ROCKET MOTOR REFERENCE FOR LIFTOFF ACOUSTICS PROGRAM Revision D

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Introduction

The liftoff acoustics program requires both the peak liftoff thrust and the exit velocity. Selection of these values requires engineering judgment since each parameter varies with time, potentially in an independent manner.

The thrust values in Table 1 and the exit velocity values in Table 2 are for sea level unless otherwise noted. Each value is an approximation.

Thrust

Table 1. Peak Liftoff Thrust

Motor	Thrust (lbf)	Appendix
SR-19	50,000 (1)	A
Castor 4B	112,000 (2)	B
MLRS	38,000 (2)	C
Peacekeeper Stage 1	570,000	D

Notes:

1. This is a conservative estimate; the thrust value is rather transient at liftoff.
2. Rounded-up.

Exit Velocity

The three parameters used to describe the exhaust velocity are:

1. Effective Exhaust Velocity
2. Characteristic Exhaust Velocity
3. Exit Velocity

The three velocity metrics are discussed in Appendix E. The Exit Velocity is the correct parameter for the liftoff program.

Table 2. Exhaust Exit Velocity		
Motor	Exit Velocity (ft/sec)	Appendix
SR-19	9254	A
Castor 4B	8475	B
MLRS	7600	C
Peacekeeper Stage 1	9060	D

References

1. TM-9613, SR19 and SR19-M57, Performance and Static Controllability Report, 1992.
2. Chris Neilly, File: MRT_I_Castor_010_Ballistics_Rev_A.xls, 2006.
3. Chris Neilly, File: Castor Exit Velocity.xls, 2006.
4. TM-9549, Castor 4A & 4B Motor Manual, 1992.
5. ME File: 020-178, TCMP3 MLRS Motor Ignition Transient, 1999.
6. Greg Wilfert & Barry Kerrigan, Private Correspondence, 2005.
7. George Sutton, Rocket Propulsion Elements, Fifth Edition, Wiley, 1986.
8. Wilby & Wilby, Predicition of External Acoustic Filed on Taurus During Liftoff, AARC Report No. 121, 1991.

APPENDIX A

SR-19 Motor Thrust

The following data is taken from Reference 1.

UNCLASSIFIED

TM-9613

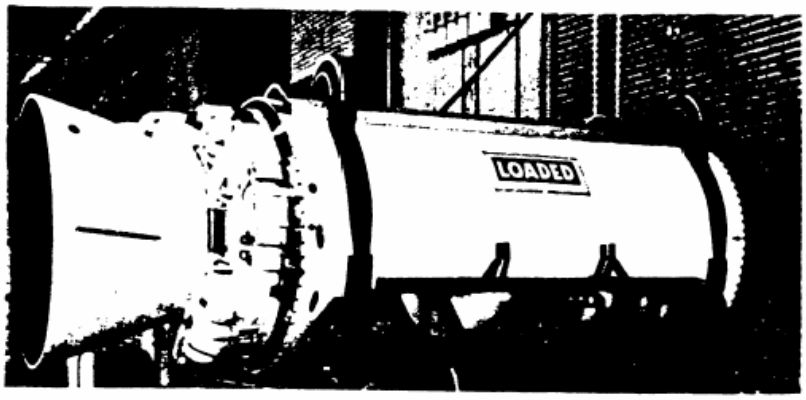
SPIA/M1
Page 1

October 1965

Unit 434
62-KS-63,000
XSR19-AJ-1
Minuteman Wing VI, 2nd stage

SOURCE:
Sponsoring agency ... USAF, BSD
Development and production agency ... Aerojet-General Corporation

APPLICATION: Propulsion unit for the second stage Minuteman Wing VI (Minuteman II, Weapon System 133B) ICBM.



I. PRINCIPAL DATA

Length (including nozzle)	162.32 in.
Diameter	
principal	52 in.
maximum	52.17 in.
Center of gravity from fwd face of igniter boss	
loaded	65.9 in.
expended	94.9 in.
Temperature limits	
operational	60 to 80 F
storage	60 to 100 F
Acceleration limits at 70 F	
axial	15 g
lateral	3 g
Maximum ignition altitude	unlimited
Nominal operating altitude	unlimited

Masses	
case assembly	432.1 lbm
nozzle assembly	378.3 lbm
insulation and liner	195.1 lbm
igniter and safearm assembly	30.2 lbm
external insulation	119.7 lbm
TVC system	575.5 lbm
other inert parts	8.6 lbm
Total inert mass	1739.5 lbm
propellant mass, m_p	13744.9 lbm
Total mass	15484.4 lbm
Propellant mass fraction	0.888
ICC Classification	
propellant	Class B
igniter	Class A
motor	Class A

II. PERFORMANCE
(70 F, vacuum)

*Pressure action time, $t_{a0.28\text{-}0p}$ (sec)	66.85
Maximum pressure, P_{max} (psia)	570
*Average pressure, $\overline{P}_{a0.28\text{-}0p}$ (psia)	445
Maximum thrust, F_{max} (lbf)	74,000
*Average thrust, $\overline{F}_{a0.28\text{-}0p}$ (lbf)	59,100
*Total impulse, $I = \overline{F}_{a0.28\text{-}0p} \times t_{a0.28\text{-}0p}$ (lbf-sec)	3,950,000
Propellant specific impulse $I_{spd} = I/m_p$ (lbf-sec/lbm)	287.4
Characteristic exhaust velocity (ft/sec)	5063

* Time interval is from 0 psia (fireswitch) to final 28 psia level; averages based on 15 samples.

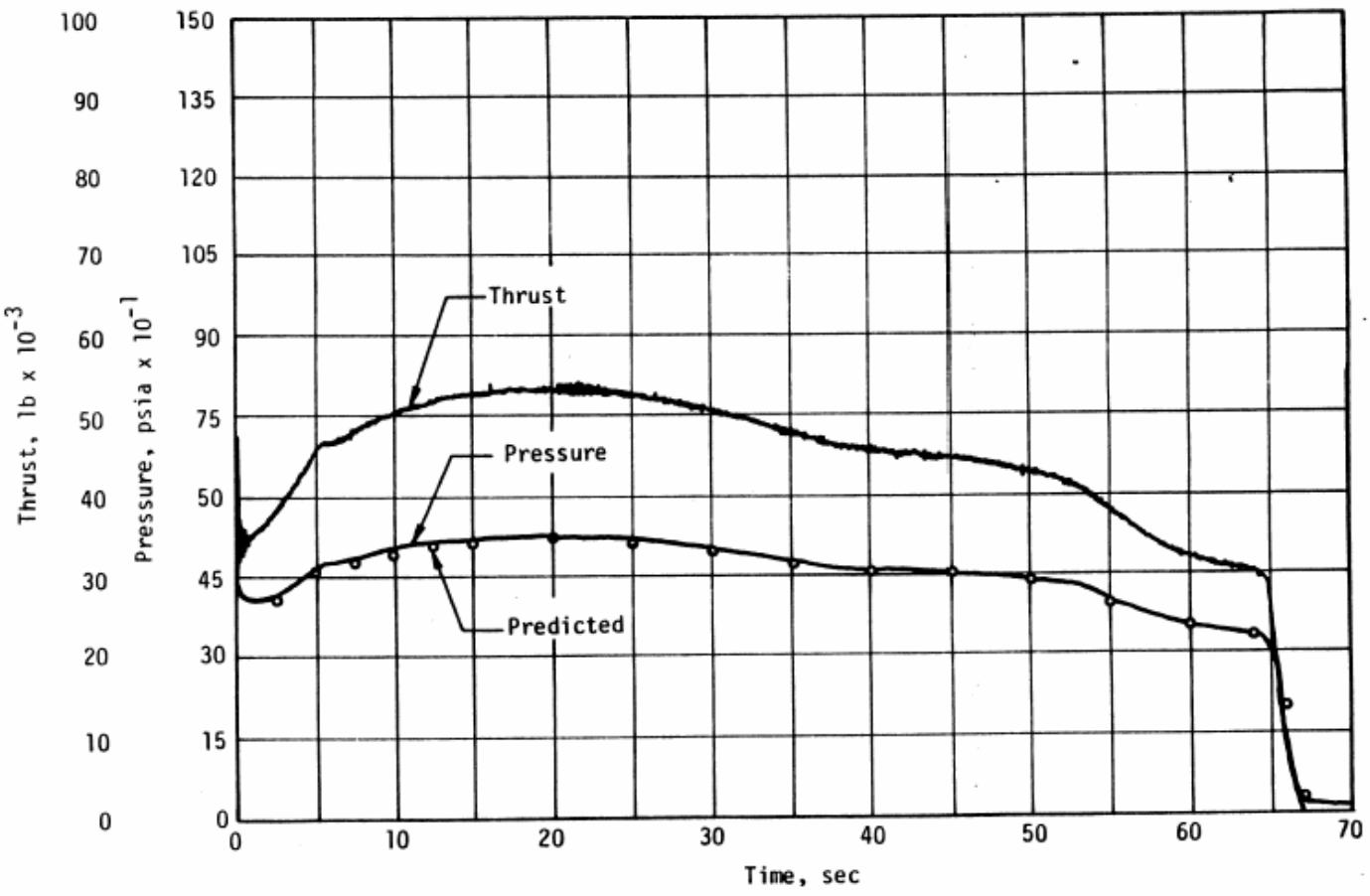


Figure 2-1. Performance Curves for SR19 with 10.02:1 Nozzle

A conservative value for the peak liftoff thrust is 50,000 lbf. Note that thrust is the far left scale.

SR-19 Exit Velocity

The specific impulse at vacuum is 287.4 lbf-sec/lbm.

The effective exhaust velocity c is

$$c = \left[287.4 \ \frac{\text{lbf - sec}}{\text{lbm}} \right] \left[\frac{32.2 \, \text{lbm}}{\text{lbf sec\^2/ft}} \right] = 9254 \ \text{ft/sec} \quad \text{(vacuum)}$$

The effective exhaust velocity is expected to be slightly lower at sea level relative to vacuum.

Assume that the sea level exit velocity is approximately equal to the vacuum effective exhaust velocity.

The value of 9254 ft/sec appears to be a reasonable velocity per the worksheet on the following page.

The peak exit velocity at vacuum is 13104 ft/sec, but this occurs at a low thrust value, so the corresponding mechanical power is low.

SR-19 Exit Velocity at Vacuum Worksheet from Chris Neilly, Received 1/25/06

10:1 Exp Ratio				W exp (oil dump)	4.0					
Prop Isp	266.944			W exp inerts	96.0	valve leakage	1.4	Throat Area	72.9	
Aexit, inches	729	0.4703		W prop	13662.0	oil SG	0.8	Gamma	1.18	
Prop Isp Rev	266.63838			t,action	64.6	duty cycle	35.0	Area Ratio	10.0	
				total prop/exp	13762.0	total in^3	132.8	Me	3.2	

Time (s)	Pressure (psia)	Pressure (pascals)	Fvac (lbf)	Prop/Exp Wgt	dWprop/dt	INT(dWprop)	Mass Flow	Pe	Ve (m/s)	Ve (ft/sec)
0	0		0	13762.0	0.0	0.0	0.0			
0.01	1	6894.8	108	13762.0	0.4	0.0	1.8	91.3	550.8	1807
0.02	45	310264.1	4862	13761.9	18.2	0.1	10.9	4109.6	3995.0	13104
0.03	18	124105.6	5186	13761.7	19.4	0.3	20.4	1643.8	2410.6	7907
0.04	56	386106.4	6051	13761.5	22.7	0.5	22.6	5114.2	2388.9	7836
0.05	90	620528.2	9725	13761.2	36.5	0.8	31.1	8219.2	2789.7	9150
0.06	183	1261740.6	19881	13760.6	74.6	1.3	57.1	16712.4	3112.8	10210
0.07	293	2020163.9	31766	13759.6	119.1	2.3	98.4	26758.2	2884.0	9459
0.08	364	2509691.7	39439	13758.3	147.9	3.6	135.1	33242.2	2608.2	8555
0.09	413	2847534.8	44732	13756.7	167.8	5.2	159.4	37717.2	2506.9	8223
0.1	441	3040588.0	47758	13754.9	179.1	7.0	175.0	40274.3	2437.8	7996
0.15	464	3199167.4	49536	13745.7	185.8	16.1	184.0	42374.7	2401.4	7877
0.6	438	3019903.7	48199	13662.5	180.8	98.6	184.8	40000.3	2333.0	7652
1.01	430	2964745.6	48674	13587.4	182.5	173.0	183.2	39269.7	2383.2	7817
3.03	445	3068167.0	53010	13199.1	198.8	558.2	192.2	40639.6	2485.1	8151

APPENDIX B

Castor 4B Thrust

The peak liftoff thrust of the Castor 4B is 125,711.7 lbf in a vacuum per Reference 2.

The nozzle exit diameter is 35.5 in.

The nozzle exit area is 990 in^2.

Assume sea level atmosphere at 14.7 psi.

The atmospheric force is the area times the ambient air pressure. The resulting force is 14,566 lbf.

The peak thrust value corrected for sea level is 111,145 lbf.

Castor 4B Exit Velocity

The exit velocity at liftoff is 8475 ft/sec for sea level altitude per Reference 3.

As a check, the specific impulse at vacuum is 266 lbf-sec/lbm per Reference 4.

The effective exhaust velocity is

$$\left[266\ \frac{\text{lbf - sec}}{\text{lbm}}\right]\left[\frac{32.2\,\text{lbm}}{\text{lbf sec\^2/ft}}\right] = 8565\ \text{ft/sec} \quad \text{(vacuum)}$$

The effective exhaust velocity is expected to be slightly lower at sea level relative to vacuum.

The sea level exit velocity is thus approximately equal to the vacuum effective exhaust velocity.

Castor 4B Exit Velocity at Sea Level Worksheet from Chris Neilly, Received 1/20/06

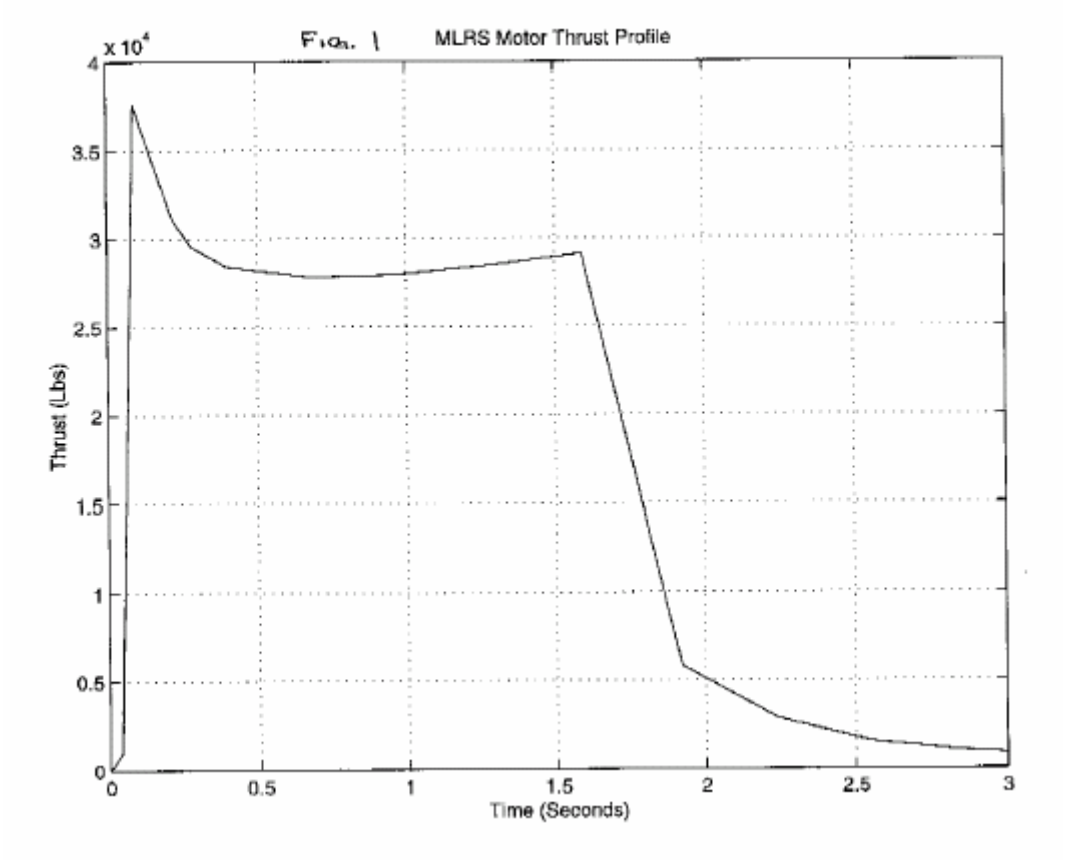
Castor Estimated Exit Velocity

Time (sec)	Pressure (psia)	Thrust (lbf)(vac)	Mdot (lbm/sec)	Prop Mass (lbm)	V Exit Vacuum (ft/sec)	Vexit Sea Level (ft/sec)
0.00	14.34	1775.10	9.36	0.14	6104.69	5747.37
0.02	14.98	14744.70	77.77	1.31	6104.92	6055.65
0.04	56.59	14936.30	70.84	3.03	6789.71	6600.39
0.06	174.52	21683.40	85.81	4.23	8136.74	7662.20
0.08	446.16	79789.50	293.83	9.34	8743.85	8387.39
0.10	611.76	114137.10	422.03	15.25	8708.50	8367.92
0.11	721.38	101387.70	365.35	20.00	8935.89	8474.35
0.12	767.91	112860.30	408.27	25.30	8901.27	8461.29
0.14	777.86	126020.80	459.85	31.74	8824.30	8427.95
0.16	750.08	123280.90	450.44	43.09	8812.74	8422.46
0.18	730.10	124488.60	456.40	49.48	8783.00	8407.81
0.20	699.74	123972.60	456.14	60.88	8751.54	8391.48
0.22	690.75	121317.00	446.00	67.12	8758.72	8395.28
0.24	674.75	125654.10	464.53	78.58	8710.07	8368.80
0.26	670.88	120070.30	442.21	84.77	8743.11	8386.96
0.28	667.50	126241.90	467.40	96.27	8697.06	8361.42
0.30	665.31	118237.20	435.17	102.36	8748.91	8390.05
0.32	652.18	122429.50	452.96	113.62	8703.34	8365.00
0.34	645.43	113173.70	416.00	119.45	8760.08	8396.00
0.36	639.44	119039.60	440.06	130.42	8710.35	8368.96
0.38	638.04	111018.40	407.78	136.13	8766.41	8399.30
0.40	633.89	117774.90	435.30	147.11	8712.00	8369.89
0.42	630.23	109235.60	401.09	152.72	8769.57	8400.93
0.44	622.14	114570.60	423.10	163.52	8719.49	8374.10
0.46	618.73	107239.10	393.76	169.03	8769.60	8400.95
0.50	611.67	106714.60	392.07	185.19	8764.21	8398.15
0.60	604.37	106565.30	391.91	225.17	8755.52	8393.61
0.80	593.80	107243.50	395.30	306.21	8735.70	8383.00
1.01	585.25	106216.40	391.70	386.11	8731.60	8380.75
1.21	578.50	105422.80	388.93	464.29	8728.17	8378.87
1.41	573.16	104835.60	386.90	542.05	8725.10	8377.17
1.61	568.61	104387.30	385.38	619.90	8722.03	8375.48
1.81	564.74	104010.70	384.11	697.10	8719.35	8373.99
2.01	561.39	103655.50	382.89	774.06	8717.23	8372.82

APPENDIX C

MLRS Motor

The thrust-time curve is taken from Reference 5.



The exhaust exit velocity is taken from Reference 6.

APPENDIX D

Peacekeeper Stage 1

This motor is used as Taurus Stage 0.

The following is taken from Reference 8.

Thrust:	570,000 lb (2,534,000 N)
Weight Flow:	2026 lb/sec (918 kg/s)
Exhaust Velocity:	9060 ft/sec (2760 m/s)
Nozzle Exit Diameter:	60.9 inches (1.547 m)

APPENDIX E

Velocity Metrics

Again, there are three parameters used to describe the exhaust velocity.

4. Effective Exhaust Velocity
5. Characteristic Exhaust Velocity
6. Exit Velocity

The Exit Velocity is the correct parameter for the liftoff program.

Effective Exhaust Velocity

The effective exhaust velocity c is the average equivalent velocity at which propellant is ejected from the vehicle. It is defined as

$$c = I_S g_o = \frac{F}{\dot{m}} \qquad \text{(E-1)}$$

where

I_S = Specific impulse

g_o = Gravity

F = Thrust

$\dot{m}$ = Mass flow rate

Characteristic Exhaust Velocity

The characteristic exhaust velocity c^* is defined as

$$c^* = \frac{I_S g_o}{C_F} = \frac{c}{C_F} \qquad \text{(E-2)}$$

where C_F is the thrust coefficient.

The thrust coefficient is

$$C_F = \frac{F}{A_t\, p_1} \qquad \text{(E-3)}$$

where

A_t = Throat area

p_1 = Chamber pressure

Typically, $1.0 < C_F < 2.3$ per Reference 7, Figure 3-7.

Exit Velocity

The exit velocity V_e is calculated from the thrust F.

$$F = \dot{m} V_e + (p_e - p_o)\, A_e \qquad \text{(E-4)}$$

where

A_e = Nozzle exit plane area

p_e = Pressure at the exit plane

p_o = Ambient air pressure

$$\dot{m} V_e = \left[F - (p_e - p_o)\, A_e \right] \qquad \text{(E-5)}$$

$$V_e = \left[\frac{1}{\dot{m}}\right]\left[F - (p_e - p_o)\, A_e \right] \qquad \text{(E-6)}$$

$$V_e = \left[\frac{1}{\dot{m}}\right]\left[F + (-p_e + p_o)\, A_e \right] \qquad \text{(E-7)}$$

Recall

$$c = \frac{F}{\dot{m}} \tag{E-8}$$

The exit velocity V_e is related to the effective exhaust velocity c by

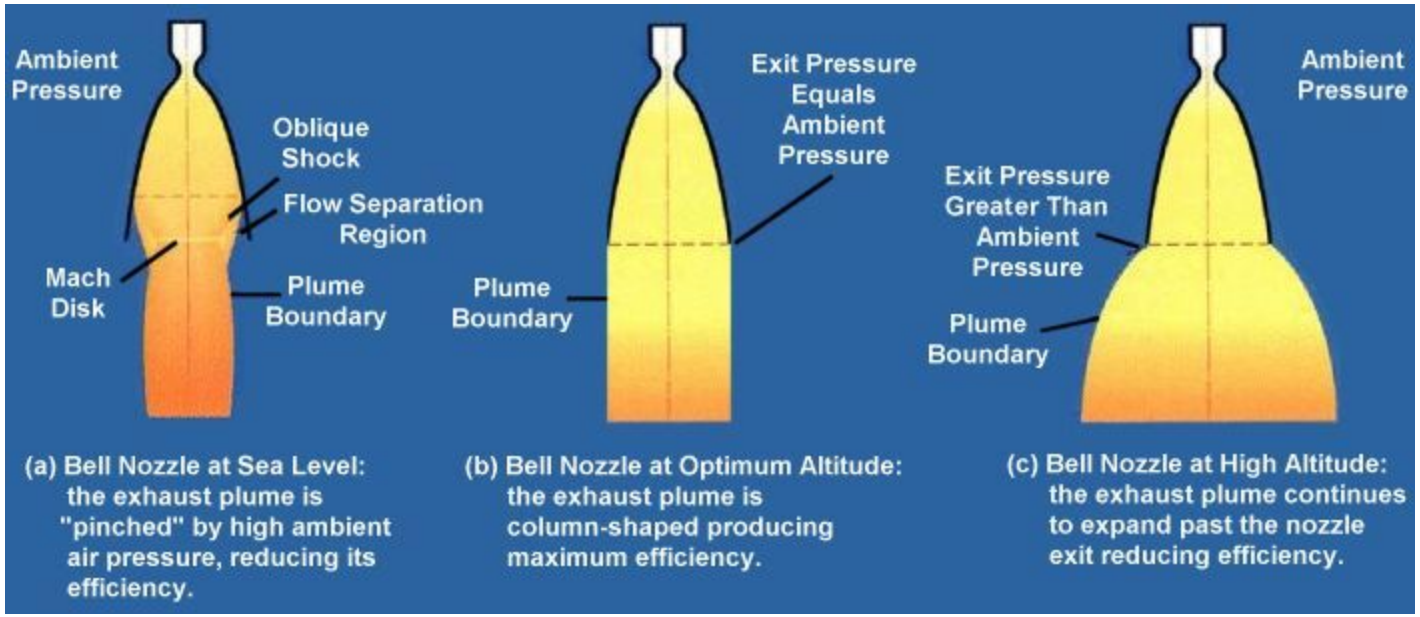
$$V_e = c + \left[\frac{1}{\dot{m}}\right]\left[(-p_e + p_o)\, A_e\right] \tag{E-9}$$

The exit velocity V_e is related to the characteristic exhaust velocity c^* by

$$V_e = c^* C_F + \left[\frac{1}{\dot{m}}\right]\left[(-p_e + p_o)\, A_e\right] \tag{E-10}$$

Table E-1. Nozzle Conditions

Nozzle Condition	Pressure	Velocity	Columns
Over-expanded	$p_e < p_o$	$V_e > c$	Oblique shock waves form. A rocket nozzle is typically over-expanded at liftoff.
Ideally Expanded	$p_e = p_o$	$V_e = c$	Optimum expansion.
Under-expanded	$p_e > p_o$	$V_e < c$	The expansion waves equalize the pressure.



Difference in flow behavior between (a) overexpansion, (b) ideal expansion, and (c) underexpansion

http://www.aerospaceweb.org/question/propulsion/q0220.shtml