TRANSMISSION LOSS OF A PLANE WAVE WITH NORMAL INCIDENCE THROUGH A MULTI-LAYER PARTITION

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Consider plane wave propagation at normal incidence through the system in Figure 1.

The particle displacements are

\[ u_1 = A_i \exp(j(\omega t - k_1x)) + A_r \exp(j(\omega t + k_1x)) \] (1)
\[ u_2 = B_i \exp(j(\omega t - k_2x)) + B_r \exp(j(\omega t + k_2x)) \] (2)
\[ u_3 = C_i \exp(j(\omega t - k_3(x - L_1))) + C_r \exp(j(\omega t + k_3(x - L_1))) \] (3)
\[ u_4 = D_i \exp(j(\omega t - k_4(x - L_{12}))) + D_r \exp(j(\omega t + k_4(x - L_{12}))) \] (4)
\[ u_5 = A_i \exp(j(\omega t - k_5(x - L_{13}))) \] (5)

The corresponding velocities are

\[ \frac{\partial}{\partial t} u_1 = j\omega A_i \exp(j(\omega t - k_1x)) + j\omega A_r \exp(j(\omega t + k_1x)) \] (6)
\[ \frac{\partial}{\partial t} u_2 = j\omega B_i \exp(j(\omega t - k_2x)) + j\omega B_r \exp(j(\omega t + k_2x)) \] (7)
\[ \frac{\partial}{\partial t} u_3 = j\omega C_i \exp(j(\omega t - k_3(x - L_1))) + j\omega C_r \exp(j(\omega t + k_3(x - L_1))) \] (8)
\[ \frac{\partial}{\partial t} u_4 = j\omega D_i \exp(j(\omega t - k_4(x - L_{12}))) + j\omega D_r \exp(j(\omega t + k_4(x - L_{12}))) \] (9)
\[ \frac{\partial}{\partial t} u_5 = j\omega A_i \exp(j(\omega t - k_5(x - L_{13}))) \] (10)
Figure 1.

Note that $\omega$ is the frequency in (rad/sec) and that $k$ is the wavenumber.
The derivates with respect to distance are

\[
\begin{align*}
\frac{\partial}{\partial x}u_1 &= -k_1A_t \exp j(\omega t - k_1x) + k_1A_r \exp j(\omega t + k_1x) \\
\frac{\partial}{\partial x}u_2 &= -k_2B_t \exp j(\omega t - k_2x) + k_2B_r \exp j(\omega t + k_2x) \\
\frac{\partial}{\partial x}u_3 &= -k_3C_t \exp j(\omega t - k_3(x - L_1)) + k_3C_r \exp j(\omega t + k_3(x - L_1)) \\
\frac{\partial}{\partial x}u_4 &= -k_4D_t \exp j(\omega t - k_4(x - L_{12})) + k_4D_r \exp j(\omega t + k_4(x - L_{12})) \\
\frac{\partial}{\partial x}u_5 &= -k_5A_t \exp j(\omega t - k_5(x - L_{13}))
\end{align*}
\] (11-15)

The particle displacements are equal at the \( x = 0 \) interface.

\[A_t + A_r = B_t + B_r\] (16)

The particle displacements are equal at the \( x = L_1 \) interface.

\[B_t \exp j(-k_2L_1) + B_r \exp j(k_2L_1) = C_t + C_r\] (17)

The particle displacements are equal at the \( x = L_{12} \) interface.

\[C_t \exp j(-k_3L_2) + C_r \exp j(k_3L_2) = D_t + D_r\] (18)

The particle displacements are equal at the \( x = L_{13} \) interface.

\[D_t \exp j(-k_4L_3) + D_r \exp j(k_4L_3) = A_t\] (19)
The pressures are equal at the $x = 0$ interface.

\[ B_1 \frac{\partial}{\partial x} u_1 = B_2 \frac{\partial}{\partial x} u_2 \]  

(20)

\[-B_1 k_A i + B_1 k_A r = -k_2 B_2 B_t + k_2 B_2 B_r \]  

(21)

\[-B_1 k_A i + B_1 k_A r + k_2 B_2 B_t - k_2 B_2 B_r = 0 \]  

(22)

The pressures are equal at the $x = L_1$ interface.

\[ B_2 \frac{\partial}{\partial x} u_2 = B_3 \frac{\partial}{\partial x} u_3 \]  

(23)

\[-k_2 B_2 B_t \exp(j(-k_2 L_1)) + k_2 B_2 B_r \exp(j(k_2 L_1)) = -B_3 k_3 C_t + B_3 k_3 C_r \]  

(24)

\[-k_2 B_2 B_t \exp(j(-k_2 L_1)) + k_2 B_2 B_r \exp(j(k_2 L_1)) + B_3 k_3 C_t - B_3 k_3 C_r = 0 \]  

(25)

The pressures are equal at the $x = L_{12}$ interface.

\[ B_3 \frac{\partial}{\partial x} u_3 = B_4 \frac{\partial}{\partial x} u_4 \]  

(26)

\[-B_3 k_3 C_t \exp(j(-k_3 L_2)) + B_3 k_3 C_r \exp(j(k_3 L_2)) = -B_4 k_4 D_t + B_4 k_4 D_r \]  

(27)

\[-B_3 k_3 C_t \exp(j(-k_3 L_2)) + B_3 k_3 C_r \exp(j(k_3 L_2)) + B_4 k_4 D_t - B_4 k_4 D_r = 0 \]  

(28)

The pressures are equal at the $x = L_{13}$ interface.

\[ B_4 \frac{\partial}{\partial x} u_4 = B_5 \frac{\partial}{\partial x} u_5 \]  

(29)

\[-k_4 D_t \exp(j(-k_4 L_3)) + k_4 D_r \exp(j(k_4 L_3)) = -k_5 A_t \]  

(30)

\[-k_4 D_t \exp(j(-k_4 L_3)) + k_4 D_r \exp(j(k_4 L_3)) + k_5 A_t = 0 \]  

(31)
Assemble the equations from the boundary conditions in matrix form.

\[
\begin{bmatrix}
A_i \\
A_r \\
B_t \\
B_r \\
C_t \\
C_r \\
D_r \\
D_t \\
A_t
\end{bmatrix}
= \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

(32)

The \( M \) coefficient matrix is shown on the next page.

Note that \( A_i \) is given the value of 1.

The intermediate goal is to obtain the ratio \( \frac{A_t}{A_i} \).
\[
[M] = \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \exp j(-\hat{k}_2) & \exp j(\hat{k}_2) & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \exp j(-\hat{k}_3) & \exp j(\hat{k}_3) & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \exp j(-\hat{k}_4) & \exp j(\hat{k}_4) & -1 & \end{bmatrix}
\]

where

\(\hat{k}_2 = k_2L_1, \quad \hat{k}_3 = k_3L_2, \quad \hat{k}_4 = k_4L_3\)
A single, closed-form equation could be derived from equation (32). This derivation is pending. In the mean time, equation (32) may be solved for each frequency of interest via Matlab script.

The sound energy transmission coefficient $\alpha_t$ is

$$\alpha_t = \frac{R_1}{R_5} \left[ \frac{A_t}{A_i} \right]^2 \quad (34)$$

The transmission loss TL as a function of frequency $f$ is

$$TL(f) = 10 \log[\alpha_t] \quad \text{dB} \quad (35)$$

Note that this approach does not account for the transmission loss in the vicinity of the critical frequency.

References

2. T. Irvine, Semi-Definite System Examples, Vibrationdata, 20
APPENDIX A

Sample Problem

The partition consists of two aluminum panel separated by an air gap. Each panel is 2 mm thick. The air gap is 10 mm thick. Each panel may be considered to have infinite height and width with no mechanical connection to each other.

The transmission loss was calculated using the equations in the main text as implemented in Matlab script: transmission_five_media.m

Note that the dilatational, or longitudinal, natural frequency is 365 Hz. This is confirmed via the hand calculation in Appendix B.

A practical system would have mechanical connections between the two panels, thereby suppressing the effect of the dilatational frequency.

Figure A-1.
The “Two Panels” curve is the same as the curve in Figure A-1.

The Single Panel curve represents a 2 mm thick aluminum panel.

The Two Panel configuration offers superior attenuation except at the dilatational frequency.
The first natural frequency is zero, which corresponds to rigid-body motion.

The second natural frequency $f_2$ is given by

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{k}{m_1 m_2}} \quad (B-1)$$

For $m = m_1 = m_2$,

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{2k}{m}} \quad (B-2)$$

The spring element is the air gap for the sample problem.
Equation (B-2) can thus be rewritten as

\[
f_2 = \frac{1}{2\pi} \sqrt{\frac{2(B/L_s)}{\rho L_m}}
\]  

(B-3)

Note that the area terms have been omitted from Equation (B-3) because they cancel out one another, leaving a ratio of unity.

The parameters for the example in Appendix A are given in Table B-1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Bulk Modulus of Air</td>
<td>1.42e+05 Pa</td>
</tr>
<tr>
<td>L_s</td>
<td>Air Gap Thickness</td>
<td>10 mm</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Aluminum Panel Density</td>
<td>2700 kg/m³</td>
</tr>
<tr>
<td>L_m</td>
<td>Panel Thickness</td>
<td>2 mm, each</td>
</tr>
</tbody>
</table>

The natural frequency is thus

\[
f_2 = \frac{1}{2\pi} \sqrt{\frac{2 \text{ (1.42e+05 Pa / 0.010 m)}\text{ (2700 kg/m}^3\text{ (0.002 m)})}{\rho L_m}}
\]

(B-4)

\[
f_2 = 365 \text{ Hz}
\]

(B-5)