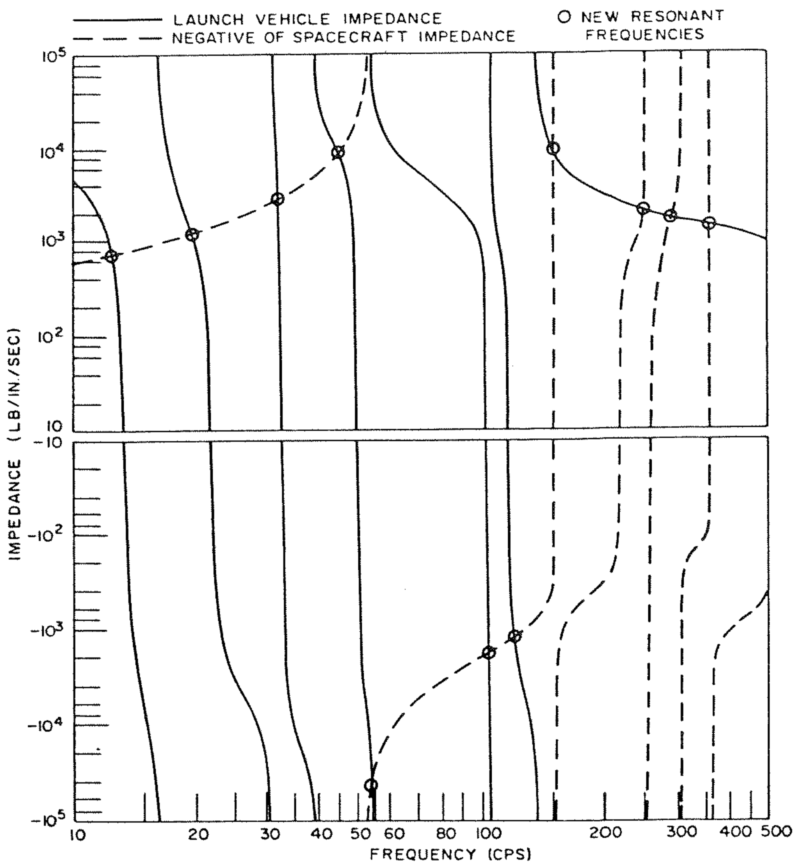

Fig. 3 - Undamped direct impedance curves for typical spacecraft and launch vehicle showing new resonant frequencies of combined system

available for the interface, this becomes the most convenient location to apply the excitation.

The frequency range of greatest interest for this discussion lies in the vicinity of the first spacecraft antiresonance, where an amplification of 27 occurs during a motion input test (see Fig. 2). This is the only spacecraft antiresonance in the lower frequency range at which significant amplification occurs.

The response curves for this discussion are most conveniently presented in terms of mobility, which is the inverse of mechanical impedance. Mobility, or the response to a unit exciting force, is plotted in curve 1 of Fig. 4 for the typical launch vehicle in the range around 53 cps, the first spacecraft antiresonant frequency. The 50-cps launch vehicle resonance is seen as a peak in the mobility curve.

If this response is now regarded as measured vibration data, and the conventional approach to establishing a conservative vibration test is followed, a straight line would be drawn as shown by curve 2 of Fig. 4, through or above the peak vibration levels observed. This would then become the test input motion for the spacecraft.

The response at the top of the spacecraft to this input motion is plotted in the third curve of Fig. 4, and it is seen that the spacecraft vibration environment becomes 27 times as severe as the original launch vehicle environment. It is not at all uncommon to proceed from this point by assuming that the spacecraft represents a large impedance to all subsystems mounted within it, and to provide an envelope of the levels measured during the vibration test as an input to these subsystems. In this manner, it is possible to "pyramid" the overall amplifications to extremely high and generally unrealistic values.

Next, it is necessary to determine the actual spacecraft response when the coupled

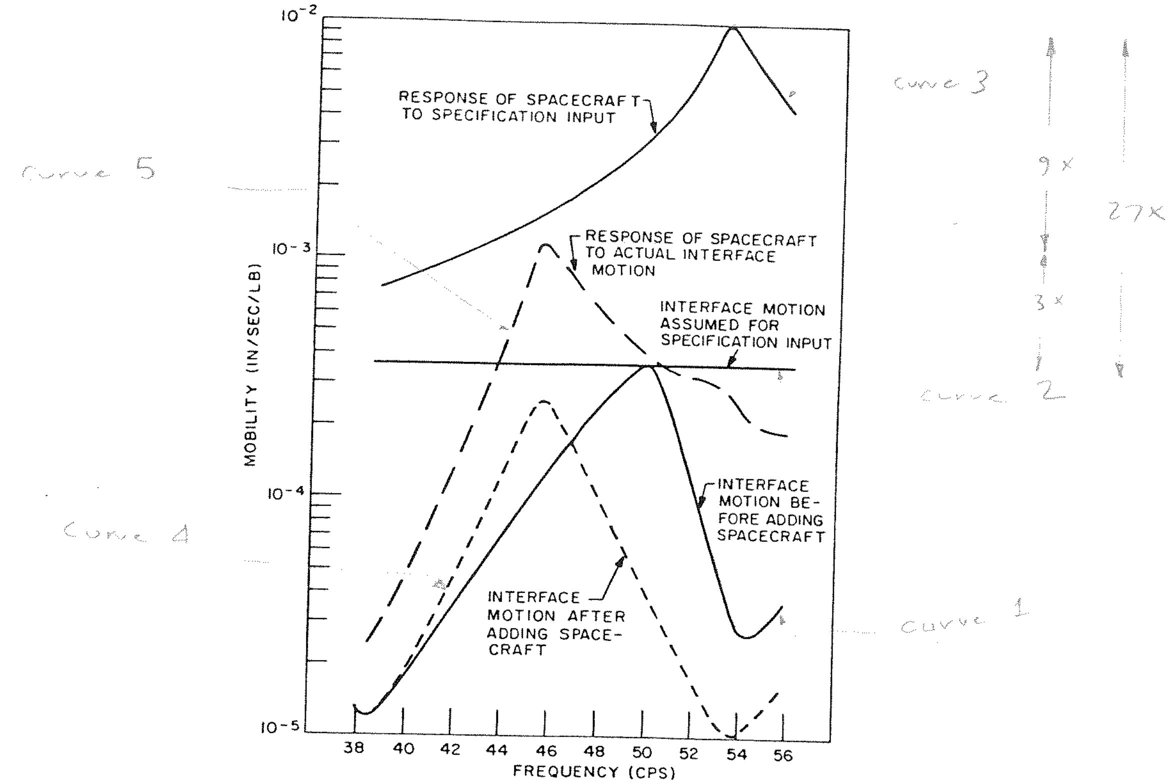

Fig. 4 - Spacecraft and launch vehicle response (mobility) curves

spacecraft-launch vehicle system is acted on by the same excitation that caused the interface motions shown by curve 1, Fig. 4. Prior to the attachment of the spacecraft, the launch vehicle impedance at the interface is

$$z_1 = \frac{F}{\dot{q}} . \tag{4}$$

After the spacecraft is attached, the interface impedance becomes the sum of the spacecraft and launch vehicle impedances (from Eq. (2)), that is,

$$z_{TOT} = z_1 + z_2 = \frac{F}{\dot{q}'} , \tag{5}$$

where $\dot{q}'$ is the new interface motion. Dividing Eq. (4) by Eq. (5) and rearranging, yields the new interface response in terms of the original interface response and the impedances of the spacecraft and launch vehicle,

$$\dot{q}' = \frac{z_1}{z_1 + z_2} \dot{q} . \tag{6}$$

If this calculation is carried out, with due regard for the phase angles of the impedances, the interface response shown in curve 4, Fig. 4 results. It is seen that at the 53 cps spacecraft antiresonance the interface vibration level has been reduced appreciably from an already low values, and that a new resonant peak has appeared at 45.5 cps.

The response at the top of the spacecraft corresponding to this new interface motion is shown in curve 5 of Fig. 4. The maximum spacecraft vibration levels are seen to be significantly lower on the actual coupled system than they are during the vibration test. For the coupled system the most severe spacecraft vibration levels are only 3 times the highest original launch vehicle levels, as compared to 27 times these levels during the vibration test. The motion input test in this case is therefore conservative by a factor of nine.

A widely held belief which has been demonstrated to be valid for a simple system[6] is that the spacecraft will experience the most severe vibration if a coincidence occurs between a spacecraft antiresonance and a launch vehicle resonance. In order to consider the result of this eventuality, the previous calculations were repeated with the spacecraft impedance curve shifted slightly to the left to bring about a coincidence between the 53-cps spacecraft antiresonance and the 50-cps launch vehicle resonance. The set of curves corresponding to this calculation are shown in Fig. 5. The interface vibration levels for the coupled system now show a pronounced dip at what was previously the frequency of maximum vibration, the new response being only 1/25 of the original. On both sides of this dip are the new resonant frequencies of the coupled system. The response at the top of the spacecraft is again much less on the coupled system than it is during the vibration test, and a comparison of the spacecraft vibration levels in Figs. 4 and 5 discloses that there is little difference between the two. Thus, even in the "worst case" the motion input test remains conservative by a factor of nine for this spacecraft-launch vehicle combination.

A MODIFIED TEST PROCEDURE

It has been shown in the previous sections that the impedance of a test specimen must be small compared to that of its supporting structure, if a motion input requirement is to be a valid test. If the impedance of a specimen is significant, the reaction forces generated by its vibration can have an appreciable effect on both the resonant frequencies and the vibration levels of its supporting structure. Under these conditions it is no longer realistic to consider an

[6]Blake, R. E., and Ringstrom, T., "The Influence of Mass and Damping on the Response of Equipment to Shock and Vibration," Shock and Vibration Bulletin No. 28, Part IV (August 1960).

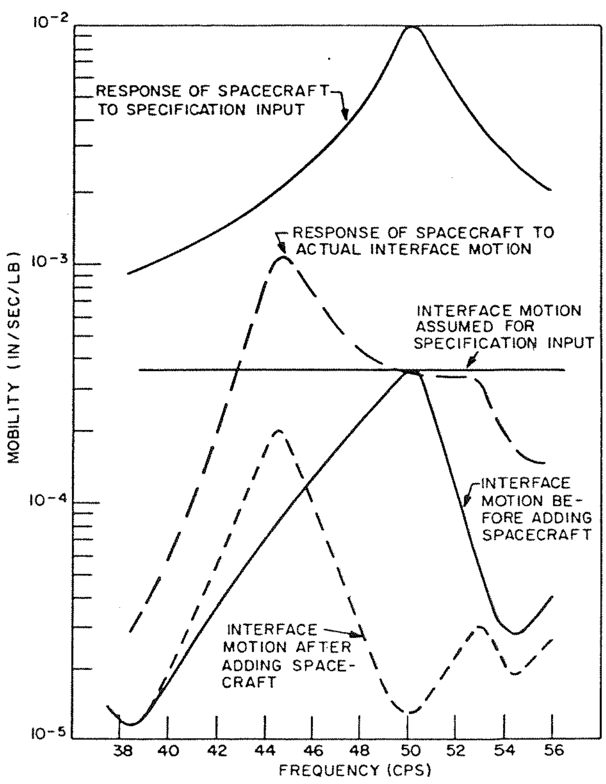

Fig. 5 - Spacecraft and launch vehicle response (mobility) curves for case of coincident spacecraft antiresonant and launch vehicle resonant frequencies

envelope of previously measured interface motions as "inputs" to the test specimen, since these motions are now part of the response of a new coupled system. Assuming broadband excitation, this response occurs primarily at the resonant frequencies of the coupled system, and not at either the resonant frequencies of the supporting structure alone or the antiresonant frequencies of the test specimen alone.

The use of a conventional motion input test results in the highest vibration levels occurring at the specimen antiresonant frequencies, whereas for specimens with significant impedance there is no reason to expect the highest levels to occur at these frequencies in the operational environment. Furthermore, a specimen with significant impedance loads down its supporting structure at its antiresonant frequencies and causes a reduction in the "input" motion. For the spacecraft-launch vehicle combination shown in Fig. 5 a reduction by a factor of over 25 was observed at the first spacecraft antiresonant frequency. No allowance is made for this effect in a conventional motion input test.

One modification of the motion input test that is commonly used permits a reduction in the input at critical antiresonant frequencies to avoid unrealistically high vibration levels or structural loads. An alternate and more straightforward approach is to abandon the motion input altogether, define instead an envelope of the anticipated response on the primary structure of the specimen in its operational environment, and use this envelope to specify the response on the primary structure of the specimen during its vibration test. It is important to point out that in using this method, the response cannot be defined for any single monitoring point, since at the antiresonant frequencies associated with the point chosen, large amplifications may occur on the test specimen in the same manner as they do at the driving point antiresonances during a motion input test. To avoid this condition, it is necessary to define the test envelope by the levels of several monitoring points distributed over the primary structure. The envelope may be defined as either the average vibration level of all the monitoring points, or the highest level shown by any single point. Reasonable arguments exist for using either of these approaches, however, it is not clear at present which is the most desirable. If the maximum level is used, care must be exercised to avoid having the test unduly influenced by very localized response. On the other hand, if the average level is used, the possibility exists of unrealistically high vibration levels over a small portion of the structure. In either case, the levels used should be representative of vibration on primary structure, and large amplifications occurring during the test on secondary structure should not be arbitrarily considered as grounds for reducing the overall test levels, since these amplifications may actually be representative of the operational environment.

For large impedance specimens the response envelope will probably not be appreciably above the envelope of measured data conventionally used as an input, and will not be subject to variations as large as those experienced by the interface in the frequency ranges near the specimen antiresonances. In the case of the spacecraft-launch vehicle combinations considered in Figs. 4 and 5 the maximum spacecraft response was approximately three times that observed at the 50-cps launch vehicle resonance before the spacecraft was attached. However, the response of the launch vehicle to the same excitation at some of its other resonant frequencies (e.g., 21 and 115 cps) would be even higher than the spacecraft response occurring in the coupled system in the 50-cps range, so that a broad flat envelope through the peaks in the launch vehicle response would also include the maximum spacecraft response for this critical frequency range. For this case, therefore, a vibration envelope at or slightly above the levels conventionally used as inputs would be suitable to define the spacecraft response.

Since only one case has been investigated to a limited extent in this paper, considerably more work is needed before general rules can be developed for establishing response envelopes for a large variety of specimens. Qualitatively, it is evident that as the specimen impedance becomes smaller the response envelope for testing must be moved further and further above the envelope of measured data until eventually it becomes more realistic to return to a motion input test. Until general rules can be developed it may be necessary to do simple impedance calculations of the type shown in this paper for each individual case before a conservative response envelope can be established with confidence.

The specification of response, rather than input, has several decided advantages over a conventional motion input test for specimens having significant impedance. First, it does not produce greatly increased response levels at the specimen antiresonant frequencies; rather it subjects the specimen to more nearly the same intensity of vibration throughout the frequency range, thus providing more equal assurance of acceptability for resonances occurring at any frequency. Second, by defining the response, the degree of conservatism in the test becomes more predictable, since variations in peak response are less pronounced than variations in the "input." Finally, generally higher vibration levels can be used throughout the test range, since the difficulties associated with large antiresonant amplifications are eliminated. Some increased difficulty in instrumentation and shaker control will inevitably occur in using this test procedure, but this will not be serious enough to outweigh the benefits obtained from more realistic vibration tests.

It is recognized that this discussion leaves many questions unanswered, some of which must remain unanswered until new test techniques are actually tried in the laboratory. It is hoped that others in the field will be encouraged by this paper to present their own views and suggestions, and that ultimately, more realistic vibration test and design procedures for structures with appreciable impedance will result from these discussions.

* * *