VIBRATION, SHOCK, AND ACOUSTICS

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Section 1
INTRODUCTION

1-1 PURPOSE

A comprehensive manual of standard engineering tools has been assembled herein for the vibration, shock, and acoustics technologies. The purpose of the manual is twofold:

a. Design solutions, test results, and analytical techniques often apply to a variety of space vehicle and missile system designs. This manual eliminates repetition of the same basic derivations for each new system, and standardizes their format and use throughout Development Engineering.

b. Increased engineering work loads necessitate a significant increase in the number of dynamicists and acousticians within the several divisions. Based on past experience, the large percentage of new hires in this field are recent college graduates. Because a formal college education does not usually provide direct applicable experience in the aerospace discipline, job experience and training are required. Because of the physical separation of divisions, personal contact with specialists is not always feasible. This manual minimizes the amount of training time required and also the necessity for extensive direction from experienced personnel.

1-2 SCOPE

The manual is a compilation of current techniques for analysis, a summary of commonly used reference data, and includes indices of local reference material. Specifically, it:

a. Includes a summary of commonly used reference data pertinent to vibration, shock and acoustics;

b. Contains the best and most current descriptions (either experimental or analytical) of usual forcing functions.

c. Describes the currently preferred (optimum) techniques for predicting response to the forcing functions.

d. Presents simplified graphs, charts, equations, nomographs, etc. which provide quantitative estimates.

e. Summarizes proven results from special studies which may find future application.

f. Describes current capability for computer usage as applied to the technologies (briefly describing each applicable program).

g. Defines statistical quantities and accuracies used in describing random processes.

h. Contains terminology and biographical data pertinent to the technologies.

1-3 ANALYSIS PROCESS

A comprehensive analysis process is available by which flight hardware or equipment can be assessed for its ability to withstand dynamic environments. It is the performance of this analysis process which comprises the activity of the Vibration, Shock and Acoustics Branch (VS&A).

The process itself has many aspects including test planning and direction, acquisition and analysis of test data, and performance of various dynamic analyses. A block diagram of the major activities and the usual feed-back loops is shown on Figure 1-1. Only two of the major blocks are performed almost independently of VS&A (i.e., the actual design and flying of the vehicle). All the rest depend heavily on guidance and/or contributions by the VS&A technologies. Specifically, the activities of a VS&A engineer may include, but are not restricted to, the following tasks:

a. Establish, by analysis and test, design and test criteria for structure, equipment, subassemblies, assemblies, and man in the areas of vibration, acoustics, shock, blast, and other dynamic environmental phenomena.

b. Calculate dynamic response of vehicles, structure, and equipment to all dynamic environments to provide other technologies with dynamic characteristics, natural frequencies and mode shapes, dynamic loads and/or stresses for local structure (for use in design and stress analysis), and sonic fatigue data.

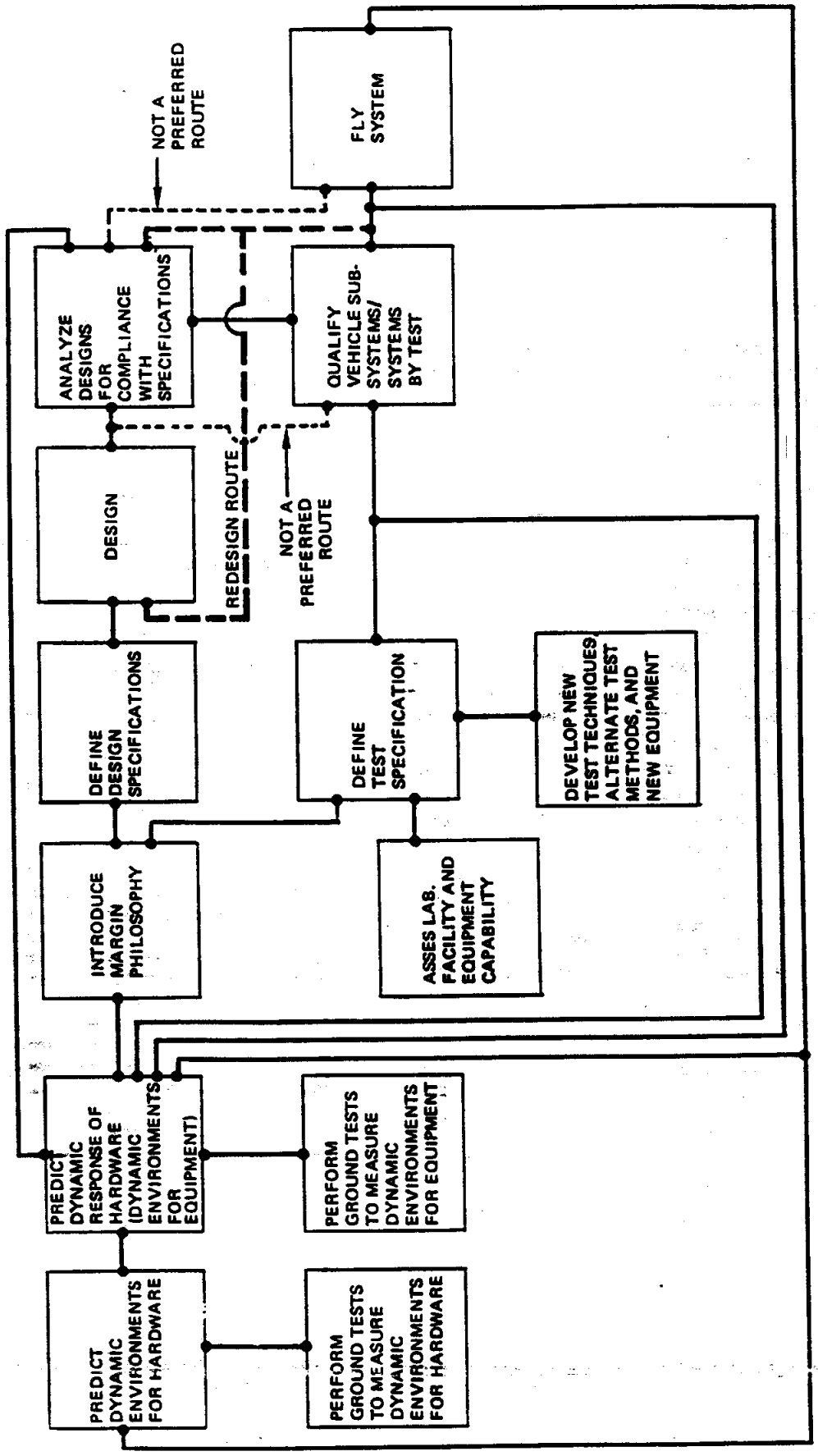


Figure 1-1. Dynamic Analysis Process in Design

c. Review designs and offer the design departments drawing approval to ensure that proper consideration has been given to dynamic loads in the design of structure, bracketry and subsystems.

d. Support Flight Mechanics in the development of structural design requirements and analyses for airborne support equipment in the area of vibration, shock and acoustics.

e. Plan, perform, and participate in development qualification, and acceptance tests for dynamic environments; analyze the results, and in case of failures, determine the cause of failure and recommend design changes accordingly.

f. Determine and define the effects of dynamic environments on man.

g. Determine and assist in the design of control measures to enhance the performance of vehicle systems from the standpoint of dynamic environments.

h. Provide technical consultation service and act as coordinating agency for the Structures Department on all matters relating to shock, blast, vibration and acoustics (including aerodynamically-induced dynamic forcing functions).

i. Support the preparation of proposals on, and the performance of, research and development projects which contain consideration of dynamic environments.

j. Recommend optimum instrumentation, test, and data reduction equipment and techniques (anticipating the need for new facilities and/or methods) in the areas of vibration, shock and acoustics.

k. Maintain technical capability comparable or better than the current "state-of-the-art" in the technologies of vibration, shock and acoustics.

l. Collaborate with cognizant design and project engineers to ensure a clear understanding of system and/or customer requirements with regard to dynamic phenomena.

To support these activities requires considerable versatility of the technical personnel, and a diversity of technical skills. Any or all of the dynamic forcing functions commonly encountered by a missile or space vehicle (i.e., transportation and handling induced environments of shocks, sinusoidal and random vibration, or acoustic fields, dynamic environments prior to launch such as ground shock or acoustic fields, launch and flight exterior or internal acoustic fields, ignition overpressures, launch, maneuver, or separation shocks, aerodynamically-induced fluctuating pressures from turbulent boundary layers, separated flow, cavities, base turbulence, and shock distributed flow, mechanically-induced vibration, vehicle oscillation – POGO, jet impingement, shock waves either endo- or exo-atmospheric, and panel flutter) requires: the ability to assess equipment and techniques used in data acquisition and handling; experience in mathematical specialities such as statistics, complex algebra, and calculus; basic physics, etc. Figure 1-2 indicates the wide spectrum of knowledge required in the subject technology.

No single job necessitates consideration of all the possibilities, indeed, from the viewpoint of VS&A requirements, flight articles may be generally classified on the basis of the technical emphasis. For example, large diameter structure, spacecraft, etc., which are weight critical, may be constrained in their design by the dynamically-induced loads. This would probably not be the case for small, stiff structure exposed (as in the case of high performance vehicles) to high static loads.

Equipment mounted in the large, slow-moving vehicle may be easily designed to survive dynamic environments by the simple expediency of moving them away from the positions of high-level environments, introducing isolation, or mismatching the equipment resonances with the high-level, low-frequency structural response. However, equipment mounted in small high-performance vehicles is exposed to considerable energy in the same frequency domain as the resonances of small parts and pieces. Equipment, therefore, becomes a more serious problem in the smaller vehicles.

Plumbing lines or other ducting is susceptible to the large relative deflections between parts often representative of large vehicle assemblies exposed to dynamic environments. Hence, design and test of plumbing is much more influenced by vibration on large vehicles as compared to that measured on small vehicles.

In general, the large vehicle (booster, spacecraft, etc.) requires detailed analyses of structural integrity, and extensive testing and analysis of large installations or plumbing, while small vehicles necessitate development of component fragility limits and testing for fragility or spurious response during operation. Intermediate size vehicles may entail both large and small vehicle problems if flight goals necessitate lightweight construction.

Because of this need for the VS&A engineer to possess a rather comprehensive capability, this manual was prepared to simplify the routine tasks for which standards (e.g. techniques, quantities, proportionalities, etc.) have been established. The information contained herein is to be considered as preferred by MDAC–WD for use in the VS&A analysis process until such time that a revision alters it. Whenever applicable, therefore, the contents of this manual should be utilized in determining VS&A output.

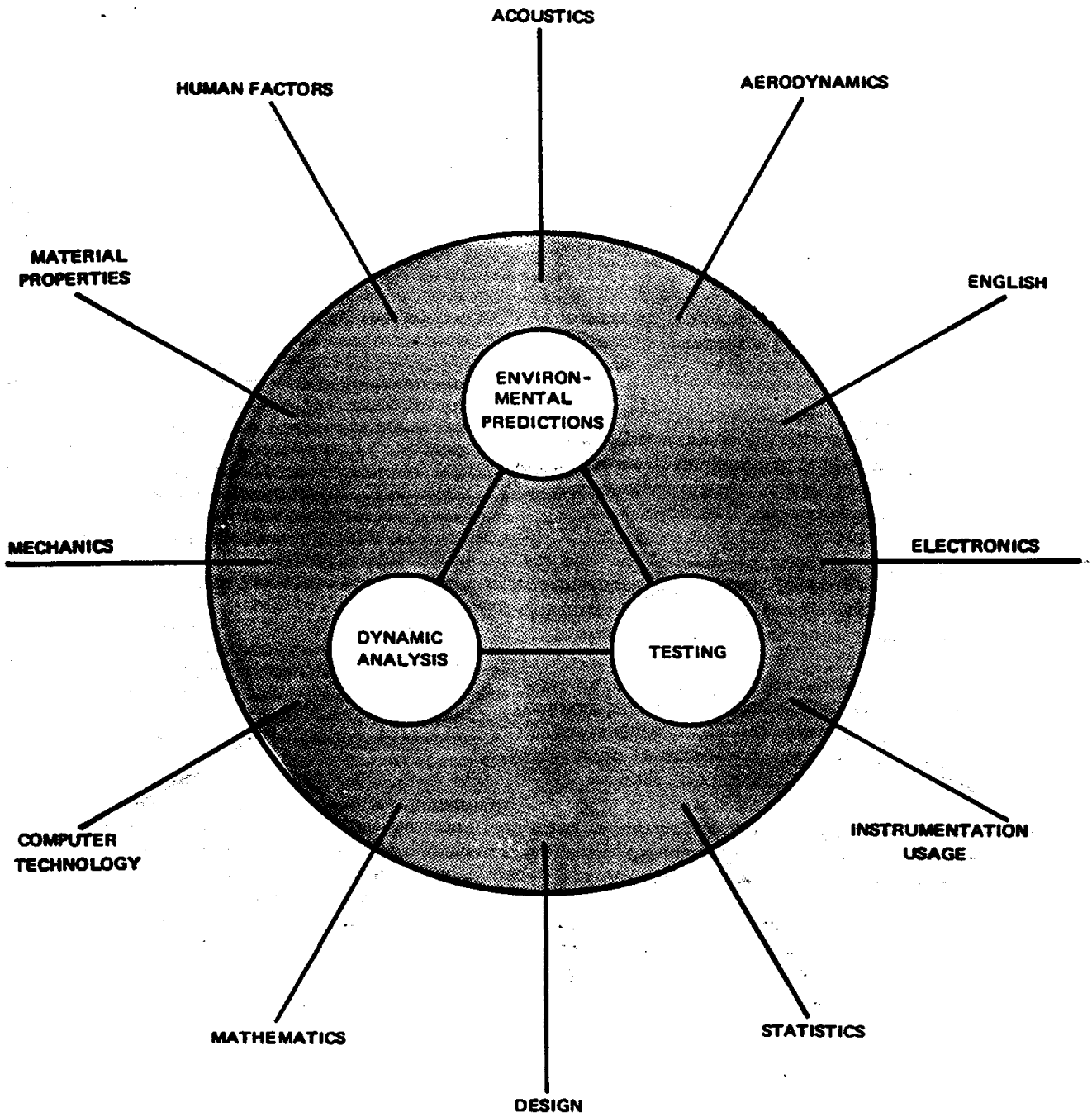


Figure 1-2. VS&A Sphere of Interaction

1-4 DOCUMENTATION

1-4.1 Test Control Drawing Specifications for Vibration, Shock, & Acoustical Tests

Complete definition of a vibration, shock, or acoustical test is required on the Test Control Drawing (TCD) to enable the Dynamics Branch of Engineering Laboratories and Support to determine test feasibility and practicality, to estimate the time and expense required, and to plan, design, and carry out the objectives of the test without delay or confusion.

To assist design engineers in meeting these requirements a review of items to be included in the TCD (as referenced to the checklist shown in Figure 1-3) follows. Reference documents delineating phases of these requirements include Drafting Manual Section 11.6, TOP 072, memorandum A3-K332-M-32 and OMP MSSD-200-8.

1.0 OBJECTIVE OF TEST

A clearly stated objective is necessary to enable proper test planning and performance leading to meaningful data.

2.0 TEST PART(S) DESCRIPTION

A description and/or drawing and part number of the specific part or parts under test (including a functional description) is necessary to complete the objective.

3.0 RESPONSIBILITIES

3.1 Test Director

The test director is the source of information, clarification, and test plan approval. This function is usually retained by the Design Group in which case the director should be named. If this function is to be left unassigned, the responsible group should be named in the TCD. Dynamics and Acoustics Laboratory may be named to assume this function if desired. The test director does not participate directly in the operation of the test.

Only one group may be named as test director.

3.2 Test Conductor

The test conductor is charged with the planning, overall coordination, direct supervision of the test and with preparing the laboratory test report. The conductor is named by the laboratory section chief at the time of TCD release.

3.3 Other Responsibilities

Other responsibilities which will be assumed by the Dynamics and Acoustics Laboratory personnel unless previously negotiated with the cognizant design group and specifically stated within the TCD, include:

3.3.1 Test Specimen Procurement

The test specimen consists of all vehicle hardware and equipment to be included in the test set up. This is supplied in a complete and finished form by the cognizant design groups (usually the group charged with test direction).

3.3.2 Test Fixture Design

Dynamics and Acoustics Laboratory will be responsible for all phases of preparing the test specimen for mounting or suspending, the fixturing required, and all other miscellaneous hardware or facility buildup required. However, fixture design must be approved by the test director before release for fabrication.

3.3.3 Instrumentation

Dynamics and Acoustics Laboratory will design and install the proper instrumentation to meet data requirements.

3.3.4 Data Recording, Reduction, and Preparation

These functions are the responsibility of Dynamics and Acoustics Laboratory. Data is transmitted on 8 1/2 x 11" vellum data sheets in accordance with drawing number 1809647.

3.3.5 Report

A Laboratory Test Report is written by the test conductor. This report must detail the who, how, what, and when of the test, and include the reduced data.

4.0 TEST SPECIMEN DEFINITION

The test specimen is defined as all vehicle parts to be included in the test set up. Brackets, frame and skin sections, supporting plumbing, miscellaneous hardware, etc. must be specified. A drawing and description of the test specimen is required, showing minimum areas of vehicle structure to be included, terminations of plumbing and hardware, and end zones of the test specimen available to

TCD _____
EO _____

TCD Returned for Clarification or Inclusion of:

1.0	TEST OBJECTIVE
2.0	TEST PART DESCRIPTION
3.0	RESPONSIBILITIES
3.1	Test Director
3.2	Test Conductor
3.3	Other Responsibilities
3.3.1	Test Specimen Procurement
3.3.2	Test Fixture Design
3.3.3	Instrumentation
3.3.4	Data Recording, Reduction, & Preparation
3.3.5	Report
4.0	TEST SPECIMEN DEFINITION
5.0	CONTROL POINT LOCATION
6.0	INSTRUMENTATION
	a. Locations (transducers)
	b. No. of channels (transducers)
	c. Freq. Response (transducers)
	d. Environment (transducer)
	e. Accuracies (transducers + system)
	f. Non-vibration measurements required
7.0	DATA REQUIREMENTS
8.0	VIBRATION TEST
8.1	Sine Sweep Test
8.2	Sine Dwell Test
8.3	Random Noise Test
8.4	Mixed Sine & Random Noise Test
9.0	SHOCK TEST
10.0	ACOUSTICAL TEST
10.1	Sine Sweep Test
10.2	Sine Dwell Test
10.3	Random Noise Test
10.4	Mixed Sine & Random Noise Test
11.0	DEFINING RESONANCE
12.0	SUPPORTIVE ENVIRONMENTS

SAMPLE

Figure 1-3. TCD Checklist (Sample)

fixture designers as areas where the test specimen ends and hard mounting of the specimen to the fixture may begin. (See Figure 1-4.) Installation drawings should be included for reference.

If the test part (see 2.0) is the total test specimen, the normal mounting or attachment provisions of the part or other convenient means of attachment to the fixture will be used. This type of test set up should be called out in the TCD as hard mount test part(s).

5.0 CONTROL POINT LOCATION (vibration and shock only)

Dynamic input to the specimen must be monitored to provide a feedback signal from which to control that input. It is essential for both the test plan and fixture design that the control point be clearly defined. Several approaches to the problem of control points follow.

5.1 Input Through the Fixture

A "rigid" fixture is provided on which the test specimen is attached. The control point is the fixture structure, which provides essentially equal inputs to the mounting surfaces of the specimen. Since the control point is ridigly tied to the shaker, this is the only method for which the shaker force ratings are directly applicable. This is the most common approach.

5.2 Input on the Specimen

The specimen is mounted as in 5.1 however, a point is selected on the specimen from which to monitor the input. Because of specimen frequency response, the shaker will invariably be providing an input to the fixture (and hence to the specimen mounting surfaces) quite different from that seen at the control point must be explicitly identified on the TCD.

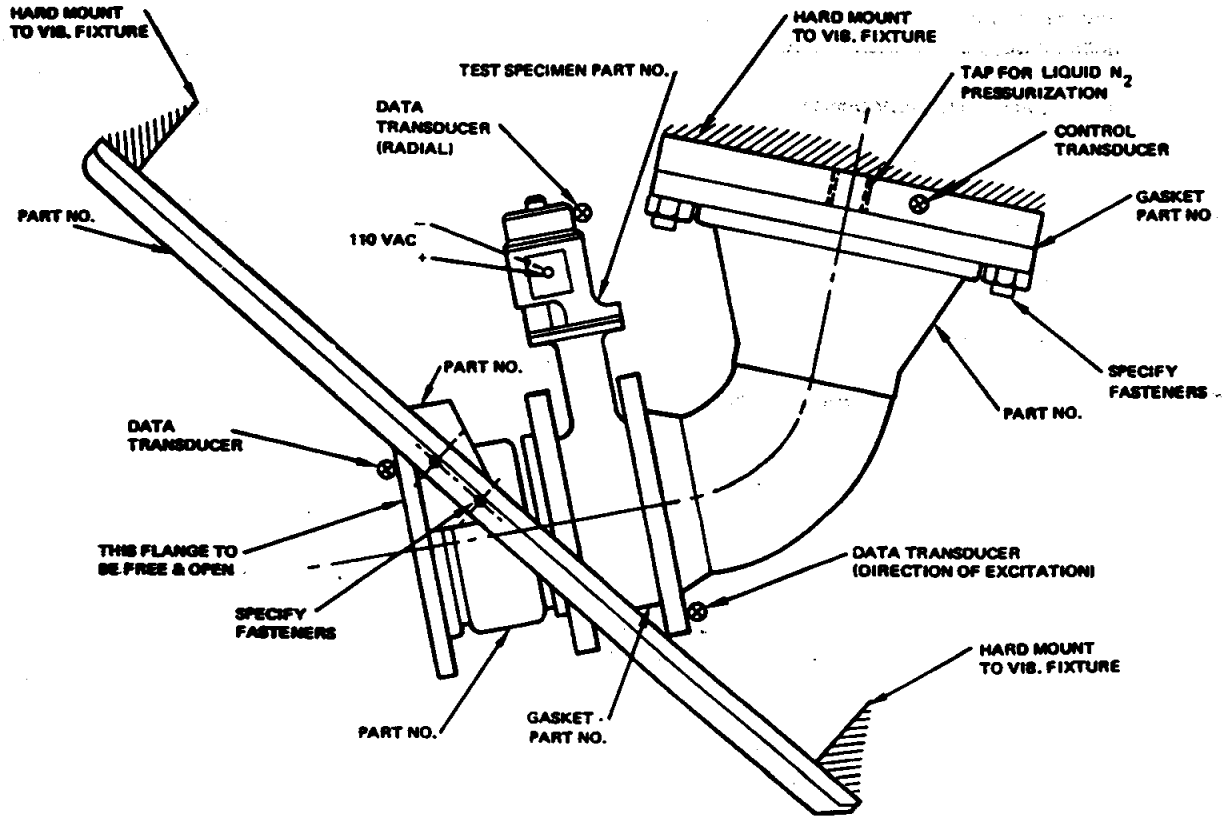


Figure 1-4. Typical Vibration Specimen

5.3 Multiple Control Points

In certain cases the input at several locations is critical. Multiple control points are sometimes used to provide adequate control. Where multiple control points are used, the locations must be defined and a single control parameter chosen, i.e., control to highest control point signal, control to lowest control point signal, or control to average of control point signals.

6.0 INSTRUMENTATION

A generalized description of the measurements desired should be supplied to the Dynamics and Acoustics Group, the instrumentation engineer and test conductor will then utilize the furnished information to predetermine the date channel configurations. If the requesting group desires to assume this function, a detailed sketch, and/or accurate description of the desired data acquisition locations, as well as a description of the data desired should be included in the TCD. The detailed description shall include:

a. Locations (transducers)

b. Number of channels (transducers)

c. Frequency response (transducers)

d. Environment (transducer)

e. Accuracies (transducers + system)

f. Non-vibration measurements required

g. Recording channel assignment and permissible phase differences (if correlation studies are to be performed)

If transducers are not required, but other data-gathering instrumentation such as slow-motion photographs, or contact closure monitoring is necessary, the desired information shall be specified.

In all cases, the type of data or data objective shall be specified rather than the equipment to be used in acquisition. The instrumentation engineer and the test conductor shall determine the optimum "hardware" to be used for a specific test; due to the rapidly changing state-of-the-art, and the experience required to design and assemble a data system, the system is the responsibility of the Dynamics and Acoustics Laboratory Section.

Standard data value accuracies shall be assumed, per the Douglas Instrumentation Test Standards (DITS) manual and applicable test specification drawing, unless otherwise stated in the TCD.

7.0 DATA REQUIREMENTS

If special data or data in forms other than that defined in drawing 1B09647 are required, the special requirements must be discussed with Dynamics and Acoustics Laboratory personnel prior to writing the TCD, and detailed specifications resulting from the discussion must be included within the TCD.

8.0 VIBRATION TEST

The items following each type of test listed below are necessary to completely define the dynamic environment. Tolerances of the specifications should be included if other than standard tolerances are desired.

8.1 Sine Sweep Test

a. Sweep frequency range (Hz-Hz)

b. Logarithmic sweep rate (octaves/min. or min./sweep)

c. Duration of test if other than one sweep (min.)

d. Acceleration ($G's_{o\text{-}p_k}$) or displacement (inches p_k-p_k) levels required over specific frequency bands.

8.2 Sine Dwell Test

a. Frequency of dwell (Hz or method of determination)

b. Duration of dwell (min.)

c. Acceleration ($G's_{o\text{-}p_k}$) or displacement (inches p_k-p_k) level

8.3 Random Noise Test

a. Frequency range (Hz-Hz)

b. Duration of test (min.)

c. Acceleration (g^2/Hz) levels over specitic frequency bands and acceleration roll off (dB/octave) as necessary

8.4 Mixed Sine and Random Noise Test

a. Sine sweep or dwell specification in 8.1

b. Random noise specification in 8.3

9.0 SHOCK TEST

a. Wave form (half sine, sawtooth, triangular, etc.)

b. Acceleration level ($G's_{o\text{-}pk}$)

c. Rise time to peak level (MS)

d. Pulse duration (MS)

e. Shock spectrum (in place of a. - d.) though this is not preferred.

10.0 ACOUSTICAL TEST

10.1 Sine Sweep Test

a. Sweep frequency range (Hz-Hz)

b. Logarithmic sweep ratio (octaves/min. or min./sweep)

c. Duration of test if other than one sweep (min.)

d. Sound levels (dB) required

10.2 Sine Dwell Test

a. Frequency of dwell (Hz or method of determination)

b. Duration of dwell (min.)

c. Sound levels (dB) required

10.3 Random Noise Test

a. Frequency range (Hz-Hz)

b. Duration of test (min.)

c. Sound levels (dB) over specific frequency bands

10.4 Mixed Sine and Random Noise Test

a. Sine sweep or dwell specification in 10.1

b. Random noise specification in 10.3

11.0 DEFINING RESONANCE

Resonance of complex structure is not an easily defined condition. To ask for "all conditions of resonance" is meaningless. If major structural resonances or resonance of a particular part relative to another is desired, the resonance specification should include that description. Also, if more than a major resonance is desired, the number of harmonics to be noted should be named, e.g. "1st 3 structural resonances" or "major resonance of P/N relative to its supports", etc.

If notching of resonances is desired, specific information must be included in the TCD identifying the resonances to be notched and the amount of notching required.

12.0 SUPPORTIVE ENVIRONMENTS

Non-dynamic environments such as temperature, pressure, etc., desired during the vibration and shock tests should be specified.

Additional information assistance can be obtained from Dynamics and Acoustics Laboratory Section.

1-4.2 Measurement Request Drawings

Measurement Request Drawings (MRD) are for the purpose of initiating the complete sequence of events that result in a measurement of data. The sequence includes acquisition and calibration of a sensing system, installation of the properly-oriented system, on-board hookup and checkout, and some sort of monitoring of the expected signal. The initiator of an MRD should therefore be cognizant of each of these major steps, and provide sufficient information for other technologies to perform their functions in a manner suitable to achieve a successful measurement.

Instructions for usage and completion of MRD Form No. 60-1407 are included in Drafting Manual Section 11.801. Figure 1-5 is a sample fill-in of this form with specific application to Vibration Shock and Acoustical requirements such as:

a. Full definition of the variable to be measured

1. Dynamic range of interest

2. Frequency range (realizing that care must be exercised to avoid overdriven channels or fold back from unwanted frequency content by appropriate signal conditioning).

3. Phase requirements, if any.

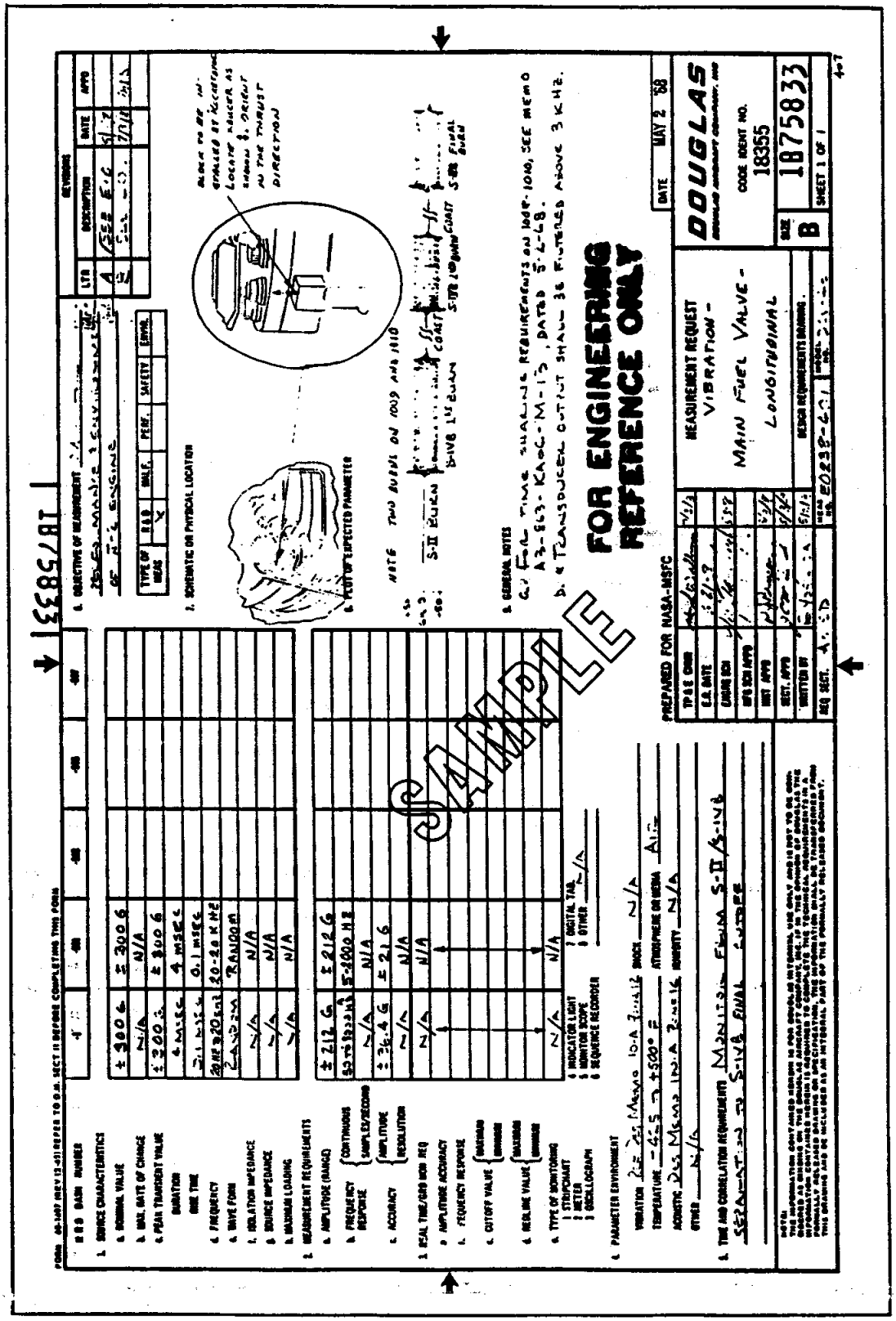


Figure 1-5. Typical Measurement Request Drawing

4. Time duration, or specific times of measurement

5. Accuracy requirements (linearity, recovery time, frequency response, etc.)

b. Full definition of other environments which are capable of affecting the sensing system (such as temperature, particle impingement, humidity, wind, other dynamic environments, electrical fields, radiation, etc.), and the permissible error introduced by them.

c. Complete description of measurement location

1. Orientation (using an accepted frame of reference)

2. Exact location (include sketch whenever possible) accounting for installation and clearance considerations.

d. Complementary measurements or data required (e.g. internal static pressure, temperature, wind speed, humidity, vehicle velocity, time, etc.)

e. Calibration requirements, both as sensing system specifications and as pre- and post-test capability.

For further convenience a sample fill-in of the required companion EO is shown in Figure 1-6.

The MRD Form (60-1407) is a 'B' size drawing, and the original vellum is issued and controlled by the Records Department through Drawing Request Form 60-685, in accordance with Drafting Manual Section 7.2.

A sample fill-in of Form 60-685, as applied to MRD's for Vibration, Shock and Acoustics usage, is shown in Figure 1-7.

After properly filling out the 60-685 form, make two copies; one for the numbers clerk and one for Vibration, Shock and Acoustics records. Next, present the original and one copy to the numbers clerk, whereupon a drawing number will be stamped on the 60-685 form and as many MRD forms as are required. This assigns a specific number to each title, and a log of these related descriptors is maintained by records and is available on request.

1-4.3 Data Books

The term data book describes a specialized type of publication. Such a book does not include any data analysis, but must include sufficient information for an engineer to perform one at a later date. Hence, a data book is a means of storing data for future retrieval. It should include the following:

a. The entire data package.

b. A discussion and estimate of the error in the data.

c. A sufficient description (or reference to a description) of the test history, the test setup, and calibration procedures, so that an assessment of data validity and meaning can be made.

d. Any photographs or other records (in other media) which provide redundant or supplemental information to the data.

e. All pertinent references (including documented references, identification of sources for information, and the critical key nomenclature which uniquely determines the data such as names, dates, places, etc.).

f. At a minimum, the table of contents should include the following:

1. Introduction (purpose and objectives)

2. Test setup description

3. Test hardware description

4. Instrumentation

5. Test history (sequence of actual events)

6. Data reduction – Techniques used, and an error/accuracy discussion

1-4.4 Technical Memoranda or DAC Reports

These documents are formal presentations of information which may be transmitted outside of the and as such, are referable. As a consequence, a more formal approval cycle is required. A sample of a DAC Report Preparation Schedule (see Figure 1-8) indicates the necessary time to complete the approval cycle. In addition to allowing adequate time for approvals, the preparation of a report should entail consideration of:

a. Logic – The reader must be informed of what is intended, and how it is to be done. Then, after such a "logic road map" has been presented, periodic signposts should reestablish contact with the route chosen so that the reader is still attending when the conclusion is reached.

b. Accuracy – Any technical reviewer will seek errors, and finding one, whether in logic or in the data will probably discount the whole report. Hence, it is advantageous to be correct even in secondary aspects. This is not to say that only positive results should be reported or that data which were proven to be in error should not be mentioned. On the contrary, full recognition of error, when it occurs, can be beneficial in establishing the integrity of the report, and negative results are useful in determining what is not true.

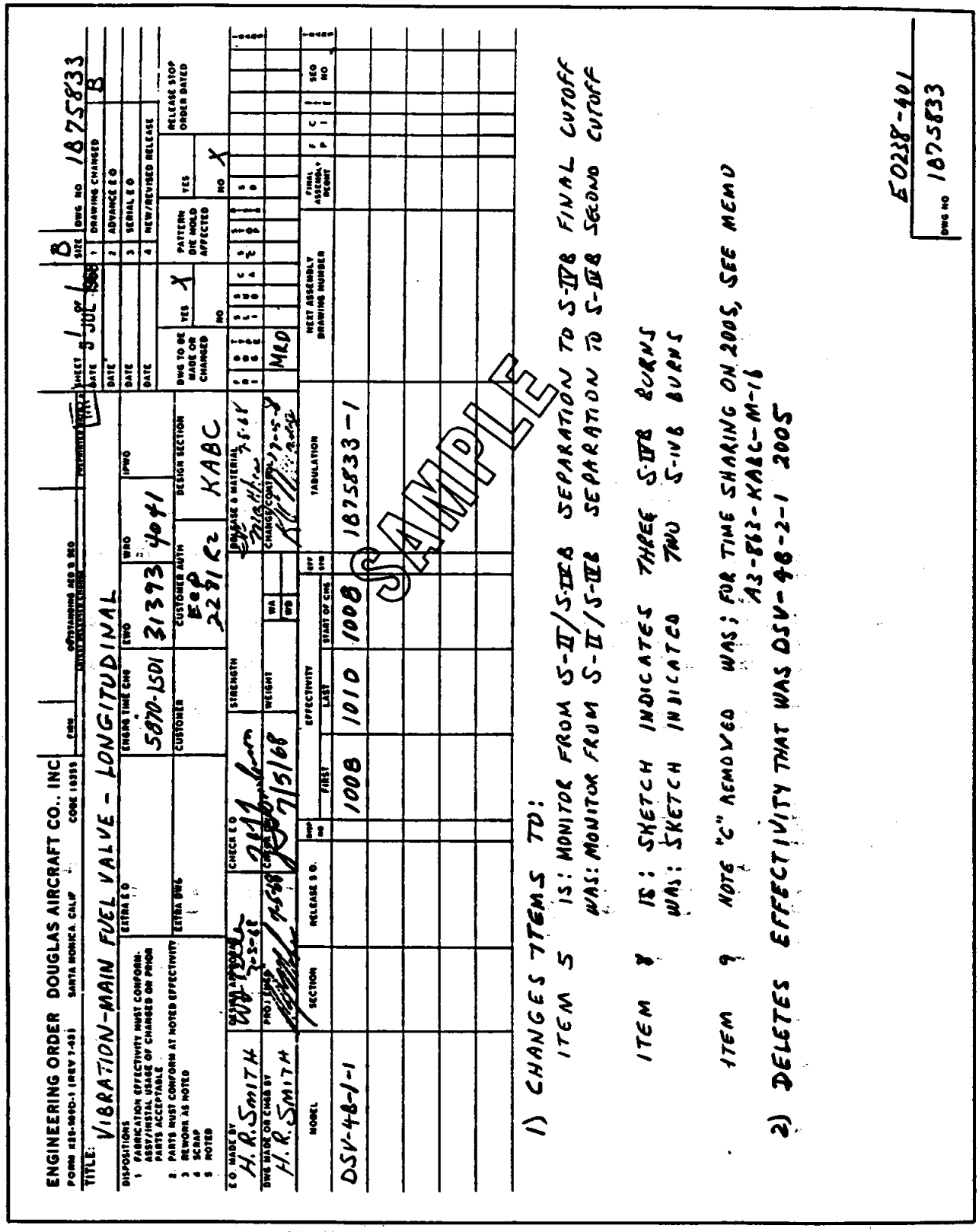


Figure 1-6. Measurement Request Drawing EO – Sample

FORM 60-885 (REV. 7-66)

DRAWING REQUEST FORM

SIZE	DRAWING NUMBER		MODEL	SECTION NUMBER	DESIGN SECTION	SECURITY CLASSIFICATION
B	1D75832	1	SATURN DSV-4B		A&D	UNCLASSIFIED X / CONFIDENTIAL / SECRET

SAMPLE

2 TITLE VIBRATION — MAIN FUEL VALVE — LONGITUDINAL

3 DATE	EWO/MEMO	ENGR & PHONE		
5 2 68	31393	EVOLUTION X2630(A3)	SPECIFICATION CONTROL / SOURCE CONTROL / DESIGN PROCUREMENT	FORGING, CASTING, EXTRUSION, MOLDING / OTHER MRD X / NOT TO BE RELEASED LAYOUT, SKETCH, WTM, ETC.

DRAWING TYPE

SCHEDULES APPROVAL N/A

NOMENCLATURE APPROVAL N/A

OTHER C. M. Ackman for J. C. McClymonds

DESIGN DETAILS

MATERIAL TYPES	
JOINING METHODS	WELD BRAZE ADHESIVE MECHANICAL SOLDER
OPERATING TEMP	____ °F MIN TO ____ °F MAX
CRITICAL ENVIRONMENT	(EG: AIR, LOX, LH_2, ETC) INTERNAL EXTERNAL
LUBRICATION REQD	YES ___ NO ___
SEALING REQD	PRESSURE ___ PSI. HERMETIC YES ___ NO ___
SPEC COVERAGE	PROCESS, MATERIAL, FINISH ADEQUATE ___ INADEQUATE ___
PROD. TEST REQD	YES ___ NO ___

Figure 1-7. Drawing Request Form (Sample) — For Measurement Request Drawing

In addition to maintaining accuracy in all statements, every report should include an error discussion. If analytical, the report should assess the importance of the assumptions made. If experimental, the report should evaluate the probable error or some other statistical parameter which defines the confidence in the data.

c. Title — The title should be as short as possible but should still differentiate the contents from all other reports; ambigious or abstract words should be avoided.

d. Contents — The report must include a summary or abstract which, when read, reveals the intent, premise and conclusions. A table of contents and a list of figures must be included, and they must agree with the labeling and page numbering of the report. Besides the introduction and conclusions sections, major subject matter should be partitioned to simplify review by those with limited interest.

e. Sequence — Graphs, tables, figures, etc. are preferably included when mentioned by the text. However, if desired, all such material may be assembled at the end of the document, all tables being placed sequentially before the figures. References are placed at the end of the written text (after the conclusions but before tables and figures if these latter items are assembled after the text). All appendices are attached after the compilation of text, references, tables and figures.

DAC REPORT PREPARATION SCHEDULE

	Maximum Allowed Time
1. OUTLINE	R* + 2 days
a. Section Designation	
b. Paragraph Designation	
c. Review with Group Engineer	
d. Review with Supervision	
2. ANALYSES/TEST/CRITERIA/LOADS	R + 7 days
a. Prepare all final figures to be included in report	
b. Outline analyses/test results/criteria analyses in detail from 2a.	
c. Review with Group Engineer	
d. Review with Supervision	
3. PREPARE REPORT (allow 2 days typing time)	R + 20 days
4. REVIEW/APPROVAL OF ROUGH DRAFT (Complete Document)	
a. By Group Engineer(s)	R + 22 days
b. By Supervision	R + 24 days
5. INCORPORATE COMMENTS OF SUPERVISION INTO ROUGH DRAFT	R + 26 days
6. REVIEW/APPROVAL OF FINAL ROUGH DRAFT BY SUPERVISION	R + 27 days
7. FINAL TYPE	R + 30 days
8. APPROVAL OF FINAL DRAFT	
a. Review/approval by Group Engineer (initial by author's signature)	R + 31 days
b. Review/approval by Section Chief	R + 32 days
c. Review/approval by Branch Chief	R + 33 days
d. Review/approval by Structures Representative (program)	R + 34 days
e. Review/approval by Chief Engineer of Structures Department**	R + 35 days
f. Review/approval by Chief Engineer (Program)	
g. Review/approval by Program Director	

SAMPLE

* R - represents time at which flight/ground test is completed and valid test data is in section; loads/criteria analyses are completed, i.e., calculations from computer runs or environmental derivations are performed.

** NOTE: Author(s) responsibility extends through 8e.

Figure 1-8. DAC Report Preparation Schedule

Section 2
CRITERIA

2-1 ACOUSTICAL PHYSIOLOGICAL AND PSYCHOLOGICAL CRITERIA

2-1.1 Noise Criteria Curves

The noise criteria (NC) curves shown in Figure 2-1 and the information in Table 2-1 represent sound pressures which provide minimum requirements for communication and comfort, but do not represent noise tolerance safety requirements.

2-1.2 Limit Curves

The long-term noise-tolerance criteria are shown in Figure 2-2. Shorter duration (less than 8 hours per day) noise tolerance may be derived from Figure 2-3 or Figure 2-4. Criteria for very short-duration impulsive noise (less than 1 second within 20 dB of peak level) can be obtained from Figure 2-5.

2-1.3 Hearing Damage or Loss Criteria

Whenever the criteria of paragraph 2-1.2 are exceeded, a potential for hearing damage exists, and hearing protection must be specified. The protectors may be ear plugs, ear muffs, or a combination of both. Attenuation characteristics of some of these devices may be obtained from Figure 2-6.

NOTE:

At no time may personnel be exposed to overall sound pressure levels exceeding 150 dB, regardless of hearing protection worn.

2-1.4 References

1. MIL-STD-1472, 9 Feb. '68, Human Engineering Design Criteria for Military Systems, Equipment and Facilities

2. HEL Standard S-1-63B June '65 Maximum Noise Level for Army Material Command Equipment

3. MSFC-STD-267A, 23 September 1966, George C. Marshall Space Flight Center, National Aeronautics and Space Administration, Standard, Human Engineering Design Criteria

4. Air Force Regulation No. 160-3B, 7 February 1967

5. NASA SP-3006 – Bioastronautics Data Book

2-1.5 Noise Parameters

a. *Impulse Noise:* A non-periodic variation in atmospheric pressure which may be completely described by its pressure versus time history. It has a positive pressure envelope duration of less than 1,000 milliseconds, and a peak to root mean square value greater than 10 dB.

b. *Positive-Peak Sound-Pressure Level:* The highest instantaneous pressure achieved (expressed in dB or in PSI).

c. *Positive Pressure Duration:* The time required for the pressure wave to rise to its first positive peak and to return momentarily to ambient.

d. *Positive Pressure Envelope Duration:* The time required for the pressure wave to rise to its highest positive peak and to decrease to, and remain 20 dB below, this highest peak.

To be acceptable positive-peak sound-pressure level versus positive *pressure* duration shall be below limit "A" of Figure 2-5 and positive-peak sound-pressure level versus *positive pressure* envelope duration shall be below limit "B".

2-2 VIBRATION

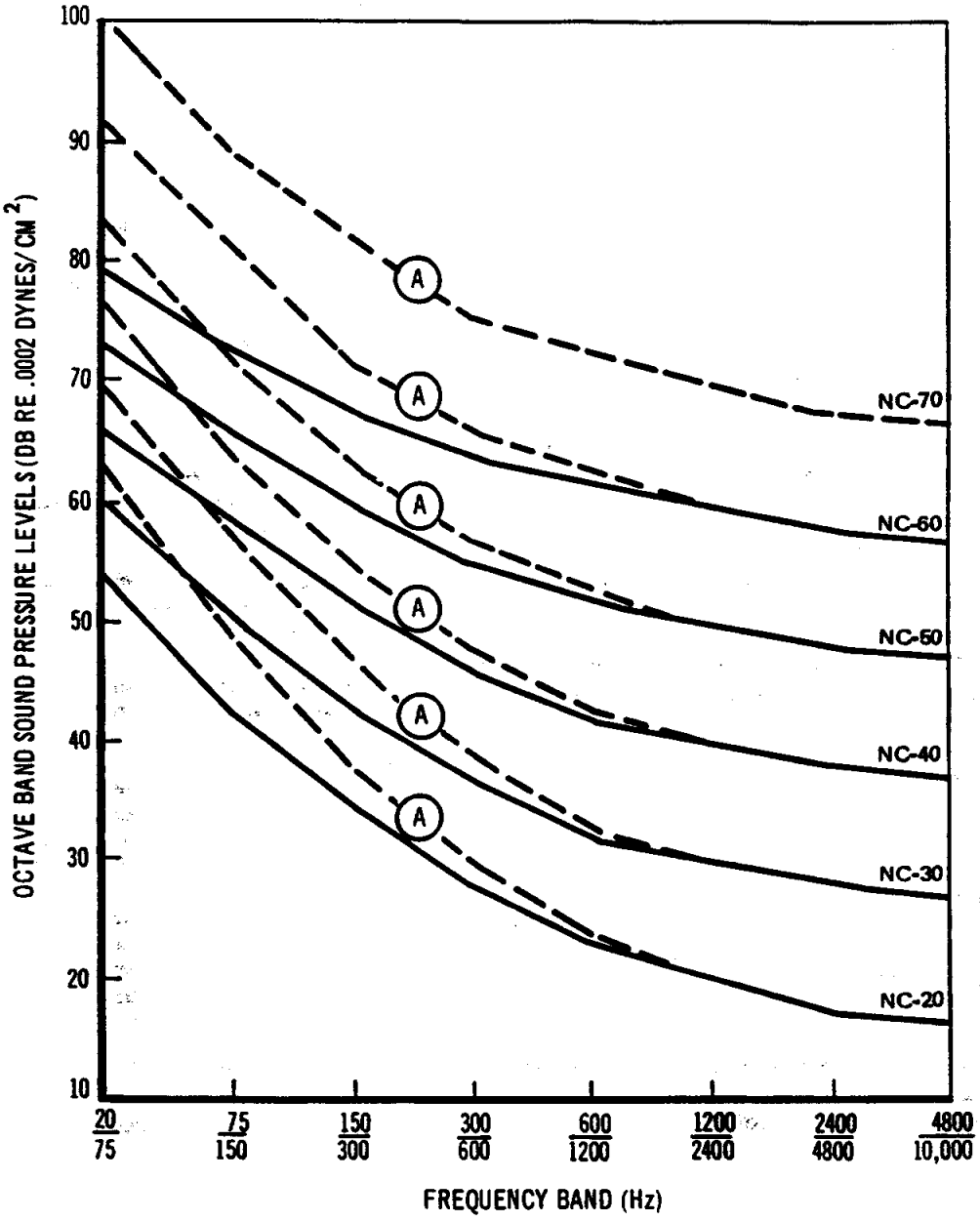
2-2.1 Good Design Practices

Experience and good engineering judgement are invaluable in the design of vibration-resistant equipment and supporting structures. The design/development process includes:

a. Definition of vibration environments and test criteria.

b. Evaluation of good, and rejection of bad dynamic designs.

c. Evaluation of the effects of vibration on both equipment and supporting structures.

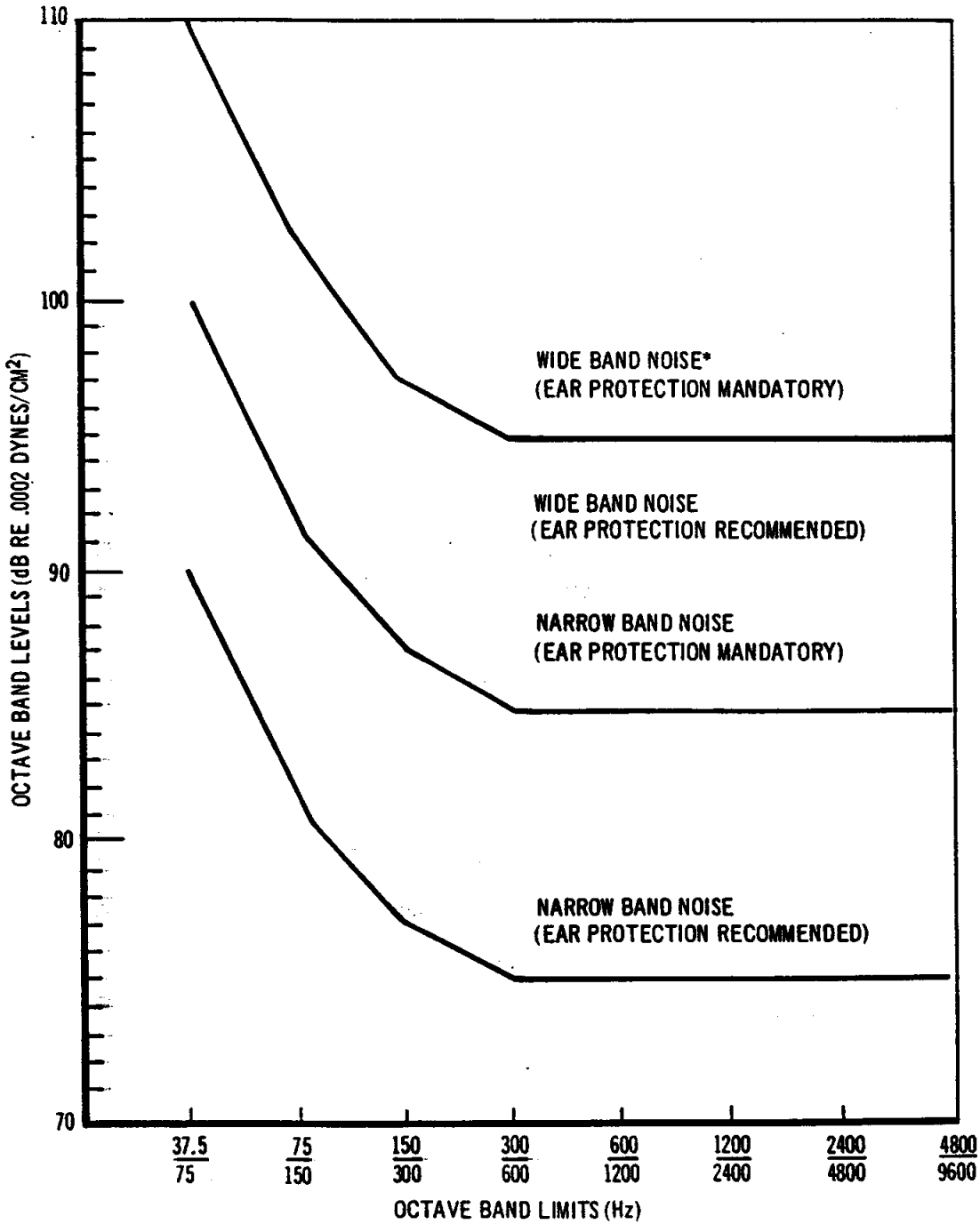


NOTE: These criteria curves are to be used with Table 2-1 in determining the permissible sound pressure levels in 8 octave frequency bands. The NC curves (Solid Lines) are recommended for specifications whenever a favorable relation between the low and the high frequency portion of the spectrum is desired. The NCA curves are the maximum recommended deviation from the NC curves whenever economy dictates a maximum compromise and where, in addition, the noise is steady and free of beats between low frequency components.

Figure 2-1. Noise Criteria Curves.

Table 2-1. NC for Offices and Workspaces

OFFICES	
NOISE MEASUREMENTS MADE FOR THE PURPOSE OF COMPARING THE NOISE IN AN OFFICE WITH THESE CRITERIA SHOULD BE PERFORMED WITH THE OFFICE IN NORMAL OPERATION, BUT WITH NO ONE TALKING AT THE PARTICULAR DESK OR CONFERENCE TABLE WHERE SPEECH COMMUNICATION IS DESIRED (ie, WHERE THE MEASUREMENT IS BEING MADE). BACKGROUND NOISE WITH THE OFFICE UNOCCUPIED SHOULD BE LOWER, SAY BY 5 TO 10 dB.	
NC (OR NCA) CURVE	COMMUNICATION ENVIRONMENT
NC–20 TO NC–30	VERY QUIET OFFICE – TELEPHONE USE SATISFACTORY – SUITABLE FOR LARGE CONFERENCES.
NC–30 TO NC–35	"QUIET" OFFICE, SATISFACTORY FOR CONFERENCES AT A 15–FT TABLE; NORMAL VOICE 10 TO 30 FT; TELEPHONE USE SATISFACTORY
NC–35 TO NC–40	SATISFACTORY FOR CONFERENCES AT A 6 TO 8 FT TABLE; TELEPHONE USE SATISFACTORY; NORMAL VOICE 6 TO 12 FT.
NC–40 TO NC–50	SATISFACTORY FOR CONFERENCES AT A 4 TO 5 FT TABLE; TELEPHONE USE OCCASIONALLY SLIGHTLY DIFFICULT; NORMAL VOICE 3 TO 6 FT; RAISED VOICE 6 TO 12 FT.
NC–50 TO NC–55	UNSATISFACTORY FOR CONFERENCES OF MORE THAN TWO OR THREE PEOPLE; TELEPHONE USE SLIGHTLY DIFFICULT: NORMAL VOICE 1 TO 2 FT; RAISED VOICE 3 TO 6 FT.
ABOVE NC–55	"VERY NOISY", OFFICE ENVIRONMENT UNSATISFACTORY; TELEPHONE USE DIFFICULT.
	WORK SPACES, SHOP AREAS
NC–60 TO NC–70	PERSON-TO-PERSON COMMUNICATION WITH RAISED VOICE SATISFACTORY TO 1 TO 2 FT; SLIGHTLY DIFFICULT 3 TO 6 FT. TELEPHONE USE DIFFICULT.
NC–70 TO NC–80	PERSON-TO-PERSON COMMUNICATION SLIGHTLY DIFFICULT WITH RAISED VOICE 1 TO 2 FT; SLIGHTLY DIFFICULT WITH SHOUTING 3 TO 6 FT. TELEPHONE USE VERY DIFFICULT.
ABOVE NC–80	PERSON-TO-PERSON COMMUNICATION EXTREMELY DIFFICULT. TELEPHONE USE UNSATISFACTORY



NOTE: DIFFERENCES IN STANDARDS AND CRITERIA FOR NOISE TOLERANCES ARE QUITE WIDE AND NO SIMPLE CRITERION IS PRESENTLY AGREED TO BE VALID BY ALL INVESTIGATORS.

* TYPICAL STATE NOISE CONTROL SAFETY ORDERS

Figure 2-2. Long Term Damage Risk Criteria (8 Hours Per Day for 25 Years).

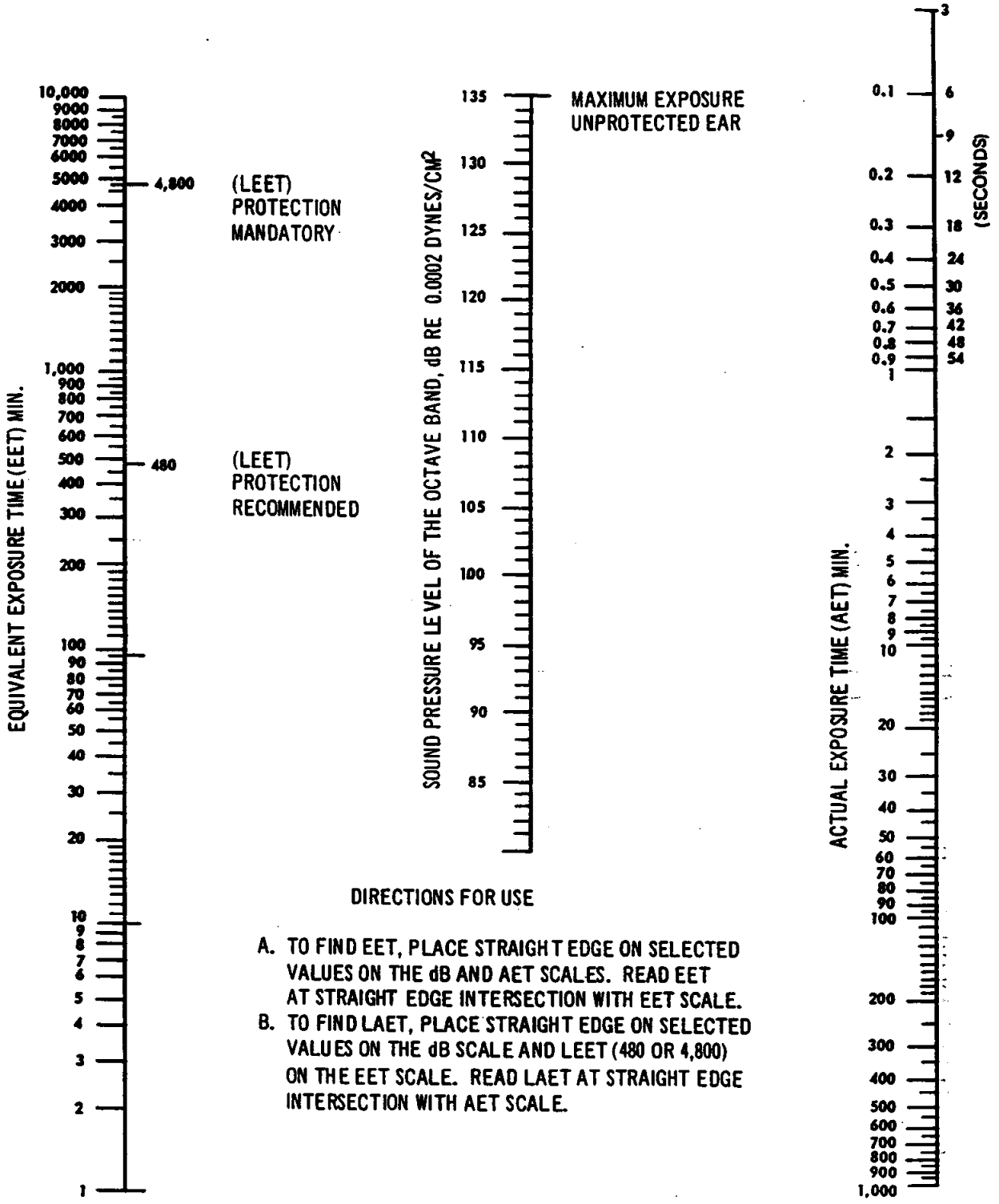


Figure 2-3. Nomogram for Computing Equivalent Exposure Time (EET) and Limiting Actual Exposure Time (LAET).

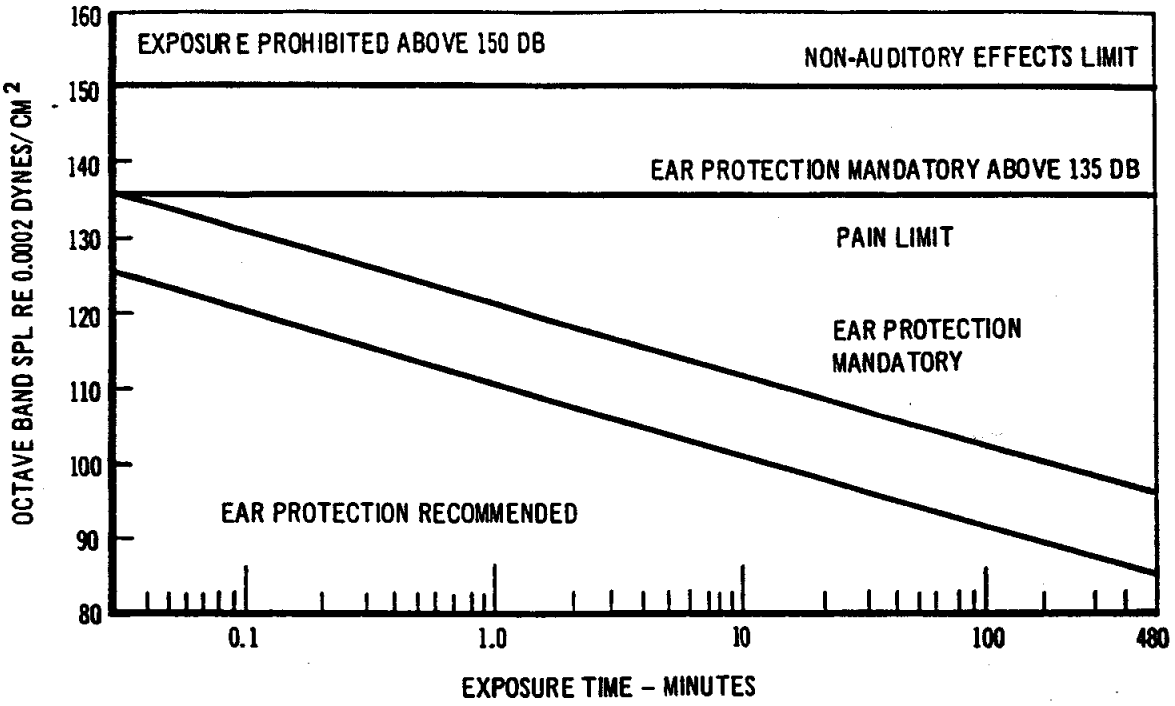


Figure 2-4. Short Term Damage Risk Criteria (Wide Band Noise).

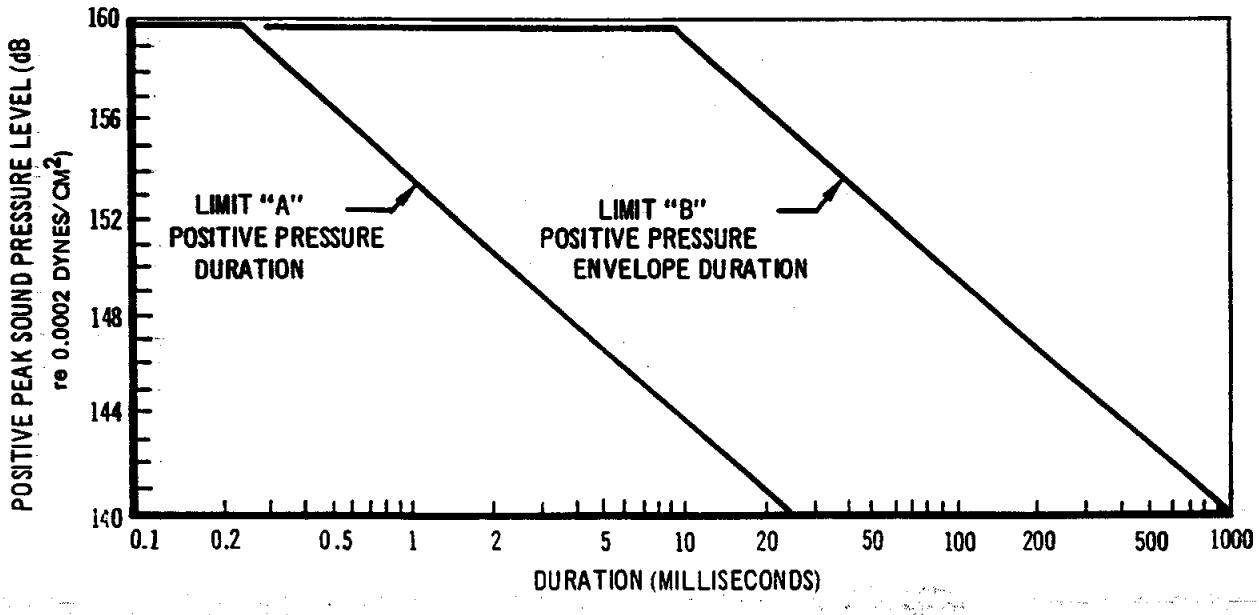
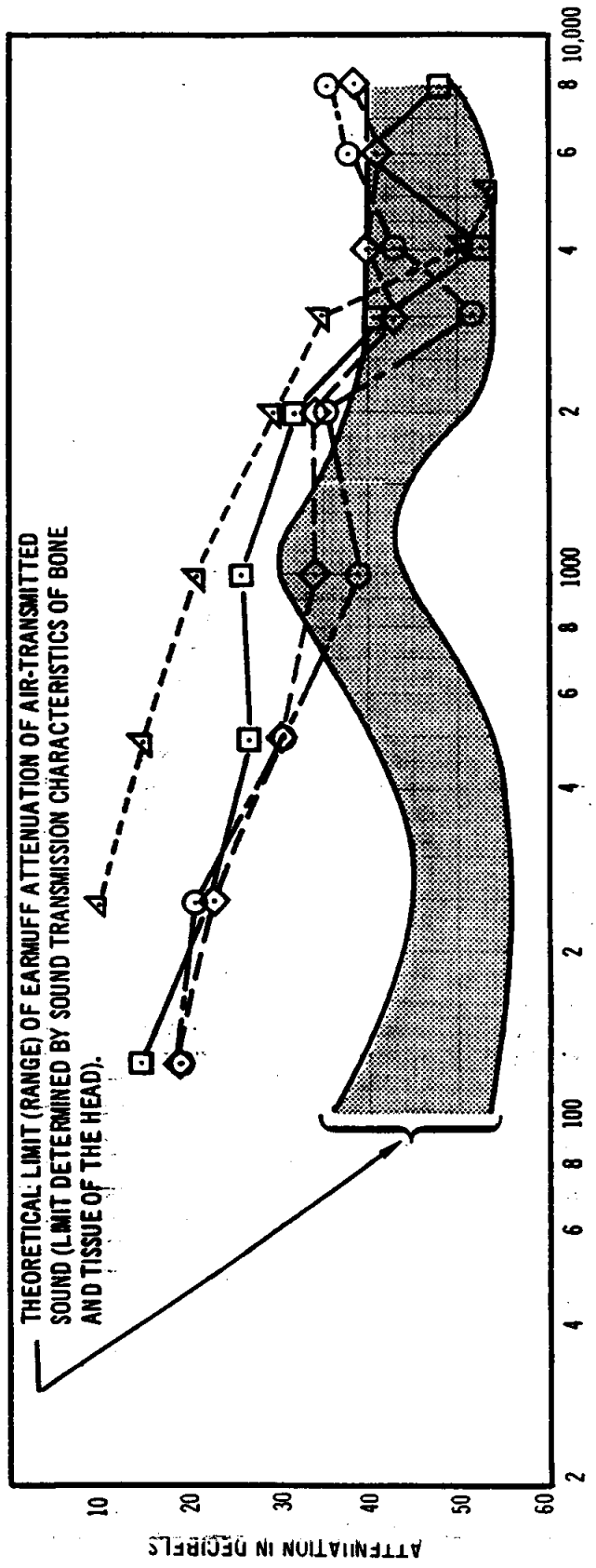


Figure 2-5. Maximum Acceptable Impulse Noise Parameters for Small Arms and Similar Impulsive Noise.



MAKE OF EAR PROTECTOR	TOTAL WT. GRAMS	HEADBAND*	EAR CUSHION*	EARPHONES*	MICROPHONE●
D-8 FLITE-SOUND HELMET	439	A	R	AV	AV
AMERICAN OPTICAL 372-8	339	A,W	R	AV	AV
MINE SAFETY MARK II	389	A,W,R	R,W	AV	AV
WILSON NO. 258					

* A = ADJUSTABLE, W = WASHABLE, R = REMOVABLE
● AV = AVAILABLE

Figure 2-6. Comparison of a Helmet and Earmuff-Type Ear Protectors.

Designing for vibration should include: dynamic analyses, defining dynamic loads, dynamic tests, consideration of damping or isolation, decoupling of frequencies, estimating fragility levels, knowledge of properties of materials, and review of assembly configurations. The degree of consideration varies with each type of flight vehicle. The greatest consideration is required for vehicles with weight limitations since optimized designs are required that may dictate flexible primary structures.

Generally, primary structures are not designed to vibration requirements, but are designed to static loads. Recent trends are to verify primary structures prior to flight by analytical methods and if necessary, to modify structures to meet the dynamic environmental requirements.

Within weight limitations, primary structure should be relatively strong and stiff (to withstand flight loads and to provide a satisfactory support for airborne equipment). Rigidity for primary structures may be provided by ring frames, intercostals and stringers. Intercostals and ring frames should be used for equipment support with the intercostals and ring frames providing a box-type primary support whenever possible. Some needed damping, to minimize vibration, may be provided by built-up assemblies using rivets and other fasteners. Large panels in primary structure should be avoided. In cases where fragility levels of equipment are exceeded, isolation or additional damping may be required.

An ideal design will uncouple mechanical systems at critical frequencies. In addition, mechanical systems should be designed and constructed to produce low values of transmissibility. The importance of uncoupling is easily demonstrated since two coupled systems have a combined transmissibility, $H_c(f)$, which may equal the product of their individual transmissibilities. Assuming a transmissibility of 5 for 2 individual systems (e.g., a black box and its supporting panel), then the combined transmissiblity, $H_c(f)$, at resonance is equal to 25. This accounts for excessively high vibration levels measured on poorly-supported parts. These damaging motions can be attenuated by detuning the systems.

Equipment parts should be small, light, and stiff for good dynamic design. Miniaturized parts are desirable, since they are generally stronger than larger parts. The natural frequency of the part and its mounting should be higher than the natural frequency of the black box. Electrical parts (condensors, resistors, and the like) should never be supported by their leads. The part and the lead should be secured firmly to a terminal strip or circuit board. Cables and wires should be supported or held firmly in place to prevent damaging relative motion.

The use of suitable cements and potting compounds is recommended in vibration-resistant designs. The use of flexible terminal strips and circuit boards should be avoided. Parts should never bridge two different terminal strips. If possible, heavy parts should not be mounted in the middle of unsupported panels. Better locations for heavy parts are the rigid corners of a chassis. Parts should not be cantilevered. Center of gravity should be as close to the support point as possible. Brackets for parts should not be allowed to cantilever and gusset supports should be added to prevent large low-frequency motions. A chassis should be designed to have box-like construction and should have stiffeners across large panels.

Black boxes should be rigid, should have stiffeners and gussets, and should be secured firmly to supporting panels or structures, with many fasteners or bolts uniformly located around the box (to uniformly distribute the load). Sharp bends and cutouts should be avoided on supporting bracketry.

Equipment panels and supporting bracketry should be as rigid as possible within weight limitations. Equipment panels should be constructed of materials with low transmissibilities at resonance. Panels should have high natural frequencies, but be uncoupled from modules and parts.

Materials suitable for panels are glass fiber honeycomb, metal honeycomb, and end grain cuts of balsa wood encased in metal frames. Whenever possible, equipment weight should be distributed uniformly over the equipment panel.

2-2.2 Vibration Isolators

The application of vibration isolators is both an art and a science. The required mathematics for analysis of isolator systems are available in the literature, but experience, empirical data, and practical considerations are essential to the final selection of optimum isolation design. There is a vast array of commercial isolators available, however, it is imperative that analysis be performed and isolators be sized to the specific hardware installation whether commercial isolators are used or specific isolators are designed. This section contains information on classical vibration-isolation systems with references for supporting mathematics.

Usually, technical data are available from isolator vendors and manufacturers. However, such data should be used with caution since the data may be general and may not apply to specific isolators. Also the data usually apply to the isolation of a rigid body which is seldom the case in actual practice.

The primary function of vibration isolators is to protect equipment from vibrations in specific frequency ranges. Vibration isolators must be selected and used with discrimination based on knowledge of equipment

fragilities, the expected vibration environment, and the isolator system characteristics. In general, the isolation system frequency should be in a range where the environment is known to be minimum. This is usually above 30 Hz and below 100 to 200 Hz. All vibration isolation designs should be confirmed by test.

Figure 2-7 presents five different isolation system configurations: one base-mounted system and four center-of-gravity (c.g.) systems. The c.g. systems are preferred, but because of practical limitations, the base-mounted systems are used in many installations. Negligible rocking occurs in correctly-designed c.g. systems. Figure 2-8 (also see Reference 1) shows schematic diagrams of vibration-isolation systems employing six types of vibration isolators. The various vibration isolators differ from each other primarily in the mechanisms by which the isolator stiffness and damping are provided.

Experimental (absolute) transmissibility curves for these vibration-isolation systems are presented in Figure 2-9 (also from Reference 1). Vibration-isolation systems such as those shown in Figure 2-9(a) and 4(d) have been used by MDAC-WD for airborne applications with excellent results.

A minimum effort solution for an isolator system design is:

a. Establish that vibration isolators are required to protect the airborne equipment.

b. Establish f_{min}, the lower limit of acceptable natural frequencies. This value corresponds to a minimum stiffness for isolators and is a function of 1. the maximum permitted displacement, d_{max}, across the isolators and 2. the largest peak acceleration, $\ddot{x}_{max}$, of the supporting structure.

   For a sample problem, d_{max} is given as 0.1 inches and $\ddot{x}_{max}$ is given as 10 g's peak. This information is used in the following equation to find f_{min} (References 2 and 3).

$$f_{min} = 3.13 \sqrt{\frac{\ddot{x}_{max}}{d_{max}}} = 31 \text{ Hz} \qquad (1)$$

c. Use a c.g. vibration-isolation system whenever possible.

d. Design or select a rigid (strong) supporting structure (high impedance) to avoid coupling structural resonances with isolation system resonances.

e. Select vibration isolators to attenuate high-frequency vibrations associated with known equipment fragilities.

f. Consider environmental vibration levels and vibration test levels when selecting isolators so as to avoid, if possible, high vibration inputs at isolation system natural frequencies.

g. When available, use empirical vibration test data on isolators to establish system transmissibilities.

h. When empirical data are not available on vibration isolators, use the classical curves of Figure 2-10 to obtain an approximation of system transmissibilities.

i. Consider all temperature or inertial acceleration influences which could modify the operating chracteristics of the isolator and design accordingly.

Using the above rules, a satisfactory isolation system could probably be selected. However, analytical methods are also available, and prove to be more useful in some instances provided the isolator can be considered a single degree-of-freedom (SDF) system (which is, by definition, one in which the motion in time can be completely described by the use of a single coordinate—the height of the mass with respect to the fixed support).

Figure 2-10 presents a family of transmissibility curves and basic mathematical relationships for sinusoidally-excited SDF systems. (Section 5 of this manual discusses SDF systems in much greater detail). The transfer function $H_\infty(f)$ for the SDF system excited sinusoidally is given by:

$$H_\infty(f) = \frac{\text{response}}{\text{input}} = \left\{ \frac{1 + 4\left(\frac{c}{c_c} \times \frac{f}{f_n}\right)}{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + 4\left(\frac{c}{c_c} \times \frac{f}{f_n}\right)^2} \right\}^{\frac{1}{2}} \qquad (2)$$

Here, f is the frequency at which the system is being exicted, c is the damping, c_c is the critical damping, and f_n is the natural frequency of resonance.

The subscript "∞" denotes an infinite source mechanical impedance, Z_∞, or infinitely rigid structure at the point or points where the input excitation, I(f), is applied.

The response of a mechanical system to random vibration may be described in any of the units: g_{rms}, g^2/Hz, or $g/\sqrt{Hz}$. For an applied spectral density that is constant with frequency, the response of a simple resonant member is proportional to $\sqrt{H_Q f_n}$, where H_Q is

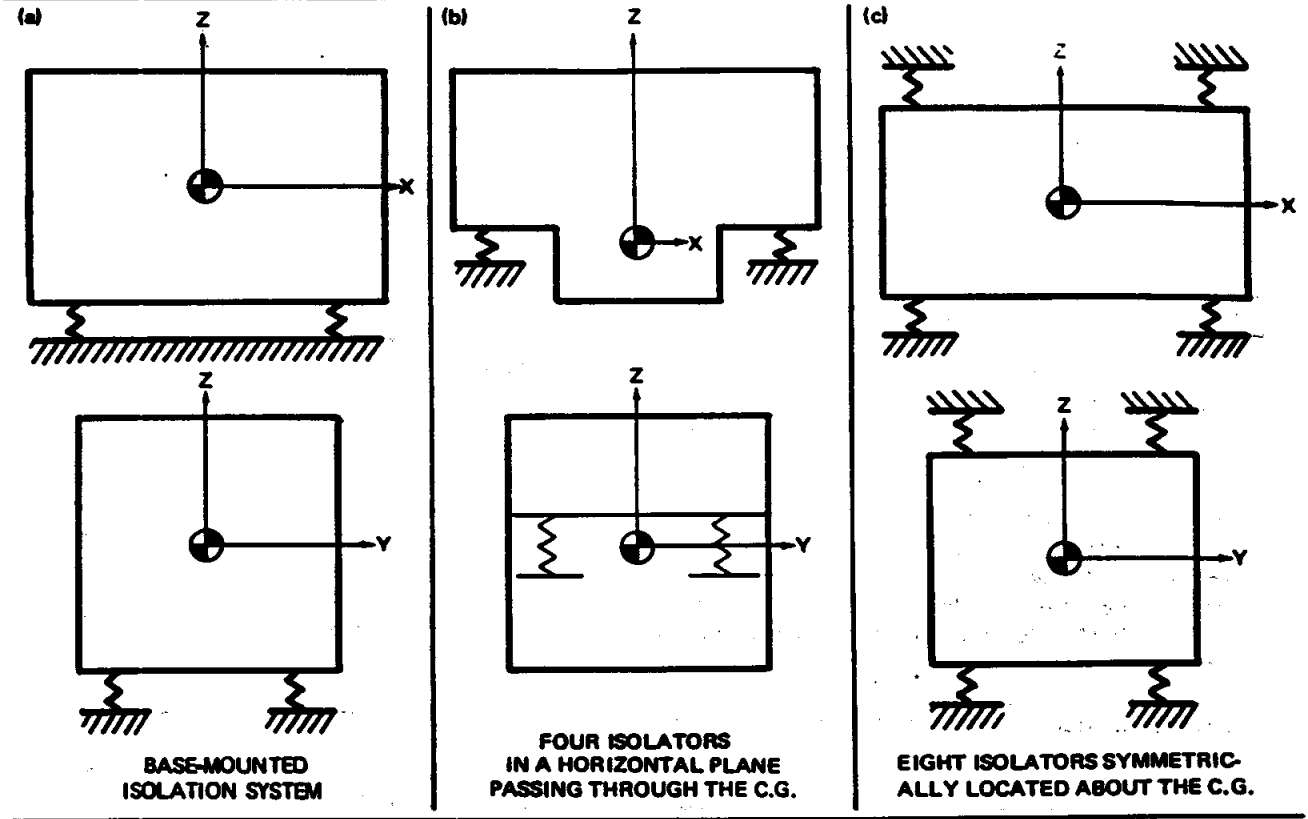
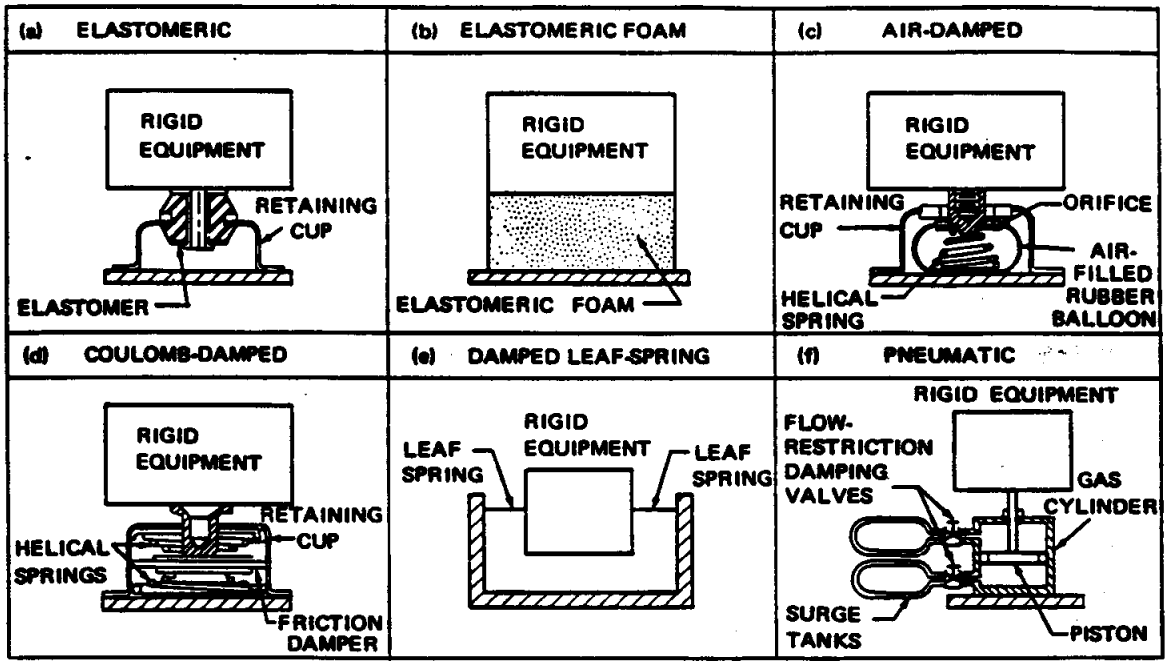


Figure 2-7. Isolator Configurations.



Elastomeric and Elastometric-Foam Vibration Isolators are Self-Damped Resilient Elements, whereas Helical-Spring, Leaf-Spring and Pneumatic Resilient Elements Require Auxiliary Air. Coulomb (dry-friction) or Viscoelastic Damping Mechanisms.

Figure 2-8. Schematic Diagrams of Rigid-Equipment Vibration-Isolation Systems Employing Various Types of Vibration Isolators.

the transmissibility at resonance, f_n is the frequency of resonance as before, and the response is stated in terms of rms acceleration. This is contrasted to the sinusoidal case, where the response of a simple resonant member is proportional to H_q. The acceleration variance or the mean square acceleration, σ_r^2, of the response to random excitation is defined as follows (Reference 6):

$$\sigma_r^2 = \omega \frac{G(\omega)}{8\left(\frac{c}{c_c}\right)} \qquad (3)$$

Where $\omega = 2\pi f_n$ and $G(\omega)$ is the power-spectral-density acceleration in g^2/Rad/sec. Since the transmissibility, H_q, at resonance is equal to $1/2(c/c_c)$, equation 3 can be changed to:

$$\sigma_r^2 = G(\omega)\frac{2\pi f_n}{4} \times \left(\frac{1}{2\frac{c}{c_c}}\right) \qquad (4)$$

$$= G(\omega) H_q \frac{2\pi f_n}{4} = G(f) H_q \frac{f_n}{4}$$

where G(f) is the power spectral density of the acceleration in g^2/Hz.

Then, σ_r or the rms acceleration of the response becomes:

$$\sigma_r = 1/2\sqrt{G(f) H_q f_n} \qquad (5)$$

The power spectrum of the response, $G_r(f)$, is obtained by multiplying the power spectrum of the excitation, G(f), by the square of the magnitude of the system's complex frequency response, H(f):

$$G_r(f) = |H(f)|^2 G(f) \qquad (6)$$

Basic SDF equations and the curves of Figures 2-10, 2-11 and 2-12 can be used as design guides to determine desirable frequencies for equipment and structures. The ideal design will place equipment natural frequencies in the low transmissibility (say less than 0.1) region of Figure 2-10.

A resiliently-supported mechanical system may have from one to six degrees-of-freedom, depending on the mounting arrangement. A system with six degrees-of-freedom has three rotational (yaw, pitch, and roll) and three translational (vertical, lateral, and longitudinal) natural modes of vibration. Rotational

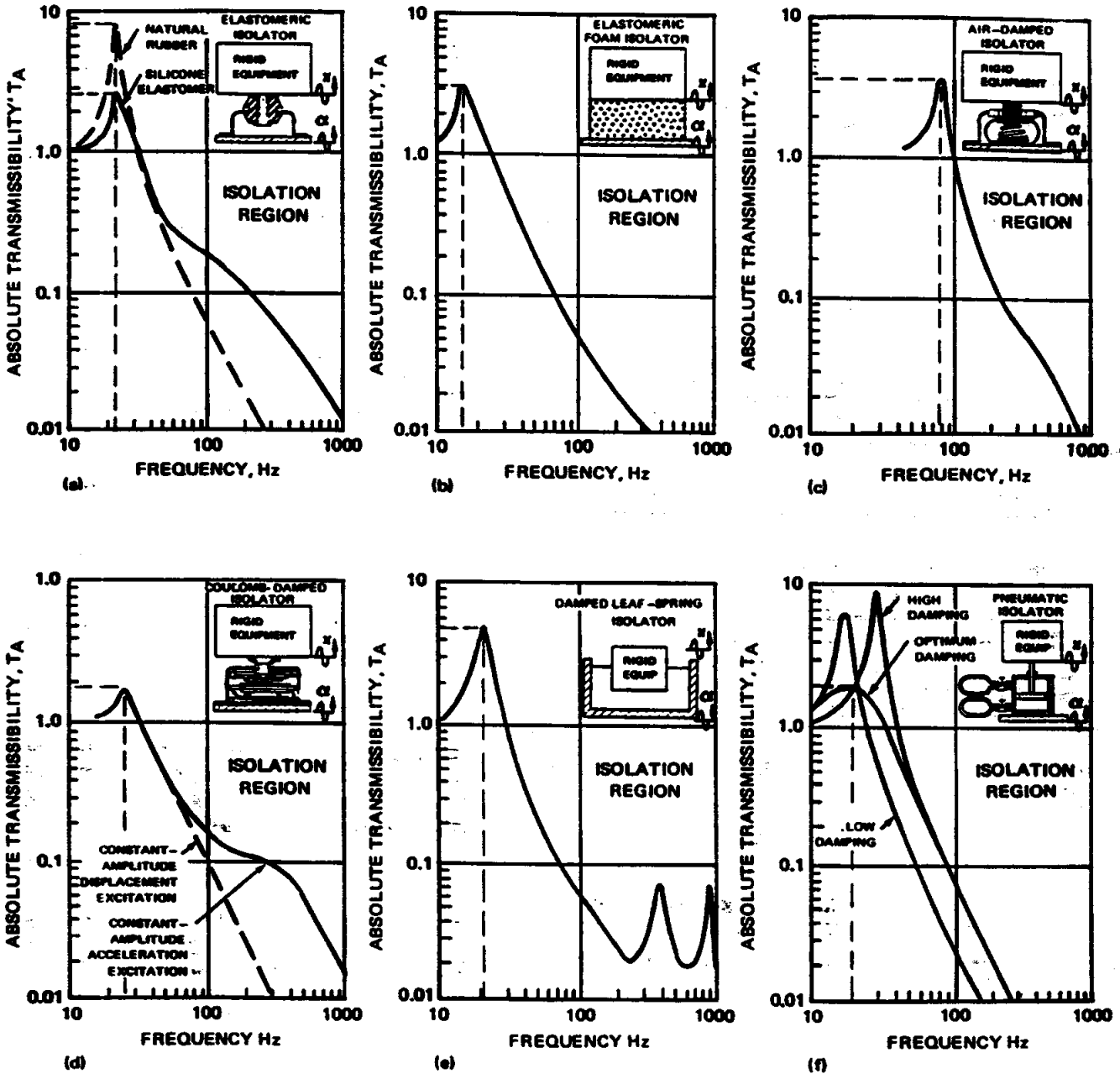


Figure 2-9. Experimental Absolute-Transmissibility Curves for Various Rigid-Equipment Vibration-Isolation Systems (Room Temperature).

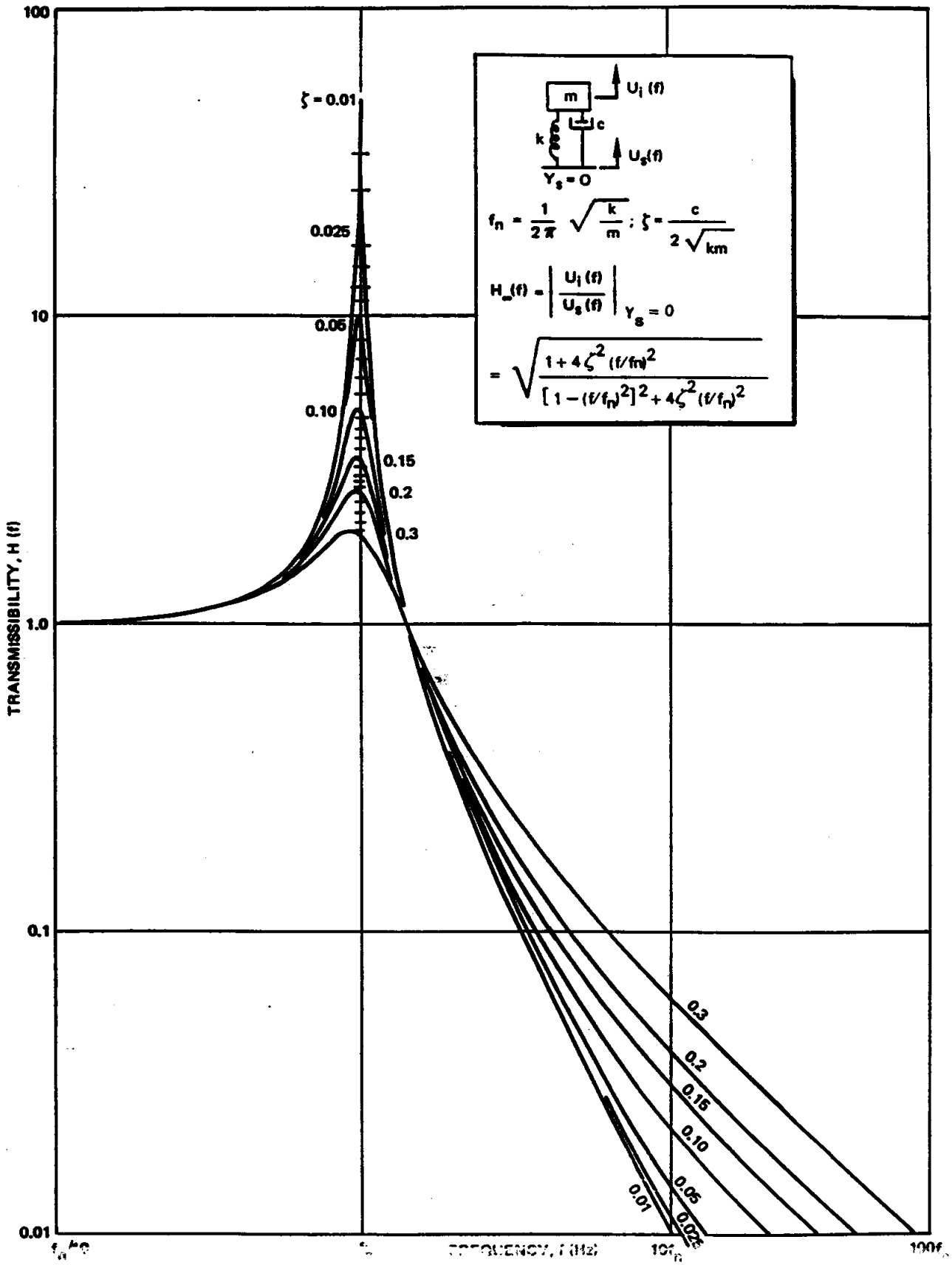


Figure 2-10. Family of Single-Degree-of-Freedom Transmissibility Curves.

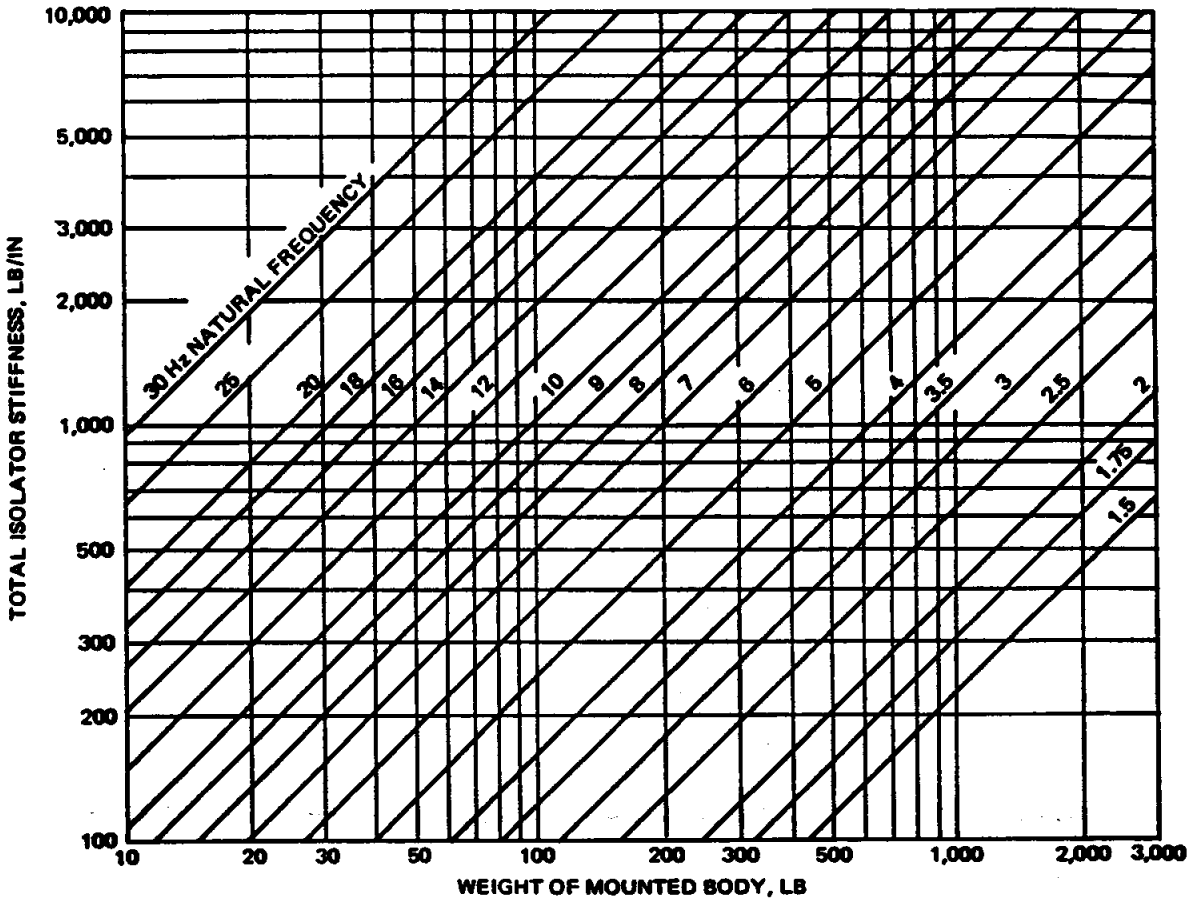


Figure 2-11. Natural Frequency of a Single-Degree-of-Freedom System as a Function of Weight of Mounted Body and Total Isolator Stiffness.

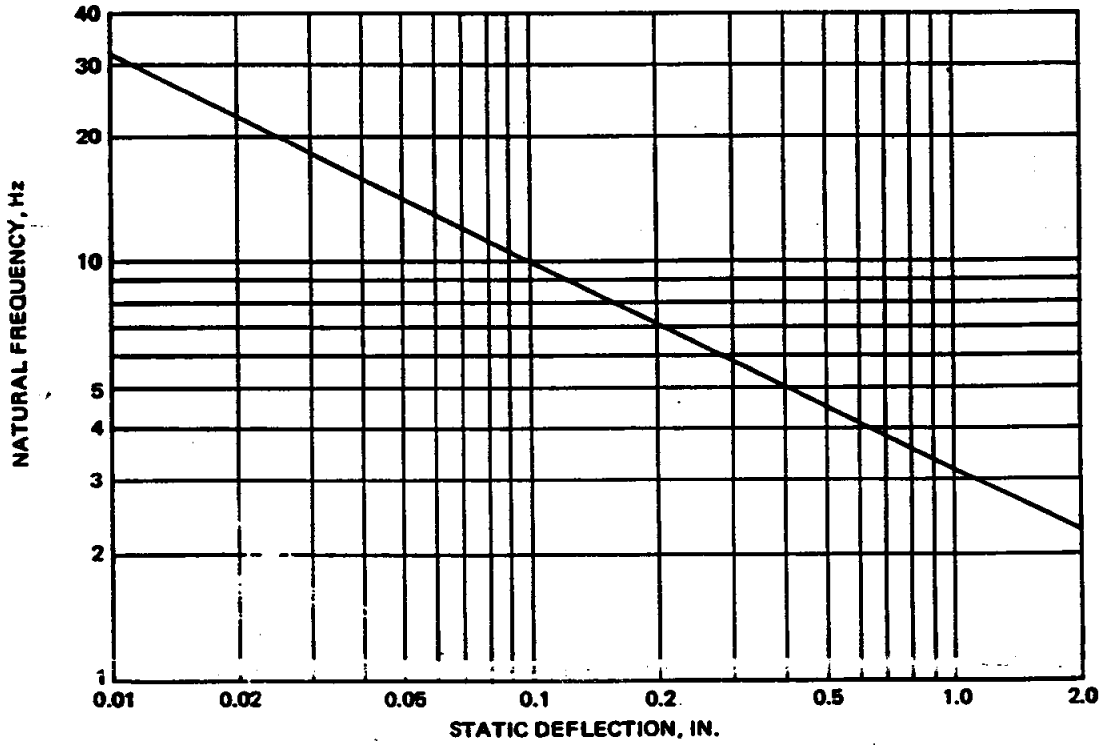


Figure 2-12. Relation Between a Natural Frequency and Static Deflection for a Linear, Single-Degree-of-Freedom System.

natural frequencies for isolation systems located symmetrically about the center of gravity (cg), can be approximated by using the following equations (Reference 2):

$$f_{n\alpha} = \frac{1}{\rho_x}\sqrt{f_{ny}^2 a_z^2 + f_{nz} a_y^2}$$

$$f_{n\beta} = \frac{1}{\rho_y}\sqrt{f_{nx}^2 a_z^2 + f_{nz} a_x^2} \qquad (7)$$

$$f_{n\gamma} = \frac{1}{\rho_z}\sqrt{f_{nx}^2 a_y^2 + f_{ny}^2 a_x^2}$$

where f_{nx}, f_{ny}, and f_{nz} are the translational natural frequencies in the x, y, and z directions, respectively; ρ_x, ρ_y and ρ_z are the radii of gyration of the equipment about the x, y, and z axes, respectively; and a_x, a_y, and a_z are distances from the c.g. of the equipment to the elastic centers of the individual isolators in the x, y, and z directions, respectively. Assuming the mass of the mounted equipment to be uniformly distributed throughout its volume, the radius of gyration about an axis through the center of gravity is defined by:

$$\rho = \sqrt{\frac{d^2 + e^2}{12}} \quad \text{in inches.}$$

where d and e are the vertical and horizontal dimensions of the equipment about each axis.

The calculation of natural frequencies for the base-mounted isolation system of Figure 2-7 has been presented in Reference 4 and will be summarized in this section. Vibration in the horizontal translatory mode degenerates to a coupled mode in which the mounted body vibrates in rotation with respect to an axis below the cg of the body. The rotational vibration degenerates to coupled vibration with respect to an axis above the cg. The horizontal and rotational modes are thus coupled and vibration in one of these modes cannot exist independently of vibration in the other mode. Equation 8 and Figure 2-13 present the relationship between two coupled or degenerate modes and the decoupled natural frequency:

$$\frac{\Omega_c}{\Omega_y} \times \frac{\rho_z}{b} = \frac{1}{\sqrt{2}}\sqrt{\frac{\eta\rho_z^2}{b^2}\left(1+\frac{a^2}{\rho_z^2}\right)+1 \pm \sqrt{\left[\frac{\eta\rho_z^2}{b^2}\left(1+\frac{a^2}{\rho_z^2}\right)+1\right]^2 - \frac{4\eta\rho_z^2}{b^2}}} \qquad (8)$$

where Ω_c is the natural frequency for the two coupled modes, Ω_y is the decoupled SDF vertical natural frequency, a and b are dimensions shown in Figure 2-13, ρ_z is the radius of gyration about the z axis, and η is the dimensionless stiffness ratio k_x/k_y. Thus, the two coupled natural-frequencies may be determined from equation 8 or Figure 2-13 if the following information is available:

a. Weight of the mounted equipment (pounds).

b. Stiffness of the isolators in x and y axes (pounds per inch).

c. Radius of gyration (inches) or moment of inertia (I) of the mounted equipment.

Where:

$$I = m\rho^2 = W\rho^2/g \qquad (9)$$

d. The location of the center of gravity of the mounted equipment.

e. The spacing between isolators.

It is assumed that there are two planes of symmetry, XY and ZY, and that the vertical translatory mode of vibration is decoupled. (To use Figure 2-13 the dimensionless ratios $\rho\sqrt{k_x/k_y}$ /b and a/ρ are calculated.)

Figure 2-13 is entered on the horizontal scale at the value of $\rho\sqrt{k_x/k_y}$ /b and vertical intersections are made with two curves marked a/ρ. These two intersections are used to find two values of the dimensionless ratio ρ/b x Ω_x/Ω_y from the vertical scale at the left hand margin.

Next, these two values are divided by ρ/b and multiplied by Ω_y to determine the two coupled natural-frequencies. To avoid resonances and ensure a low transmissibility, the forcing frequency should be substantially greater than the highest coupled natural frequency.

The preceding discussion has provided a means of determining the coupled natural frequency in translation along the Y axis and the coupled natural frequencies in the XY plane. Since the YZ is also a plane of symmetry, coupled natural frequencies corresponding to those in the XY plane may be found by the same method.

The sixth remaining frequency (in radians per second), that of rotation about the Y axis, may be derived by the following method (from Reference 4):

$$\Omega_\beta = 2\sqrt{\frac{b^2 k_z + c^2 k_x}{I}} \qquad (10)$$

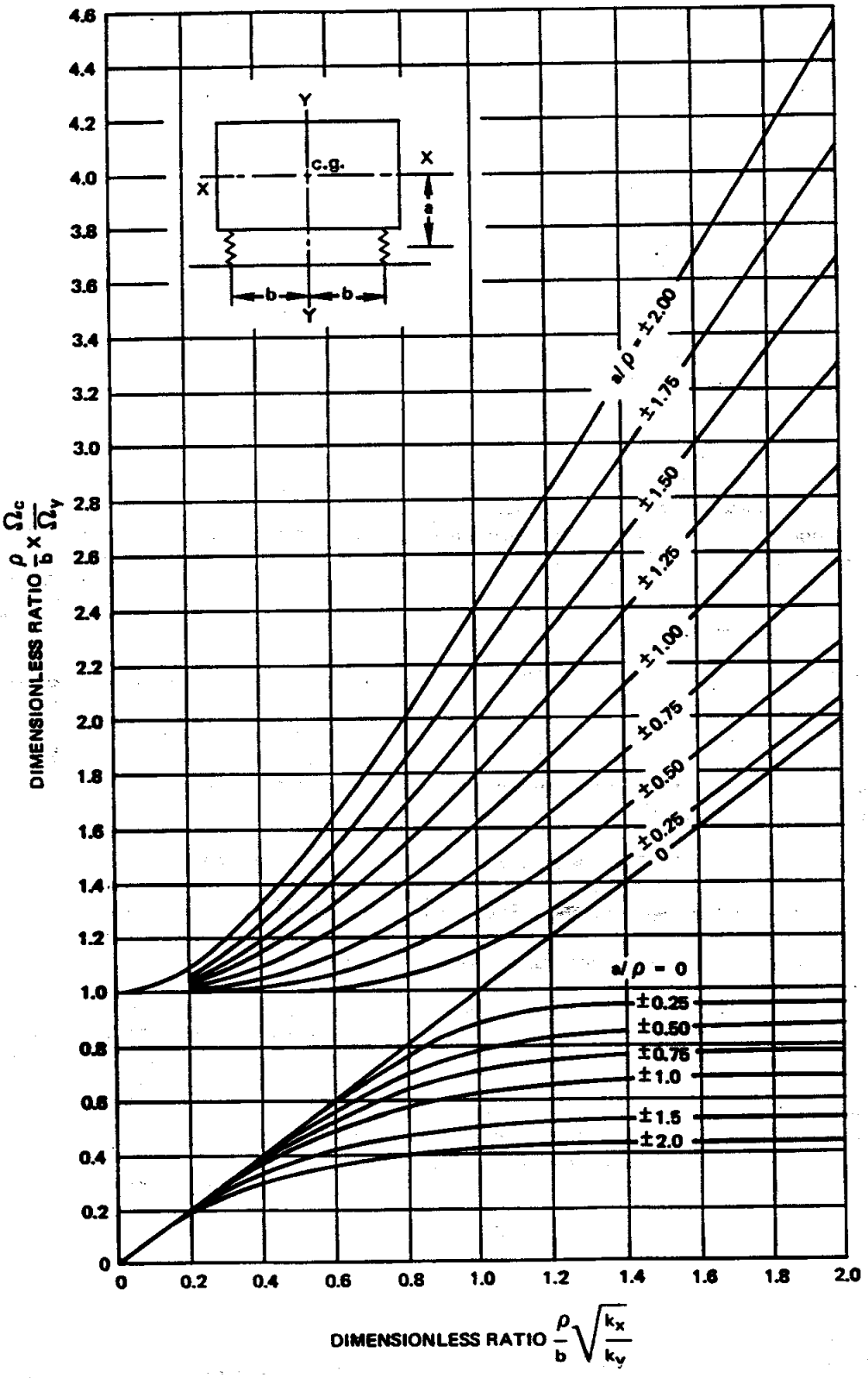


Figure 2-13. Comparison of Coupled and Decoupled Mode Frequencies.

Where c is the horizontal distance from the c.g. to the isolators in the Z axis and b is shown in Figure 2-13. Equation 10 can be put in terms of the translational natural frequency in the Y direction by substituting $k_x = k_z = k = \eta k_y$ and $I_y = m\rho_y^2$

$$\frac{\Omega_\beta}{\Omega_y} = \sqrt{\frac{\eta (b^2 + c^2)}{\rho_y^2}} \qquad (11)$$

and

$$\Omega_y = \sqrt{\frac{4 k_y}{m}} \qquad (12)$$

The application of mechanical admittance (reciprocal of impedance) data to the numerical solution of a vibration-isolation problem is treated in Reference 2. A complete explanation of this procedure is beyond the scope of this manual; however, the principles are reviewed.

The procedure is limited to the selection of cg isolation systems and is based upon the assumptions that:

a. The equipment behaves as a rigid mass.

b. The supporting structure is not rigid.

c. The vibration isolators are linear, massless, spring-dashpot elements.

d. There is mainly translational vibration in each of three mutually-perpendicular directions. The susceptibility to failure of the equipment can be described by random and/or sinusoidal fragility curves.

The data listed in Figure 2-14 are required to carry out all steps of the procedure. The weight of the equipment, the fragility levels of the equipment, the free vibration of the supporting structure, and the mechanical admittance of the supporting structure are required to determine the need for isolation. The last four items in Figure 2-14 are needed only if it is established that isolation is required. The fragility of the equipment, the free vibrations of the supporting structure, and the mechanical admittance of the supporting structure may be presented in graphical form.

The first step in the procedure is the calculation and the plotting of the real and imaginary normalized admittances $\overline{A}_s$ and $\overline{B}_s$ as a function of frequency (f). This is accomplished by inserting the weight of the equipment, W, along with the values of the original admittance data into the equations:

$$\overline{A}_s(f) = 2\pi W f\, A_s(f)/g \qquad (13)$$

$$\overline{B}_s(f) = 2\pi W f\, B_s(f)/g \qquad (14)$$

The next step in the procedure is the calculation and the plotting of the maximum-allowable transfer function, H_m. This transfer function is an upper limit based on the fragility vibration magnitude which the equipment can withstand.

$$H_{ms}(f) = \left| \frac{U_f(f)}{U_s(f)} \right| \qquad \text{for the Sinusoidal Case} \qquad (15)$$

$$H_{mr}(f) = \sqrt{\frac{S(U_f, f)}{S(U_s, f)}} \qquad \text{For the Random Case} \qquad (16)$$

Where:

$U_f(f)$ is the amplitude of sinusoidal input displacement (in inches) expressing the fragility limit of the equipment.

$U_s f)$ is the amplitude of sinusoidal output displacement (in inches) of the unloaded vibration source.

$S(U_f, f)$ is the spectral density of the displacement (in^2/Hz) which the equipment can withstand as a function of frequency.

$S(U_s, f)$ is the spectral density of the displacement (in^2/Hz) of the unloaded source.

NOTE:
The fragility of the equipment and the free vibration of the source may also be expressed in terms of velocity or acceleration provided that the same units are used consistently in the above equations.

The H_m curve, plotted as a result of the above procedure, is a composite which exhibits, at each frequency, the smaller of the two ordinates of the H_{ms} or H_{mr} curves.

Another transfer function, H_o, is then calculated and plotted on the same graph with H_m. H_o is defined as the absolute value of the ratio of the velocity of the load mass to the free velocity of the source when the mass is attached directly to the source or:

$$H = \left\{ \left[1 - \overline{B}_s(f) \right]^2 + \overline{A}_s^2(f) \right\}^{-\frac{1}{2}} \qquad (17)$$

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WESTERN DIVISION

VIBRATION, SHOCK, AND ACOUSTICS

BY _____
DATE _____
PAGE ___ OF _____

DESIGNATION OF EQUIPMENT: _____

WEIGHT OF EQUIPMENT: _____

DATA*		X-DIRECTION	Y-DIRECTION	Z-DIRECTION
FRAGILITY OF EQUIPMENT	SINUSOIDAL			
	RANDOM			
FREE VIBRATION OF SUPPORTING STRUCTURE	SINUSOIDAL			
	RANDOM			
MECHANICAL ADMITTANCE OF SUPPORTING STRUCTURE				
**				
MAXIMUM LOW-FREQUENCY ACCELERATION				
MAXIMUM ALLOWABLE RELATIVE DISPLACEMENT				
MINIMUM MEAN TIME BETWEEN EXCESSIVE DISPLACEMENTS				
RADIUS OF GYRATION				

SAMPLE

*IF ANY DATA REQUIRE AN EXTENSIVE DESCRIPTION OR A PLOT, NOTE THE DATA SHEET NUMBER WHICH CONTAINS THIS INFORMATION.

**DATA BELOW THIS LEVEL REQUIRED ONLY AFTER THE NEED FOR ISOLATION IS ESTABLISHED.

SKETCH OF EQUIPMENT OUTLINE AND MOUNTING, INCLUDING THE LOCATION OF THE C.G. AND THE PRINCIPAL AXES X, Y, Z:

COMMENTS:

Figure 2-14. Data Sheet for Design Procedure.

Figure 2-15 (see also Reference 2) is provided for calculating H_o values. The peaks and valleys in Figure 2-15 are determined by inspection of the normalized mechanical-admittance curves and are located where $B_s(f)$ equals one. If $H_o(f)$ is less than $H_m(f)$ at all frequencies for each of the x, y, z directions, no isolation is required and the procedure stops. If $H_o(f)$ is greater than $H_m(f)$ at any frequency for any of the x, y, z directions, isolation must be provided and the procedure continues.

Assuming that isolation is required, the next step in the procedure is to determine the minimum natural frequency for the isolation system (assume an SDF system). An approximation to the transfer function, H_i of the desirable isolation system is provided by the family of transmissibility curves of Figure 2-10.

Figure 2-10, in the form of an overlay, is placed on the graph of $H_o(f)$ and $H_m(f)$ to establish approximate damping values for the first peak in the transfer function curve of the isolation system. The ordinate of H equal to one for the overlay is juxtapositioned on the H equals one ordinate of the $H_o(f)$ and $H_m(f)$ graph. With this juxtaposition maintained, the overlay is moved laterally. The largest and smallest values of ζ for which all of the corresponding curve in Figure 2-10 lies below H_m are noted. These values are plotted on a graph showing ζ_i (fraction of critical damping) versus the natural frequency, f_{ni}. These damping values are corrected by using Figure 2-16 and equations 18 and 19.

$$\zeta_i(\max) = \frac{1}{2}\sqrt{\frac{1}{\bar{B}_s(f_{ni})} - 1} \qquad (18)$$

$$D\left[\zeta_i(\min), 1\right] = \frac{2\zeta_i(\min)}{1 + 4\zeta_i^2(\min)} = \bar{A}_s(f_{ni}) \qquad (19)$$

or more generally,

$$D\left(\zeta, \frac{f}{f_n}\right) = \frac{2\zeta(f_n/f)}{(f_n/f)^2 + 4\zeta^2} \qquad (20)$$

The function $D(\zeta, f/f_n)$ is displayed graphically in Figure 2-17. In equation 19 the value of f/f_n is equal to 1. Normalized admittance data are entered in Figure 2-16, and the Figure 2-16 calculations are completed to yield maximum values of damping. The ζ (min) values are obtained from Figure 2-17 by using $D = \bar{A}_s$ and $f/f_n = 1$. These new values of damping limits for the first peak in the transfer function curve of the isolation system are used to correct the graph of critical damping versus natural frequency.

Additional limiting values of ζ_i, based on the maximum permitted displacement and relative displacement across the isolators, are obtained by consulting Figure 2-18.

Figure 2-18 is utilized to permit the computations indicated in equations 21 and 22. At each value of f_{ni} the value of ζ_i must be greater than ζ_{is} or ζ_{ir}, whichever is larger. The parameter ζ_{is} limits the displacement of the equipment during sinusoidal vibration at the frequency f_{ni}. The parameter ζ_{ir} limits the displacement of the equipment during random vibration assuming the system has a single resonance at the frequency f_{ni}. For values of these particular values of ζ_i less than 0.3, the following approximate formulas are valid:

$$\zeta_{is}(f_{ni}) = \frac{\sqrt{\bar{A}_s^2(f_{ni}) + [T_{is}^2(f_{ni}) - 1]G(f_{ni})} - \bar{A}_s(f_{ni})}{2[T_{is}^2(f_{ni}) - 1]} \qquad (21)$$

$$\zeta_{ir}(f_{ni}) = \frac{E_r(f_{ni}) + \bar{A}_s(f_{ni})}{2}\left\{1 - \sqrt{1 - \frac{G(f_{ni})}{[E_r(f_{ni}) + \bar{A}_s(f_{ni})]^2}}\right\} \qquad (22)$$

where:

$$T_{is}(f_{ni}) = \frac{4\pi^2 f_{ni}^2 \zeta_m}{g\,|\ddot{U}_s(f_{ni})|} \qquad (23)$$

$$G(f_{ni}) = [1 - \bar{B}_s(f_{ni})]^2 + \bar{A}_s^2(f_{ni}) \qquad (24)$$

$$E_r(f_{ni}) = \frac{16\pi^3 f_{ni}^3 \bar{\zeta}_m^2}{g^2 S(U_s, f_{ni})} \qquad (25)$$

$$\bar{\zeta}_m^2 = \frac{\zeta_m^2}{4.61 \operatorname{Log}_{10}(2 f_{ni} \Delta t)} \qquad (26)$$

$\bar{\zeta}_m^2$ = mean squared displacement

ζ_m = maximum permitted displacement

Δt = mean time between successive occurrences of the maximum displacement

$$H_o = \left\{ \left[1 - \bar{B}_s(f) \right]^2 + \bar{A}_s^2(f) \right\}^{-1/2}$$

A. DETERMINATION OF FREQUENCIES AT WHICH TO CALCULATE H_o:

f_ℓ = f_u =

MULTIPLES OF CENTER COLUMN					PEAKS AND VALLEYS	MULTIPLES OF CENTER COLUMN				
0.1	0.3	0.5	0.7	0.9		1.1	1.4	2	6	50

SAMPLE

B. VALUES OF H_o AT ABOVE FREQUENCIES:

① f (Hz)	② $\bar{A}_s(f)^*$	③ $\bar{B}_s(f)$	④ $\lvert 1 - ③ \rvert^{**}$	⑤ $④^2$	⑥ $②^2$	⑦ ⑤ + ⑥	⑧ $1/\sqrt{⑦}$

SAMPLE

* COLUMN ⑧ = 1/ ② FOR CHECKED FREQUENCIES

** IF $\bar{A}_s$ <0.1, COLUMN ⑧ = 1/ ④

Figure 2-15. Calculation of H_o.

$$\zeta_i\,(\max) = \frac{1}{2}\sqrt{\frac{1}{\bar{B}_s\,(f_{ni})} - 1}$$

$$D\,[\,\zeta_i\,(\min),\ 1\,] = \frac{2\,\zeta_i\,(\min)}{1 + 4\,\zeta_i^2\,(\min)} = \bar{A}_s\,(f_{ni})$$

① f_{ni} (Hz)	② $\bar{B}_s(f_{ni})$	③ 1/②	④ ③ − ①	⑤ $\sqrt{④}/2$ ζ_i (max)	⑥ $\bar{A}_s(f_{ni})$	⑦ ζ_i (min)*

SAMPLE

*If $\bar{A}_s\,(f_{ni}) \leq 0.15$, $\zeta_i\,(\min) = \dfrac{\bar{A}_s(f_{ni})}{2}$

If $\bar{A}_s\,(f_{ni}) > 0.15$, USE FIGURE 2-17

If $\bar{A}_s\,(f_{ni}) > 0.44$, INSERT "NO LIMIT"

Figure 2-16. Limiting Values of ζ_1 Based on Reliability with Spring-Dashpot Isolators.

The values of ζ_{is} and ζ_{ir} are used to again modify the graph of critical damping versus natural frequency for the first peak in the transfer function curve of the isolation system. All values of ζ_i for each f_{ni}, which occur below ζ_{is} or ζ_{ir}, do not meet the displacement limitations on the equipment and are not acceptable values for the isolation system.

The preceding analysis has provided damping values for low frequencies in the area of the first peak of the transfer function curve of the isolation system. Equations 27 through 30 (from the procedure) are used to establish f_n and ζ for the isolator-equipment combination. Data from the preceding analysis are entered into equations 27 through 30. Figure 2-19 from the procedure, is provided to facilitate the calculations.

$$\bar{A}_i\,(f_{ni}) = D(\zeta_i,1) \cdot \bar{A}_s\,(f_{ni}) \qquad (27)$$

$$\bar{B}_i\,(f_{ni}) = \frac{1}{2\zeta_i}\ D(\zeta_i,1) \cdot \bar{B}_s\,(f_{ni}) \qquad (28)$$

$$f_n = \sqrt{\frac{\bar{B}_i\,(f_{ni})}{\bar{A}_i^2\,(f_{ni}) + \bar{B}_i^2\,(f_{ni})}}\,(f_{ni}) \qquad (29)$$

$$\zeta = \frac{1}{2}\,\frac{f_n}{f_{ni}}\,\frac{\bar{A}_i\,(f_{ni})}{\bar{B}_i\,(f_{ni})} \qquad (30)$$

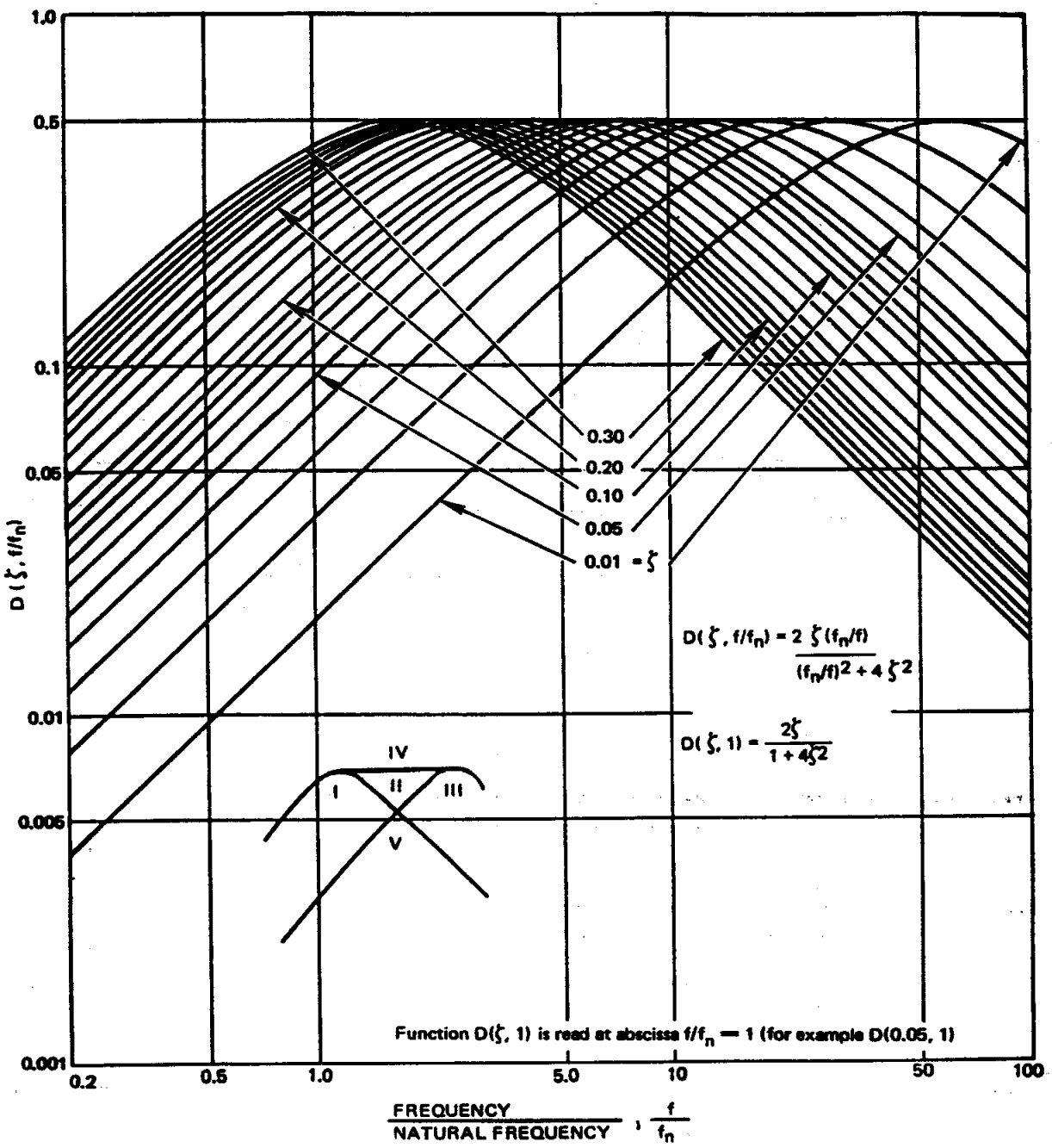


Figure 2-17. Plot of Function D (ζ, f/f_n) for Constant ζ.

$$\zeta_{is}(f_{ni}) = \frac{\sqrt{\bar{A}_s^2(f_{ni}) + \left[T_{is}^2(f_{ni}) - 1\right] G(f_{ni})} - \bar{A}_s(f_{ni})}{2\left[T_{is}^2(f_{ni}) - 1\right]}$$

$$\zeta_{ir}(f_{ni}) = \frac{E_r(f_{ni}) + \bar{A}_s(f_{ni})}{2} \left\{ 1 - \sqrt{1 - \frac{G(f_{ni})}{\left[E_r(f_{ni}) + \bar{A}_s(f_{ni})\right]^2}} \right\}$$

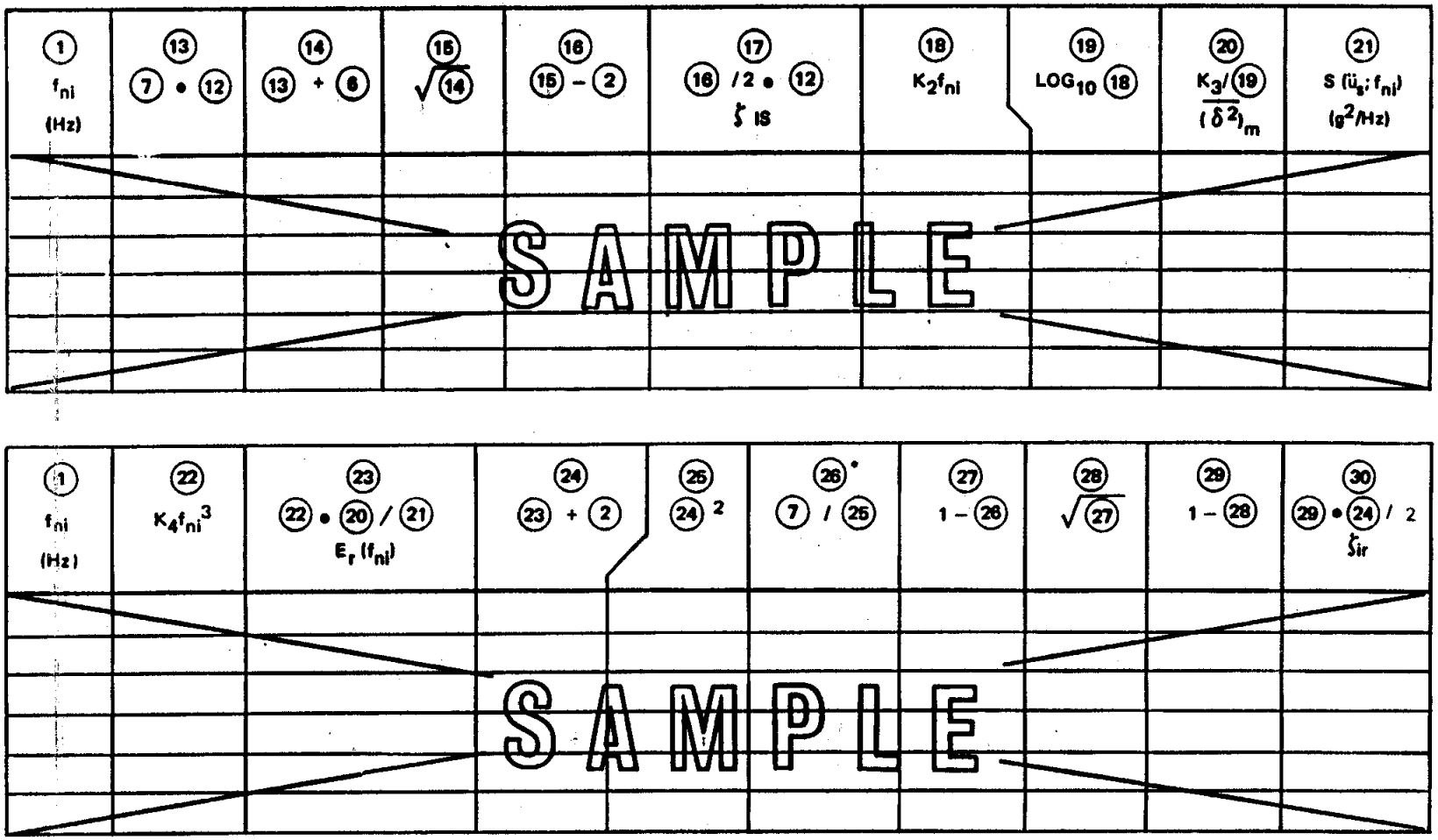
δ_m = _____ $K_1 = 4\pi^2 \delta_m/9 = 0.102\,\delta_m$ = _____ $K_3 = \delta_m^2/4.61$ = _____

Δt = _____ $K_2 = 2\,\Delta t$ = _____ $K_4 = 16\pi^3/g^2 = 0.00333$

(1) f_{ni} (Hz)	(2) $\bar{A}_s(f_{ni})$	(3) $\bar{B}_s(f_{ni})$	(4) $\lvert 1 - (3) \rvert$	(5) $(4)^2$	(6) $(2)^2$	(7) (5) + (6) $G(f_{ni})$	(8) $U_s(f_{ni})$ (g)	(9) $K_1 f_{ni}^2$	(10) (9) / (8) $T_{is}(f_{ni})$	(11) $(10)^2$	(12) (11) − (1)

SAMPLE

Figure 2-18. Limiting Value of ζ_1 Based on Displacement Limitations (Sheet 1 of 2).



*IF COLUMN ㉘ ≤ 0.15, ㉙ = ㉘/2

Figure 2-18. Limiting Value of ζ_1 Based on Displacement Limitations (Sheet 2 of 2).

$$\bar{A}_i(f_{ni}) = D(\zeta_i, 1) - \bar{A}_s(f_{ni});$$

$$\bar{B}_i(f_{ni}) = \frac{1}{2\zeta_i} D(\zeta_i, 1) - \bar{B}_s(f_{ni})$$

$$f_n = \sqrt{\frac{\bar{B}_i(f_{ni})}{\bar{A}_i^2(f_{ni}) + \bar{B}_i^2(f_{ni})}}\, f_{ni};$$

$$\zeta = \frac{1}{2} \frac{f_n}{f_{ni}} \frac{\bar{A}_i(f_{ni})}{\bar{B}_i(f_{ni})}$$

① f_{ni} (Hz)	② ζ_i	③ $D(\zeta_i, 1)$ (FIG 2-17)	④ $\bar{A}_s(f_{ni})$	⑤ ③ − ④ $\bar{A}_i(f_{ni})$	⑥ ③/2 · ②	⑦ $\bar{B}_s(f_{ni})$	⑧ ⑥ − ⑦ $\bar{B}_i(f_{ni})$	⑨ ⑤²	⑩ ⑧²	⑪ ⑨ + ⑩	⑫ ⑧/⑪	⑬ $\sqrt{⑫}$	⑭ ① · ⑬ f_n	⑮ ⑬ · ⑤/2 · ⑧ ζ

SAMPLE

Figure 2-19. Boundary Values of f_n, ζ Parameters.

Where $\bar{A}_i$ (f_{ni}) and $\bar{B}_i$ (f_{ni}) are the real and imaginary parts of the normalized admittance of the isolators themselves at the frequency f_{ni}. The natural frequency of the isolator-equipment combination is defined by f_n in equation 29 and the fraction of critical damping for the isolator-equipment combination is expressed in equation 30. The f_n and ζ values from Figure 2-19 are plotted on a graph of ζ vs. f_n and the graph is titled: "Region of Permissible Parameters f_n, ζ. For the Isolation System". Whereas the isolator parameters found above have been in the area of the first peak, it becomes necessary to investigate the higher frequencies. The possibility that the transfer function H_i exceeds H_m, the maximum transfer function, is examined. The upper bound to the transfer function H_i is given by equation 31:

$$H_m(f) = \frac{1}{\bar{A}_i(f) + \bar{A}_s(f)} = \frac{1}{(f/f_n)^2\, D(\zeta,\ f/f_n) + \bar{A}_s(f)} \quad (31)$$

Figure 2-20 has been set up to facilitate the solution of equation 31 for the function of $D(\zeta, f_c/f_n)$ as shown at the top of the table. The determination of D in Figure 2-20 should be accomplished at several frequencies, f_c, to check the isolator system design in the higher frequencies. Information in column 10 is found by using

$$D(\zeta, f_c/f_n) = \frac{f_n^2}{f_c^2}\left[\frac{1}{H_m(f_c)} - \bar{A}_s(f_c)\right]$$

① f_c (Hz)	② $H_m(f_c)$	③ 1/②	④ $\bar{A}_s(f_c)$	⑤ ③ − ④	⑥ f_n	⑦ f_c/f_n	⑧ ⑤ / ⑦^2 D	⑨ REGION FIG. 2-17	⑩ SATISFACTORY ζ

SAMPLE

Figure 2-20. Limits of Suspect f_n, ζ Designs.

the regions of column 9 according to the following scheme:

Column 9 Region in Figure 2-17	Column 10 Satisfactory	
I	> _____	Values of ζ from 2-17
II	___ to ___	
III	< _____	
IV	None	
V	All	

The satisfactory data are used to correct the graph of f_n versus ζ.

Next, a trial design is selected within the region of permissible parameters that allows a variation in natural frequency of at least ±20%, that permits a variation of ±20% in fraction of critical damping, and is feasible by the current state-of-the-art in the production of vibration isolators.

It is necessary to check this trial design to see that the isolator transfer function does not exceed H_m, the maximum allowable transfer function. The check of the nominal design is accomplished by using Figure 2-21 and Figure 2-22. The peaks and valleys of the isolator transfer function are determined by using Figure 2-21 as an overlay on the plot of normalized mechanical admittance. The frequencies at the peaks and valleys of the $\overline{H}_i$ are found at the frequencies where $1-\overline{B}_i(f)$ equals $\overline{B}_s(f)$.

Figure 2-22 provides a convenient form for efficiently computing H_i, the transfer function of the isolator system:

$$H_i(f) = \left| [1 - \overline{B}_s(f) - \overline{B}_i(f)]^2 + [\overline{A}_s(f) + \overline{A}_i(f)]^2 \right|^{-1/2} \quad (32)$$

If all values of $H_i(f)$ are less than the maximum allowable transfer function H_m at all frequencies, the trial design is considered satisfactory. The trial design may be checked for rotational natural frequencies by using equation 6. Two criteria from the procedure are applied to these rotational natural frequencies. These rotational frequencies are considered harmful if they fall within $0.7\ (1 - \Delta f/f_n)$ to $1.25\ (1 + \Delta f/f_n)$ of any critical frequency or if they fall below $f_n(\text{min})$. This completes the summary of the numerical procedure presented in Reference 2.

2-2.2.1 References

1. Ruzicka, J. E., "Vibration Control, Electro-Technology Science and Engineering Series," August 1963.
2. "Design of Vibration Isolation Systems," Advances in Engineering, Volume 3, SAE Committee G-5 Aerospace Shock and Vibration, Society of Automotive Engineers, Inc., 1962.
3. C. T. Molloy, et. al., "A Rational Procedure for the Design of Vibration Isolation Systems," SP-223, Society of Automotive Engineers, Inc., October 1961.
4. C. E. Crede, "Vibration and Shock Isolation," John Wiley and Sons, Inc. New York, 1951.
5. C. E. Crede, "Vibration and Shock Isolators", Machine Design August 1964.
6. C. T. Morrow and R. B. Muchmore, "Shortcomings of Present Methods of Measuring and Simulating Vibration Environments," Journal of Applied Mechanics, September 1955.

2-2.3 Damping Materials and Definition

See Tables 2-2 through 2-5, and Figure 2-23.

2-2.4 Damping Nomographs

a. The nomographs (Figure 2-24 through 2-29) presented in this section, have been derived for the purpose of obtaining the reduction in response of strips, flat plates, and curved plates resulting from the addition of damping compounds. The response parameters considered are displacement, velocity and acceleration while the forcing functions are both broad-band and discrete.

   These graphs do not apply to a mass-loaded plate and should not be used when this situation exists.

b. Instructions

   1. Enter the nomograph on the δ scale (line A) with the damping ratio of the basic plate. (C/Critical = δ).
   2. Trace left on a horizontal line from δ to the value of Young's Modulus (E_m) of the basic structure.
   3. From the intersection of E_m and δ draw a line parallel to the slanting lines up to vertical line B.
   4. Trace horizontally to the left from this point (found in step 3) to the desired reduction expressed as a ratio (q_1/q_2, $\dot{q}_1/\dot{q}_2$ $\ddot{q}_1/\ddot{q}_2$, $\overline{q}_1/\overline{q}_2$ $\overline{\dot{q}}_1/\overline{\dot{q}}_2$ or $\overline{\ddot{q}}_1/\overline{\ddot{q}}_2$ depending on the graph used).
   5. From this new point, draw a line parallel to the slanted lines down to line C.

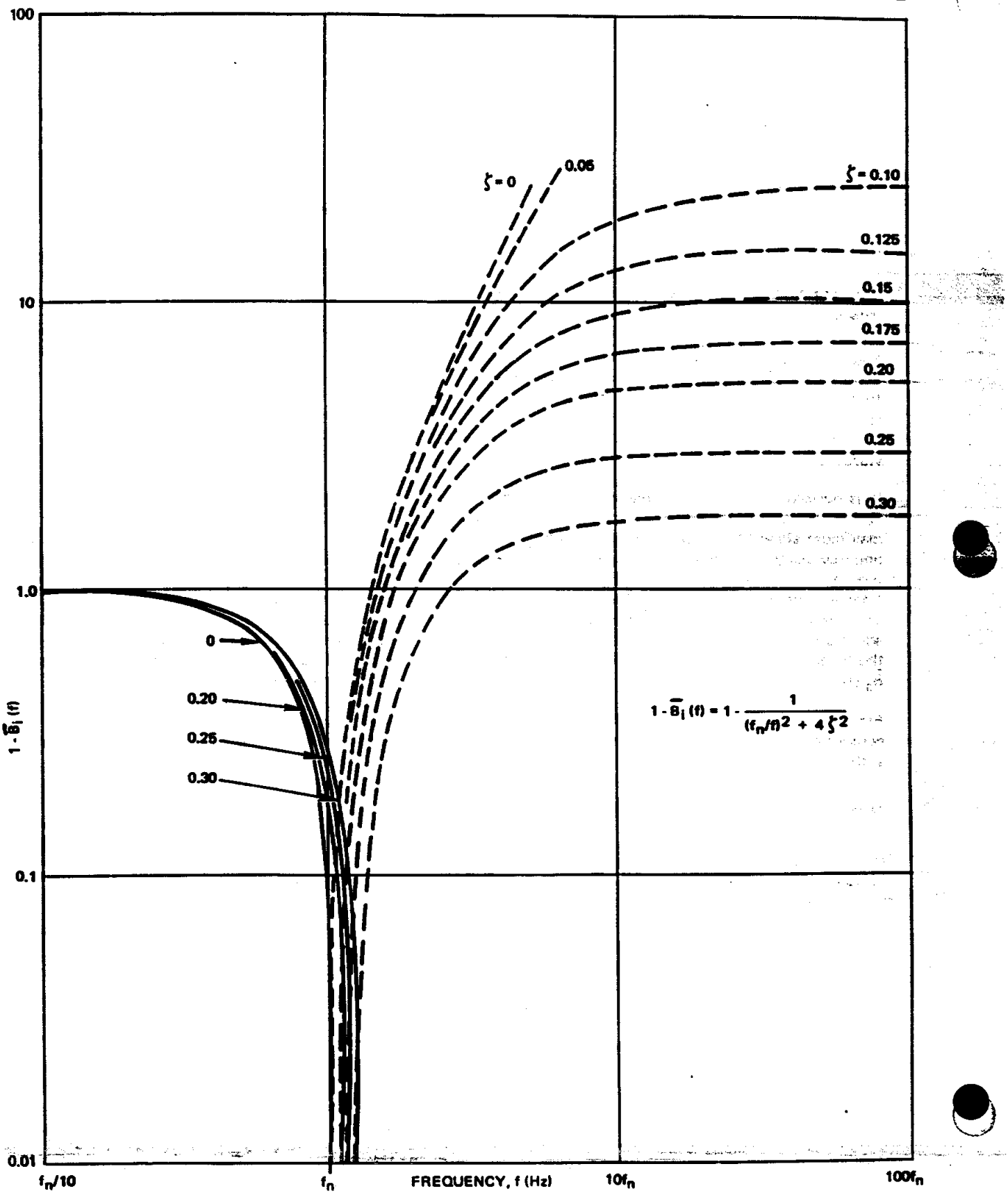


Figure 2-21. Plot of $1-\bar{B}_i$ (f), Where B_i (f) is the Normalized Value of the Imaginary Part of an Isolator's Admittance

$$\bar{A}_i(f) = \frac{f^2}{f_n^2} D(\zeta, f/f_n); \qquad \bar{B}_i(f) = \frac{1}{2\zeta} \frac{f}{f_n} D(\zeta_i, f/f_n)$$

$$H_i(f) = \left| \left[1 - \bar{B}_s(f) - \bar{B}_i(f) \right]^2 + \left[\bar{A}_s(f) + \bar{A}_i(f) \right]^2 \right|^{-1/2}$$

ISOLATION SYSTEM DESIGN: f_n = _____ H_z ζ = _____

A. DETERMINATION OF FREQUENCIES AT WHICH TO CALCULATE H_i:

f_u = _____ f_ℓ = _____

MULTIPLES OF CENTER COLUMN					PEAKS AND VALLEYS	MULTIPLES OF CENTER COLUMN				
0.1	0.3	0.5	0.7	0.9		1.1	1.4	2	6	50

SAMPLE

B. VALUES OF H_i AT ABOVE FREQUENCIES:

(1) f (Hz)	(2) f/f_n	(3) D	(4) (2)^2·(3) $\bar{A}_i$	(5) $\bar{A}_s$	(6)* (4) + (5) $\bar{A}_s + \bar{A}_i$	(7) (2)(3)/2ζ $\bar{B}_i$	(8) $\bar{B}_s$ (FIG.1)	(9) (8) + (7) $\bar{B}_s + \bar{B}_i$	(10) $\lvert 1 - (9) \rvert$**	(11) (10)^2	(12) (6)^2	(13) (11) + (12) $1/H_i^2$	(14) $1/\sqrt{(13)}$ H_i

SAMPLE

*COLUMN (14) = 1/(6) FOR CHECKED FREQUENCIES

**IF $(\bar{A}_s + \bar{A}_i) < 0.1$, COLUMN (14) = 1/(10)

Figure 2-22. Calculation of H_1.

Table 2-2. Types of Damping Phenomena

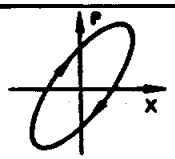
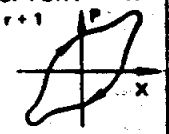
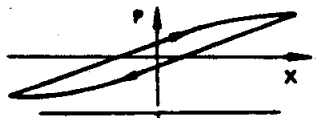
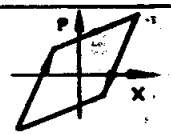
HEADING		RATE-DEPENDENT DAMPING: TYPE R		RATE-INDEPENDENT DAMPING: TYPE S		
1. DISTINGUISHING CHARACTERISTICS OF CYCLIC P-X LOOP.		DEPENDS ON LOAD RATE $\dot{P}$, $\ddot{P}$, ETC. DISPLACEMENT RATE $\dot{X}$. ALSO DEPENDS ON CYCLIC LOAD AMPLITUDE. P-X LOOP ELLIPTICAL OR DEGENERATE FORMS THEREOF.		DEPENDS ONLY ON CYCLIC LOAD AMPLITUDE. INDEPENDENT OF RATE OF LOAD OR DISPLACEMENT. P-X LOOP POINTED AT MAXIMUM VALUES AND MAY HAVE OTHER SHARP CORNERS.		
2. NATURE OF P-X LAWS		INCLUDE TIME DERIVATIVES OF P OR X.		NON-LINEAR EQUATION WITHOUT TIME DERIVATIVES OF P OR X.		
3. OTHER NAMES USED TO IDENTIFY TYPE OF DAMPING.		DYNAMIC HYSTERESIS, RHEOLOGICAL DAMPING, ANELASTICITY, INTERNAL FRICTION, COMPLEX DAMPING.		STATIC HYSTERESIS, HYSTERETIC DAMPING, PLASTIC STRAIN DAMPING.		
4. EXAMPLES OF MICRO-MECHANISMS AND MACRO-MECHANISMS WHICH DISSIPATE ENERGY.	(a) IN MATERIALS AND COMPOSITES.	IN METALS: ANELASTIC PHENOMENA (SOLUTE ATOM, GRAIN BOUNDARY AND THERMAL CURRENT EFFECTS) IN POLYMERS: FLOW AND INTERACTIONS OF SEGMENTED MOLECULES		IN METALS: PLASTOELASTICITY, MAGNETOELASTICITY, MICROSLIP AND SLIDE AT INCLUSIONS. IN NON-METALS: SLIP AND SLIDE BETWEEN MICRO- AND MACROCONSTITUENTS.		
	(B) IN STRUCTURES.	FLUID FLOW (ACOUSTICAL AND VISCOUS EFFECTS). ELECTROMAGNETIC EFFECTS, EDDY CURRENTS.		SLIP AND SLIDE AT INTERFACES IN STRUCTURAL AND MACHINE JOINTS. IMPACT EFFECTS IN JOINTS HAVING GAPS		
5. SPECIAL CASES AND VALUE OF n IN $D = J\sigma_a^n$		RATE-DEPENDENT PROPORTIONAL OR QUADRATIC. TYPE RP (n = 2)	RATE-DEPENDENT NON-PROPORTIONAL DAMPING. TYPE RN (n ≠ 2)	RATE-INDEPEN. PROPORTIONAL, QUADRATIC. TYPE SP (n = 2)	RATE-INDEPENDENT, NON-PROPORTIONAL DAMPING. TYPE SN (n ≠ 2)	
6. CHARACTERISTIC SHAPES OF HYSTERETIC LOOP AND TYPICAL VALUES FOR n.		 ELLIPSE FOR LINEAR DAMPING.	LOOP FOR $\dot{P}^r \propto \dot{X}^r$ $n = r + 1$  ELASTOMERS AT LARGE ϵ_a MAY HAVE ELLIPTICAL LOOP.	MATERIALS  n FOR PLASTOELASTICITY. 2–3 IF $\sigma < \sigma_g$ 2–30 IF $\sigma > \sigma_g$	n FOR MAGNETOELASTICITY. 3 IF $\sigma < \sigma_h$ 0 IF $\sigma > \sigma_h$	STRUCTURES  LOOP FOR PARTICULAR BEAM (n = 3)
7. ENGINEERING EXAMPLES OF PARTICULAR TYPE OF DAMPING.		METALS AT LOW STRESS DUE TO ANELASTICITY PHENOMENA. POLYMERS AT LOW AND INTERMEDIATE STRAIN.	POLYMERS AND ELASTOMERS AT HIGH STRAIN. STRUCTURES AND MACHINES HAVING RATE-DEPENDENT COMPONENTS.	METALS IN INTERMEDIATE STRESS. STRUCTURAL JOINTS HAVING CERTAIN TYPES OF INTERFACE SLIDE AND SLIP	METALS AT HIGH STRESS DUE TO PLASTOELASTIC MICROSLIP METALS HAVING MAGNETOELAS EFFECTS	STRUCTURAL JOINTS HAVING CERTAIN TYPES OF INTERFACE SLIDE AND SLIP. CONTACT STRESS ACROSS INTERFACE. ROLLING "FRICTION"
8. EFFECTS OF NUMBER OF FATIGUE CYCLES		NO EFFECT.	CHANGE IN D PROBABLE.	PROBABLY NO CHANGE.	LARGE CHANGE IF $\sigma > \sigma_h$	CHANGES DUE TO DETERIORATION OF INTERFACES.
9. EFFECT OF STRESS ON LOSS COEFFICIENT. TYPICAL VALUES OF η.		$\eta \neq f(\sigma)$ ANELASTICITY η = 0.01 TO 0.1 VISCOELASTICITY η = 0.1 TO 1.5	$\eta = f(\sigma)$ η USUALLY INCREASES AS σ INCREASES. η USUALLY GREATER THAN FOR TYPE RP DAMPING.	$\eta \neq f(\sigma)$ η = 0.001 TO 0.05	FOR PLASTOELASTICITY: η INCREASES, PARTICULARLY ABOVE σ_g. η = 0.001 TO 0.05 AT σ_g η = 0.001 TO 0.1 FOR σ_g FOR MAGNETOELASTICITY: η INCREASES UP TO σ_h, THEN DECREASES η = 0.01 TO 0.1 AT σ_h η = 0.001 TO 0.05 FOR $\sigma > \sigma_h$ STRUCTURES. η = 0.001 TO 0.2	

Table 2-5. Characteristics of Aquaplas

% OF VIBRATION REDUCTION	TEMPERATURE RANGE (°F)	FREQUENCY RANGE (Hz)	RATIO OF COATING THICKNESS / METAL THICKNESS
80	100	400 – 10,000	1.3
80	68 – 194	400 – 10,000	1.8
80	86 – 158	40 – 8,000	1.8
80	32 – 200	40 – 10,000	2.2
50	100	400 – 10,000	.75
50	68 – 194	400 – 10,000	1.0
50	86 – 158	40 – 8,000	1.0
50	32 – 200	40 – 10,000	1.25
50	14 – 200	20 – 10,000	1.6

NOTE: THE MANUFACTURER HAS INDICATED THE ABILITY TO SHIFT THE OPTIMUM TEMPERATURE RANGE BY VARYING THE KIND AND QUAN TITY OF FILLER USED IN THE COMPOUND.

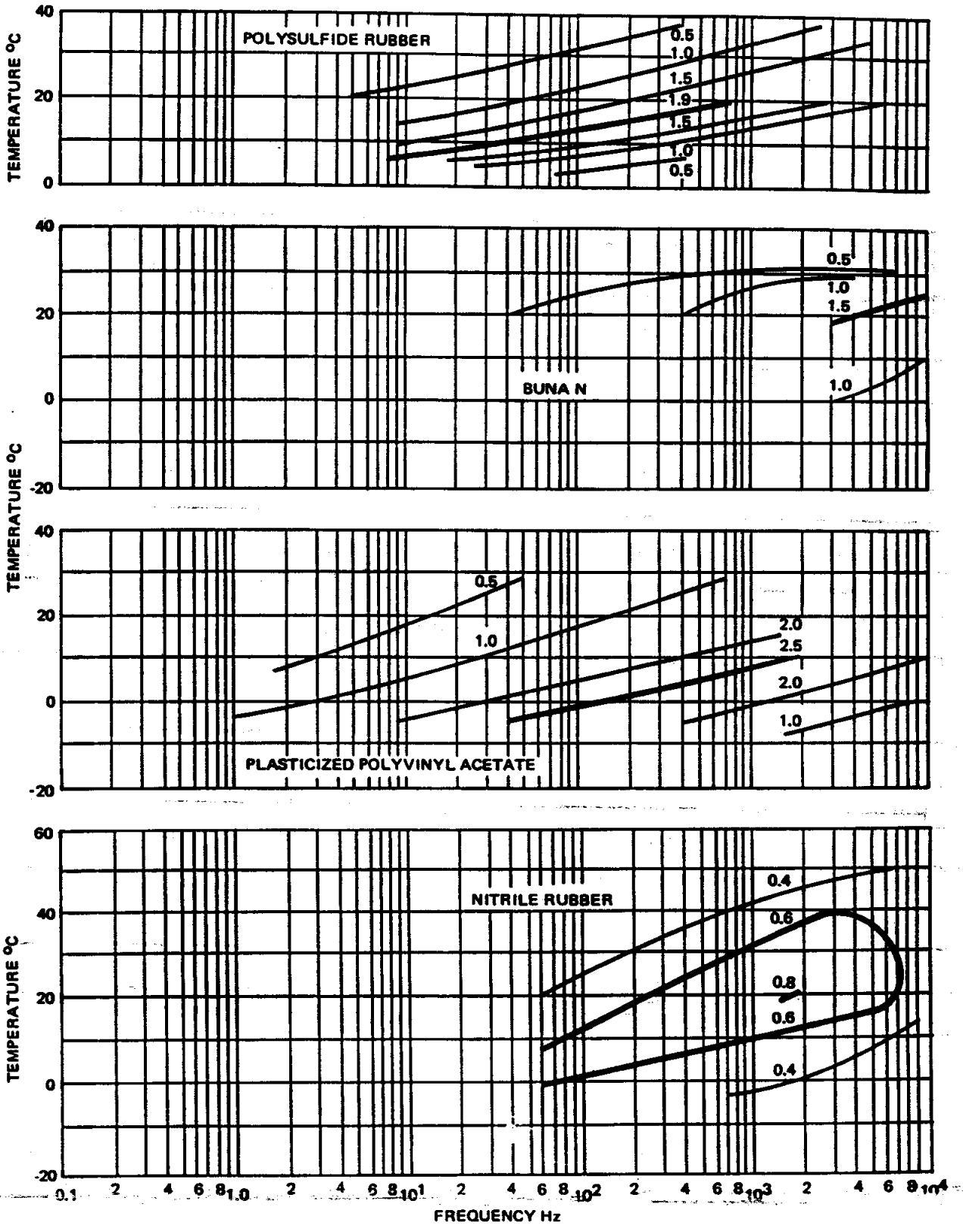


Figure 2-23. Contour Plots for High-Damping Materials (Sheet 1 of 5).

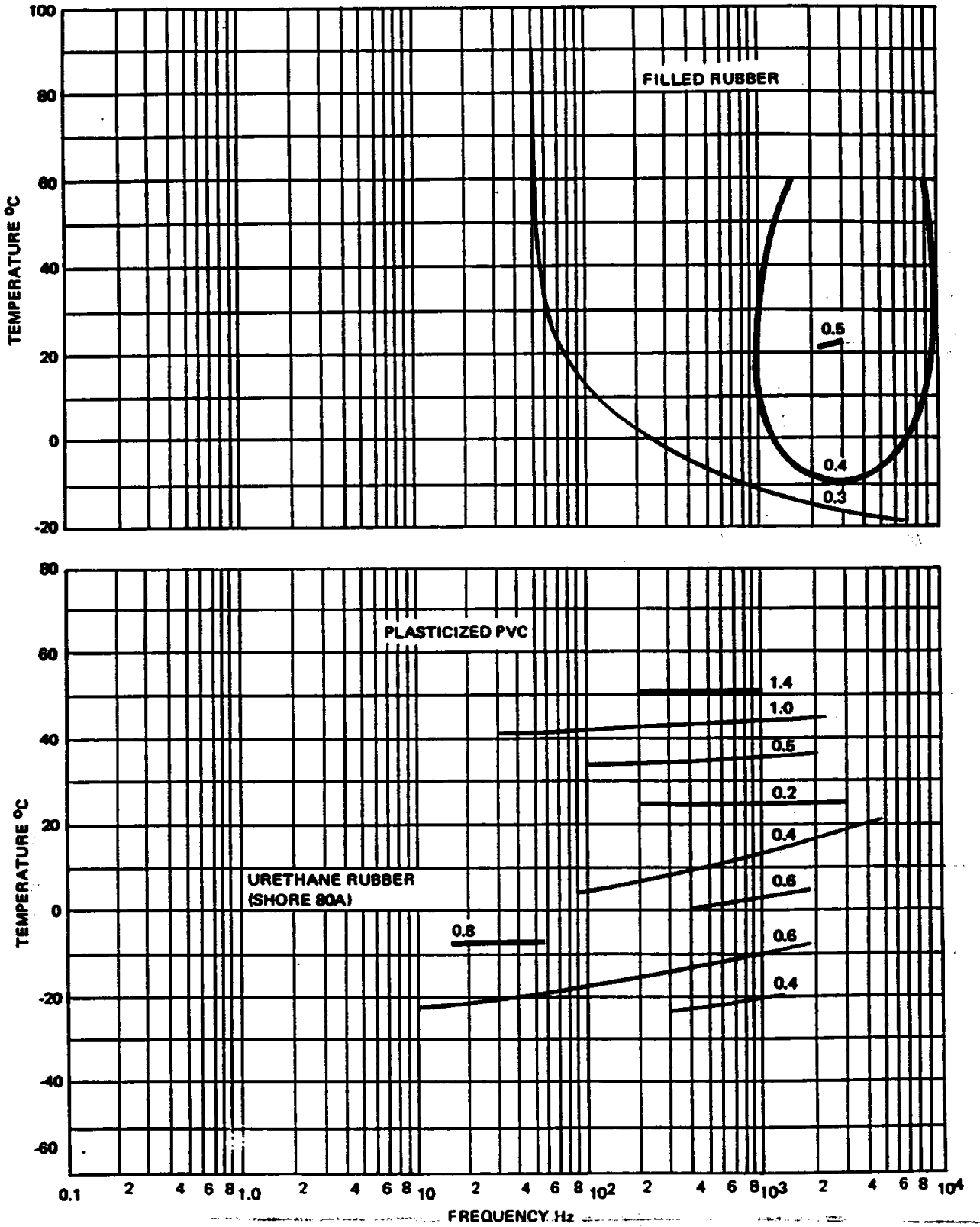


Figure 2-23. Contour Plots for High-Damping Materials (Sheet 2 of 5).

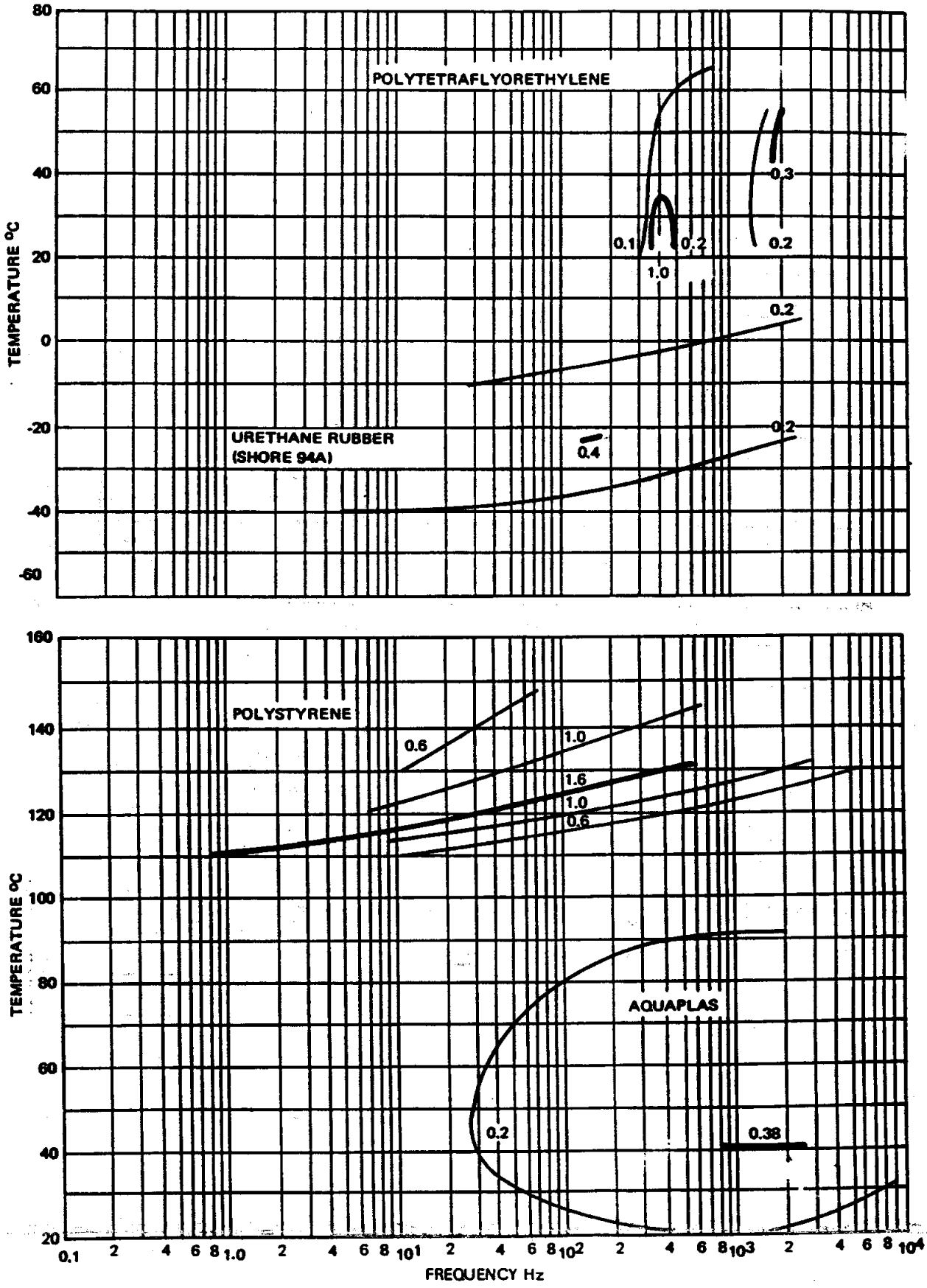


Figure 2-23. Contour Plots for High-Damping Materials (Sheet 3 of 5).

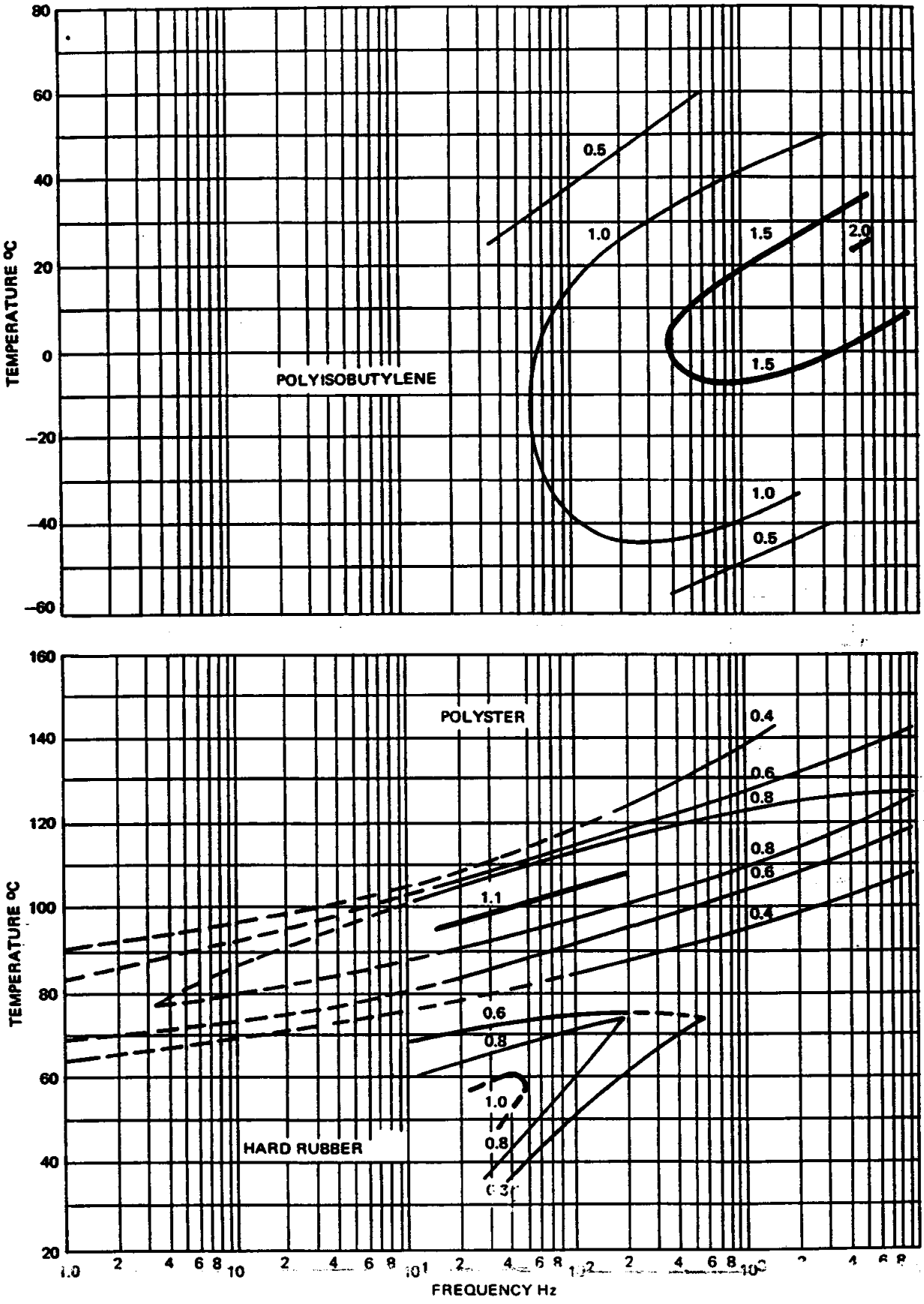


Figure 2-23. Contour Plots for High-Damping Materials (Sheet 4 of 5).

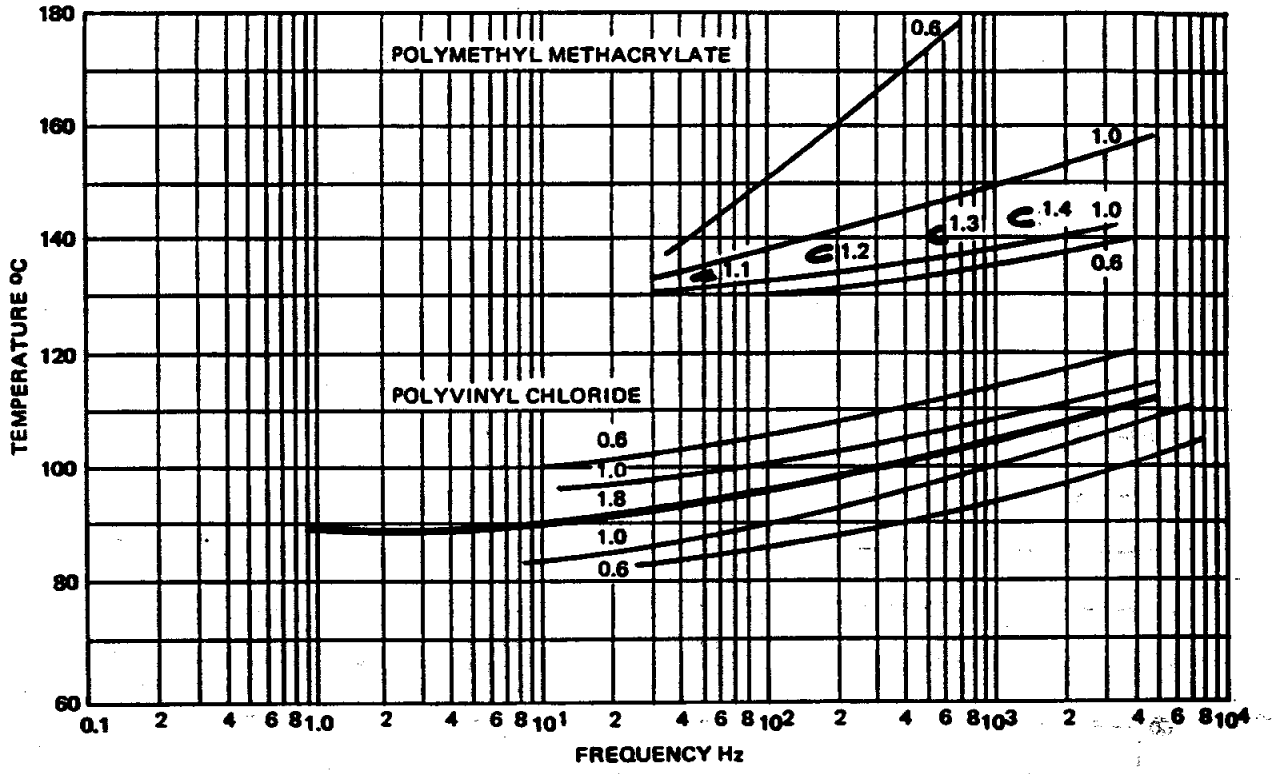


Figure 2-23. Contour Plots for High-Damping Materials (Sheet 5 of 5).

6. From the point on line C found in step 5, draw a perpendicular line down to the bottom of the answer graph.

7. Any point selected on the line drawn in step 6 (between lines C and D) corresponds to a compound thickness divided by the plate thickness (the "a" lines) for a specific value of $E_d\eta_d$ (Young's Modulus x Loss Factor for a damping compound). Some damping compounds and their $E_d\eta_d$ are listed in Tables 2-6 and 2-7.

CORRECTION 1 (Stiffness Correction)

8. From the point chosen in step 7, draw two lines, a horizontal line to the right until line F is reached, and a vertical line down to the line G.

9. From the two points found in 8, draw a vertical and horizontal line respectively to a point of intersection.

10. Draw a short line through this point parallel to the slanted lines.

11. Then draw a line from the same point used in step 10 to the point Z.

12. Enter the correction 1 graph on the E_m scale (line H) and follow the associated E_m line to the value of η_d (loss factor) for the damping compound chosen. (See Table 2-6).

13. From this point draw a line parallel to the X lines up to the line J.

14. From the intersection on the line J found in step 13, draw a line parallel to the line drawn in step 11 extending it until the short slanted line drawn in step 10 is intersected.

15. From the point of intersection draw a vertical line up to the line F and a horizontal line to the left to the line G. From the two points on line F and G, draw a horizontal (to the left) and a vertical (upward) lines respectively to a point of intersection.

Table 2-6. Dynamic-Property Data for Wide Range of Temperature and Frequency

(SUMMARIZES MOST USEFUL DATA FROM THE CONTOUR PLOTS)				
MATERIAL	MAX LOSS FACTOR, η_d	POSITION OF βMAX		SHEAR† MODULUS G_1 AT βMAX, PSI
		FREQ., Hz.	TEMP, °C	
1. PLASTICIZED POLYVINYL ACETATE	2.6	50	6	2,180
2. POLYSTYRENE	>2.0	>2000	~ 140	14,800
3. POLYISOBUTYLENE	>2.0	>6000	25	290
4. POLYSULFIDE RUBBER (THIOKOL RD.	1.9	7	5	10,100
5. POLYVINYL CHLORIDE	1.8	20	92	3,330
6. BUNA N (TYPE B-1), VULCANIZED	1.5	4000	20	1,480
7. POLYMETHYL METHACRYLATE	1.5	1200	142	14,800
8. PLASTICIZED PVC (KOROSEAL)	1.45	660	50	3,470
9. POLYESTER	1.1	200	108	10,100
10. POLYTETRAFLUOROETHYLENE (TEFLON-TFE)	1.0	400	23	5,800
11. HARD RUBBER	1.0	40	60	10,100
12. NITRILE RUBBER	0.8	1600	20	16,900
13. URETHANE RUBBER (SHORE 60A)	0.8	30	–8	2,220
14. FILLED RUBBER	0.5	2600	22	33,300
15. URETHANE RUBBER (SHORE 94 A)	0.4	140	–23	43,800
16. AQUAPLAS (WATER SOLUBLE)	0.38	1000	40	24,200

†TO GET TENSILE MODULUS E_1, MULTIPLY G_1 VALUES BY 3.

Table 2-7. Dynamic-Property Data Available at Low Frequencies Only

(CONTOUR PLOTS NOT YET AVAILABLE FOR THESE MATERIALS)				
MATERIAL	MAX LOSS FACTOR, η_d	TEMP RANGE, °C (FOR $\beta \geq 0.75\ \beta$MAX)	TEST FREQ Hz	SHEAR† MODULUS G_1 AT βMAX, PSI
POLYVINYLCARBAZOL	>1.6	200 TO 220	10.0	1,200
POLYVINYL-N-BUTYL ETHER	>1.6	–40 TO –20	0.8	1,500
POLYVINYL-ISOBUTYL ETHER	>1.6	–15 TO 10	1.2	1,800
POLYVINYL-TERT-BUTYL ETHER	>1.6	70 TO 80	1.7	1,800
BUTYL RUBBER (UNVULCANIZED)	1.2	–20 TO 20	1.0	18,000
NEOPRENE GN-60 (VULCANIZED)	0.6	–20 TO 5	~100.0	5,800
NATURAL RUBBER (HIGH MOL. WT)	~ 0.6	–70 TO –60	<10.0	1,800
GR-S RUBBER	~ 0.5	–60 TO –40	<10.0	7,200
POLYTRIFLUOROCHLOROETHYLENE (EG. FLUOROTHENE)	0.42	90 TO 120	3.3	11,000
POLYVINYLFLUORIDE	0.36	20 TO 60	1.7	5,800
POLYETHYLENE	0.23	30 TO 80	12.0	2,900
BUNA N (B-6), CARBON-FILLED, VULCANIZED	0.16	0 TO 10	80.0	29,000

†TO GET TENSILE MODULUS E_1, MULTIPLY G_1 VALUES BY 3.

16. The new point found in 15 gives a corrected value for "a", the ratio of thicknesses of the damping layer and the plate, for the damping compound selected.

CORRECTION 2 (Mass Correction)

17. Enter the Correction 2 graph with the value for ρ_d, the density of the basic structure in gr/cm^3 (line K).

18. From ρ_m trace a horizontal line to the left until the ρ_d line, (the density of the damping material) is reached.

19. From the intersection found in 18, draw a vertical line down to the L line. This provides a value for b.

20. Introduce the value of b found in 19 on line M.

21. Trace to the left, following the curves, until the "a" is reached which was chosen in step 7 of the instruction, or in step 15 of correction 1 if the latter was utilized.

22. Trace horizontally and to the left from this point to the P line.

23. From the point Y, (on the same line P as the point just located in step 2) draw a line to the previously determined point found in the answer graph (step 7 or step 16) and then a short vertical line through the point.

24. From the point found in step 22, draw a line parallel to the line drawn from point Y in step 23 until it intersects the vertical line drawn in step 23.

25. This new point found in 24 gives a corrected value for "a", the thickness ratio, for the damping compound chosen earlier.

2-2.5 Stability of Columns Under Varying Axial Loads

2-2.5.1 Reference

1. Timoshenko, S.P., Gere, J.M., Theory of Elastic Stability (1961)

2. Bolotin, V.V., The Dynamic Stability of Elastic Systems

A colut subjected to an oscillating end load of the form:

$$P_o + P_t \cos \theta t \qquad (33)$$

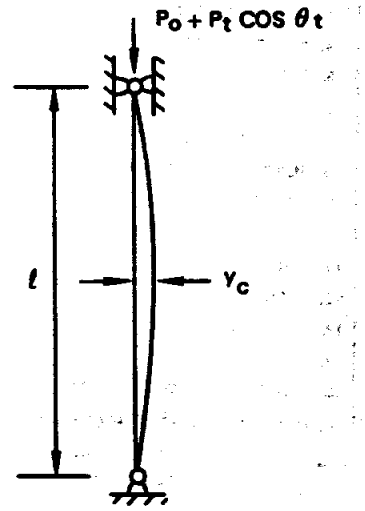
is investigated in Reference 2 to determine the transverse vibration of the column.

The differential equation of the system, when damping is neglected, can be written as:

$$EI \frac{\partial^4 y}{\partial x^4} + (P_o P_t \cos \theta\ t) \frac{\partial^2 y}{\partial x^2} + m \frac{\partial^2 y}{\partial t2} = 0 \quad (34)$$

The solution can be written in the form:

$$y = f(t) \sin (\pi x/\ell) \qquad (35)$$



Substituting equation 35 into equation 34 yields:

$$\frac{d^2 f}{dt^2} + \frac{\pi^2}{m\ell^2} \left[\frac{\pi^2 EI}{\ell^2} - P_o\right] f(t) - \left[\frac{\pi^2}{m\ell^2} P_t \cos \theta t\right] f(t) = 0 \qquad (36)$$

If

$$\Omega = \omega_o \sqrt{1 - \frac{P_o}{P_e}} \qquad (37)$$

and

$$\mu = \frac{P_t}{2(P_e - P_o)}, \qquad (38)$$

then equation 36 can be written as:

$$\frac{d^2 f}{dt^2} + \Omega^2 (1 - 2\mu \cos \theta t) f = 0 \qquad (39)$$

This is the Mathieu Equation which governs many physical systems and its properties are discussed in

several texts. For certain values of Ω and μ the solution of the equation is unbounded. Thus, for some combinations of the parameters the vibratory motion, f(t), will be divergent and theoretically grow to infinity.

The regions of instability are therefore of interest to define areas which should be avoided in design. The stability boundaries are given approximately as:

$$\text{1st } \frac{\theta}{\Omega} = 2\sqrt{1 \pm \mu}$$

$$\text{2nd } \frac{\theta}{\Omega} = \sqrt{1 + \tfrac{1}{3}\mu^2}$$

$$\frac{\theta}{\Omega} = \sqrt{1 - 2\mu^2}$$

$$\text{3rd } \frac{\theta}{\Omega} = \frac{2}{3}\sqrt{1 - \frac{9\mu^2}{8 \pm 9\mu}} \qquad (40)$$

It is interesting to note that by letting P_o and P_t in equation 4 be zero the differential equation:

$$\frac{d^2f}{dt^2} + \frac{\pi^4 EI}{m\ell^4} f = 0$$

is obtained. This is the equation of a system with a natural frequency of:

$$\omega^2 = \frac{\pi^4 EI}{mL^4}$$

which is recognized as the natural frequency of a beam identical to the column being considered.

If on the other hand only P_t is zero, then:

$$\frac{d^2f}{dt^2} + \frac{\pi^2}{m\ell^2}\left[\frac{\pi^2 EI}{\ell^2} - P_o\right] f = 0$$

which for:

$$P_o < \frac{\pi^2 EI}{\ell^2}$$

Gives a resonance frequency:

$$\omega^2 = \frac{\pi^2}{m\ell^2}\left[\frac{\pi^2 EI}{\ell^2} - P_o\right]$$

The Euler load is obtained by setting the coefficient of f to zero, i.e.,

$$\frac{\pi^2}{m\ell^2}\left[\frac{\pi^2 EI}{\ell^2} - P_o\right] = 0$$

or

$$P_e = \pi^2 EI/\ell^2$$

The first three regions of instability are plotted in Figure 2-30a from equation 40. It is noted that additional regions could be defined for lower values of the parameter θ/Ω

Damped System – A more practical condition is the study of the system with damping included. Equation 39 is modified by including a term which accounts for viscous damping, i.e.

$$\frac{d^2f}{dt} + 2\epsilon\frac{df}{dt} + \Omega^2(1 - 2\mu\cos\theta t)f = 0 \qquad (41)$$

This equation has been solved in Reference 1 and the equations for the stability boundaries are found. The boundaries for the case when the constant load (P_o) is zero are given approximately as:

$$\text{1st } \frac{\theta}{\Omega} = 2\sqrt{1 \pm \sqrt{\frac{P_t^2}{4P_e^2} - 4\zeta^2}}$$

$$\text{2nd } \frac{\theta}{\Omega} = \sqrt{1 - \frac{P_t^2}{4P_e^2} \pm \sqrt{\frac{P_t^4}{16P_e^4} - (2\zeta)^2\left(1 - \frac{P_t^2}{4P_e}\right)}}$$

$$\text{3rd } \frac{\theta}{\Omega} = \frac{2}{3}\sqrt{1 - \delta}$$

$$\delta = \frac{\frac{8}{9}\left(\frac{P_t}{2P_e}\right)^2 \pm \sqrt{\left(\frac{P_t}{2P_e}\right)^6 - (2\zeta)^2\left(\frac{64}{81} - \frac{2}{12}\frac{P_t^3}{P_e^2}\right)}}{\frac{64}{81} - \left(\frac{P_t}{2P_e}\right)^2} \qquad (42)$$

where ζ is the ratio of damping to critical damping.

It is shown that the minimum points corresponding to the zero in the undamped case lie near the frequency:

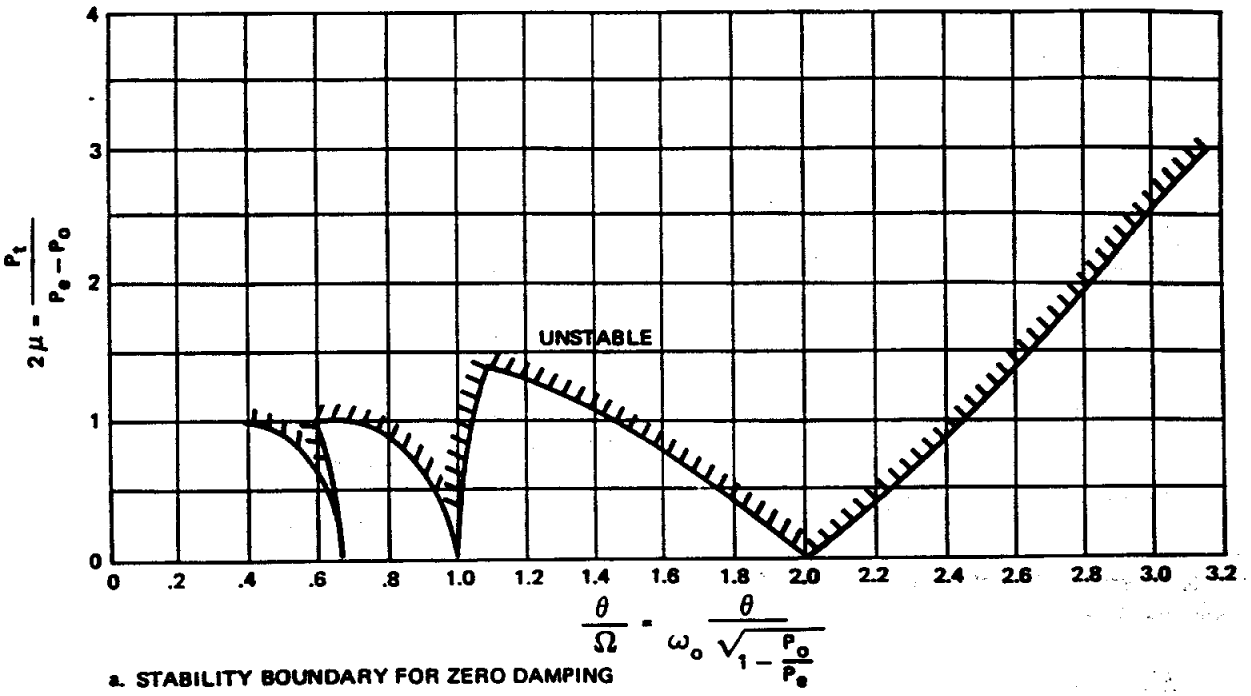
$$\theta/\Omega = 2/k \quad k = 1,2,3,...$$

and that the value of the point is:

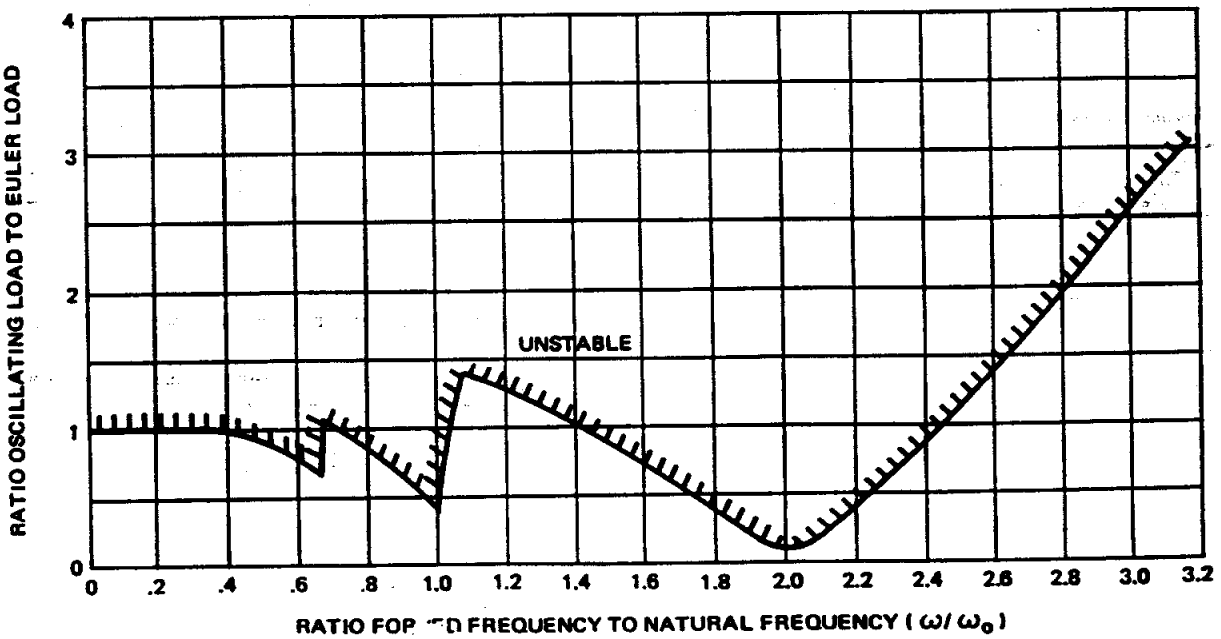
$$P_t/P_e = m2z$$

The stability boundaries are plotted in Figure 2-30b for a damping ratio of $\zeta = .02$ and for the constant load $P_o = 0$.

The minimum location on the stability boundary is noted to be at a frequency twice the natural frequency. This is in contrast to the usual vibration response which



a. STABILITY BOUNDARY FOR ZERO DAMPING

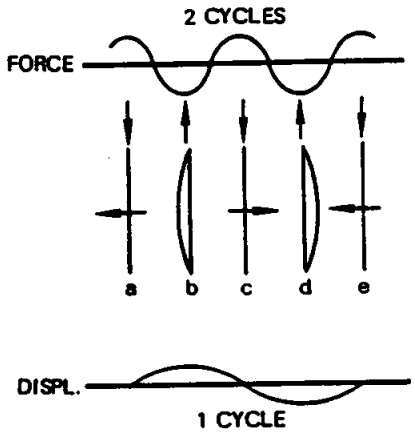


b. STABILITY BOUNDARY FOR $\zeta = .02, P_o = 0$

Figure 2-30. Column Dynamic Stability Boundary.

is maximum when the forcing and natural frequencies coincide.

By referring to the sketch it is seen that the load is always acting to force the column in the direction in which it is moving. This is the condition in which the greatest excitation will take place and in which the maximum amount of work will be done on the column. Thus, intuitively, it would appear that the most critical frequency would be when the forcing frequency is twice the natural frequency of the column.



2-3 SHOCK

2-3.1 Good Design Practices

Aerospace vehicle response to shocks can overstress or overdeflect structural or mechanical parts causing failure, or can cause malfunction of hydraulic, mechanical, and electromechanical components in either the main operational system or in subsystems. To avoid this problem, aerospace vehicle designers must consider the effects of each (of the many) shock-loading conditions.

Shock sources of significance are: manufacturing, handling, transportation, engine transients, separation, explosive devices, docking, maneuvering, and landing. There are two basic classes of shocks: low-frequency mechanical shocks (such as engine transients) and high-frequency shocks (from explosive devices). Low-frequency shocks may, by exciting fundamental modes of the structure, cause failures, while the high-frequency explosive shocks may cause equipment failures or malfunctions. Damage resulting from shock loads is usually confined to local structure and/or to electromechanical components.

Common practice is to use static or quasi-static loads in the initial design of a vehicle, with added margins to account for dynamic loads. The final design is then examined as to its ability to withstand the various dynamic and static loads. Shock-load effects can be determined by analysis or by ground tests (after the shock input has been determined). Analytical methods are available for establishing the response of vehicle structure to low-frequency mechanical shock (see Section 5, Analysis) but no valid method is available for analyzing the response of structures to high-frequency explosive shock.

When shock induces a critical design condition, consideration should be given to reducing the shock source, to altering the structure, or, in the case of delicate airborne equipment, to relocating or isolating the equipment. In addition, combined stresses should be accounted for in the design, including the superposition of all concurrent dynamic and static loads (e.g., acoustic, staging or thrust-transient, thermal, and vibratory loads).

Major steps in the design process are:

a. Collect and analyze measured input and response data on similar vehicles to obtain appropriate shock definition.

b. Devise an adequate mathematical model (only for low-frequency mechanical shock) and select or devise an accurate and efficient computational method.

c. Analyze response of the model to the low-frequency shocks.

d. Modify the design as required.

e. Conduct tests on the equipment, segments of the structure, or on the complete structure to verify the design.

Explosive devices, such as linear-shaped charges, explosive bolts, explosive nuts, squibs, pin pullers, and strap cutters, are used to perform such functions as stage separation, shroud separation, valve operation, antenna erection, satellite ejection, release of a hatch, or release of a fairing. The shocks generated by these explosive devices often measure thousands of g's. Fortunately, these shocks are attenuated rapidly through structure and through structural discontinuities. Damage to structure occurs only near the explosive device and this damage can be eliminated with proper design of the ordnance system.

The greatest problem with explosive shocks has been the failure and malfunction of delicate equipment in the immediate region of the explosion. Delicate equipment should be located away from the explosive area or should be mounted on dynamic isolators.

Figures 2-31 and 2-32 show measured shock spectra caused by a primacord detonation on a space vehicle.

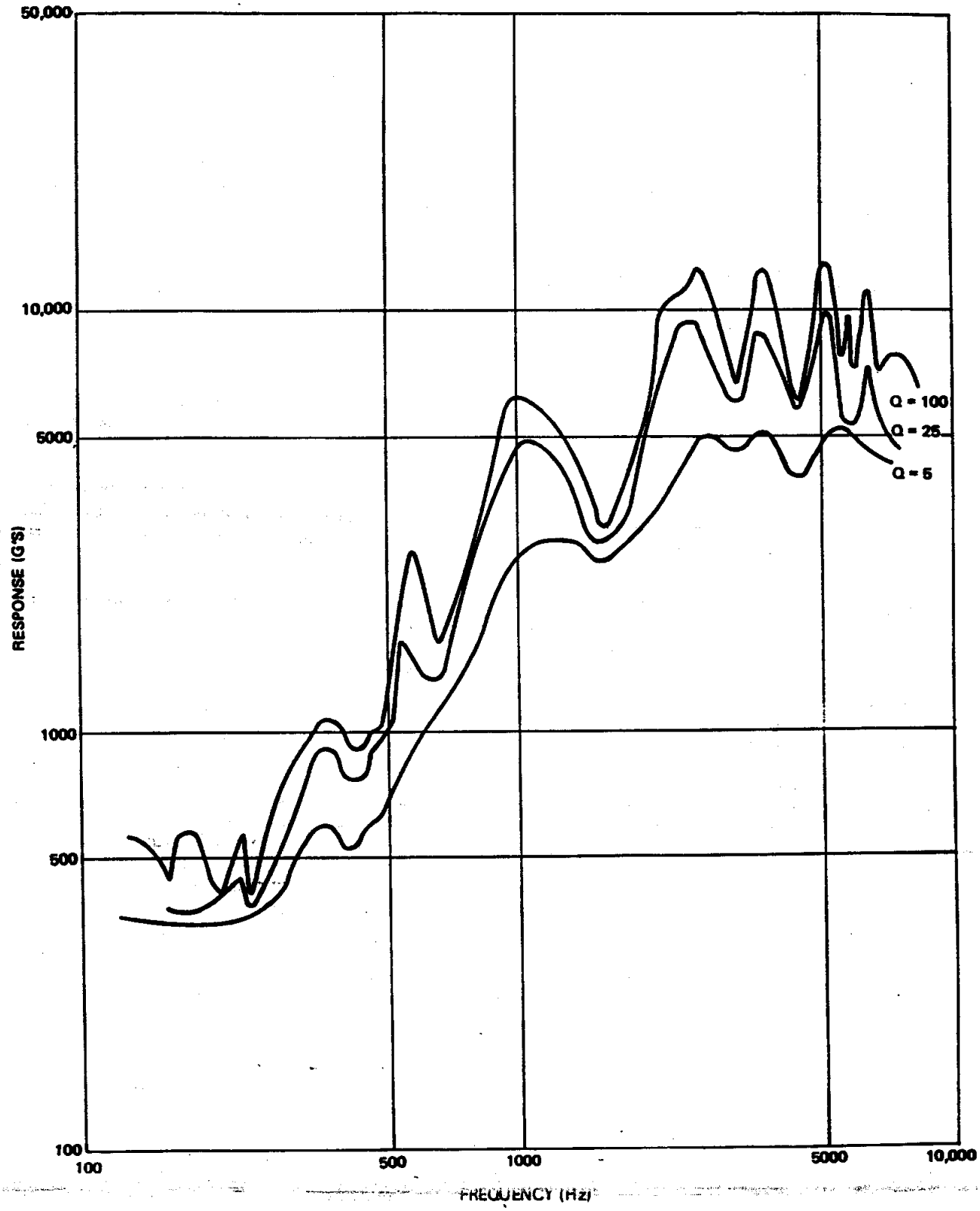


Figure 2-31. Shock Spectra Data Meas. V6 — (Lateral Engine-Cone-Skin Primacord Detonation).

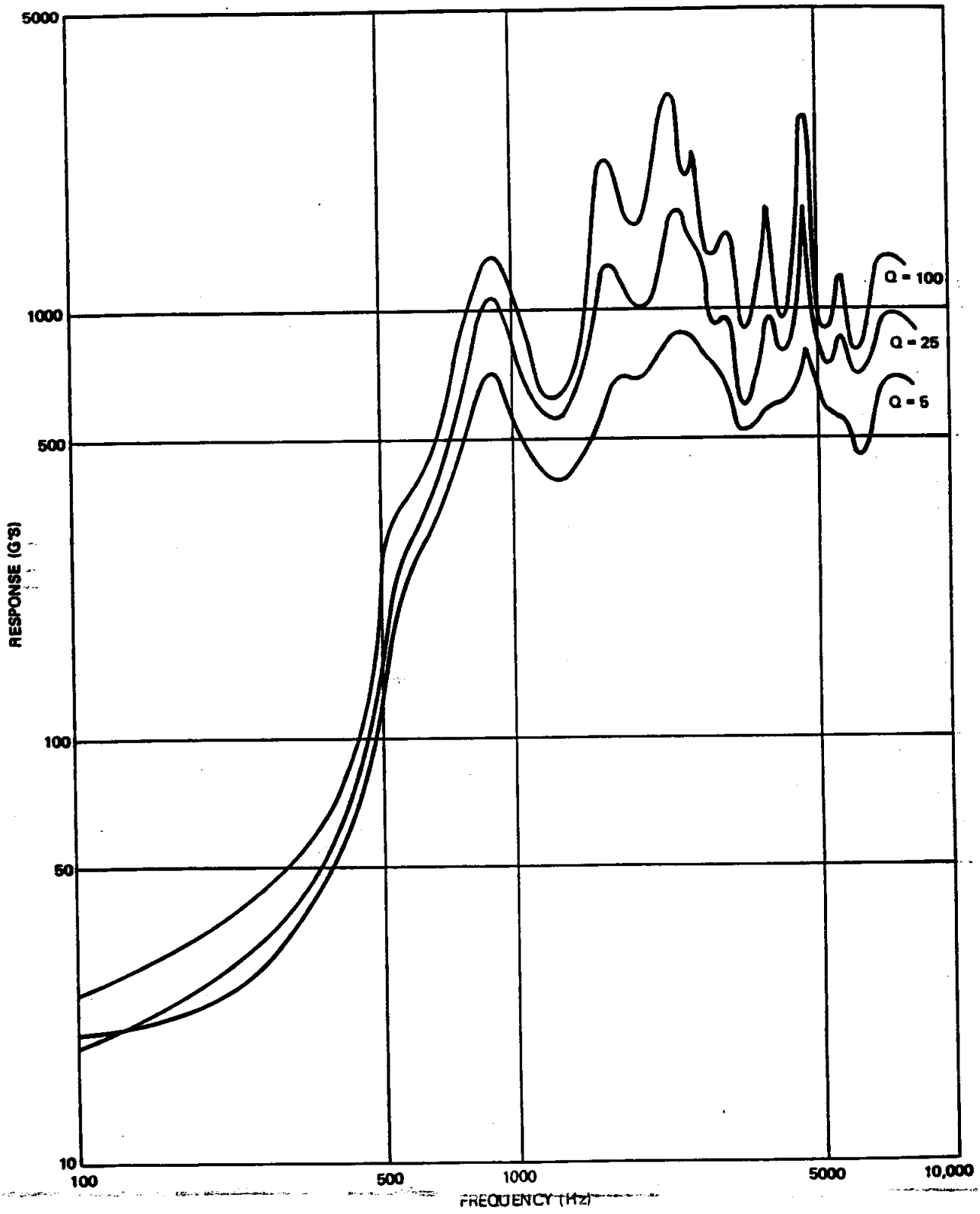


Figure 2-32. Shock Spectra Data Meas. V21 (Lateral) Engine-Mount-Ring (Sta.4-11) Primacord Detonation.

The spectral analyses assumed damping ratios of 0.10 (Q=5), 0.02(Q=25), and 0.005(Q=100). An examination of Figure 2-31 data (5" from the separation plane) and Figure 2-32 data (26" from the separation plane) shows the rapid attenuation of the explosive shock with distance through a structure.

Figure 2-33 (see also Reference 1), displays a compilation of explosive shock measurements versus distance in inches. The illustration shows almost no shock at a distance of 100 inches from the explosive device.

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When expected shocks exceed the maximum safe value, a shock isolator must be incorporated in the product between the shock-sensitive elements and the point of impact. In a strict sense, all (real) elastic structures between the shock sensitive elements and the point of impact are shock isolators. The major practical problem is trade-off between isolator deflection and the maximum shock load that can be accepted. Increased margin of safety requires increased allowance for movement.

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Drop height can usually be determined with fair accuracy. The shock-rise time is dependent on the elasticity of the striking mass, resilience of impact surface, and extent of contact area, which may be a flat or a curved surface, an edge, or a point. Shock-rise time has been determined for a variety of conditions by test. For point contact between a rigid steel frame and a concrete surface, assuming that the mass of the struck surface is at least 10 times the mass of the striking body, the rise time is 1 to 2 millisec; for a flat face contacting soft earth or sand, the rise time is about 6 millisec.

Impact on less rigid surfaces such as wood, mastic, compact earth, or sand produces rise times of 5 to 15 millisec (Table 2-8). A 3-ft drop to mastic or wood with a rise time of 5 millisec will produce 85 g; against a sand or mud surface with a rise time of 10 to 15 millisec, 25 to 40 g. A unit slipping from the hands of an operator and dropping 4 in. onto a felt pad 1 in. thick will have a shock-rise time of about 30 millisec and receive a shock of 2 to 4 g. The shock responses are assumed to be half sines, and the shock rise time t is the time from 0 g (impact) to maximum acceleration (maximum displacement). The frequency of the system is:

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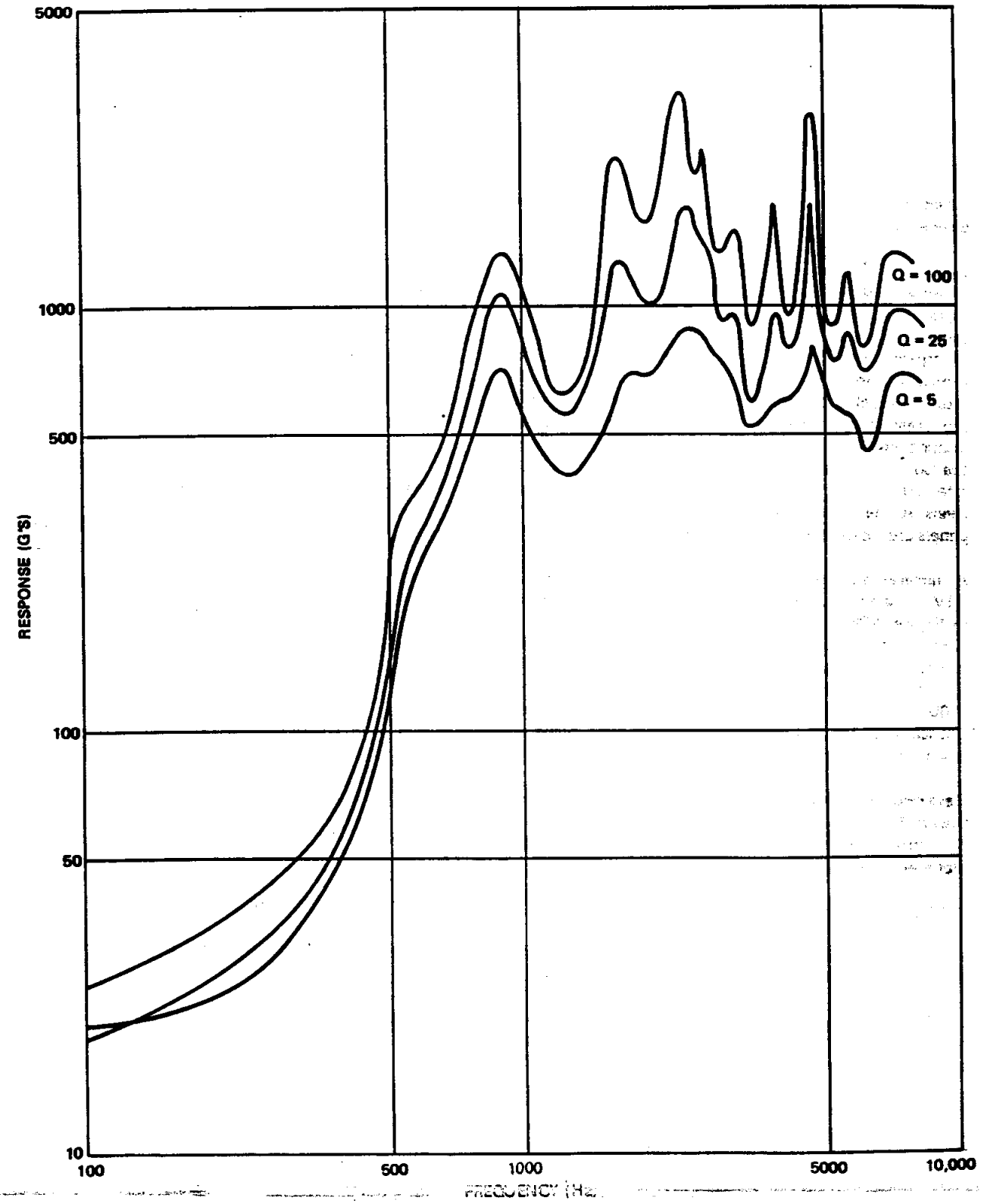


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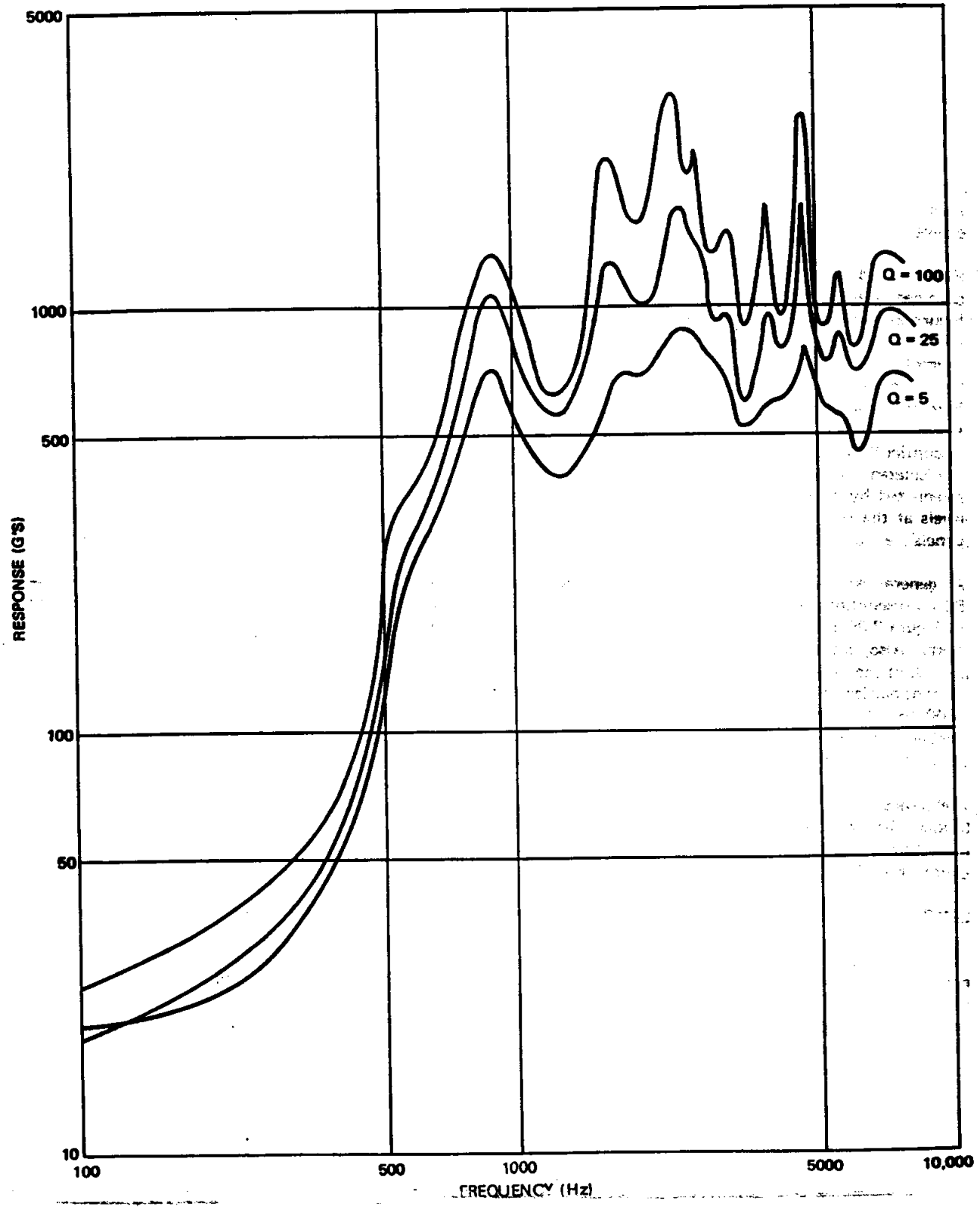


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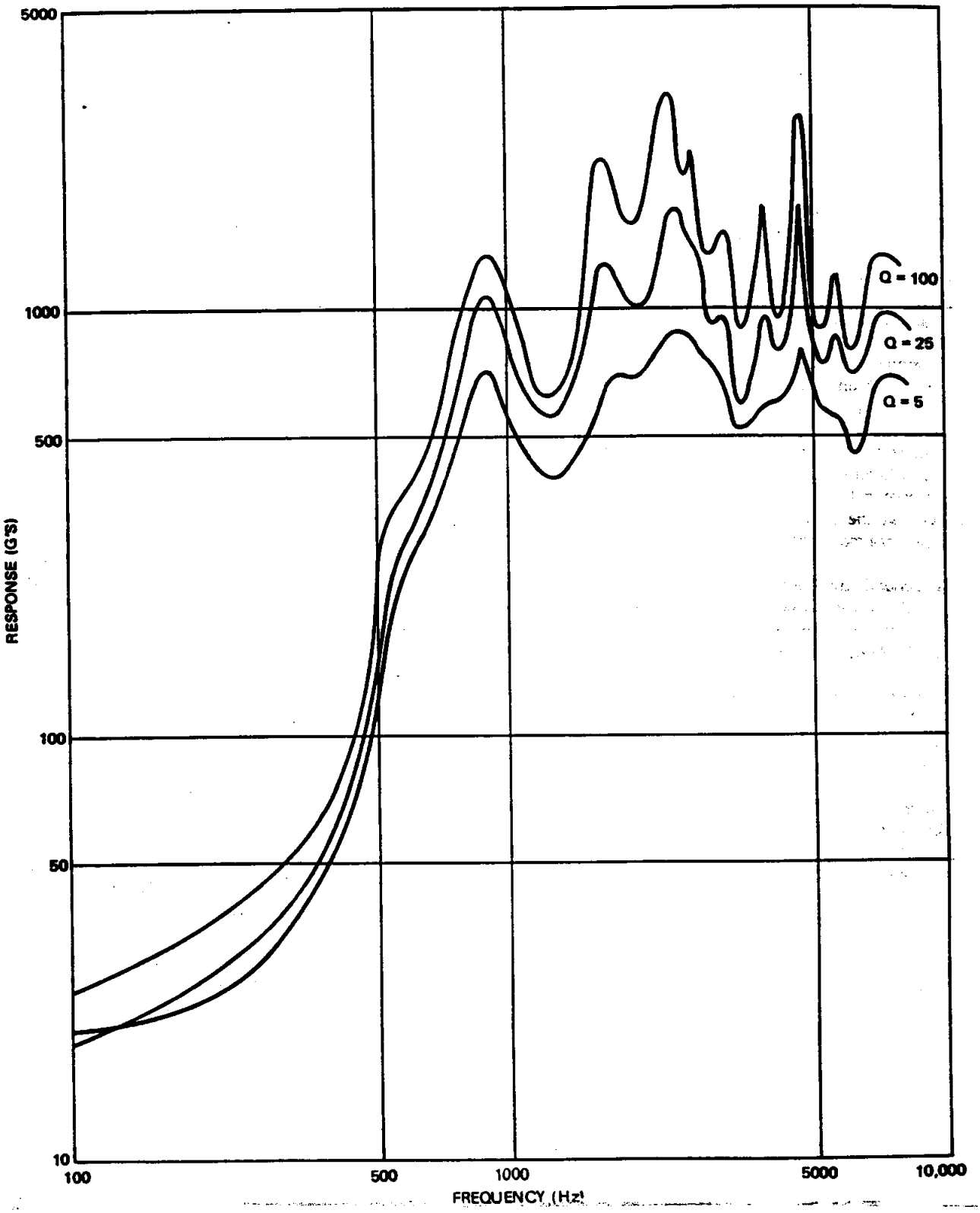


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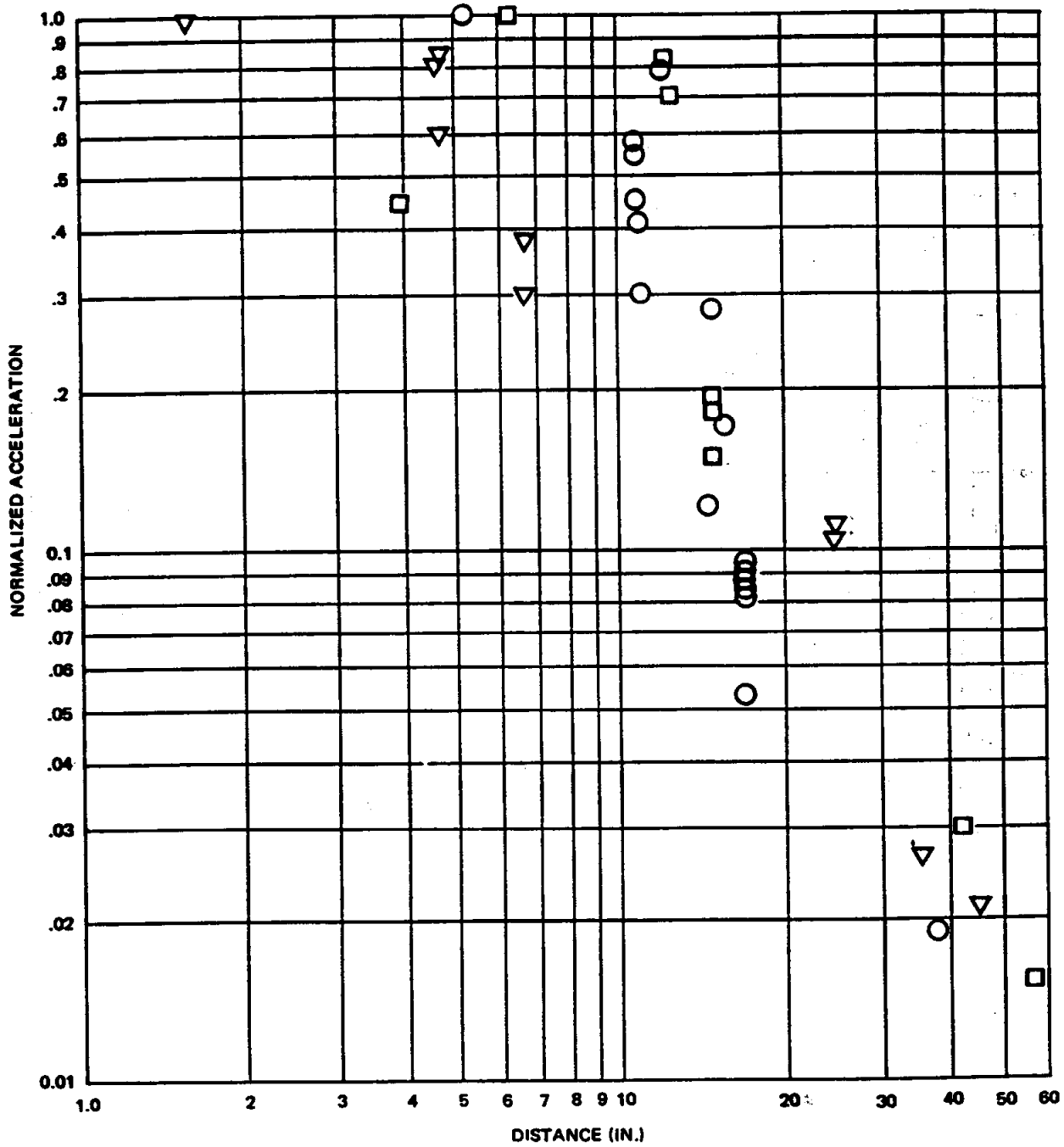


Figure 2-33. Distance Attenuation.

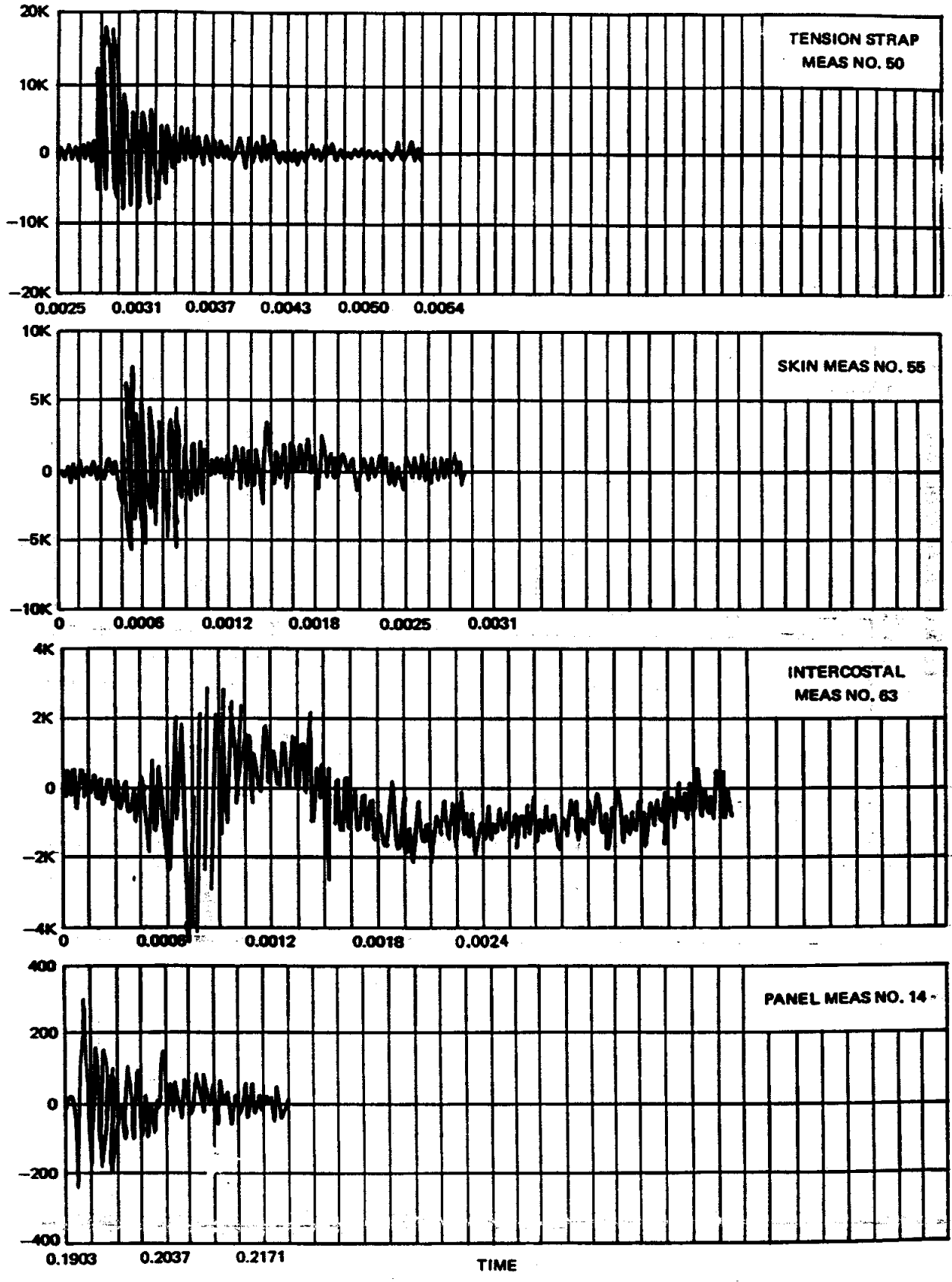


Figure 2-34. Samples of Oscillograms.

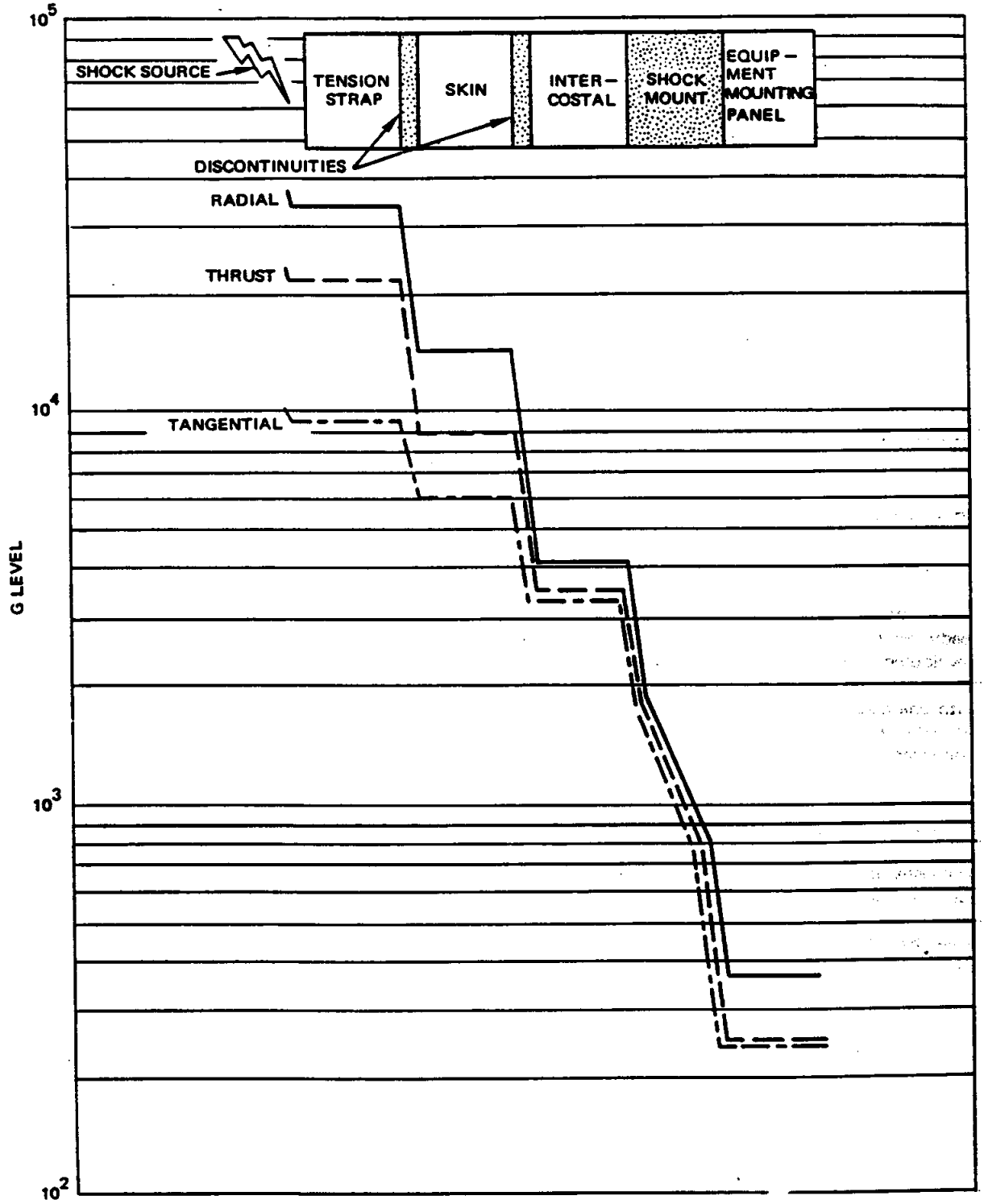


Figure 2-35. Approximate Shock Gradient.

Table 2-8. Approximate Shock Rise-Times

CONDITION	FLAT FACE RISE TIME (MSEC)	POINT RISE TIME (MSEC)
RIGID STEEL AGAINST CONCRETE	1	2
RIGID STEEL AGAINST WOOD OR MASTIC	2-3	5-8
STEEL OR ALUMINUM AGAINST COMPACT EARTH	2-4	6-8
STEEL OR ALUMINUM AGAINST SAND	5-6	15
PRODUCT CASE AGAINST MUD	15	20
PRODUCT CASE AGAINST 1-IN. FELT	20	30

NOTE: MASS OF SHOCK SURFACE IS ASSUMED TO BE AT LEAST 10 TIMES THE STRIKING MASS. POINT CONTACT WITH SPHERICAL RADIUS OF 1 IN.

The final velocity (V_f) of a dropping object, in units of "G msec" for convenience, is:

$$V_f = 72\sqrt{h}$$

The nomogram shown on Figure 2-36 gives a quick and ready answer for preliminary design, particularly in developing clearance requirements and isolator capacity. To simplify the various charts, a linear undamped system with one degree of freedom is assumed; results given are conservative, particularly if such design refinements as a nonlinear deflection rate and some damping are added.

For a drop of 24 in. with a maximum transmitted shock or 33 g, the isolator deflection will be 1.65 inches.

The right half of the chart shows the specific spring rate for the selected isolator deflection. Multiplying the specific spring rate by the weight carried by each isolator gives the deflection rate (in pounds per inch) that should be specified for the isolator.

where

h = drop height (inches)

The velocity step in 1/2 of a half-sine step is:

$V = 2Gt/\pi$

G = max acceleration (g)

t = shock rise time (msec)

V = velocity (G msec)

Equating the two yields:

$$G = 113\sqrt{h}/t$$

This final equation is the simple working equation which is plotted in Figure 2-37.

Continuing the example, the specific spring rate is 19 for 33 g and 1.65 inches deflection. If the equipment weighs 25 pounds and the load is divided equally between four isolators, the spring rate for each isolator will be 19 x 25/4 = 118 lb/in.

An isolator with a nonlinear deflection rate (stiffness increasing with deflection) will reduce the isolator deflection for a given transmitted acceleration, and will also give added protection against bottoming under a shock of higher intensity than anticipated. Addition of damping for control of excursion at the resonance frequency will also reduce isolator deflection by a small amount.

Most mounting systems provide six degrees-of-freedom; they are free to move in vertical, lateral, and longitudinal axes and to rotate about these axes. The deflection rate of an isolator may be substantially different in vertical and horizontal directions if this is required by expected service conditions.

The function of a shock mount is to provide enough protection to avoid damage under expected conditions, but overdesign can be costly, both in the design of the product and in the shock-mount components. To avoid underdesign and service failures on one hand, and overdesign and added cost on the other, shock-mount specialists should be asked for their assistance and recommendations before the design is frozen.

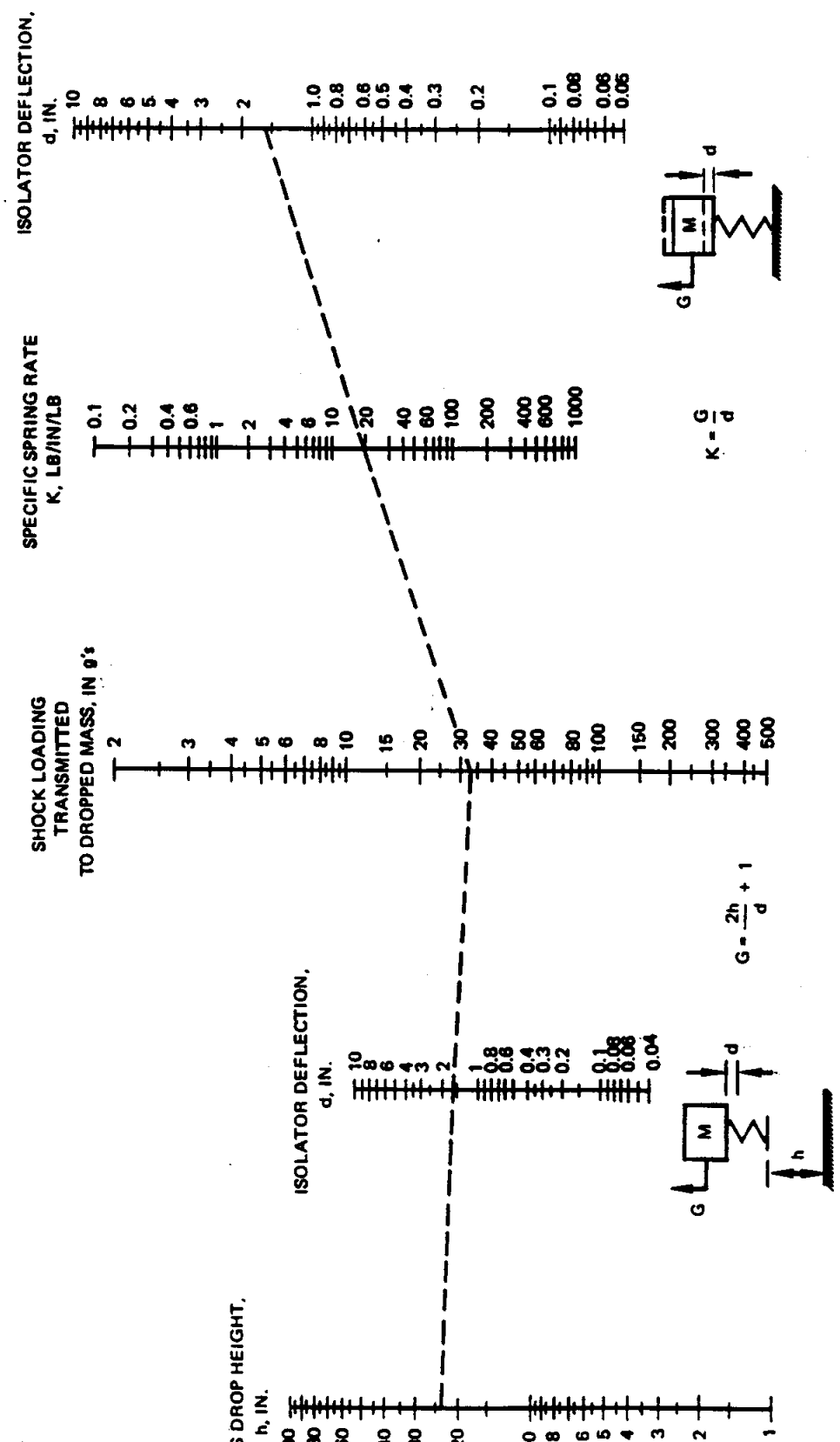


Figure 2-36. Nomogram for Isolator Design Characteristics.

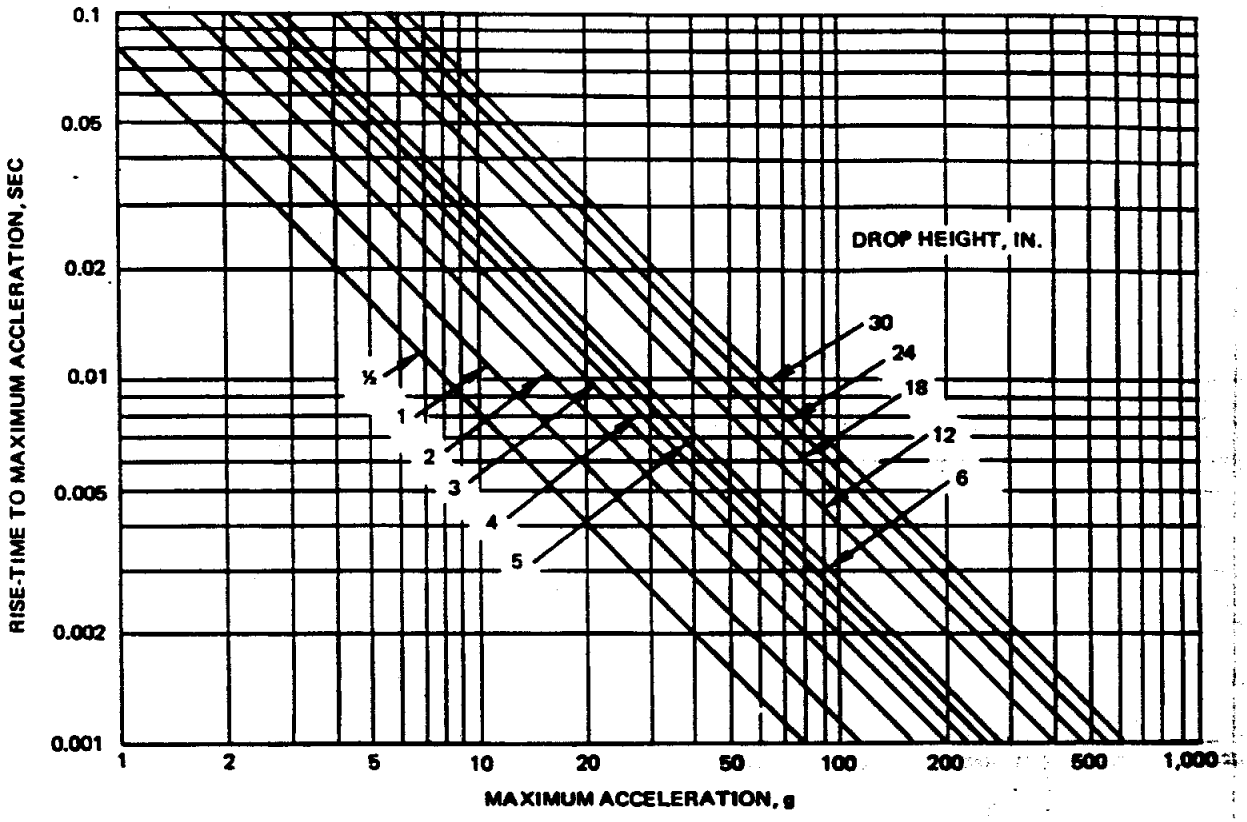


Figure 2-37. Drop-Height vs Rise Time Graph.

2-3.3 Shock-Cushioning Materials

There are many materials which may be used to cushion fragile items from shock. Some are permanently deformed by the shock while others are elastic enough to return to their original configuration after the shock. Obviously, to use crushable materials necessitates a single event. If more than one shock is expected, non-crushable (resilient) materials must be employed. The following list of shock-cushioning materials is not intended to be complete, but is representative of the diversity of materials available for this application (depending on the load and shock strength).

a. Crushable Materials

Styrofoam

Cardboard & cardboard structures

Glass fiber (e.g., Fiberglas PF600 Series)

Popcorn

Wax combs

Hollow glass balls

b. Resilient Materials

Rubber

Silicone Plastics

Vinyls

Sponge or resilient foams

Air-filled bags

Tennis balls

2-4 PANEL FLUTTER

Panel flutter is a self-excited oscillation of an external skin panel exposed to aerodynamic flow. It is often violent, with failure occurring a few seconds from the onset of flutter. If the edges of the panel have sufficient support, the flutter divergency is limited due to membrane tension; thus in some cases panel flutter may be a non-destructive vibration of low amplitude.

2-4.1 Criteria for Flat Panels

Figures 2-38 and 2-39 present flutter criteria based on experimental and flight data. The limitations of these empirical criteria are:

a. The curves become meaningless near Mach one.

b. The boundary layer was relatively thin.

c. Differential pressure across the panel was zero.

A flutter design criterion has been derived from Figures 2-38 and 2-39 and is presented in Figure 2-40.

2-4.2 Corrections for Curvature

For a curved panel of radius R, the ratio of flat to curved flutter parameters is:

$$\left(\frac{\text{F.P.FLAT}}{\text{F.P.CURVED}}\right)^{1} = 1 + \frac{12(1-\mu^2)L^4}{\pi^4 t^2 R^2} \cdot \frac{(n\frac{L}{W})^2 \left[8 + 5(n\frac{L}{W})^2\right]}{\left[5 + 2(n\frac{L}{W})^2\right]\left[4 + (n\frac{L}{W})^2\right]^3 \left[1 + (n\frac{L}{W})^2\right]^2}$$

The most severe case is for a minimum value of F.P. Flat/F.P. Curved. This occurs for axial mode number, m=1 and the circumferential mode number, n = value found by repetitive trial using the above equation.

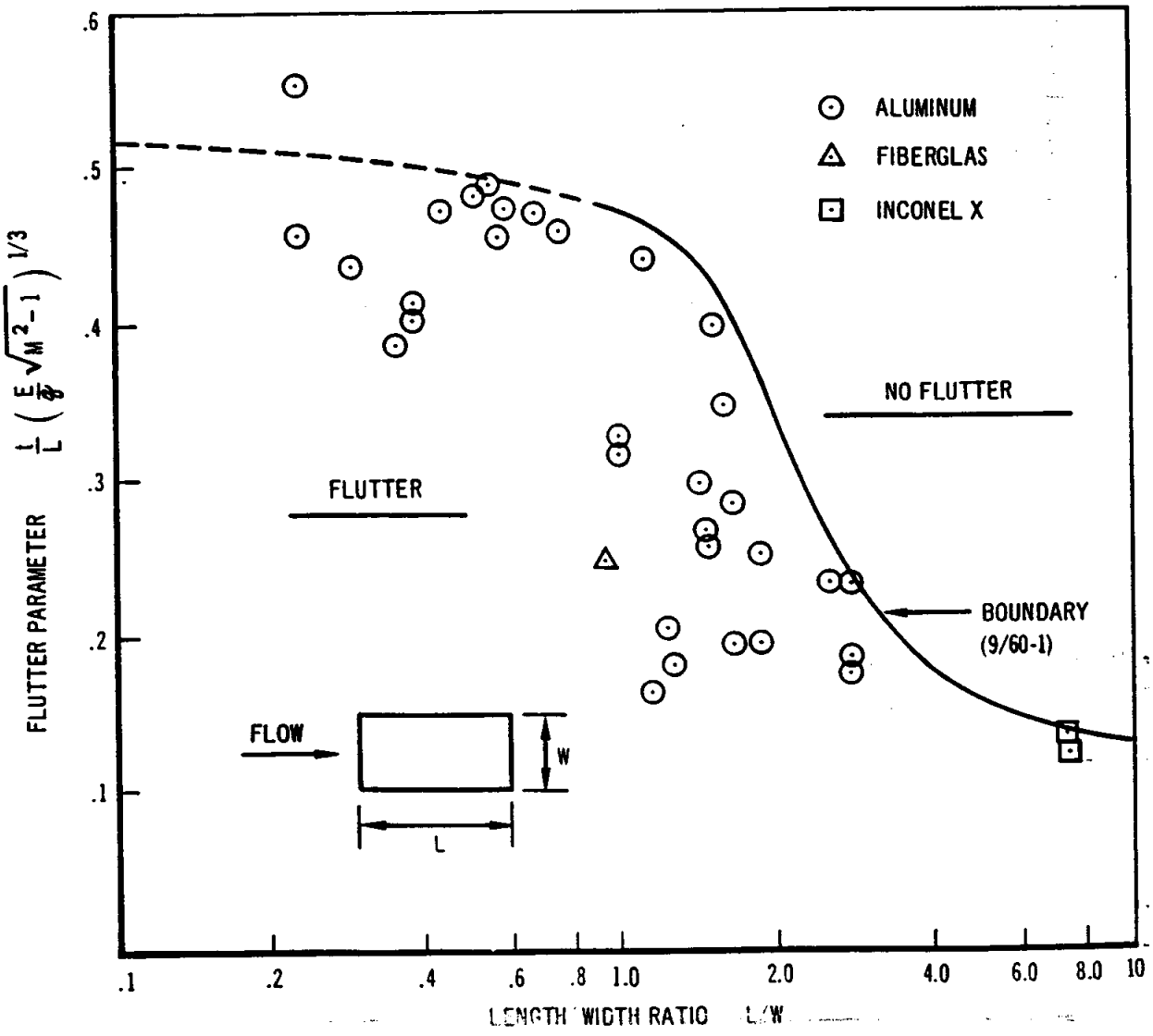


Figure 2-38. Flight Test Results Compiled by Kordes and Noll.

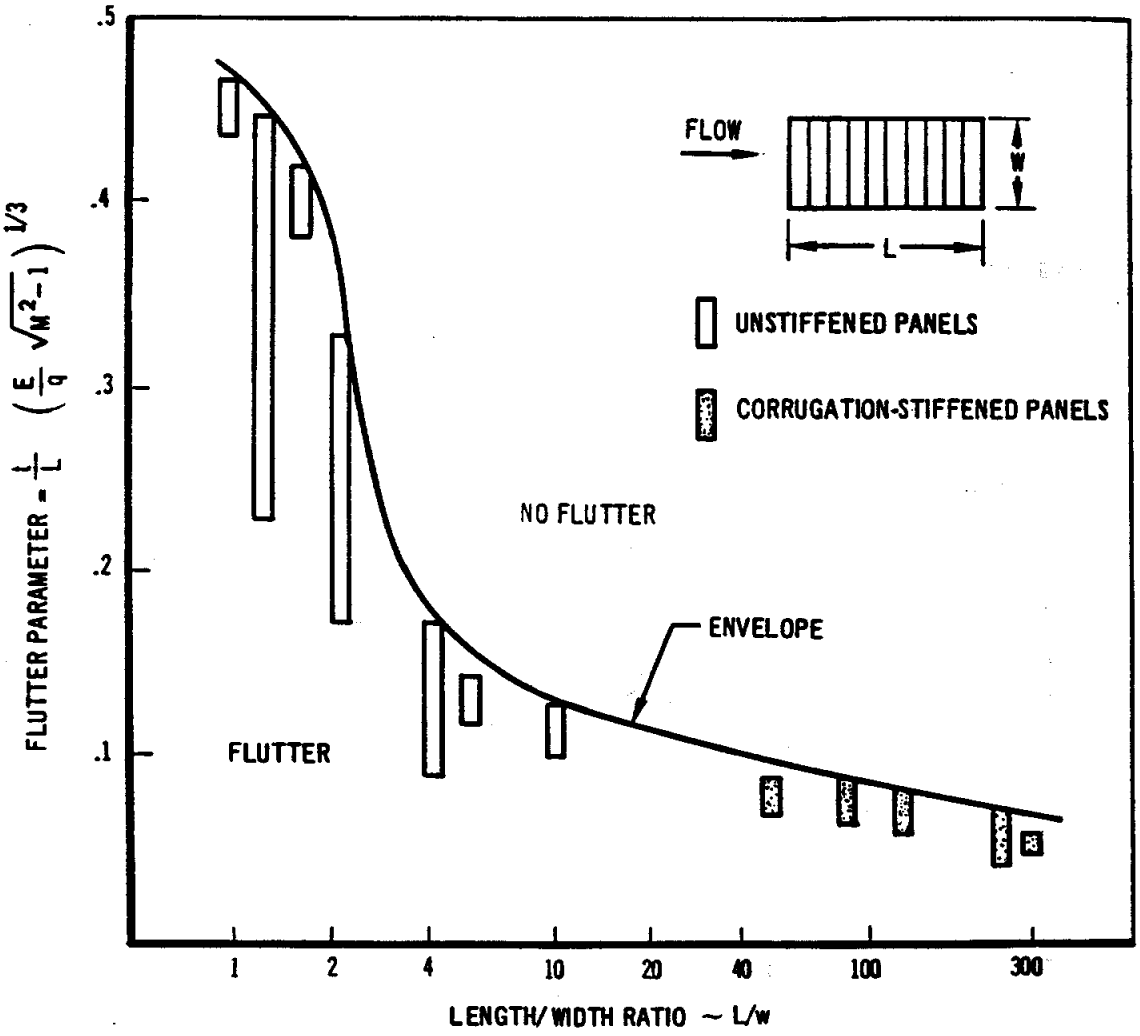


Figure 2-39. Experimental Results of Kordes, Tuovilla, and Guy.

For a cylindrical shell with m = 1, n ≥ 2, M ≥ $\sqrt{2}$,

$$(F.P.)^3 = \left(\frac{t}{R}\right)^2 \frac{n^4}{C}$$

where:

C = 180 for both ends simply supported

C = 240 for one end simply supported and the other end clamped

C = 360 for both ends clamped

and:

$$n = \sqrt{\frac{R}{L}\left(5.69\sqrt{\frac{R}{t}} - \pi^2\frac{R}{L}\right)}$$

Figure 2-41 presents the approximate effect of Mach No. on the panel thickness required to prevent flutter.

For a thick boundary layer, the flutter parameter is smaller than for a thin boundary layer, but the data are insufficient to present a mathematical relationship.

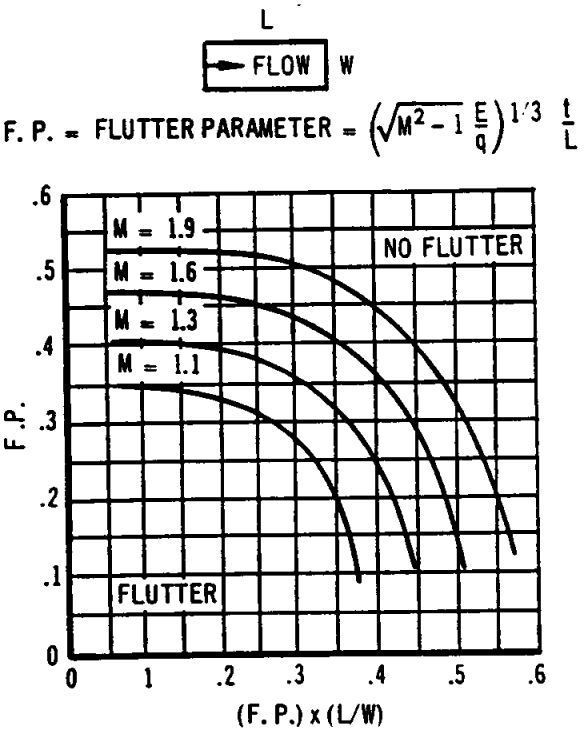


Figure 2-40. Flutter Design Criteria.

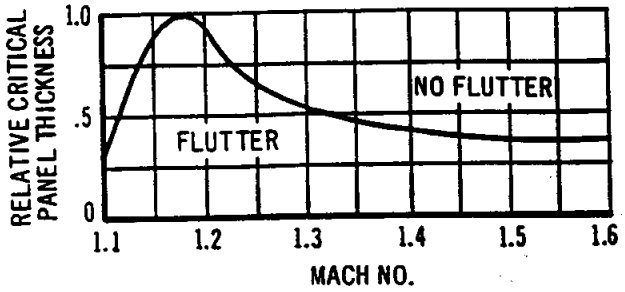


Figure 2-41. Experimental Effect of Mach No. on Flutter.

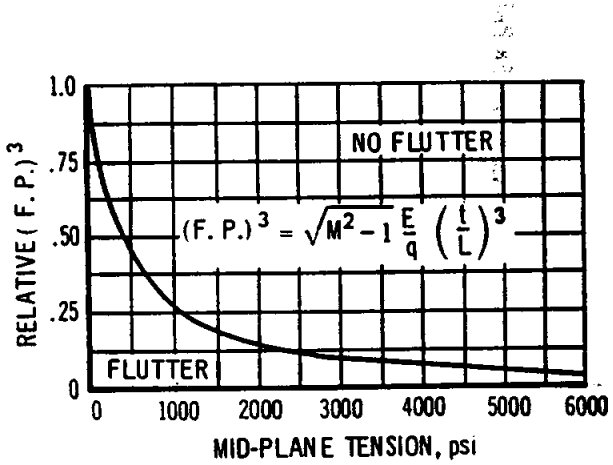


Figure 2-42. Experimental Effect of Stress on Flutter.

Figure 2-42 presents, for the case of M = 1.3 and L/w = 1.45, the effect of mid-plane tension on the flutter parameter.

For a buckled skin panel, the flutter parameter is approximately 1.6 that for an unbuckled skin panel, in the range of L/w from 2 to 10.

Figures 2-43 and 2-44 present, for a particular edge condition, the effects of differential pressure as a function of Mach No. and aspect ratio.

For further information, see Lemley, C. E., and N. H. Zimmerman, "Design Criteria for the Prediction and Prevention of Panel Flutter" Vol I and "Criteria Presentation" Vol II, February 1968, McDonnell Aircraft Co.

2-5 SONIC FATIGUE

2-5.1 Miles' Theory for Random Fatigue

For preliminary design requirements, Miles' linear single-degree-of-freedom approach yields results compatible with the overall accuracy of the problem. The basic equation, Reference 1 is for a lightly damped system in a fairly-uniform random-vibration field. As applied to a skin panel subjected to acoustically-induced vibration, the equation is:

$$\overline{\sigma}^2 = \frac{\pi}{4\frac{c}{c_{CR}}}\, f_o\, \phi(f_o)\, {\sigma_o}^2$$

where:

$\overline{\sigma}$ is RMS stress response (psi)

c/c_{CR} is system damping as a function of critical damping

f_o is fundamental panel resonance frequency (Hz)

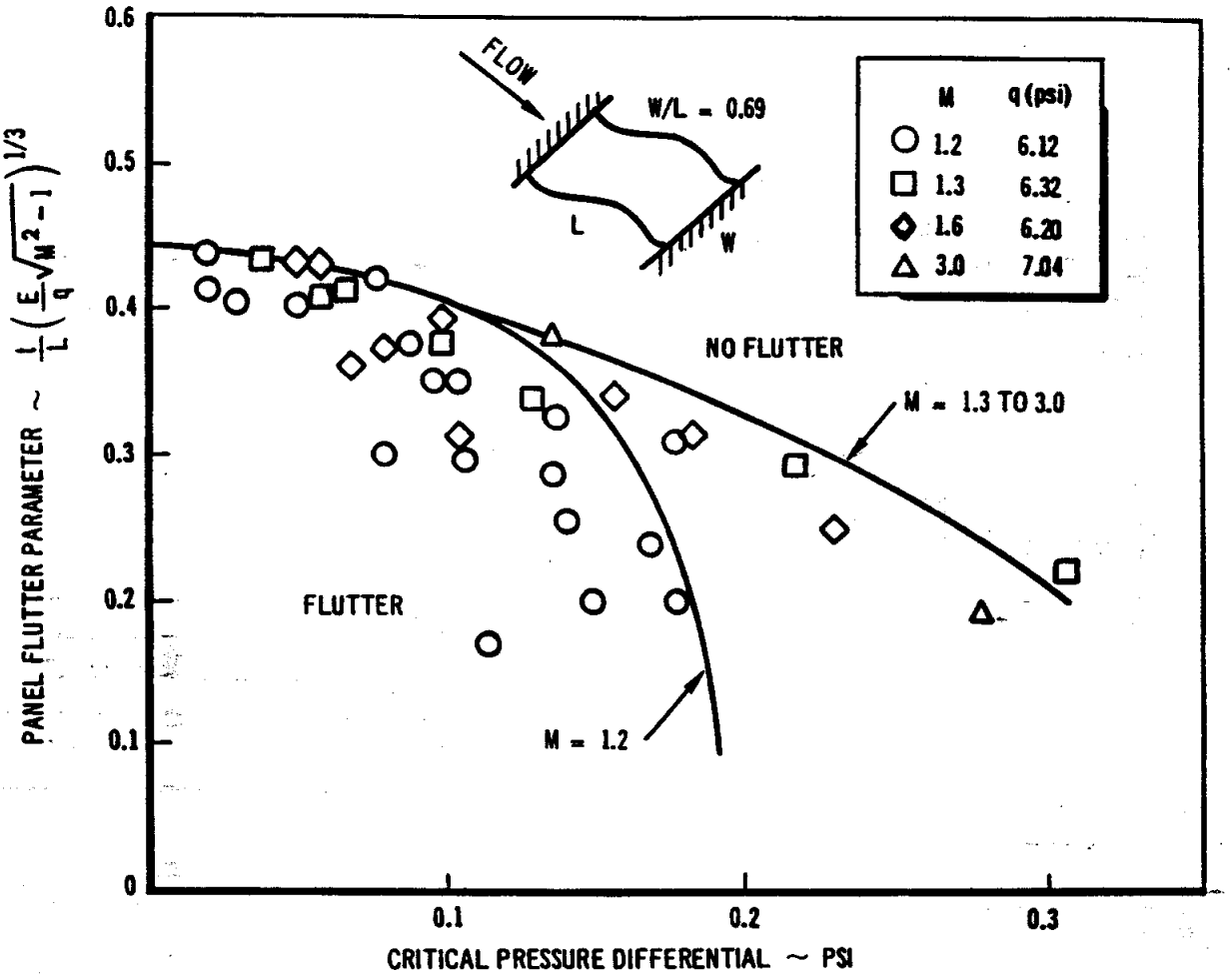


Figure 2-43. Experimental Results of Flutter in Buckled Panels.

$\phi(f_o)$ is power spectral density of the noise (psf^2/Hz)

σ_o is approximately the static stress response to a unit load (psi/psf)

$\phi(f_o)$ may also be in units of g^2/Hz and σ_o in units of psi/g.

It is preferable to use design charts based on test data, as presented in paragraphs 2-5.2 and 2-5.3, where applicable, for final design.

Reference 1: "On Structural Fatigue Under Random Loading" by J. W. Miles, Jour. Aero. Sci. Vol. 21 Nov. 1954

2-5.2 Design Nomographs*

The relationship between the various factors is most usefully presented for design purposes in the form of a nomograph. Nomographs for the various structural configurations are presented on the following pages. In these nomographs, the sound pressure level is indicated in decibels/Hz (Reference: 0.0002 dynes/cm^2), the

*Reference ASD-TDR-63-820.

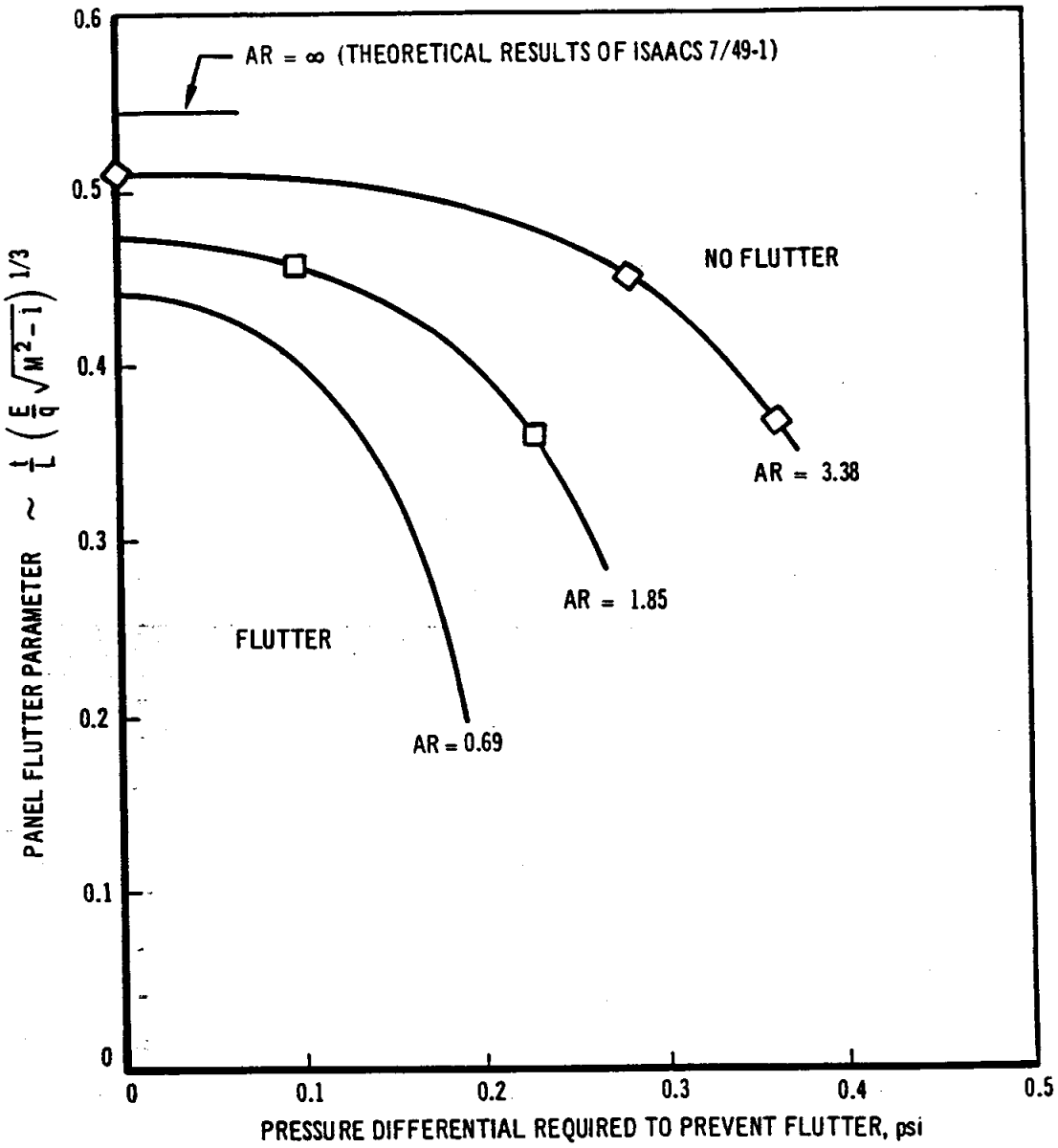


Figure 2-44. Effect of Aspect Ratio on Flutter of Buckled Panels.

panel dimensions in inches and the life in number of cycles. The stress-life curve is presented for only a single material.

Sketches of the structural configuration for each nomograph are included.

For materials other than those shown in the design nomograph, the following procedure can be used to substitute the desired new material. Random S-N data are presented for several materials in Figure 2-45. This illustration indicates a relative strength, expressed in decibels, for various materials and random life values. The difference in relative strength of the new material is determined for various lengths of life. These points are then plotted against the "relative strength" scale of the design nomograph to locate the new curve.

As an alternate method for quick single point comparison, the allowable DB_R can be determined for

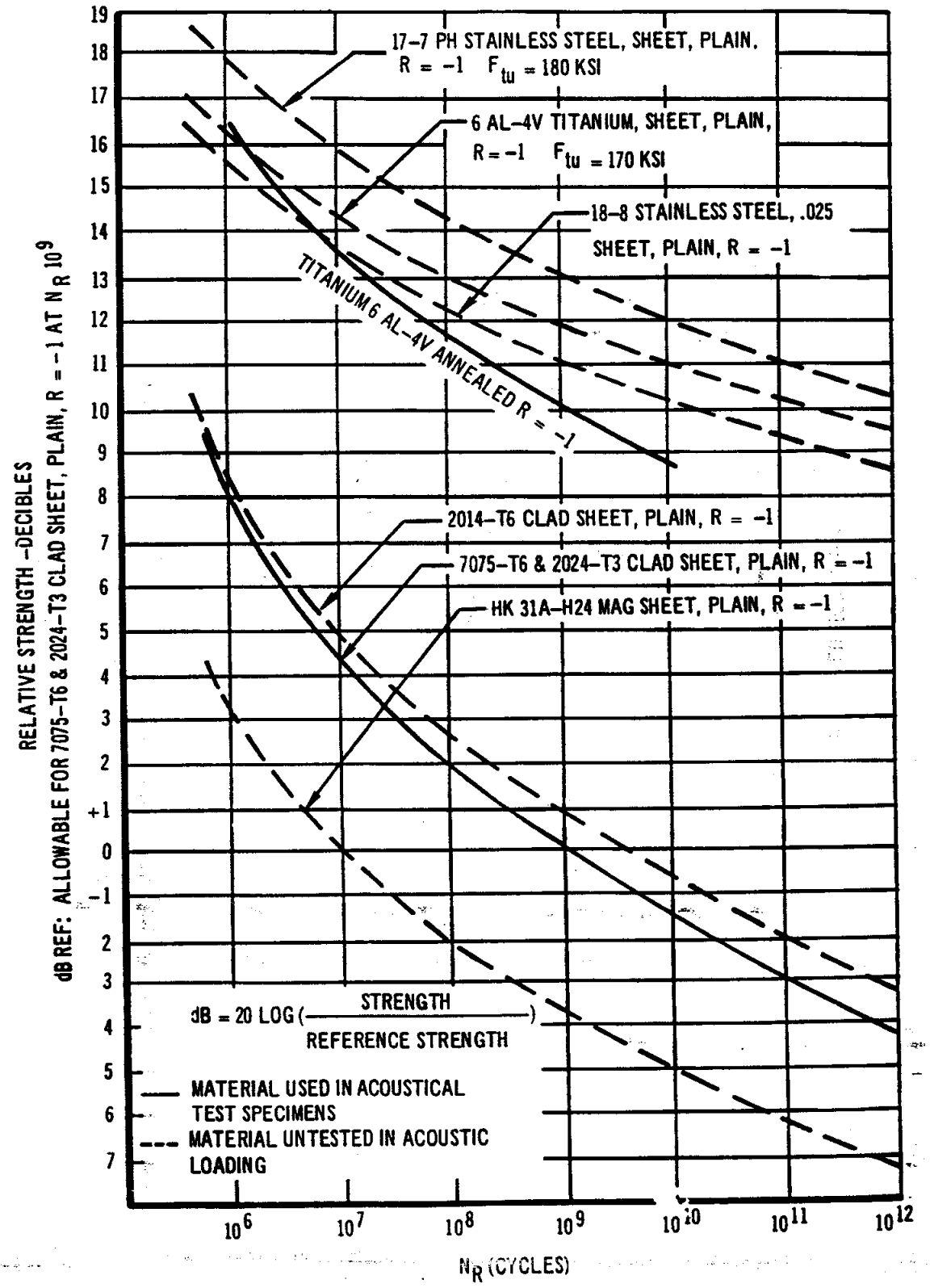


Figure 2-45. Relative Fatigue Strengths of Materials Under Random Excitation.

the material in the design nomograph and the relative strength difference at the appropriate life added to the answer.

a. *Skin and Rib Construction* – This structural configuration (Figure 2-46) is critical at either the bend radius of the rib, or in the skin at the rivet row on the side next to the heel of the rib, or the skin to rib attachments may fail in tension. Improved rib cap designs are shown and can be used when rib gage becomes excessive.

Example

A skin on rib structure of 2024-T3 material is required to withstand an estimated spectrum level, dB_R, of 118-1/2 dB at the resonance frequency of the structure. The design life is 1000 hours at this load or $N_R \simeq 10^9$ cycles. Following through the chart as indicated by the arrows to an assumed rib spacing, S = 4, a skin gage, t = .025 and a rib gage, t_r, of .038, is found, (use t_r = .040). The approximate lowest resonance frequency is found to be 250 Hz. At this frequency, the spectrum level plot is checked for agreement with the assumed spectrum level of dB_R = 118-1/2 and a check is made for agreement of N_R and the assumed value of 10^9 cycles. If necessary an iteration is made to obtain agreement.

b. *Skin and Rib with Bonded Doubler* – The purpose of scalloped bonded doublers (Figure 2-47) is to provide increased strength at the skin to rib attachment with a minimum increase in weight. Doublers are scalloped to effect a stiffness taper and prevent premature cracking at the edge of the doubler.

Example

A skin, doubler and rib structure of 2024-T3 material is required to withstand an estimated spectrum level, dB_R, of 124 dB at the resonance frequency of the structure. The design life is 1000 hours at this load or $N_R \simeq 10^9$ cycles. Following through the chart as indicated by the arrows to an assumed rib spacing, S = 4-1/4, a skin gage, t = .023, and a rib gage, t_r of .043 is found. The approximate resonance is 325 Hz. The assumed values of dB_R and N_R are checked as in the example under a (skin and rib construction). The value of N_R is found to be slightly over 10^9 cycles but not enough to affect the results. Thus, the structure would be: S = 4-1/4, t = .025, t_d = .6 (.026) = .016, t_r = .050.

c. *Attachment Requirements* – Skin to rib attachments (Figures 2-48 through 2-54) may be critical in tension if their spacing is large or if they are small in diameter. Figure 2-48 gives the upper limit for attachment spacing for a particular attachment and structural configuration. Exceeding this limit will result in the attachment being of lower strength than the skin and ribs.

Example

A structure consisting of .063 skin supported by ribs at 5 inch spacing has been selected as satisfactory for acoustic loading. Skin to rib attachments are required to develop full strength of the skin and ribs.

Entering the chart at S=5 and following through to an assume diameter of 5/32 and AD rivet, a maximum spacing of .75 is found.

2-6 DESIGN NOMOGRAPHS FOR FATIGUE OF SECONDARY STRUCTURE EXPOSED TO RANDOM EXCITATION

Design of a large number of structural brackets to withstand structural fatigue can be aided by the nomographs presented in this paragraph. The brackets considered in these nomographs are hat sections, shelf brackets, and flat plates.

The parameters to be entered into these graphs are dimensions which are shown in the sketches accompanying each graph, the allowed rms stress (F) and Young's modulus (E) for the bracket material, the environmental level described as a power spectral density (PSD) (given in terms of g^2/Hz), the weight (W) of the package on the bracket, and the amplification factor (Q) for the shelf bracket.

When shelf brackets are used to support wire bundles, a Q of 5 should be used while a Q of 20 is more appropriate for component-supporting brackets. The weight in the case of the wire-bundle support should be half of the weight of the bundle between the two adjacent supports. The PSD is obtained from either specifications or criteria once the frequency (f) for the loaded secondary structure has been estimated. The estimated value of f is also a variable that must be entered into the shelf-bracket nomograph. The random fatigue strength (F) is the required life expressed in terms of allowed root-mean-stress for the material.

This value is related to the standard (sinusoidal) fatigue strength (F_{sin}) and the recipical slope ($-\alpha$) of the standard fatigue curve (S-N curve) when it is plotted on Log-Log paper. The relationship can be written as:

$$F_{sin} = A(\alpha) F$$

The value of α varies from 6 to 8 for aluminum. In this range, an A (α) = 2 can be used. The values for A for other materials can be obtained from Figure 2-55. The value for F_{sin} can be obtained from standard fatigue data where the number of cycles is the product of the required life and the resonance frequency of the bracket. (See Section 12 for further details of sine/random equivalence.)

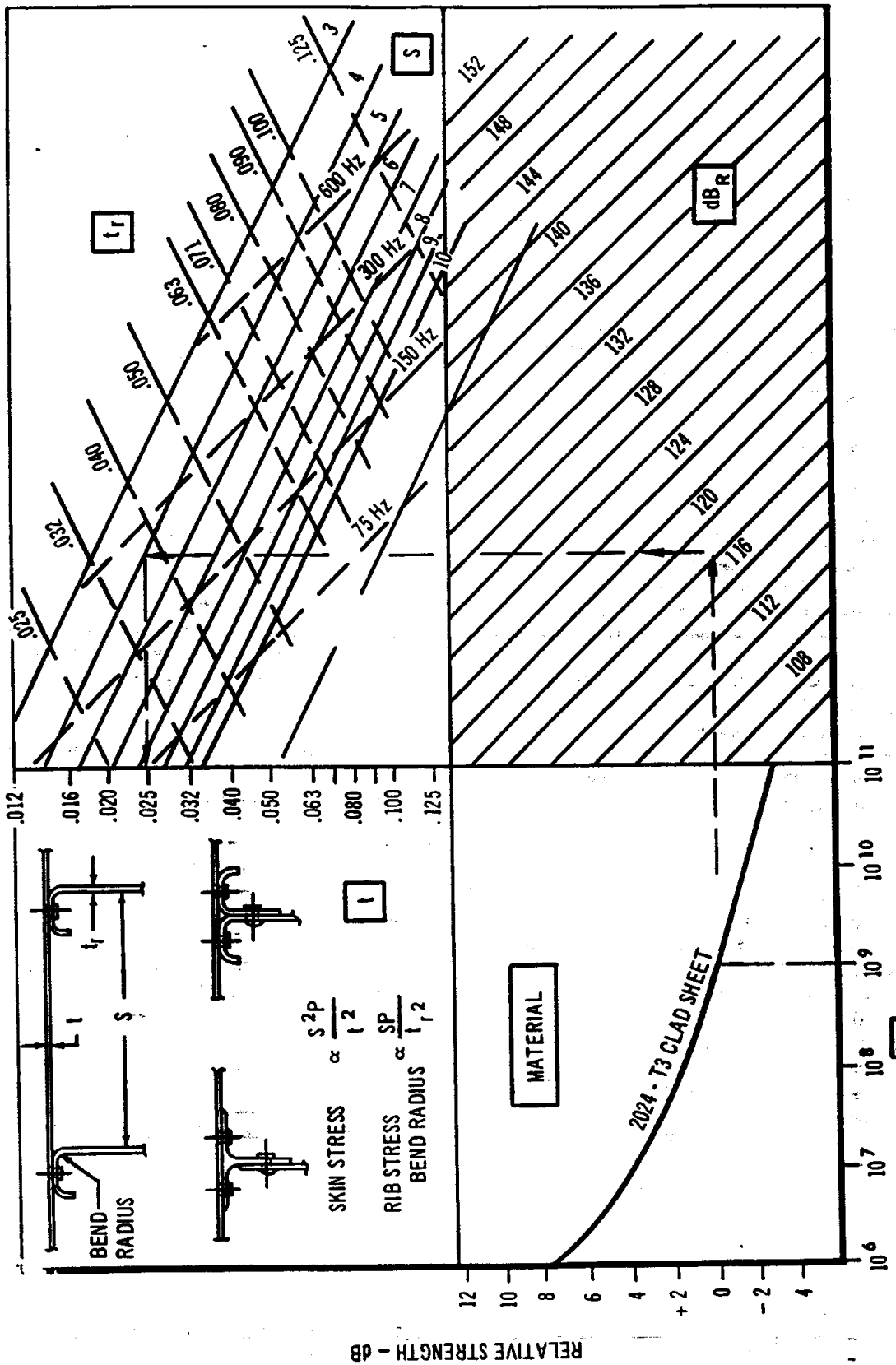


Figure 2-46. Skin and Rib Construction.

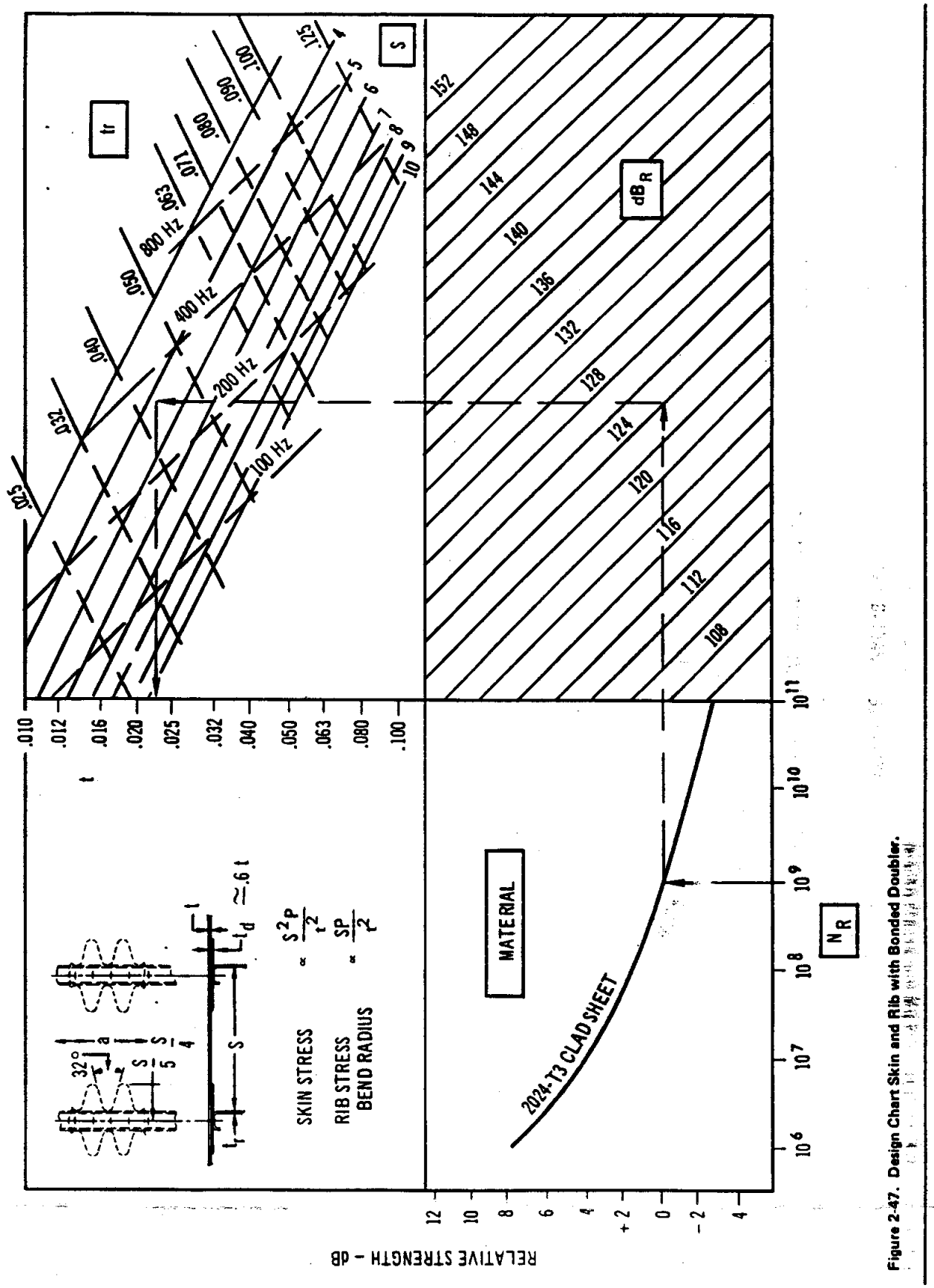


Figure 2-47. Design Chart Skin and Rib with Bonded Doubler.

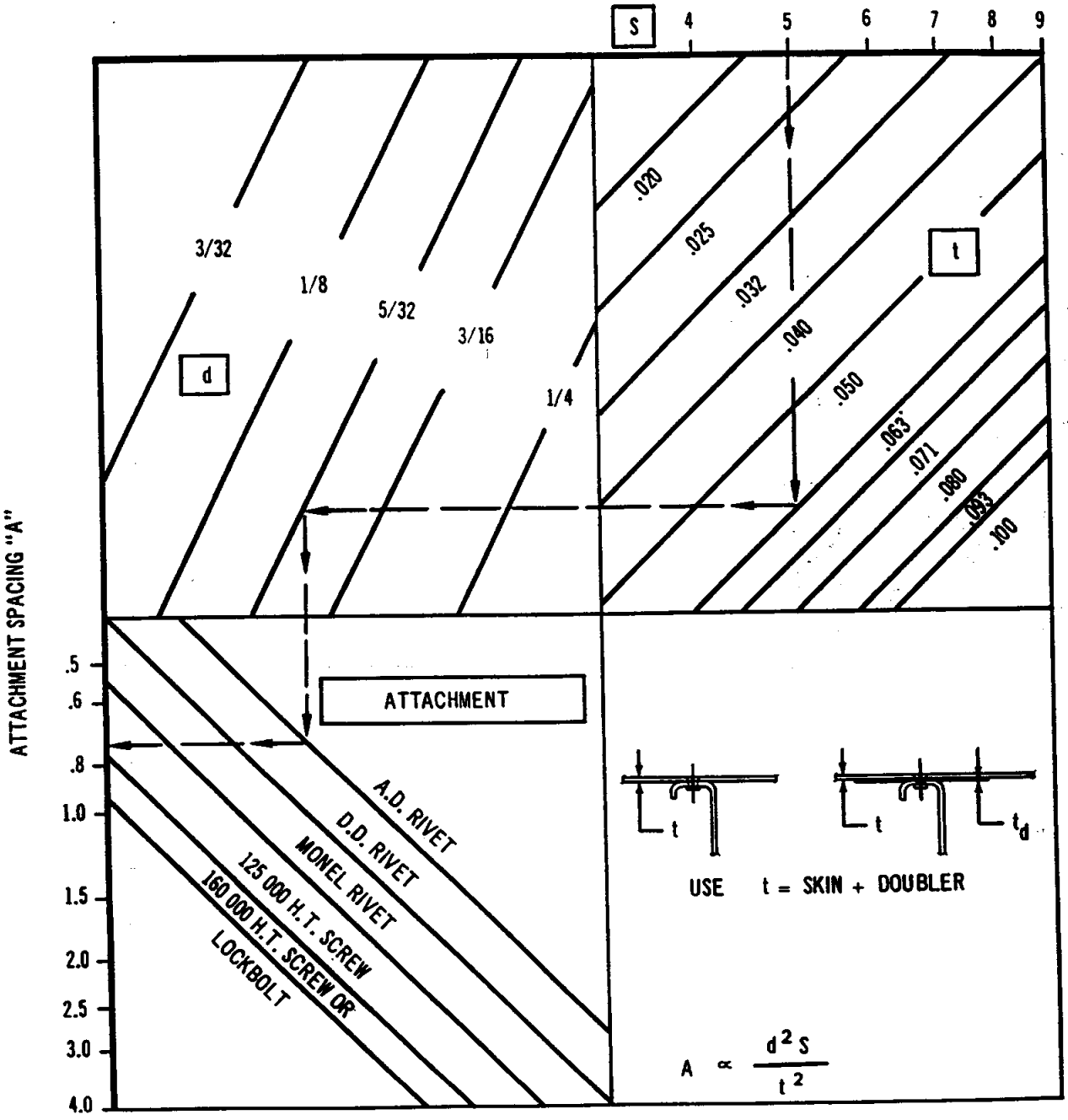


Figure 2-48. Design Chart for Attachments.

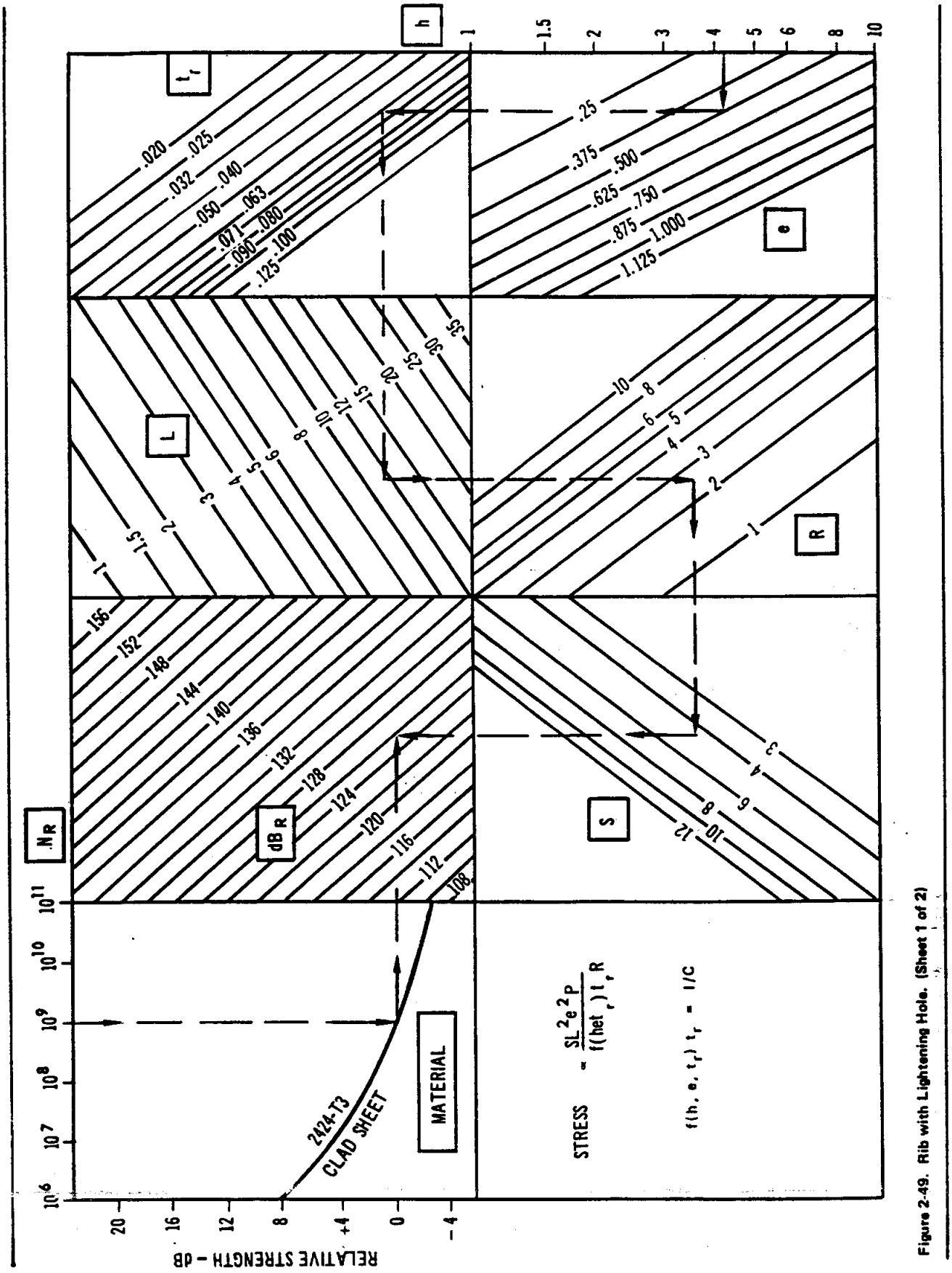


Figure 2-49. Rib with Lightening Hole. (Sheet 1 of 2)

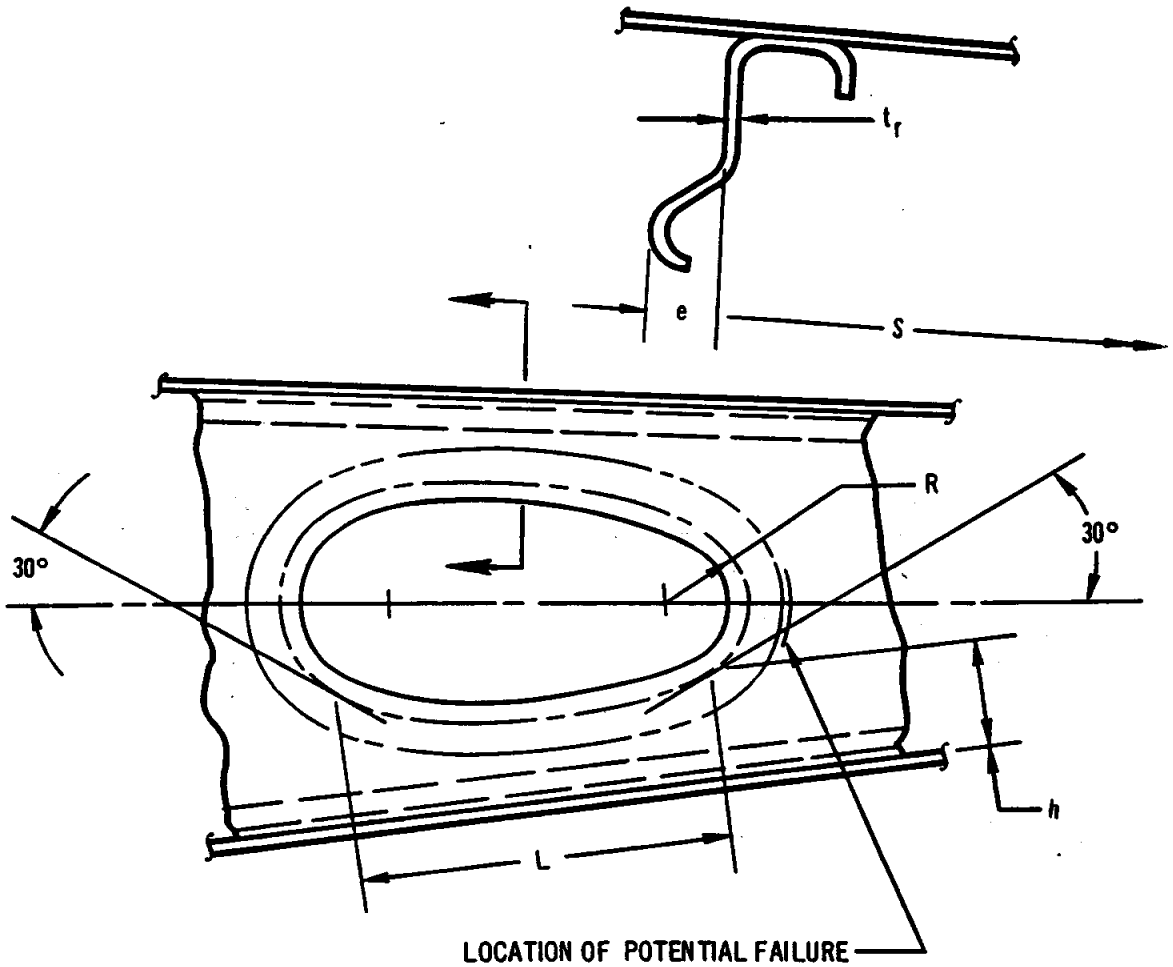


Figure 2-49. Rib with Lightening Hole. (Sheet 2 of 2)

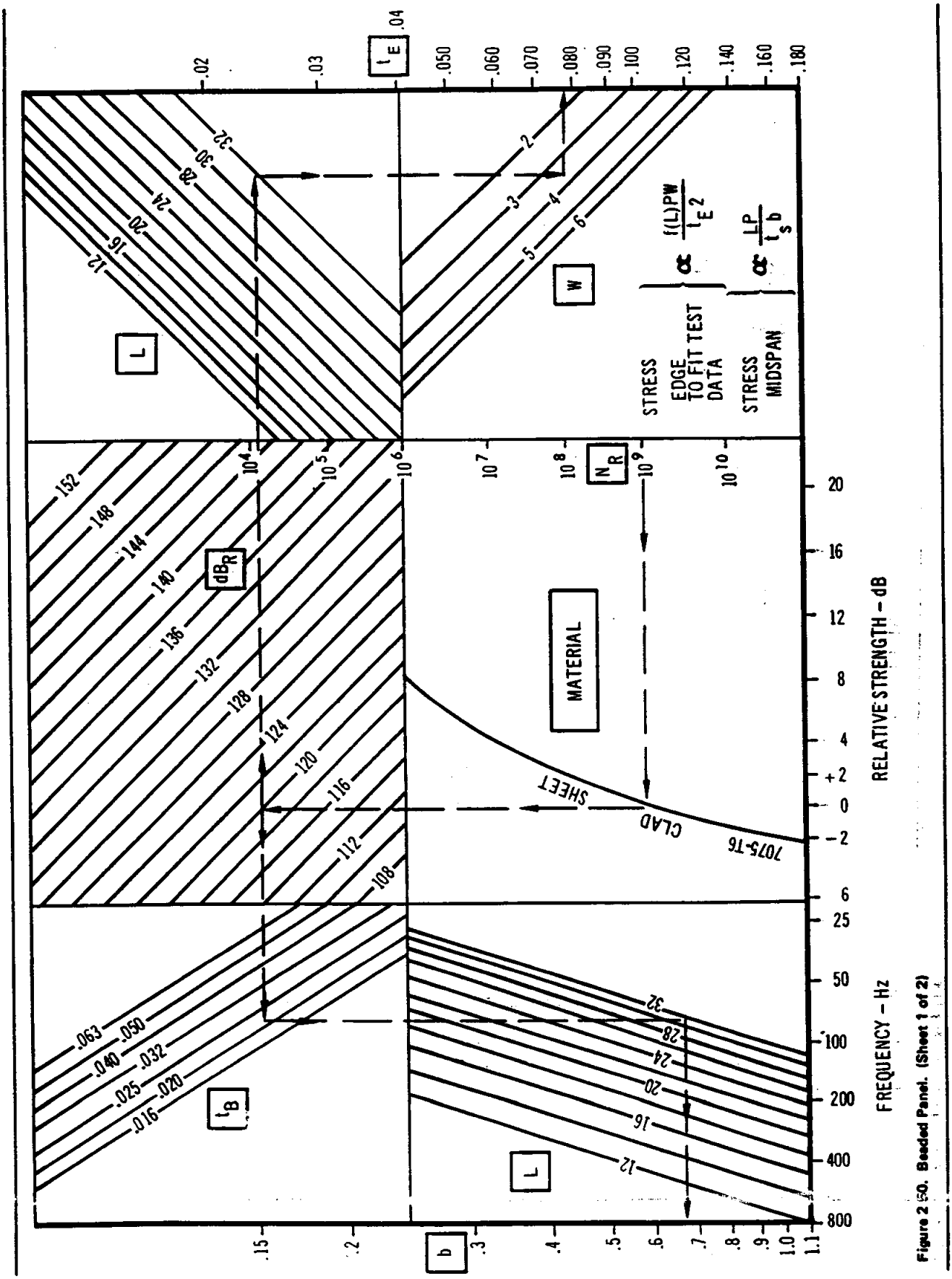


Figure 2-50. Beaded Panel. (Sheet 1 of 2)

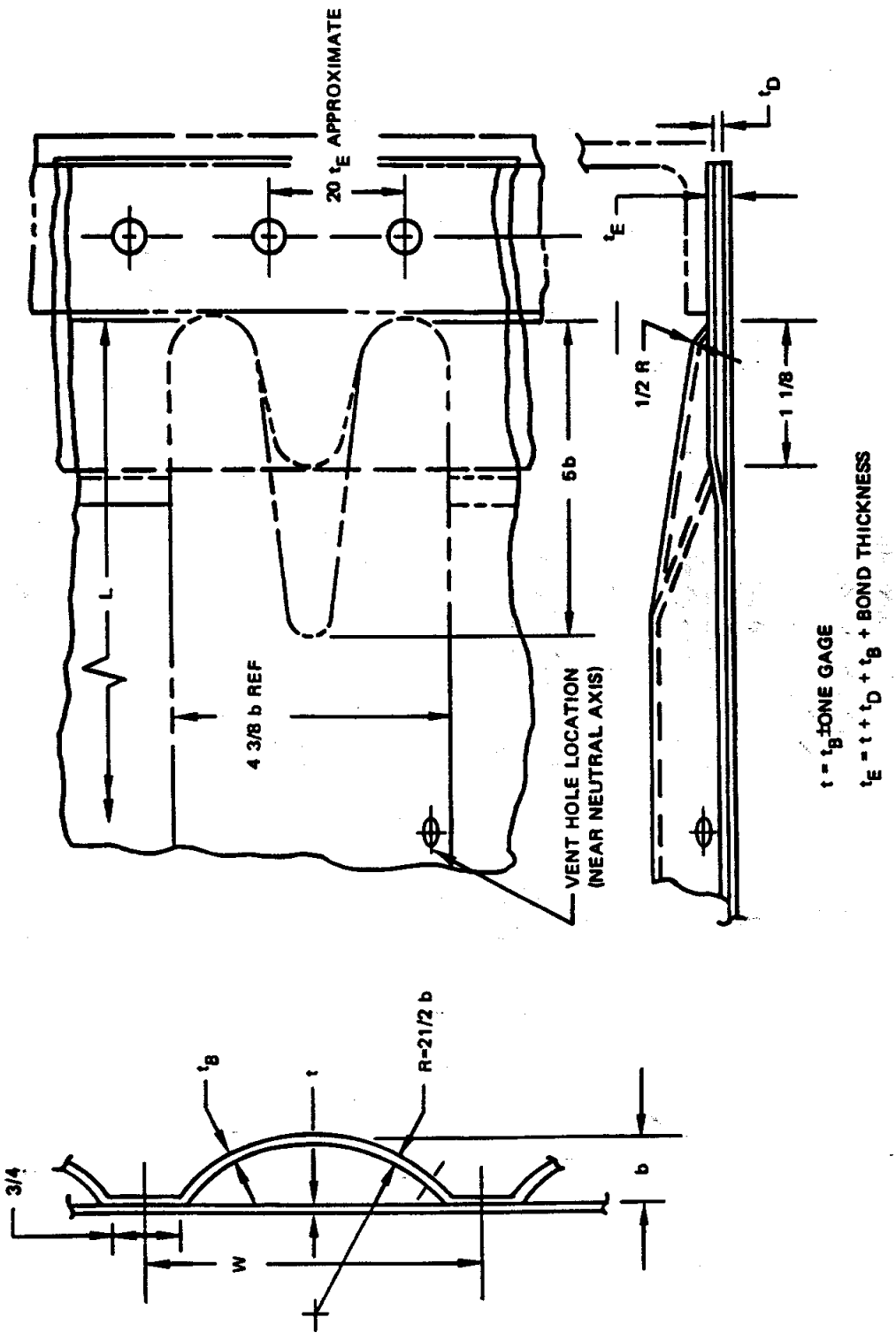


Figure 2-50. Beaded Panel. (Sheet 2 of 2)

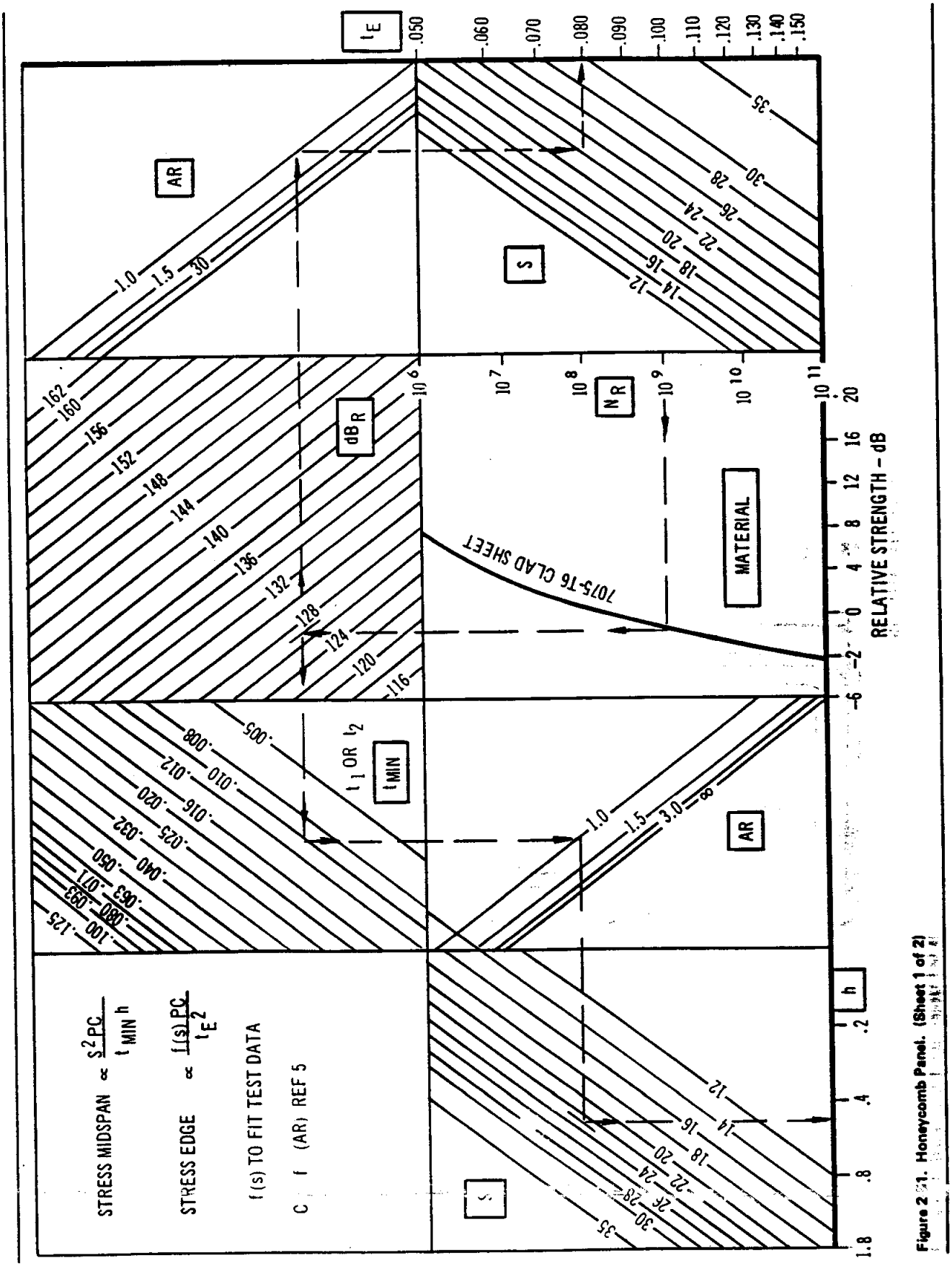


Figure 2 51. Honeycomb Panel. (Sheet 1 of 2)

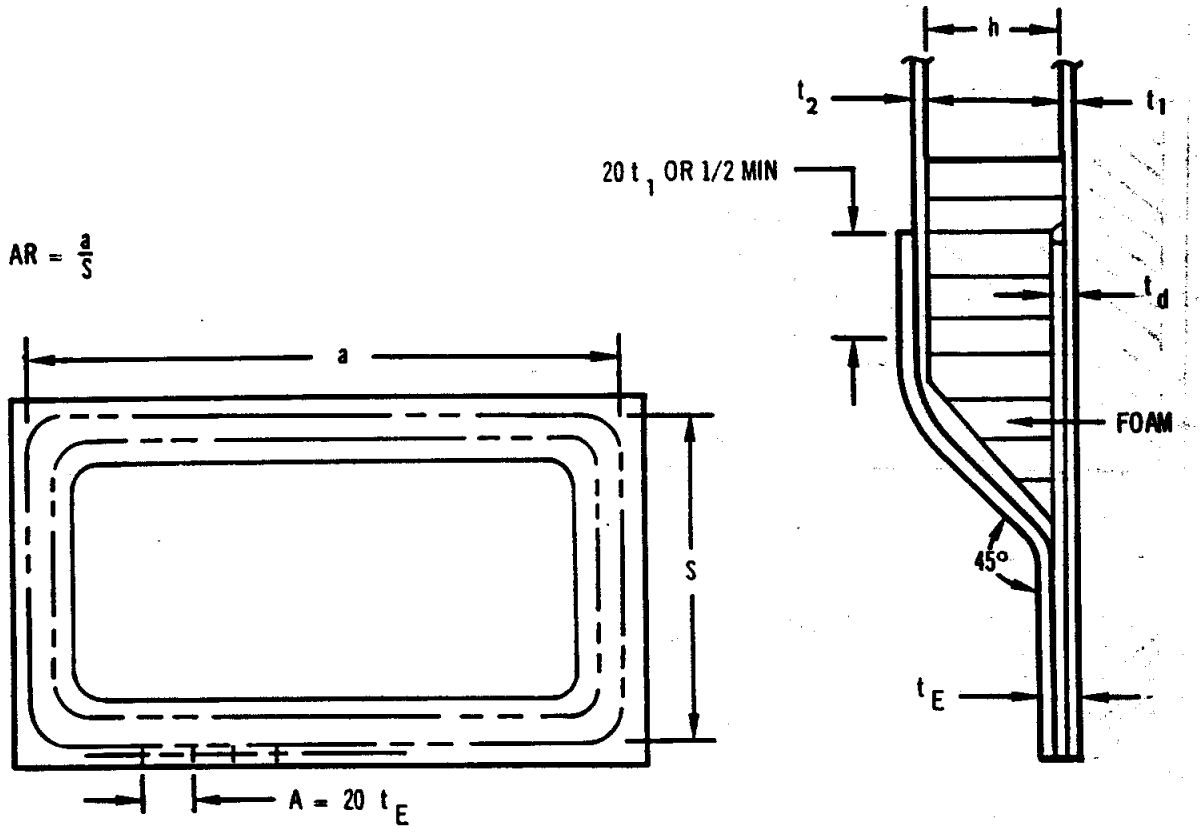
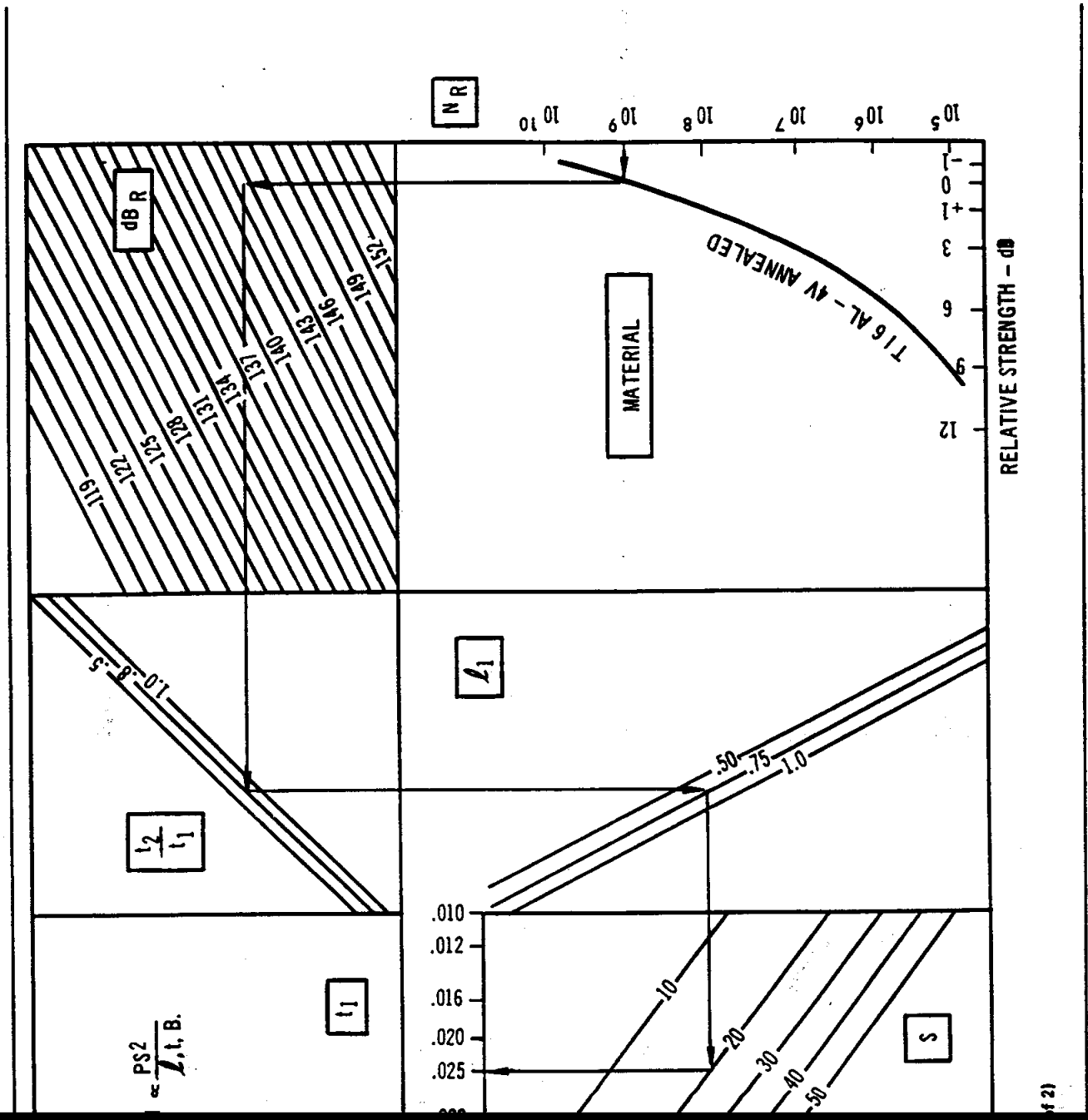
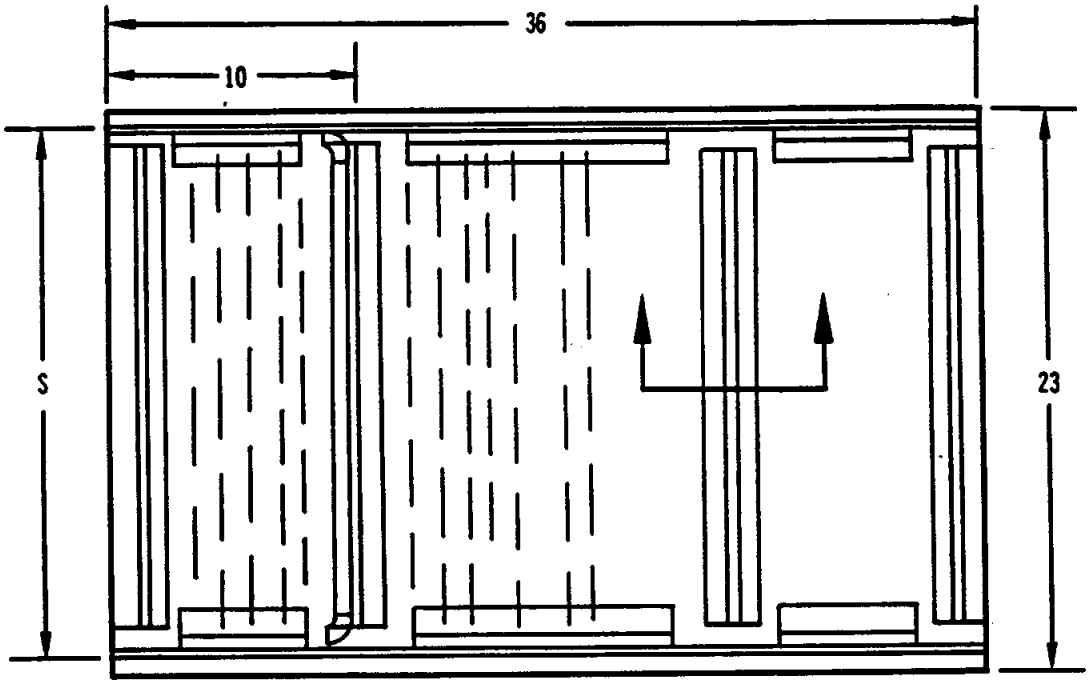
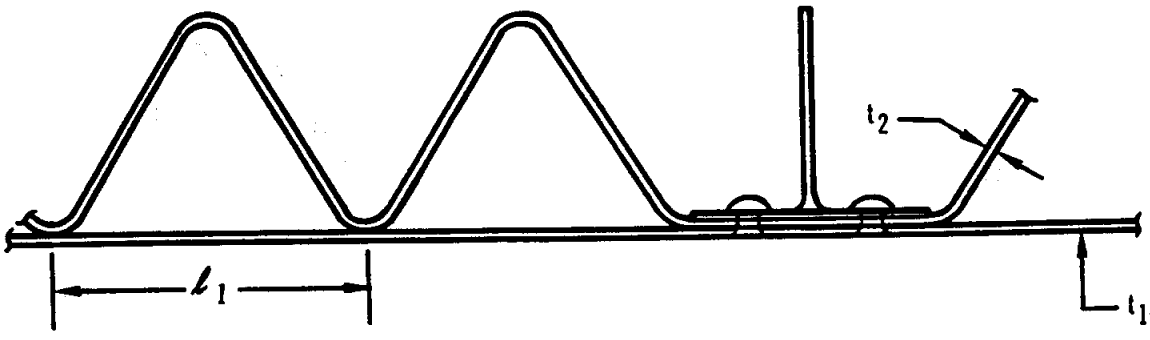


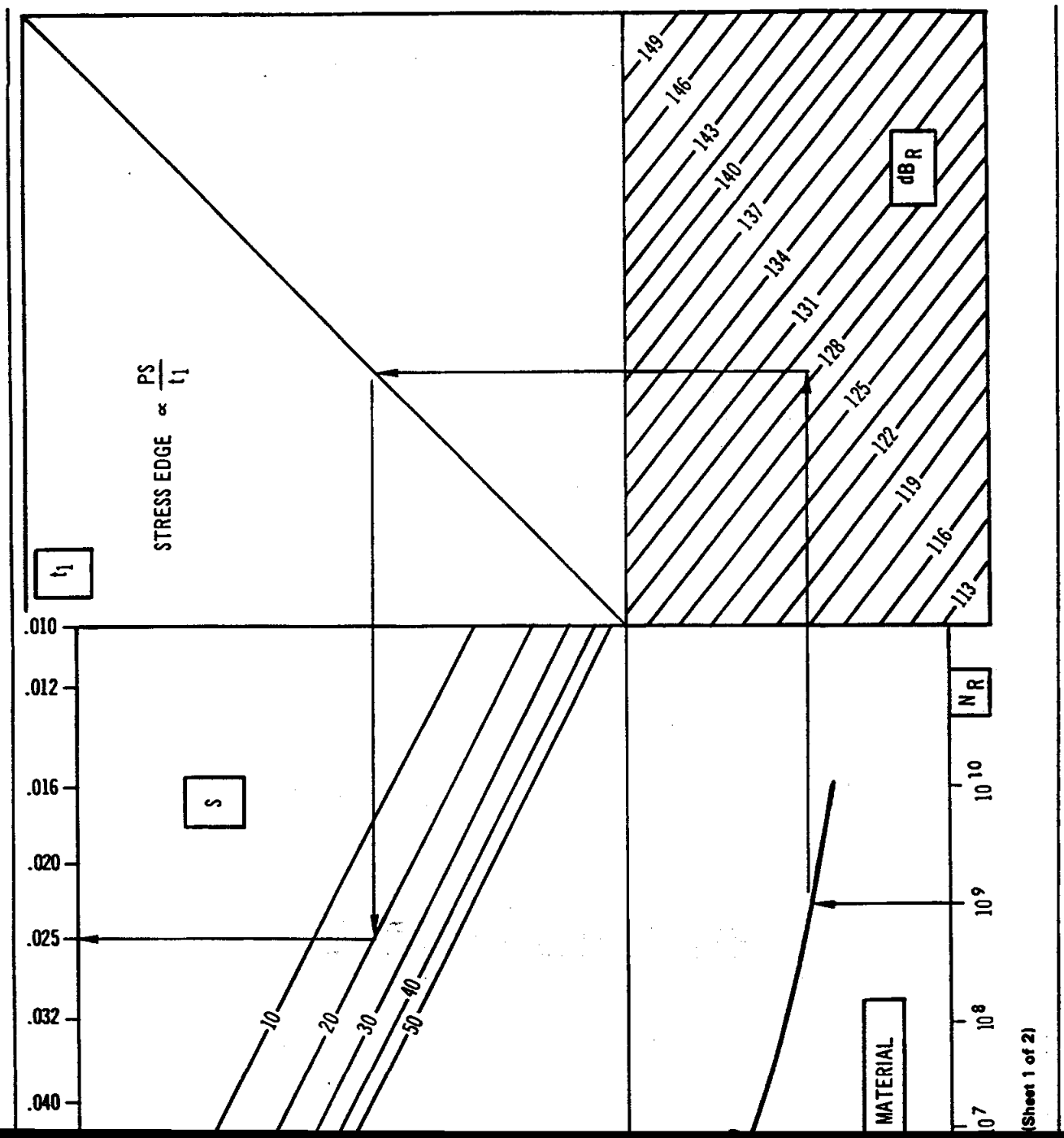
Figure 2-51. Honeycomb Panel. (Sheet 2 of 2)


N R
10 10
10 9
10 8
10 7
10 6
10 5
dB R
152
149
146
143
140
137
134
131
128
125
122
119
MATERIAL
T 6 AL - 4V ANNEALED
-1
0
+1
3
6
9
12
RELATIVE STRENGTH - dB
ℓ_1
.50
.75
1.0
$\frac{t_2}{t_1}$
1.0
.8
.5
.010
.012
.016
.020
.025
10
20
30
40
50
S
t_1
$\propto \frac{PS^2}{\ell, t, B.}$

VIBRATION, SHOCK, AND ACOUSTICS





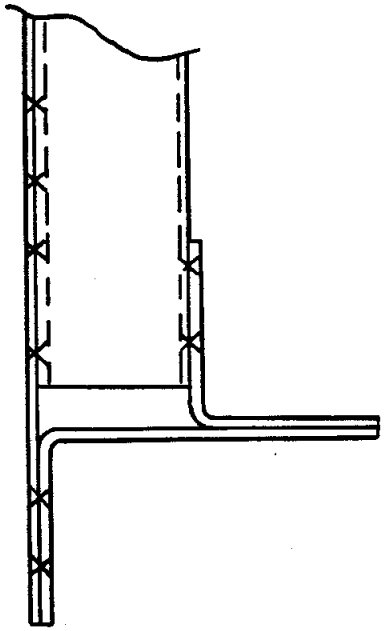

STRESS EDGE ∝ PS/t1
t1
.010
.012
.016
.020
.025
.032
.040
S
10
20
30
40
50
149
146
143
140
137
134
131
128
125
122
119
116
113
dB R
NR
10^10
10^9
10^8
10^7
MATERIAL

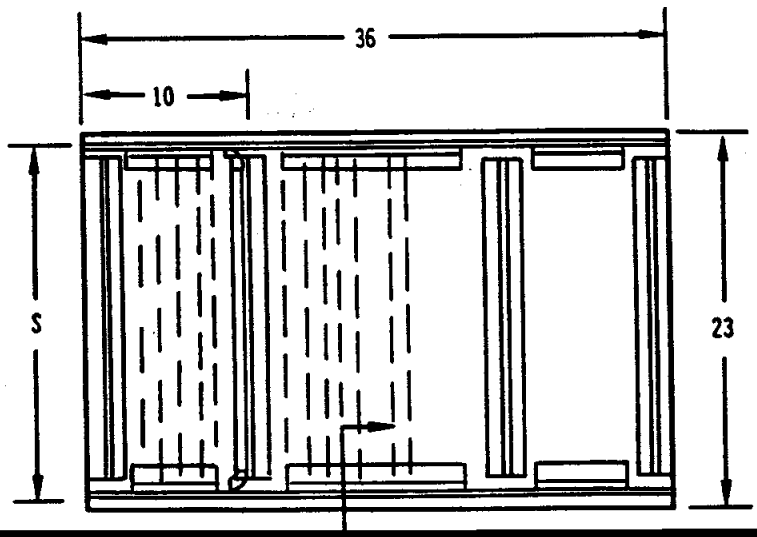
(Sheet 1 of 2)

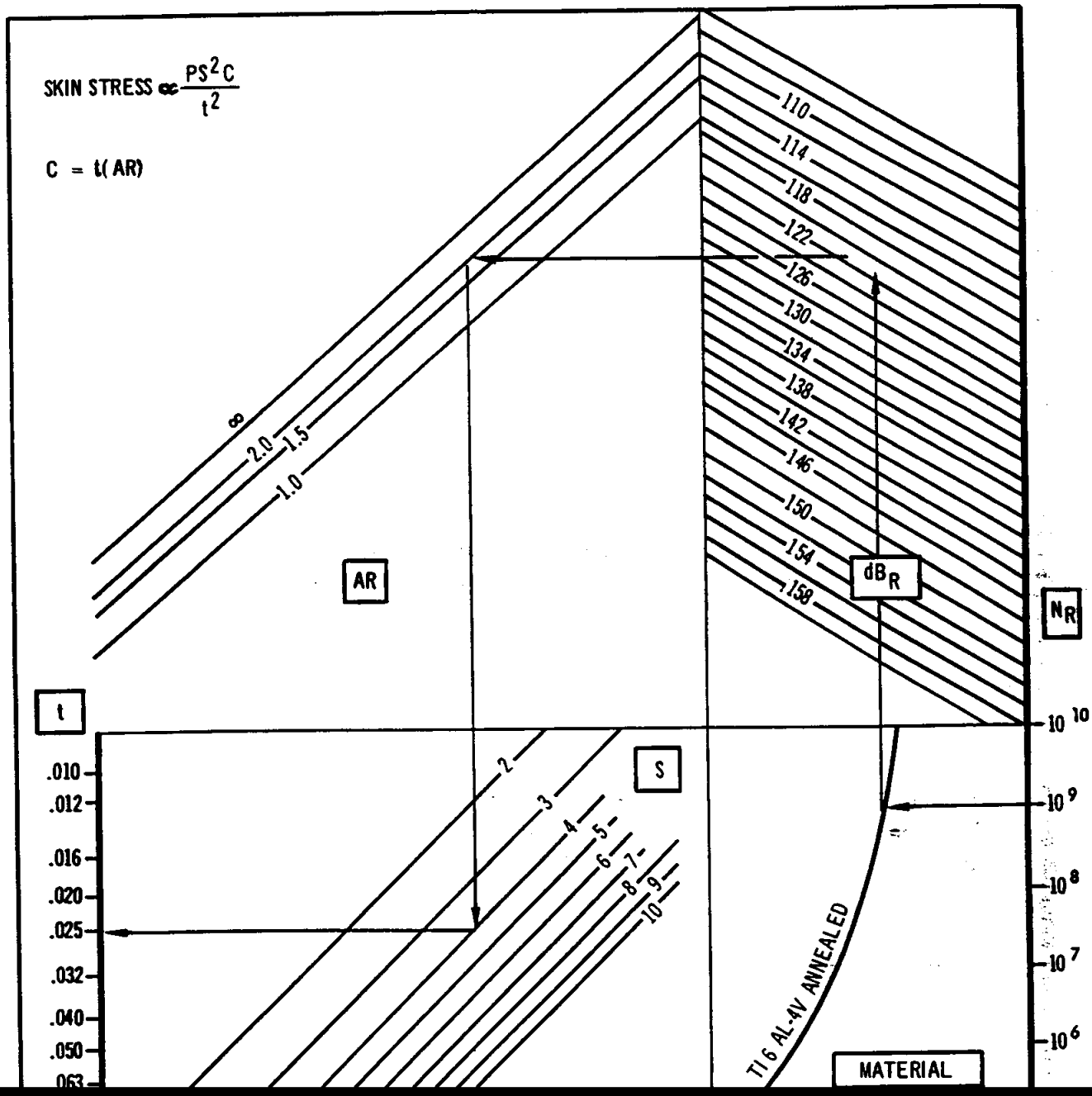
MCDONNELL DOUGLAS ASTRONAUTICS COMPANY

WESTERN DIVISION

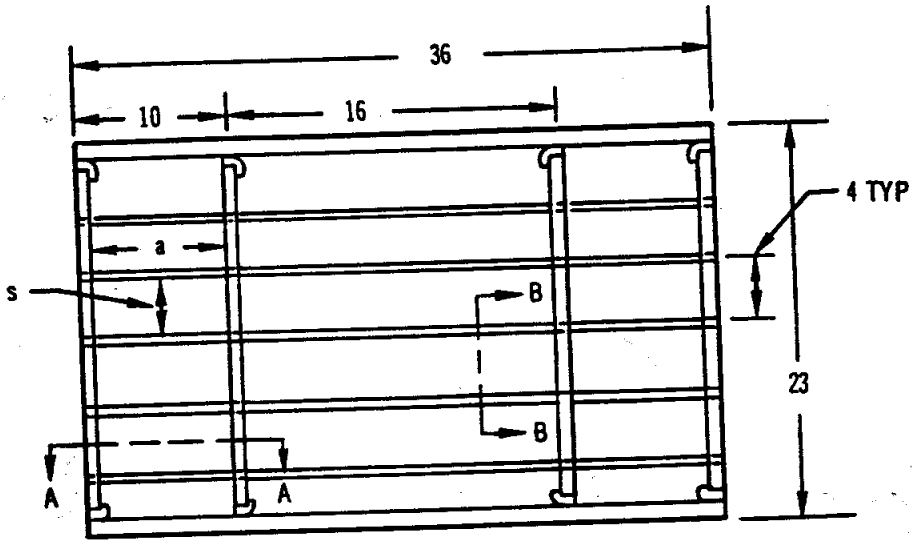
VIBRATION, SHOCK, AND ACOUSTICS



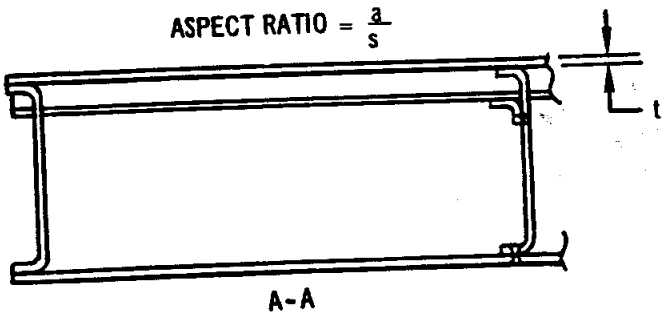



SKIN STRESS $\propto \frac{PS^2C}{t^2}$
C = t(AR)
∞
2.0
1.5
1.0
AR
110
114
118
122
126
130
134
138
142
146
150
154
158
dB R
NR
t
.010
.012
.016
.020
.025
.032
.040
.050
.063
2
3
4
5
6
7
8
9
10
S
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
TI 6 AL-4V ANNEALED
MATERIAL

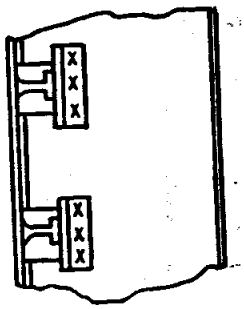
VIBRATION, SHOCK, AND ACOUSTICS



COVER NOT SHOWN



A-A



B-B

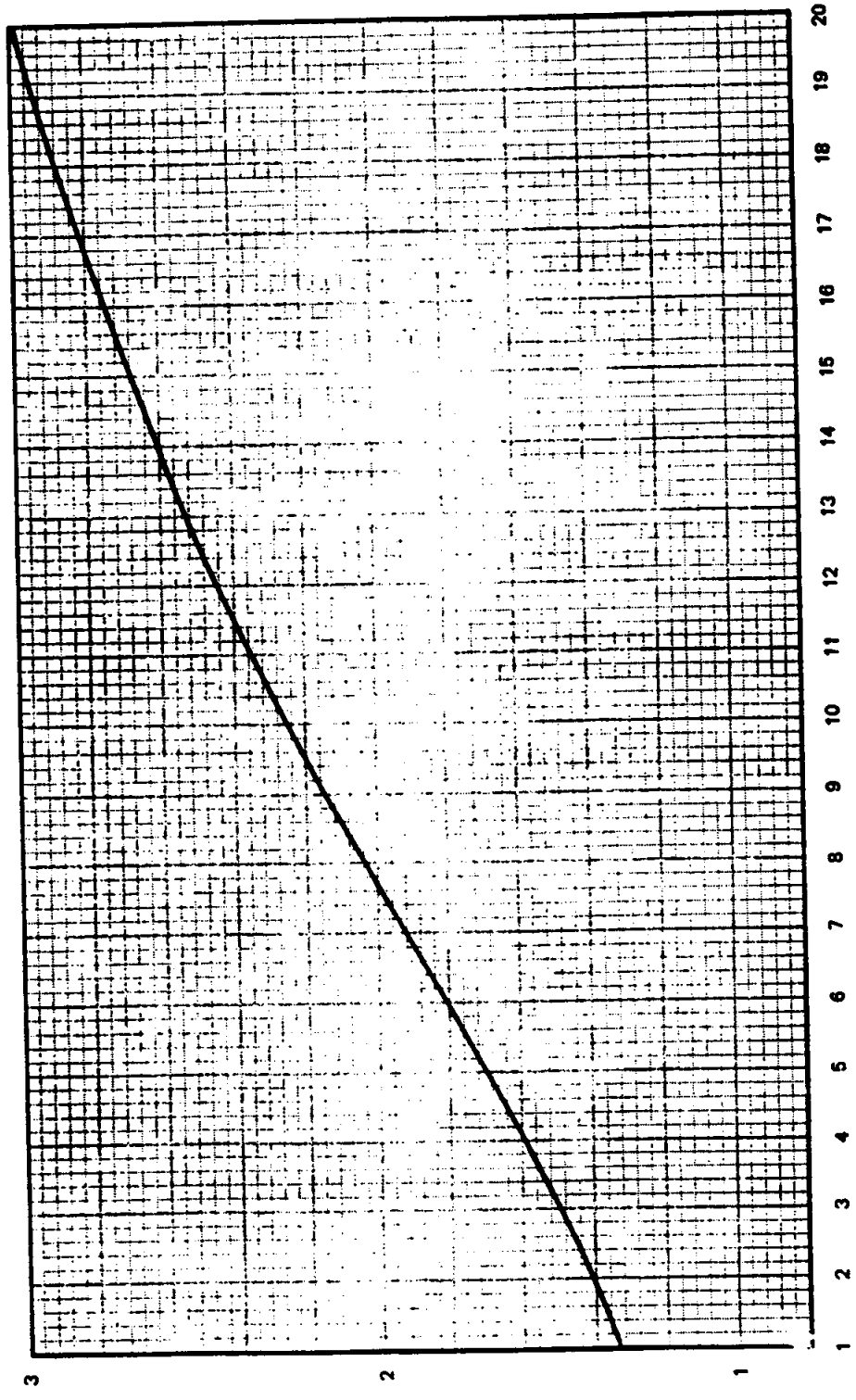


Figure 2.55. Sinusoidal Random Conversion Factor.

The nomographs are used to determine the material gage (h) needed to avoid structural failure due to fatigue by tracing through the graph from parameter to parameter as indicated on each graph. These graphs are all based on logarithimic scales. This fact can be used to obtain more accuracy when interpolation is required.

2-6.1 Hat Section

The following instructions apply in using the nomograph to size a mass-loaded hat section (Figure 2-56) to avoid structural failure due to fatigue.

The bracket skin gauge h (in inches) is the output.

a. Enter the nomograph (Figure 2-57) in the upper left-hand corner with the weight of the load in pounds.

b. Draw a horizontal line to the correct intersecting line which has the value ℓ the hat section depth in inches (Figure 2-56).

c. Draw a vertical line down from the point found in step b to the value of c, the hat section width in inches.

d. Draw a horizontal line to the right from the point found in step c to the value of a (height to c.g.) in inches.

e. Draw a vertical line up to the value of F (the allowable stress) in

f. Draw a horizontal line to the right to intersect a representative value for the PSD in g^2/Hz for the frequency range of hat-section response.

g. Draw a vertical line from the point found in step f downward to the h scale. The value of h found in this manner is the required skin gauge.

2-6.2 Shelf Bracket

The following instructions apply in using the nomograph to size a mass-loaded shelf bracket (Figure 2-58) to avoid structural failure due to fatigue.

The bracket skin gauge h (in inches) is the output.

a. Enter the nomograph (Figure 2-59) on the W scale with the weight of the load in pounds.

b. Draw a horizontal line to the right until intersecting the line with the correct value of a (the center of gravity height) in inches.

c. Draw a vertical line downward from the point found in step b to a line representing the correct value of ℓ (the critical bolt to pivot dimension in inches)

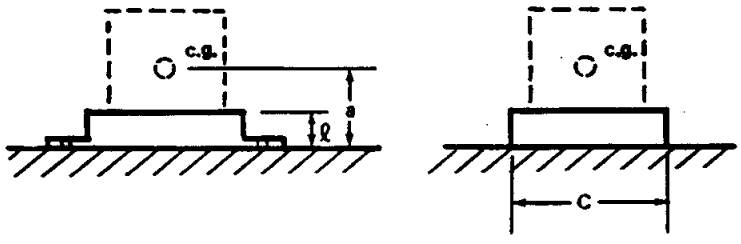


Figure 2-56. Mass Loaded Hat Section.

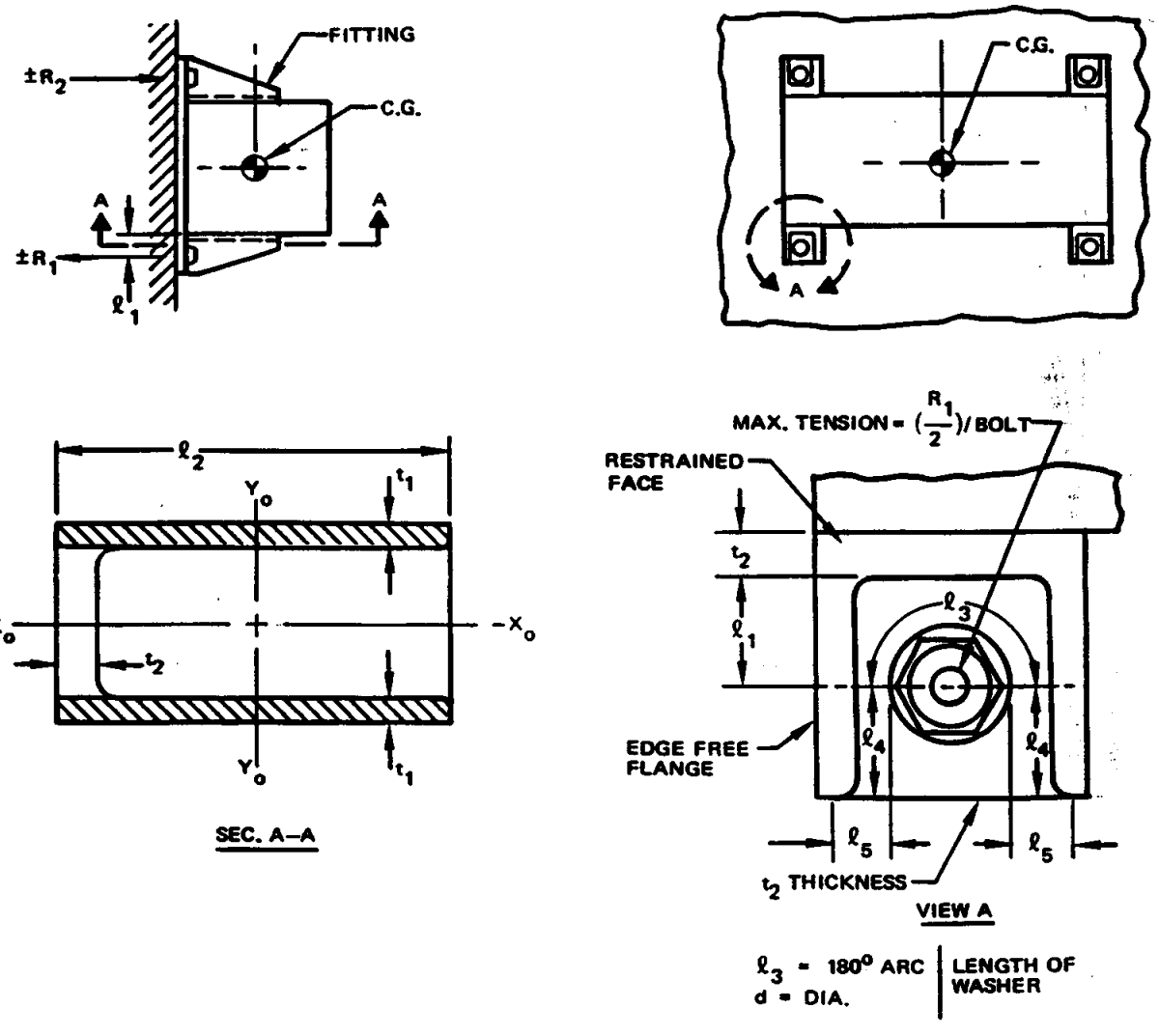


Figure 2-63. Equipment With Fitting Attachments – Hard-Mounted.

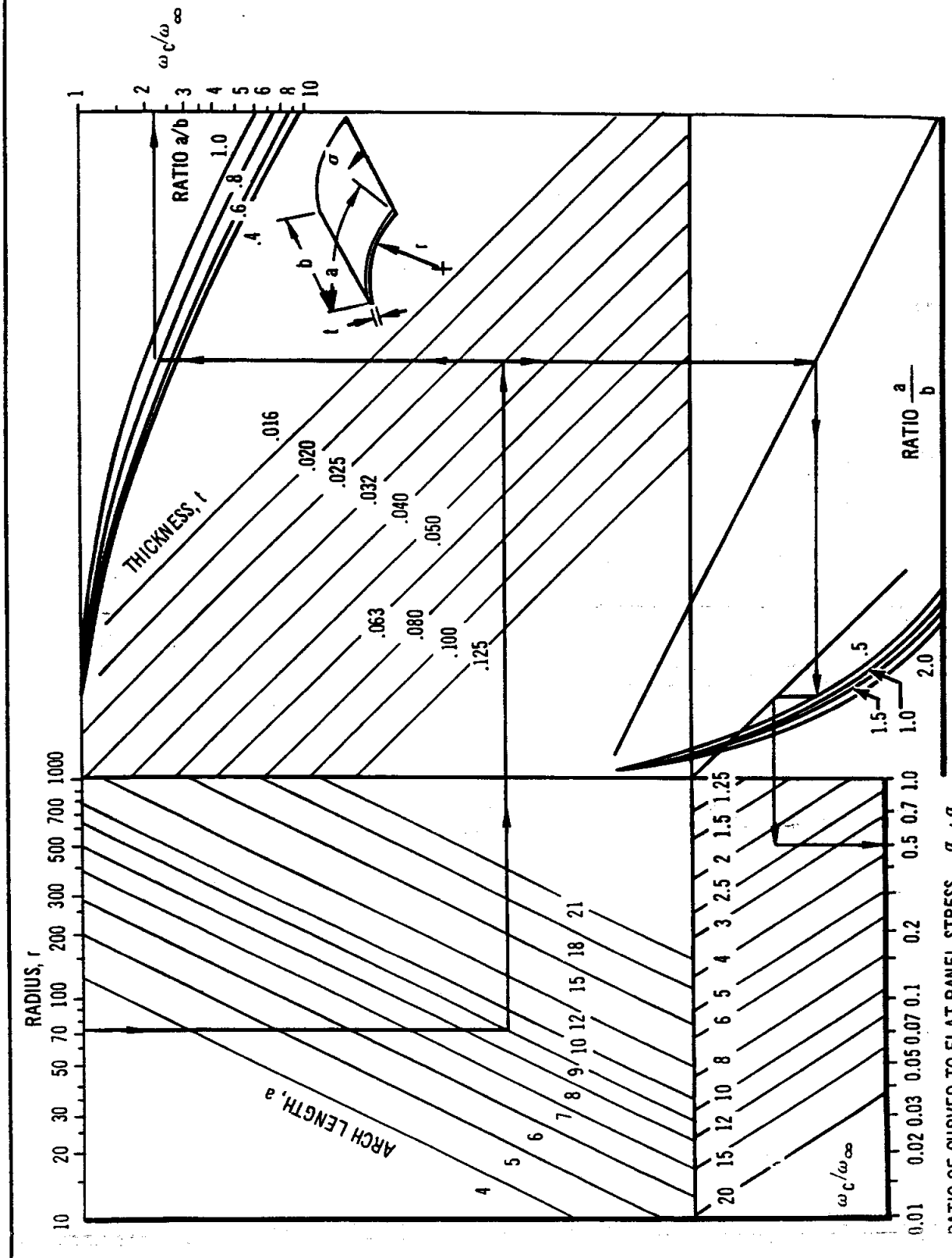


Figure 2-64. Effect of Curvature on Stress Ratio for Skin – Stringer Panels with Random Loading

2-7 BUFFET STABILITY

Vehicles having blunt noses and flared interstages can become unstable during atmospheric flight because of buffet pressures (static pressures which alternate between two amplitudes in the time and space domains). Although the large variety of geometric configurations employed on launch vehicles leads to an almost equal number of types of buffeting flow, it is possible to generalize to five primary types which could cause instability:

a. Shock-boundary-layer interaction.

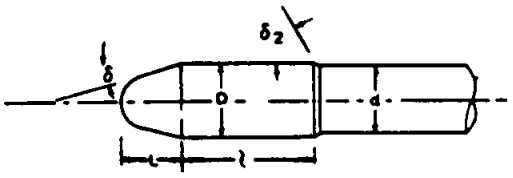
b. Separated wake.

c. Blunt body separation.

d. Separation ahead of a blunt configuration feature.

e. Separated wake impingement.

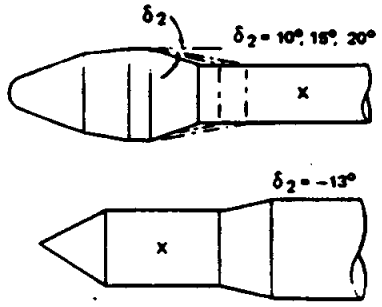
The first type is characterized by a highly localized region of fluctuating pressures near the foot of the shock wave (Figure 2-65). The second results when a disturbance is set up at the nose of the vehicle which causes the reminder of the body to fly in the resulting separated wake.



$d = 0.9$ TO $1.1\ D$ OR $\frac{D}{d} = 0.91$ TO 1.11
$L \geq 0.80\ D$
$\delta_1 \leq 15^{\circ}$
δ_2 NOT CRITICAL
$\ell > 1.5\ D$
REFERENCES:
1) NASA TM X-503 DATED MARCH 1961
2) NASA TM X-646 DATED MARCH 1962
3) NASA TM X-778 DATED DEC 1962

Figure 2-65. Typical Buffet-Free Geometry.

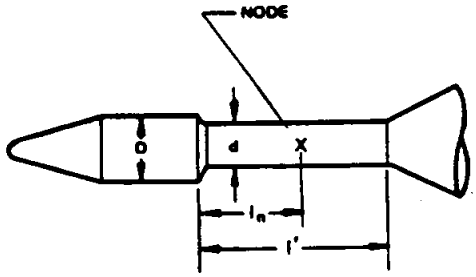
Blunt-body separation buffeting may occur if the vehicle configuration has an abrupt convex profile-change. Subsonically, the flow is not able to negotiate the corner and will separate at the shoulder. Unsymmetric reattachment points or body pressures under the separated flow can cause buffeting. However, as the flight speed increases there will be a forward velocity at which the boundary layer remains attached around the corner, and above that forward velocity buffeting will not occur. A fourth type occurs when a separated flow region builds up ahead of a blunt configuration feature, such as a steep flare or forward-facing step (Figure 2-66). If either the separated region or the resulting shock wave formation is unsteady, buffeting will result.



REFERENCES:
1. STL REPORT TM-60-0000-19075, 8/60
2. NASA TM X-787
3. NASA TM X-110

Figure 2-66. Typical Buffet-Prone Geometry.

Finally, the fifth type of buffeting takes place if separated wake turbulence (caused by a nose disturbance) impinges at some distance aft on the body (Figure 2-67). Assuming there is a self-excited oscillation of the structure in one of its natural frequencies due to aerodynamic effects, a portion of the buffet force which is dependent on the nose position will have the same frequency as the vibration structure. If this force is in phase with the vehicle motion at the point of application, the motion will be amplified and the vehicle will absorb energy from the flow. (This phenomenon is similar to flutter in that respect.) If the energy absorption, also called negative aerodynamic damping, exceeds the structural damping, an unstable condition will occur. It is primarily this latter type of buffet stability (type e.) that is considered in detail analytically, as it is the most common.



$\frac{D}{d} = 1$ TO 1.5
$\ell' \geq 2.8\ D$
$l_n \geq 1.6\ D$

REFERENCES:
1) NASA TM X-503, 3/61
2) NASA TM X-646, 3/62
3) NASA TM X-778, 12/62
4) NASA TM X-779, 1/63
5) NASA TM-D-1633
6) SR RFT TM-60-0000-AS-131

Figure 2-67. Multiple Buffet-Prone Geometry.

L. E. Ericsson et al,* have developed an analytical method for determining stability (for the type e. problem) which utilizes rigid body wind tunnel data and the modal properties of the system. The resultant criteria for stability assumes that:

a. The effect of drag is negligible.

b. The system is moving with constant speed through air of constant density.

c. Linearized methods can be used.

d. The oscillations are slow, i.e., of low reduced frequency so that quasi-steady theory is valid.

e. The lift distribution over the vehicle can be represented by discrete forces.

f. The lift force consists of two components:

   1. Local lift, i.e., lift due to local motion.

   2. Induced lift, i.e., lift induced by the wake created at the nose.

The modal response can be expressed as:

$$\ddot{q}(t) + 2\zeta\,\omega\,\dot{q}(t) + \omega^2 q(t) = F(\dot{q}) + F'(q)$$

This can be rewritten as:

$$\ddot{q}_L(t) + D_L\,\dot{q}_L + \nu_L^2\,q_L = 0$$

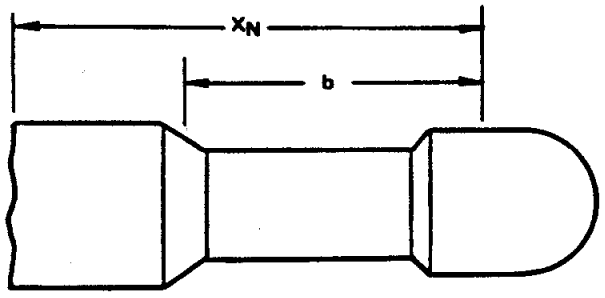
This equation (and therefore the mode) is stable if D_L and ν_L^2 are positive.

The stability criteria becomes:

$$0 < e + C_{L\alpha}(X_n) + \Delta^i\,C_{L\alpha}(X_n - b)\,\frac{F(X_n - b)}{F(X_n)} \cdot \frac{\sqrt{1+\sigma^2}}{G}\,\sin\left(\frac{\nu b}{V} + \text{ARC TAN }\sigma\right)$$

where:

e	=	$2V\delta'/B$
V	=	free stream velocity
δ'	=	% critical damping of the mode
B	=	$(\rho V^2 S/2MM)[F(X_N)]^2$
ρ	=	density
MM	=	Modal Mass
$F(X_N)$	=	Value of mode shape at tip (see sketch)
S	=	Reference area
$C_{L\alpha}(X_N)$	=	Normal lift derivative (at tip)
$\Delta^i C_{L\alpha}(X_N - b)$	=	Induced flare force Derivative measured statically if the displacement of the flare is small, or the wave length of a disturbance generated at the nose is large compared to the flare.
$F(X_N - b)$	=	Mode shape at flare
F'	=	dF/dx
ν	=	Frequency (rad/sec) of mode including aerodynamic stiffness $\simeq \omega$ for most cases



2-7.1 Clean Body of Revolution

The configuration shown in Figure 2-65 is considered free of buffeting except for highly-localized pressure fluctuations, i.e., a configuration with nose shapes having minimum theoretical incompressible-flow static-pressure-gradients in the following range. See appendix for definition of terms.

*Ericsson, L. E., et al. "A Mechanism for Self-Excited Oscillations of 'Hammer-Head' and Other Blunt-Nose Missiles" Proc. of the 6th Ballistic Missile and Aerospace Tech. Symposium, Vol IV, 1961.

$$\left[\frac{\Delta p}{q}\right]_{MIN} = 0 \text{ to } -0.14 \quad \text{and} \quad \left[\frac{d\left(\frac{\Delta p}{q}\right)}{d\left(\frac{X}{D}\right)}\right]_{MAX} = 0 \text{ to } 0.2$$

2-7.2 Buffet-Prone Bodies of Revolution

Separated flow—unstable configurations.

$$\left[\frac{d\left(\frac{\Delta p}{q}\right)}{d\left(\frac{X}{D}\right)}\right]_{MAX} = 0.2 \text{ to } 1.8$$

Figure 2-66 illustrates examples of undesirable configurations in this class. See appendix for definition of terms.

2-7.3 Multiple Buffet-Prone Body of Revolution

Separated flow impingement.

$$\left[\frac{d\left(\frac{\Delta p}{q}\right)}{d\left(\frac{X}{D}\right)}\right] > 1.8$$

Configurations having these pressure parameters or which fall in the configuration category shown in Figure 2-67 require wind tunnel evaluation to assess stability. See appendix for definition of terms.

2-7.4 Incompressible Flow—Static Pressure Coefficient

$$\frac{\Delta p}{q}(X) = -\frac{\partial}{\partial x}\int \frac{RR'}{\sqrt{(X-X')^2 + R^2}}\, dX$$

where:

X = the longitudinal coordinate of the pressure point

X' = the longitudinal coordinate of the body

R = the radius at station X'

R' = dR/dX'

D = the maximum diameter of the nose section

See Figure 2-68.

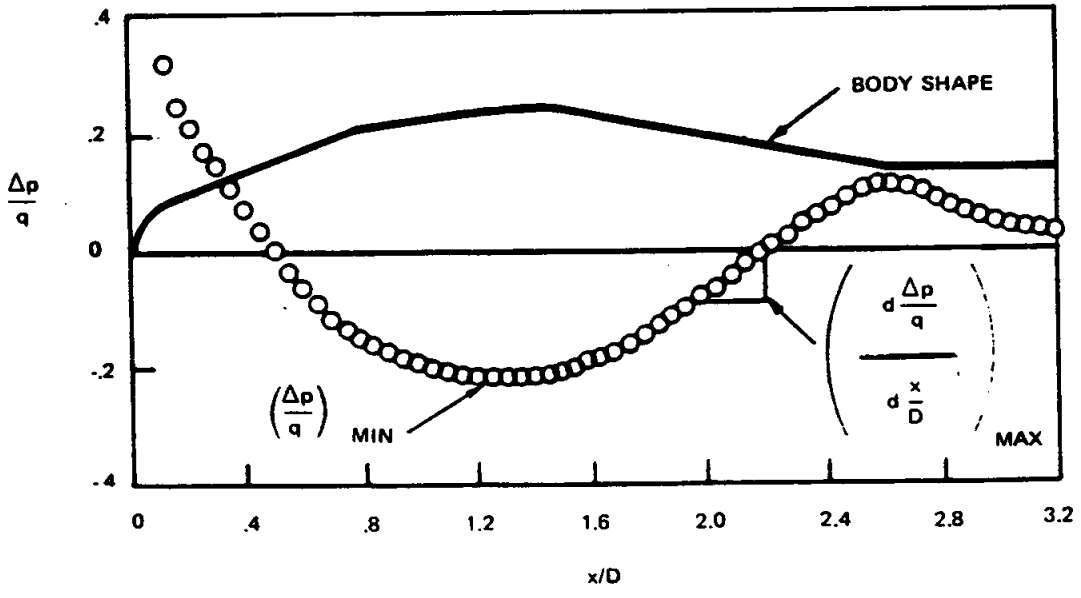


Figure 2-68. Typical Static-Pressure Coefficient Profile.

Section 3
ENVIRONMENTS

3-1 SOUND FIELDS

3-1.1 Typical Overall Sound-Pressure-Levels for Common Noise Sources

Typical sound-pressure-levels (SPL) for common noise sources are as shown in Table 3-1.

3-1.2 Sound Power From Rocket Engines

For a free jet from a single nozzle, acoustic power $\cong 0.5\%$ mechanical power. Acoustic power in dB re 10^{-13} watts

$$= 10 \log_{10} (21.83 F \times Isp \times 0.005 \times 10^{13})$$
$$= (10 \log_{10} F \times Isp) + 120.4$$

F = thrust in pounds
Isp = specific inpulse in seconds

For multiple jets, a correction may be in order to account for jet interference (a decreased power level).

For a deflected jet, a correction may be in order for a large generation of turbulence due to the geometry.

For choked or pulse jets, additional noise will be generated and this equation does not apply.

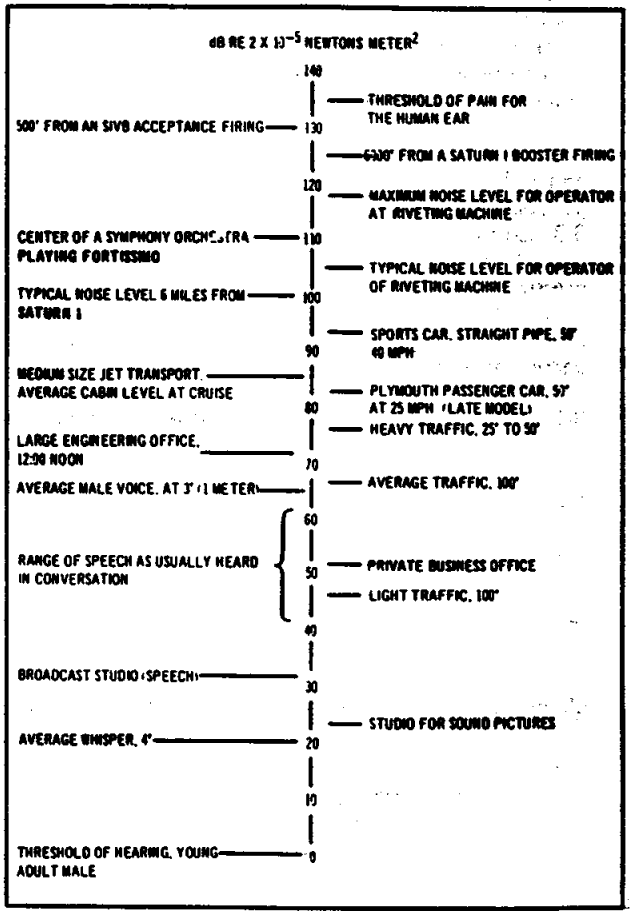
3-1.2a Rocket-Launch-Induced Sound-Pressure Level

Based on measured data from nine missiles and space systems, the maximum overall sound-pressure-level (OASPL in dB re $20 \mu N/m^2$) on the ground during a rocket launch, regardless of launch pad configuration, is:

$$OASPL = 117 + 10 \log_{10} F + 10 \log_{10} I_{sp} - 20 \log_{10} R$$

where R is the distance in feet from the ground point of interest to the launch pad center line. This equation is valid in the range of R from 5 to more than 5,000 nozzle exit diameters (much beyond a distance of 5,000 nozzle exit diameters requires the use of attenuation and/or focussing correction factors). This empirically derived expression is within 1 dB of the theoretical expression that can be found if the entire source can be considered at the center line of the launch pad and radiating into free space. Hence, to a first approximation, the distributed nature of the source seems to offset reflection, interference, or even near field effects.

Table 3-1. Typical Overall-Sound-Pressure Levels for Common Noise Sources



3-1.3 Directivity Maxima Versus Exit Velocity

Maximum radiation of sound pressure occurs at an angle (measured from the direction of the exhaust) that is 90° minus the Mach angle associated with the local highest velocity (for a free jet that is shock free). For a deflected exhaust, this is still approximately valid.

A distribution of acoustic pressures just outside the shear layer of any rocket exhaust is shown in Figure 3-1.

3-1.4 Sound Power Versus Altitude

There are various ways of characterizing the sound output of a source. In this survey three measures are considered: total power radiated, pressures near the source, and pressures far from the source. The manner in which these measures of output depend on density ρ and speed of sound c differs according to source type. It also depends on the sound frequency and the size of the source.

The ranges of application of the various approximate expressions given in Table 3-2 are defined by the values of kr and ka, where r is the distance between the source and the observation point, a is the effective radius of the radiator and k is the propagation constant equal to $2\pi d,\lambda$ being the wavelength of sound at the altitude under consideration. Low and high frequency refer to cases with small and large values of ka. The near field, or "near the source," refers to small values of (kr − ka), while that difference is large for a point in the far field.

A summary of the dependence of sound outputs upon ρ and c for a variety of sources is given in Table 3-2. To get dependence on altitude, the curves in Figures 3-2 and 3-3 can be used. These give the change with altitude of output in decibels when the power (Figure 3-2) or pressure (3-3) has the ρc dependence marked on the curves. The curves were derived from data for the ICAO Standard Atmosphere. The choice of curves for any particular source is indicated in the Table 3-2.

3-1.5 Jet-Exhaust Sound-Pressure-Levels Versus Mach Number

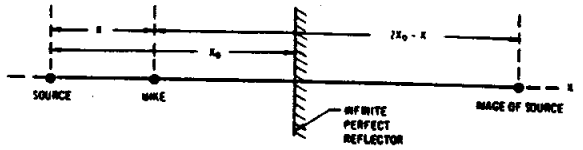
See Figure 3-4.

3-1.6 Acoustic Interference Effects

3-1.6.1 Reflection From a Nearby, Infinite, Perfect Reflector

Many acoustical measurements should ideally be made in a "free-field" . . . i.e., in a region in space completely free of all reflections. Practical measurements, however, must be made under conditions which are far from this ideal, just how far depending on the degree of accuracy desired. In particular, measurements of sound pressure levels (SPL's) in the acoustic field around large sound sources are often conducted out in the open over a plane surface (for example, an airplane on an extended concrete apron).

It sometimes happens that nearby buildings or walls introduce reflections which alter the measured SPL sufficiently to degrade the accuracy of the measurement to an intolerable degree. The accompanying curves permit an estimate of the magnitude of the error caused by a single, infinite, rigid wall, distant X_o from a source of sound, upon SPL measurements made on the source side of the wall at a distance X from the source. The spectrum of the source is assumed to be such that the direct and reflected sound *energies* (not pressures) add directly; i.e., random noise.



For random noise: $p(x)^2_{\text{total}} = p(x)^2 + p^2\,(2x_o\text{-}x)$

$$\text{Ratio} = \frac{\text{total}}{\text{direct}} = \frac{\dfrac{1}{(2x_o\text{-}x)^2} + \dfrac{1}{x^2}}{\dfrac{1}{x^2}} = 1 + \frac{x^2}{(2x_o\text{-}x)^2};$$

$$\text{Error in dB} = 10 \log_{10} \left[1 + \frac{x^2}{(2x_o\text{-}x)^2}\right].$$

This curve is shown in Figure 3-5.

3-1.6.2 Absorbent Surfaces Interference Effects

See Figure 3-6.

3-1.6.3 Interference Patterns Near Reflecting Surfaces

It is of interest to pursue the problem of acoustical reflection further than was done in paragraph 3-1.6.1, which dealt in an approximate way with sound from a point source,* and 3-1.6.2 which concerned absorbent surfaces.

If a plane progressive sound wave is reflected specularly by an infinite, flat reflector of zero absorption, the following mathematical solutions for the resultant interference patterns apply. Good acoustical judgement should be used in adapting these results to an actual situation.

NOTE

Good acoustical judgement consists mainly of keeping in mind the distinction between the mathematical model, in terms of which an analysis is made, and the actual physical situation to which it refers. Otherwise, misleading results may be obtained from the mathematical treatment.

The curves shown in Figures 3-5 and 3-6 are valid only for $(x_o\text{-}X) < \lambda/2$; otherwise interference effects invalidate even the rough approximation those curves yield.

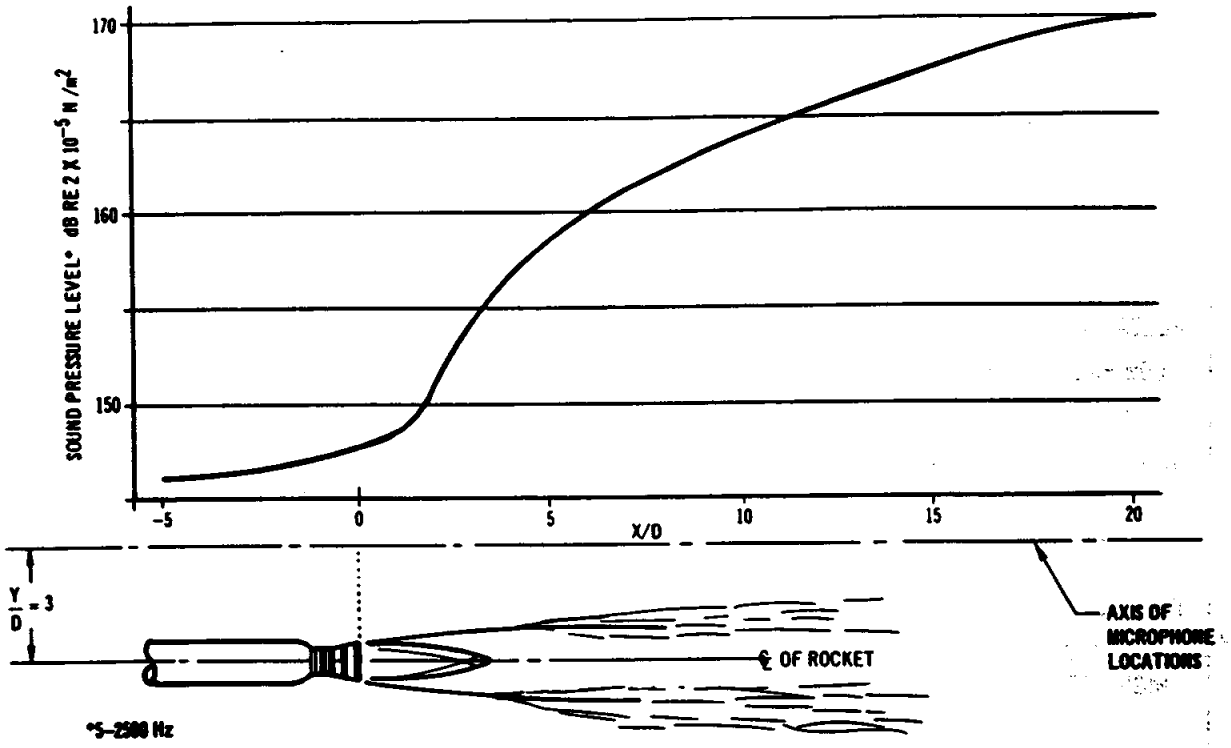


Figure 3-1. Distribution of Acoustic Pressures Along Rocket Exhaust (Free Field)

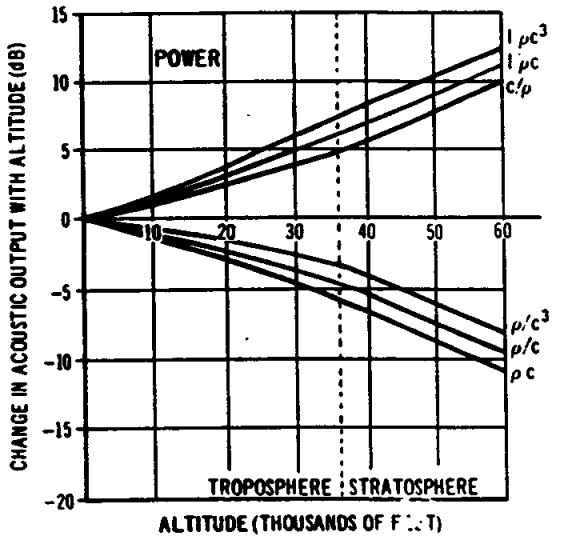


Figure 3-2. Total Radiated Sound Power Dependence on Altitude

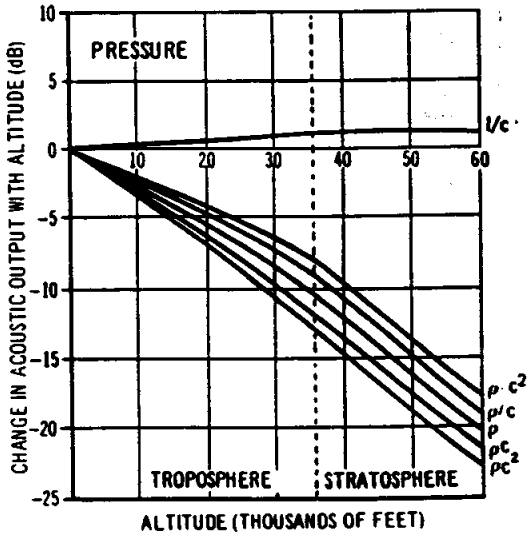


Figure 3-3. Radiated Sound Pressure Dependence on Altitude

Table 3-2. Dependence on Air Density and Speed of Sound, of Sound Pressures, and Power Radiated by Various Sources

TYPE OF SOURCE	NEAR FIELD PRESSURE $(kr-ka) << 1$	FAR FIELD PRESSURE $(kr-ka) >> 1$	TOTAL RADIATED POWER
	"CONSTANT VELOCITY" SOURCES		
PULSATING SPHERE			
LOW FREQ ($ka << 1$)*	ρ		ρ/c
HIGH FREQ ($ka >> 1$)*	ρc	ρc	ρc
VIBRATING SPHERE			
LOW FREQ ($ka << 1$)	ρ	ρc	ρ/c^3
HIGH FREQ ($ka >> 1$)	ρc	ρc	ρc
BAFFLED PISTON			
LOW FREQ ($ka << 1$)	ρ	ρ	ρ/c
HIGH FREQ ($ka >> 1$)	ρc	ρ	ρc
PISTON IN SMALL CAVITY	ρc^2	—	—
	"CONSTANT FORCE" SOURCES		
PULSATING SPHERE			
LOW FREQ ($ka \ll 1$)*	INDEPENDENT	INDEPENDENT	$1/\rho c$
HIGH FREQ ($ka >> 1$)*	INDEPENDENT	INDEPENDENT	c/ρ
VIBRATING SPHERE			
LOW FREQ ($ka << 1$)	INDEPENDENT	$1/c$	$1/\rho c^3$
HIGH FREQ ($ka >> 1$)	INDEPENDENT	INDEPENDENT	c/ρ
BAFFLED PISTON			
LOW FREQ ($ka << 1$)	INDEPENDENT	INDEPENDENT	$1/\rho c$
HIGH FREQ ($ka >> 1$)	INDEPENDENT	$1/c$	$1/\rho c$
PISTON IN SMALL CAVITY	INDEPENDENT	—	—
	AERODYNAMIC SOURCES		
TURBULENT JETS, ETC	—	ρ/c^2	ρ/c^5
TURBULENT BOUNDARY LAYER	ρ	—	—

*PRACTICAL LIMITS FOR ka MAY BE TAKEN AS ka LESS THAN 0.3 FOR LOW FREQUENCIES AND ka GREATER THAN 3 FOR HIGH FREQUENCIES WITH LESS THAN 1 dB ERROR.

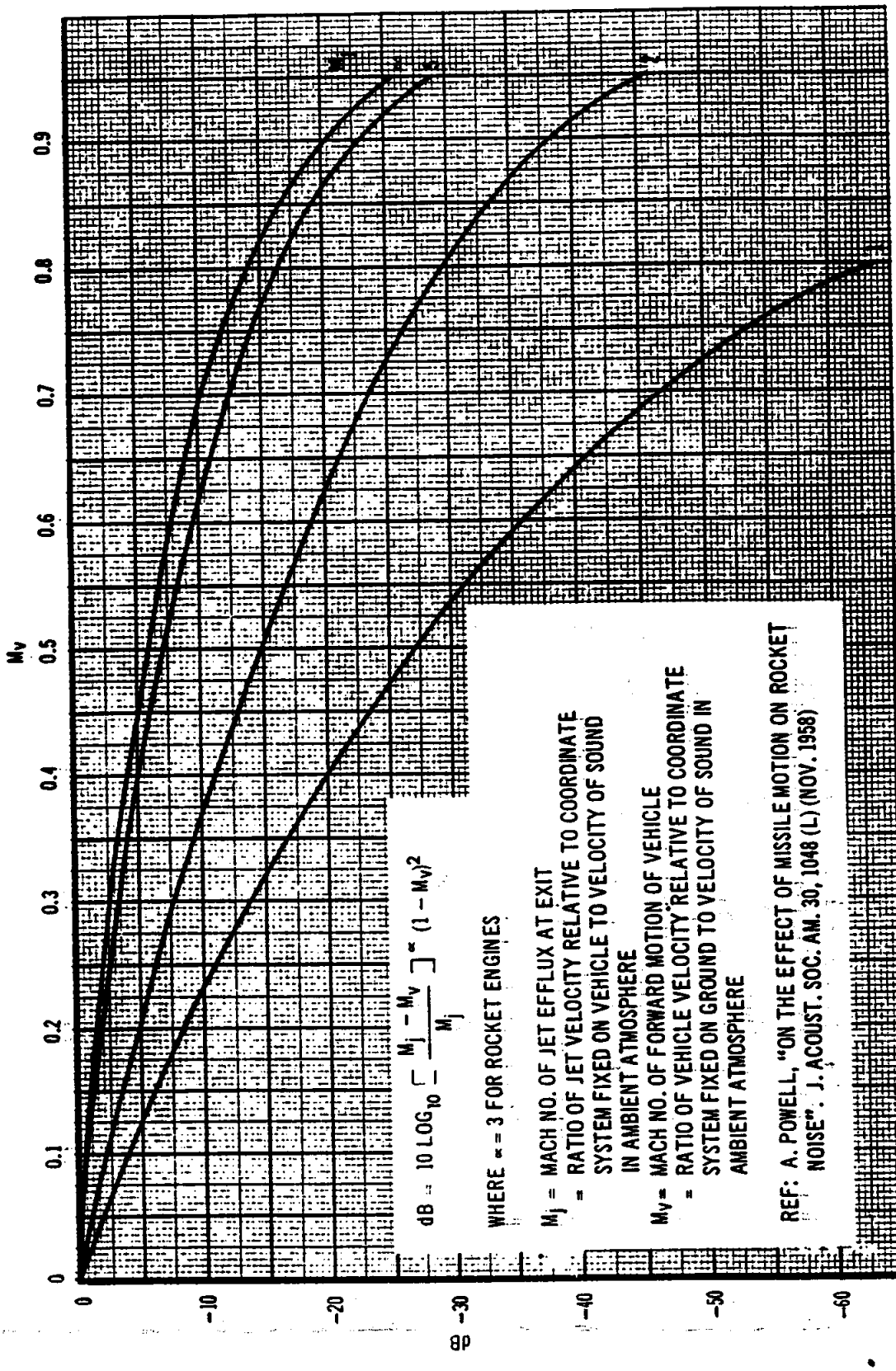


Figure 3-4. Reduction of Rocket Exhaust Noise Level — at Any Point on a Vehicle Forward of the Nozzle Exit Plane in Terms of Vehicle Mach Number

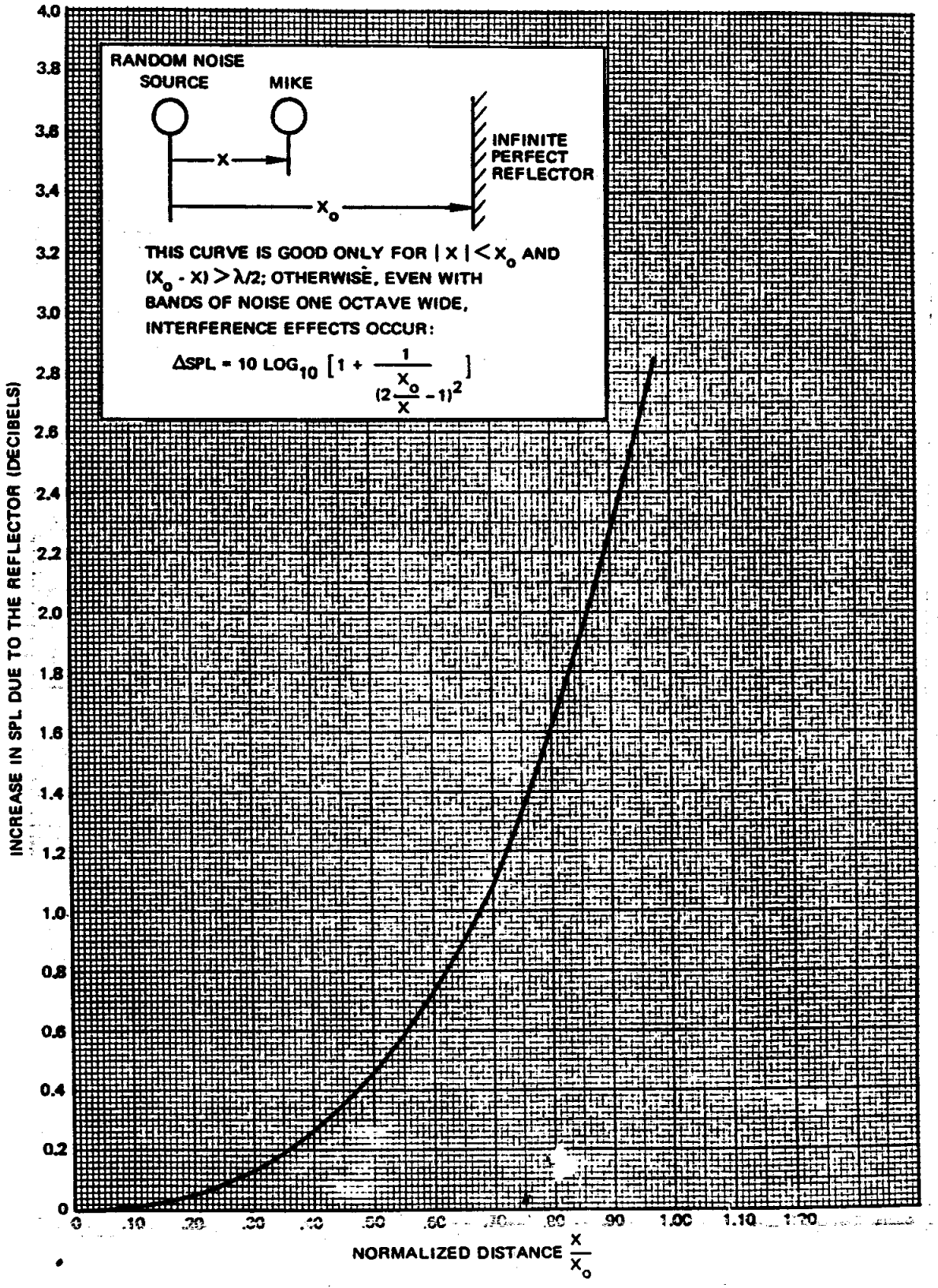


Figure 3-5. Effect of a Distant Reflector on SPL Measurements Error vs. Distance of Microphone from Source

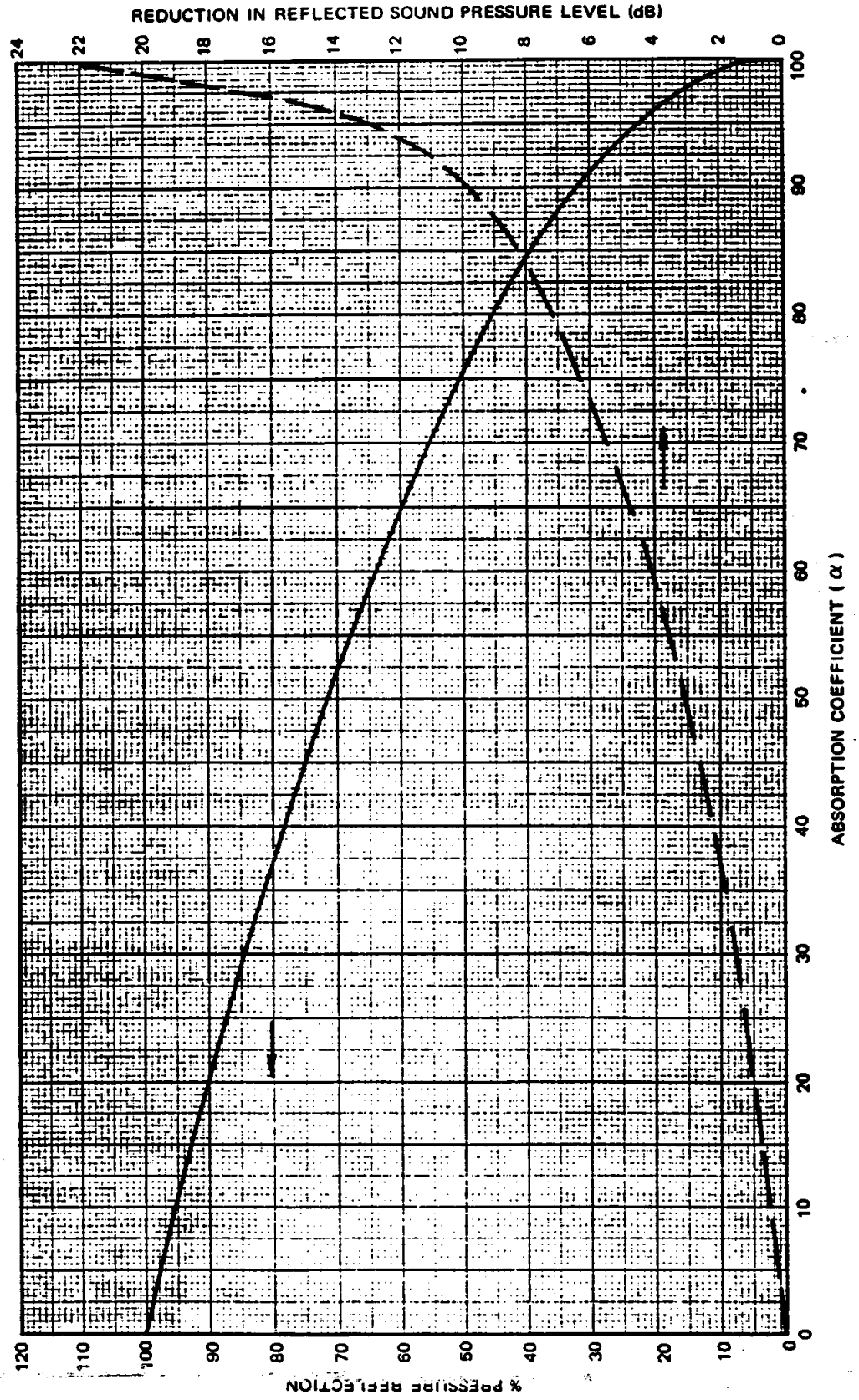


Figure 3-6. Effect of Energy Absorption by Reflector on Reflected Component of Pressure

For example, the "pressure doubling" phenomenon is well known: the pressure measured exactly at the surface of a large obstacle in a sound field will be double that which *would* be measured at the same point in the absence of the obstacle; the presence of the obstacle raises the free-field pressure by 6 dB. Reference to the field requirement, however, indicates that as a microphone is moved indefinitely far from a plane surface in a free field, it will register a level only 3 dB below the pressure at the surface. The reason for this apparent discrepancy is that the mathematical treatment from which the curve is derived assumes an *infinite* flat surface and a plane wave filling *all the space* on one side of the surface.

If we imagine a plane wave in the absence of the surface, it will fill *all* of space. Associated with it will be a sound pressure and an energy density whose (time) average values are constant throughout space. Now when the surface is introduced, all the energy which formerly would have passed beyond that surface is now reflected into the half-space on one side of it. This doubles the energy density there and therefore raises the average pressure on that side by 3 dB. Then, as a microphone is moved toward the surface, the pressure rises another 3 dB as indicated by the curve. This makes a total of 6 dB increase over the free-field case with no obstacle.

In practice one never encounters an infinite surface or an infinite plane wave; but when one moves far enough away from a large obstacle in an extended free-field, the sound field is very similar to what it would be in the absence of the obstacle. Thus the pressure here is 6 dB lower than on the surface of the obstacle.

In an actual situation, therefore, the theoretical curve on the monograph for free-field should not be used for values of Kx cos θ greater than about 7, unless both the surface and the effective beam of sound are extremely large and the sound wave nearly plane.

The reverberant case is one in which the mathematical model corresponds much more closely to the physical situation. Here, the increase of pressure near a surface, above the average pressure out in the middle of the room, actually is only 3 dB.

The difference between the proper uses of the several curves in this section of the manual must be kept in mind:

The curves in Figure 3-5 are for the purpose of estimating the error of a measurement made near to a source in a nearly free-field due to *complete* reflection from a rather distant obstacle. They are not to be used for calculating pressure errors near the reflecting surface.

The curves of the nomographs in Figures 3-7 and 3-8, on the other hand, are to be used mainly for estimating the pressure *on* a surface when the pressure has actually been measured a short distance away from that surface. Again the surface is supposed to be perfectly reflecting. Only plane and nearly-plane waves are considered.

The curves in Figure 3-6 represent surfaces which are *not* perfect reflectors, but which absorb a portion of the acoustic energy striking them. Moreover, they do not predict a pressure which a microphone might measure (i.e., the vector sum of the incident and reflected pressure waves at some point), but concern only the reflected component, expressed as a percentage of the incident pressure. For example, if the surface absorbs half the incident energy (α = 0.50), the reflected pressure component is 70.7% of the incident pressure; that is, 3 dB below what would have been reflected by a perfect reflector. If the surface absorbs 99% of the incident energy (α = 0.99), reflected pressure is 10% of the incident pressure, a 20 dB reduction.

For a single frequency of wave number $k = 2\pi/\lambda$ the mean squared pressure at a point a distance x from a flat reflecting surface in an otherwise free-field is:

$$\overline{p^2} = 1 + \cos(2k \times \cos\theta)$$

where θ is the angle the plane wave of incident sound makes with a perpendicular to the reflecting surface.

For an octave band of frequencies, this becomes

$$\overline{p_b^2} = \frac{K}{\sqrt{2}}\left[1 + J_0\left(\frac{K}{\sqrt{2}} x \cos\theta\right) \cos\left(\frac{3}{\sqrt{2}} Kx \cos\theta\right)\right]$$

where K is the geometric mean frequency of the octave band, and $J_0(z)$ is the zero order spherical Bessel function:

$$J_0(z) = \frac{\sin z}{z}$$

For 1/3-octave bands, a similar expression results:

$$\overline{p_{b/3}^2} = 0.2311\ K\ [1 + J_0\ (0.2311\ Kx \cos\theta) \cdot \cos(2.0130\ Kx \cos\theta)]$$

Neither of these two "wide-band" expressions can be plotted against Kx cos θ (their most usable form) unless normalized for wave number i.e., p_b^2/K. Therefore, the normalized curve is plotted on the nomograph shown in Figure 3-7 and attention is called to the care required in its use.

Making comparisons of the normalized pressures at different distances for some fixed wave number or frequency becomes especially simple, since in such a ratio or comparison, $\overline{p_b^2}/K$ has the same wave number and the normalizing factor is cancelled. If it is desired to allow K to vary, the ordinate $\overline{p_b^2}/K$ must be adjusted appropriately to get $\overline{p_b^2}$ for each K.

The same wide-band pressure parameters can be calculated for a Reverberant Sound Field. (This assumes that our attention is confined to points near the center of a reflecting surface and not too close to a rectangular edge or corner; i.e. The point on the surface, perpendicularly opposite the point of measurement, should be at least 10K times x distance away from either an edge or a corner, where K and x retain their former definitions.)

For a single frequency in a reverberant environment.*

$$\overline{p_b^2} = 1 + \frac{\text{Sin } 2\,Kx}{2\,Kx}.$$

For an octave band, this becomes

$$\overline{p_{br}^2} = \frac{K}{\sqrt{2}} + \frac{1}{2x}\left[\text{Si}\left(2\sqrt{2}\,Kx\right) - \text{Si}\left(\sqrt{2}\,Kx\right)\right]$$

where $\text{Si}(t) \equiv \int_0^t \frac{\sin z}{z}\,dz.$

For 1/3-octave bands,

$$\overline{p_{br^3}^2} = 0.2311\,K + \frac{1}{2x}\left[\text{Si}\left(2.2444\,Kx\right) - \text{Si}\left(1.7820Kx\right)\right]$$

Again, these expressions lend themselves to a better representation in a normalized form for the reason mentioned in the free-field case.

If the normalized forms of these equations are plotted against Kx cos θ and Kx, for free or reverberant fields respectively, and scaled in decibels, they become particularly useful in making approximate corrections for interference effects in SPL measurements taken near a reflecting surface. Nomographs (Figures 3-7 and 3-8) assist in the use of Interference Pattern Curves for the four cases of particular interest: octave bands and 1/3 octave bands in both free-field and reverberant-field environments. They can be used for two purposes:

a. To find the variation of SPL with distance from a large surface for a fixed midpoint frequency.

b. To find the *difference* between the SPL's at any two fixed distances from the surface as a function of frequency.

They cannot be used directly to determine the variation with frequency of SPL at some fixed distance x from the reflecting surface because of the normalizing factor K included in the decibel value of the ordinate.

The nomographs are constructed so that by entering with a given midpoint frequency, the known distance from the reflecting surface, the angle of incidence of the plane waves of sound on the surface, and the ambient temperature, a correction may easily be found with which to estimate the SPL at the surface from an SPL measured out in front of the surface.

To illustrate the use of the nomographs a sample problem follows:

An SPL is measured at a position one foot from a large reflecting surface in an otherwise free field. The sound wave is known to be approximately plane. The measurement is made in an octave band whose geometric mean frequency is 700 Hz. The sound arrives at the surface with normal incidence ($\theta = 0°$). The temperature is 86°F.

SOLUTION:

1. Enter the free-field, octave-band nomograph at 70 Hz on the frequency scale located in the lower left-hand corner and travel horizontally to the right until the $\theta = 0°$ line is reached. Then move straight up to intersect the "f cos θ" line. (Note that in a reverberant field, the "cos θ" factor drops out. Therefore, on those nomograph one would enter directly on the bottom horizontal frequency scale).

2. Proceed upward and to the left on the page from this intersection, paralleling the slanting lines to a point horizontally opposite the "distance from the surface", which is read on the scale at the left.

3. From this point, move vertically upward to intersect the "60°F" line of the temperature correction chart.

4. Move upward or downward from the 60° line, parallel to the slanting lines, to a point opposite the correct temperature, 86°F.

5. Move vertically up to intersect the Interference Pattern Curve.

6. The decibel scale at the left will show the number of decibels which must be *added* to the SPL measured 12 inches from the surface to obtain the SPL at the surface (3 dB in this example).

*R. V. Waterhouse, "Interference Patterns in Reverberant Sound Fields." The Journal of the Acoustical Society of America, Vol 27, No. 2, page 247.

The negative sign of the decibel correction scale indicates that the pressure *at* the reflecting surface is always larger than at any position out in front of it.

3-1.6.4 Reflection of Sound by Finite Panels

The reflection of plane sound waves by infinite, non-absorptive surfaces was discussed in paragraph 3-1.6.3 and the resultant interference effects calculated. In this paragraph, reflective panels of finite size, which are flat or nearly flat, are considered.

The ratio of the pressure on the surface of an irregularly-shaped, flat object to the free-field pressure at the same position when the object to the free-field pressure at the same position when the object is absent is approximated mathematically for a single frequency by*.

$$\left|\frac{p}{p_o}\right| = \left|1 - \frac{i\,k\cos\theta}{2\pi}\int_o^{2\pi}\int_o^{\rho} e^{ikr\,(1-\sin\theta\cos\phi)}\,dr\,d\phi\right|$$

- p = pressure at point P on the surface of the object
- p_o = free-field pressure incident at the origin (not yet defined)
- θ = angle the incident sound makes with a perpendicular to the surface
- k = wave number of the incident sound of frequency f. $k = 2\pi f/c = 2\pi/\lambda$
- c = speed of sound
- i = $\sqrt{-1}$
- r = distance from a small element of the surface to the point of measurement P
- ρ = value of r at the boundary of the surface
- ϕ = the angle which fixes the position of the point P on the surface

For certain symmetrical objects, the expression is greatly simplified. By determining the value of the integral for several of these objects (disk, end of a cylinder, flat square plate, face of a cube, ellipse, and a sphere), a result was obtained which can be used as an approximation for any flat or nearly flat irregularly-shaped object. The mathematical expressions were evaluated over an octave-band of frequencies to obtain curves of $|p/p_o|$ representing octave-band interference patterns at the *center* of the reflecting surface.

In Figure 3-9 these have been plotted for each octave band for the case of normal incidence, as a function of one-half the minimum dimension (measured at the geometrical center of the object). This curve is correct within ± 2 dB. A sample curve (dashed line) for the first octave band demonstrates the effect of extreme curvature of the reflecting surface——when the radius of curvature is approximately equal to the wavelength of the incident sound. Results for obstacles with intermediate curvatures fall between these two curves.

Correction curves, Figure 3-10, illustrate an estimate of $|p/p_o|$ for positions on the surface other than the center; and for sound having other than normal incidence ($\theta \neq 0°$). The probable accuracy for each of the correction curves is listed below the abscissa in each case. (No correction has been included for variation in the speed of sound $k \propto 1/c$.)

3-1.7 Mass-Law Noise Reduction

The acoustical mass law provides a means of estimating the noise reduction** suffered by a plane wave incident upon a wall, panel or similar structure under the following restrictions (see Figure 3-11):

a. The wall thickness must be small in comparison with the wave length of sound *in the wall*, so that sound transmission occurs only by gross motion of the wall and not by wave conduction through the wall material.

b. The wall is assumed to be infinite in extent and to radiate plane waves into semi-infinite space on the side away from the source. This implies that, when the formula is applied to an actual, finite wall, the effect of boundary conditions at the edges of the wall must be negligible (no stiffness effects); also, the law predicts the SPL on the "quiet side" of the wall only in the near field, that is, very close to the

*G. G. Muller, R. Black, and T. L. Davis; Jour. of Ac. Soc. of Am., Vol. 10, No. 1 (1938) p.6.

**The difference (in decibels) in the SPL from one side to the other side of a wall or panel; L. L. Beranek, Acoustics, McGraw-Hill, New York (1954), page 326.

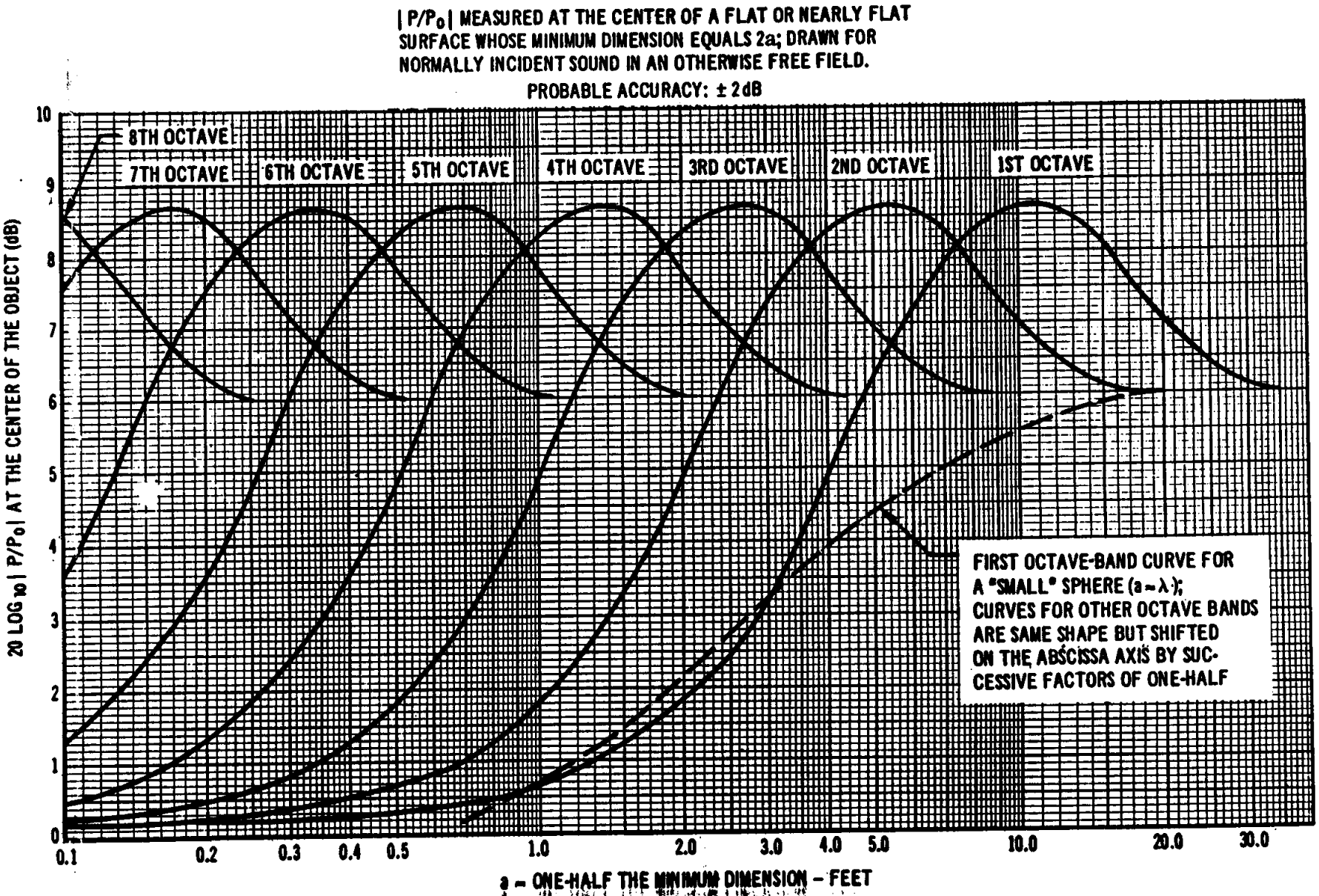
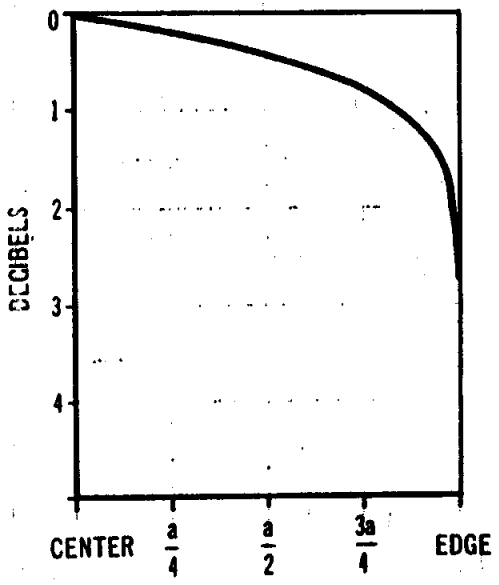


Figure 3-9. Octave Band Analysis of the Ratio of Pressure on the Surface of a Finite Flat (or Nearly Flat) Object to the Free Field Pressure at That Same Point as a Function of One-Half the Minimum Dimension of the Surface

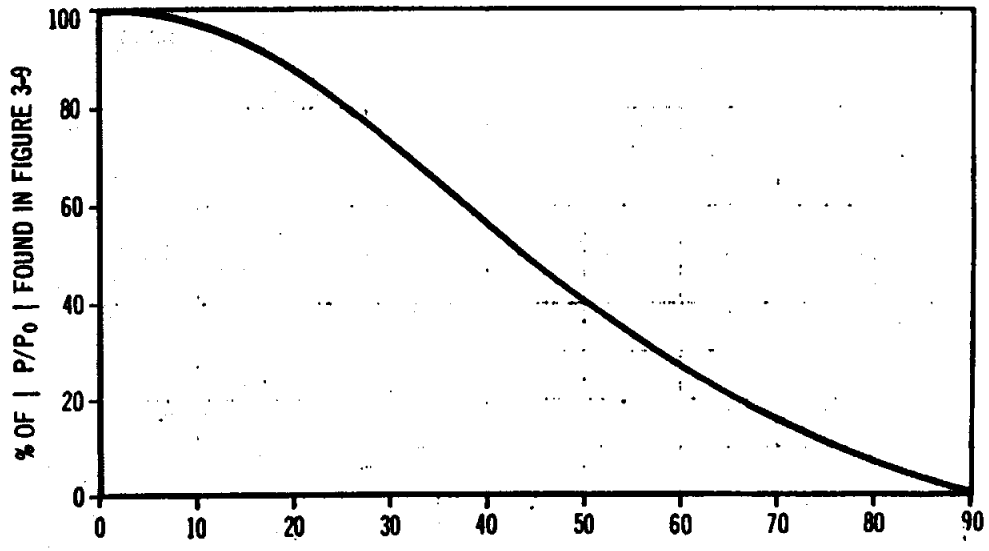
AMOUNT TO SUBTRACT FROM VALUE FOUND IN FIGURE 3-9 IF | P/P$_0$ | IS MEASURED AT OTHER THAN THE CENTER OF THE SURFACE



PROBABLE ACCURACY (dB)	±0	±1.0	±2.0	±2.5	±3.5

NOTE: USE THIS CURVE FOR Ka > 1
FOR Ka < 1, THE VARIATION FROM CENTER TO EDGE IS LESS THAN 1 dB.

% OF | P/P$_0$ | VALUE FOUND IN FIGURE 3-9 IF THE INCIDENT SOUND MAKES AN ANGLE OTHER THAN 0^0 WITH A PERPENDICULAR TO THE SURFACE



PROBABLE ACCURACY (%)	±0	±5	±10	±20	±20	±20	±20	±10	±5	±0

Figure 3-10. Correction Curves for Angle-of-Incidence and Position on the Surface to be Applied to Figure 3-9.

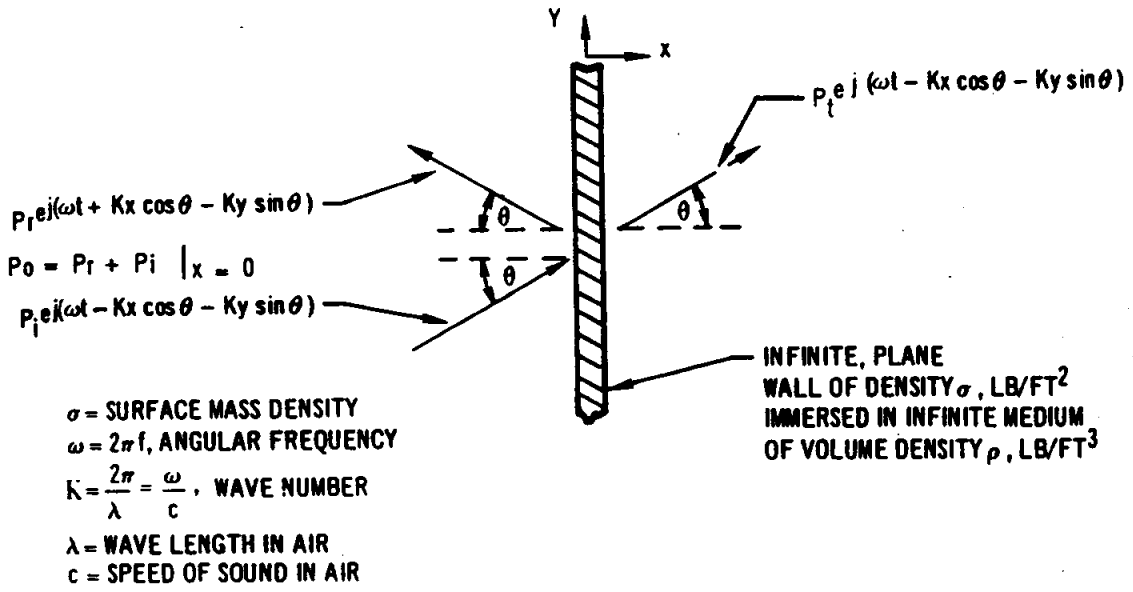


Figure 3-11. Diagram for Noise Reduction Calculations

wall and near its center. *The effects of radiation into a reverberent space or to a large distance in a free field are much more complicated and must not be attempted using only this formula.*

The assumption that stiffness effects are negligible eliminates any concern with wall resonances and with damping terms; in effect, if we regard the actual wall as a "simple oscillator" comprising mass, stiffness and damping, the "no stiffness" assumption restricts our attention to frequencies so high that the inertia (mass) term in the differential equation predominates over the stiffness and damping terms (hence the name "mass law"); that is, to frequencies well above the resonance frequency of the wall.

The resultant Noise Reduction (NR) can be shown mathematically (under the above assumption) to be equal to:

$$NR = 20\log_{10}\sqrt{1+\left(\frac{k\sigma\cos\theta}{\rho}\right)^2}$$

$$= 10\log_{10}\left[1+\left(\frac{k\sigma\cos\theta}{\rho}\right)^2\right]$$

$$= 10\log_{10}\left[1+\left(\frac{\omega\sigma\cos\theta}{\rho c}\right)^2\right]$$

This analysis includes in the p_r term (and hence also in the p_o term) a component of pressure on the source side of the wall which is contributed by the motion of the wall and is exactly equal to p_t. This is immaterial to the use of the foregoing equation in NR calculations, since it is the *existing* (measured) p_o and p_t which are involved. It is, in fact, the "p_t" term on the source side which accounts for the difference between p_i and p_r:

$$p_i - p_r = p_t$$

If Transmission Loss (TL) is to be computed rather than NR, we would compare p_i and p_t, rather than p_o and p_t. This leads to a slightly different result:

$$TL = 10\log_{10}\left[1+\left(\frac{k\sigma\cos\theta}{2\rho}\right)^2\right]$$

$$= 10\log_{10}\left[1+\left(\frac{\omega\sigma\cos\theta}{2\rho c}\right)^2\right]$$

Thus, a general relation between TL and NR for frequencies high enough (or for panels heavy enough) that

$$\frac{k\sigma\cos\theta}{\rho} = \frac{\omega\sigma\cos\theta}{\rho c} >> 1$$

is NR = TL + 6 dB.

Because of the appearance in these equations of physical parameters characterizing the medium (ρ and c) which change with altitude, both the NR and TL are functions of altitude for a given wall. The set of curves shown in Figure 3-12 includes an inset which indicates the increase in attenuation (either NR or TL if the NR = TL + 6 dB is a valid statement) as the altitude is increased.

3-1.8 Transmission Loss of Aircomb Panels*

The TL of an aircomb or honeycomb panel is controlled by stiffness. This results in poor acoustic attenuation properties.

The TL for randomly-incident sound can be forced into the mass-controlled region by filling or partially filling the core with granular material. The nomograph shown in Figure 3-13 can be used to determine the mass of fill required to obtain a mass-controlled panel as well as the approximate TL for an unfilled aircomb panel. This approximation is not necessarily valid for special panels of unusual size, shape, or surface area and should be used with caution.

3-1.9 Scaling Laws for Sound

See Table 3-3.

Table 3-3. Acoustical and Vibration Scaling Relationships (Discrete Frequency Components)

PARAMETER	PROTOTYPE	MODEL	CGS UNITS	REMARKS
CHARACTERISTIC DIMENSION	L	$\bar{L} = \frac{L}{s}$	cm	STROUHAL NUMBER CONSTANT
FREQUENCY	f	$\bar{f} = sf$	Hz	
WAVELENGTH	$\lambda(f)$	$\bar{\lambda}(sf) = \frac{1}{s}\lambda(f)$	cm	
LINEAR VELOCITY	V	$\bar{V} = V$	cm/sec	
STRESS	$\sigma(f)$	$\bar{\sigma}(sf) = \sigma(f)$	dyne/cm^2	
RADIATED PRESSURE[a]	$p_r(R,f)$	$\bar{p}_r(\frac{R}{s}, sf) = p_r(R,f)$	dyne/cm^2	
POWER	P(f)	$\bar{P}(sf) = \frac{1}{s^2}P(f)$	dyne - cm sec	
REVERBERANT PRESSURE	$p_{rv}(f)$	$\bar{p}_{rv}(sf) = p_{rv}(f)$	dyne/cm^2	
FORCE	F(f)	$\bar{F}(sf) = \frac{1}{s^2}F(f)$	dyne	
SPECIFIC ACOUSTIC IMPEDANCE	z(f)	$\bar{z}(sf) = z(f)$	dyne - sec/cm^3	
MECHANICAL IMPEDANCE	Z(f)	$\bar{Z}(sf) = \frac{1}{s^2}Z(f)$	dyne - sec/cm	
MASS	M	$\bar{M} = \frac{1}{s^3}M$	gm	
SPRING CONSTANT	k	$\bar{k} = \frac{k}{s}$	dyne/cm	
FREQUENCY RATIO	f/f_n	$\bar{f}/\bar{f}_n = f/f_n$	—	
QUALITY FACTOR FOR STRUCTURAL ACOUSTIC AND COULOMB DAMPING	Q(f)	$\bar{Q}(sf) = Q(f)$	—	
QUALITY FACTOR FOR VISCOUS DAMPING		$\bar{Q}(sf) = \frac{1}{s}Q(f)$	—	CONFLICTS WITH THE ABOVE SCALING LAWS

[a]THE SYMBOL R STANDS FOR RANGE.

3-1.10 Spherical Divergence Losses and Air Attenuation

See Figure 3-14.

3-2 AERODYNAMICALLY-INDUCED FLUCTUATING PRESSURES

3-2.1 Prediction of Fluctuating Pressure Levels

a. The overall Fluctuating Pressure Level (FPL) induced at a wall by a turbulent boundary layer should be predicted by use of Lowson's equation:

$$FPL\ (dB\ re\ 0.0002\ dynes/cm^2) = 83 + 20 \log_{10} q_\ell$$

$$-20 \log_{10} (1 + 0.04\, M_\ell^2) + X$$

(With limited confidence for $M_\ell > 5$)

where:

q_ℓ = local dynamic pressure in psf

M_ℓ = local Mach number

X = number of dB to be added due to disturbances such as steps, shock waves, surface roughness, angle of attack (impinging flow) etc.

to determine X, the following rules apply:

1. For undisturbed, slowly growing, well-developed, turbulent boundary layers on smooth surfaces, X = 0.

2. To account for surfaces that are not smooth (exposed stringers, beading, bolt heads protruding, etc.) add up to 6 dB for roughness that is up to 1/10 of the boundary layer thickness in height (above the surface). Above that height, consider the roughness to be a protuberance.

3. Directly under a region of an attached shock wave, the number X is found by

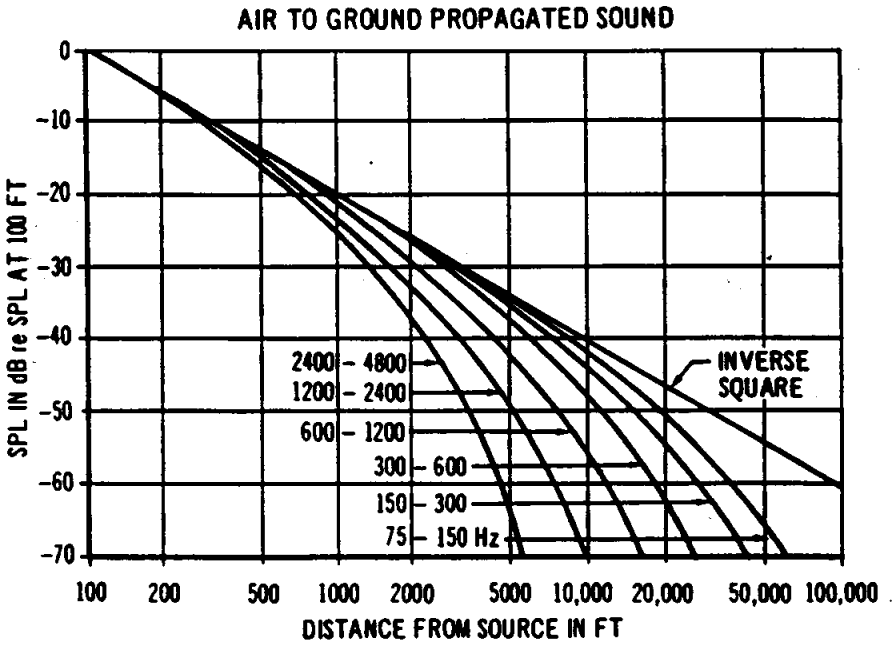
$$X \cong 24\,(P_p/P_o) - [P_p/P_o - 0.9]^4 \cdot 24$$

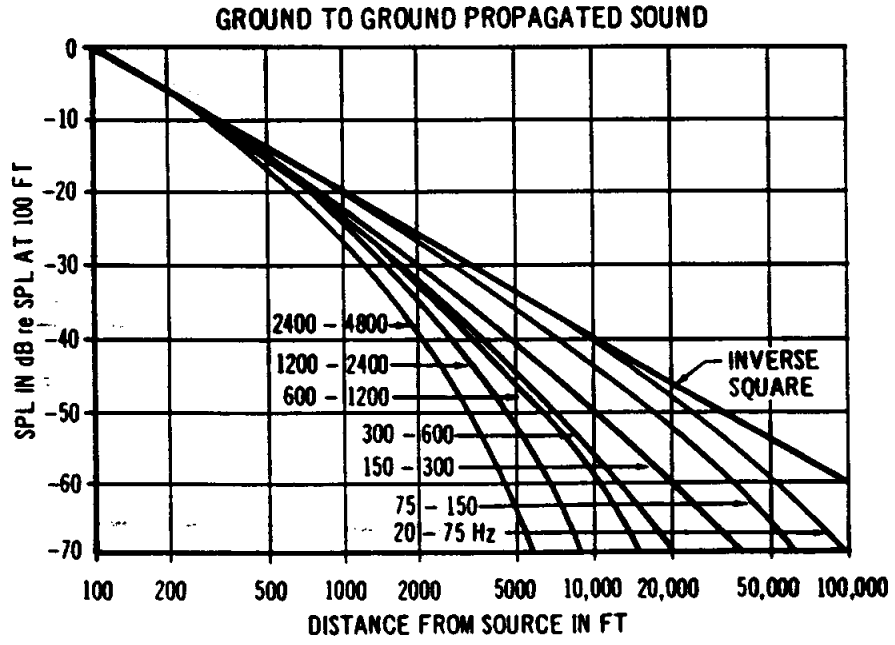
(for $p_p/p_o \leq 3.0$)

Where:

P_p/P_o = shock strength (P_p is the plateau static pressure downstream of the shock and P_o is the upstream static pressure)

Ailman, C. M., "Study of Sound Transmission Loss Characteristics for Douglas Aircomb Panels" SM Report No. 39069, Douglas Aircraft Company, December 1961.





SOURCE: WADC TR 57-354

NOTE: More accurate air-to-ground reduction of SPL may be obtained from the LB-31954 Report

Figure 3-14. Reduction of Sound Pressure Levels With Distance

In the separated flow region caused by a forward facing step (or similar flow-causing mechanisms), the FPL's maintain a nearly constant level (between the high levels associated with the shock at the position of separation and directly in front of the forward facing step). Figure 3-14a estimates the FPL for this region as an increase over the undisturbed FPL upstream of separation. The estimate is based on the increase in static pressure for that region as compared to the static pressure upstream of the initial flow disturbance (position of separation).

4. Directly in front of a broad-front forward-facing step, use the nomograph in Figure 3-15.

5. Behind a broad aft-facing step whose streamwise length to height ratio (fineness ratio) is greater than 3, X at the position of reattachment (approximately located [5.8-0.86 M_∞] feet downstream of the step face where h is the step height in feet) is a function of Mach number and free stream dynamic pressure. A conservative estimate for the increase in decibels (X) over the FPL's under an undisturbed turbulent boundary layer (which either occurs on top of a long aft-facing step or upstream of separation for a step-spoiler) may be derived from

$$X = 19 - 1.35\,(2.25 - M_\infty)^3 \text{ in dB for } 0.5 \leq M_\infty \leq 4.5$$

6. For increases in FPL's in the vicinity of three-dimensional protuberances (flow going around as well as over the protuberance), extrapolation from measured data on similar geometries is recommended.

7. When mild static pressure gradients are expected, the fluctuating pressure gradients may be derived from Figure 3-16. The approximate mathematical fit is

$$\frac{\Delta FP}{\Delta X} = 27.5\,\frac{\Delta SP}{\Delta X} - 2.5$$

where:

FP = fluctuating pressure ratio = $10^3\ p_{rms}/q_\infty$

SP = static pressure ratio = p_{static}/p_∞

X = surface dimension

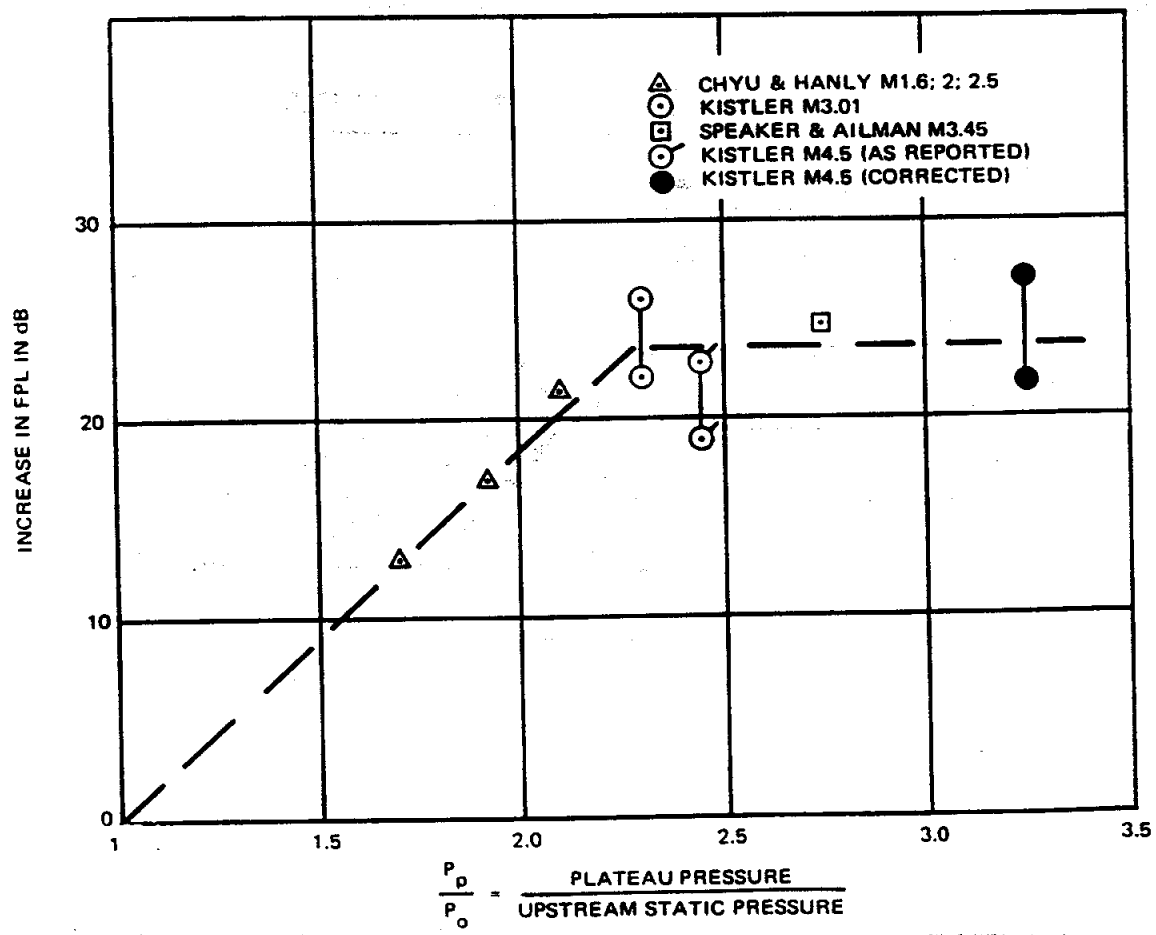
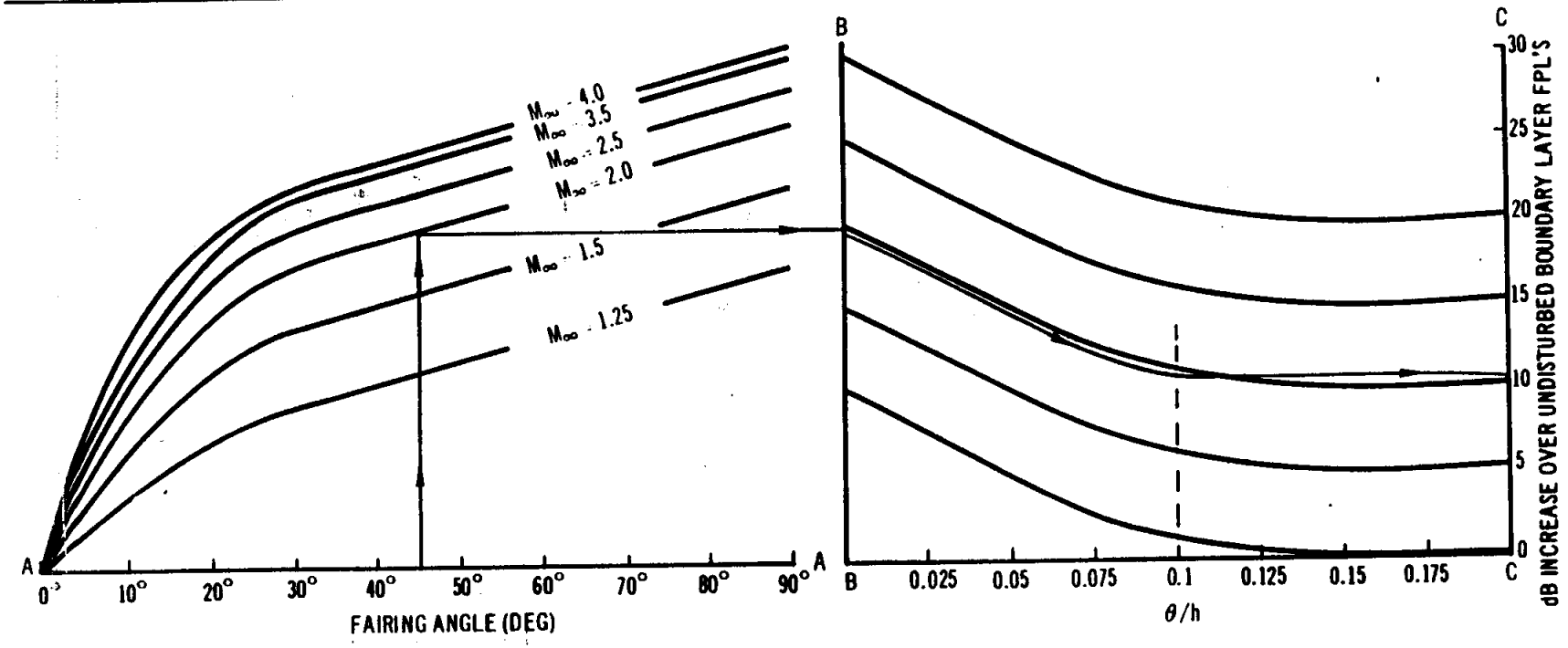


Figure 3-14a. Separated Flow Increase in FPL Versus Static Pressure Ratio



EXAMPLE:

GIVEN: A 45° FAIRING, 0° ANGLE OF ATTACK, M = 2.0 WITH AN UNDISTURBED BOUNDARY LAYER MOMENTUM THICKNESS TO MAXIMUM STEP HEIGHT RATIO OF 0.1 ($\theta/h = 0.1$). FIND THE INCREASE IN FLUCTUATING PRESSURE (OVER THE UNDISTURBED LEVEL) IN DECIBELS FOR THE REGION RIGHT IN FRONT OF THE BROAD FRONT OR TWO-DIMENSIONAL FAIRING.

STEP 1. ENTER THE FIGURE ON LINE A AT THE APPROPRIATE LEADING EDGE ANGLE OF THE FAIRING (45°).
STEP 2. DRAW A PERPENDICULAR TO A UNTIL IT INTERSECTS THE APPROPRIATE MACH NUMBER LINE ($M_\infty = 2.0$).
STEP 3. DRAW A HORIZONTAL LINE (PARALLEL TO LINE A) FROM THE POINT FOUND IN STEP 2 UNTIL LINE B IS INTERSECTED.
STEP 4. FROM THE POINT FOUND IN STEP 3 ON LINE B, FOLLOW THE PARALLEL CURVES TO THE APPROPRIATE θ/h VALUE ($\theta/h = 0.1$).
STEP 5. READ OFF THE NUMBER OF DECIBELS ON LINE C TO BE ADDED TO THE UNDISTURBED FPL'S TO PREDICT THE FPL JUST IN FRONT OF THE FAIRING AT THE POINT FOUND IN STEP 4 (10.5 dB).
STEP 6. FOR THE APPROXIMATE FREQUENCY DISTRIBUTION OF ENERGY, ADD THE NUMBER OF DECIBELS FOUND IN STEP 5 TO EACH OCTAVE BAND LEVEL OF THE UNDISTURBED BOUNDARY LAYER FPL.

Figure 3-15. Relative Increase in Fluctuating Pressure Level Just in Front of a Broadfaced Fairing. FPL's at Supersonic Speeds and Zero Degree Angle of Attack as a Function of Fairing Angle, Free Stream Mach Number, and θ/h Ratio

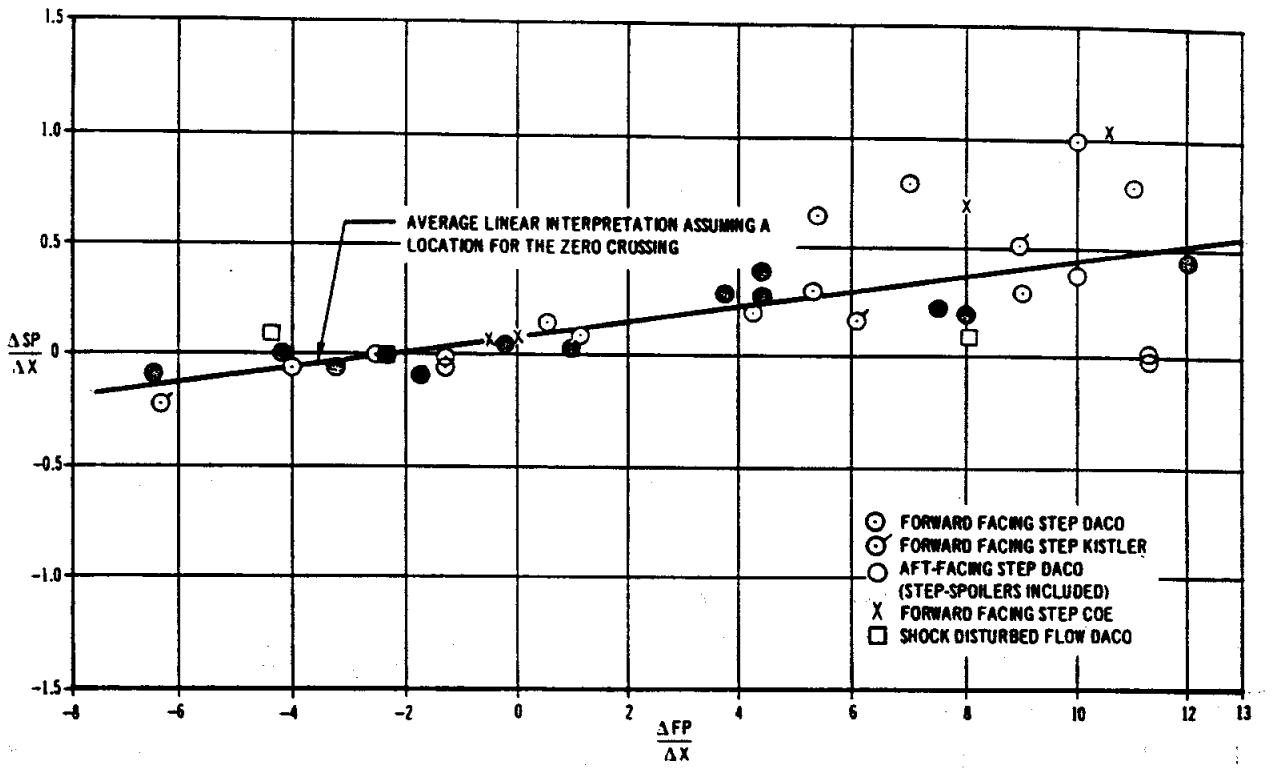


Figure 3-16. Comparison of Spatial Gradient for Static Pressures vs. Fluctuating Pressures

b. The spectral distribution of the FPL's whose overall magnitude has been determined above, have considerable variation in many instances. For a relatively crude approximation (for better predictions, the appropriate references should be consulted) the following rules apply:

1. For the undisturbed turbulent-boundary-layer-induced FPL's, or those under a thickened boundary layer (downstream of disturbances or in the presence of surface roughness), the octave band spectrum is obtained by calculating

$$f_c \cong 8 \times 10^{-2} (U_\infty/\theta)$$

where U_∞ is the free stream velocity and θ is the local boundary layer momentum thickness. Let this f_c be the upper limiting frequency of an octave band (OBT) whose level is 4 or 6 dB below the overall level previously estimated depending on whether $M_\infty > 0.6$ or < 0.6 respectively. The octave band levels on either side of OBT are found by substracting 6 dB or 2 dB per octave for successive octave bands of higher frequencies than OBT depending on whether the free stream Mach number is greater than 0.6 or less than 0.6 respectively, and by substracting 2 dB per octave for each successively lower band than OBT.

An equation which predicts the octave band FPL for undisturbed turbulent-boundary layers where the $M_\infty > 0.6$ is:

$$OBFPL = 80.0 + 20 \log_{10} q_\ell - 20 \log_{10} M + 6.67 [\log_{10} f_n + \log_{10} \theta - \log_{10} U_\ell]$$

where:

OBFPL = Decibels re 2×10^{-5} N/m^2

q_ℓ = Local dynamic pressure in psf

U_ℓ = Local velocity in ft/sec

θ = Momentum thickness in inches

M = $(1 + 0.04\, M_\ell^2)$ and is dimensionless

M_ℓ = Local Mach number

f_n = Geometrical mean frequency of the octave band of interest in Hz

This equation can be obtained by combining the equation for 3-2.1 a.1. with that of 3-2.1 b.1.

2. Directly under a shock wave, the power spectrum has a peak at approximately $f_o = 10^{-3}$ U_∞/θ. The decreasing slope on either side of this peak appears to be 5 $[P_p/P_o - 1]$ decibels per octave. It should be noted that the spectrum varies rapidly downstream of a shock wave, and appropriate references should be examined for details.

   The spectral content of the FPL's within the separated flow region in front of a forward facing step (the plateau static pressure region) may be obtained from Table 3-3a (derived from measured data).

3. Just in front of a broad-front, forward-facing step, the power spectrum is a double humped curve. The variation in power level with frequency is as much as 8 dB. However, an octave band spectrum would show much less variation. To a first approximation therefore, an octave band spectrum in decibels may be obtained by adding the number of decibels for the increase in overall FPL to each octave band of the FPL's predicted for the undisturbed flow upstream of the step. The cutoff frequency for this resulting octave band spectrum should be $f_c = 0.2\ U_\infty/\theta$.

Table 3-3a. Variation in Octave Band Levels Within Separated Flow

Octave Band below f_c*	Change in Level from A** (indB)
11	+2
10	+3.5
9	+4
8	+4
7	+3.5
6	+3
5	+2
4	+0.5
3	−1
2	−4
1	−6.5

*Frequency limits of OB = 1 are:

$f_{lower} = f_c/2$ and $f_{upper} = f_c$

where f_c is the cutoff frequency of the undisturbed turbulent-boundary-layer spectrum given in 3-2.1 b.1

**A is the increase in overall FPL given in 3-2.1 or Figure 3-14a

   Above this frequency, the octave band levels decrease by approximately 5 dB and 13 dB in the next two higher octave bands respectively. For spectra further from the step face, less energy is apparent at the high frequency maximum (about $f_c = 0.02\ U_\infty/\theta$). The low frequency maximum remains, but its position in the frequency domain probably depends on the distance from the step face.

4. Similar to the conditions of forward-facing steps, the power spectra for aft-facing steps vary markedly in the region of separation/reattachment beyond. For the maximum levels predicted in paragraph a.5, however, the spectrum is relatively flat with a cutoff frequency of approximately $f_c = 0.03\ U_\infty/\theta$. Using this as an upper frequency for an octave band, that octave band level would be 5 dB below the predicted maximum overall level. The next higher octave band above this "top" would have a level 8 dB below the overall.

   For successively lower octave bands below the "top" octave band, the decibel decreases (below the top octave band level) are 0 (two adjacent octave bands have the same level), -2.5, -5.0, -7.5, -10.0, etc., respectively. For shapes of power spectra away from the maximum level, see the appropriate references.

c. Convection velocity (U_c) of the FPL's under a turbulent boundary layer varies with distance from an upstream point of reference. This is due to a shifting of the dominant sources (eddies) away from the wall as they progress downstream. Average velocities reach 0.8 to 0.95 of the free stream velocity (U_∞). However, in regions over which good spatial correlation exists (small spatial separations) the dominant sources for the FPL's at the wall are convected at a velocity only 0.5 to 0.6 of U_∞ (very near to the shear layer). To be rigorous, since large regions over which the FPL's are poorly correlated and travel fast (nearly free stream velocity) *do* contribute to the forcing function that drives the structure into vibratory response, cognizance of, and mathematical inclusion of, a variable U_c should be made. (Actually, in most current theoretical work, U_c is assumed constant and given a value of approximately 0.6.)

d. The spatial extent of correlation depends on the kind of flow, the location within the flow, the characteristics of the flow (thickness parameters, velocity, etc.) and the frequency of interest. Under the circumstances (of so many influencing factors and too little data) no generalizations can be made at this time. Where similar flow characteristics exist to those for which experimental data exist, comparable results are expected. Hence, wind tunnel test data should be consulted where available and applicable.

3-2.2 Base Pressure Fluctuations

The fluctuating pressures which are in evidence at the base of a blunt-ended vehicle, usually called base pressure fluctuations, are a result of a wake flow and the

turbulent eddies in the boundary layer (or generated by vortex shedding at the lip) acting on the trapped recirculating air behind or downstream of the base. To a first approximation, the overall FPL (in decibels re 0.0002 dynes/cm^2) is found by:

$$FPL = 85 + 20 \log_{10} q_\infty + 20 \log_{10} \left(\frac{4.6x}{r} + 1\right)$$

for free stream Mach numbers (M_∞) less than 1.0. Here x is the distance from the center of the base to the position of interest on the base measured in the same units as r the base radius. The last term in the expression assumes a linear extrapolation of the existing data for the observed increase in the FPL when traversing the base from the center out to the edge.

The distribution in the frequency domain at the center of the base is as follows:

$$p^2(f) \text{ in } [(psf)^2/Hz] = 5 \times 10^{-5} \rho_\infty^2 \, U_\infty^3 r \text{ to } \frac{f_r}{U_\infty} = 0.1$$

Above 0.1 = f_r/U_∞ the normalized power spectrum decreases at 6 dB/octave (i.e., the slope is proportional to

$$\left(\frac{f_r}{U_\infty}\right)^{-2}$$

For locations closer to the edge (away from the center) all the increase indicated by the last term in the expression for the overall FPL occurs at frequencies below

$$\frac{f_r}{U_\infty} = 0.25$$

For further information on subsonic base pressure fluctuations, see JASA Vol 33 No. 1 pages 59-63, January 1961.

No data exist to suggest that the preceding method for predicting subsonic base pressure fluctuations is valid at supersonic speeds. On the contrary, if turbulent boundary layer data trends are indicative, a strong Mach number dependence should be introduced in addition to the existing dependence on q_∞ (or p_∞). Until such data are available, the recommended procedure for predicting base pressure fluctuations at supersonic speeds is to extrapolate aft-facing step data. (See also J. C. Houbolts method "On the Estimation of Pressure Fluctuations in Boundary Layers and Wakes" ARAP Report No. 90, dated June 1966.)

3-2.3 Equivalent SPL's for FPL's

Sound pressures are the result of a wave phenomenon whereas convected-turbulence-induced fluctuating pressures are not. Even though the sound pressures are also fluctuating pressures, generally, most of the pertinent characteristics of FPL's and SPL's differ (e.g., velocity of propagation of sound compared to the convection velocity of the FPL's, areas over which each of the fluctuating pressures are correlated – in phase, amount of damping provided by each when structures are driven into vibration, etc.). A vehicle flying within a dense atmosphere is exposed to many fluctuating pressure environments. Typical examples are those induced by turbulent boundary layers or wake flows.

The same vehicle might be qualified for such a flight environment during ground testing by exposure to an acoustic pressure field. It is therefore necessary to develop an equivalence for the two, different, forcing functions.

The difficulty arises in the diversity of each. The turbulent boundary layer might be separated, attached, disturbed by a shock wave, or thickened by a roughened surface. Wake flow might be influenced by an angle of attack or geometry. The acoustic field might be in a free space, a progressive wave tube, a shroud, or a reverberant room.

To introduce still more complexity, each type of forcing function is dependent on a number of its own variables and parameters. With such a large number of factors, the results are bound to present diverse comparisons. Thus, it is fairly surprising to find that a definite trend is visible when all the available data are compiled. Figure 3-17 shows a relative efficiency between rocket noise and boundary-layer pressure fluctuations. Recent Saturn data (from the forward interstage of the S-IVB-501 flight) have been added which further confirm the data trend, in inducing structural vibration. The spread in the data is attributable to the factors discussed above. The efficiency is plotted versus a frequency which has been normalized to the fundamental bending frequency of the local structure.

The results may be summarized as:

a. Below the natural frequencies of local panels, acoustic energy is more efficient in transferring energy into major structural modes (probably because of its better correlation over large areas).

b. Above the fundamental frequencies of resonance for the local panels, convected turbulence is more efficient in transferring energy into structural response (probably because of longer correlation lengths at high frequencies and higher convection velocities during supersonic flight, matching flexural wave speeds of the structure to excite coincidence).

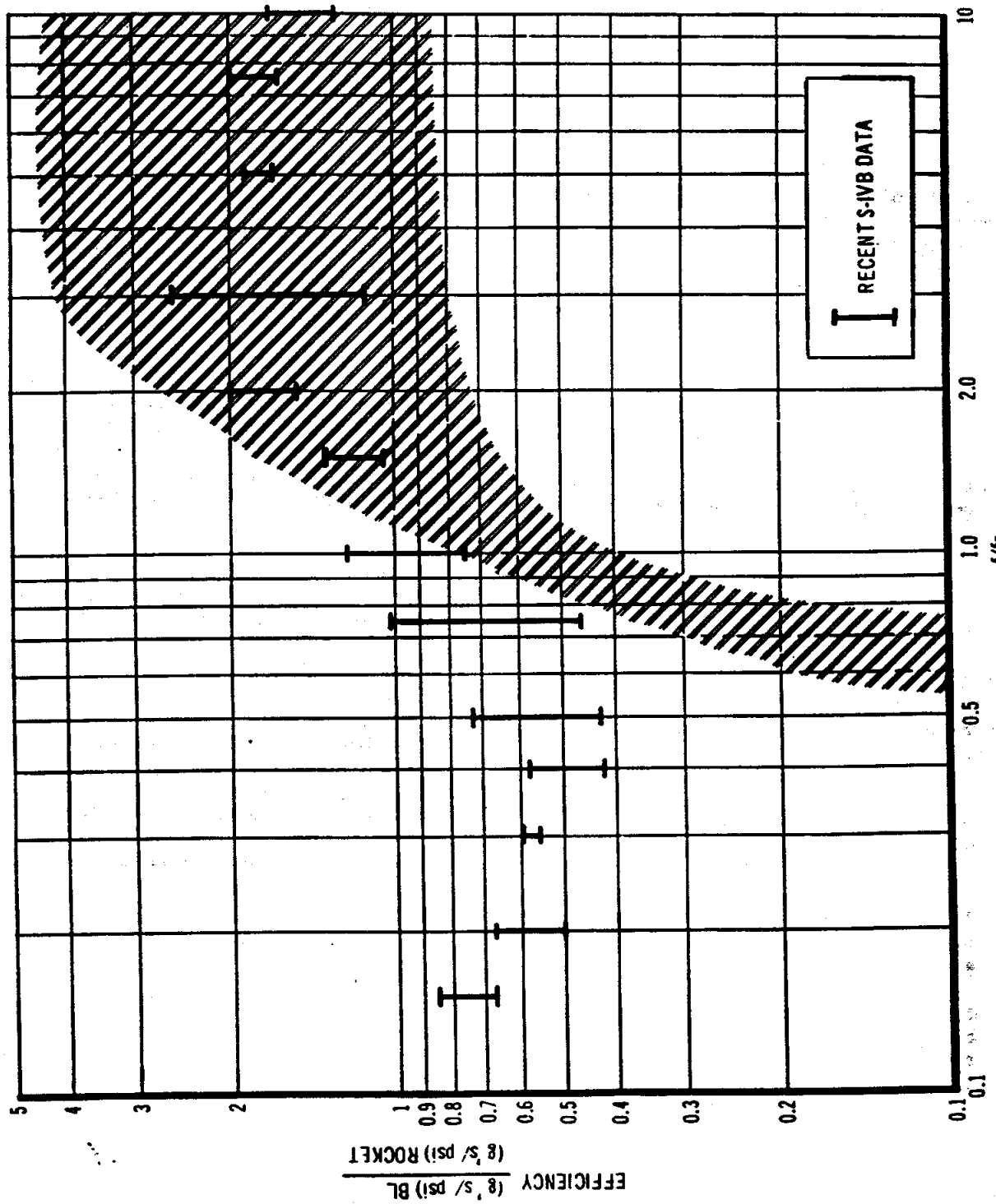


Figure 3-17. Summary of Data on the Relative Efficiency of "Rocket" Noise and Boundary-Layer Pressure Fluctuations in Inducing Structural Vibration

c. Depending on the combined effects of the many factors involved, the degree of efficiency of energy transfer may vary considerably during one flight (e.g., a difference of more than a factor of 2 at one frequency between Mach 1, a shock wave phenomenon, and maximum dynamic pressure, an undisturbed turbulent boundary layer phenomenon, on the Saturn forward interstage).

The recommended practice is:

d. At frequencies below or equal to local panel resonance frequencies use FPL's directly as equivalent SPL's. Even though the data indicate such a one-to-one relationship provides an overly conservative ground test, two additional factors provide the basis for this recommendation:

   1. Below panel resonances, structure is stiffness controlled and inefficiently excited into vibration.

   2. The acoustic environment at liftoff is usually the predominant influencing factor of the low frequency criteria – probably overriding FPL's.

e. Above the fundamental frequency of resonance of the local structure, add (to the FPL's) 3 dB in the next higher octave band and 6 dB thereafter to estimate SPL's that will produce equivalent response in structure where the acoustic field has a preferred direction (sound field from a jet exhaust, progressive wave tube, or a shroud test). For equivalent SPL's resulting from a reverberant room, 8 to 10 dB should probably be added to the FPL's in the higher octave bands (instead of 6).

3-2.4 Buffeting

As defined in paragraph 3-2.1, the phenomonon of buffeting is describable as unsteady aerodynamics, because it pertains to flow conditions which vary with time in somewhat random way. The variations may result in major changes in static pressure over a large portion of a vehicle's surface. Such a condition is conducive to total vehicle movement and to excitation of major body bending or breathing modes. Buffeting is usually observable as a variation in static pressure between two relatively fixed levels in an alternating (low frequency of occurrence) random way.

To predict the possibility of unstable flight due to buffeting forces which drive the vehicle in a divergent body bending mode (resulting in unstable flight and possible break up of the vehicle) consult the NASA Space Vehicle Design Criteria SP-8001 entitled "Buffeting During Launch and Exit." Also, see Section 2, paragraph 2-7 for additional buffet stability criteria.

3-2.5 Scaling Laws for FPL's

It is recommended that the following scaling laws be utilized when fluctuating pressure data are scaled. For turbulent-boundary-layer-induced fluctuating pressures, levels are scaled from one trajectory to another at the same Mach number on the basis of

$$FPL_2 = 20\,LOG_{10}\left[\frac{(q_\ell)_2}{(q_\ell)_1}\right] + FPL_1$$

where the q_ℓ's are local dynamic pressures and the FPL's are in decibels re 0.0002 dynes/cm^2.

Frequencies for similar flow conditions are scaled as

$$\frac{\omega_2}{\omega_1} = \frac{\theta_1}{\theta_2} \cdot \frac{(U_\ell)_2}{(U_\ell)_1}$$

where the θ's are boundary-layer momentum thicknesses and the U_ℓ's are local velocities.

Only if the two configurations are identical, except for size (and thus have identical boundary layer velocity profiles resulting in $\theta_2/\delta_2 = \theta_1/\delta_1$ where δ is the turbulent boundary layer thickness), can frequencies be scaled according to:

$$\frac{\omega_2}{\omega_1} = \left(\frac{D_1}{D_2}\right)^{4/5} \frac{(U_\ell)_2}{(U_\ell)_1} \left[\frac{(R_e)_2}{(R_e)_1}\right]^{1/5}$$

Where the D's are the comparable linear dimensions (e.g., the vehicle diameter or the vehicle length), and the R_e's are the free stream Reynolds Number per unit length.

Correlation lengths and areas are presumed to scale on the basis of boundary-layer momentum thickness. If Λ's and ℓ's are the areas and lengths over which the fluctuating pressures at a wall beneath a turbulent

boundary layer are correlated where L can be either perpendicular to, or parallel with, the flow, then:

$$\frac{\Lambda_2}{\Lambda_1} = \frac{\theta_2^2}{\theta_1^2} \text{ and } \frac{L_2}{L_1} = \frac{\theta_2}{\theta_1}$$

For buffet pressures (defined here as randomly-varying static pressure between two basic levels — usually caused by oscillating shocks or an unstable condition in a region of separated flow), the amplitude scales proportionally to the P_∞ ratio (where P_∞ is free-stream static pressure) if the Mach numbers, gas media and geometries are identical. For identical geometry of differing sizes, the frequency appears to scale as:

$$\frac{\omega_2}{\omega_1} = \frac{D_1}{D_2} \frac{(U_\infty)_2}{(U_\infty)_1}$$

Where the D's are vehicle diameters at the same relative location.

3-3 SHOCKS

3-3.1 Prediction of Leading and Trailing-Edge Shock Pressures

To predict the fly-through shocks for multiple launches, sonic boom theory may be used. For the near field, effects of nose, f in-fuselage junctions, protuberances, and boat-tail surfaces preclude hand calculations. (See Figure 3-18a.) Refer to NASA TND-3082, where a numerical method is presented for use with a digital computer.

Reference: "A Numerical Method for Calculating Near-Field Sonic Boom Pressure Signatures," by W. D. Middleton and H. W. Carlson, NASA TND-3082, November 1965.

For the far-field, directly to the side of, or below the flight path:

$$\frac{\Delta P \text{ Max}}{P} \frac{(h/\ell)^{3/4}}{K_r \beta^{1/4}} = \frac{1.19\gamma}{\sqrt{\gamma+1}} C$$

See Figure 3-18b.

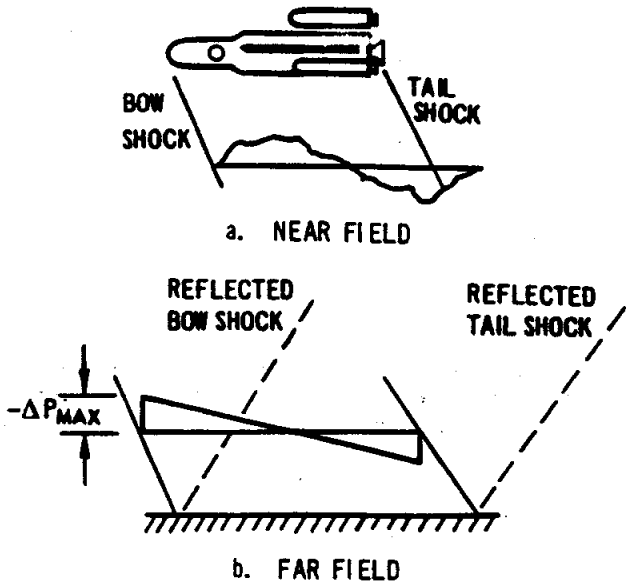


Figure 3-18. Leading and Trailing Edge Shocks

where:

ΔP Max is max value of overpressure (lb/Ft^2)

P is average ambient air pressure (lb/Ft^2)

h is distance from vehicle (Ft)

ℓ is length of vehicle (Ft)

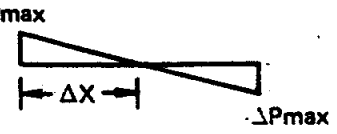
K_r is reflection factor (if any)

β is $\sqrt{M^2 - 1}$

γ is ratio of specific heats = 1.4 for air

C is effective area factor dependent upon the distributions of cross-sectional area and lift.

To determine the time duration of the pulse,



$$\frac{\Delta x}{\ell} \left(\frac{h}{\ell}\right)^{-1/4} = 1.19 \sqrt{\frac{\gamma+1}{\gamma+1}} \frac{M^2}{\beta^{3/4}} C$$

$$\text{SLOPE} = \frac{\Delta p}{\Delta x} = K\gamma \frac{p}{h} \frac{\beta}{M^2} \frac{\gamma}{\gamma+1}$$

Assuming a classic N-shaped wave $\Delta t = 2\,\Delta x/V$

where:

Δt is time interval between bow shock and tail shock

V is velocity of vehicle

K_γ = 1.0 in air

= 2.0 at hard surface

= 1.8 at sand or plywood surface

For zero angle of attack ($\alpha = 0$) wind tunnel tests at M = 2 show that:

$\frac{1.19\gamma}{\sqrt{\gamma+1}}$ = 0.12 for parabolic body of revolution

= 0.13 at side for wedge shape at $h/\ell \geq 5$ and 0.14 below

= 0.08 at side for rectangular wing of aspect ratio 0.5 and 0.13 below with 12.5% thick parabolic-arc section at $h/\ell \geq 7$

= 0.15 for 60° delta wing at $h/\ell \geq 8$

= 0.08 for supersonic bomber at $h/\ell \geq 50$

= 0.06 for supersonic transport design for reduced sonic boom at $h/\ell = 50$

= 0.24 for 60° delta wing at $\alpha = 5°$ at $h/\ell \geq 20$

= 0.30 for 60° delta wing at $\alpha - 10°$ at $h/\ell \geq 16$

As a first approximation, use $P = \sqrt{p_a\, p_g}$

Where:

p_a is ambient pressure at the vehicle

p_g is ambient pressure at the point of interest

Tests show that propagation losses, temperature variations, and winds can bend the path of the shock so as to reduce or eliminate it at a great distance from the source.

Turns or accelerations can cause an overpressure buildup factor of 2 to 4 over the steady state value.

References:

NASA TRR 213, "Correlation of Sonic-Boom Theory With Wind Tunnel and Flight Measurements," by H. W. Carlson, December 1964.

NASA TND-48, "Ground Measurements of the Shock-Wave Noise From Airplanes in Level Flight at Mach Numbers to 1.4 and at Altitudes to 45,000 ft," by D. J. Maglieri, H. H. Hubbard, and D. L. Lansing, September 1959.

NASA TND-2370, "Sonic Booms From Aircraft in Maneuvers," by D. J. Maglieri and D. L. Lansing, July 1964.

3-3.2 Overpressures From Explosives

Aerospace boosters are a potential explosive hazard. As a first approximation, an exploding booster is assumed to cause an explosion with characteristics similar to those of a certain predetermined amount of TNT. First, determine the relative explosive "effectiveness" of the booster fuel, compared to TNT; in general, most chemical fuels (meaning fuel/oxidizer mixture) are assumed to be equivalent to TNT on a weight basis. Then determine the total weight of fuel actually exploding (generally much less than the total fuel available). Explosion-induced shock wave overpressures can be obtained from Figure 3-19.

NOTES:

1. As in all acoustic phenomena, variations will occur at long distances.
2. Damage is sometimes caused by wind (drag) on the structure. This wind is a result of the shock wave and can be best described by its dynamic pressure ($1/2\,\rho V^2$ where ρ is the air density behind the shock and V is the wind velocity behind the shock). For details of this other phenomenon, or more information concerning the damage potential of overpressures, see "The Effects of Nuclear Weapons." Samuel Glasstone, editor, U. S. AEC, April, 1962.

3-3.3 Separation Shock Amplitude Versus Distance

A strictly mathematical solution for the explosive-shock problem has not been proven practical. However, tests show that the shock severity generally decreases with distance from the source, especially in complex systems. Beyond the first foot or so, Figure 3-20 may be used to estimate shock attenuation.

3-3.4 Transportation and Handling Environments

Shock and vibration environments have been measured extensively for the four major transportation modes (truck, rail, air and ship). The results of these measurement programs have been reviewed, summarized and rereferenced (e.g., see References 1 and 2). A discussion of representative shock and vibration data for the four modes is presented in the following paragraphs.

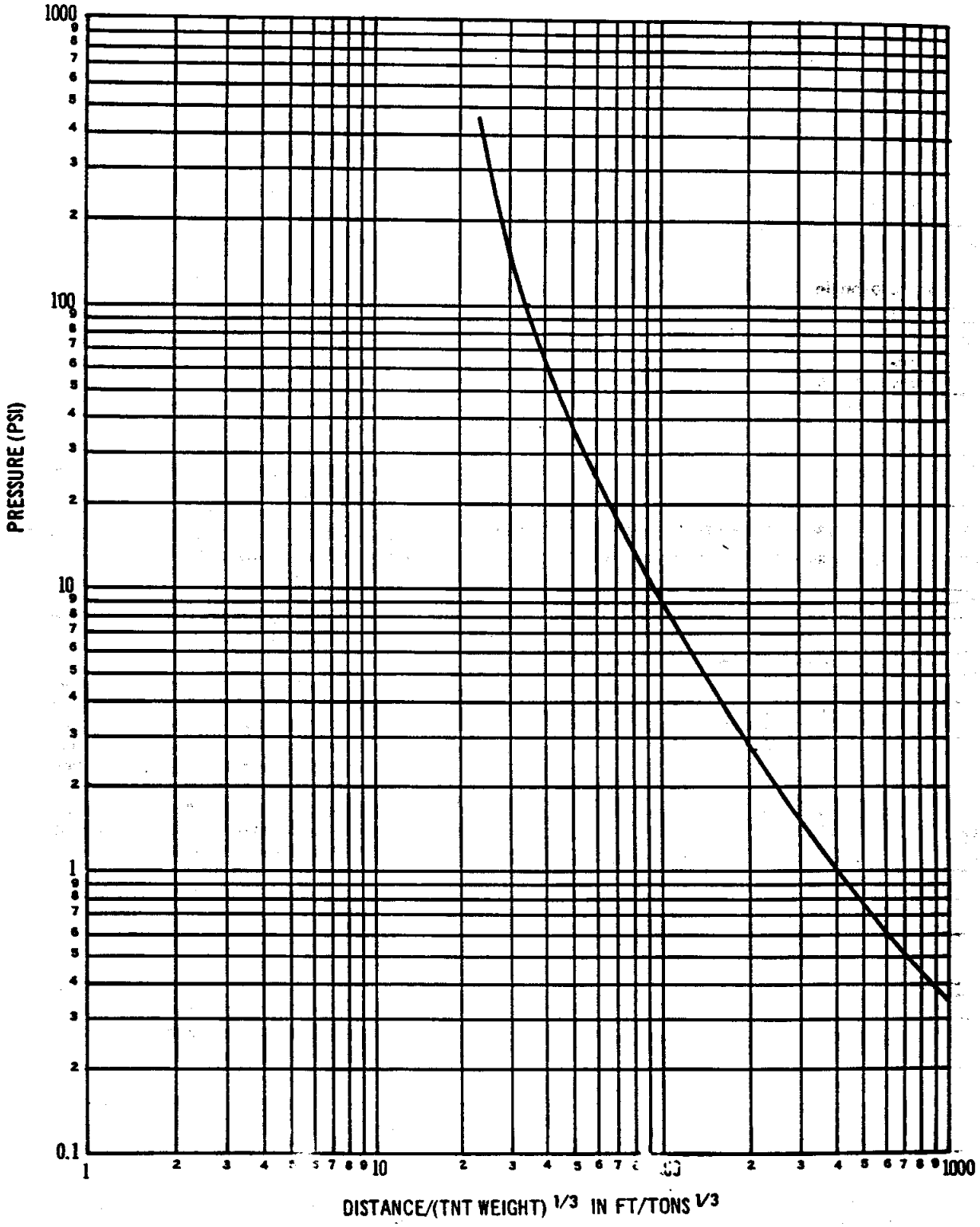


Figure 3-19. Peak Overpressures vs. Scaled Distance for TNT Explosions in Half-Space

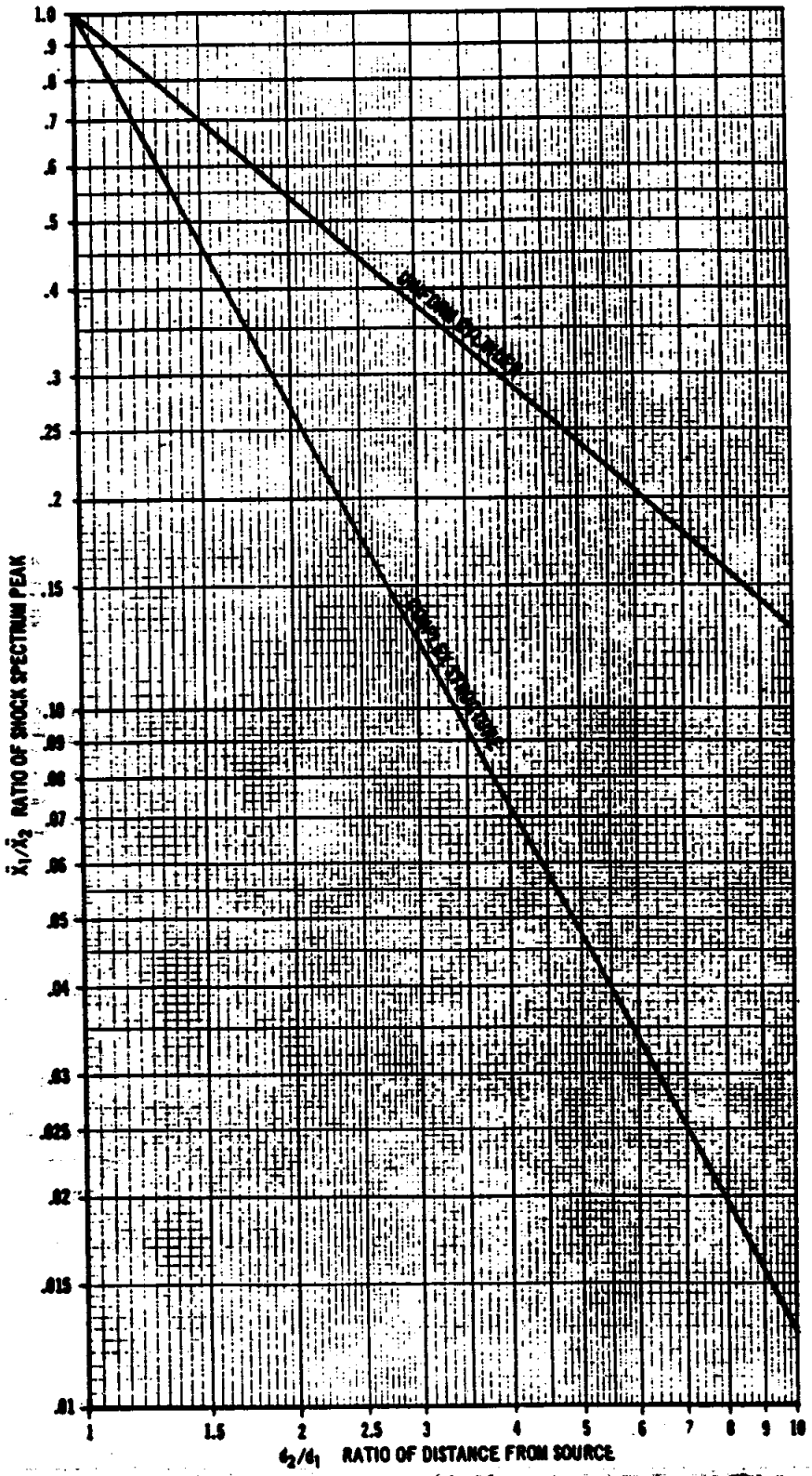


Figure 3-20. Shock Attenuation

3-3.4.1 Transportation

a. By Truck – Response frequencies in motor trucks depend on the natural frequency of the spring system, and the natural frequencies of body structure. The vibration amplitudes depend on road conditions and the speed of travel. Intermittent road shocks of high magnitude can occur, resulting in a severe shock environment for unlashed cargo. Vibrations, due to the truck engine and transmission system, are relatively insignificant in the cargo area.

Reference 1 presents vibration data representative of trucks traversing rough roads, ditches, potholes, railroad crossings, bridges and paved roads. These data were obtained from three standard commercial semi-trailers, each having one of three basic suspension systems (air-ride tandem suspension, stable-ride single suspension, and single-axle spring suspension). The tests were run at two load conditions, empty and full. The data show that the vibration levels are almost unaffected by load in the lower frequency ranges. Higher frequency components, however, are reduced on loaded trucks.

Data are also included to show the effect of orientation. The curves show that the vibration levels are highest in the vertical direction over wide ranges of frequency. The lateral and fore-and-aft directions have almost equal vibration levels, but lower than the vertical direction.

Reference 2 summarizes recent data recorded on the cargo floor of a semi-trailer during a transcontinental shipment (Reference 3). Only the summary plots are presented in Reference 2. Data for specific road speeds, road types and their frequency of occurrence can be obtained from Reference 3.

Reference 4 presents shock and vibration data from an air-ride suspension van recorded during rough road and smooth highway operations. The amplitudes of the van floor rarely exceeded 1-g peak. At low frequencies the center of the van floor lengthwise and widthwise is less severe.

b. By Rail – The following paragraphs describe the railroad shock and vibration environment. These data are categorized into two major classifications: over-the-road operation and coupling. The over-the-road environment includes all data except the shock motions associated with coupling or humping operations.

Railroad (Over-the-Road) – Reference 1 presents an extensive analysis and summary of these data. The data were enveloped for all types of suspension systems, road conditions, directions, and speeds, and therefore reflect the highest levels of vibration which would normally be measured.

Curves are presented showing the effect of direction during normal running conditions. These curves show that the most severe environment over wide frequency ranges is caused by vertical vibrations followed by that caused by vibration in the lateral direction. The fore-and-aft direction indicates the lowest level of continuous vibration.

The effect of train speed on vibration levels is shown for a number of train speeds (20, 40, 73, 80 MPH). The data include measurements on cars with various types of suspension systems. On the soft ride car, for Minuteman, the longitudinal levels were so low (approximately 0.10g) that humping was considered the only significant longitudinal environment.

2. Railroad Coupling – The shocks which occur during coupling operation are generally considered to be the most damaging rail environment. The data which describe these shocks are usually presented in terms of shock spectra.

Reference 1 presents summaries of data for coupling and humping operations in terms of shock spectra. Data are presented for both standard and high-capacity draft gears for a variety of impact velocities.

c. By Air – Reference 1 presents summaries of the vibration environments occurring during air transportation. Composite curves are shown for propeller, helicopter, and jet aircraft. The vertical, lateral and fore-and-aft components are shown for C-123, 377PG, C-130, C-133, KC-135, and H-37 aircraft. The data presented are composites of data obtained under all operating conditions. The diagrams show that the vibration levels are highest for the helicopter and lowest for the jet. The only aircraft showing a significant effect of orientation on the environment is the Pregnant Guppy (377PG). For this aircraft, the vertical vibrations are highest, and the fore-and-aft vibrations are lowest.

d. By Ship – Data from many sources have been used in Reference 1 in constructing composite diagrams for shipboard vibration. The data have been separated into continuous and transient vibrations. Transient vibrations are defined as those which occur during emergency maneuvers and by slamming (the impacting of the ship on the water after the bow has left the water.) Continuous vibrations are defined as those which occur during normal operations, including operations in rough seas.

At the present time, measurements in the cargo areas of ships are insufficient to define the environment. Fantail levels have usually been monitored because they are the most severe. Since cargo area data are meager, the fantail data can be used as an upper bound.

In Reference 1, composite data are presented for slamming, emergency maneuvers, and continuous vibrations. The continuous vibration-composite envelope includes data recorded under extremely rough sea conditions.

Diagrams are presented showing the effect of the sea state for two different ship lengths (L = 820 ft. and L = 380 ft.). The accelerations for the smaller ship (a destroyer) are almost twice as large as those for the large ship (an aircraft carrier). For each class of ship, the acceleration increases by a factor of two when the ship operates in a rough rather than a smooth sea.

The higher vibration frequencies (> 10 Hz) are due to machinery vibration and are seen to be less a function of sea state than the lower frequency rigid-body motions.

Data are also presented showing the effects of orientation and maneuvers on a 572 ft. single-screw cargo ship. Examination of these curves show that the vertical vibration environment is the highest, followed by the lateral and fore-and-aft directions. Maneuvers generate higher levels of vibration than straight runs.

Reference 5 presents data measured on an ocean-going barge used in transporting the S-IV stage. For this case, the primary environment was caused by gentle swells causing low-level accelerations in the range from 4-12 Hz. For one shipment, strong winds produced an 8-10 ft. chop. The shallow draft of the barge was disadvantageous as it began to "spank." The spanking induced responses at around 9 Hz.

In general, normal sea transportation provides a very smooth mode of transportation, even including rough sea conditions. Sea transportation is comparable to normal highway travel - assuming that proper stowage and tie-down precautions are employed.

3-3.4.2 Handling Shock Environment

Shocks are generated during handling operations such as loading and unloading, bench handling, hoisting and jacking. These shocks are, in general, more severe than those which occur during transportation. Recently, extensive measurement programs have been initiated to define these shocks more accurately.

Reference 2 presents summaries of the available handling shock data. These data are reported in terms of drop height, zones of shock peak accelerations, or shock spectra.

The data reported in terms of drop height have been organized to show the drop height probability, the number of drops likely to occur during a trip, and the distribution of the drops over the faces, corners and edges. The effect of the distribution system, handholds and labels on the package, and the effect of package size and weight are also described.

Data are also presented in terms of zones of shock from 1 to 5 with the 5th zone representing the severest shock. These data provide information on the relative severity of the transportation and handling environment as determined from a study of commerical impact recorders mounted in wooden boxes and shipped as ordinary products. Data are also given from laboratory tests performed to correlate instrument readings with drop heights. The correlation is difficult to establish because of the wide spread in the data.

A shock spectrum is presented which was computed from data recorded during a transfer of the Saturn rocket stage. The shock was produced when the forward end of the stage dropped from a height of three inches. The plot envelops the shock spectra at four locations on the rocket.

3-3.4.3 References

1. Ostrem, F. E., and Rumerman, M. L., *Shock and Vibration Transportation Environmental Criteria,* Contract NAS 8-11451, Final Report MR 1262, General American Research Division, Niles, Ill, September 21, 1965.

2. Ostrem, F. E., and Rumerman, M. L., *Transportation and Handling Shock and Vibration Environmental Criteria,* contract NAS 8-11451, Final Report MR 1262-2, General American Research Division, Niles, Ill., April 28, 1967.

3. Foley, J. T., *The Environment Experienced by Cargo on a Flatbed Tractor-Trailer Combination,* Research Report SC-RR-66-677, Sandia Corporation, December 1966.

4. Schule, J. W., *The Dynamic Environment of Spacecraft Surface Transportation,* Tech Report No. 32-876 Jet Propulsion Laboratory, March 15, 1966.

5. Trudell, R. W., and Elliott, K. E., *The Dynamic Environment of the S-IV Stage During Transportation,* The Shock and Vibration Bulletin, No. 33, Part IV, March 1964.

3-3.5 Scaling Laws for Shock

Test data indicate that a separation shock amplitude is proportional to the thickness of structure cut, if the skin has a constant thickness from the separation plane to the location of interest (for skin discontinuities, see Figure 3-21). The shock is independent of charge size for linear shaped charge. For cylinders, the frequency of maximum amplitude (shock spectrum) of the shock is assumed to be inversely proportional to the diameter.

When separating around the circumference of a vehicle with a continuous system, the acceleration levels are assumed to be inversely proportional to the diameter. For point sources the acceleration is assumed to be inversely proportional to the diameter raised to the three-halves power, and to be proportional to the number of point explosives situated in a plane normal to the skin, provided the point of interest is a distance from the separation plane greater than half the distance between point explosive locations.

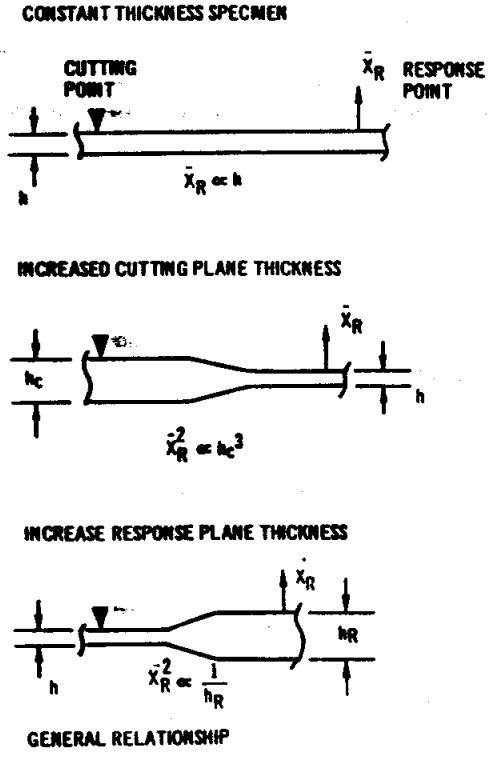


Figure 3-21. Scaling Explosive-Skin-Cutting Shock-Amplitudes

3-3.6 Motor Ignition Overpressure

Due to the short rise-time to achieve full chamber pressure for certain solid and liquid rocket engines, a shock wave is generated which can be compared to a TNT explosion shock wave. However, the motor ignition overpressure pulse will be ragged in amplitude and in the near-field (say 100 ft.) will have directionality.

The peak value of chamber pressure and the positive phase duration can only be determined by pressure transducers mounted on the rocket chamber wall during tests of the actual engine. In the far-field the overpressure will follow standard explosive scaling laws.

3-3.7 Motor Ignition and Shut-Down Transients

Rapid changes in acceleration forces during motor ignition or shut-down can lead to low-frequency transient loads going into the missile structure, and to decaying sinusoidal responses in vehicle compression and bending modes.

The magnitude and duration of these shock response loads can only be determined by test. Design shock loads may be obtained by scaling from a similar, known vehicle, or by dynamic analysis using the force transients when they are well defined.

For large liquid–propellant vehicles, an interaction between the propulsion system and vehicle structure may result in "Pogo" (low frequency longitudinal) oscillations which can be more severe than ignition or shut-down transients. See paragraph 3-4.2.

3-3.8 Shock (Response) Spectra

The Shock Spectrum is a plot or listing of the envelope of the maximum absolute values of response to a shock of an ensemble of single degree-of-freedom systems having natural frequencies which cover the frequency range of interest. The responses may be expressed in terms of accelerations, velocities or displacements, and may be in the form of maximax, maximax relative, or residual shock spectra. The maximax and maximax relative titles are self explanatory; the residual shock spectrum is the maximum response which occurs after the exciting shock has ended. Shock spectra are usually given as a family of curves where each curve represents a specified constant value of damping for the entire ensemble.

Shock Spectra are used:

a. As a measure of the severity of a shock as a function of frequency.

b. To define design and test criteria.

c. To determine the maximum response calculated by equations having single degree-of-freedom forms.

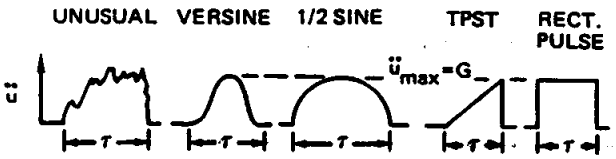
Maximax shock spectra for some basic shock pulses are presented in Figures 3-22 through 3-28. They are plotted as a response/input ratio versus the time duration of the pulse (τ) times the frequency (in Hz). Further discussions of shock spectra are given in chapters 8 and 23 of the "Shock and Vibration Handbook," by Harris and Crede, and in an article in Sound and Vibration Vol 4 Nos. 8 and 9 Aug and Sept 1970 by J. E. Ruzicka entitled "Passive Shock Isolation".

3-3.9 Drop Height (Inches) Required to Obtain the Same Velocity Step as a Single Shock Pulse

The total velocity step (V) of a pulse is given by:

$$V = \int_0^\tau \ddot{u}\, dt$$

where $\ddot{u}$ is the acceleration and τ is the total time duration for the pulse. For some known pulses (e.g., 1/2-sine, terminal peak sawtooth, etc.) this integration is straightforward. For an unusual pulse, the integration must be performed by some means. Typical examples are:



$$V = \int_0^\tau \ddot{u}\, dt \quad = \frac{G\tau}{2} \quad = \frac{2}{\pi} G\tau \quad = \frac{1}{2} G\tau \quad = G\tau$$

For all pulses,

$$\ddot{x} = \omega \int_0^\tau \ddot{u}\, dt = \omega V$$

where $\ddot{x}$ is the total acceleration in units of inertial acceleration (in/sec^2) and ω is frequency in radians per second.

The terminal velocity (V′) just prior to impact of a dropped item is given as:

$$h = \frac{(V')^2}{2g}$$

where V′ is in inches/sec, h is the drop height in inches, and g is the acceleration due to gravity. Assuming the mass upon which the item is dropped is much greater than that of the item, implies that V′ = V. Hence:

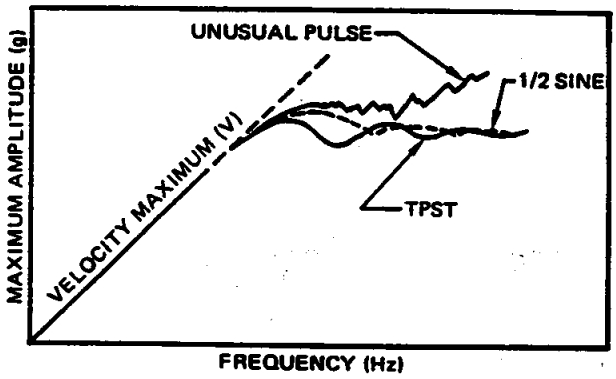
$$h = V^2/2g.$$

At a frequency of 10 Hz, or 20π radians per second, a frequency low enough that all the shock spectra of interest are on the constant velocity curve,

$$V = \ddot{x}/20\pi \text{ or } V = ag/20\pi$$

where a is acceleration in units of g, and $h = 0.049\, a^2$. This provides a quick way to predict the height of a drop which will cause a certain g level (comparable to a given shock pulse) at 10 Hz. For example, a 150 g, 2 msec TPST has a shock spectrum that crosses 10 Hz at 9.42 g. Thus, a comparable drop height to provide an equivalent velocity step is 4.35 inches.

In addition, the expressions given above for simple pulse shapes plus the relationship $\ddot{x} = \omega V$ provides a quick method for locating shock spectra on graph paper. A value for $\ddot{x}$ at some convenient frequency is quickly found, and a 45° line drawn through that point on shock spectrum paper provides the constant velocity portion of the spectrum for lightly damped systems as shown in the following sketch.



3-4 VIBRATION

3-4.1 Prediction of Random Vibration Environment Due to Fluctuating Pressure Excitation

Fluctuating pressures, impinging on a launch vehicle or aircraft, induce random vibration in the vehicle surface skin. The two basic sources of these fluctuating pressures are:

a. The rocket motor or jet engine noise occuring at liftoff and during subsonic flight.

b. The fluctuating pressures occuring beneath the turbulent boundary, or in a wake, during supersonic flight.

Paragraphs 3-1 and 3-2 have presented the techniques for establishing the acoustic levels for these noise sources. This paragraph will start with the acoustic environment and show the methods for deriving the random vibration environment.

Many methods of predicting the random vibration have been advanced, and may generally be grouped in the following categories: a classical or multiple-input approach, an extrapolation approach, and a statistical energy approach. The classical or multiple-input approach consists of mathematically modeling the structure (either distributed or lumped parameter) and the forcing functions and solving the equations of

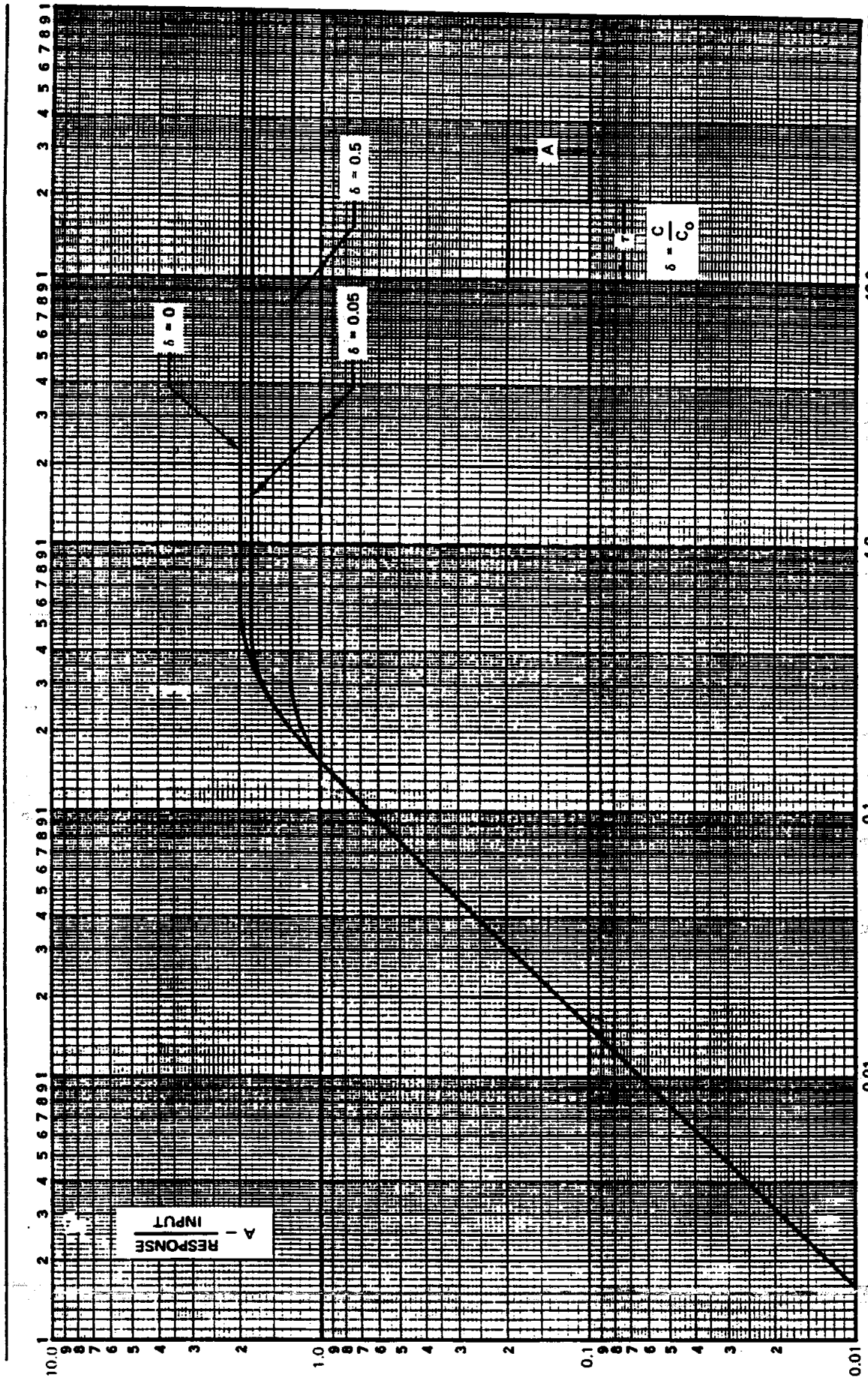


Figure 3-22. Maximum Shock (Response) Spectrum for a Step Function

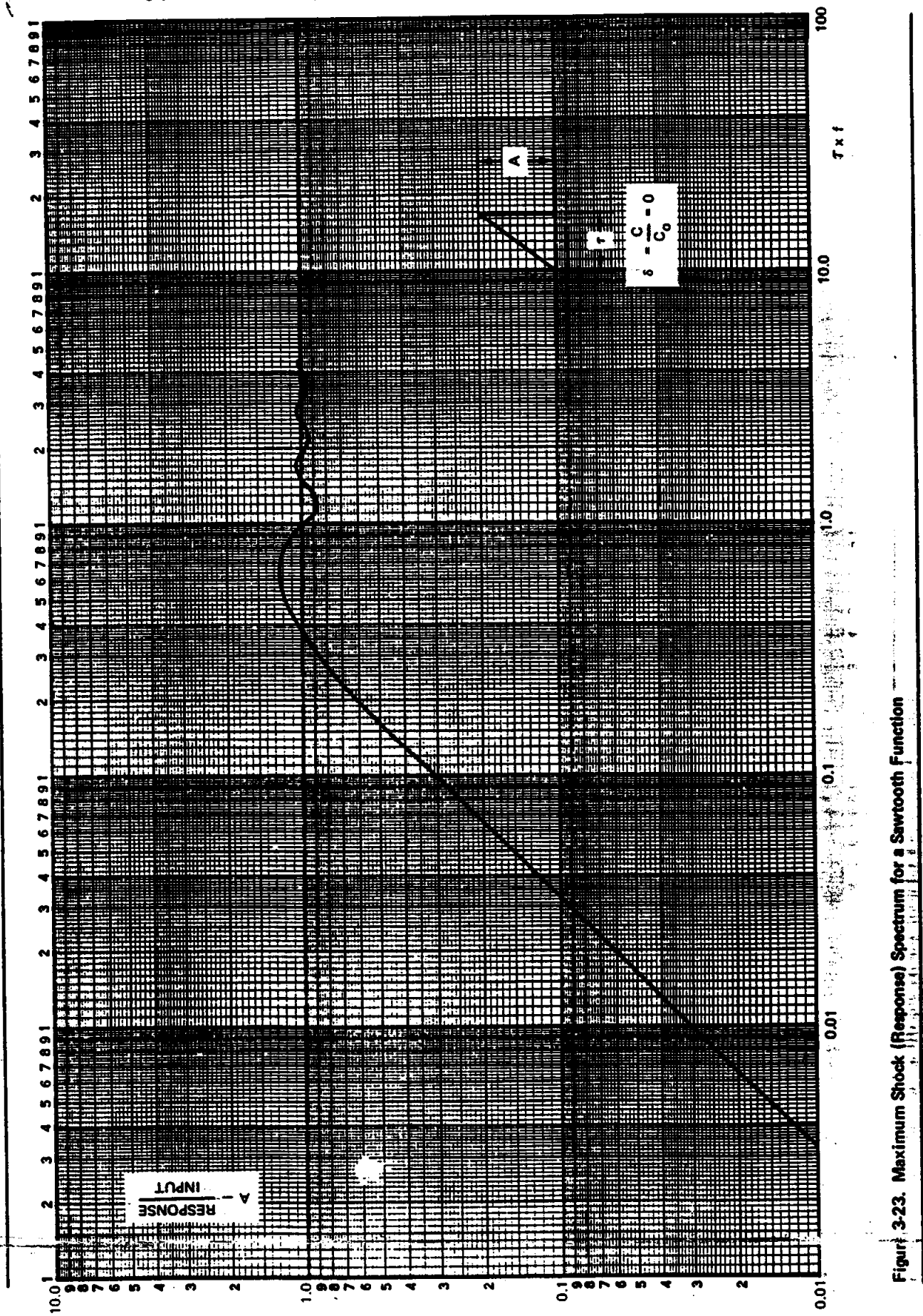


Figure 3-23. Maximum Shock (Response) Spectrum for a Sawtooth Function

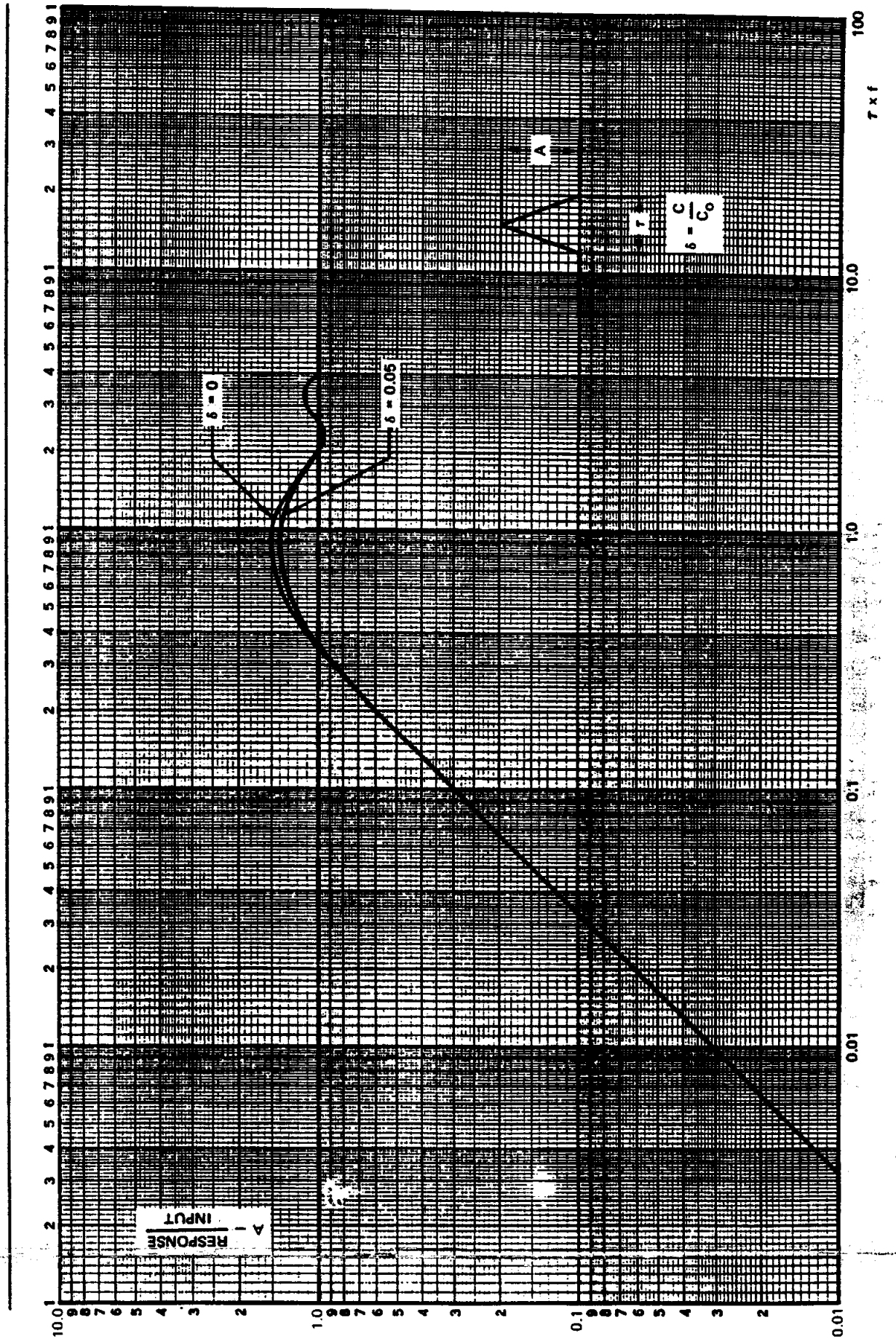


Figure 3-24. Maximum Shock (Response) Spectrum for a Triangular Pulse

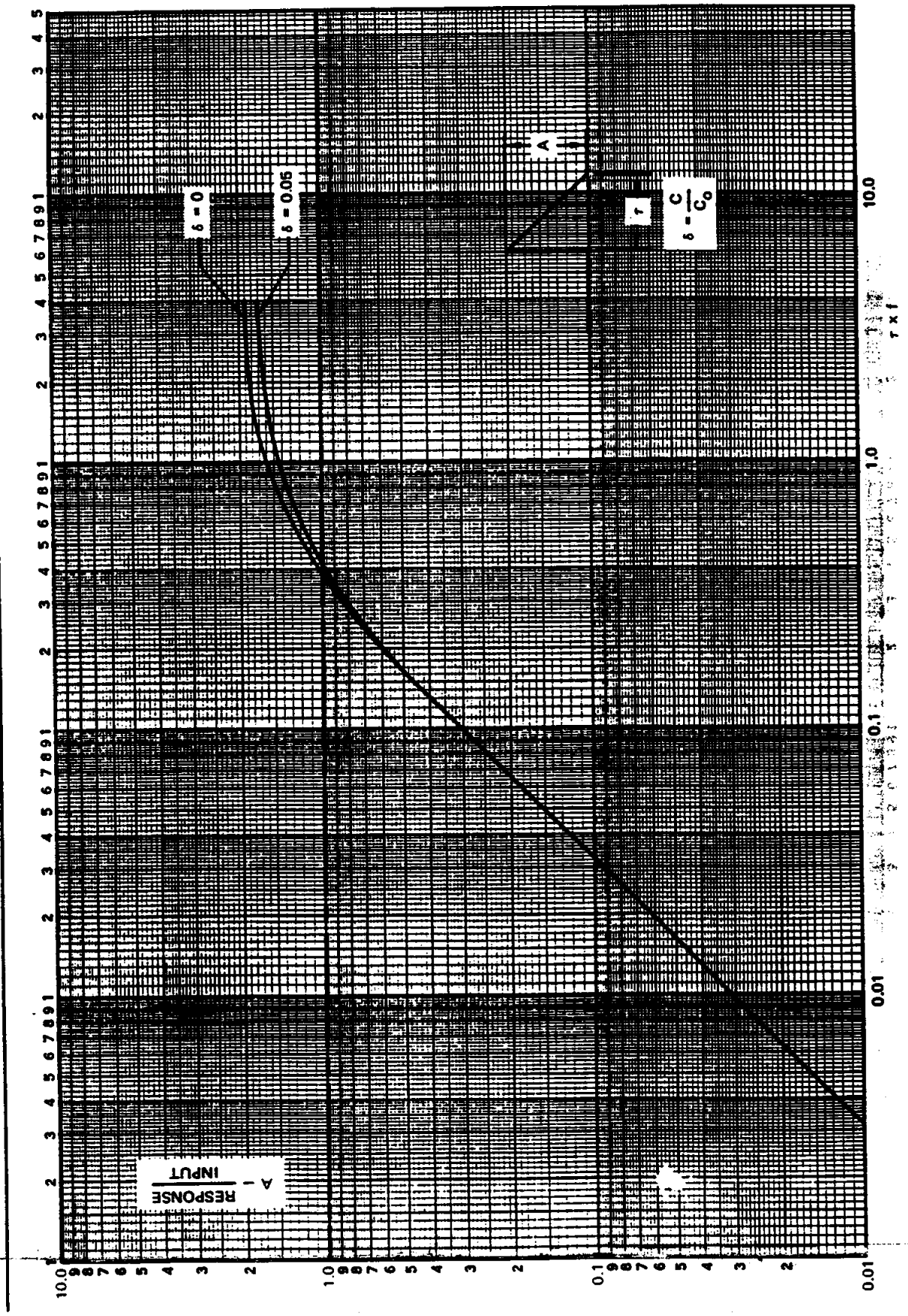


Figure 3-25. Maximum Shock (Response) Spectrum for a Reverse Sawtooth Pulse

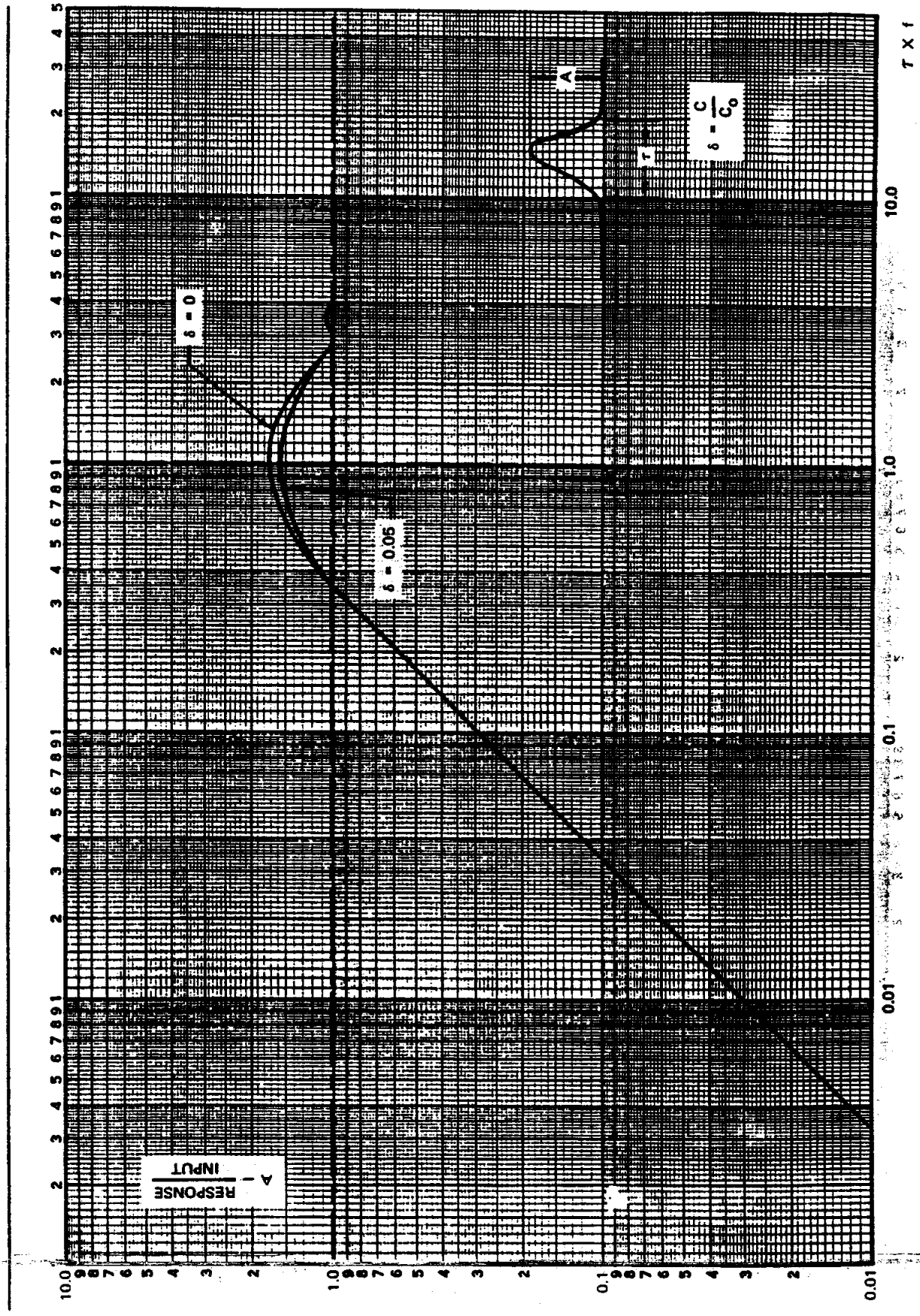


Figure 3-26. Maximum Shock (Response) Spectrum for a Haversine Function

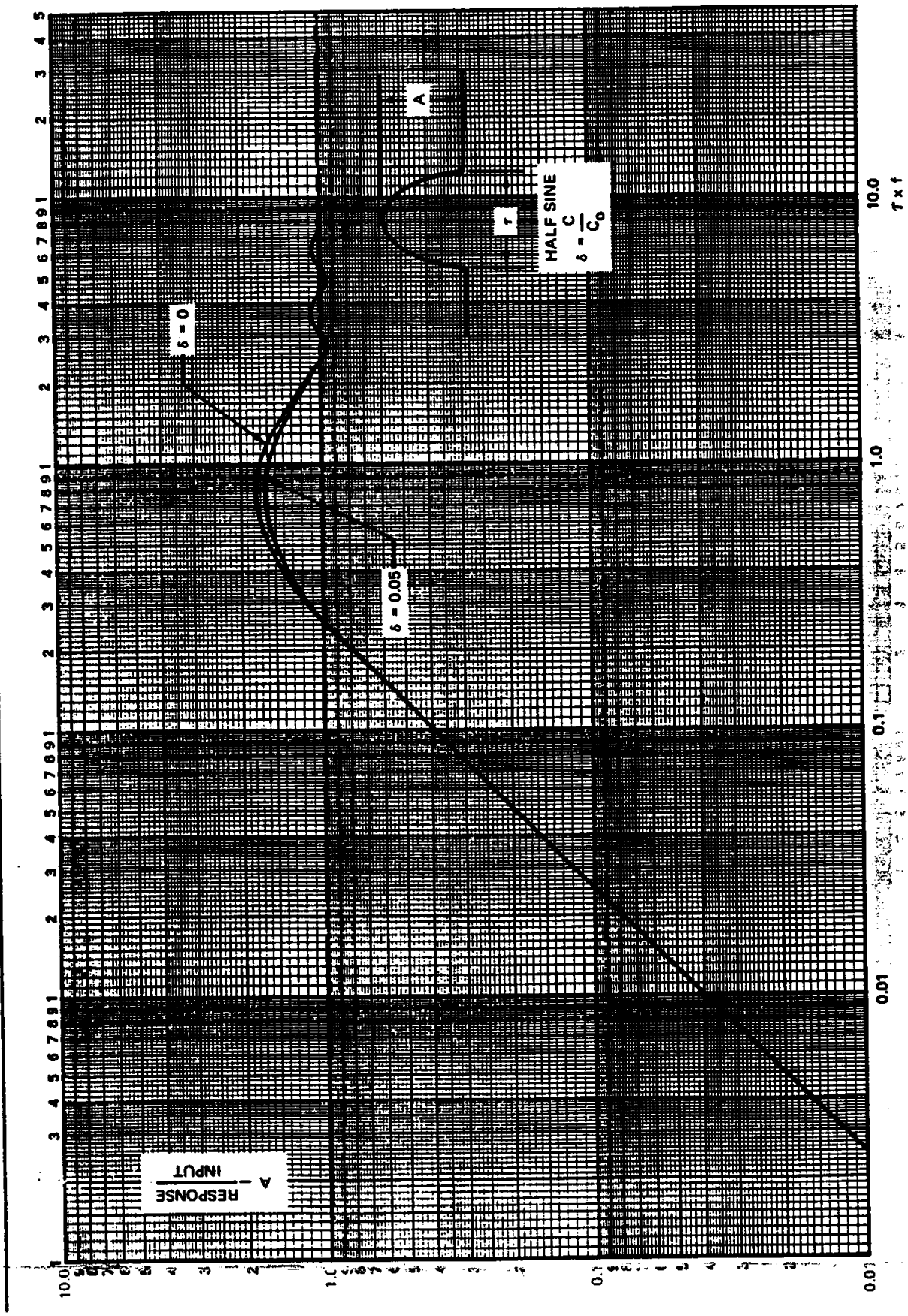


Figure 3-27. Maximum Shock (Response) Spectrum for a Half-Sine Pulse With Low Damping

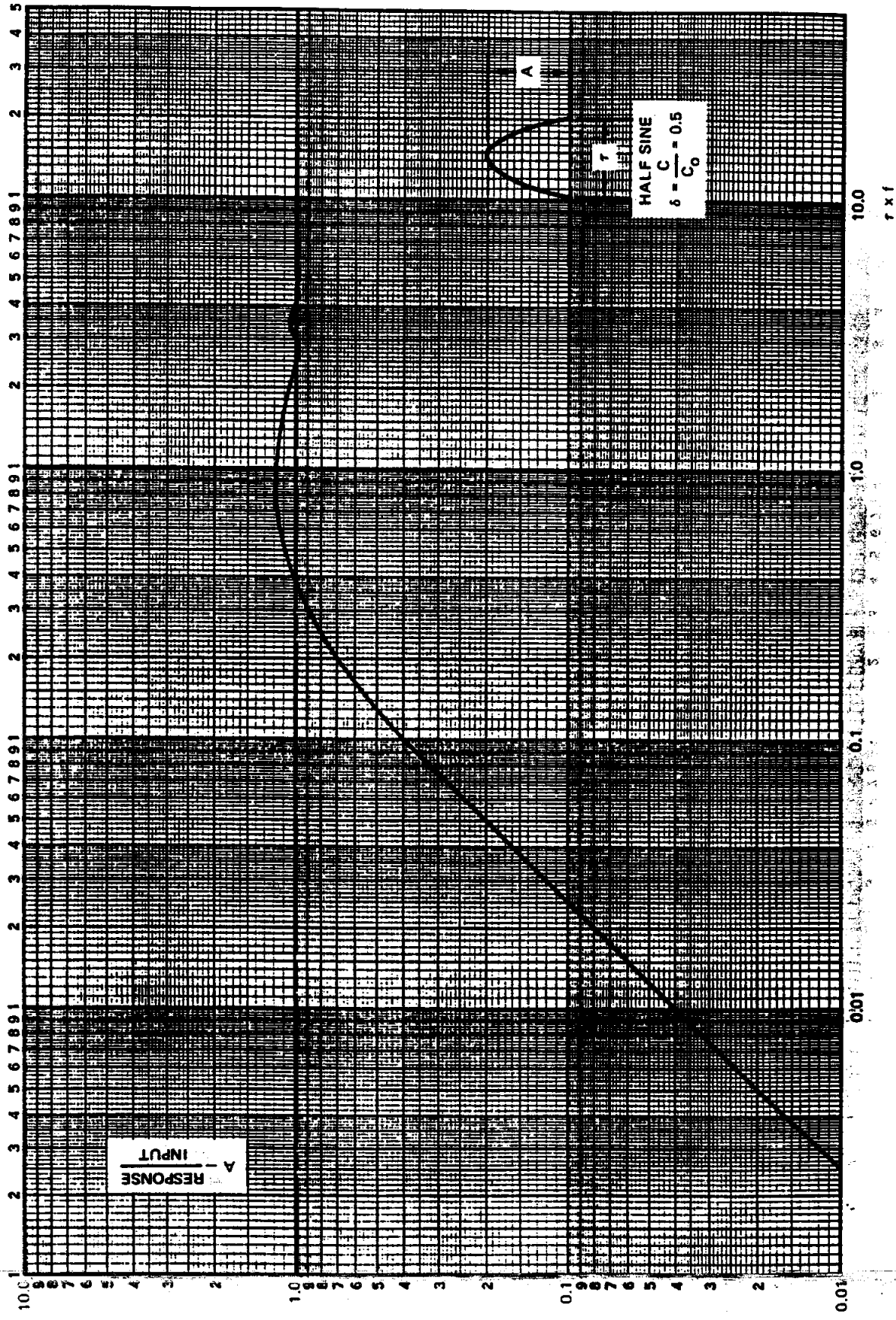


Figure 3-28. Maximum Shock (Response) Spectrum for a Half-Sine Pulse With High Damping

motion. The extrapolation approach is based on use of empirical data measured on similar vehicles and scaled to the particular vehicle being considered. The statistical energy approach assumes the motion of the structure is dominated by the resonance response rather than the forced non-resonance response, and uses a statistical description of the structure. The statistical energy method is not sufficiently developed to be considered a usable (handbook-type) engineering tool.

Of these categories, the extrapolation approach is the most commonly used, and since it is amenable to presentation in this manual, it is presented in detail. References 6 through 12 present various classical techniques.

Fundamentally the extrapolation technique consists of using measured data and scaling relationships to transform the measured data to the vehicle being considered. Therefore the data being used as the basis for the prediction of a random vibration environment should be obtained from measurements made on a vehicle with structural similarities (vehicle diameter, skin, stringers and frame spacing) as close as possible. Thor, Delta, Saturn, or Spartan flight data should be used when it is required to predict a random vibration environment on the respective or similar vehicles.

3-4.1.1 Scaling Relationships

The general scaling procedure may be summarized in the following steps:

a. Obtain a baseline measured random-vibration-spectrum induced in a vehicle, similar to the one being considered, at a known acoustic environment.

b. Adjust the baseline random-vibration-spectrum to account for the difference in acoustics between the two vehicles.

c. Adjust the baseline random-vibration-spectrum for the structural differences between the two vehicles.

The general scaling relationship for random vibration may be expressed as follows:

$$G_n(f) = G_d(f) \left(\frac{W_d}{W_n}\right)^2 A_R$$

where:

subscript n denotes new vehicle

subscript d denotes the data vehicle

G(f) = power spectral density for the acceleration response (g^2/Hz)

W = average surface weight density of vehicle structure (lb/ft^2)

A_r = acoustical ratio.

The acoustical ratio is defined as the ratio of the power spectral densities for the pressure excitation as follows:

$$A_r = \frac{G_{pn}(f)}{G_{pd}(f)}$$

where G_p (f) = power spectral density for the pressure excitation (psi^2/Hz).

In most instances the pressure excitation is defined in terms of octave-band sound-pressure-levels (OBSPL). When this is the case, and the power spectrum is not available, the OBSPL's can be used by converting to a pressure spectrum (S_p in dB) by:

$$S_p(f) = \text{OBSPL} - 10 \log \Delta f$$

where Δf = octave band frequency bandwidth. Assuming a smooth spectrum using S_p, A_r becomes:

$$A_r = \frac{p_n^2(f)}{p_d^2(f)}$$

Where p (f) is the pressure excitation in force per unit area (units calculated from S_p).

The preceding scaling relationship assumes that there are not significant differences in local lumped masses between the "data" and "new" vehicles. To account for local mass effects (which tend to attenuate the skin random vibration environment), two techniques are available, with the latter being the preferred method.

Method 1. Attenuation due to mass loading,

$$\frac{G_L}{G_U} = \frac{W_n}{W_n + W_c}$$

where:

subscript L denotes the mass-loaded structures
subscript U denotes the unloaded structure

W_n = weight of structure in area where the mass is to be mounted (lb)

W_c = weight of the component to be mounted

Method 2. Attenuation due to mass loading,

This technique, of correcting the unloaded structural response for local mass-loading effects, is based on a characteristic of the random vibration

response to pressure fluctuations; i.e., the response maximizes in the high modal-density region of the frequency domain. Modal-density may be defined as the number of modes (of resonance) per unit bandwidth. In this frequency region the effect of boundary conditions and shape of the structural panels on the response of structure is minimized so that the panels may be approximated by infinite flat panels. With this approximation the attencuation factor may be based on equating the impedance looking into an infinite panel to the impedance for a rigid mass, as follows:

$$\frac{G_L}{G_U} = \frac{Z_s^2}{Z_s^2 + (\omega M)^2}$$

where:

subscript L denotes loaded panels
subscript U denotes unloaded panels

$Z_s = \frac{4}{\sqrt{3}} \rho C_\ell t$, impedance of an infinite flat panel

C_ℓ = wave velocity in the material (ft/sec); (in aluminum $C_\ell \cong 17{,}000$ ft/sec)

ρ = surface density (slug/ft^2)

t = total skin thickness (ft)

M = local mass (slugs)

ω = circular frequency (sec $^{-1}$)

3-4.1.2 Empirical Prediction Techniques

When data are not available on a vehicle similar to the one being considered, empirical data presented in a general form may be used to estimate the random vibration response.

Many such techniques have been presented in the literation. The Mahaffey and Smith Method, Reference 12, was designed to predict the acoustically-induced vibration environment of new, jet-powered vehicles by use of a response function developed from data measured on the B-58 airplane. The Mahaffey and Smith method should therefore be used when aircraft are being considered. However, due to this limited application the method is not presented in this manual.

For missiles and launch vehicles the most generally accepted empirical technique is the Franken Method, Reference 13. This method is designed to predict acoustically-induced free skin vibration levels for cylindrical vehicles based on external sound pressure levels, vehicle diameter and average surface weight density of the vehicle skin. A vibro-acoustical transfer function has been established from measured data and is presented in Figure 3-29. Therefore, the transfer function must be translated by dividing the abscissa by the diameter of the vehicle prior to reading the ordinate. The transfer function (TF), the ordinate, may be transformed into the following equation to calculate the induced vibration power spectral density directly:

$$\tilde{G} = \frac{\log^{-1}(0.1[SPL + TF])}{\Delta f_n W^2}$$

where:

$\tilde{G}$ = the average vibration power spectral density (g^2/Hz)

SPL = external sound pressure level (dB)

TF = vibro-acoustical transfer function (dB) read at the geometrical mean frequency of Δf.

Δf = bandwidth of SPL, (Hz)

W = average surface weight density (lb/ft^2)

This technique, therefore, predicts an average vibration level over the bandwidth used to average the sound pressures. If the SPL is an octave band SPL (OBSPL) then the bandwidths are octave bandwidths and $\tilde{G}$ is the average value over that Δf. The abscissa is expressed in frequency times the vehicle diameter. The TF is a mean of the data on which it is based, and a 95 percent confidence band is TF ± 3 dB.

An interesting footnote is that the transfer function peaks near the so-called ring frequency f_R:

$f_R = C_\ell/\pi D$

where:

c_ℓ = Speed of sound in the surface material or approximately equal to $\sqrt{E/\rho}$.

D = Vehicle diameter as before.

E = Young's Modulus of the surface material.

ρ = mass density of the surface material.

For most metals, f_R is approximately 5200/D, or $f_R D$ is 5200.

The related characteristic of rms strain, ϵ, induced by the TF-predicted acceleration a, can be found using the equation:

$\epsilon = KC\, a\, [2\pi f c_\ell]^{-1}$

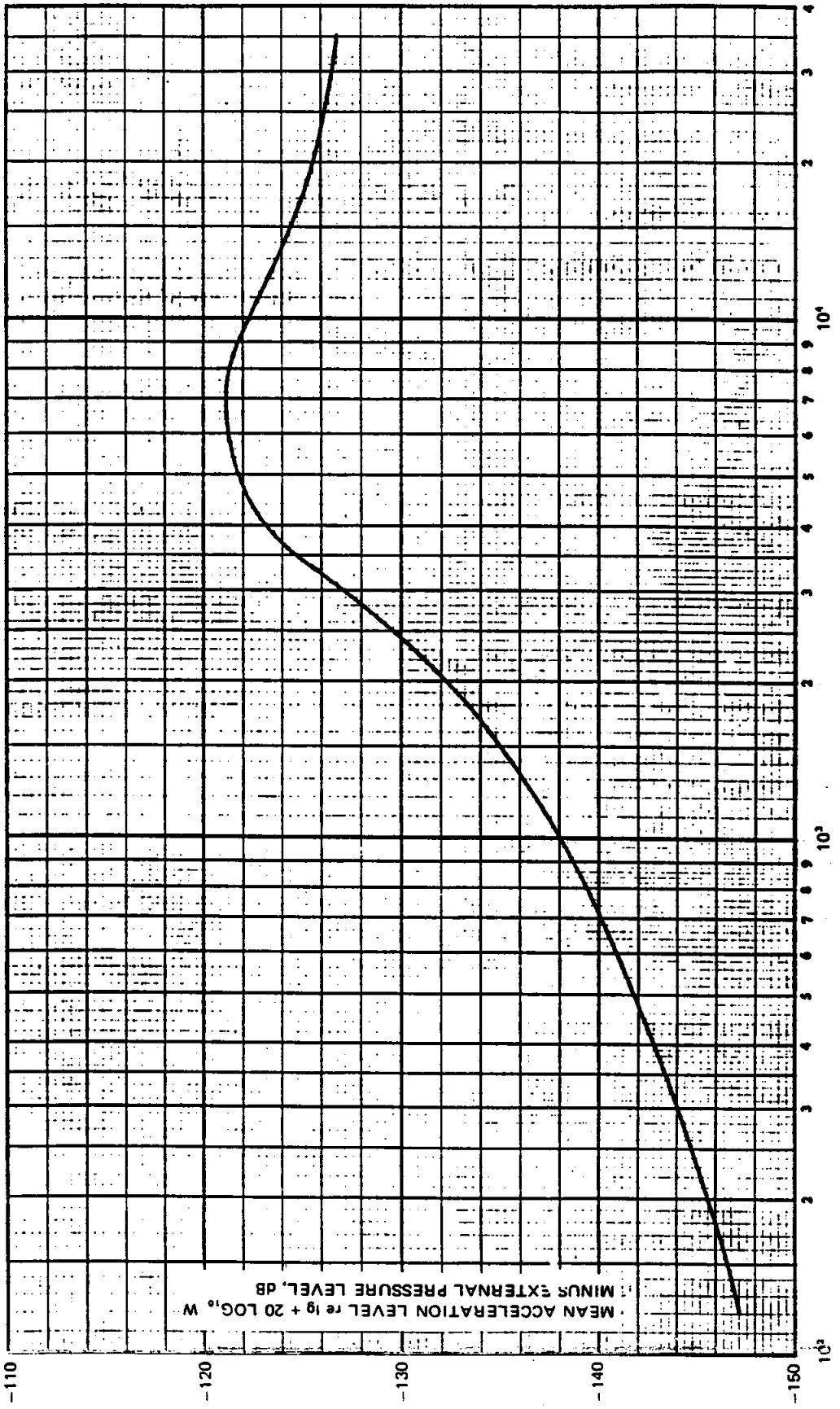


Figure 3-29. Francken Transfer Function

where:

K = Constant determined by type of wave propagation and structural shape.

= $\sqrt{3}$ for straight-crested flexural wave traveling in a plate.

C = Strain concentration factor

= 1 if no strain/stress concentration.

a = Acceleration predicted by Franken curve.

f = Frequency.

c_ϱ = Speed of sound in the material.

The above TF, developed for cylindrical vehicles, may be used to estimate the vibration environment in the aft end of conical vehicles by using a diameter equal to 2/3 of the base diameter.

3-4.2 Prediction of Sinusoidal Vibratory Response

From launch to reentry, most of the flight dynamic environmental loads in a missile or spacecraft are essentially random in nature. However, there are sources of sinusoidal vibration which may be of importance (see paragraph 3-3.7): Motor ignition, motor screeching, rotating mechanical devices, motor shut-down, transient payload response to buffeting in the body bending modes, and propulsion system/structure interaction of large liquid engine vehicles.

The latter phenomenon, called Pogo oscillation, is a temporary but severe longitudinal low-frequency sinusoidal oscillation, and has been observed on Thor, Titan, and Saturn vehicles.

A method of predicting whether Pogo oscillations will occur is discussed in an AIAA Paper by Sheldon Rubin from the "AIAA Symposium on Structural Dynamics and Aeroelasticity", Boston August 30-September 1, 1965.

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Section 4
LOADS

This section of the manual contains procedures for establishing test requirements, design requirements, and structural dynamic loads. These procedures are recommended for use unless specific customer requirements are imposed by contract.

4-1 DERIVATION OF TEST REQUIREMENTS

The derivation or establishment of test requirements depends upon the test specimen level of assembly, the type of test program, and the test objectives. These items in turn depend upon overall program test philosophy, reliability requirements, and available funding and facilities. It is suggested that in planning test requirements, the specification writer consult Section 7 for recommendations regarding test specimen definition, test set up, test fixtures, and test instrumentation. The test requirement documents for various MDAC-WD programs, listed in Table 4-1, should be reviewed for format.

Table 4-1. Summary of Design and Test Criteria Documents

PROGRAM	TYPE OF VEHICLE	MDAC-WD REFERENCE DOCUMENT
DELTA	SPACE BOOSTER	DWG. 1B07587
MOL	MANNED ORBITING LABORATORY	MEMO: A3-260-ABD2-277
SATURN S-IVB	UPPER-STAGE SPACE BOOSTER	RPT. 56824
UPSTAGE	ANTI-MISSILE MISSILE	DWG. 1D11764
SPARTAN	ANTI-MISSILE MISSILE	APPENDIX D TO SS00002 (DESIGN) DWG 1B14229 (TEST)
NIKE ZEUS	ANTI-MISSILE MISSILE	MEMO: A2-260-ABD1-310

The following paragraphs contain descriptions of various types of dynamic test programs and their objectives, together with recommended methods for achieving these objectives. Guidelines for establishing the test specimen level of assembly are also presented where applicable.

a. Development Tests – Development tests are performed to achieve one or more of the following objectives:

1. To obtain data on dynamic characteristics of an item, e.g., frequencies and mode shapes.
2. To obtain parametric design data, e.g., spring force required to prevent valve chatter under vibration.
3. To establish the fragility of an item.
4. To investigate the feasibility of using a particular design concept in a particular environment.
5. To finalize a design for a specific environmental requirement.
6. To demonstrate readiness for a qualification test.
7. To develop test techniques for qualification and acceptance tests.

Depending upon the particular test objectives, the environments applied during development testing might include one or more of the following:

1. Sinusoidal vibration to establish critical frequencies, transfer functions, and damping for either design or failure diagnosis.
2. One or more critical qualification environments including sinusoidal vibration, random vibration, acoustic noise, or shock, to demonstrate that the item is ready for its qualification test.
3. Random vibration fragility tests to determine failure modes and vibration allowables for use in designing support structures whose resonance frequencies will not couple with critical component frequencies.

Development tests are usually performed on a single test specimen hardmounted to a shaker or shock testing device and the test inputs include estimated bracket amplification factors. However, acoustical or pyrotechnic-shock development tests may also be performed on large subsystems or systems using either dummy or live components to obtain environmental data (including bracket dynamic response characteristics) for use in specifying component design and qualification requirements. Serious consideration should be given to the potential cost savings that could be achieved by the latter approach as a result of having a more precise definition of design criteria early in the program. The same test bed might be used later for qualification tests of the system or subsystem. This approach is particularly attractive for missiles and space payload/shroud combinations and will also be used for many spacecraft programs as a result of the development of many large acoustical-test-facilities.

Another advantage of the system acoustical development-test is that it provides valuable information that can be used to extrapolate flight vibration measurements to predict responses in zones where there are no flight measurements. This is possible because inflight vibration is a direct result of an acoustic pressure field or boundary-layer-induced pressure fluctuations which are assumed to be simulated resonably well in an acoustical test facility.

b. Qualification Tests — Qualification tests are performed to demonstrate to the customer or using agency that an item, as designed and manufactured, is capable of satisfying the applicable contract requirements. Some programs include both qualification and formal qualification tests. Qualification tests are usually performed on a single specimen of any item judged to be susceptible to damage or malfunction due to flight dynamic environments. Formal qualification tests are usually performed on several specimens of the most flight-critical items and require a greater degree of customer participation in test planning and evaluation. The test specimen configurations and environments are usually the same for qualification and formal qualification tests with specimen complexity ranging from single components to full systems.

Component vibration and shock qualification tests should utilize flight mounting-brackets that are in turn installed on sufficient back-up structure to ensure that the correct bracket dynamic-amplification or attenuation occurs at the proper frequencies and to ensure the structural integrity of the brackets.

The shock and vibration test requirements are specified at the interface between the brackets and the support structure. Hardmount qualification tests are not recommended; however, if the bracket amplifications and structural integrity have been previously demonstrated during development tests and the vibration levels at the component/bracket interface are conservatively enveloped, then hardmount tests are acceptable. It should be noted, however, that hardmount tests are more severe because of the enveloping required and because of impedance differences between the shaker and the vehicle. When there is dynamic interaction between components in a subsystem, then the qualification test specimen must include all such components with all connecting lines and wirebundles.

System level qualification tests may result in significant cost and schedule savings for some missile and small spacecraft programs. The environments that should be considered are acoustic noise, pyrotechnic shock, and possibly low-frequency sinusoidal vibration and shock. The advantages are greater simulation of actual environments and responses plus the elimination of many component tests that require special test fixtures and test facilities.

For large spacecraft, the disadvantages of completely relying on system level tests would appear to outweigh the advantages. One disadvantage is that unless sufficient development testing has been performed, there is a strong possibility of being overwhelmed with failing components late in the development program because the tests cannot be performed until all systems become available. Another disadvantage is the huge amount of equipment required to monitor the system outputs for functional failures. A third disadvantage is that if more than one equipment specimen or retesting of redisigned hardware is required, repeated testing may produce fatigue failures of the vehicle structure.

It is recommended that for large spacecraft, a good compromise is to place heavy reliance on subsystem tests, e.g., large equipment mounting panels with many installed components qualified by one test. If part of the subsystem is not available it can be replaced with dynamically-similar dummies, and then tested later in a second test. This approach was used successfully on the Saturn S-IVB program.

The recommended approach for deriving test requirements is to predict the acoustic, shock, and vibration environments to which the missile or space system will be exposed (level and durations), using the procedures indicated in Section 3, and then establish a minimum number of test environments that will simulate (with a reasonable factor of safety) the maximum levels and the cumulative fatigue damage caused by these environments.

A safety factor of 1.41 (3 dB) on test level and at least 2.0 on duration is recommended to account for:

1. Inaccuracies in predicting or measuring the operational environment.

2. Inaccuracies in simulating the operational environment in the laboratory.

3. The probable scatter in strength, damping, or performance capability of the component to be tested in view of the small number of specimens tested.

If the 3 dB factor is added to the maximum expected flight environment (usually the 97.5 percent confidence estimate which is approximately 3 dB above the mean environment), the qualification test requirements will be sufficiently high to ensure that production acceptance testing of flight hardware at the mean flight levels will not weaken the hardware to the point of failure prior to flight such that it fails during flight. A 3 dB safety factor will also ensure that the lower tolerance on the random vibration spectrum (-3dB) will still encompass the maximum expected flight environment.

c. Reliability Tests — Reliability tests are performed later in the program than qualification tests to obtain additional test data for a reliability assessment, or to demonstrate a required level of reliability. The first objective may be attained with fewer specimens, whereas a large number of specimens are required for the second objective. Reliability tests should be performed on randomly selected specimens of certain flight-critical components.

To properly assess the reliability by comparing component capability with actual flight requirements, the test spectrum shape should reflect all available flight data and bracket amplification data obtained during development or qualification tests. Overstress testing is recommended as compared to long duration testing because it is desirable to accumulate failure information in the shortest possible time and because many failure modes are dependent upon test level rather than duration.

d. Production Acceptance Tests — Production acceptance tests a· performed to detect workmanship or material defects which could result in failure during exposure to flight dynamic environments. Production acceptance tests can include temperature, pressure, and dynamic environments as well as visual inspections and functional tests. It is recommended that dynamic acceptance tests be performed on flight hardware only when they are deemed to be the best method of exposing quality defects. This is because dynamic tests are the most difficult to control, and usually expose the component to the greatest risk of being damaged. For example, dynamic environments can cause screws to loosen if self locking devices are omitted, and these screws may come completely loose during the later exposure to the flight environment. Also, inadvertant overtesting can ruin an otherwise good component.

In spite of these disadvantages, some components require dynamic acceptance tests to expose hidden weaknesses. If the 6 dB factor between qualification test levels and production acceptance test levels is maintained as mentioned previously under "Qualification Tests," then these risks will be minimized. As a further safety measure, it is recommended that qualification tests be periodically repeated to ensure that there has been no overall reduction in component quality and that the original 6 dB factor is maintained.

Hardware items to be considered for dynamic, production, acceptance tests should be flight critical and sensitive to dynamic environments. This would include:

1. Fragile components such as electronic parts (tubes, diodes, etc.,)

2. Dynamic systems such as switches, relays, cantilevered elements, and equipment with moving internal elements and spring-loaded elements, i.e., valves, timers, gyros, and rotary inverters.

Complexity of the design (e.g., an internal arrangement which precludes visual inspection) may also be justification for selection for acceptance tests. An additional consideration may be where the "failure mechanism" is dynamically sensitive and the hardware itself is not (e.g., loose parts, contamination, etc.).

The selection of hardware for production acceptance testing will differ from program to program as a result of differences in hardware in-flight functions and differences in ground test experience. Therefore, it is not satisfactory or practical to establish a universal dynamically-critical hardware list based only on categorical type.

From the standpoint of providing the most ideal installation configuration relative to flight, and of detecting the maximum number of production and assembly defects, it would be desirable to test at the highest level of assembly (e.g., the complete vehicle

or system level). Considerations such as test facility limitations, functional checkout capability and complexity, difficulty of detecting defects, and cost of removing and/or replacing internal components which malfunction generally make system level dynamic tests impractical for all but small missile systems and spacecraft.

For large booster and spacecraft programs at MDAC-WD, component testing is most prevalent and subsystem/subassembly testing to a lesser degree. The Saturn S-IVB Stage also has the benefit of an all-systems static-firing test that produces dynamic environments in some portions of the stage that are more severe than flight and in other portions less severe. Missile, reentry vehicle, and small spacecraft assemblies are commonly tested at the component, subsystem and/or systems level.

MDAC-WD policy as stated in SP 10.024-CD (dated June 18, 1969) requires that production acceptance tests be performed at levels 6 dB lower than qualification test levels to minimize the fatigue damage accumulated in flight hardware. If the qualification test levels are established as recommended in this manual, the acceptance test levels will be equivalent to the nominal or mean expected flight environment, and thus high confidence can be attained that the component will perform satisfactorily in flight.

Figure 4-1 defines a test time factor from which the allowable number of retests can be determined, as a function of the ratio of the qualification test time to the acceptance test time, without exceeding a fatigue damage factor of 15 percent.

Defects detected in MDAC-WD production acceptance testing have included the complete spectrum — production deficiencies, such as cold solder joints, stress incompatibility, wire/terminal wrapping, and delamination, material defects; out-of-tolerances, such as contact pressures, etc.; and contamination. Many defects detected in these programs assuredly have precluded mission failure.

e. WD-MDAC Margin Philosophy — Refer to SP 10.024-CD dated 6-18-69 (at end of this section).

4-1.1 Acoustical Test Requirements

a. Components Requiring Test — Acoustical qualification tests of components should not be required as a routine supplement to vibration and shock tests because, in general, the response of the vehicle primary structure to the acoustic environment causes a vibration input to a component through its attach points that is more severe than that resulting from direct acoustic excitation of the component. Acoustical qualification tests are generally required only for the following types of components:

1. Components suspected of having significant resonance frequencies above the upper frequency limit of vibration exciters (i.e., above 2,000 Hz).

2. Components with diaphragm like elements such as circuit boards, structural skin panels, pressure switches, and pressure transducers.

3. Components mounted on large equipment mounting plates, especially if the plates are attached to primary structure with vibration isolators.

Acoustical qualification tests are not required for components located in an environmental zone where the overall SPL is less than 135 dB or for components not susceptible to acoustic excitation (those with a high mass to volume ratio, small surface area, high internal damping, or those with encapsulated parts).

Components are selected for development, reliability, and production acceptance tests according to the above guidelines but the specific objectives of each of these tests place further restrictions on the number of components as indicated in paragraph 4-1.

In general, acoustical tests are not performed in combination with other environments such as vibration, high or low temperature, or static acceleration.

b. Test Conditions — The first step in deriving acoustical qualification test criteria for components and subsystems is to establish environmental zones on the external surface and internal regions of the missile or space vehicle. A zone is an area or region in which the sound pressure level is expected to be the same at all locations within the zone boundary at any given time during the mission.

In establishing zone boundaries, consideration must be given to the effects of the distance from rocket engines, boundary layer thickness, shell noise reduction characteristics, internal acoustic-absorption characteristics, and proximity to external protuberances or changes in diameter of the vehicle. Time histories of the acoustic environment are then predicted for each zone using the procedures given in Section 3.

If acoustical tests are to be used to simulate fluctuating pressures in the boundary layer, the test sound spectrum should be corrected as shown in

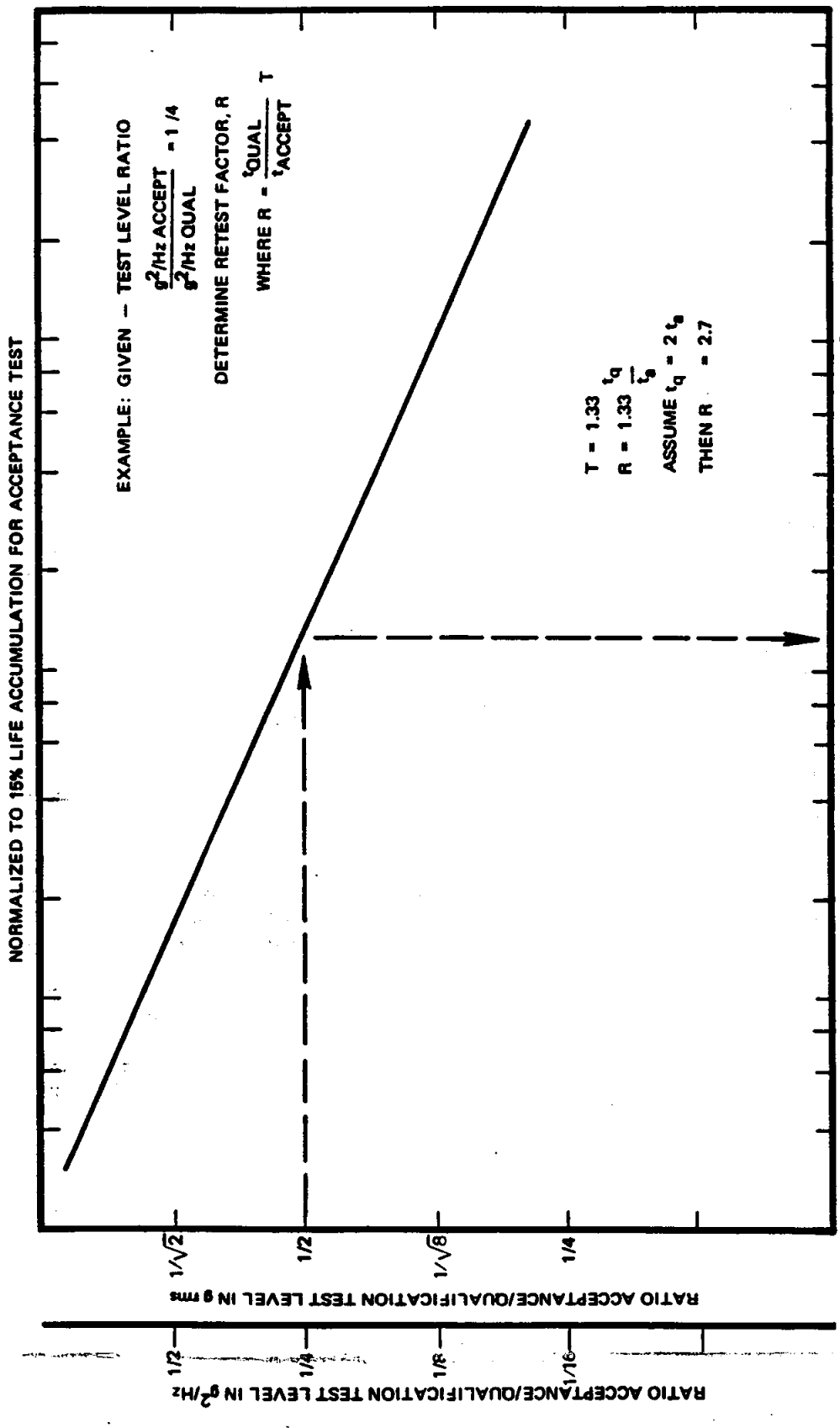


Figure 4-1. Ratio of Test Amplitude to Test Time Factor

Section 3, paragraph 3-2.3. Test conditions (usually limited to one or two) are then established by grouping together time periods that have similar sound spectra. The test spectrum for each condition is established by enveloping the maximum expected sound spectrum (the 97.5 percent confidence estimate or 3 dB above the expected mean sound spectrum, whichever is greater) and then increasing the spectrum level by an additional 3 dB safety factor to account for the items mentioned in paragraph 4-1.

This safety factor is also sufficient to ensure that production acceptance tests can be performed at the mean flight levels (6 dB below the qualification levels) without damaging flight hardware. The recommended test duration is two times the duration of the environment being simulated unless a longer time is required for adequate test control or specimen check out or to simulate fatigue damage caused by lower level, longer duration, environments. The type of test facility must be specified as a test requirement, i.e., reverberent room, progressive wave tube, etc.

Test conditions for development, reliability, and production acceptance tests are generally expressed as some percent of (or otherwise related to) the qualification test conditions depending upon the considerations expressed in paragraph 4-1.

c. Specification Tolerances – Acoustical test requirements must include specified tolerances on the test levels. It is common practice at MDAC-WD to specify +4 dB and -0 dB in the frequency range from 100 to 2,000 Hz. Below 100 Hz and above 2,000 Hz it is common practice to specify +10 dB and -0 dB because of inherent difficulty in controlling the test in these ranges.

On the Saturn S-IVB program the specified tolerance below 100 Hz and above 2,000 Hz is +0 dB and the lower limit is the capability of the testing facility; except that the lowest level shall not be less than -10 dB below the specification. This deviation was necessary because the available test facilities were unable to meet the specification.

The deviation was acceptable because the low frequency range was adequately covered by vibration tests and the facility capability in the high frequency range was sufficient to cover the measured flight environment even though it was below the nominal test specification.

d. Facilities – An excellent description of acoustical test facilities, procedures, and instrumentation is presented in Section 2 of the MDAC-WD Testing Manual. MDAC-WD and outside test facilities are described. This information should be reviewed before issuing a new specification to ensure that the specified tests can be performed.

If a good simulation of the desired acoustic environment cannot be attained in existing random noise test facilities, it may be necessary to restrict the test bandwidth to achieve higher levels by making maximum use of the available energy. This type of compromise can be used only when a thorough evaluation of the test specimen indicates that no significant resonances of the specimen will be omitted during the test. The possibility of missing important modal coupling must be recognized in determining whether a test with reduced bandwidth is acceptable.

4-1.2 Shock Test Requirements

a. Components Requiring Test – Shock/qualification tests should be performed on all components and subsystems judged to be susceptible to damage or malfunction as a result of shock. In general, this will include at least the following types of components:

1. Electromechanical components ("black boxes," relays, switches, batteries, pin connectors, and antennas).

2. Propellant management and pneumatic components (ducts, valves, bellows assemblies, regulators, flow meters, and small pneumatic lines attached to components mounted on isolators).

3. Hydraulic components (pumps, reservoirs, valves, and representative samples of hydraulic lines, especially those that are attached to components mounted on vibration isolators).

4. Propulsion units (retrorocket, ullage rocket, and control rocket motors).

5. Ordnance components (exploding bridgewire units, explosive bolts, and safe and arming devices).

It is sometimes possible to eliminate the shock qualification-test requirement by showing that the dynamic loads on the elements of a component resulting from the shock test are less than those resulting from the specified static-acceleration test or the vibration test. However, for complex devices having many resonances it must be shown that the response to the shock test is less at all frequencies than the corresponding response to the vibration test. This substitution of one test for another is possible because the failure mode in a shock test is a single excursion beyond the ultimate strength or

maximum allowable deflection of an element within the components and is generally not time dependent as in the case of fatigue, leakage, or wear out modes of failure. The possibility of eliminating the shock test requirements should be investigated for large complex test specimens, especially when it becomes necessary to use a special test set-up, other than that used for the vibration test.

It is significant to note that throughout the entire S-IVB and S-IV qualification test programs only three shock failures occured. On the Saturn S-IVB Program the shock test specified for components was a 20 g half sine shock with a 10 ± 2 msec duration. On the S-IV Program, the shock duration was based on the natural frequency of the component being tested (to ensure maximum response) and the shock levels varied from 35 to 80 g.

b. Test Conditions – The first step in deriving shock test criteria is to establish environmental zones similar to those established for acoustical test requirements. Sub zones are also required for components having similar weight because the weight of the component will influence the shock levels at the point where the component mounting bracket attaches to the primary structure. The time history of the shock environment in each zone and sub zone must be determined by using the procedures specified in Section 3.

Shock test conditions must then be established by grouping together shocks that have similar pulse shapes or shock spectra. In establishing these conditions, the type of shock (mechanical or explosive), the pulse shapes and shock spectra, and the other environments that could combine with the shock to deteriorate the performance of the component must all be considered. Good judgment must be used to minimize the number of test conditions because of cost and schedule considerations.

When the shock being simulated is a mechanical shock requiring simulation on a shock machine or vibration exciter, a factor of safety of 1.4 should be used. When the shock being simulated is a handling shock or explosive shock where a good simulation can be created in the test laboratory, no factor of safety is required. When pyrotechnic shocks are produced in the laboratory, adequate consideration must be given to the structural joints and splices that occur in the vehicle, and to the test set up as well as the distance between the input to the component and the pyrotechnic charge.

Joints splices and distance significantly attenuate a pyrotechnic shock. For qualification tests it is recommended that the test specimen be installed on its production mounting brackets which are, in turn, mounted on back-up structure that is dynamically similar to that in the flight vehicle installation. All lines, ducts, or wire bundles, or other components that might influence dynamic response must also be included. This approach is necessary to ensure the structural integrity of the installation and to ensure that any coupling between component internal resonances and bracket or line resonances is adequately simulated.

Detailed knowledge of mounting-bracket transmissibility (which can be determined during the qualification test) is also necessary to determine component margins of safety from in-flight shock measurements (because the latter measurements are usually made on vehicle primary structure).

Components mounted on vibration isolators should be qualified on the isolators for the same reasons. When isolators are used, and the flight shocks being simulated occur during a period of high static acceleration, the static acceleration forces should be applied to the specimen by a bungee or centrifuge to ensure that the isolators do not bottom out and cause severe shocks.

Extreme temperature or differential pressure conditions should be specified for application during shock tests where applicable. Extreme temperature can cause thermal stresses and material property changes that, when combined with shock, can result in significant failure modes that might go undetected in separate tests. Differential pressure can cause static stresses and frequency shifts that could influence the ability of the component to withstand shock.

If a suitable shock pulse can be supplied by a vibration exciter, then the cost and schedule impact of combining temperature and pressure environments will be minimized because the shock test can be performed when the vibration test set up is still intact.

When analysis indicates that the shock environment may be critical, shock test inputs should simulate the maximum expected environment and include a safety factor to account for uncertainties in the test simulation. When the shock requirement is an arbitary shock pulse based upo. Military Standard or other standard reference, alternate pulse shapes may be used if dictated by facility limitations, so long as the shock spectrum of the alternate shock equals or exceeds that of the specified shock. A compilation of shock spectra is contained in Section 4.

Development, reliability, and production-acceptance tests are usually performed by hardmounting the component on a rigid test fixture. If these tests simulate vehicle environments, the dynamic amplification of all supporting or mounting brackets must be included in the test specification. The amplification factor must be estimated with sufficient conservatism to account for inaccuracies in the estimate or else must be determined by testing dynamically-similar dummy components on the vehicle mounting-brackets.

The number of test shocks should be two times the number of shocks that occur in the service life of the component.

c. Specification Tolerances – Shock test requirements must include specified tolerances on the test level and duration. It is recommended that the following tolerances be specified:

1. Shock Pulse

Amplitude: ±10 percent

Duration: ±10 percent

2. Shock Spectrum

Amplitude: No maximum, -0 percent

d. Facilities – An excellent description of MDAC-WD shock test facilities is presented in section 2 of the MDAC-WD Testing Manual. This information should be reviewed before issuing a new specification to ensure that the specified test can be performed.

4-1.3 Vibration Test Requirements

a. Components Requiring Test – Vibration tests should be performed on all components and subsystems judged to be susceptible to damage or malfunction as a result of vibration. In general, this will include at least the following types of components:

1. Electromechanical components ("black boxes," relays, switches, batteries, pin connectors, antennas, and representative samples of wire bundles).

2. Propellant management and pneumatic components (ducts, valves, storage tanks, bellows assemblies, regulators, and flowmeters).

3. Hydraulic components (pumps, reservoirs, valves, and representative samples of hydraulic lines).

4. Propulsion units (retrorocket, ullage rocket, and control-rocket motors).

5. Ordnance components (exploding bridgewire units, explosive bolts, and safe and arming devices).

b. Test Conditions – The recommended approach for establishing test criteria is to simulate the maximum expected vibration environments rather than to establish arbitrary requirements based on past experiences with other programs or military standards.

The first step in deriving vibration test criteria is to establish environmental zones similar to those established for acoustical test requirements. Sub zones are also required for components having similar weight because the weight of the component will influence the vibration levels at the point where the component mounting-bracket attaches to the primary structure. A procedure for determining weight correction-factors is presented in Section 3, paragraph 3-4.

Next, the time history of the vibration environment in each zone and sub zone must be determined by using the procedures specified in Section 3, paragraph 3-4. Vibration test conditions must then be established by grouping together time periods that have similar vibration spectra. In establishing these conditions, the type of vibration (random or sinusoidal), the spectrum shapes, and the other environments that could combine with the vibration to deteriorate the performance of the component must all be considered. Good judgment must be used to minimize the number of test conditions because of cost and schedule considerations.

For qualification tests it is recommended that the test specimen be installed on its production mounting bracket which is in turn mounted on backup structure that is dynamically similar to that of the flight vehicle installation. All lines, ducts, or wire bundles that might influence dynamic response must also be included. This approach is necessary to ensure the structural integrity of the installation and to ensure that any coupling between component internal-resonances and higher-order bracketry or line resonances is adequately simulated. Detailed knowledge of mounting-bracket transmissibility, which can be determined during the qualification test, is also necessary to determine component margins-of-safety from inflight vibration measurements because these measurements are usually made on vehicle primary structure.

For the same reasons, components mounted on vibration isolators should be qualified on the isolators. When isolators are used, and the period of maximum vibration coincides with a period of high static acceleration, forces should be applied to the

specimen by bungee or centrifuge to ensure that the isolators do not bottom out and cause severe vibration.

Extreme temperature or differential pressure conditions should be specified for application during vibration tests where applicable. Combined temperature and vibration can result in significant failure modes that might go undetected in separate tests. Differential pressure can cause static stresses and frequency shifts that could influence the ability of the component to withstand vibration. Differential pressure can also influence the dynamic response of valves and poppets as well as leakage rates.

It is also important that oxygen or nitrogen not be substituted for hydrogen or helium where leakage is a significant failure mode because of the large difference in the "leakability" of the gases. Also, in the liquid state, the large differences in fluid mass could significantly affect the dynamic response of a duct, pneumatic line, or bellows assembly.

For most missile or space vehicle applications, a random vibration test should be used to simulate the vibration that occurs during launch and flight.

A sinusoidal vibration test may be required if:

1. A component is located near a piece of rotating equipment such as a turbopump.

2. There is significant low-frequency vehicle-response in body bending modes or as a result of a "POGO" effect, and if significant specimen resonances exist in the frequency range of this excitation.

3. There is significant response induced by aircraft propellers or ship engines during transportation.

4. Random test facilities are not available and it is necessary to determine an "equivalent" sinusoidal test.

5. It is desirable to perform a diagnostic sweep test to define resonances or to determine the frequencies at which malfunctions such as leakage or relay chatter are induced.

Development tests, fragility tests, and reliability verification tests are usually performed by hard mounting the component on a rigid test fixture. For such tests the dynamic amplification of all supporting or mounting brackets must be included in the test specification. The amplification factor must be estimated with sufficient conservatism to account for inaccuracies in the estimate or else must be determined by testing dynamically-similar dummy-components on flight mounting brackets.

c. Random-Vibration Test Levels and Durations – A random-vibration test spectrum is established by enveloping the maximum expected vibration-spectra (the 97.5-percent confidence estimate or 3 dB above the expected mean vibration-spectrum, whichever is greater), and then increasing the spectrum level by an additional 3 dB safety factor to account for the items mentioned in paragraph 4-1. This safety factor is also sufficient to ensure that production acceptance tests can be performed at the mean flight levels (6 dB below the qualification levels) without damaging flight hardware.

   The recommended test duration is two times the duration of the period during which the actual maximum expected Grms vibration level is greater than 0.707 times the maximum value. Vibration levels below this value do negligible fatigue damage compared to the maximum level. When applying this rule, care should be taken to ensure that no significant changes in spectrum shape occur because a portion of the spectrum might change more rapidly than the Grms value.

d. Sinusoidal-Vibration Test Levels and Durations – The sinusoidal test spectrum should be derived by enveloping the maximum expected sinusoidal-environments and applying the same safety factor used for the random test. If the sinusoidal test is intended to simulate the damage caused by a random test no additional safety factors should be applied over those already considered in establishing the random test specification.

   Because of the difficulty in detecting the most critical resonances, sinusoidal resonance-dwell testing is not recommended. A test yielding much more repeatable results is the sinusoidal sweep test using a logarithmic sweep rate expressed in octaves per minute. The logarithmic sweep rate ensures that the same amount of time is spent sweeping through each resonance having the same Q. The relationship between sweep rate, Q, and time spent at resonance is presented in Section 12.

   Because the time spent at any one resonance is usually of the order of a few seconds, an extremely large number of sweeps may be required to simulate the fatigue damage caused by exposure to a relatively long-duration environment. It may be desirable to exaggerate or increase the test level to produce the same fatigue damage in fewer sweeps. (It may also be desirable to exaggerate a random test level if the duration of the environment is relatively long.)

It is recommended that test levels never be exaggerated beyond the maximum level expected in service (including safety factor if applicable) because malfunctions may occur that are related to level and not dependent upon fatigue considerations. Also, nonlinear effects may become important.

e. Exaggeration Factors – Exaggeration factors are most often used when determining the additional test time required to simulate the fatigue damage caused by long duration exposure to vibration environments that have levels well below the maximum expected levels (i.e., transportation environments).

In the case of an inadvertent overtest, these data can also be used to determine the additional safety factor that was demonstrated relative to that which would have been demonstrated at the proper test level.

To exaggerate either a sinusoidal or random test level it is necessary to know the slope of the fatigue curve for the weakest element in the component to be tested and to equate the fatigue damage done by a given vibration level and duration to that done by a higher level in a shorter duration. This relationship is given by the equation:

$$\frac{S_2}{S_1} = \frac{g(o\text{-}p)_2}{g(o\text{-}p)_1} = \sqrt{\frac{(g^2/Hz)_2}{(g^2/Hz)_1}} = \left(\frac{N_1}{N_2}\right)^{\frac{1}{\alpha}}$$

where:

S_2 = Peak stress producing failure in N_2 cycles

S_1 = Peak stress producing failure in N_1 cycles

$g(o\text{-}p)$ = Peak sinusoidal vibration level producing stress (S)

g^2/Hz = Acceleration spectral density producing stress (S)

N = Number of stress cycles producing failure at stress (S) as indicated by S-N diagram

$\frac{1}{\alpha}$ = -1 times the slope of a fatigue curve on a log-log plot

Figure 4-2 shows a group of S-N curves for typical materials used in aerospace components. Since the slopes of the curves vary with the number of stress cycles, the slope is indicated for each cycle range. For specific applications, where the exact S-N curve is known for the critical element in a component, α should be determined from the exact curve taking into account the effect of temperature and stress ratio R.

Figure 4-3 presents exaggeration factor versus cycle ratio for a range of α's.

f. Random and Sinusoidal Equivalence – When random test facilities are not available and it becomes necessary to specify an "equivalent" sinusoidal test or when it is necessary to evaluate a sinusoidally-tested component for use in a random environment, the random to sinusoidal equivalence specified in Section 12 should be used. A detailed discussion is also presented in Chapter 24 of reference 1.

g. Specification Tolerance – Vibration test requirements must also include specified tolerances on the test levels. It is recommended that the following tolerances be specified:

Random Vibration

Amplitude:

Hard mounted and less than 50 pounds: ±1.5 dB when measured with a 50 Hz or narrower filter, from 10 to 500 Hz and ±3 dB from 500 to 2,000 Hz.

Other

±3 dB when measured with a 50 Hz or narrower filter, from 10 to 2,000 Hz.

Frequency:

±2 percent

1. Sinusoidal Vibration

Amplitude:

±10 percent

Frequency:

±2 percent

h. Facilities – An excellent description of vibration test facilities is presented in Section 2 of the MDAC-WD Testing Manual. This information should be reviewed before issuing a new specification to ensure that the specified test can be performed.

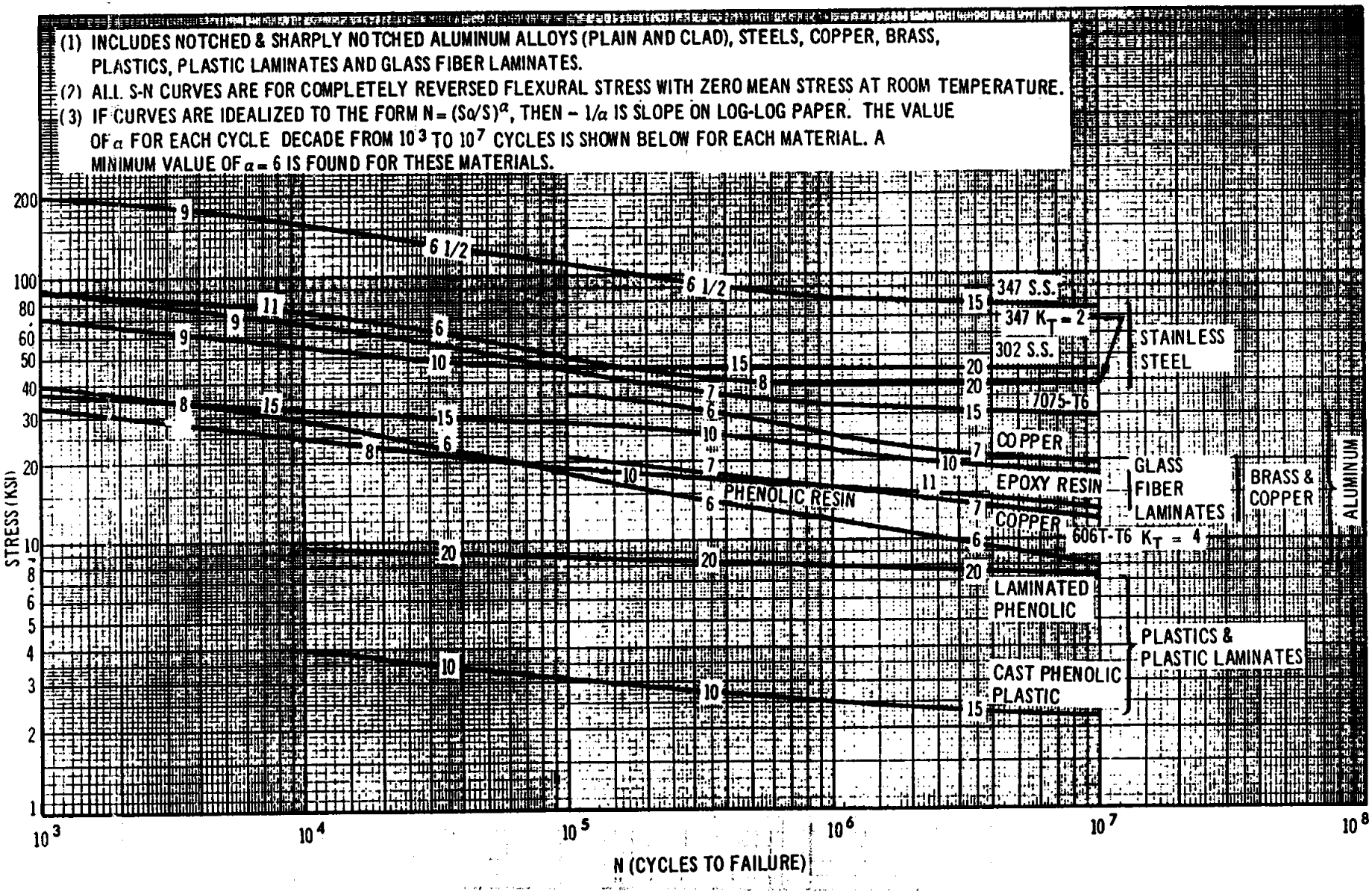


Figure 4-2. Representative Fatigue Curves For Certain Structural Materials

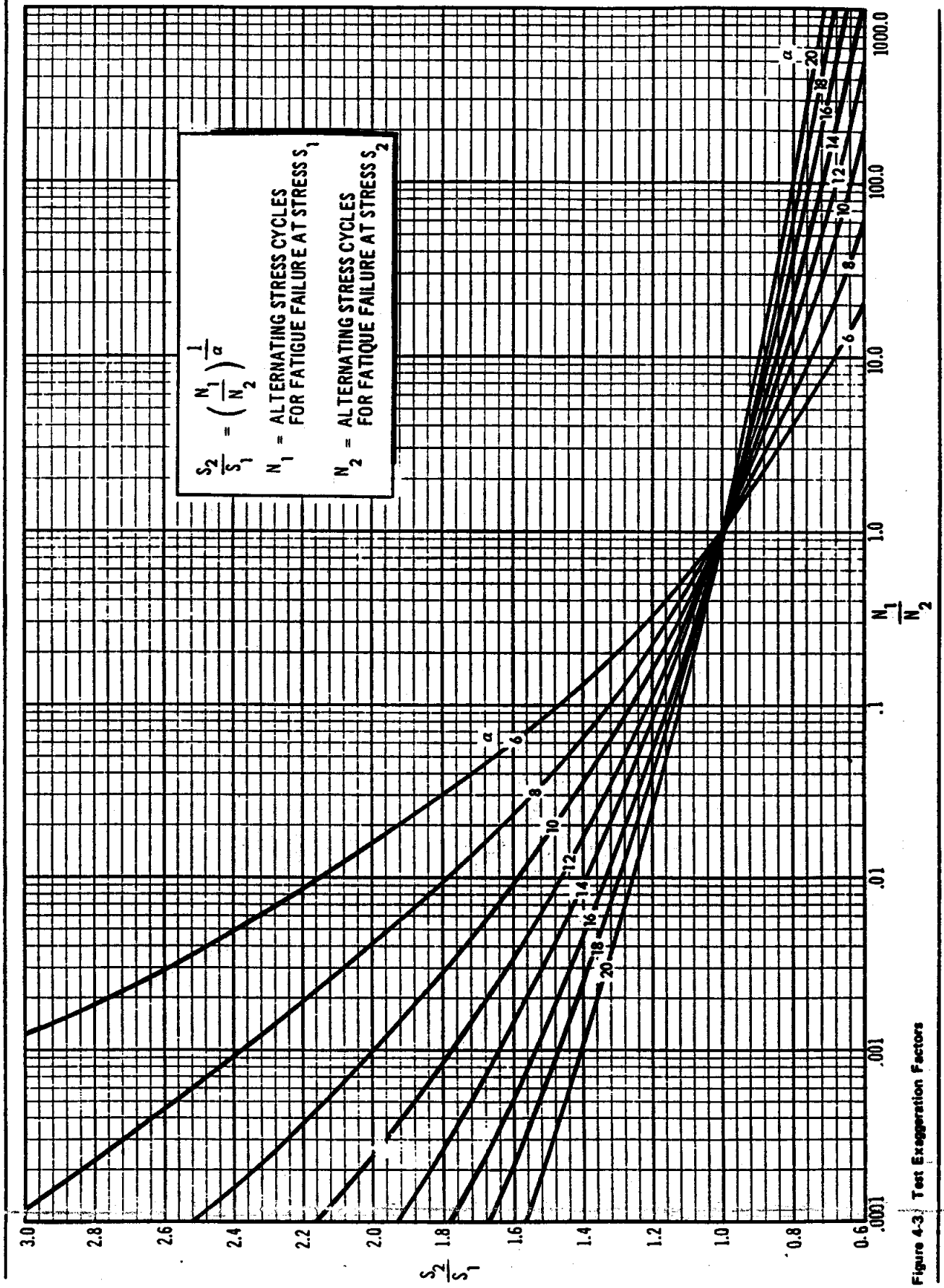


Figure 4-3 Test Exaggeration Factors

4-2 DERIVATION OF DESIGN REQUIREMENTS

Design requirements for dynamic environments are imposed on MDAC-WD components and subsystem hardware from one or more of three sources. These are:

a. Contractual in nature as defined by the customer requirements and/or specifications (generally specified as obligatory test requirements).

b. Internally-generated environmental analysis which incorporated MDAC-defined margins of safety and design philosophy.

c. Military – standard specifications which are generally specified as obligatory test conditions.

The following paragraphs provide the methodology to be used in the derivation of dynamic design requirements by MDAC personnel for use on those programs in which MDAC is responsible for an integrated system design. The methodology to be used is common for each of the three major design environments (acoustics, vibration, and shock). First a compilation of pertinent environmental sources is prepared. Sources of data, either analytical or empirical are then defined. Where analytical or empirical data are not readily available, development tests must be specified and implemented early in the program such that data gathered from the test can be used to specify final requirements. Based on the confidence level attached to the environmental predictions, a factor of safety (which accounts for environmental data scatter and hardware variations) is applied to the predicted environments. Duration of exposure to the environments are realistically defined.

Other environments and loads which combine with the dynamic environment to produce a composite stress field on the hardware should then be evaluated and defined. Typically these include static loads/accelerations, and thermal conditions. The ultimate demonstration of satisfactorily meeting design dynamic requirements is by ground test. In performing such a test, it is often necessary to simplify the actual service environments (to account for test capability of laboratory equipment) and to arbitrarily redefine boundary conditions of the flight system (to meet test constraints). For these reasons, the final design requirements for flight (service) environments may be influenced by ground simulation techniques employed for design verifications.

4-2.1 Acoustical Design Requirements

Acoustical design requirements influence the design of lightweight airframe structure, the random vibration requirements for vehicle subsystems, and the location, design and performance of equipment in man-rated flight systems and ground facilities. The applicable acoustical design requirements are therefore dictated by the vehicle (system) under consideration. Anyone or all of the design areas, structure, equipment or personnel, can be influenced by acoustic phenomena.

a. Acoustic Sources Definition – Identification of all acoustic-like sources represents the initial step in the definition of acoustical design requirements. A listing of all the potential sources in chronological sequence is first required. Such acoustic sources include:

   1. Rocket exhaust
   2. Aerodynamic flow/shock
   3. Machinery/equipment

b. Environmental Definition – To establish applicable design requirements, a definition of the statistical properties in addition to spectral characteristics is required. Rocket-exhaust loads can be established from the literature, scale-model tests, ground-test firings and published flight-test data from similar propulsion systems. Aerodynamic flow/shock phenomena can be defined from published flight-data on similar systems (vehicle geometry) and appropriately-sized, scale-model, wind-tunnel test data.

c. Design/Test Margins – Based on the confidence level associated with the environmental prediction, appropriate margins are incorporated to account for a 2-sigma service operation. Depending on the ultimate use of the acoustic load data, i.e., structural design or subsystem vibration requirements, added margins will be used to account for material properties, vibration, fabrication variances and often structural requirements. In general, environment variations of ± 3 dB are used about a mean flight-load prediction. Material and structural variations are based on another ± 3 dB factor (see Figure 4-4). The combination of these two, a 6 dB (factor of 2 in level), results in a 99.8 percent confidence level or approximately 3 sigmas as shown in Figure 4-5.

d. Design Requirement – Structure – acoustic-load requirements are specified in terms of pressure-spectrum levels vs. frequency, test/flight duration with a minimum factor of 2 in time, temporal and spatial properties when known from empirical data.

e. Equipment – Acoustical-design requirements are specified in terms of octave band or 1/3-octave-band sound-pressure-levels vs. frequency.

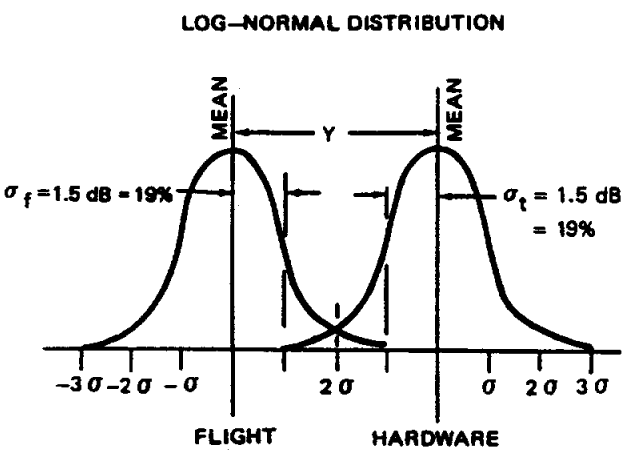


Figure 4-4. Log – Normal Distribution

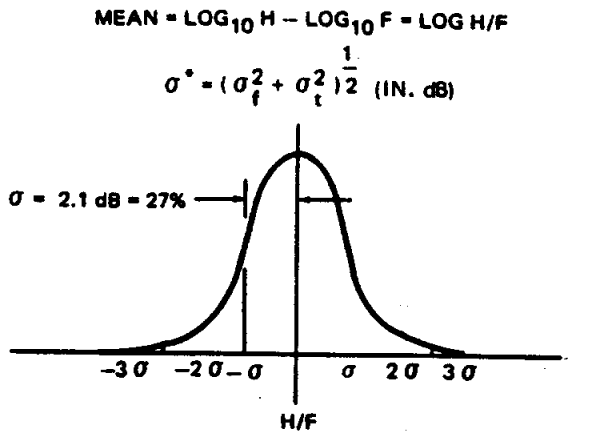


Figure 4-5. Total Risk Distribution is Log – Normal

f. **Machinery/Personnel** – Acoustical-design requirements for manned areas will be specified by the appropriate noise-criteria curves and allowable acoustic-power levels as determined under ground-test conditions (free field, anechoic chamber, etc.).

g. Static Load/Thermal Requirements – In evaluating the acoustical design requirement on structure and equipment, appropriate combined environments should be specified. Appropriate test durations will also be specified for each of the above considerations.

4-2.2 Shock Design Requirements

Design requirements for transient accelerations are generally specified in one of three formats.

a. Acceleration time-histories.

b. Shock-response spectra.

c. Simple test pulses.

Shock phenomena can be generally classified into two categories; "low-frequency" shocks (which are characterized by a considerable quantity of energy below 200 Hz and are generally produced by thrust ignition/shutdown or transportation conditions), and "high-frequency" shocks (generated by ignition overpressures, ordnance devices, explosive bolts, squib switches, FLSC, MDF explosives, explosive nuts, etc.). The latter phenomena have energy concentrated in the frequency range above 1 KHz. Early development tests should be performed to assess local structural responses to these transient forces such that a realistic design requirement may be obtained for critically-affected components.

Methodology in establishing shock design requirements are:

4-2.2.1 Source Definition

In the early stages of design, a list of all potential shock sources is mandatory. Typical sources include:

a. Transportation irregularities.

b. Thrust transients.

c. Separation ordnance.

d. Electronic/Ordnance switches.

4-2.2.2 Environmental Definition

Low-frequency transient loads can be derived by normal-mode analysis techniques from which either acceleration time-histories or shock spectra are computed. High-frequency shocks should be based on development test data since local stiffness and mass properties control shock levels and are not readily amenable to accurate structural analysis.

For each condition, define the number of events which produce the shock environments. The low-frequency forced-response analysis should use +2-sigma force input. High-frequency shock-test data should be acquired in

sufficient quantity to define data variances. Factors of 2 to 4 are not uncommon in high-frequency, data scatter.

4-2.2.3 Design Margin

A 3 dB factor of safety is added to the 2 sigma environment to obtain shock design requirements. Depending on the selected pulse shape for simulation, the design and test environment should be at least twice the number of actual occurrences expected in the field.

4-3 CONVERSION OF DESIGN REQUIREMENTS TO STRUCTURAL LOADS

To utilize dynamic requirements for a structural design, the loads or stresses due to the dynamic design environments must be determined. This calculation may be separated into two distinct parts:

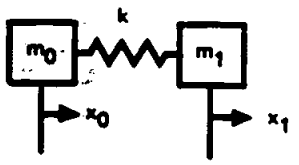
a. Determination of the deflection response of the structure due to the given excitation.

b. The determination of the loads in the various members of the structure associated with that response.

The calculation of the displacement and acceleration response to several classes of exciting forces is treated extensively in Section 5, and will not be considered here.

Two different "loads" are commonly used in describing the dynamic forces in a structure. The "equivalent static load" is defined as the external force that will produce a static deformation identical to the dynamic deformation. These equivalent static loads are sometimes a useful tool in determining the internal load.

A simple example illustrates this calculation.



The response calculation would either yield x_1 and x_0 at some time or in the case of random excitation, would yield a root mean square (rms) or power spectral density (PSD) response for each point. The equivalent static load on the structure would then be given by:



Next one may (trivially for this sample case) section the spring and determine the internal forces due to this static loading to be $k(x_1-x_0)$.

For more complex systems, it is advantageous to use a matrix formulation. The equivalent static load on a structure $\{F\}$ may be expressed as a function of the structural stiffness $[K]$ and displacement $\{x\}$:

$$\{F\} = [K]\{x\}$$

The internal loads $\{Q\}$ may then be calculated using a matrix $[L]$ of the internal forces in a structure due to unit external loading:

$$\{Q\} = [L]\{F\}$$

In general, x, F, and Q are functions of time, and these may be calculated for each time point. One would then find the maximum Q(t) for each member to determine the critical load in that member. Note that there is no general way to determine at what time the maximum internal load occurs without calculating the load for all time points.

One procedure to find times with high internal loads is: Compute the response (η) in modal coordinates and rearrange the other calculations so that:

$$\{Q\} = [LK\phi]\{\eta\}$$

where $[LK\phi]$ is the internal force due to modal displacements of one.

$$[LK\phi] = [L]\,[K]\,[\phi]$$

where ϕ is defined as the mode shape:

$$x = [\phi]\{\eta\}$$

From examination of $[LK\phi]$, determine in which mode the internal force in a particular member is a maximum. Then use the modal displacements when that mode's response is maximum to calculate the internal forces. When a particular mode (or modes) is dominating the response, then only consider times of maximum modal displacement for that mode. Note that these approaches bypass computation of an equivalent static force.

The problem of obtaining the [L] matrix remains. Basically, the procedure is one of considering the equilibrium equations at each mass or collocation point. For small simple systems, this can be trivial as in the example. For larger systems with more complex structural elements, the computation may become quite complex. Further information may be obtained from a structures text such as reference 1.

Various MDAC computer programs are available to aid in making the foregoing calculations. Programs SA48 and SA40M are available for the matrix computation needed. Both are flexible enough to build a structural model and loads matrix if it is desired. For structural modeling (including the loads matrix), the Redundant Force Analysis portion of DEMON and of FORMAT III are available for general structures. SABOR is available for axisymmetric shells of revolution. These structural programs are lengthy and should be used only with the advice and assistance of a cognizant user.

Program DAO2 will assist in modeling beam-type structures in bending, axial, and torsional directions. In addition, DAO2 computes the loads due to a modal displacement. This may be used to good advantage if care is taken to calculate response (η) in modal coordinates. Then:

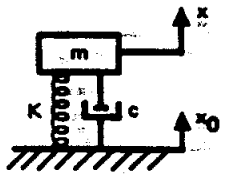
$$Q = [LK\phi] \{\eta\}$$

where $[LK\phi]$ is the modal force matrix calculated in DAO2. Program DAO2 allows the calculation of response (modal or real) from any forcing function with a history that may be described in either real or modal coordinates.

Keep in mind that these loads are subject to the same assumptions and restrictions as the displacement response. In addition, the force obtained is an "averaged" value for the spring section in question. If the forcing function had a large frequency component greater than that of the model frequencies, then local effects might not be accurately included.

A quick, but inconclusive, check for this accuracy is to calculate the internal loads in adjacent sections. Unless there is a drastic change in section properties, the forces should be approximately equal for the two sides of a collocation point.

The following subparagraphs present a method for calculating the equivalent static load for a simple mechanical oscillator.



$y = x - x_0$

4-3.1 Deterministic Response of a Simple Mechanical Oscillator

This subject is treated extensively in Section 5. Therefore, it is assumed that the relative displacement response of the oscillator y(t) has been determined. The force in the spring is simply the spring constant times the relative deflection:

$$F(t) = K\, y(t)$$

The equivalent static load is the greatest force which occurs in the spring:

$$F_s = K \left| y(t) \right|_{max}$$

This is true whether the response of the oscillator is steady-state or transient. In other words, the excitation at the base of the oscillator may be a continuous sinusoidal input or a shock pulse.

4-3.2 Random Response of a Simple Mechanical Oscillator

This discussion is restricted to random inputs at the base of the oscillator which are stationary, ergodic, and Gaussian. If it is further assumed that the power spectral density of the base acceleration is a constant (white noise), then the following equation can be derived:

$$y_{rms} = \left(\frac{\pi}{2} Q f_n S_0\right)^{1/2} \Big/ \omega_n^2 = \left(\frac{Q S_0}{32 \pi^3 f_n^3}\right)^{1/2}$$

y_{rms} = root mean square value of the relative displacement

$Q = \frac{1}{2\zeta}$ where $\zeta = \frac{C}{C_c}$ = fraction of critical damping

$f_n = \omega_n / 2\pi$ = natural frequency (Hz)

$\omega_n = \sqrt{\frac{K}{m}}$ = natural circular frequency (rad/sec)

S_0 = Power spectral density of the base acceleration (acceleration^2/Hz)

The rms force in the spring is simply the spring constant times the rms relative displacement:

$$F_{rms} = K\, y_{rms}$$

The equivalent static load in the spring is defined as the four sigma spring force:

$$F_s = 4\, F_{rms}$$

Only 0.134 percent of the actual peak loads will exceed F_s. This is based upon the fact that the relative displacement is a narrow-band normal process, and therefore the peaks have a Rayleigh distribution.

References:

1. Przemieniecki, J. S.; *Theory of Matrix Structural Analysis* McGraw Hill, 1968

MCDONNELL DOUGLAS ASTRONAUTICS COMPANY
WESTERN DIVISION
MCDONNELL DOUGLAS CORPORATION

STANDARD PRACTICE

NO: SP 10.024-CD
PAGE 1 OF 4
DATE: 18 June 1969
SUPERSEDES:
New

ORGANIZATION: Division-wide Engineering

SUBJECT: DYNAMIC ACCEPTANCE TESTING

A. SUMMARY:

Specifies the policy and requirements for performing dynamic acceptance testing. Includes the criteria for selecting the hardware to be dynamically tested and for establishing the levels and duration of test.

B. APPLICABLE TO:

Santa Monica Development Engineering (SMDE), Huntington Beach Development Engineering (HBDE), Advance Systems & Technology (AS&T), and Development Engineering at Field Centers

C. DEFINITIONS: None

D. REGULATIONS:

1. Certain components and segments of aerospace equipment that heavily influence flight reliability are sensitive to vibration and/or shock. To improve and maintain the flight reliability of vehicle systems, a comprehensive program of production acceptance testing of selected vehicle components and subassemblies is required. Production acceptance tests using simulated service environments for production hardware will be implemented to achieve this goal. The vibration technology and cognizant design technology, in conjunction with the program design office, are responsible for implementation of, and compliance with, this SP.

2. This policy does not replace acceptable inspection methods or quality control procedures. It is intended to supplement such methods and procedures, particularly where they are unable to fully uncover defects in workmanship and material.

3. Dynamically sensitive production hardware on the component/module, subsystem, or section level that is critical in meeting mission objectives, and whose integrity cannot be determined by other means of inspection, is subjected to simulated service dynamic environments while being functionally monitored to detect any out-of-tolerance performance.

4. Production hardware that should be dynamically tested in accordance with REGULATIONS 3 above shall be selected by the design and vibration technologies of the respective programs. The selection will be based on the following criteria:

   a. Vibration critical hardware, as delineated by analysis, ground test, or related flight-test experience

      Example:

      (1) Electronic, electromechanical, and mechanical flight or ground hardware that are essential in meeting primary mission objectives,

      (2) Dynamically sensitive

hardware; e.g., dynamic systems and fragile components, such as switches and relays with cantilevered elements; valves, timers, and gyros with spring-loaded elements; and inverters and gyros with moving internal elements.

b. Complexity in internal arrangement which precludes detailed visual inspection at final assembly; e.g., the effect of dynamic failure modes (loose particles) and defective wiring terminations, connectors, etc. of otherwise nondynamic sensitive hardware; should be considered.

5. Dynamic testing of hardware will be done at the highest practical deliverable level of assembly, as determined by the following considerations of effectiveness and cost.

a. Ability of test facilities to simulate the service environment

b. Complexity of functional checkout

c. Ease of locating defects

d. Cost of replacing internal components or modules that malfunction. The testing of dynamically sensitive parts/components (relays, gyros, valves, switches, etc.) should be considered . the "purchased parts" level of assembly, even if they are to be included in subsequent higher-level-of-assembly tests.

6. The hardware will be subjected to the most critical types of dynamic service environments which were of significant consideration in the hardware design. These environments can include sinusoidal vibration, random vibration, mechanical shock, and acoustic noise, applied either singly or as combined environments where appropriate. The simulated service environment shall follow the characteristics of the service environment.

7. Substitution of equivalent sinusoidal vibration for the service random vibration is not desirable and should only be considered if random vibration testing cannot be achieved, or if random testing results in meaningless data. Deviation from service environment simulation shall require approval of the vibration technology.

8. Test amplitudes and durations shall be based on optimizing reliability through maximum defect detection, commensurate with maintaining the prescribed residual useful life. In achieving this goal, the following criteria shall apply:

a. Dynamic acceptance test amplitudes (levels) shall be at least equal to service amplitudes, provided an acceptable demonstrated margin exists between acceptance test amplitudes and qualification test amplitudes. The minimum margin between acceptance and qualification test amplitudes (qualification being of the higher level) shall be as follows:

(1) Random vibration - down 6 dB, i.e., a factor of 1/4 in PSD (g^2/Hz)

(2) Sinusoidal vibration - down 6 dB, i.e., a factor of 1/2 in acceleration (g)

(3) Mechanical shock - down 6 dB, i.e., a factor of 1/2 in peak g

(4) Acoustic noise - down 6 dB, i.e., a factor of 1/2 in sound pressure

If present qualification levels are such that the above criteria cannot be met (i.e., the qualification test amplitude is not a minimum of 6 dB greater than the service amplitude), requalification of the hardware shall be considered. Pending requalification, production hardware shall be acceptance tested at an environmental level 6 dB below the known qualification levels. It should be recognized, however, that this results in testing which is lower than service environmental levels and the acceptance test is considerably degraded.

b. Dynamic acceptance test duration shall be kept to a minimum, so that the test will not exceed the service environment duration or the minimum duration required adequately to function the production hardware.

c. Not more than 15% of the demonstrated fatigue life of the hardware is to be expended, after which an engineering evaluation must be performed to determine the disposition of the hardware before testing is to be continued.

9. The specific number of allowable shock and/or vibration tests and their durations, corresponding to the criteria of REGULATIONS 8 above, shall be specified in the test requirements for each item. This specified number of allowable tests shall include retest considerations and will take into account relative test durations and test amplitudes.

10. Recording of the duration, amplitude, and frequency spectrum of all dynamic acceptance tests is required. The requirements for this recording, and the requirement for the record to accompany the hardware and to be filed with the record for the end item in which it is ultimately used, will be specified in the applicable test requirements, test procedures, and in drawing and specifications used for procurement. The appropriate drawings for the hardware will include a requirement for serialization (in accordance with program serialization requirements to ensure positive identification of the item to its record and traceability to the using end item.

11. The dynamic acceptance testing policy set forth in this SP shall be complied with, except when superseded by program requirements that are contractually binding; or by specific customer direction after he has been adequately presented with the MDAC-WD position on dynamic acceptance testing. If

it is wise to deviate from this acceptance test policy for other reasons, based on a thorough hardware, environment, and quality control review, the intent and justification for deviation must be submitted to the appropriate Development Engineering Subdivision head for approval.

12. The test levels specified herein are intended as minimum requirements, and more severe requirements may be used when officially requested by or negotiated with the customer.

E. PROCEDURES: None

F. REFERENCES: None

W. H. P. Drummond
Vice President
Santa Monica Development Engineering

Section 5

BASIC DYNAMIC RESPONSE ANALYSIS TECHNIQUES

In this section, some of the theory and mathematical tools available to the MDAC-WD VS&A engineer are described. Two basic categories are developed: calculations performed completely by the engineer, and calculations programmed on the computer requiring only input data and output usage by the engineer. Knowledge of when to apply the information contained in the following subsections is assumed. Hence, only the technique is described in detail.

Numerous references and texts are available to elaborate on any particular point of interest, and the user of this manual is urged to refer to them when the material contained herein is not directly applicable to a problem.

5-1 SIMPLE ANALYSES NOT REQUIRING COMPUTER

The material in this section is in a form which is best suited to hand calculations. In some limited instances existing computer capability is redundant, but for the most part, the information contained in this section is useable directly.

5-1.1 Spring Mass Systems (or The Single Degree-of-Freedom Oscillator)

The equation of motion for a single degree-of-freedom system (Figure 5-1) can be expressed as

$$M\ddot{X} + C\dot{X} + KX = F \quad \Longleftrightarrow \quad \ddot{X} + \frac{C}{M}\dot{X} + \frac{K}{M}X = \frac{F}{M}$$

Define:

$$\omega^2 = \frac{K}{M}$$ where ω is called the

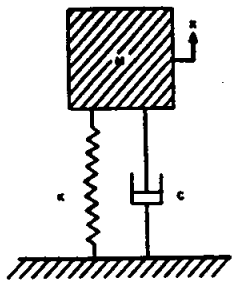


Figure 5-1. Single Degree of Freedom Oscillator

undamped natural frequency of the oscillator. Its units are radians/second

$f = \frac{\omega}{2\pi}$ is the natural frequency in units of Hertz (Hz)

$C_c = 2\sqrt{KM}$, and is defined as the critical damping value (lb-sec/in)

$\delta = C/C_c$, is the fraction of critical damping (unitless)

Now

$$\frac{C}{M} = \frac{2 \cdot C}{2\sqrt{KM}} \sqrt{\frac{K}{M}} = 2\delta\omega$$

Therefore, the differential equation may be written:

$$\ddot{X} + 2\omega\delta \dot{X} + \omega^2 X = \frac{F}{M} = \omega^2 \frac{F}{K}$$

This form is much more convenient to use.

5-1.1.1 Static Solution

When $F(T) = F_o$, a constant, it is called the static problem. The solution is $X(T) = F_o/K$, since the velocity and acceleration are zero. This value of X is called the static deflection.

For the special case where the force is due to gravity, a relationship between frequency and static displacement can be derived ($f \approx 3.13/X^{1/2}$). This relationship is shown in the nomograph of Figure 5-2.

5-1.1.2 Homogeneous Solution

The homogeneous solution to the differential equation (zero forcing function) is

$$X(T) = e^{-\omega\delta T} \left[C_1 \cos \omega \sqrt{1 - \delta^2}\, T + C_2 \sin \omega \sqrt{1 - \delta^2}\, T \right]$$

For damping less than critical ($\delta < 1$), the solution is oscillatory, while for $\delta > 1$ the solution is a straight exponential decay. Since the differential equation is linear, the homogeneous solution can be added to any particular solution and the constants of the homogeneous solution can be varied to match the initial conditions of the problem. The special case of zero force (which has a zero particular solution) is called an initial value problem.

If:

$$X(0) = X_o \quad \text{and} \quad \dot{X}(0) = \dot{X}_o$$

then:

$$X(T) = e^{-\omega\delta T} \left[X_o \cos \omega \sqrt{1 - \delta^2}\, T + \frac{\dot{X}_o + \omega\delta X_o}{\omega\sqrt{1 - \delta^2}} \sin \omega \sqrt{1 - \delta^2}\, T \right]$$

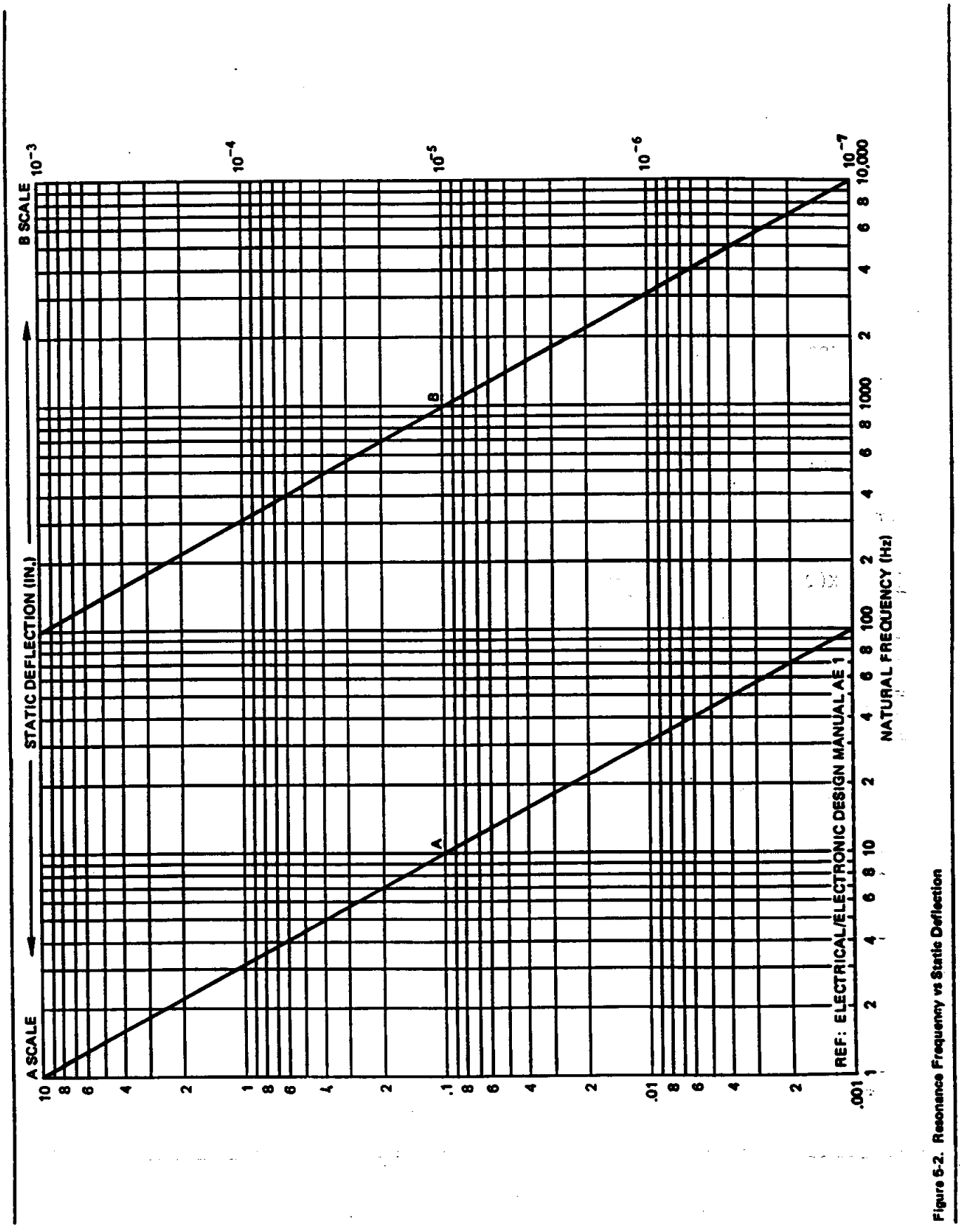


Figure 6-2. Resonance Frequency vs Static Deflection

The logarithmic decrement Δ is defined as the log to the base e of the ratio of the amplitude at one peak to the amplitude one cycle later. The initial value solution consists of the product of an exponential decay and a periodic function. Obviously after one cycle, the periodic function has the same value so the ratio only involves the exponential portion. Hence:

$$\Delta = \frac{2\pi\delta}{\sqrt{1-\delta^2}} \qquad \text{or for} \quad \delta \ll 1, \quad \Delta \approx 2\pi\delta$$

5-1.1.3 Steady-State Sinusoidal Solution

The steady-state solution provides a great deal of information as to the nature of resonance response. The particular or steady state solution to a sinusoidal forcing function is the real part of the solution using:

$$F(T) = F_o \cos(\omega_o T + \theta)$$

or

$$X(T) = \frac{(F_o/K)\cos(\omega T + \theta - \phi)}{\left[\left(1 - \frac{\omega^2}{\omega_N^2}\right)^2 + 4\delta^2 \frac{\omega^2}{\omega_N^2}\right]^{1/2}}$$

where

$$\phi = \tan^{-1}\left(\frac{2\delta \frac{\omega}{\omega_N}}{1 - \frac{\omega^2}{\omega_N^2}}\right)$$

For ω_o small, the phase angle

$$\phi \approx \tan^{-1} 0 = 0, \pi$$

We must pick the zero value because the π value would imply a displacement in the direction opposite of the applied load. For ω increasing, ϕ increases. As ω approaches ω_N the argument becomes infinite and $\phi = \pi/2$. Further increasing ω changes the sign, and the angle increases. These relationships are presented in the graphs of Figures 5-3 and 5-4. The amplification function is the ratio of the amplitude of the vibration to the zero frequency (static) amplitude. Figure 5-5 presents the amplification function for displacement, velocity and acceleration.

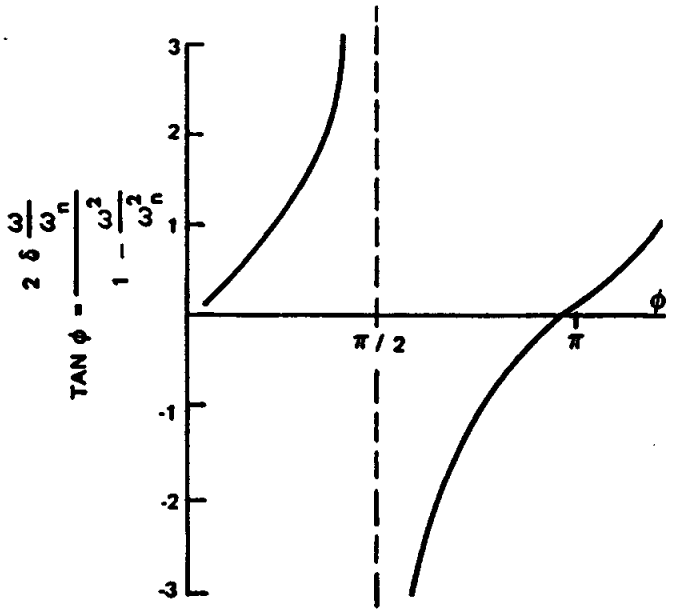


Figure 5-3. Phase Relationships

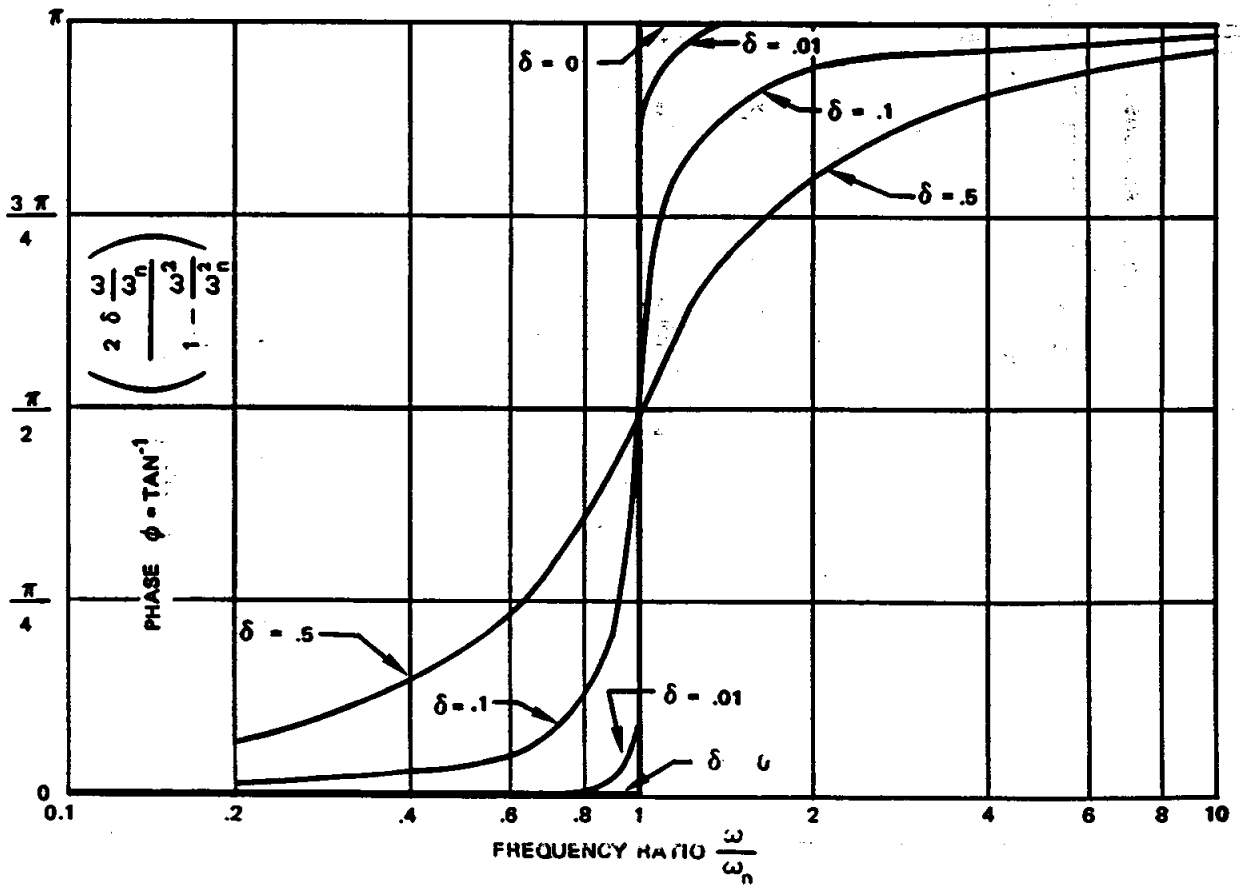


Figure 5-4. Phase vs Forcing Frequency

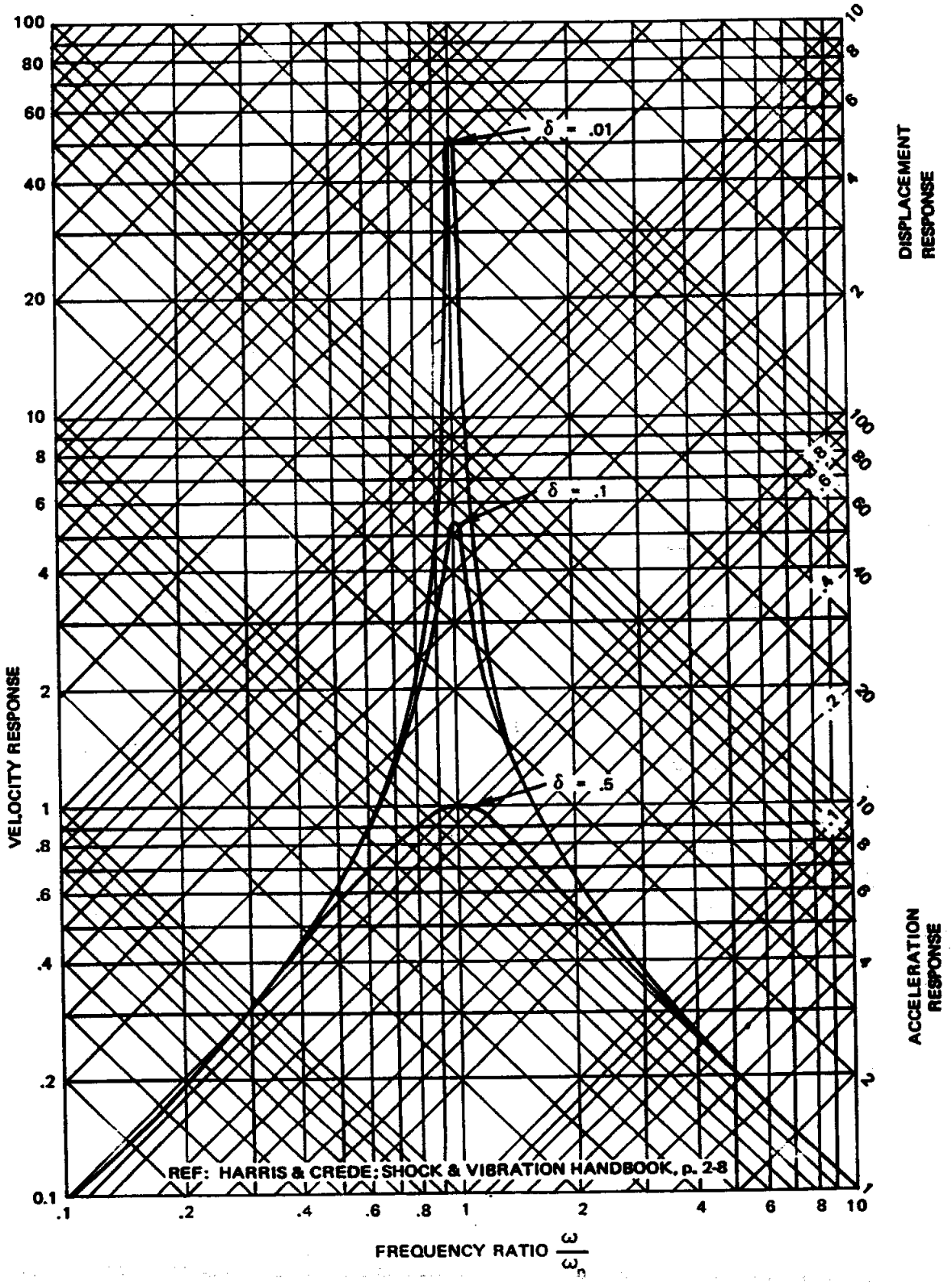


Figure 5-5. Amplification Function

The mathematical form of these three functions are:

$$A_{DISP} = \frac{1}{\left[\left(1 - \frac{\omega^2}{\omega_N^2}\right)^2 + 4\delta^2 \frac{\omega^2}{\omega_N^2}\right]^{1/2}} = \frac{X_{PEAK}}{F_o/K}$$

$$A_{VELOCITY} = \frac{\omega/\omega_N}{\left[\left(1 - \frac{\omega^2}{\omega_N^2}\right)^2 + 4\delta^2 \frac{\omega^2}{\omega_N^2}\right]^{1/2}} = \frac{\dot{X}_{PEAK}}{F_o/\sqrt{KM}}$$

$$A_{ACCELERATION} = \frac{\omega^2/\omega_N^2}{\left[\left(1 - \frac{\omega^2}{\omega_N^2}\right)^2 + 4\delta^2 \frac{\omega^2}{\omega_N^2}\right]^{1/2}} = \frac{\ddot{X}_{PEAK}}{F_o/M}$$

Note that at $\omega_o = \omega_N$ the displacement is 1/2 δ; however, this is not the maximum displacement. The maximum displacement (A_{MAX}) is:

$$A_{MAX} = \frac{1}{2\delta\sqrt{1 - \delta^2}} \approx \frac{1}{2\delta} \text{ when } \delta << 1$$

The frequency associated with the maximum displacement is called the resonance frequency, and for small damping:

$$\omega_{RES} \approx \omega_N, \quad Q \approx \frac{1}{2\delta}$$

For small damping ($\delta < 1$) the frequency difference between the 1/2 power (or $1/\sqrt{2}$ of peak amplitude) frequencies is:

$$\frac{\omega}{\omega_{NH}} - \frac{\omega}{\omega_{NL}} \approx \frac{\omega_N}{Q} \approx 2\delta\,\omega_N$$

The response is called stiffness controlled when $\omega << \omega_N$. In this case

$$A_{DISP} \approx \frac{F_o}{K} .$$

When $\omega \approx \omega_N$ the response is near resonance and is damping controlled,

$$A_{VEL} \approx \frac{F_o}{C} .$$

For $\omega >> \omega_N$ the response is mass controlled,

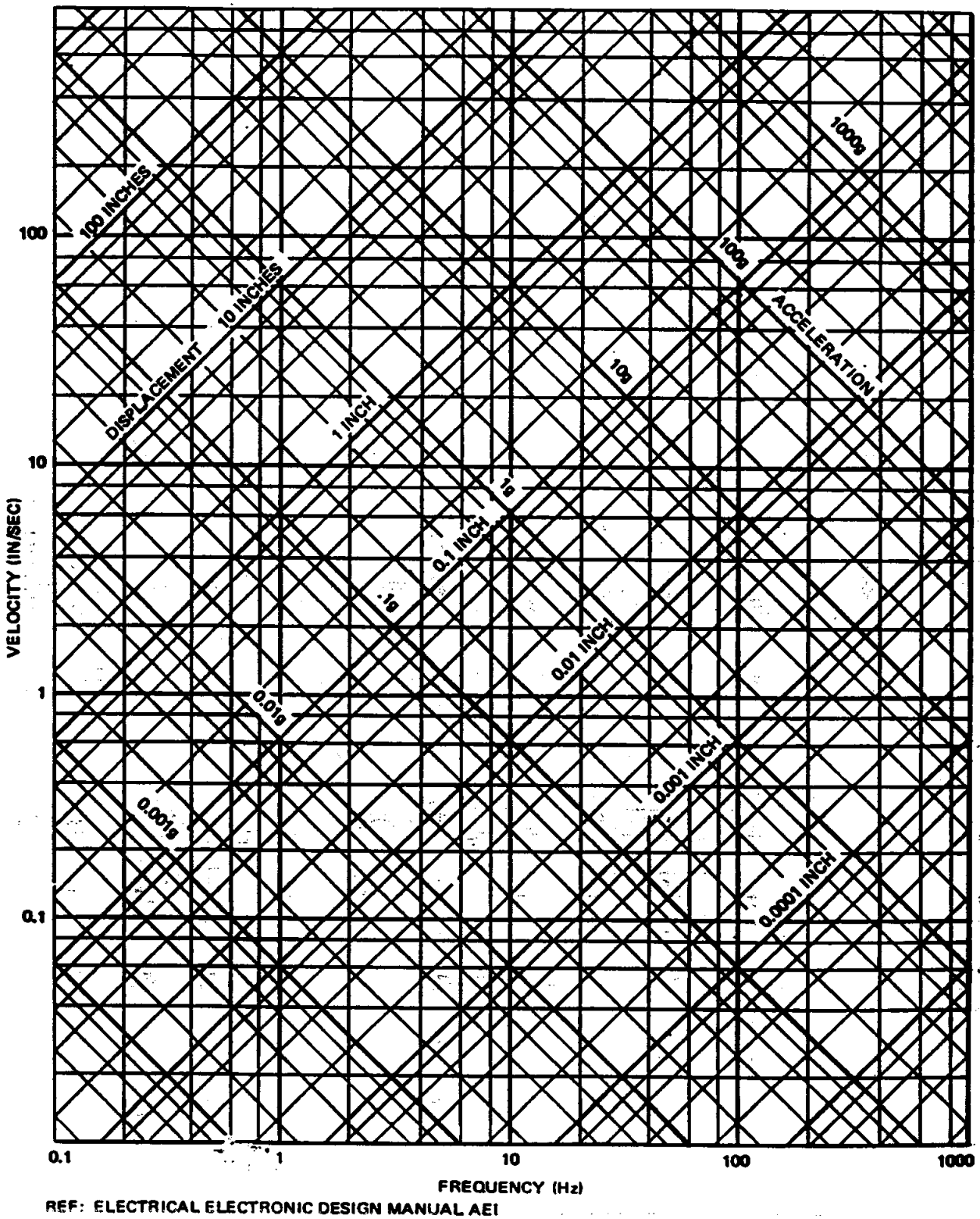
$$A_{ACCELERATION} \approx \frac{F_o}{M} .$$

These results correspond to all of the other terms of the differential equation vanishing except the controlling term.

For sinusoidal motion, peak velocity is ω times peak displacement, and peak acceleration is ω^2 times peak displacement. These relationships are shown in Figure 5-6.

5-1.1.4 Vibration Isolation

The single degree-of-freedom oscillator can also be excited by motion of its support. Figure 5-7 presents the transmissibility of such a system, which is defined as the ratio of the amplitude of the system displacement to the ground displacement. It should be noted that, for frequencies greater than $\sqrt{2}\ \omega_N$, increasing the damping increases the transmissibility. However, some damping is important to protect against the resonance-type phenomenon.



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Figure 5-6. Relationship Between Derivatives

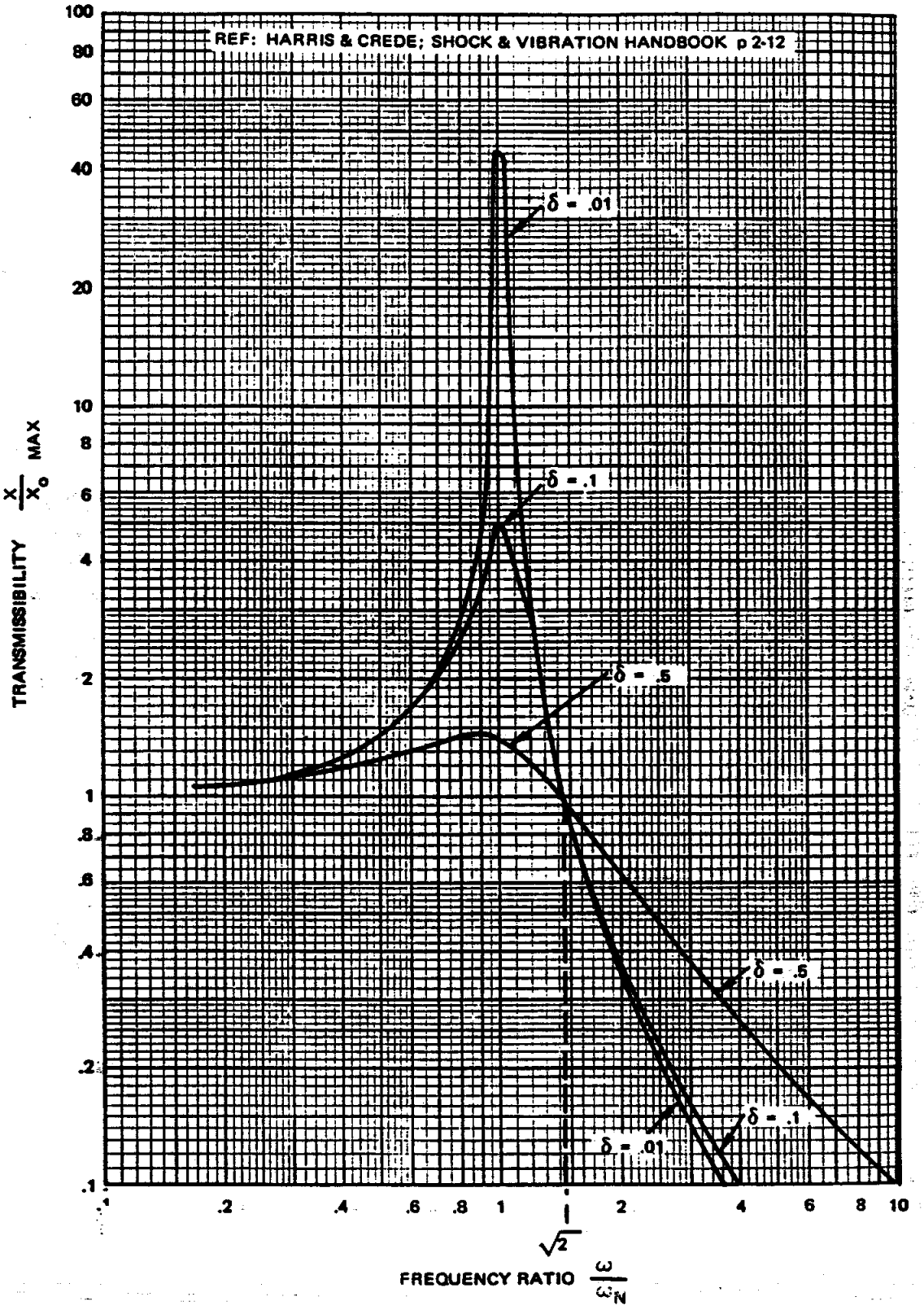


Figure 5-7. Transmissibility Curves

5-1.1.5 Response To A Step Function

A step is a forcing function whose value is zero for all time before T_o and is F_o for all time after T_o. The solution before T_o is clearly zero. At T_o the problem reduces to a static force F_o and zero initial conditions.

$$X(t) = \frac{F_o}{K}\left\{1 - e^{-\omega\delta\,(T - T_o)}\left[\cos\,\omega\sqrt{1 - \delta^2}\,(T - T_o) + \frac{\delta}{\sqrt{1 - \delta^2}}\sin\,\omega\,\sqrt{1 - \delta^2}\,(T - T_o)\right]\right\}$$

The solution is plotted in Figure 5-8. The static portion of the response is also plotted for reference. Notice that for long times, the response approaches the static solution.

5-1.1.6 Response To An Impulse

An impulse is an instantaneous force of infinite amplitude. The integral of the force over the time it acts is called the impulse of the force and is finite for an impulsive load. An impulsive force whose integral is F_o acting at T_o is written $F_o\,\delta'\,(T - T_o)$ where δ' is the Dirac Delta function. The solution for an impulse can be derived in many ways. One way is to consider an impulse to be the limiting case of a square wave as its duration approaches zero, but its integral remains constant.

An alternative method of calculating the impulse response is to directly integrate both sides of the differential equation. This shows that the impulse response reduces to a velocity, initial-value problem. The solution is:

$$X(t) = \frac{F_o}{K}\,e^{-\omega\delta\,(T - T_o)}\,\frac{\omega}{\sqrt{1 - \delta^2}}\,\sin\,\omega\sqrt{1 - \delta^2}\,(T - T_o)$$

or

$$= \frac{\omega F_o}{K\sqrt{1 - \delta^2}}\,e^{-\omega\delta\,(T - T_o)}\,\sin\,\omega\sqrt{1 - \delta^2}\,(T - T_o)$$

or

$$= \frac{F_o}{\omega M\sqrt{1 - \delta^2}}\,e^{-\omega\delta\,(T - T_o)}\,\sin\,\omega\sqrt{1 - \delta^2}\,(T - T_o)$$

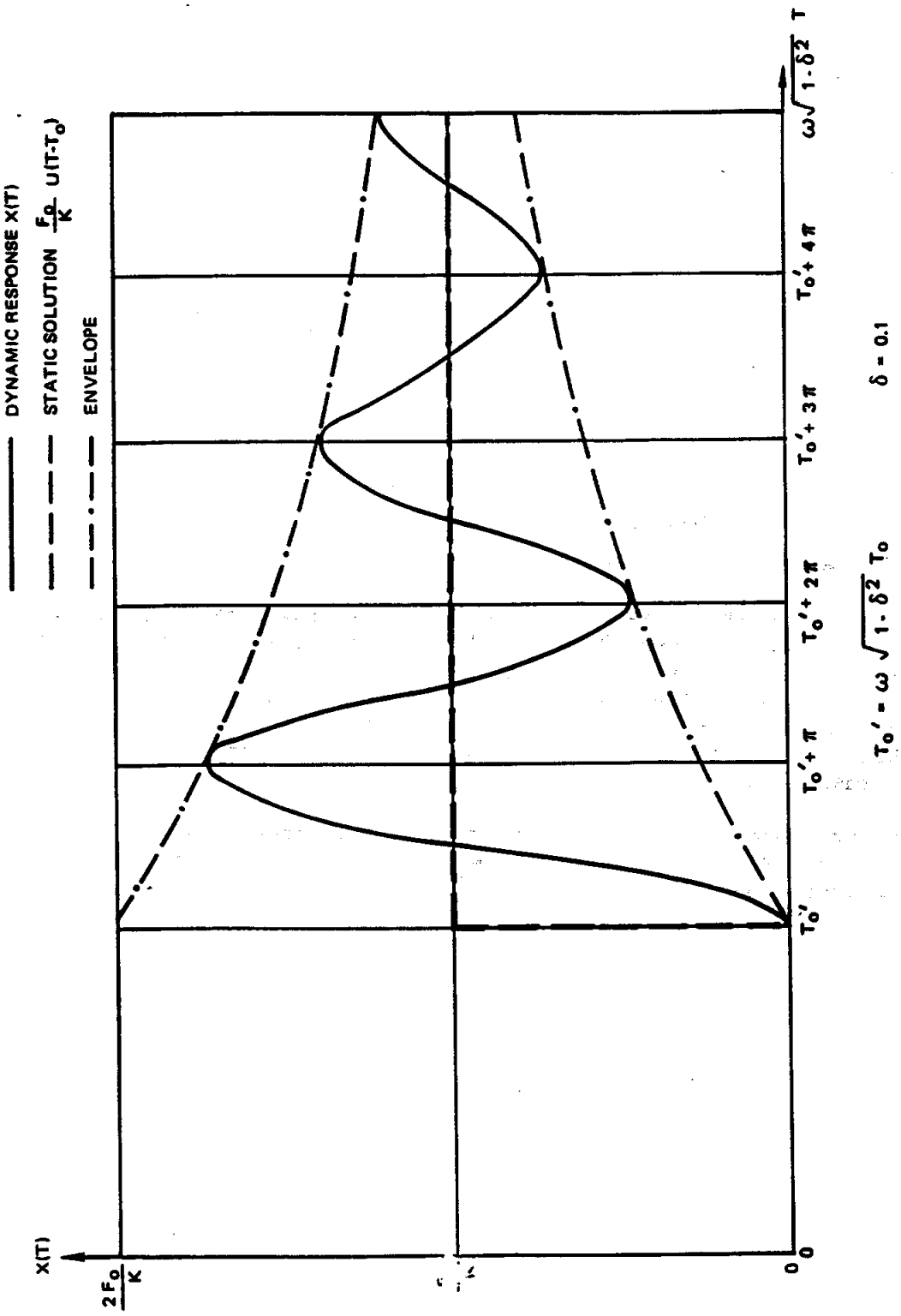


Figure 5-8. Response of an S.D.O.F. System to a Step Function

This provides a very convenient way to calculate the response to an arbitrary force. If the arbitrary force is visualized as a continuum of impulses, the force is just the integral of this. Similarly, the response is just the integral of the impulse responses. This integral is called the convolution integral.

$$F(T) = \int_{-\infty}^{T} f(\tau)\, \delta'(T - \tau)\, d\tau$$

$$\Longrightarrow \quad X(T) = \int_{-\infty}^{T} f(\tau)\, \frac{\omega\, e^{-\omega\delta\,(T - \tau)}}{K\sqrt{1 - \delta^2}} \sin \omega \sqrt{1 - \delta^2}\, (T - \tau)\, d\tau$$

5-1.1.7 Random Response

The random response of a single degree-of-freedom system cannot be directly solved using the convolution integral, since any particular time history will not necessarily be representative of any other (a random signal being defined as one whose value cannot be predicted at any time). Hence, statistical quantities are used to describe both the forcing function and the response thereto. The response of a system to a random force can be calculated using advanced calculus. The exact solution for the mean squared response to white noise, a forcing function with a constant spectral density [$G_X(\omega)$ = constant, which means equal energy in a one Hertz band throughout the frequency domain], and the approximate solution for lightly damped systems is:

$$\bar{X}^2 = \int_{-\infty}^{\infty} |\, H(\omega)\, |^2\, G_X(\omega)\, d\omega$$

where

$\bar{X}^2$ = mean squared response

and

$$|\, H(\omega)\, |^2 = \text{Transfer Function} = \frac{\omega_N^4}{\left(\omega_N^2 - \omega^2\right)^2 + 4\, \omega^2\, \omega_N^2\, \delta^2}$$

For $\delta << 1$ and $G_x(\omega)$ = constant.

$$\overline{X}^2 = \frac{\pi}{2} G_x(\omega_N) \omega_N \cdot Q$$

or

$$\overline{X} = \left[\frac{\pi}{2} \cdot Q \cdot f \cdot G_x(f_N)\right]^{1/2}$$

5-1.1.8 Proportionalities

Summarizing proportionalities for a linear, forced, single-degree-of-freedom system.

In:

$$m\ddot{x} + b\dot{x} + kx = F(t)$$

where:

m = mass;

b = damping;

k = stiffness

let:

$$\frac{b}{m} = \beta \quad \text{and} \quad \frac{k}{m} = \omega_o^2$$

then:

$$\ddot{X} + \beta \dot{X} + \omega_o^2 X = \frac{F(t)}{m}$$

Let:

$$Q = \frac{\omega_o}{\beta} = \frac{\sqrt{k}}{\sqrt{m}} \cdot \frac{m}{b} = \frac{\sqrt{km}}{b}$$

and:

$$D(\omega_o) = \text{spectral density of } \frac{\text{Force}}{\text{Stiffness}}$$

$$= \left(\frac{F}{k}\right)^2 \text{ per Hz} = \frac{D_F}{k^2}$$

then:

a. Mean Square Deflection

$$\bar{X}^2 = D(\omega_o) \times Q^2 \times \frac{\pi}{2} \beta$$

$$= \frac{D_F}{k^2} \times \frac{km}{b^2} \times \frac{\pi}{2} \frac{b}{m}$$

$$= \frac{D_F}{k} \times \frac{1}{b} \times \frac{\pi}{2} = D_F \frac{\pi}{2} \times \frac{1}{kb}$$

$$\therefore \bar{X}^2 \propto \frac{1}{\text{stiffness x damping}}, \text{ independent of mass.}$$

b. Mean Square Velocity

$$\bar{\dot{X}}^2 = \bar{X}^2 \omega_o^2 = D_F \frac{\pi}{2} \times \frac{1}{kb} \times \frac{k}{m} = D_F \frac{\pi}{2} \times \frac{1}{mb}$$

$$\therefore \bar{\dot{X}}^2 \propto \frac{1}{\text{mass x damping}}, \text{ independent of stiffness.}$$

c. Mean Square Acceleration

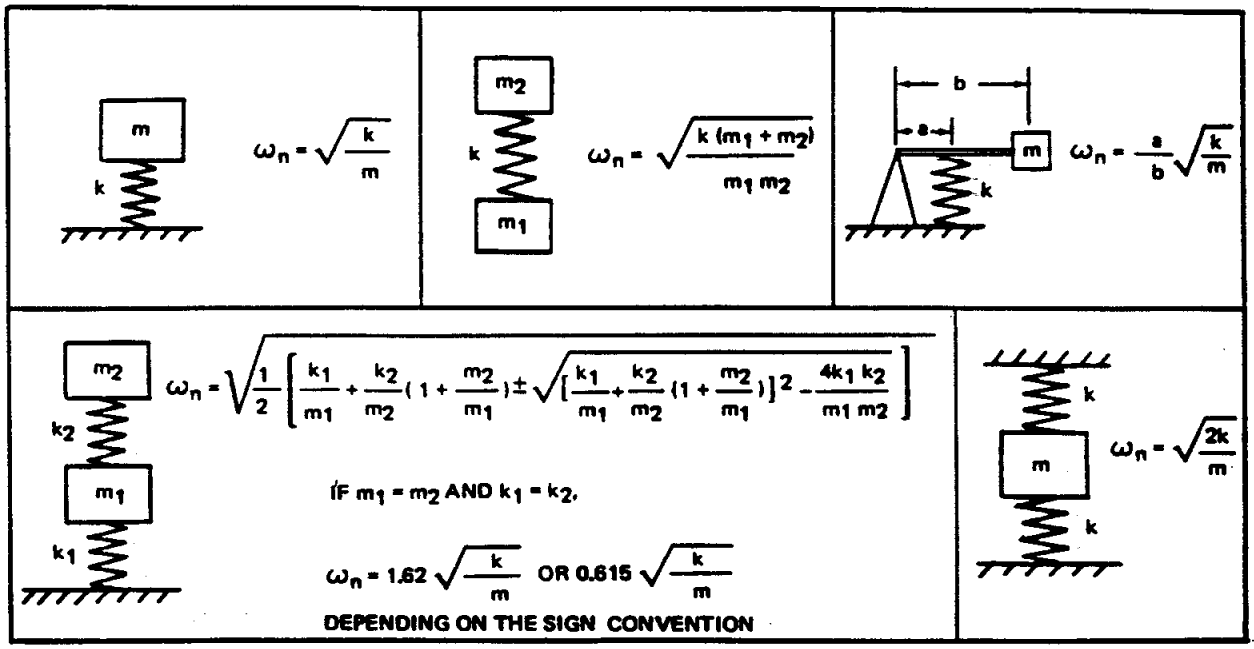
$$\bar{\ddot{X}}^2 = \bar{\dot{X}}^2 \omega_o^2 = D_F \frac{\pi}{2} \times \frac{1}{mb} \times \frac{k}{m} = D_F \frac{\pi}{2} \times \frac{k}{m^2 b}$$

$$\therefore \bar{\ddot{X}}^2 \propto \frac{\text{stiffness}}{\text{mass}^2 \text{ x damping}}, \text{ dependent on k, m, b}$$

5-1.2 Natural Frequency of Commonly Used Systems

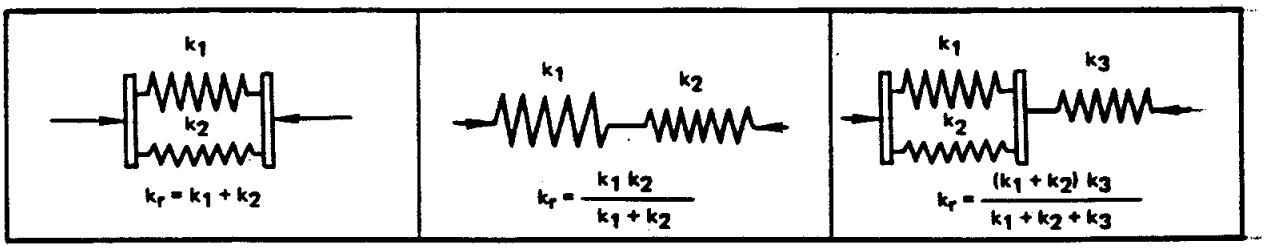
MASS – SPRING SYSTEMS IN TRANSLATION
(RIGID MASS AND MASSLESS SPRING)

k = SPRING STIFFNESS, LB/IN.
m = MASS, LB-SEC^2/IN.
ω_n = ANGLULAR NATURAL FREQUENCY, RAD/SEC



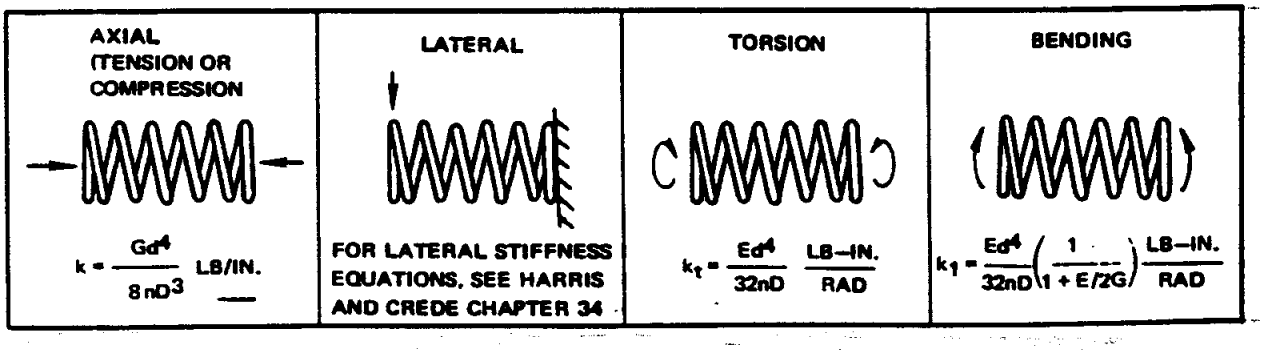
SPRINGS IN COMBINATION

k_r = RESULTANT STIFFNESS OF COMBINATION



HELICAL SPRINGS

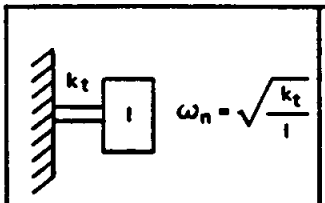
d = WIRE DIAMETER, IN.
D = MEAN COIL DIAMETER, IN.
n = NUMBER OF ACTIVE COILS
E = YOUNG'S MODULUS, LB/IN.^{2}
G = MODULUS OF ELASTICITY IN SHEAR, LB/IN.^{2}



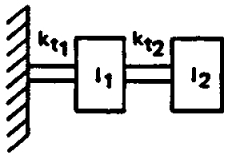
ROTOR–SHAFT SYSTEMS

(RIGID ROTOR AND MASSLESS SHAFT)

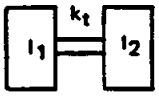
k_t = TORSIONAL STIFFNESS OF SHAFT, LB–IN./RAD
I = MASS MOMENT OF INERTIA OF ROTOR, LB–IN.–SEC^2
ω_n = ANGULAR NATURAL FREQUENCY, RAD/SEC



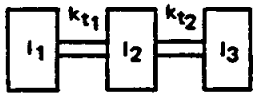
$$\omega_n = \sqrt{\frac{k_t}{I}}$$



$$\omega_n = \frac{1}{2}\left[\frac{k_{t1}}{I_1}+\frac{k_{t2}}{I_2}\left(1+\frac{I_2}{I_1}\right) \pm \sqrt{\left[\frac{k_{t1}}{I_1}+\frac{k_{t2}}{I_2}\left(1+\frac{I_2}{I_1}\right)\right]^2 - \frac{4k_{t1}k_{t2}}{I_1 I_2}}\right]$$

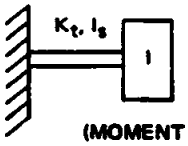


$$\omega_n = \sqrt{\frac{k_t(I_1+I_2)}{I_1 I_2}}$$



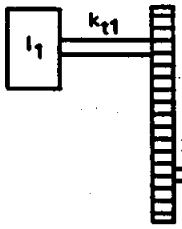
$$\omega_n = \frac{1}{2}\left[B \pm \sqrt{B^2 - \frac{4k_{t1}k_{t2}}{I_1 I_2 I_3}(I_1+I_2+I_3)}\right]$$

$$\text{WHERE } B = \frac{k_{t1}}{I_1} + \frac{k_{t2}}{I_3} + \frac{k_{t1}+k_{t2}}{I_2}$$



(MOMENT OF INERTIA IF SHAFT = I_s)

$$\omega_n = \sqrt{\frac{k_t}{I + \frac{1}{3} I_s}}$$



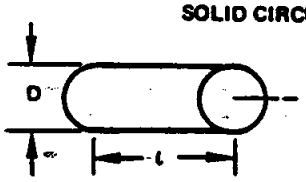
GEARED SYSTEM WITH MASSLESS GEARS (SPEED OF ROTOR 2 IS n TIMES SPEED OF ROTOR 1)

$$\omega_n = \sqrt{\frac{k_1 k_2 (I_1 + n^2 I_2)}{I_1 I_2 (n^2 k_2 + k_1)}}$$

STIFFNESS OF SHAFTS IN TORSION

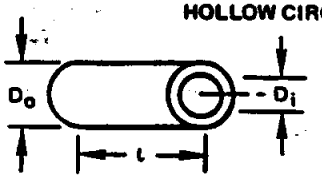
G = MODULUS OF ELASTICITY IN SHEAR, LB/IN.^{2}
ℓ = LENGTH OF SHAFT, IN.
I_p = POLAR MOMENT OF INERTIA OF SHAFT CROSS-SECTION, IN.^{4}

SOLID CIRCULAR



$$k_t = \frac{\pi G D^4}{32\ell}$$

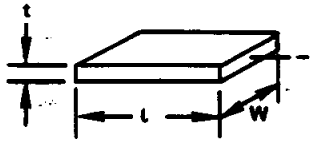
HOLLOW CIRCULAR



$$k_t = \frac{\pi G (D_o^4 - D_i^4)}{32\ell} \quad \text{IN-LB/RAD}$$

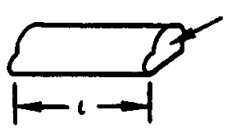
OR $= K_t/R^2$ LBS/IN.

RETANGULAR



$$k_t = \frac{GWt^3}{3\ell}$$

ANY SOLID SECTION

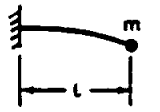
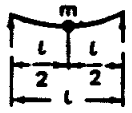
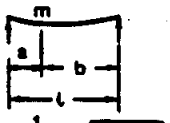
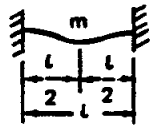
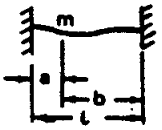


$$k_t = \frac{GS^4}{4\pi^2 \ell I_p}$$

VIBRATION, SHOCK, AND ACOUSTICS

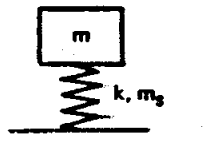
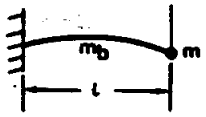
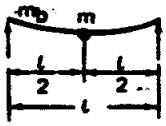
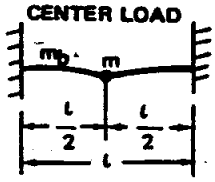
MASSLESS BEAMS WITH CONCENTRATED MASS LOADS

m = MASS OF LOAD, LB–SEC^2/IN.
ℓ = LENGTH OF BEAM, IN.
I = AREA MOMENT OF INERTIA OF BEAM CROSS SECTION, IN.^{4}
E = YOUNG'S MODULUS, LB/IN.^{2}
ω_n = ANGULAR NATURAL FREQUENCY, RAD/SEC

FIXED–FREE END LOAD	HINGED–HINGED CENTER LOAD	HINGED–HINGED OFF–CENTER LOAD	FIXED–FIXED CENTER LOAD	FIXED–FIXED OFF–CENTER LOAD
				
$\omega_n = \sqrt{\frac{3EI}{m\ell^3}}$	$\omega_n = 4\sqrt{\frac{3EI}{m\ell^3}}$	$\omega_n = \frac{1}{ab}\sqrt{\frac{3EI\ell}{m}}$	$\omega_n = 8\sqrt{\frac{3EI}{m\ell^3}}$	$\omega_n = 4\sqrt{\frac{3EI}{mab\ell}}$

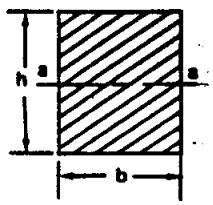
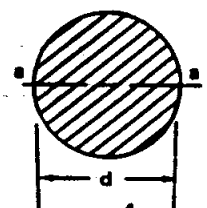
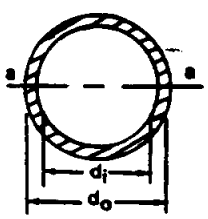
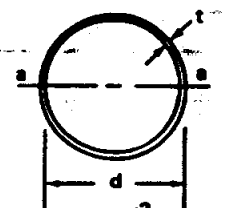
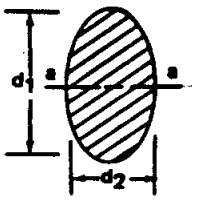
MASSIVE SPRINGS (BEAMS) WITH CONCENTRATED MASS LOADS

m = MASS OF LOAD, LB–SEC^2/IN.
$m_s(m_b)$ = MASS OF SPRING (BEAM), LB–SEC^2/IN.
k = STIFFNESS OF SPRING LB/IN.
ℓ = LENGTH OF BEAM, IN.
I = AREA MOMENT OF INERTIA OF BEAM CROSS SECTION, IN.^{4}
E = YOUNG'S MODULUS, LB/IN.^{2}
$^1\omega_n$ = ANGULAR NATURAL FREQUENCY, RAD/SEC

MASS–HELICAL SPRING	FIXED–FREE END LOAD	HINGED–HINGED CENTER LOAD	FIXED–FIXED CENTER LOAD
			
$\omega_n = \sqrt{\frac{k}{(m + \frac{m_s}{3})}}$	$\omega_n = \sqrt{\frac{3EI}{\ell^3(m + 0.23m_b)}}$	$\omega_n = \sqrt{\frac{48EI}{\ell^3(m + 0.5m_b)}}$	$\omega_n = 14\sqrt{\frac{EI}{\ell^3(m + 0.375m_b)}}$

WHEN m = 0, USE $0.406m_s$
(MYKLESTAD PG 39)

AREA MOMENT OF INERTIA OF BEAM SECTIONS
(WITH RESPECT TO AXIS a – a)

RECTANGLE	CIRCLE	HOLLOW CIRCLE	THIN WALL CIRCLE	ELLIPSE
				
$I = \frac{bh^3}{12}$	$I = \frac{\pi d^4}{64}$	$I = \frac{\pi}{64}(d_o^4 - d_i^4)$	$I = \frac{\pi d^3 t}{8}$	$I = \frac{\pi d_2 d_1^3}{64}$

5-1.3 Beams

5-1.3.1 Frequency Charts

See Figure 5-9

5-1.3.2 Bending Vibration of Beams

The theory underlying the data given for the natural frequencies of beams assumes small deflections and neglects rotary inertia and shear effects.

The natural frequency of the uniform beam in the bending mode of vibration is directly proportional to the radius of gyration, r, of the beam. Similarly, the frequencies of the rectangular beam and the uniform rod are directly proportional to the depth and diameter, respectively. Note that the frequency is independent of width, because the width adds as much stiffness as it does mass.

The method is based on the use of a frequency factor which is defined as:

λ_f = f/r for uniform beams of any cross-section, and

λ_f = f/h for uniform rectangular beams

where

f = natural frequency, Hz

r = radius of gyration = $\sqrt{I/A}$, inches

h = beam depth, inches

Figures 5-10 and 5-11 give the values of the frequency factor for various beams of engineering interest, for the first mode of vibration. These values are based on a density of 0.097 lb/cu in. and a Young's modulus of 10.4×10^6 psi for aluminum, and cover a range of 1 to 100 inches.

The graphs can be expanded to other ranges by using the relationship that the natural frequency is inversely proportional to the square of the beam length.

Figure 5-12 gives the correction factors for use with non-aluminum structures.

It is of some interest to compare the fundamental mode frequencies for the different boundary conditions given in Figure 5-10. The ratios as compared to the clamped-free beam (B) are:

$\frac{A}{B} = 0.7$, $\frac{C}{B} = 1.59$, $\frac{D}{B} = 2.81$, $\frac{E}{B} = 4.39$, and $\frac{F}{B} = 6.37$.

ANGULAR NATURAL FREQUENCY $\omega_n = A\sqrt{\frac{EI}{\mu \ell^4}}$ RAD/SEC

WHERE E = YOUNG'S MODULUS, LB/IN^2
I = AREA MOMENT OF INERTIA OF BEAM CROSS SECTION, IN.^{4}
ℓ = LENGTH OF BEAM, IN.
μ = MASS PER UNIT LENGTH OF BEAM, LB–SEC^2/IN.^{2}
A = COEFFICIENT FROM TABLE BELOW

NODES ARE INDICATED IN TABLE BELOW AS A PROPORTION OF LENGTH ℓ MEASURED FROM LEFT END

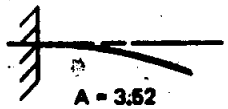
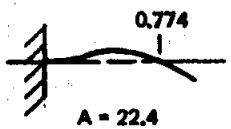
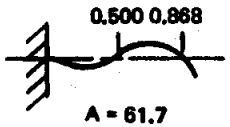
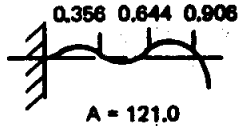
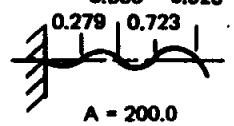
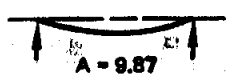
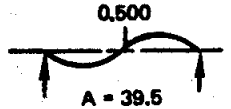
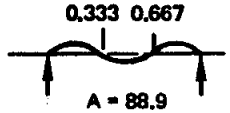
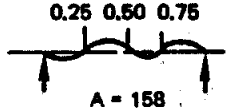
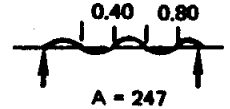
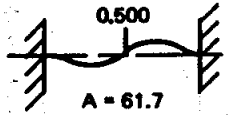
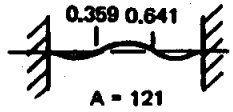
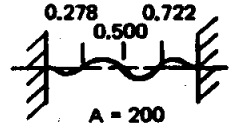
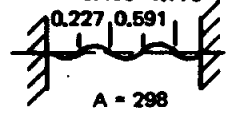
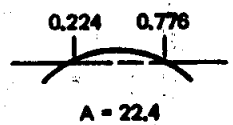
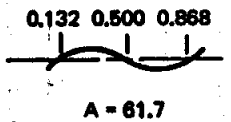
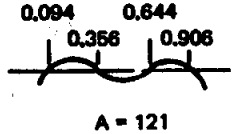
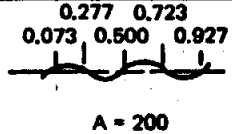
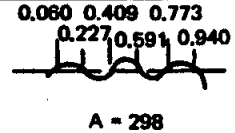
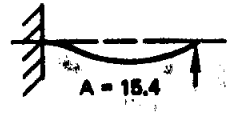
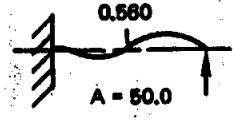
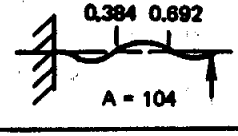
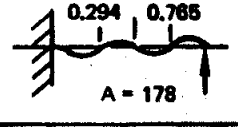
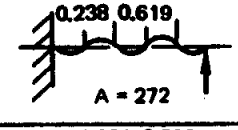
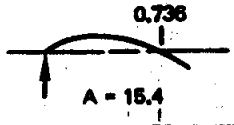
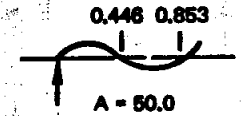
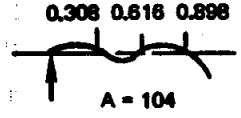
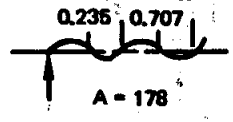
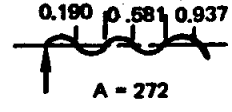
FIXED–FREE (CANTILEVER)	 A = 3.52	 0.774 A = 22.4	 0.500 0.868 A = 61.7	 0.356 0.644 0.906 A = 121.0	 0.500 0.926 0.279 0.723 A = 200.0
HINGED–HINGED (SIMPLE)	 A = 9.87	 0.500 A = 39.5	 0.333 0.667 A = 88.9	 0.25 0.50 0.75 A = 158	 0.20 0.60 0.40 0.80 A = 247
FIXED–FIXED (BUILT–I)	 A = 22.4	 0.500 A = 61.7	 0.359 0.641 A = 121	 0.278 0.722 0.500 A = 200	 0.409 0.773 0.227 0.591 A = 298
FREE–FREE	 0.224 0.776 A = 22.4	 0.132 0.500 0.868 A = 61.7	 0.094 0.644 0.356 0.906 A = 121	 0.277 0.723 0.073 0.500 0.927 A = 200	 0.060 0.409 0.773 0.227 0.591 0.940 A = 298
FIXED–HINGED	 A = 15.4	 0.560 A = 50.0	 0.384 0.692 A = 104	 0.529 0.294 0.765 A = 178	 0.429 0.810 0.238 0.619 A = 272
HINGED–FREE	 0.736 A = 15.4	 0.446 0.853 A = 50.0	 0.308 0.616 0.898 A = 104	 0.471 0.922 0.235 0.707 A = 178	 0.381 0.763 0.190 0.581 0.937 A = 272

Figure 5-9. Beams of Uniform Section and Uniformly Distributed Load

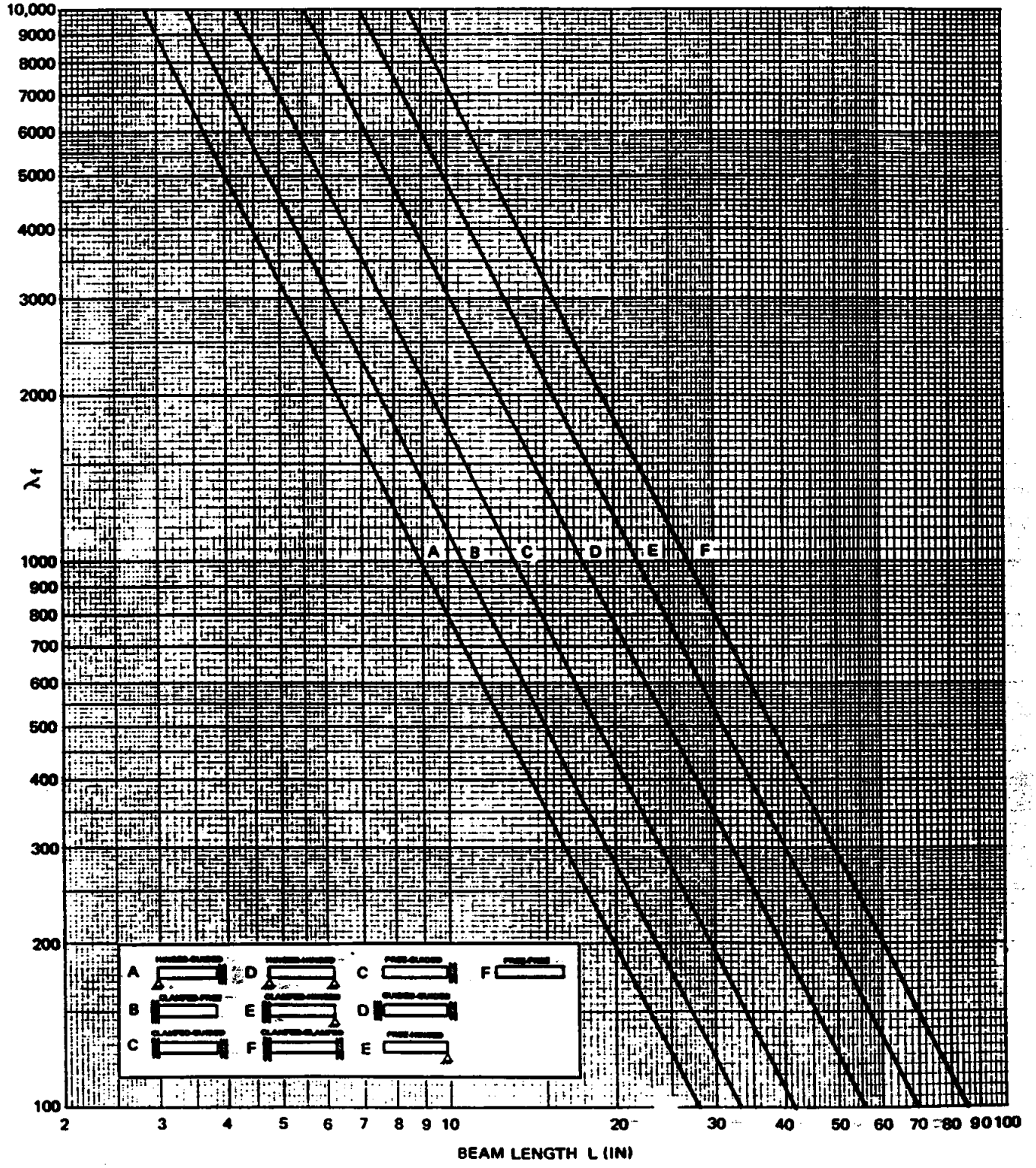


Figure 5-10. Uniform Aluminum Beams of any Cross-Section

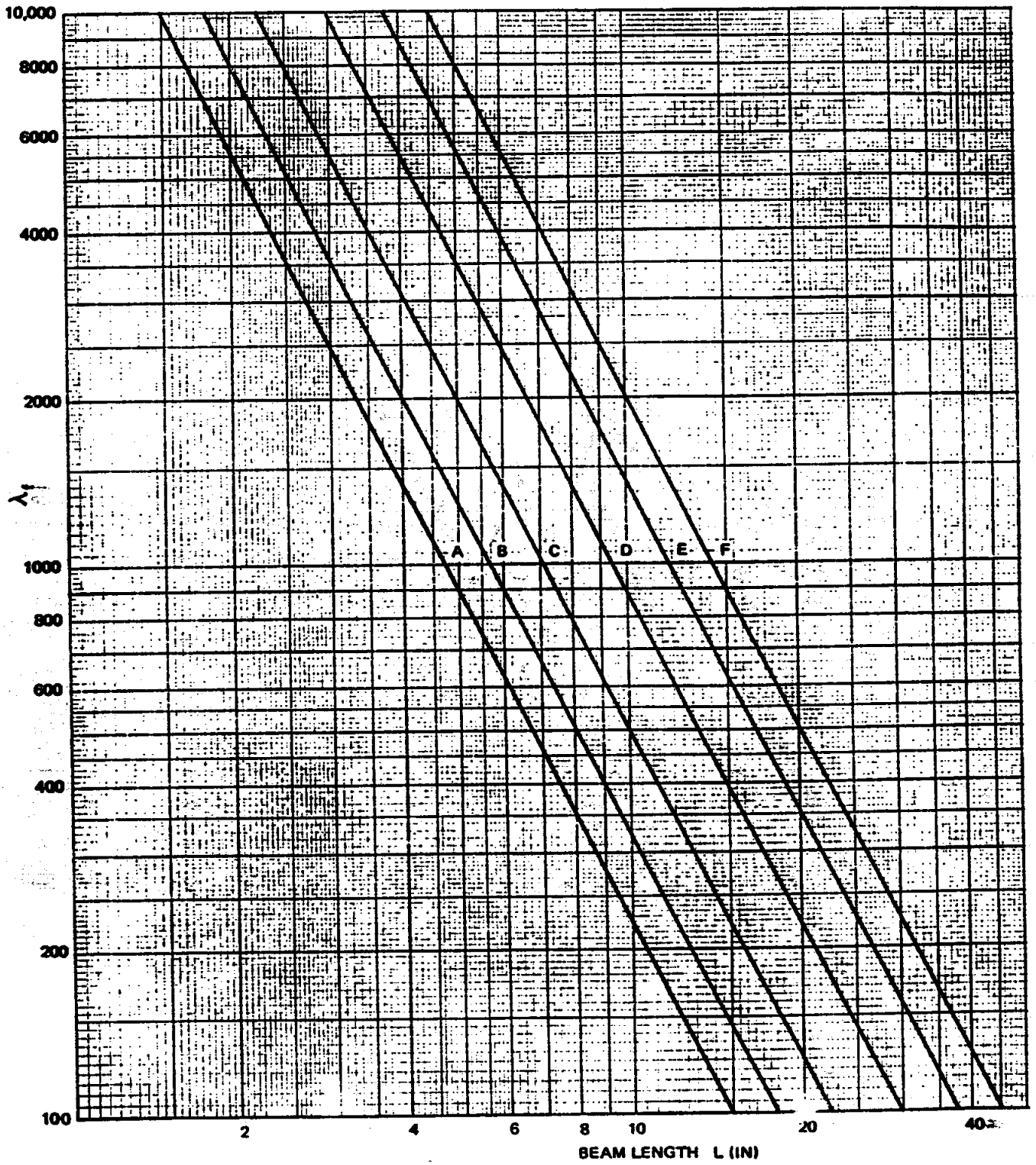
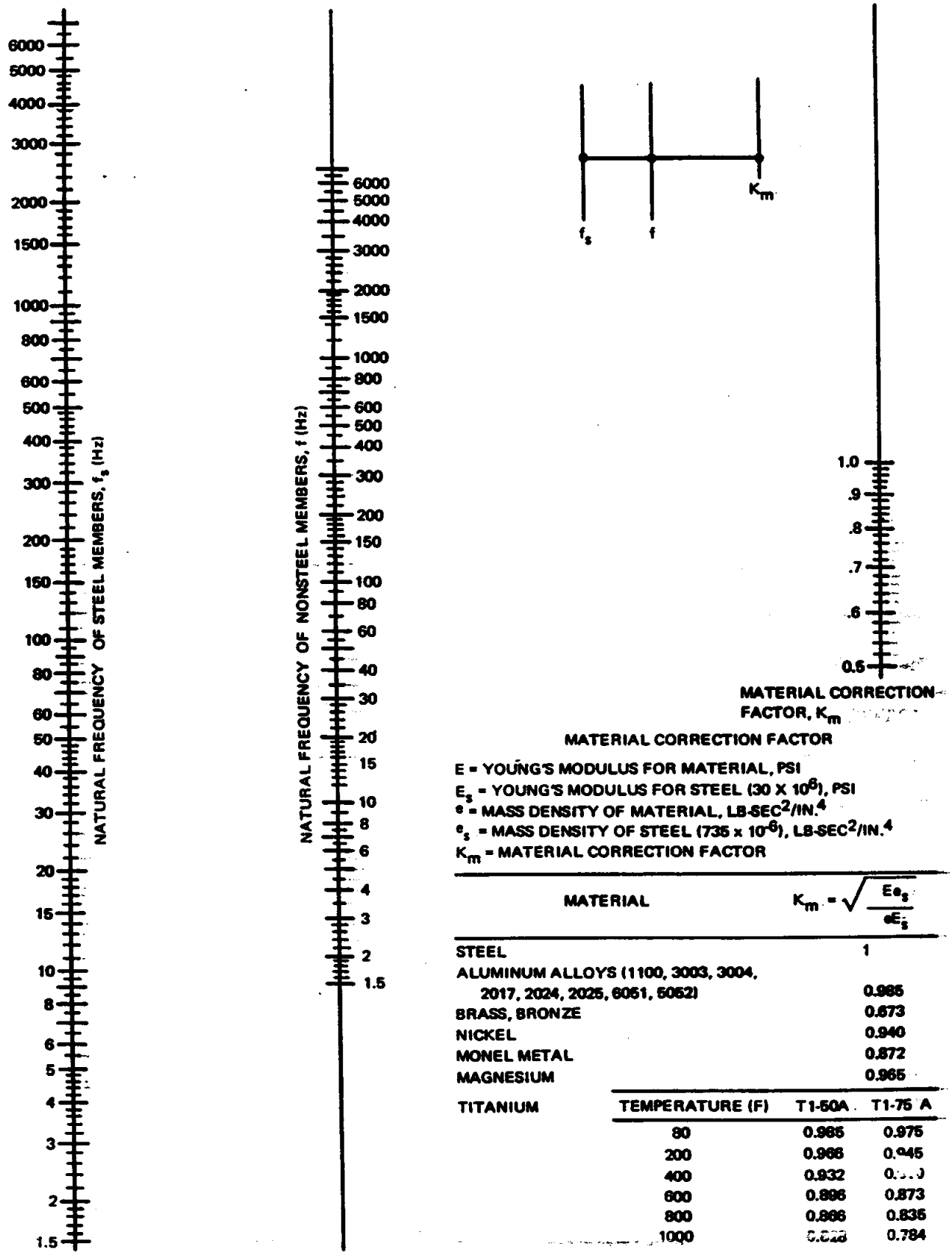


Figure 5-11. Uniform Aluminum Rectangular Beams



MATERIAL CORRECTION FACTOR

E = YOUNG'S MODULUS FOR MATERIAL, PSI
E_s = YOUNG'S MODULUS FOR STEEL (30 X 10^6), PSI
ρ = MASS DENSITY OF MATERIAL, LB-SEC^2/IN.^{4}
ρ_s = MASS DENSITY OF STEEL (735 x 10^{-6}), LB-SEC^2/IN.^{4}
K_m = MATERIAL CORRECTION FACTOR

MATERIAL	$K_m = \sqrt{\frac{E\rho_s}{\rho E_s}}$	
STEEL	1	
ALUMINUM ALLOYS (1100, 3003, 3004, 2017, 2024, 2025, 6051, 5052)	0.985	
BRASS, BRONZE	0.673	
NICKEL	0.940	
MONEL METAL	0.872	
MAGNESIUM	0.965	
TITANIUM	TEMPERATURE (F)	T1-50A T1-75 A
	80	0.986 0.975
	200	0.966 0.945
	400	0.932 0.[illegible]
	600	0.896 0.873
	800	0.866 0.835
	1000	0.828 0.784

Figure 5-12. Correction Table and Nomogram for Determination of Factor K_m and Natural Frequency f for Nonsteel Structures

5-1.3.3 Higher Mode Frequencies

The ratio of higher mode frequencies to the fundamental (found from Figures 5-10 or 5-11) is given in Table 5-1.

Table 5-1. Multiplication Factors.

CASE	SYSTEM	MODE NUMBER					
		1	2	3	4	5	n
A	HINGED-GUIDED	1	9.00	25.0	49.0	81.0	$(2n-1)^2$
B	CLAMPED-FREE	1	6.27	17.5	34.4	56.8	
C	CLAMPED-GUIDED (FREE-GUIDED)	1	5.40	13.3	24.8	39.8	
D	HINGED-HINGED (GUIDED-GUIDED)	1	4.00	9.00	16.0	25.0	
E	CLAMPED-HINGED (FREE-HINGED)	1	3.24	6.76	11.6	17.6	
F	CLAMPED-CLAMPED (FREE-FREE)	1	2.76	5.40	8.93	13.3	$\left(\frac{2n+1}{3}\right)^2$

The difference between the beam frequencies becomes less with increasing mode number because the boundaries become less and less significant. The higher mode frequencies converge to the frequency of the simply-supported beam.

5-1.3.4 Relation Between Natural Frequency and Beam Cross-Section

The general expression for the bending natural frequency of beams is:

$$f_n = a_n \left(\frac{E}{e}\right)^{1/2} (r/L^2)$$

This expression indicates the direct relationship between frequency and radius of gyration.

The radius of gyration values for the more common beam sections are given in Table 5-2. Note the relation between the rod diameter, d, and r. Figure 5-10 can be used for solid rods by using the following relationship:

$$\lambda_f = \tfrac{1}{4} f/d.$$

Various cross-sectional shapes, commonly encountered in structural design, are compared in Table 5-3.

Table 5-2. Comparison of Various Cross-Sections.

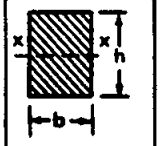
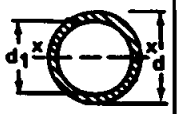
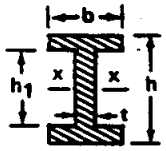
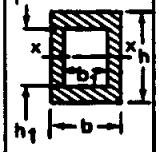
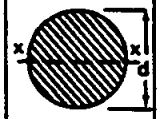
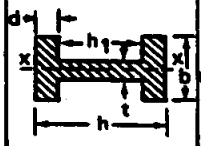
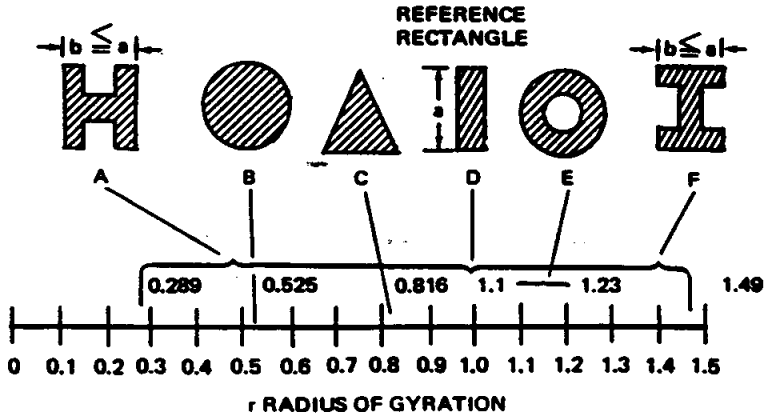
SECTION	RADIUS OF GYRATION	SECTION	RADIUS OF GYRATION
	$\frac{h}{\sqrt{12}} = 0.289h$		$\frac{\sqrt{d^2 + d_1^2}}{4}$
	$\frac{h}{\sqrt{18}} = 0.236h$		$\sqrt{\frac{bh^3 - h_1^3(b-t)}{12[bh - h_1(b-t)]}}$
	$\sqrt{\frac{bh^3 - b_1h_1^3}{12(bh - b_1h_1)}}$		
	$\frac{d}{4}$		$\sqrt{\frac{2db^3 + h_1t^3}{12[bh - h_1(b-t)]}}$

Table 5-3. Comparison of Structural Shapes with the same Cross-Sectional Area.



Note that the I-beam is the most efficient uniaxial member if a good portion of its area is located away from the neutral bending axis. The upper limit condition for the I-beam, which is 50 percent higher than the reference rectangle, applies when all the area is concentrated in the flanges and the web has zero thickness. The lower limit condition occurs when the area is all concentrated in the I-beam web, which, in effect, is the reference rectangle where r = 1.

Comparison of Table 5-3 B and E indicates the desirability of using hollow tubes rather than solid round bars if bending loads predominate in the structure. A hollow tube is the most efficient omniaxial member.

Because the radius of gyration of the hollow tube is proportional to the sum of the squares of the internal and external diameters, the natural frequency will increase with decreasing wall thickness of the tube. Rectangles in the form of box beams can also be used with high efficiency (same range as I-beam).

If the cross-section is to be varied to optimize the design of a cantilevered beam, or the beam is supported at several places, see References 1 and 2 for further frequency information.

5-1.3.5 References

1. Barre, R. J., *Bending Vibration of Beams*, Aerospace Division, Westinghouse Defense and Space Center, Westinghouse Electric Corporation.

2. Macduff, J. N. and R. F. Felgar, *Vibration Frequency Charts*, Machine Design, Vol. 29, No. 3, 2-7-57, pp 109-115 (ASME Trans. Vol. 79, No. 7, Oct 57, pp 1459-1465).

5-1.4 Plates

A general equation for the frequency of resonance of a simply-supported homogeneous, uniform, rectangular plate is:

$$\omega_{mn}^2 = \frac{g}{W_o}\left\{\pi^4 D\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2 + \pi^2\left[N_x\left(\frac{m}{a}\right)^2 + N_y\left(\frac{n}{b}\right)^2\right]\right\}$$

where:

ω_{mn} = natural frequency (in rad/sec) of the mn mode,

g = gravitational constant (in/sec^2),

W_o = surface weight (lbs/in^2),

$D = E h^3/\{12 (1 - \sigma^2)\}$,

E = modulus of elasticity (lbs/in^2),

h = plate thickness (in.),

σ = Poisson's ratio,

a = length of plate (in.),

b = width of plate (in.),

m,n = mode numbers in a and b directions respectively,

N_x and N_y are the normal loading in the a and b directions respectively. If no preloading of the plate is in evidence, and no large deflections cause plate stretching, the last two terms involving N_x and N_y approximately equal zero.

Bisplinghoff's correction for plate curvature (from Reissner's shallow-shell theory) may be stated as:

$$\omega_{mn}^2 = \frac{g}{W_o R^2}\left[\frac{D}{R^2}\left(\lambda_n^2 + \mu_m\right)^2 + \frac{Eh\,\lambda_n^4}{\left(\lambda_n^2 + \mu_m^2\right)^2}\right]$$

where:

R = radius (inches),

$$\lambda_n = \frac{n\pi R}{a},$$

$$\mu_m = \frac{m\pi}{\beta},$$

and

β = angle decrement of the shell segment = b/R,

and the other symbols are as listed above.

5-1.4.1 Natural Frequency Calculations

5-1.4.1.1 Uniform Flat Plates

a. Notation

a	longer dimension of plate (in.)
b	shorter dimension of plate (in.)
t	thickness of plate (in.)
E	Young's modulus (lbf/in^2)
w	weight per unit volume of plate material (lbf/in^3)
g	acceleration due to gravity (in/sec^2)
f_n	natural frequency of vibration of plate (Hz)
K_n	natural frequency parameter (in/sec)

Suffixes

1, 2, 3	in first, second and third modes

b. Introduction

This paragraph gives the first 3 natural frequencies of vibration of initially unstressed, uniform, rectangular plates with either simply-supported or fixed edges.

The natural frequencies are given by:

$$f_n = K_n \frac{t}{b^2}$$

Figures 5-13, 5-14 and 5-15 give curves of K_{n1}, K_{n2} and K_{n3} respectively against a/b for plates with all edges either simply-supported or fixed.

The parameter K_n is proportional to $(Eg/w)^{1/2}$ and in deriving the curves a value of 200,000 in/sec was assumed. This value is sufficiently accurate for all common structural metallic materials.

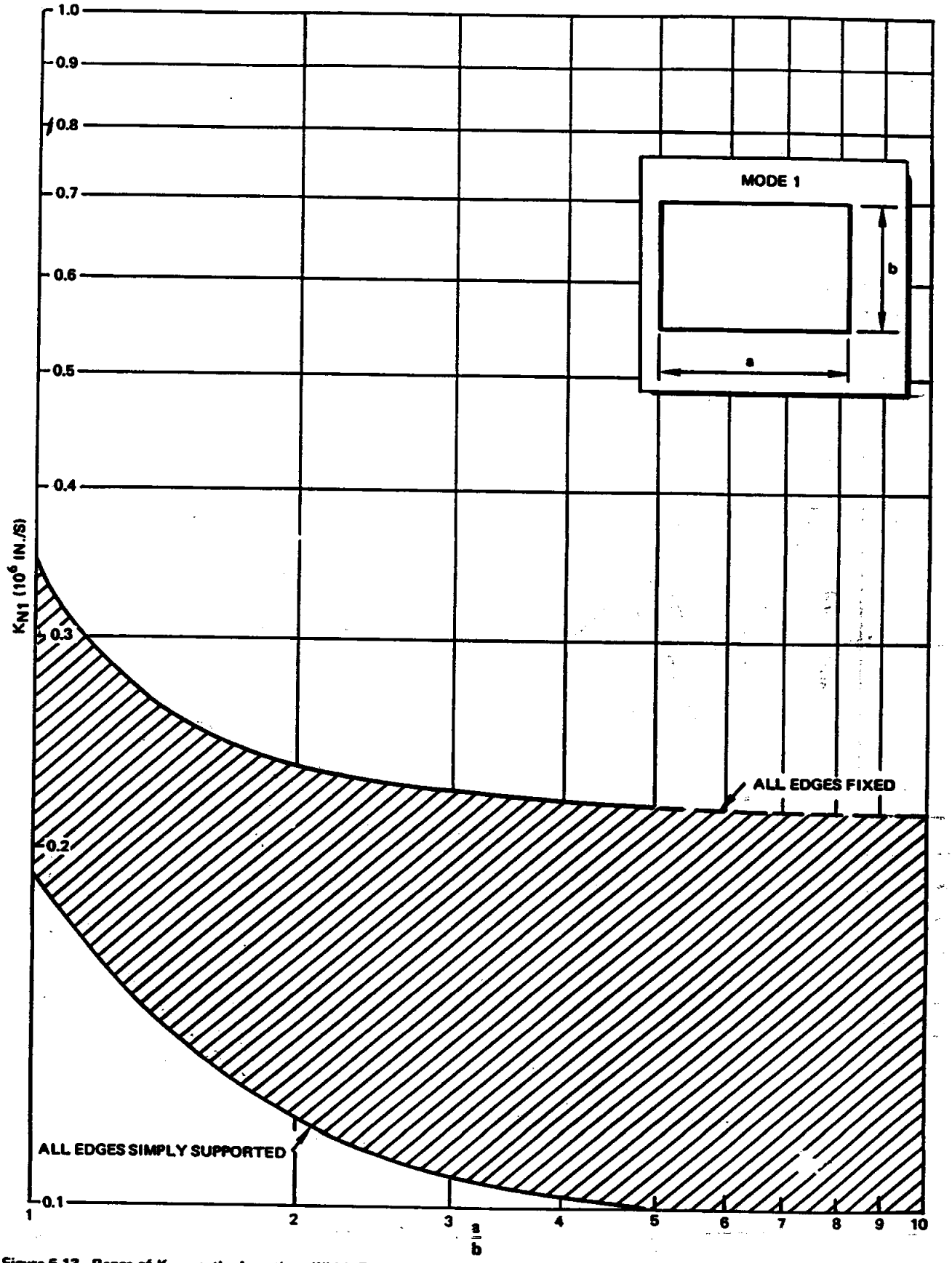


Figure 5-13. Range of K_{n1} vs. the Length to Width Ratio

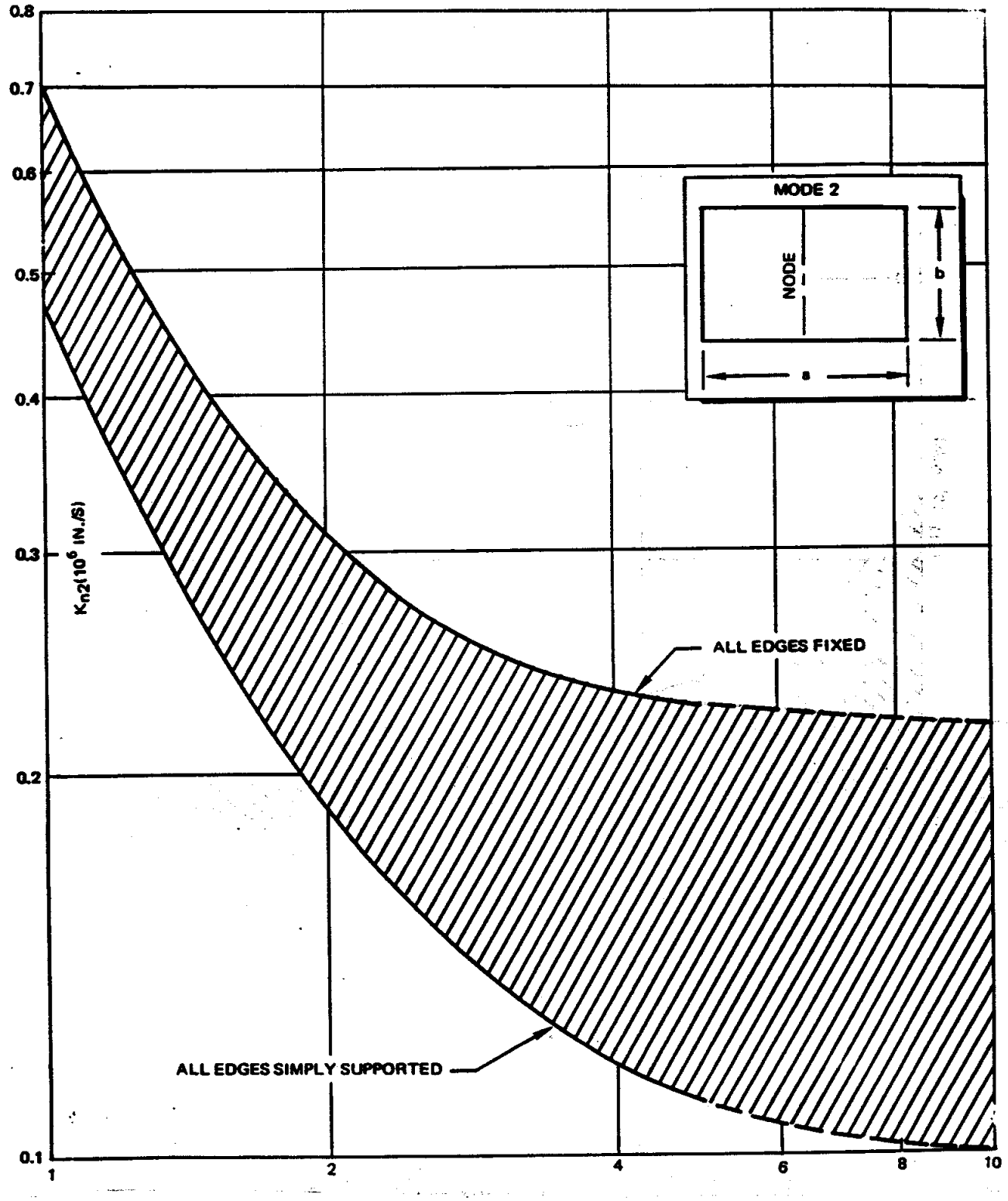


Figure 5-14. Range of K_{n2} vs. the Length to Width Ratio

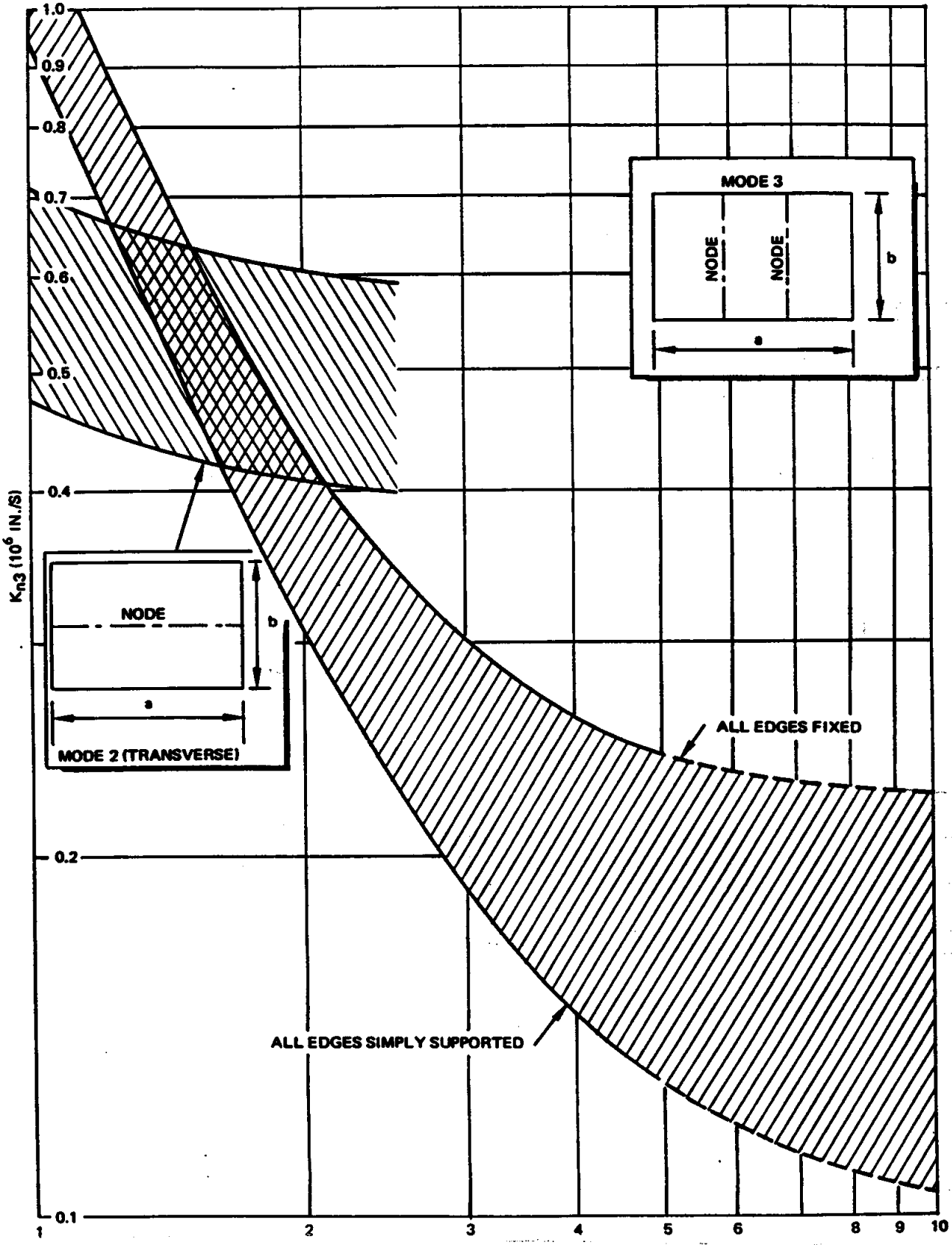


Figure 5-15. Range of K_{n3} vs. the Length to Width Ratio

c. Application to Aerospace Structural Panels

In typical aerospace construction, stiffening members such as ribs, frames and stringers divide the panels into plates. Owing to the mechanical coupling between the plates, normal modes result in which groups of plates vibrate. The number of plates showing predominant vibration at any one natural frequency of the group depends on the relative dimensions of the plates.

The lowest natural frequency is generally associated with a mode of vibration in which the response of adjacent plates is out of phase. If the stiffeners are of low torsional stiffness, the frequency for the panel will be close to the value indicated by this data sheet for a single plate with simply-supported edges. With increasing torsional stiffness the lowest resonance frequency of the group rises to approach the value indicated by this data sheet for a single plate with fixed edges.

The highest natural frequency of the panel in the fundamental mode is associated with a mode of vibration in which adjacent plates are in phase with each other. This fixes the upper bound of the natural frequency of the panel since this behaviour corresponds with fixed edge conditions.

d. Derivation

1. Warburton, G. B., *The Vibration of Rectangular Plates*. Proc. Instn Mech. Engrs, Vol. 168, pp. 371-384, 1954.

2. Lin, Y. K., *Free Vibrations of Continuous Skin-Stringer Panels*. J. Appl. Mech., Vol. 27, pp. 669-676, December 1960.

3. Clarkson, B. L., *The Design of Structures to Resist Jet Noise Fatigue*. Jl R. Aeronaut. Soc., Vol. 66, pp. 603-616, October 1962.

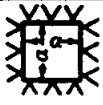
5-1.4.1.2 Natural Frequencies of Square and Round Flat Plates

See Figure 5-16.

NATURAL FREQUENCIES OF THIN FLAT SQUARE AND ROUND PLATES OF UNIFORM THICKNESS

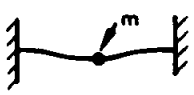
$$\omega_n = B\sqrt{\frac{E t^2}{\rho a^4 (1-\nu^2)}} \quad \text{RAD/SEC}$$

E = YOUNG'S MODULUS, LB/IN.^{2}
t = THICKNESS OF PLATE, IN'
ρ = MASS DENSITY, LB-SEC^2/IN.^{4}
a = DIAMETER OF CIRCULAR PLATE OR SIDE OF SQUARE PLATE, IN.
ν = POISSON'S RATIO

SHAPE OF PLATE	DIAGRAM	EDGE CONDITIONS	VALUE OF B FOR MODE:							
			1	2	3	4	5	6	7	8
CIRCULAR		CLAMPED AT EDGE	11.84	24.61	40.41	46.14	103.12			
CIRCULAR		FREE	6.09	10.53	14.19	23.80	40.88	44.68	61.38	69.44
CIRCULAR		CLAMPED AT CENTER	4.35	24.26	70.39	138.85				
CIRCULAR		SIMPLY SUPPORTED AT EDGE	5.90							
SQUARE		ONE EDGE CLAMPED-THREE EDGES FREE	1.01	2.47	6.20	7.94	9.01			
SQUARE		ALL EDGES CLAMPED	10.40	21.21	31.29	38.04	38.22	47.73		
SQUARE		TWO EDGES CLAMPED-TWO EDGES FREE	2.01	6.96	7.74	13.89	18.25			
SQUARE		ALL EDGES FREE	4.07	5.94	6.91	10.39	17.80	18.85		
SQUARE		ONE EDGE CLAMPED-THREE EDGES SIMPLE SUPPORTED	6.83	14.94	16.95	24.89	28.99	32.71		
SQUARE		TWO EDGES CLAMPED–TWO EDGES SIMPLY SUPPORTED	8.37	15.82	20.03	27.34	29.54	27.31		
SQUARE		ALL EDGES SIMPLY SUPPORTED	5.70	14.26	22.82	28.52	37.08	48.49		

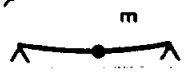
MASSLESS CIRCULAR PLATE WITH CONCENTRATED CENTER MASS

CLAMPED EDGES



$$\omega_n = 4.09\sqrt{\frac{Et^3}{ma^2(1-\nu^2)}}$$

SIMPLY SUPPORTED EDGES



$$\omega_n = 4.09\sqrt{\frac{Et^3}{ma^2(1-\nu)(3+\nu)}}$$

Figure 5-16. Natural Frequencies of Square and Round Flat Plates.

5-1.4.2 Natural Frequency of Printed Circuit Boards

Arnold, J. T. and F. P. Strother, Jr., *A Practical Approach to the Prediction of the Natural Frequency of Printed Circuit Boards*, Shock & Vib. Bul., No. 35, Part 7, pg. 137, April 1966.

$$f_n = \beta \sqrt{\frac{E t^3 A}{(1 - \nu^2) W}} \left(\frac{m^2}{c^2} + \frac{n^2}{d^2} \right)$$

β was determined by test for first 2 modes (has the dimension of $\sqrt{in/sec^2}$)

c = longest unsupported dimension (in)

d = shortest unsupported dimension (in)

A = board area (16 to 48 in^2)

E = Young's Modulus (psi)

t = board thickness (in)

W = total weight (board + components) in pounds

m,n = mode numbers in the c and d directions respectively

Screwed edges and standoffs (screws torqued to 10 in-lbs). (See Figure 5-17.)

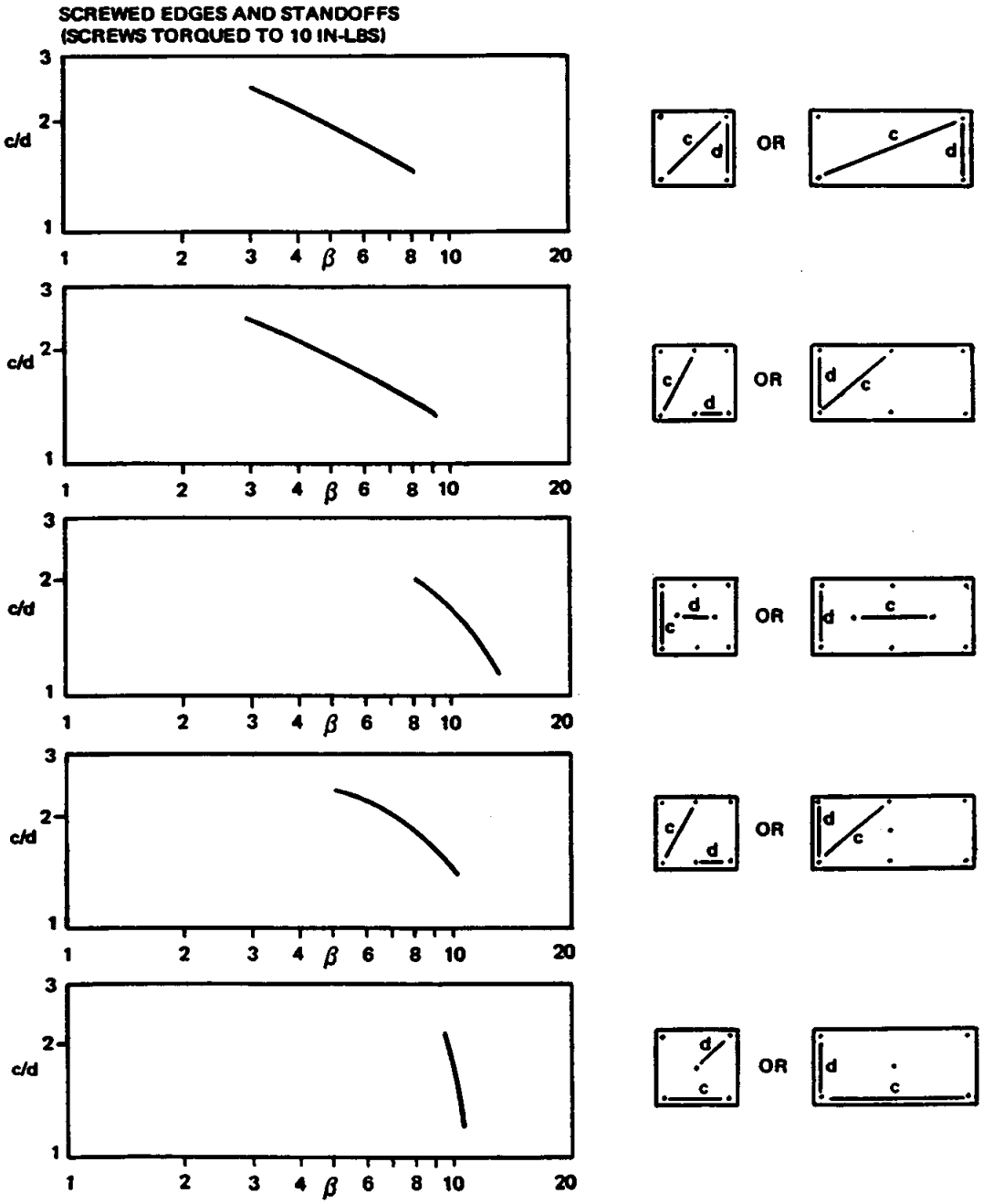


Figure 5-17. Natural Frequency of Printed Circuit Boards

5-1.4.3 Natural Frequency of Honeycomb Structures

5-1.4.3.1 Natural Frequencies of Flat Sandwich Panels with Cores of Zero Flexural Stiffness and Simply-Supported Edges

a. Notation

a	longer dimension of panel (in.)
b	shorter dimension of panel (in.)
t	thickness of each faceplate (in.)
t_c	thickness of sandwich core (in.)
E	Young's modulus of faceplates (lbf/in^2)
σ	Poisson's ratio of faceplates
G_c	effective shear modulus of core (lbf/in^2)
w	weight per unit volume of faceplate material (lbf/in^3)
w_c	weight per unit volume of core material (lbf/in^3)
g	acceleration due to gravity (in/sec^2)
m	number of half-waves in a-direction
n	number of half-waves in b-direction
λ_{am}	half wavelength in a-direction a/m (in.)
λ_{bm}	half wavelength in b-direction b/n (in.)
K, C, ψ	convenient non-dimensional parameters (see Derivation)
V	convenient parameter with dimensions of velocity (see Derivation) (in/sec)
$f_{m,n}$	natural frequency of vibration of honeycomb sandwich panel in mode characterized by half wavelengths λ_{am} and λ_{bn} (Hz)

b. Introduction

This paragraph gives a method of calculating the natural frequencies of initially-unstressed flat-rectangular panels of sandwich construction with cores of zero flexural-stiffness and simply-supported edges.

For convenience in presentation the natural frequencies are expressed in the form:

$$f_{m,n} = KCV \frac{t}{\lambda_{am}^2},$$

where K, C, and V are given in the figures.

In Figure 5-18, K is plotted against $\lambda_{am}/\lambda_{bn}$. In Figure 5-19, ψ is plotted against $(E/G_c)\,(t/t_c)\,(t_c/\lambda_{am})^2$ for a range of values of $\lambda_{am}/\lambda_{bn}$, C being plotted against ψ in Figure 5-20 for a range of values of t_c/t. In Figure 5-21, V is plotted against $(t_c/t)\,(w_c/w)$.

In deriving the curves, the flexural stiffness of the core has been neglected. The shear modulus of the core has been assumed to be the same with respect to both axes of the panel. For the faceplates $(Eg/w)^{1/2}$ has been taken as 200,000 in/sec which is sufficiently accurate for all common structural metallic materials.

For a square panel with fixed edges the natural frequency in the fundamental mode is about 35 percent higher than indicated by this data sheet. For panels with a/b greater than 5 and fixed edges, the frequency is about 75 percent higher than indicated.

c. Calculation Procedure

1. Specify the mode by choosing values of m and n.

2. Evaluate λ_{am}, λ_{bn} and $\lambda_{am}/\lambda_{bn}$ from the panel dimensions, m and n.

3. Read K for the given value of $\lambda_{am}/\lambda_{bn}$ from Figure 5-18.

4. Evaluate $(E/G_c)\,(t/t_c)\,(t_c/\lambda_{am})^2$ from the properties and dimensions of the panel. Values of G_c are obtainable from honeycomb manufacturers' data or test results.

5. Read ψ from Figure 5-19 for the given values of $(E/G_c)(t/t_c)\,(t_c/\lambda_{am})^2$ and $\lambda_{am}/\lambda_{bn}$.

6. Evaluate t_c/t from the dimensions of the panel.

7. Read C from Figure 5-20 for the given values of ψ and t_c/t.

8 Evaluate $(t_c/t)(w_c/w)$ from the dimensions and properties of the panel. Values of w_c are obtainable from honeycomb manufacturers' data or measurements.

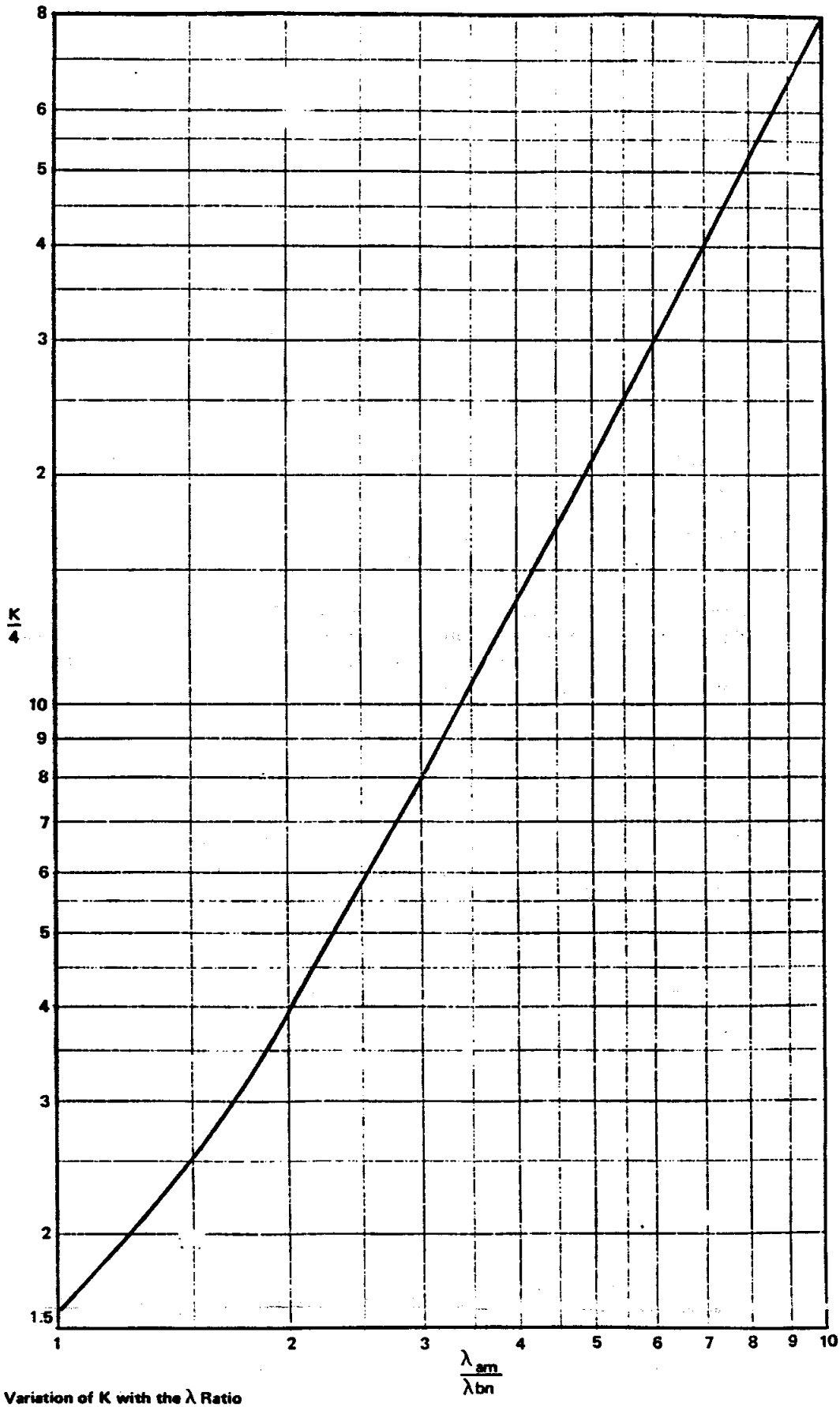


Figure 5-18. Variation of K with the λ Ratio

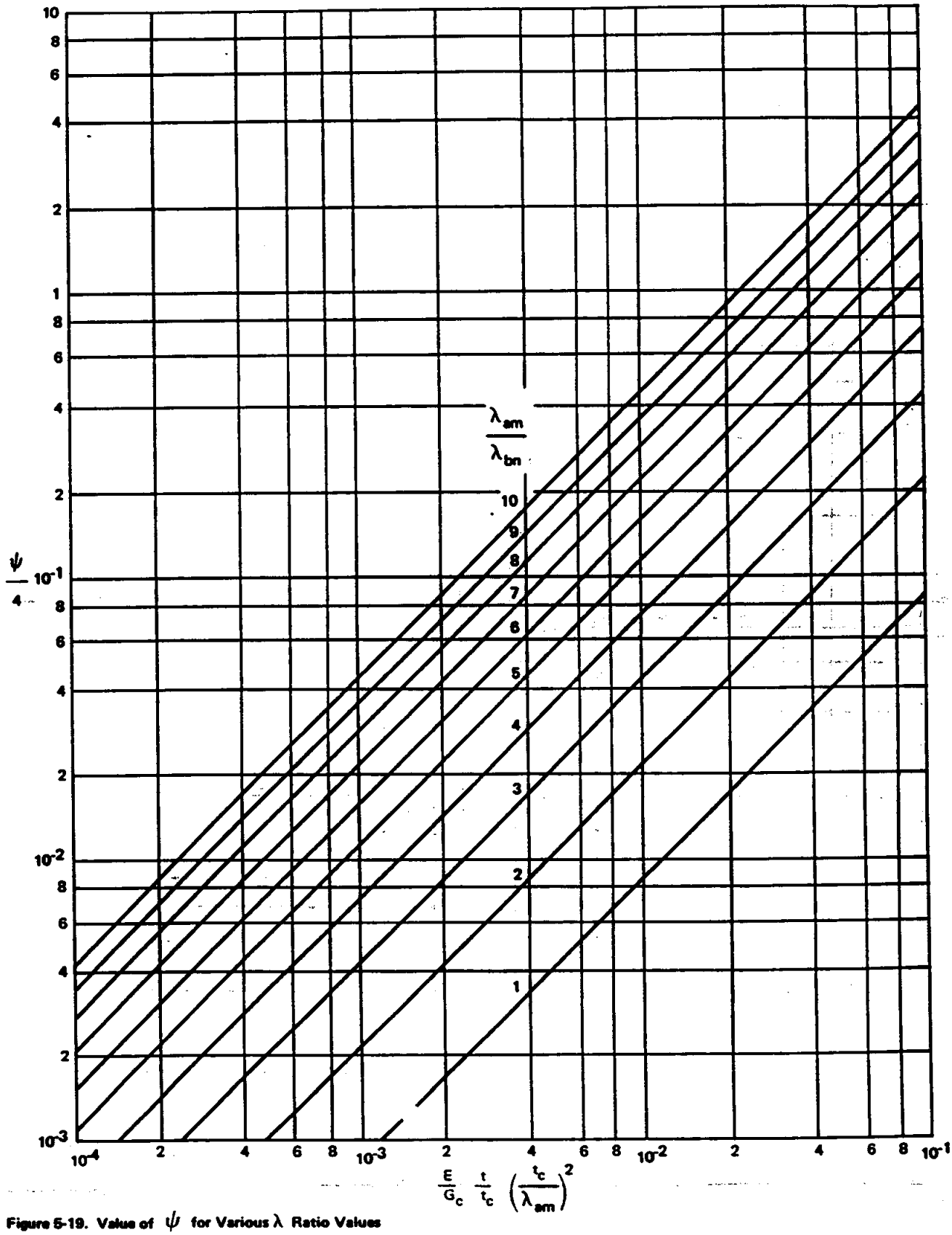


Figure 5-19. Value of ψ for Various λ Ratio Values

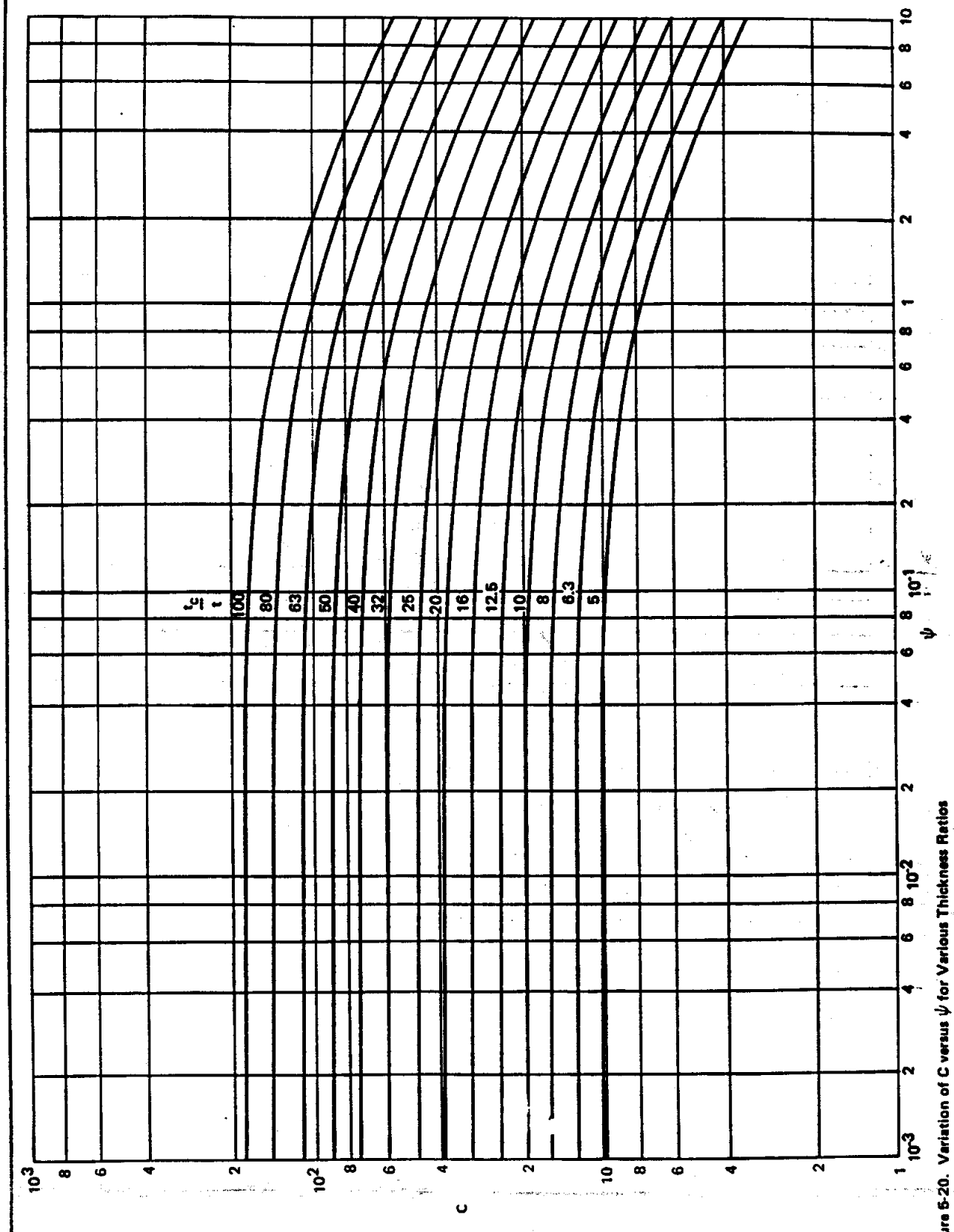


Figure 6-20. Variation of C versus ψ for Various Thickness Ratios

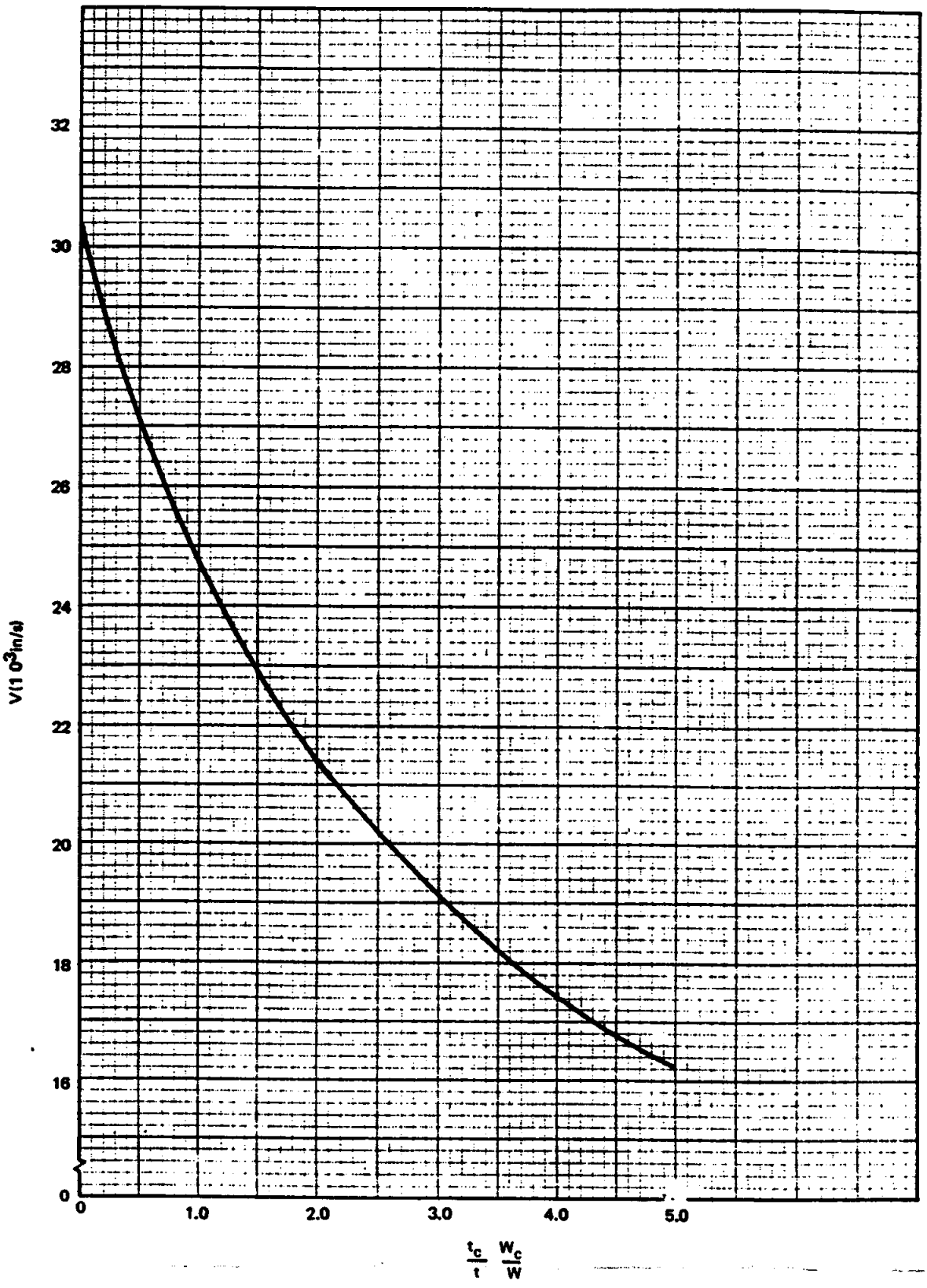


Figure 5-21. Values of V versus Thickness and Weight Ratios

9. Read V from Figure 5-21 for the given value of $(t_c/t)(w_c/w)$.

10. Substitute K, C and V obtained from the figures with t and λ_{am} into the equation in Paragraph b to determine $f_{m,n}$.

d. Derivation

The expression:

$$f_{m,n} = KCV \frac{t}{\lambda_{am}^2},$$

where:

$$K = \pi \left| 1 + (\lambda_{am}/\lambda_{bn})^2 \right|$$

$$C = \left\{ 1 + \frac{3(1 + t_c/t)^2}{1 + \psi} \right\}^{1/2}$$

$$V = \left\{ \frac{Eg}{24w\left(1 - \sigma^2\right)\left(2 + \frac{t_c}{t}\frac{w_c}{w}\right)} \right\}^{1/2}$$

and:

$$\psi = \frac{\pi}{2} \left\{ \frac{1 + (\lambda_{am}/\lambda_{bn})^2}{1 - \sigma^2} \right\} \frac{E}{G_c} \frac{t}{t_c} \left(\frac{t_c}{\lambda_{am}} \right)^2$$

e. Example

It is required to estimate the fundamental natural frequency of a honeycomb sandwich panel with simply-supported edges having the following dimensions and material properties:

a = 20 in.,

b = 10 in.,

t = 0.018 in.,

t_c = 0.5 in.,

E = 10.2×10^6 lbf/in^2,

w = 0.1015 lbf/in^3,

σ = 0.3,

G_c = 47,000 lbf/in^2,

w_c = 0.0033 lbf/in^3.

In the fundamental mode m = n = 1.

Hence:

$$\lambda_{am} = a/m = 20 \text{ in.}$$

and:

$$\lambda_{am}/\lambda_{bn} = a/b = 2.0.$$

From Figure 5-18 for $\lambda_{am}/\lambda_{bn} = 2.0$,

$$K = 3.93.$$

Now:

$$\frac{E}{G_c}\,\frac{t}{t_c}\left(\frac{t_c}{\lambda_{am}}\right)^2 = \frac{10.2 \times 10^6 \times 0.018 \times 0.5}{47{,}000 \times 20^2} = 4.88 \times 10^{-3}$$

so, from Figure 5-19 with $\lambda_{am}/\lambda_{bn} = 2.0$,

$$\psi = 1.05 \times 10^{-2}.$$

Then;

$$t_c/t = 0.5/0.018 = 27.8$$

so interpolating for this value with $\psi = 1.05 \times 10^{-2}$ in Figure 5-20 gives:

$$C = 49.5.$$

Finally;

$$\frac{t_c}{t}\,\frac{w_c}{w} = \frac{27.8 \times 0.0033}{0.1015} = 0.903$$

so, from Figure 5-21;

$$V = 25{,}120 \text{ in/sec.}$$

The fundamental natural frequency from the equation in Paragraph 6 is therefore:

$$f_{1,1} = 3.93 \times 49.5 \times 25{,}120 \times \frac{0.018}{20^2}$$
$$= 220 \text{ Hz}$$

Reference

Mead, D. J., *Honeycomb Sandwich Plates Under Random Pressure Excitation.* ARC 26942, 1965. (Unpublished.)

5-1.4.3.2 Natural Frequencies of Honeycomb Structures Compared to Steel Plates

The ratio of the frequency of resonance for a honeycomb panel to that of a solid-steel panel of the same size has been calculated and charted in Figure 5-22 for several ratios of core density to skin density.

5-1.4.4 Natural Frequency of Miscellaneous Systems

See Figure 5-23.

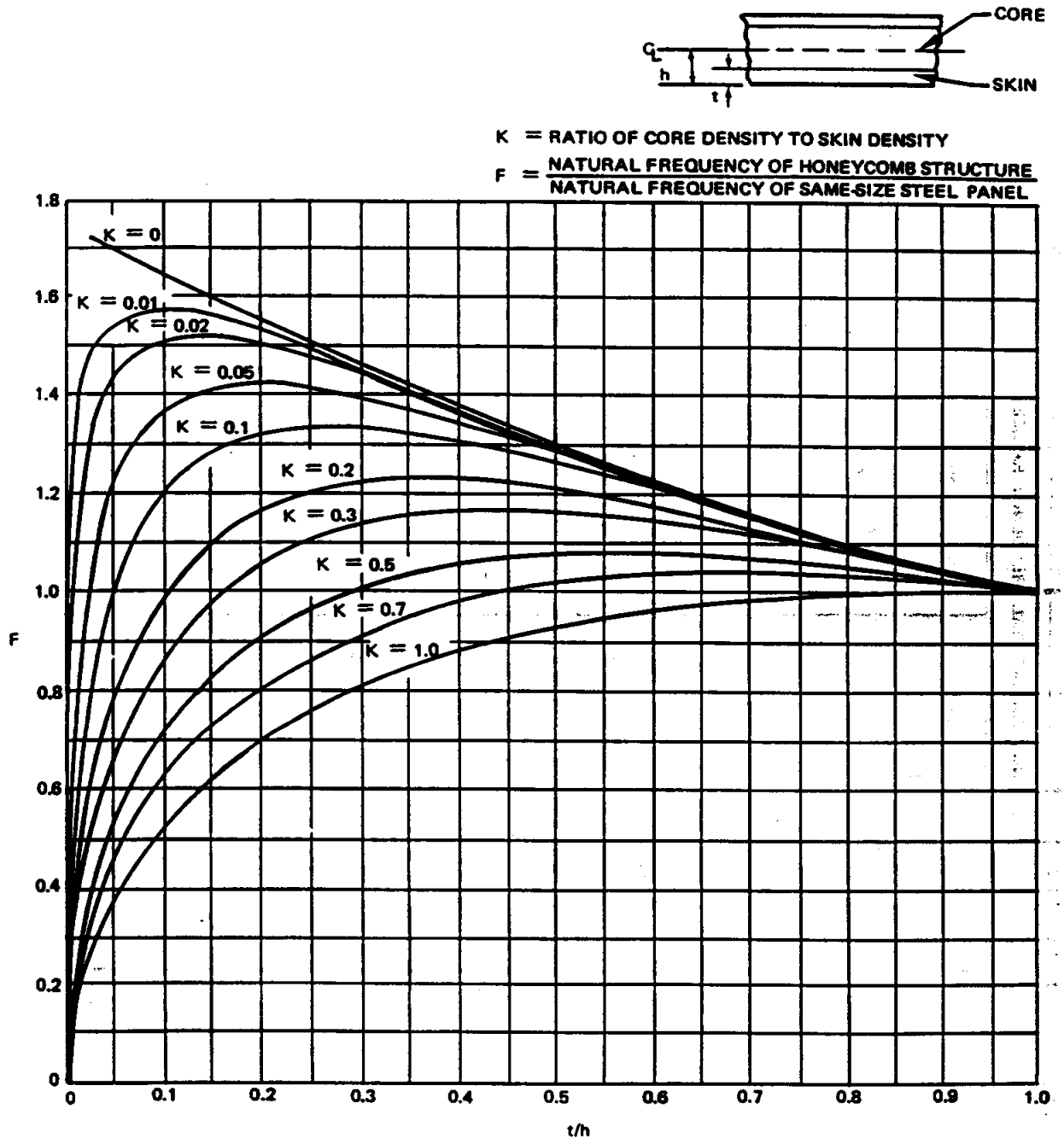
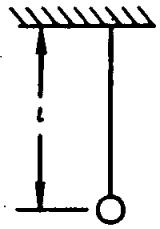


Figure 5-22. Natural Frequencies of Honeycomb Structures Compared to Steel Plates

(ω_n = ANGULAR NATURAL FREQUENCY, RAD/SEC)

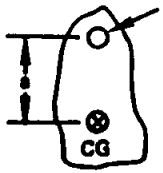
SIMPLE PENDULUM



$$\omega_n = \sqrt{\frac{g}{\ell}}$$

g = ACCELERATION OF GRAVITY
(g AND ℓ IN CONSISTENT UNITS)

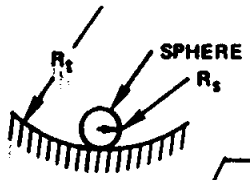
COMPOUND PENDULUM



$$\omega_n = \sqrt{\frac{ag}{\rho_o^2}} = \sqrt{\frac{aW}{I_o}}$$

ρ_o = RADIUS OF GYRATION ABOUT AXIS OF SUPPORT
g = ACCELERATION OF GRAVITY
W = WEIGHT OF PENDULUM
I_o = MOMENT OF INERTIA ABOUT AXIS OF SUPPORT

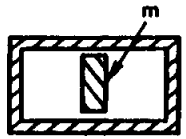
SPHERE IN CYLINDRICAL TRACK



$$\omega_n = \sqrt{\frac{5g}{7(R_t - R_s)}}$$

g = ACCELERATION GRAVITY
R_t = RADIUS OF TRACK
R_s = RADIUS OF SPHERE

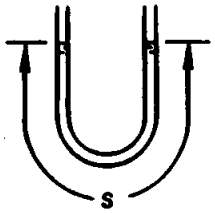
PNEUMATIC SYSTEM



$$\omega_n = \sqrt{\frac{2pS^2}{mV_0}}$$

p = PRESSURE AT EACH END OF CYLINDER, LB/IN.^{2}
S = AREA OF PISTON, IN.^{2}
m = MASS OF PISTON, LB–SEC^2/IN.
V_o = VOLUME OF EACH END OF CYLINDER, IN.^{3}

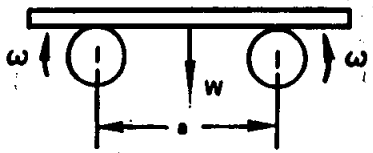
U – TUBE WITH LIQUID



$$\omega_n = \sqrt{\frac{2g}{S}}$$

g = ACCELERATION OF GRAVITY

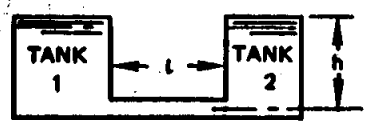
PLANK ON ROTATING DRUMS



$$\omega_n = \sqrt{\frac{2\mu g}{a}}$$

g = ACCELERATION OF GRAVITY
μ = COEFFICIENT OF FRICTION BETWEEN PLANK AND DRUM

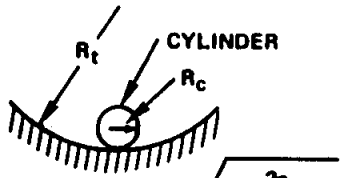
TANKS WITH CONNECTING CONDUIT



$$\omega_n = \sqrt{\frac{g(1 + S_1/S_2)}{h(1 + S_1/S_2) + \ell(S_1/S_0)}}$$

g = ACCELERATION OF GRAVITY
S_1 = AREA OF TANK 1
S_2 = AREA OF TANK 2
S_0 = AREA OF CONDUIT

CYLINDER IN CYLINDRICAL TRACK



$$\omega_n = \sqrt{\frac{2g}{3(R_t - R_c)}}$$

g = ACCELERATION OF GRAVITY
R_t = RADIUS OF TRACK
R_c = RADIUS OF CYLINDER

Figure 5-23. Natural Frequency of Miscellaneous Systems

5-1.4.5 Vibration Frequency Charts

Beams and plates are common structural elements of mechanical design. It is often necessary to design these elements so that they have either a selected natural frequency, or so that their natural frequencies are sufficiently far-removed from excitation frequencies to avoid resonance.

Tables and nomograms presented in this article provide a quick procedure for estimating natural frequencies of uniform and nonuniform beams, and uniform plates. These data enable the designer to assess the effect of changing the dimensions, type of support, and material of the structural element.

Data for the tables were obtained from readily available references. The theory underlying the data given for the natural frequencies of beams and plates assumes small deflections and neglects rotary inertia and shear effects. These restrictions must be kept in mind when determining frequencies from these data.

Method of Solution: The method is based on the use of a frequency constant defined as follows for the different types of structural elements:

Beams	$C = fL^2/k$
Square and rectangular plates	$C = fa^2/h$
Circular plates	$C = fr^2/h$

Figures 5-24 to 5-33 include definition of symbols and give values of the frequency constant C, or corresponding frequency function, for various mechanical structures and several modes of vibration. These tabulated values of C are based on the characteristic density and Young's modulus for steel.

The nomogram in Figure 5-34 may be used with the proper frequency Constant C and the characteristic dimensions to determine natural frequency directly. Nomograms in Figures 5-35 and 5-36 present an alternate method for determining the natural frequency by first determining the value of L^2/k, a^2/h or r^2/h from the nomogram in Figure 5-35 and then entering the nomogram in Figure 5-36 with this item and the frequency constant. Figures 5-34, 5-35, and 5-36 are to be used with Figures 5-24 to 5-31 in which the values of the frequency constants are tabulated.

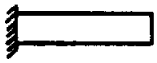
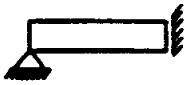
For materials other than steel, the material correction factor is obtained from the table in Figure 5-12. With this factor, and the natural frequency, f_s, of a steel member of the same dimensions, the nomogram in Figure 5-12 may then be used to determine the natural frequency.

Some of the less common mechanical structural members, such as membranes, have a frequency relation which is not defined by the foregoing nomograms. In such

C = FREQUENCY CONSTANT $k = (I/A)^{1/2}$ = RADIUS OF GYRATION, IN.

f_s = NATURAL FREQUENCY, Hz L = BEAM LENGTH in.

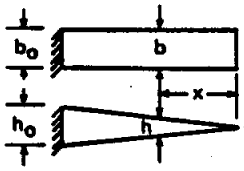
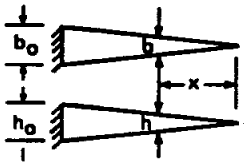
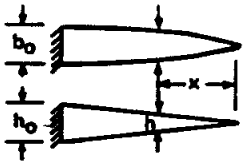
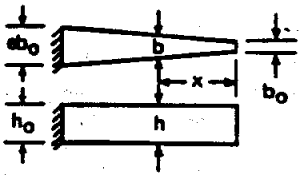
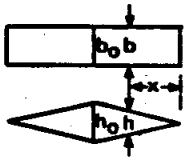
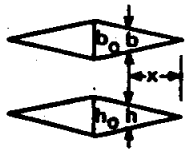
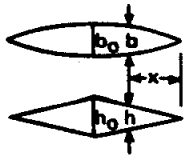
M = VIBRATION MODE NUMBER

BEAM STRUCTURE	$C/10^4 = (f_s L^2/k)/10^4$				
	M = 1	M = 2	M = 3	M = 4	M = 5
 CLAMPED-CLAMPED FREE-FREE	71.95	198.29	388.73	642.60	959.96
 CLAMPED-FREE	11.30	70.85	198.30	388.73	642.60
 CLAMPED-HINGED FREE-HINGED	49.57	160.65	335.17	573.20	874.65
 CLAMPED-GUIDED FREE-GUIDED	17.96	97.18	239.96	446.25	715.98
 HINGED-HINGED GUIDED-GUIDED	31.73	126.93	285.60	507.73	793.33
 HINGED-GUIDED	7.93	71.40	198.33	388.73	642.60

C = FREQUENCY CONSTANT $k = (I/A)^{1/2}$ = RADIUS OF GYRATION, IN.

f_s = NATURAL FREQUENCY, Hz L = BEAM LENGTH, IN.

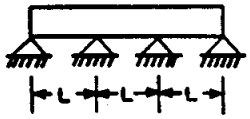
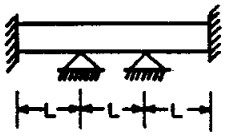
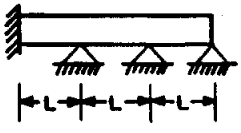
M = VIBRATION MODE NUMBER

BEAM STRUCTURE	b/b_o	h/h_o	— $C/10^4 = (f_s L^2/k) / 10^4$ — M = 1	M = 2	M = 3
	1	x/L	17.09	48.89	96.57
	x/L	x/L	26.08	68.08	123.64
	$(x/L)^{1/2}$	x/L	22.30	58.18	102.90
	$e^{x/L}$	1	15.23	77.78	206.07
	1	x/L	21.21* 35.05†	65.97	
	x/L	x/L	32.73* 49.50†	76.57	
	$(x/L)^{1/2}$	x/L	25.66 42.02†	66.06	

*Symmetric †Antisymmetric

VIBRATION, SHOCK, AND ACOUSTICS

C = FREQUENCY CONSTANT
f_s = NATURAL FREQUENCY, Hz
$k = (I/A)^{1/2}$ = RADIUS OF GYRATION, IN.
L = SPAN LENGTH, IN.
M = VIBRATION MODE NUMBER
n = NUMBER OF SPANS

BEAM STRUCTURE	n	$C/10^4 = (f_s L^2/k)/10^4$ M = 1	M = 2	M = 3	M = 4	M = 5
EXTREME ENDS SIMPLY SUPPORTED						
	1	31.73	126.94	285.61	507.76	793.37
	2	31.73	49.59	126.94	160.66	285.61
	3	31.73	40.52	59.56	126.94	143.98
	4	31.73	37.02	49.59	63.99	126.94
	5	31.73	34.99	44.19	55.29	66.72
	6	31.73	34.32	40.52	49.59	59.56
	7	31.73	33.67	38.40	45.70	53.63
	8	31.73	33.02	37.02	42.70	49.59
	9	31.73	33.02	35.66	40.52	46.46
	10	31.73	33.02	34.99	39.10	44.19
	11	31.73	32.37	34.32	37.70	41.97
	12	31.73	32.37	34.32	37.02	40.52
EXTREME ENDS CLAMPED						
	1	72.36	198.34	388.75	642.63	959.98
	2	49.59	72.36	160.66	198.34	335.20
	3	40.52	59.56	72.36	143.98	178.25
	4	37.02	49.59	63.99	72.36	137.30
	5	34.99	44.19	55.29	66.72	72.36
	6	34.32	40.52	49.59	59.56	67.65
	7	33.67	38.40	45.70	53.63	62.20
	8	33.02	37.02	42.70	49.59	56.98
	9	33.02	35.66	40.52	46.46	52.81
	10	33.02	34.99	39.10	44.19	49.59
	11	32.37	34.32	37.70	41.97	47.23
	12	32.37	34.32	37.02	40.52	44.94
EXTREME ENDS CLAMPED-SUPPORTED						
	1	49.59	160.66	335.20	573.21	874.69
	2	37.02	63.99	137.30	186.85	301.05
	3	34.32	49.59	67.65	132.07	160.66
	4	33.02	42.70	56.98	69.51	129.49
	5	33.02	39.10	49.59	61.31	70.45
	6	32.37	37.02	44.94	54.46	63.99
	7	32.37	35.66	41.97	49.59	57.84
	8	32.37	34.99	39.81	45.70	53.63
	9	31.73	34.32	38.40	43.44	49.59
	10	31.73	33.67	37.02	41.24	46.46
	11	31.73	33.67	36.33	39.81	44.19
	12	31.73	33.02	35.66	39.10	42.70

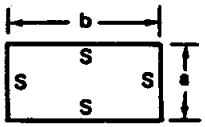
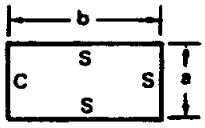
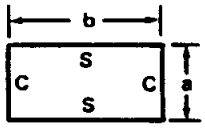
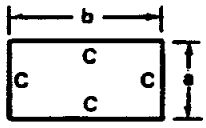
a = PLATE SIDE, IN
C = FREQUENCY CONSTANT
f_s = NATURAL FREQUENCY, Hz
h = PLATE THICKNESS, IN.
M = VIBRATION MODE NUMBER

PLATE STRUCTURE*	$C/10^4 = (f_s a^2/h)/10^4$					
	M = 1	M = 2	M = 3	M = 4	M = 5	M = 6
F / F □ F / C	2.40	8.32	20.86	26.71	30.32	
F / C □ F / C	6.77	23.43	26.07	46.75	61.44	
F / F □ F / F	13.72	19.99	23.25	34.98	59.93	60.47
S / S □ S / S	19.20	48.00	76.82	96.01	124.82	163.20
S / S □ S / C	23.01	50.28	57.06	83.79	97.58	110.13
S / C □ C / S	28.16	53.26	67.44	92.02	99.43	125.60
C / C □ C / C	35.01	71.42	105.36	128.03	128.71	160.72

*F = FREE, S = SUPPORTED, C = CLAMPED

Figure 5-27. Square Steel Plates

(FIRST VIBRATION MODE)

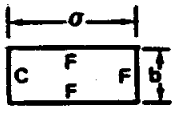
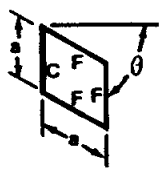
PLATE STRUCTURE*	b/a	$C/10^4$	a/b	$C/10^4$
	1.0	19.20		
	1.5	13.87		
	2.0	12.00		
	2.5	11.14		
	3.0	10.67		
	∞	9.60		
	1.0	23.01	1.0	23.01
	1.5	18.39	1.5	15.15
	2.0	16.86	2.0	12.57
	2.5	16.18	2.5	11.43
	3.0	15.82	3.0	10.84
	∞	15.01	∞	9.60
	1.0	28.16	1.0	28.16
	1.5	24.37	1.5	16.90
	2.0	23.17	2.0	13.32
	2.5	22.64	2.5	11.80
	3.0	22.37	3.0	11.05
	∞	21.76	∞	9.60
	1.0	35.00		
	1.5	26.27		
	2.0	23.90		
	2.5	23.12		
	3.0	22.56		
	∞	21.76		

*F = FREE, S = SUPPORTED, C = CLAMPED

$C/10^4 = (f_s a^2/h)/10^4$

a = PLATE SIDE, IN.
C = FREQUENCY CONSTANT
f_s = NATURAL FREQUENCY, Hz
h = PLATE THICKNESS, IN.

Figure 5-28. Rectangular Steel Plates

PLATE STRUCTURE*	$C/10^4 = (f_s a^2/h)/10^4$				
RECTANGULAR CANTILEVER PLATE					
	a/b	M = 1	M = 2	M = 3	M = 4	M = 5
	1/2	3.41	5.23	21.36	9.98	24.18
	1	3.40	8.32	20.86	28.71	30.32
	2	3.38	14.52	21.02	91.92	47.39
	5	3.36	33.79	20.94	548.60	103.03
SKEWED CANTILEVER PLATE						
	θ	M = 1	M = 2			
	15	3.50	8.63			
	30	3.85	9.91			
	45	4.69	13.38			

*F = FREE, C = CLAMPED

a = PLATE SIDE, IN.
c = FREQUENCY CONSTANT
f_s = NATURAL FREQUENCY, Hz
h = PLATE THICKNESS, IN.
M = VIBRATION MODE NUMBER
θ = SKEW ANGLE, DEG

Figure 5-29. Cantilever Steel Plates

C = FREQUENCY CONSTANT
f_s = NATURAL FREQUENCY, Hz
h = PLATE THICKNESS, IN.
r = PLATE RADIUS, IN.
m = NUMBER OF NODAL CIRCLES
n = NUMBER OF NODAL DIAMETERS

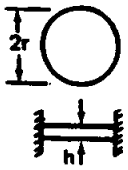
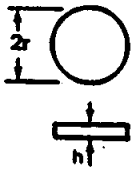
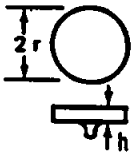
PLATE STRUCTURE*	$C/10^4 = (f_s r^2/h)/10^4$			
CIRCULAR PLATE CLAMPED AT BOUNDARY				
	n	m = 0	m = 1	m = 2	m = 3
	0	9.936	38.713	86.516	
	1	20.651			
	2	33.906			
CIRCULAR PLATE WITH FREE BOUNDARY					
	0		8.832	37.487	
	1		19.970	58.255	
	2	5.110	34.295		
	3	11.902	51.491		
CIRCULAR PLATE CLAMPED AT CENTER					
	3.649	20.349	59.053	116.490	

Figure 5-30. Circular Steel Plates

C = FREQUENCY CONSTANT
f_s = NATURAL FREQUENCY, Hz
h = RING THICKNESS, IN.
n = NUMBER OF FULL WAVES AROUND PERIPHERY
r = MEAN RING RADIUS, IN.

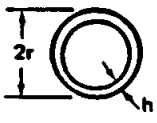
RING STRUCTURE	$C/10^4 = (f_s r^2/h)/10^4$				
	n = 2	n = 3	n = 4	n = 5	n = 6
	2.51	7.10	13.6	22.2	32.2

Figure 5-31. Steel Ring Vibrating in its own Plane

C_f = FREQUENCY FUNCTION
f_s = NATURAL FREQUENCY, Hz
h = MEMBRANE THICKNESS, IN.
m = NUMBER OF NODAL CIRCLES
n = NUMBER OF NODAL DIAMETERS
s = TENSION OF PERIPHERY, LB/IN.

MEMBRANE STRUCTURE $C_f = f_s r/(s/h)^{1/2}$

m	n = 0	n = 1	n = 2	n = 3	n = 4	n = 5
1	14.09	22.49	30.12	37.46	44.56	51.55
2	32.41	41.22	49.14	57.30	64.94	72.22
3	50.79	59.71	61.94	76.45	84.55	92.18
4	69.28	78.09	86.90	95.12	103.31	111.56
5	87.48	96.88	105.68	113.91	122.13	130.35
6	106.27	115.08	123.80	132.69	140.91	119.13
7	124.47	133.87	142.68	160.90	152.70	167.92
8	143.26	152.07	160.88	169.68	178.49	186.71

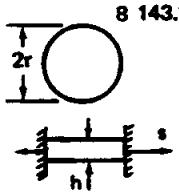


Figure 5-32. Circular Steel Membranes

C_f = FREQUENCY FUNCTION
f_s = NATURAL FREQUENCY, Hz
L = BEAM LENGTH, IN.
n = NUMBER OF HALFWAVES ALONG BEAM LENGTH

BEAM STRUCTURE $C_f/10^4 = f_s L/10^4$

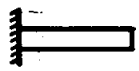
	n = 0	n = 1	n = 2	n = 3	n = 4	n = 5
 CLAMPED-FREE	5.05	15.15	25.25	35.35	45.46	55.56
 CLAMPED-CLAMPED		10.10	20.20	30.30	40.41	50.51

Figure 5-33. Longitudinal Vibration of Steel Beams

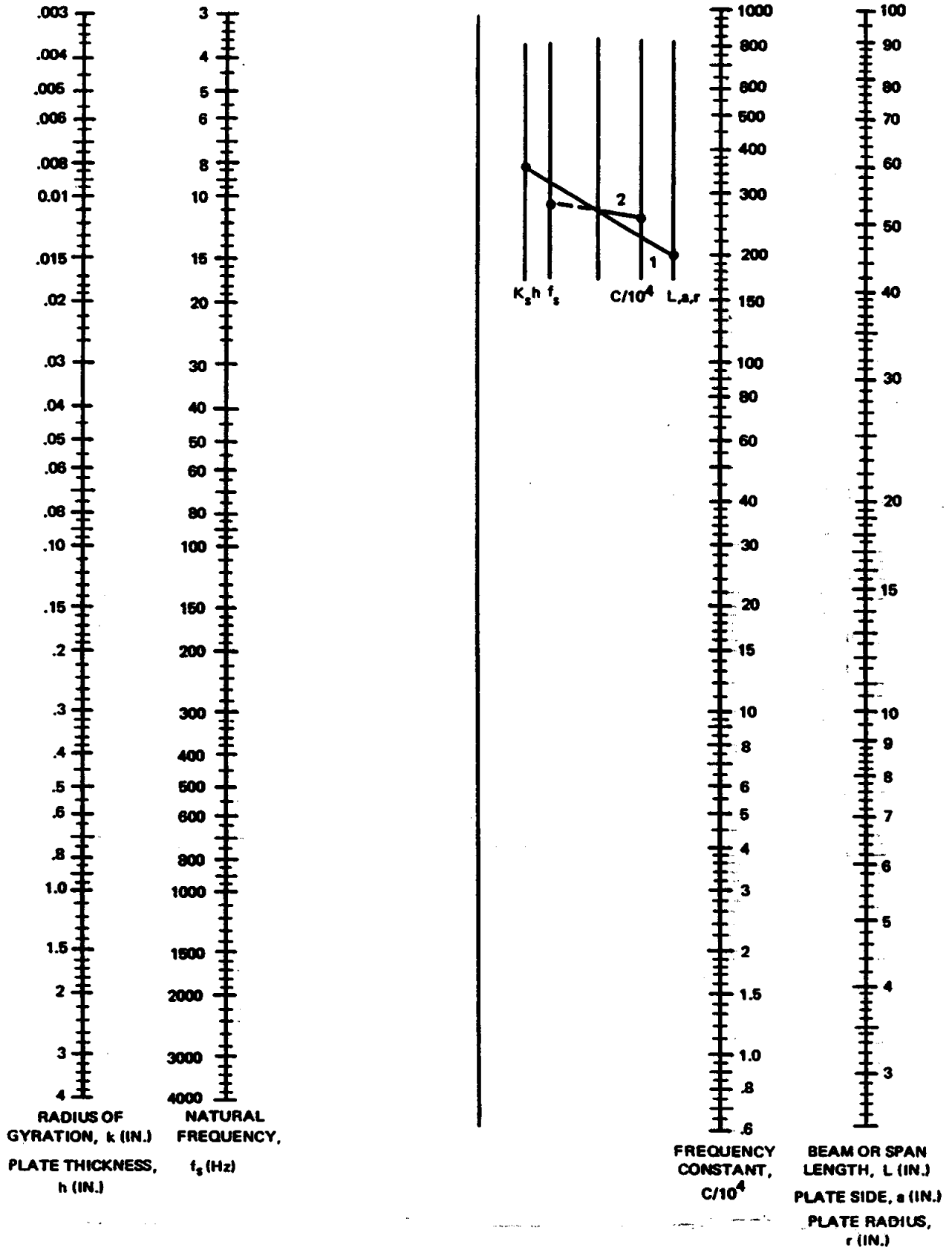


Figure 5-34. Nomogram for Determination of Natural Frequency f_2 from Frequency Constant C_1 in Figures 5-24 to 5-31.

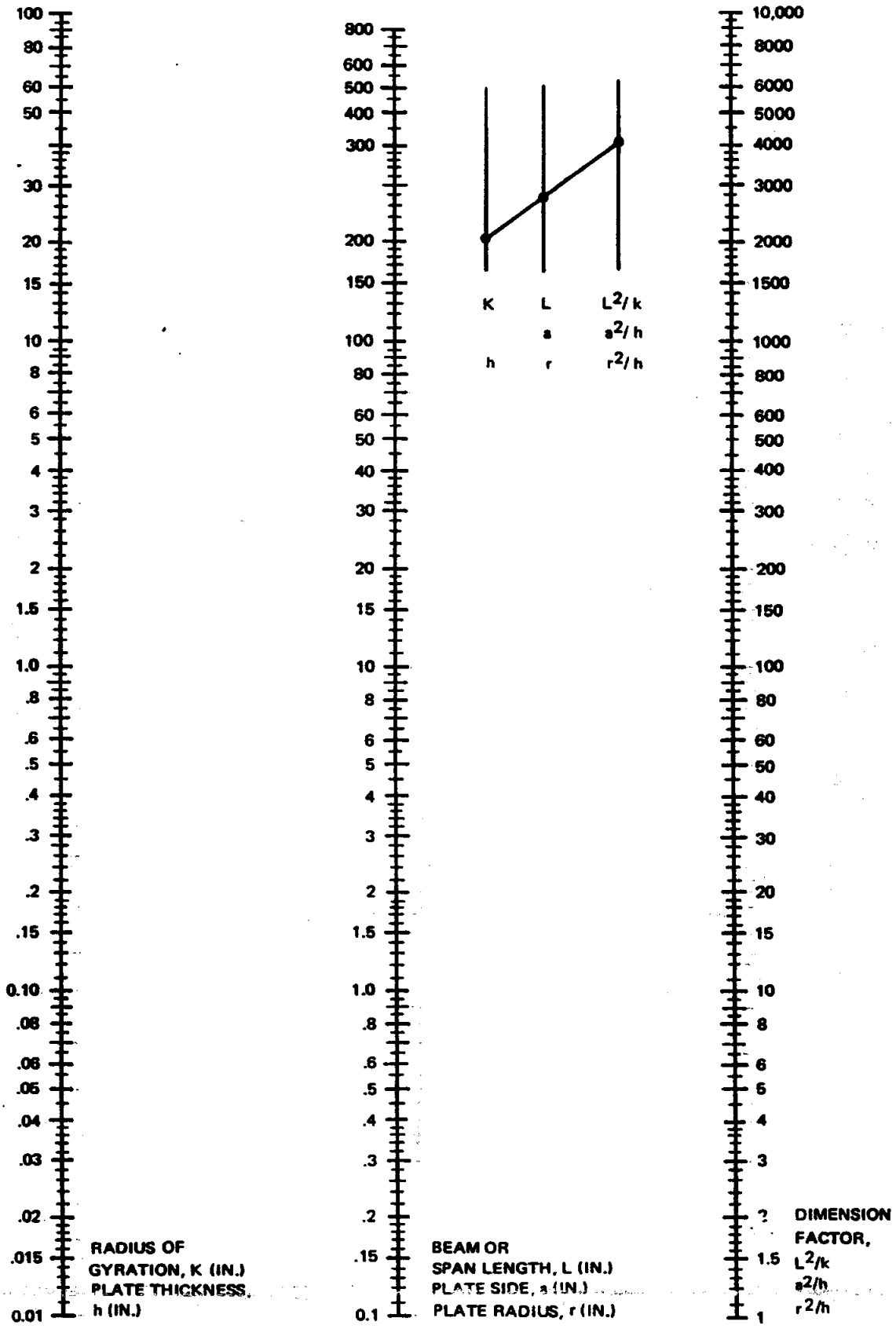


Figure 5-35. Nomogram for Determination of Dimension Factors, L^2/k_1, a^2/h and r^2/h in Frequency Constant Equations of Figures 5-24 to 31.

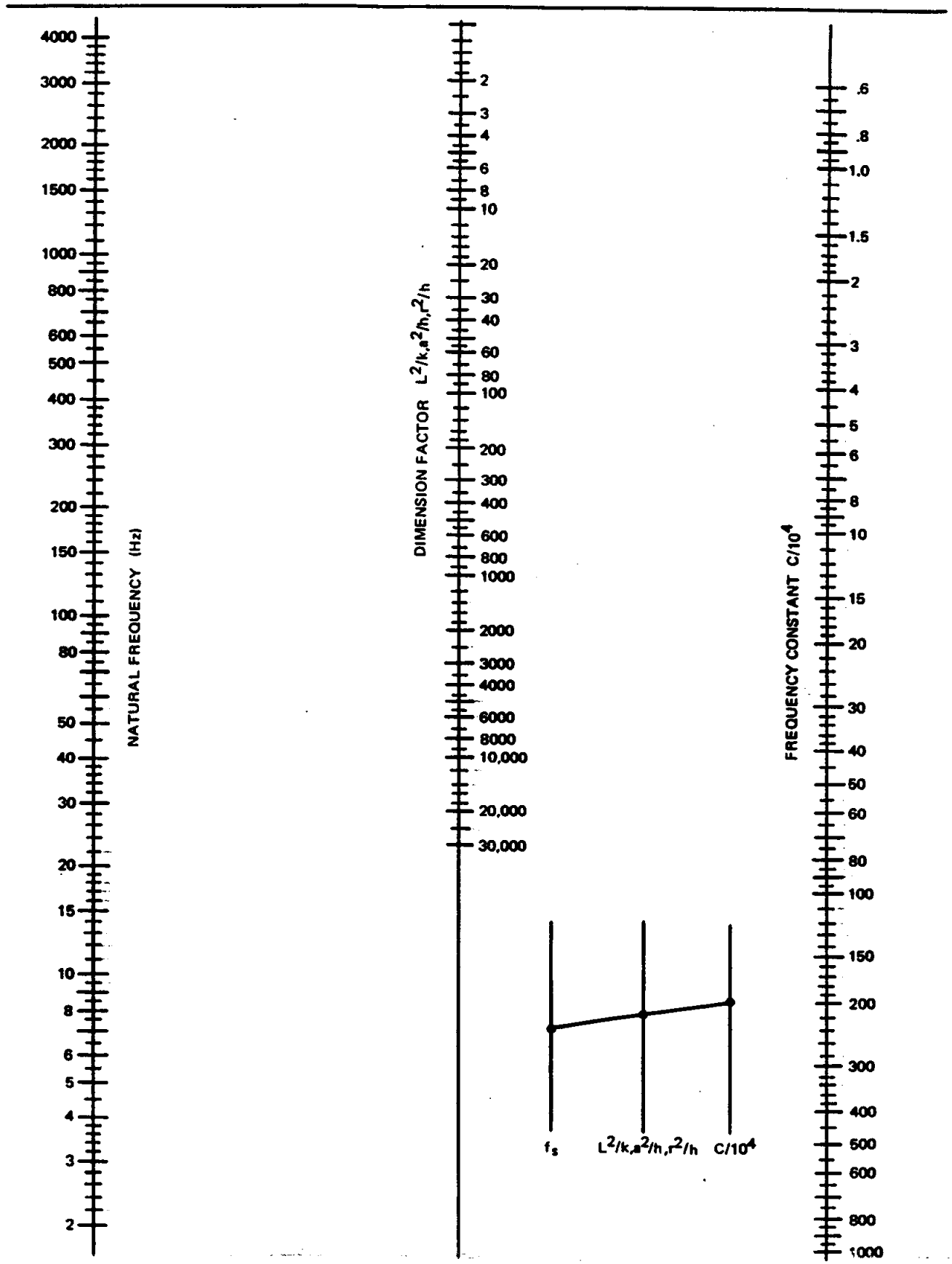


Figure 5-36. Nomogram for Alternate Solution of Natural Frequency f, from Frequency Constant C and Dimension Factor Determined from Figure 5-35.

r = 3 in.; plate thickness, h = 0.090 in.; and estimated temperature of operation is 400 F.

From Figure 5-30, $C/10^4 = 3.649$. From Figure 5-34 then, the natural frequency of a steel plate of the same dimensions is $f_s = 370$ Hz.

The result is also arrived at by the alternate procedure, Figures 5-35 and 5-36. From Figure 5-35, for r = 3 and h = 0.09, $r^2/h = 100$, and from Figure 5-36, for a steel plate of the same dimensions, $f_s = 370$ Hz.

Since plate material is titanium, final solution will be given by Figure 5-12. From the table in Figure 5-12, material correction factor for Tl-75A titanium at 400 F is $K_m = 0.910$. From the nomogram then, for this value of K_m and $f_s = 370$ Hz, frequency of the titanium plate is f = 325-350 Hz.

References:

Machine Design, v. 29, n. 3 (Feb. 7, 1957), pp. 109-115.

Trans. ASME, v. 79, n. 7 (Oct. 1957), pp. 1459-1475.

5-1.5 Shells

5-1.5.1 Frequency Calculations

A general equation for the frequencies of resonance for a simply-supported, uniform, homogeneous, cylindrical shell (Reference on page 5-73) is:

$$f_{mn} = \frac{1}{2\pi a}\sqrt{\frac{E}{\rho}}\sqrt{\frac{\lambda^4}{\left(n^2+\lambda^2\right)^2}+\frac{(h/a)^2}{12\,(1-\sigma^2)}\left(n^2+\lambda^2\right)^2+\left(\frac{P}{E}\right)\left(\frac{a}{h}\right)\left(n^2+\frac{\lambda^2}{2}\right)}$$

where:

f_{mn} = frequency of resonance for the mn mode (in Hz),

a = radius (in.),

E = modulus of elasticity (lbs/in^2),

ρ = material density ($lbs - sec^2/in^4$)

$\lambda = \frac{m\pi a}{\ell}$,

ℓ = length of cylinder (in.),

m = number of axial half waves,

n = number of circumferential waves,

σ = Poisson's ratio,

p = internal pressure (lbs/in^2),

h = wall thickness (in.),

The above equation assumes $\ell > 2a$.

On subsequent pages in this section are charts plotting frequencies for various radius to length ratios (Figure 5-38), a graph showing the effects of internal pressure, and frequency calculations for boundary conditions other than simply supported. Following that is a figure giving frequencies of flexure for rings.

5-1.5.2 Frequency Charts for Cylindrical Shells

The problem of determining the natural frequency of cylindrical shells has been studied by many authors. A brief historical review may be found in Ref. 1. So far, the frequency curves obtained by the former investigators do not cover the ranges of wave number and shell radius-thickness ratio broad enough for practical use.

In this brief note, a set of frequency curves is presented. They cover the radius-thickness ratio from 100 to 1500, the circumferential wave number from 0 to 38, and axial wave number ma π/ℓ from 0.3 to 30.

The equation of motion used was obtained in the manner of Morley[2]:

$$\nabla^4 \left(\nabla^2 + 1\right)^2 w + 4K^4 \frac{\partial^4 w}{\partial x^4} + \frac{a^4}{D} \rho h \nabla^4 \frac{\partial^2 w}{\partial t^2} = 0 \qquad (1)$$

Here w is the quantity of the radial displacement w′ divided by the shell radius a. x′ and θ are the cylindrical coordinate system and $x = x'/a$:

$$\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial \theta^2)$$

$$D = Eh^3/12\,(1 - \nu^2)$$

$$K^4 = 3\,(1 - \nu^2)\,(a/h)^2$$

E and ν are, respectively, Young's modulus and Poisson's ratio, and ρ and h are, respectively, the mass density of the material and the thickness of the shell.

For a simply supported cylindrical shell in free vibration, the radial motion is taken as

$$w = e^{i\omega t} \cdot C_{sn} \cos sx \cdot \cos n\theta \qquad (2)$$

where $s = ma/l$. The cylinder length is ℓ, and m is the number of half-waves in the x direction. Combining Eqs. (1) and (2) yields the following frequency equation:

$$\left\{\left[12\,(1-\nu^2)\cdot\left(\frac{\rho}{E}\right)\right]\cdot a^2\right\}^{1/2}\cdot\omega_{sn} = \frac{h/a}{s^2+n^2}\left[(s^2+n^2)^2\,(s^2+n^2-1)^2 + 4K^4 s^4\right]^{1/2} \qquad (3)$$

Charts based on Eq. (3) are prepared with:

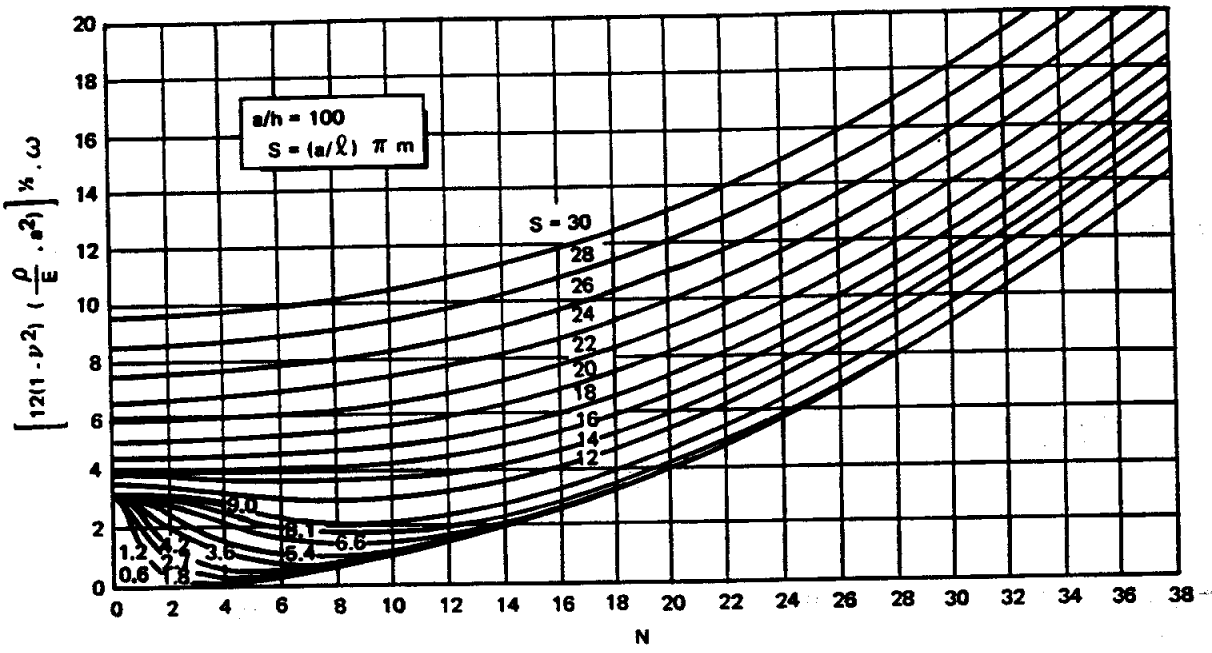
$$\left[12\,(1-\nu^2)\;\frac{\rho}{E}\;a^2\right]^{1/2}\omega_{sn}$$

as the frequency parameter; shown in Figure 5-38. It is believed that the range of the curves thus covered is broad enough for engineering uses.

References

1 Weingarten, V. I., *Free Vibration of Thin Cylindrical Shells*, AIAA J. 2, 717-722 (1964).

2. Morley, L. S. D., *An Improvement on Donnell's Approximation for Thin-Walled Circular Cylinders*, Quart. J. Mech. Appl. Math. XII, 93 (1959).



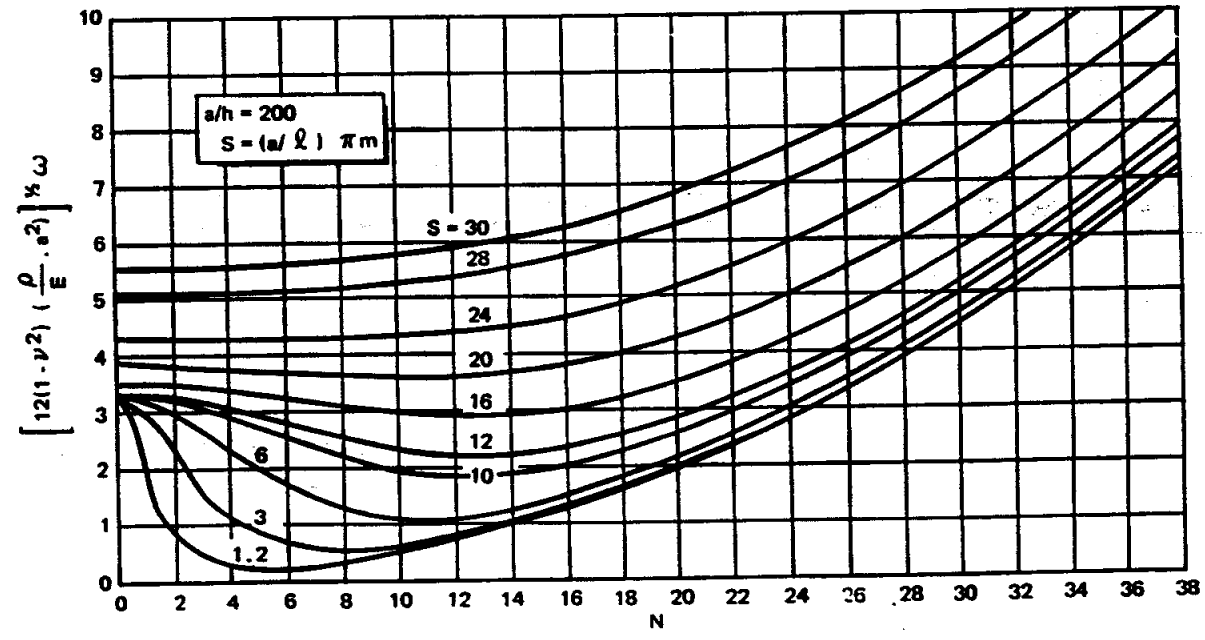
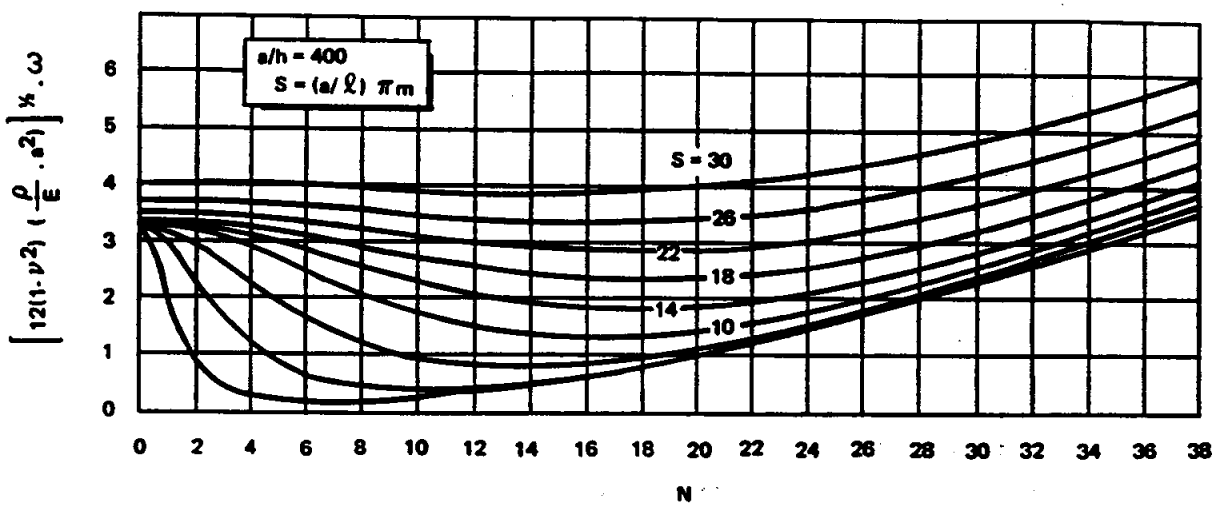


Figure 5-38. Natural Frequency of Cylindrical Shells (Sheet 1 of 4)



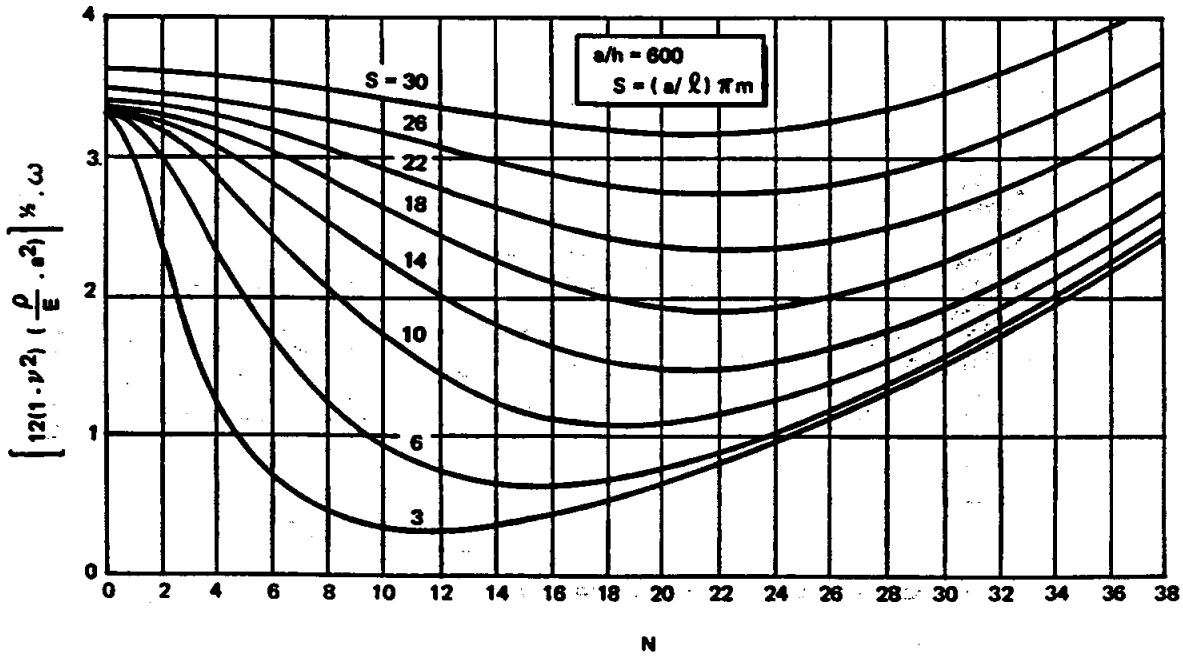
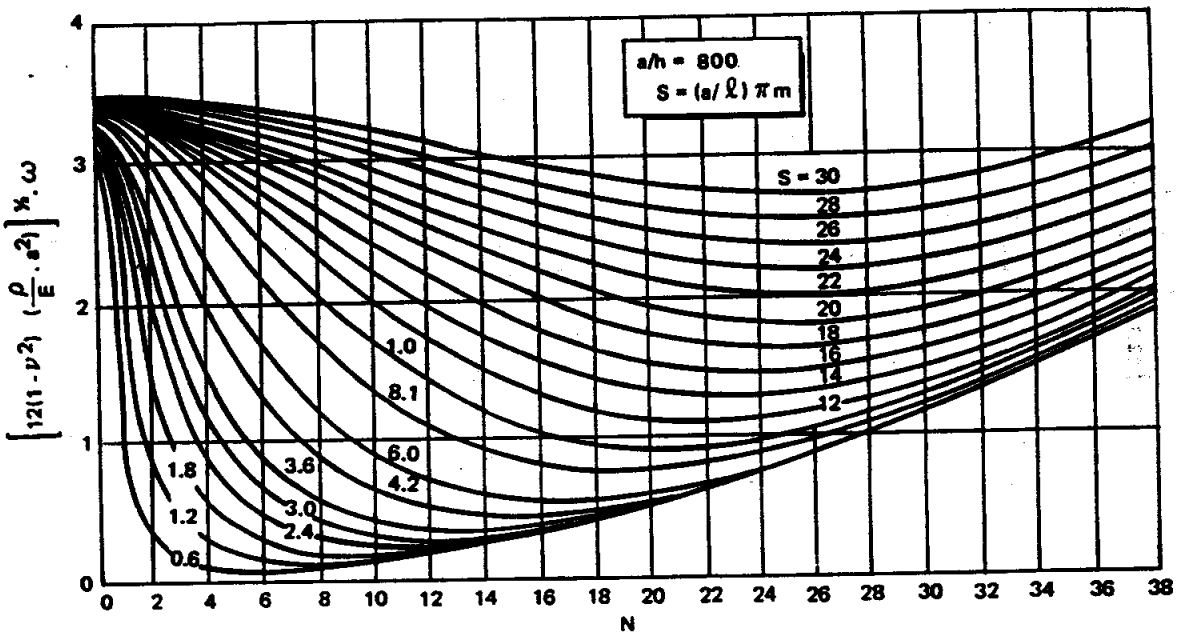


Figure 5-38. Natural Frequency of Cylindrical Shells (Sheet 2 of 4)



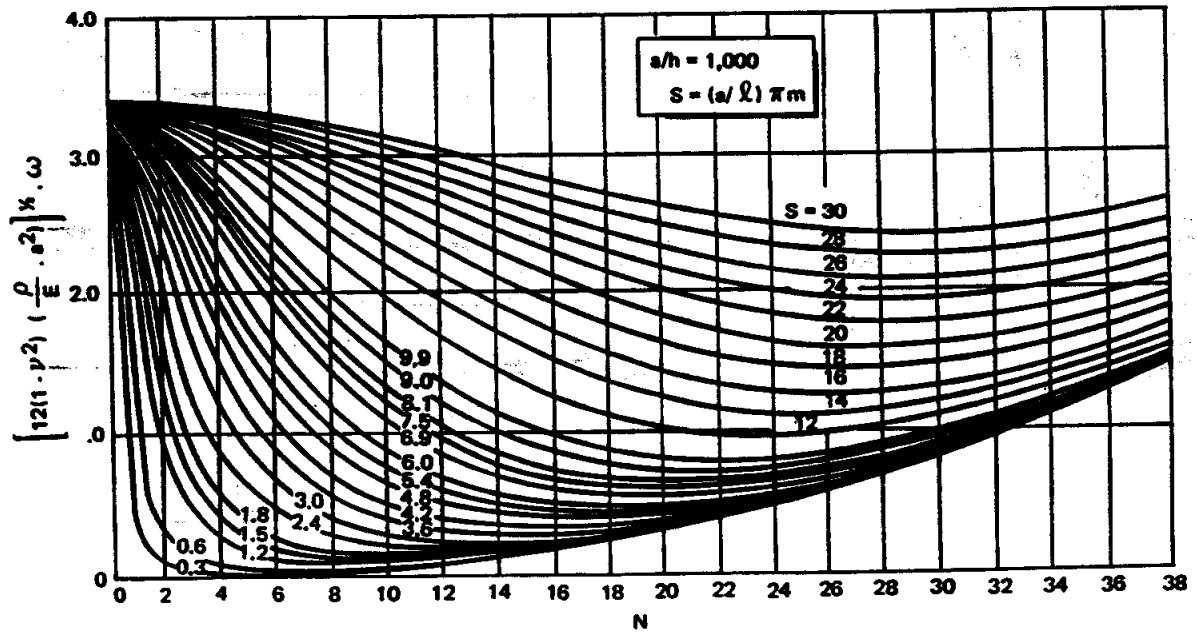


Figure 5-38. Natural Frequency of Cylindrical Shells (Sheet 3 of 4)

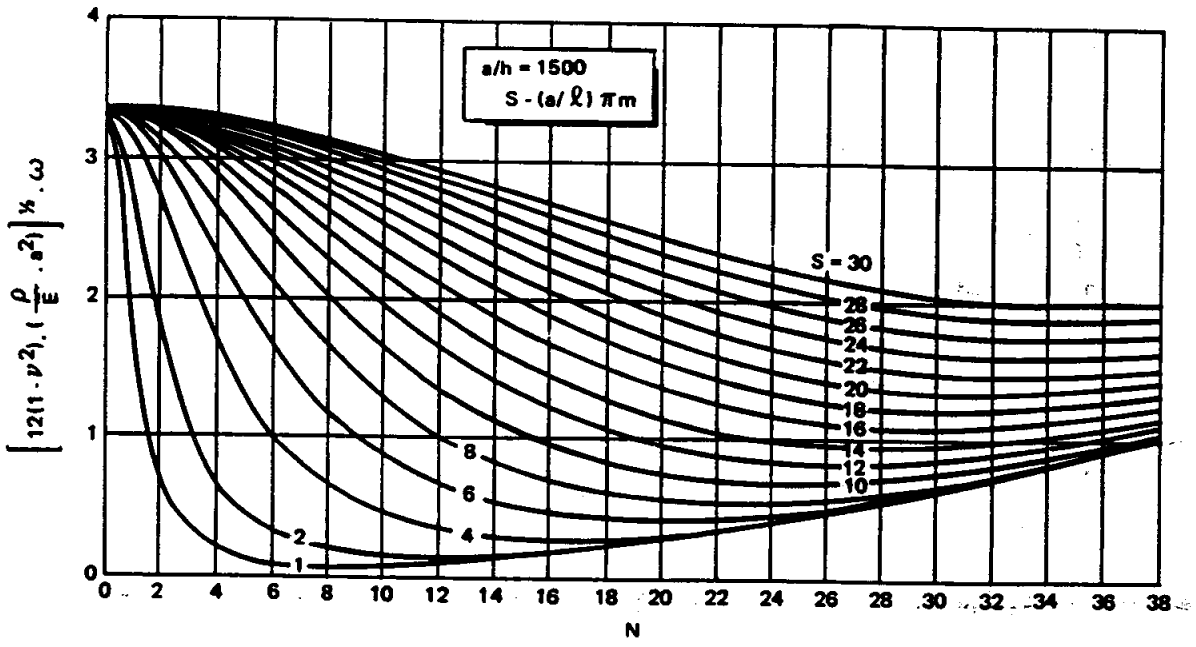


Figure 5-38. Natural Frequency of Cylindrical Shells (Sheet 4 of 4)

Modal Density of Cylinders

Reference

Ungar E. E., *Fundamentals of Statistical Energy Analysis of Vibrating Systems*, AFFDL-TR-66-52 May 1966.

$$n_{sc} = \frac{\Delta N}{\Delta f} = \frac{A_s}{2\kappa_p c_\ell} = n_{sp} \qquad \frac{f}{f_r} > 1$$

$$n_{sc} = \frac{\Delta N}{\Delta f} = \frac{A_s}{2\kappa_p c_\ell} \left(\frac{f}{f_r}\right)^{2/3} \qquad \frac{f}{f_r} < 1$$

where

n_{sc} = modal density of cylinder
n_{sp} = modal density of flat plate
ΔN = number of modes in bandwidth
A_s = surface area (in^2)
κ_p = $\sqrt{h^2/12}$ (in)
c_ℓ = $\sqrt{E/\rho\ (1 - \sigma^2)}$ (in/sec)
ρ = mass density ($lbs - sec^2/in^4$)
σ = Poisson's ratio of shell material
h = skin thickness (in)
E = Young's Modulus (lbs/in^2)
f_r = ring frequency = $c_\ell/2\pi R$ (Hz)
R = Radius of cylinder (in)

5-1.5.3 Frequency Charts for Cylindrical Shells with Internal Pressure

Theoretical results for a "freely supported" cylinder, from the equation under Paragraph 5-1.5.1, are plotted in Figure 5-39. The solid curves refer to m = 1, the dotted curves to m = 2, and the chain dots to m = 3. Frequencies for m > 3 are not plotted here. The bending modes, n = 1, are also omitted from these plots, since in bending modes the masses attached to the ends of the cylinder contribute significantly to the frequencies; whereas, for n > 2, the end masses, if rigidly connected, are of small influence.

The actual end conditions of the test specimens were somewhat between freely supported and clamped. Reference 2 shows that, for an unpressurized cylinder, $\rho = 0$, the frequencies for the clamped ends are higher than those for the freely supported ends, and these frequencies can be estimated approximately by replacing m by (m + c) in the formula for the freely supported ends. The constant, c, lies between 0.2 to 0.4. The theoretical effect of the end conditions on the frequency in the pressurized case has not yet been determined.

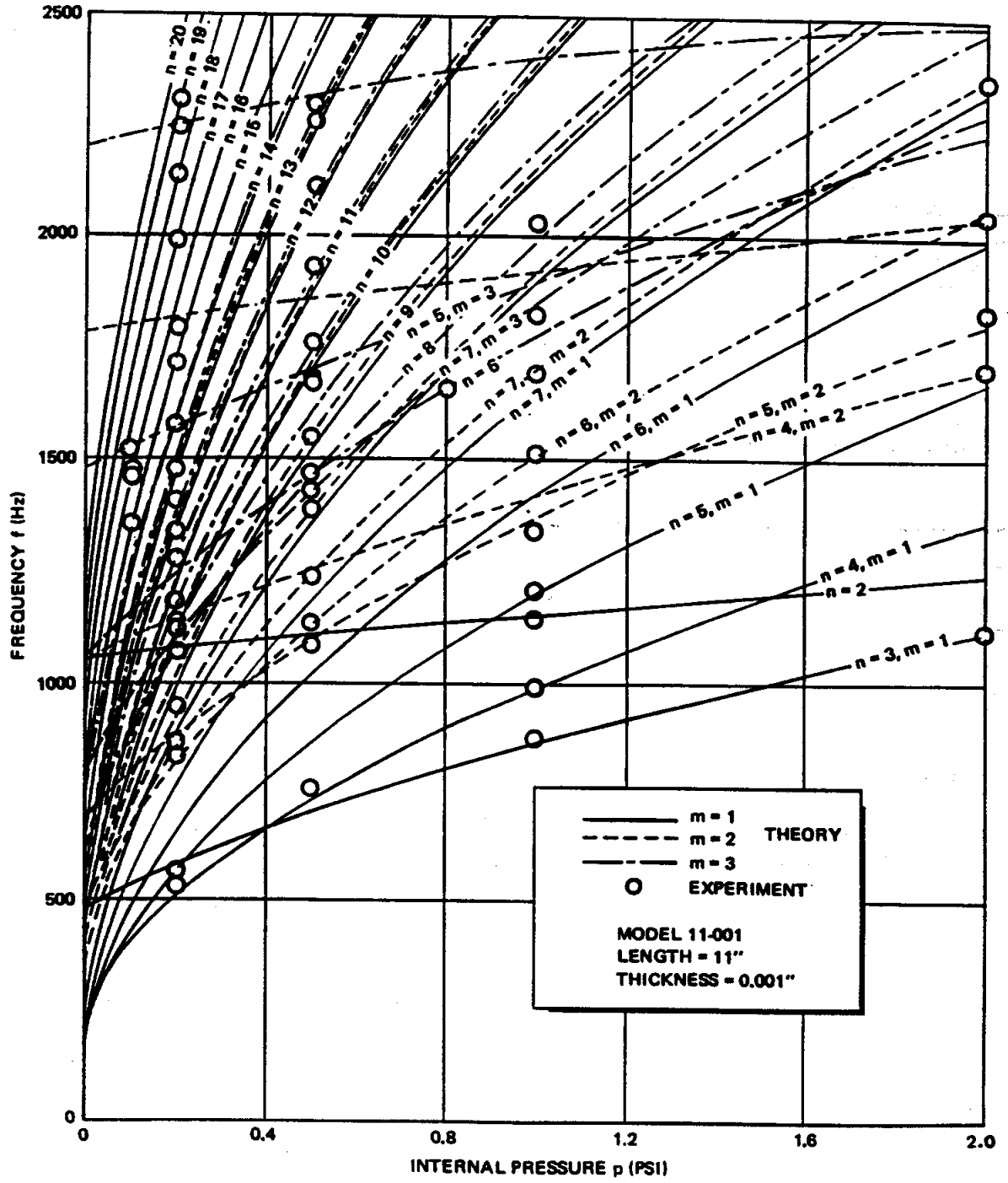


Figure 5-39. Natural Frequencies vs Internal Pressure.

It seems evident that the theoretical values for the frequencies of the test specimens should be higher than the freely supported curves shown in Figure 5-39, the error being largest for m = 1. On the other hand, the apparent mass of air was neglected in the theoretical curves. If it were included, the frequencies would be lowered. These factors of opposing influences have not yet been evaluated theoretically.

5-1.5.4 Free Vibration of Thin, Clamped, Cylindrical Shells Subject to Radial Pressure

In a recent paper, Fung, Sechler, and Kaplan[1] studied the effects of internal pressure on the breathing-mode vibrations of a simply-supported cylinder, obtaining the usual cubic equation for frequency. Also reported in this reference was a very simple expression for frequency which was obtained by Reissner[2] who combined his shallow-shell vibration theory with Marguerre's equations. In this connection, it is of interest to note that Donnell's equation leads to Reissner's result for a simply supported shell (which is not surprising since elimination of the stress function in Marguerre's linearized equations yields Donnell's equation), and that the procedure employed by Yu[3] permits one to obtain equally simple results for a cylinder with clamped and with clamped, freely supported edges.

The appropriate Donnell's equation (made nondimensional with respect to radius) is:

$$k \nabla^8 w + (1 - \nu^2) \frac{\partial^4 w}{\partial x^4} + \nabla^4 \left[N_\phi \frac{\partial^2 w}{\partial \phi^2} + \gamma \frac{\partial^2 w}{\partial t^2} \right] = 0 \qquad (4)$$

where:

$$k = \frac{h^2}{12R^2},$$

$$\gamma = \frac{\rho (1 - \nu^2) R^2}{E},$$

$$\nabla^4 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \phi^2} \right)^2$$

and

h = wall thickness,

R = shell radius,

ν = Poisson's ratio,

(x,ϕ) are shell coordinates,

ρ = density,

E = Young's modulus,

$N_\phi = p^{(1-\nu^2)} R/hE$ = hoop-stress parameter,

p = external pressure,

w = nondimensional radial deflection, and

ℓ = L/R is a nondimensional shell length.

For a simply supported shell, a solution to equation 4 is:

$$w(x,\phi,t) = W \sin \beta_m x \cos n\phi \cos \omega t \tag{5}$$

where $\beta_m = m\pi/\ell$, $m = 1,2,3, \ldots$. Substituting equation 5 into equation 4:

$$\Omega = k(\beta_m^2 + n^2)^2 + (1-\nu^2) \frac{\beta_m^2}{(\beta_m^2 + n^2)^2 - n^2 N_\phi} \tag{6}$$

where

$$\Omega = \gamma\omega^2 .$$

If it is now assumed (this is equivalent to assuming that the number of circumferential waves is large with respect to the number of axial waves) that:

$$| \beta_m/n |^2 << 1 \tag{7}$$

Equation 6 reduces to:

$$\Omega = kn^4 + (1 - \nu^2)(m\pi/n\ell)^4 - n^2 N_\phi \tag{7a}$$

which is identical to Reissner's result reported by Fung et. al. as equation 15. The first term accounts for wall bending, the second for stretching, and the last for the influence of external pressure. The frequency for an unstressed cylinder is obtained by setting $N_\phi = 0$. At the other extreme, the buckling pressure is

$$\overline{N}_\phi = kn^2 + \left[\frac{(1 - \nu^2)}{n^6}\right]\left(\frac{m\pi}{\ell}\right)^4 \tag{8}$$

For other types of end conditions, we assume that:

$$w(x,\phi,t) = W e^{\lambda x} \cos n\phi \cos \omega t \tag{9}$$

where λ must satisfy:

$$(1 - \nu^2)\lambda^4 = n^4 \left[(\lambda/n)^2 - 1\right]^2 (N_\phi n^2 + \Omega) - kn^8 \left[(\lambda/n)^2 - 1\right]^4 \tag{10}$$

Again employing the simplification of equation 7, $|\lambda/n|^2 \ll 1$, equation 10 reduces to:

$$(1 - \nu^2)\lambda^4 = n^4 (N_\phi n^2 + \Omega) - kn^8 \tag{11}$$

which has roots of the form:

$$\lambda = \pm K, \pm iK \tag{12}$$

where K is real and $K^4 > 0$. (It can be shown that $K^4 < 0$ does not lead to an oscillatory solution.) The general solution to equation 4 is thus:

$$w(x,\phi,t) = (C_1 \sin Kx + C_2 \cos Kx + C_3 \sinh Kx + C_4 \cosh Kx) \cos n\phi \cos \omega t \quad (13)$$

For a clamped shell, appropriate boundary conditions are:

$$w = \partial w/\partial x = 0 \quad \text{at} \quad x = 0,1 \quad (14)$$

Yu has shown that equations 13 and 14 lead to the characteristic equation:

$$\cos K\ell \cosh K\ell = 1 \quad (15)$$

so that:

$$K = (m'\pi/\ell),\ m' = 1.506,\ 2.500,\ 3.500,\ \ldots$$

corresponding to 1/2, 1, 1-1/2, ... axial waves. Thus, the frequency of a clamped shell subject to external pressure is:

$$\Omega = kn^4 + (1 - \nu^2)(m'\pi/n\ell)^4 - n^2 N_\phi \quad (16)$$

Limiting cases of equation 16 are:

$$\bar{N}_\phi = kn^2 + \left[\frac{(1 - \nu^2)}{n^6}\right](m'\pi/\ell)^4 \quad (17)$$

and

$$\Omega_{N_\phi = 0} = kn^4 + (1 - \nu^2)(m'\pi/n\ell)^4 \quad (18)$$

Comparing equation 17 with equation 8, one observes that a clamped shell of length L buckles under the same pressure as a simply supported cylinder of approximate length (2/3) L. This result was also obtained by Bijlaard.[4] For the oscillatory modes, the ratio for equal frequencies goes as 2/3, 4/5, 6/7, ...

For a shell with one edge simply supported and the other clamped, a similar procedure leads to:

$$\Omega + kn^4 + (1 - \nu^2)\,(m''\pi/n\ell)^4 - n^2 N_\phi \qquad (19)$$

where $m'' = 1.250, 2.250, 3.250, \ldots$ Comparing with equation 8, a clamped-supported shell of length L buckles at the same pressure as a simply supported shell of length (4/5) L. For equal frequencies, the lengths go as 4/5, 8/9, 12/13, ...

The effect of internal pressure is obtained by reversing the sign before N_ϕ. Attention is called to the linear relation between ω^2 and N_ϕ that exists for all three cases.

If all three components of translatory inertia are included, then Donnell's uncoupled equation 8 must be used (see Yu). The improved frequency equation:

$$\Omega = \frac{kn^8 + (1 - \nu^2)(\mu\pi/\ell)^4 - N_\phi n^6}{n^2\,(n^2 + 1) + [(3 - \nu)/(1 - \nu)]\,kn^6} \qquad (20)$$

is then obtained, where $\mu = m, m', \text{or } m''$ depending upon boundary conditions. Equation 20 approaches the previous results when $n^2 >> 1$. The writer[6] has shown that equation 10 agrees quite well with experimental results for n sufficiently large (see Table 5-4). The difference between equations 10, 7a, 16, and 19 is of the order of a few percent.

N = 0
I = 4

Table 5-4. Comparison of Frequencies (Hz) for Clamped Shell, h/R = 1/300

AXIAL WAVES		n 4	5	6	7	8	9	10	11	12	13	14
1/2	EQ. (20)	926	646	563	606	727	891	1,088	1,311	1,554	1,827	2,118
	EXPERIMENTAL	700	559	525	587	720	885	1,100	1,310	1,560	1,850	2,140
1	EQ. (20)	2,515	1,645	1,197	987	940	1,009	1,153	1,349	1,577	1,841	2,128
	EXPERIMENTAL	1,620	1,210	980	838	900	995	1,145	1,365	1,600	1,865	2,160

References

1. Fung, Y. C., Sechler, E. E., and Kaplan, A., *On the Vibration of Thin Cylindrical Shells Under Internal Pressure*, J. of Aeronautical Sc. Vol. 23, #9, September 1957.

2. Reissner, R., *Non-Linear Effects in Vibrations of Cylindrical Shells*, Aeromechanics Rep. No. AM5-6, Ramo-Wooldrige Corp., Aug. 1955.

3. Yu, Yi-Uuan, *Free Vibrations of Thin Cylindrical Shells Having Finite Lengths with Freely Supported and Clamped Edges*, J. App. Mech., Trans, ASME, Vol. 77, Dec. 1955.

4. Bijlaard, P. P., *Buckling Stress of Thin Cylindrical Clamped Shells Subject to Hydrostatic Pressures*, Jour. of Aeronautical Sc., Vol. 21, #21, December 1954.

5. Donnell, L. H., *Stability of Thin-Walled Tubes Under Torsion*, NACA TR 479, 1933.

6. Koval, L. R., *On the Free Fibrations of Thin Cylindrical Shells Subjected to an Initial Static Torque*, Doctorate Dissertation, Cornell University, Sept. 1961.

5-1.5.5 Generalized Flexural Frequencies of Uniform Structural Rings.

See Figure 5-40.

5-1.6 Multi-mode Response Analysis

5-1.6.1 Joint Acceptance

If the line or surface density is m, the viscous damping b (neglecting hysteretic damping) and the applied load is p (per unit length or area), the differential equation for the deflection y is:

$$m\ddot{y} + b\dot{y} + D(y) = p(t,x) \qquad (21)$$

where D is the appropriate linear operator. Introducing the natural modes

$$m\ddot{y} + D(y) = 0,$$

and letting the solution be

$$y_n = \alpha_n(x)\, e^{i\omega_{on}t}$$

1. ENTER CHART AT APPROPRIATE SECTION PROPERTY VALUE, I/A (IN^2), ON LEFT HAND SCALE.
2. MOVE HORIZONTALLY TO THE RIGHT UNTIL THE PROPER RADIUS CURVE, R (IN) SHOWN WITH A SOLID LINE, IS INTERSECTED.
3. DROP OR RISE TO DESIRED MODE, n, SHOWN WITH A DASHED LINE.
4. MOVE HORIZONTALLY TO THE LEFT AND READ FREQUENCY. NOTE: MODES SHOWN IN PARENTHESIS ( ) RELATE TO FREQUENCIES SHOWN IN PARENTHESES, I.E., FREQUENCIES BETWEEN 100 Hz AND 10,000 Hz ARE PRESENTED IN THIS MANNER.

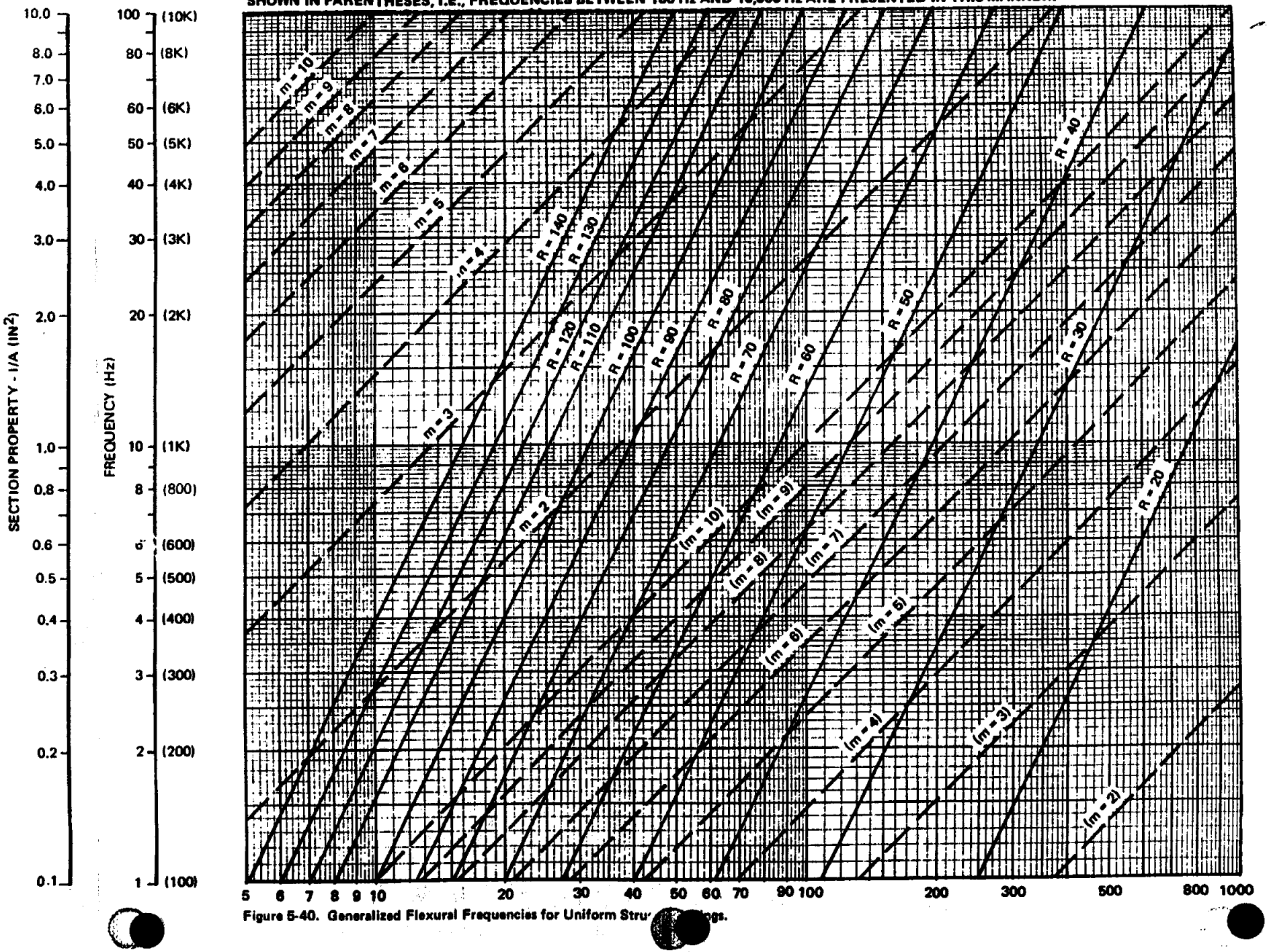


Figure 5-40. Generalized Flexural Frequencies for Uniform Stru[illegible]ngs.

where ω_{on} is the natural (angular) frequency of the n^{th} mode and $\alpha_n(x)$ describes its shape, equation 21 of the displacement, expressed in terms of natural modes, is:

$$m\sum \alpha_n \ddot{\xi}_n + b\sum \alpha_n \dot{\xi}_n + \sum \omega_{on}^2 m \alpha_n \xi_n = p(t,x) \qquad (22)$$

Natural modes are othogonal

$$\left(\int m \alpha_n(x) \alpha_m(x) dx = 0; m \neq n . \right)$$

Defining a generalized mass

$$m_n = \int m \alpha_n^2(x) dx$$

and a generalized damping coefficient (assuming no damping coupling between modes:

$$\beta_n = \frac{1}{M_n} \int b \alpha_n^2(x) dx,$$

the response equation becomes:

$$\ddot{\xi}_n + \beta_n \dot{\xi}_n + \omega_{on} \xi_n = \frac{1}{M_n} \int p(t,x) \alpha_n(x) dx \qquad (23)$$

Now, if the pressure is Fourier transformed (resolving it into frequency components):

$$S_p(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} p(t,x) e^{-i\omega t} dt ,$$

and likewise ξ_n transformed to $S_{\xi_n}(\omega)$, equation 23 becomes:

$$\left(-\omega^2 + i\,\beta\,\omega + \omega_{on}^2\right) S_{\xi_n}(\omega) = \frac{1}{M_n} \int S_p(\omega,x)\,\alpha_n(x)\,dx \qquad (24)$$

(ω_{on} still being the natural frequencies of resonance and ω being the variable of transformation).

Now

$$S_y(\omega,x) = \sum \alpha_n(x)\, S_{\xi_n}(\omega) \qquad (25)$$

(because $y = \Sigma\, \alpha_n(x)\, \xi_n(x)$).

An expression for the power spectrum then becomes:

$$W_y(\omega,x) = \frac{2}{T}\, S_x(\omega,x)\, S_y^*(\omega,x) \qquad (26)$$

where the asterisk indicates the couplex conjugate and the average time T is to be taken to the limit. Since power spectrum is a real quantity, the power spectrum for the response becomes (from equations 24-26):

$$W_y(\omega,x) = \sum_n \sum_m \frac{\alpha_n(x)}{|Y_n(\omega)|} \frac{\alpha_m(x)}{|Y_m(\omega)|} \iint \frac{2}{T}\, S_p(\omega,x)\, S_p^*(\omega,x')\, e^{i(\theta_n - \theta_m)}$$

$$\alpha_n(x)\, \alpha_m(x')\, dx\, dx' \qquad (27)$$

where:

$$|Y(\omega)|^2 = M^2 \left[(\omega_o - \omega^2)^2 + \beta^2\, \omega^2\right]$$

and:

$$\tan \theta = \frac{\beta\omega}{(\omega_o^2 - \omega^2)}$$

It should be noted that equation 27 could have been written for velocity or acceleration, for stress, for slope, or even for curvature so long as the new variable is linearly connected to displacement. This can be expressed mathematically by:

$$\chi(t,x) = \sum Y_n(x)\, \xi_n(t)$$

where χ is any quantity linearly connected to displacement, and the Y_n's are the appropriate modal coefficients.

Now the expression

$$\left(\frac{2}{T}\right) S_p(\omega,x)\, S_p^*(\omega,x')\, e^{i(\theta_n - \theta_m)}$$

in equation 27 is simply the correlation of the pressure, in a narrow frequency band (1/T), at one point x with respect to the pressure at another point x' at a time τ later, and averaged over time. This correlation can be denoted $W(\omega,x,x',\tau)$. If power spectrum results are desired, $\tau = 0$, and if space averages are performed (neglecting cross terms), equation 27 becomes:

$$\overline{W_x(\omega)} = \sum \frac{\overline{Y^2(x)}}{|Y_n|^2} \iint W_p(\omega,x,x')\, \alpha_n(x)\, \alpha_n(x')\, dx\, dx' \qquad (28)$$

where $W_p(\omega,x,x')$ has been written for $W_p(\omega,x,x',0)$ and bars over the quantities denote space averages. The neglect of the cross terms is equivalent to assuming no correlation between motions in various modes.

If the forcing function W_p is homogeneous in space (not dependent on x or x but only on $x - x' = \zeta$) then a Fourier transform of W_p expresses W_p in the wave number space

$$\Phi(\omega,k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} W_p(\omega\zeta)\, e^{-ik\zeta}\, d\zeta \tag{29}$$

similarly, the mode shapes can be expressed as

$$S_\alpha(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \alpha(x)\, e^{-ikx}\, dx \tag{30}$$

Then the generalized load becomes:

$$W_p(\omega) = \iint W_p(\omega,\zeta)\, \alpha(x)\, \alpha(x')\, dxdx'$$

$$= \frac{1}{\sqrt{2\pi}} \iiint \Phi(\omega,k)\, e^{ik(x-x')}\, \alpha(x)\, d(x')\, dxdx'$$

$$= \sqrt{2\pi} \int \Phi(\omega,k)\, |S_\alpha(k)|^2\, dk \tag{31}$$

This factor $W_p(\omega)$ divided by the power spectrum of the pressure $P_p(\omega)$ times the square of the length (or area) over which it acts (A^2) is called joint acceptance $j_n^2(\omega)$. In terms of joint acceptance, equation 28 becomes:

$$\overline{W_x(\omega)} = \sum \frac{\overline{\gamma^2(x)}}{a} P_p(\omega)\, A^2\, j_n^2(\omega)$$

The j_n^2 is a quantity that reflects the coupling of the pressure field wave numbers with the mode of vibration in question (spatial coupling), while $|Y_n|^2$ concerns the selectivity in frequency (temporal dependence).

Many j_n^2 have been calculated for specific fields acting on specific structures. Several specific examples are given in Random Vibration edited by S. H. Crandall 1958 (Chapter 8) with other calculations available in the literature.

The use of joint acceptance is beneficial if the response of a simple structure is required for which mode shapes are known, and complete correlation definition of the pressure field is known. Under these circumstances the response in each mode is a fairly simple calculation. Like any other structural response calculations, however, complicated problems require computers to handle the extensive arithmetical calculations involved.

5-1.6.2 Integral Representations—Table of Response Solutions for Beams, Plates, Cylindrical Shells Excited by Deterministic and Random Loads

This paragraph summarizes the solutions for the response of uniform simple structures to a large class of pressure fields common in applications. The solutions have been derived with the use of the response integral representations (Green's function technique). The detailed derivations of the solutions are given in Reference 1. Numerical evaluation of the solutions, parametric studies, etc. can be performed with the computer program DA24. This program has been documented in Reference 2.

5-1.6.2.1 Deterministic, Nonconvecting Pressure Fields—Response of Beams

a. Load—The mathematical description (input function) for the load is assumed to be of the form:

$$p(x,t) = p_1(x)\, p_2(t) \quad ; \quad a \le x \le b,\ 0 < t \le t_1$$
$$= 0 \quad ; \quad x < a,\ b < x,\ t > t_1 \qquad (32)$$

(x = spatial coordinate; t = time variable).

That is, the load can be written as the product of two functions, each of which depends on only one variable. Spatially the load may be truncated at a and b, and it may be truncated in time at t_1.

The response solutions which follow are applicable to all loads whose spatial and time distribution can be expressed in the form:

$$p_1(x) = i\, e^{i\kappa_1 x}\left[\beta_{1,1} + \beta_{1,2}\,(ix) + \frac{\beta_{1,3}}{2!}(ix)^2 + \frac{\beta_{1,4}}{3!}(ix)^3 + \ldots\right]$$

$$+ i\, e^{i\kappa_2 x}\left[\beta_{2,1} + \beta_{2,2}\,(ix) + \frac{\beta_{2,3}}{2!}(ix)^2 + \frac{\beta_{2,4}}{3!}(ix)^3 + \ldots\right]$$

$$+ i\, e^{i\kappa_3 x}\left[\beta_{3,1} + \beta_{3,2}\,(ix) + \ldots + \ldots\right]$$

$$+ \ldots + i\, e^{i\kappa_j x}\left[\beta_{j,1} + \beta_{j,2}\,(ix) + \ldots\right]$$

$$= i \sum_{j=1} \sum_{r=1} \frac{\beta_{j,r}}{(r-1)!}\, e^{i\kappa_j x}\,(ix)^{r-1} \qquad (33)$$

$$p_2(t) = i \sum_{i=1} \sum_{s=1} \frac{\alpha_{i,s}}{(s-1)!}\, e^{i\nu_i t}\,(it)^{s-1} \qquad (34)$$

($\alpha_{i,s}$, $\beta_{j,r}$ = constant coefficients - in general, complex; ν_i, κ_j = complex constants with positive imaginary parts; i, j, r, s = integers; n! = n (n - 1) (n - 2) ... 1.)

That is, equations 33 and 34 represent the class of loads whose dependence on the spatial coordinate and time is expressible by various combinations of exponential, trigonometric functions, and polynomials of any degree in the variables. This class of functions adequately describes most pressure field data one is likely to encounter in applications.

The Fourier transforms of equations 33 and 34 are given by:

$$\hat{p}_1(k) = \sum_{j=1} \sum_{r=1} \beta_{j,r}\,(k - \kappa_j)^{-r} \qquad (35)$$

$$\hat{p}_2(\omega) = \sum_{i=1} \sum_{s=1} \alpha_{i,s} (\omega - \nu_i)^{-s} \tag{36}$$

(k = wave number variable — Fourier conjugate of x;

ω = frequency variable — Fourier conjugate of t).

In applications, loads can be given in terms of either the(x,t) or the (k,ω) variables, depending on the form in which the data are given. In either case, to use the response solutions of this section given a particular load, one must first identify the constants which appear in equations 33, 34, or 35, 36.

b. Examples

1. Given:

$$p_2(t) = P_o e^{-0.5t} \; ; \; P_o = \text{constant} \tag{37}$$

Comparing with equation 34, s = 1, i = 1, $i\alpha_{1,1} = P_o$, ($\alpha_{1,1} = -iP_o$), $i\nu_1 = -0.5$, ($\nu_1 = 0.5i$), $\alpha_{i,s} = 0$ for all i,s other than 1.

2. Given:

$$p_1(x) = e^{-0.3x} \cos 5x \tag{38}$$

Write:

$$p_1(x) = \frac{1}{2} \left[e^{i(5+0.3i)x} + e^{i(-5+0.3i)x} \right]$$

Comparing with equation 33, r = 1, j = 1,2, $i\beta_{1,1} = 1/2$, $i\beta_{2,1} = 1/2$, ($\beta_{1,1} = \beta_{2,1} = 1/2i$), $\kappa_1 = 5 + 0.3i$, $\kappa_2 = -5 + 0.3i$ and all other coefficients $\beta_{j,r}$ are zero.

3. Given:

$$p_2(t) = P_o\, e^{-0.5t} \sin t \left(t^3 - \frac{3}{4}t + 0.03\right); \quad P_o = \text{constant}$$

Write:

$$p_2(t) = \frac{P_o}{2}\, e^{i(1+0.5i)t} \left(t^3 - \frac{3}{4}t + 0.03\right)$$

$$- \frac{P_o}{2}\, e^{i(-1+0.5i)t} \left(t^3 - \frac{3}{4}t + 0.03\right) \qquad (39)$$

Since the degree of the polynomial in equation 39 is 3, we conclude that the summation over s in equation 34 goes up to 4; equation 36 has two exponential terms, therefore, i is summed up to 2. Equation 34 now reads:

$$p_2(t) = i\, e^{i\nu_1 t}\left[\alpha_{1,1} + \alpha_{1,2}\,(it) + \frac{\alpha_{1,3}}{2!}(it)^2 + \frac{\alpha_{1,4}}{3!}(it)^3\right]$$

$$+ i\, e^{i\nu_2 t}\left[\alpha_{2,1} + \alpha_{2,2}\,(it) + \frac{\alpha_{2,3}}{2!}(it)^2 + \frac{\alpha_{2,4}}{3!}(it)^3\right]$$

Comparing with equation 36:

$$\nu_1 = 1 + 0.5i$$

$$\nu_2 = -1 + 0.5i$$

$$\alpha_{1,1} = 0.03\, P_o/2i$$

$$\alpha_{1,2} = 3\, P_o/8$$

$$\alpha_{1,3} = \alpha_{2,3} = 0$$

$$\alpha_{1,4} = 3\ P_o$$

$$\alpha_{2,1} = -0.03\ P_o/2i$$

$$\alpha_{2,2} = -3\ P_o/8$$

$$\alpha_{2,4} = -3\ P_o$$

4. Given:

$$\hat{p}_2(\omega) = \frac{\omega}{\omega^2 + 2a\omega + b}\ ;\quad a,b = \text{constants} \tag{40}$$

Write:

$$\hat{p}_2(\omega) = \frac{\omega}{(\omega - \omega_1)(\omega - \omega_2)}$$

$$\omega_1 = -a + \sqrt{a^2 - b}$$

$$\omega_2 = -a - \sqrt{a^2 - b}$$

By partial fractions,

$$\hat{p}_2(\omega) = \left(\frac{\omega_1}{\omega_1 - \omega_2}\right)(\omega - \omega_1)^{-1} + \left(\frac{\omega_2}{\omega_1 - \omega_2}\right)(\omega - \omega_2)^{-1}$$

Comparing with equation 36,

$$s = 1$$

$$i = 1,2$$

$$\alpha_{1,1} = \frac{\omega_1}{\omega_1 - \omega_2}$$

$$\alpha_{2,1} = \frac{\omega_2}{\omega_1 - \omega_2}$$

$$\nu_1 = \omega_1$$

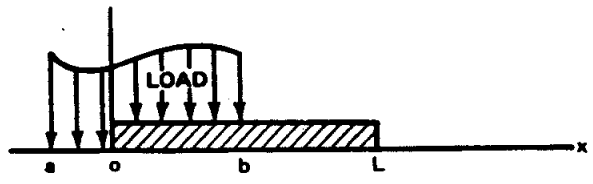
$$\nu_2 = \omega_2$$

c. Response Solutions

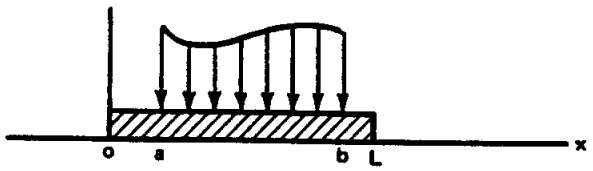
The displacement response at position x and time t of a simply-supported, uniform beam of length L, excited by any load of the class given in equations 32, 33, and 34, is given by:

For $t < t_1$

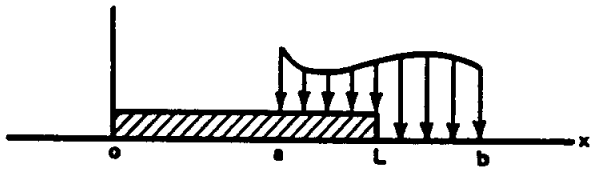
$$w_b(x,t) = \sum_n A_n \psi_n(x) C_n(t) D_n(b,0) \quad ; \quad L > b > 0 > a \qquad (41)$$



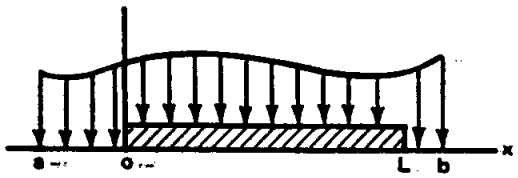
$$w_b(x,t) = \sum_n A_n \psi_n(x) C_n(t) D_n(b,a) \quad ; \quad L > b > a > 0 \qquad (42)$$



$$w_b(x,t) = \sum_n A_n \psi_n(x) C_n(t) D_n(L,a) \quad ; \quad b > L > a > 0 \qquad (43)$$



$$w_b(x,t) = \sum_n A_n \psi_n(x) C_n(t) D_n(L,0) \quad ; \quad b > L > 0 > a \qquad (44)$$



where:

$w_b(x,t)$ = normal (transverse) displacement of beam

n = 1,2,3, ... (mode numbers)

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin k_n x \quad , \quad k_n = \frac{n\pi}{L} \text{ (normalized mode shapes)}$$

$$A_n = \frac{1}{16\pi^3 i\mu\omega_n} \quad (\omega_n = \text{natural frequencies, } \mu = \text{mass density})$$

$$C_n(t) = 2\pi i \left[e^{i\Omega_2 t} \hat{p}_2(\Omega_2) - e^{i\Omega_1 t} \hat{p}_2(\Omega_1) \right.$$

$$\left. - \sum_{i=1} \sum_{s=1} \sum_{q=0}^{s-1} \alpha_{i,s} e^{i\nu_i t} \frac{(it)^q}{q!} \left\{ \left(\Omega_2 - \nu_i\right)^{q-s} \right.\right.$$

$$\left.\left. - \left(\Omega_1 - \nu_i\right)^{q-s} \right\} \right] U(t) \qquad (45)$$

$U(t) = 1, \; t > 0$

$= 0, \; t < 0$

$\Omega_1 = \omega_n + ia_n$ (a_n = structural damping in nth mode)

$\Omega_2 = -\omega_n + ia_n$

$\hat{p}_2(\Omega_1)$, $\hat{p}_2(\Omega_2)$ — given by equation 36 with $\omega = \Omega_1$, $\omega = \Omega_2$, respectively

i, s, $\alpha_{i,s}$, ν_i — load parameters, equation 34

$$D_n(y,z) = 2\pi^2\sqrt{\frac{2}{L}}\left[\sum_{j=1}\sum_{r=1}\sum_{q=0}^{r-1}(-1)^{r-q}\,\beta_{j,r}\,\frac{(iy)^q}{q!}\left\{\frac{e^{i(\kappa_j - k_n)y}}{(\kappa_j - k_n)^{r-q}}\right.\right.$$

$$\left.\left. - \frac{e^{i(\kappa_j + k_n)y}}{(\kappa_j + k_n)^{r-q}}\right\} - \sum_{j=1}\sum_{r=1}\sum_{q=0}^{r-1}(-1)^{r-q}\,\beta_{j,r}\,\frac{(iz)^q}{q!}\right.$$

$$\left.\left\{\frac{e^{i(\kappa_j - k_n)z}}{(\kappa_j - k_n)^{r-q}} - \frac{e^{i(\kappa_j + k_n)z}}{(\kappa_j + k_n)^{r-q}}\right\}\right] \qquad (46)$$

$$D_n(y,0) = -2\pi^2\sqrt{\frac{2}{L}}\left[\hat{p}_1(k_n) - \hat{p}_1(-k_n)\right.$$

$$\left. - \sum_{j=1}\sum_{r=1}\sum_{q=0}^{r-1}(-1)^{r-q}\,\beta_{j,r}\,\frac{(iy)^q}{q!}\left\{\frac{e^{i(\kappa_j - k_n)y}}{(\kappa_j - k_n)^{r-q}}\right.\right.$$

$$\left.\left. - \frac{e^{i(\kappa_j + k_n)y}}{(\kappa_j + k_n)^{r+q}}\right\}\right] \qquad (47)$$

The parameters y, z, in equations 46 and 47 stand for the constants a, b, L of equations 41 through 44; j, r, $\beta_{j,r}$, κj are the load parameters in equation 33; $\hat{p}_1(\pm k_n)$ is given by equation 35 with $k = \pm k_n$.

For $t > t_1$

The response solutions are given by equations 41 through 44 with $C_n(t)$ replaced by $E_n(t,t_1)$, where:

$$E_n(t,t_1) = 2\pi i \left[e^{i\Omega_2 t} \hat{p}_2(\Omega_2) - e^{i\Omega_1 t} \hat{p}_2(\Omega_1) \right.$$

$$- e^{i\Omega_2 t} \sum_{i=1} \sum_{s=1} \sum_{q=0}^{s-1} \alpha_{i,s} e^{-i(\Omega_2 - \nu_i)t_1} \frac{(it_1)^q}{q!} (\Omega_2 - \nu_i)^{q-s}$$

$$+ e^{i\Omega_1 t} \sum_{i=1} \sum_{s=1} \sum_{q=0}^{s-1} \alpha_{i,s} e^{-i(\Omega_1 - \nu_i)t_1}$$

$$\left. \cdot \frac{(it_1)^q}{q!} (\Omega_1 - \nu_i)^{q-s} \right] U(t) \qquad (48)$$

Example — Let us write the response solutions corresponding to any combination of truncation in the pressure field for the load of equations 38 and 39:

$$p(x,t) = P_o e^{-0.3x} \cos 5x \, e^{-0.5t} \sin t \left(t^3 - \frac{3}{4}t + 0.03 \right) \qquad (49)$$

By comparison with equations 33 and 34, we previously identified the pressure field parameters as follows:

$$\begin{array}{llll} r = 1,\ j = 1,2 & s = 1 \text{ to } 4 & \alpha_{1,1} = 0.03\, P_o/2i = -\alpha_{2,1} \\ \beta_{1,1} = \beta_{2,1} = 1/2i & i = 1,2 & \alpha_1 = 3P_o/8 & = -\alpha_{2,2} \\ \kappa_1 = 5 + 0.3i & \nu_1 = 1 + 0.5i & \alpha_{1,3} = 0 & = \alpha_{2,3} \\ \kappa_2 = -5 + 0.3i & \nu_2 = -1 + 0.5i & \alpha_{1,4} = 3P_o & = -\alpha_{2,4} \end{array} \qquad (50)$$

To obtain the response solutions, we need only substitute the above constants in equations 45 through 48, which now read:

$$C_n(t) = 2\pi i \left[e^{i\Omega_2 t} \hat{p}_2(\Omega_2) - e^{i\Omega_1 t} \hat{p}_2(\Omega_1) \right.$$

$$- \sum_{i=1}^{2} \sum_{s=1}^{4} \sum_{q=0}^{s-1} \alpha_{i,s} e^{i\nu_i t} \frac{(it)^q}{q!} \left\{ (\Omega_2 - \nu_i)^{q-s} \right.$$

$$\left. \left. - (\Omega_1 - \nu_i)^{q-s} \right\} \right] U(t) \qquad (51)$$

$$\hat{p}_2(\Omega_1) = \sum_{i=1}^{2} \sum_{s=1}^{4} \alpha_{i,s} (\Omega_1 - \nu_i)^s \quad \text{(from equation 36)} \qquad (52)$$

$\hat{p}_2(\Omega_2)$ — as in equation 52 with Ω_1, replaced by Ω_2

$$E_n(t,t_1) = 2\pi i \left[e^{i\Omega_2 t} \hat{p}_2(\Omega_2) - e^{i\Omega_1 t} \hat{p}_2(\Omega_1) \right.$$

$$- e^{i\Omega_2 t} \sum_{i=1}^{2} \sum_{s=1}^{4} \sum_{q=0}^{s-1} \alpha_{i,s} e^{-i(\Omega_2 - \nu_i)t_1} \frac{(it_1)^q}{q!(\Omega_2 - \nu_i)^{s-q}}$$

$$- e^{i\Omega_1 t} \sum_{i=1}^{2} \sum_{s=1}^{4} \sum_{q=0}^{s-1} \alpha_{i,s} e^{-i(\Omega_1 - \nu_i)t_1}$$

$$\left. \cdot \frac{(it_1)^q}{q!(\Omega_1 - \nu_i)^{s-q}} \right] U(t) \qquad (53)$$

$$D_n(y,z) = 2\pi^2 (2/L)^{1/2} \left[\sum_{j=1}^{2} (-1) \beta_{j,1} \left\{ \frac{e^{i(\kappa_j - k_n)y}}{\kappa_j - k_n} - \frac{e^{i(\kappa_j + k_n)y}}{\kappa_j + k_n} \right\} \right.$$

$$\left. - \sum_{j=1}^{2} (-1) \beta_{j,1} \left\{ \frac{e^{i(\kappa_j - k_n)z}}{\kappa_j - k_n} - \frac{e^{i(\kappa_j + k_n)z}}{\kappa_j + k_n} \right\} \right] \qquad (54)$$

$$D_n(y,o) = -2\pi^2 (2/L)^{1/2} \left[\hat{p}_1(k_n) - \hat{p}_1(-k_n) \right.$$

$$\left. - \sum_{j=1}^{2} (-1) \beta_{j,1} \left\{ \frac{e^{i(\kappa_j - k_n)y}}{\kappa_j - k_n} - \frac{e^{i(\kappa_j + k_n)y}}{\kappa_j + k_n} \right\} \right] \qquad (55)$$

$$\hat{p}_1(\pm k_n) = \sum_{j=1}^{2} \beta_{j,r} (\pm k_n - \kappa_j)^{-1} \qquad \text{(from equation 35)} \qquad (56)$$

Now, expand the sums in the above equations, substitute the values of the constants given in equation 50 and

$$\Omega_1 = \omega_n + i a_n$$

$$\Omega_2 = -\omega_n + i a_n$$

and simplify. The parameters y, z will take on the values of L, a, b, depending on which case of truncation one is interested in, equations 41 through 44.

5-1.6.2.2 Deterministic, Nonconvecting Pressure Fields——Response of Plates

a. Load——Let x_1, x_2 stand for the cartesian axes. We assume that the load can be written in the form:

$$p(x_1,x_2,t) = p_{11}(x_1)\, P_{12}(x_2)\, p_2(t) \;;$$

$$a_1 \le x_1 \le b_1,\ a_2 \le x_2 \le b_2,\ 0 < t \le t_1$$

$$= 0 \;;\; x_1 < a_1,\ x_1 > b_1,\ x_2 < a_2,\ x_2 > b_2,\ t > t_1 \qquad (57)$$

That is, the load may be truncated in both spatial directions at a_1, b_1, a_2, b_2 and in time at t_1. The class of functions $p_{11}(x_1)$, $p_{12}(x_2)$, $p_2(t)$ considered here is of the form given in equations 33 and 34:

$$p_{11}(x_1) = i \sum_{j=1} \sum_{r=1} \frac{\beta_{1,j,r}}{(r-1)!}\, e^{i\kappa_{1,j}x_1}\, (ix_1)^{r-1} \qquad (58)$$

$$p_{12}(x_2) = i \sum_{j=1} \sum_{r=1} \frac{\beta_{2,j,r}}{(r-1)!}\, e^{i\kappa_{2,j}x_2}\, (ix_2)^{r-1} \qquad (59)$$

$p_2(t)$——as in equation 34. The Fourier transforms of equations 58 and 59 are given by equation 35 with:

$$\beta_{j,r},\ \kappa_j$$

replaced by

$$\beta_{1,j,r},\ \kappa_{1,j}$$

or

$$\beta_{2,j,r},\ \kappa_{2,j}$$

respectively. Thus,

$$p(x_1,x_2,t)$$

is given as the product of functions of x_1, x_2, t each of which is identical to the one-dimensional case, save for the introduction of appropriate subscripts. Therefore, given any two-dimensional load in cartesian coordinates, the process of identifying the load parameters is identical to that of the examples in Paragraph 5-1.5.2 for each of the two dimensions.

b. Example

Given:

$$p(x_1,x_2) = e^{-a(x_1+x_2)} \left(x_1^2 + 1\right) \cos b\, x_2;\ a,b = \text{constants} \qquad (60)$$

For this case:

$$p_{11}(x_1) = e^{-ax_1} \left(x_1^2 + 1\right) \qquad (61)$$

$$p_{12}(x_2) = \frac{1}{2}\left[e^{i(b+ia)x_2} + e^{i(-b+ia)x_2}\right] \qquad (62)$$

Comparing equation 61 with equation 58:

$$j = 1$$

$$r = 1 \text{ to } 3$$

$$\kappa_{1,1} = ia$$

$$\beta_{1,1,1} = -i$$

$$\beta_{1,1,2} = 0$$

$$\beta_{1,1,3} = -i$$

Comparing equation 62 with equation 59:

$$j = 1, 2$$

$$r = 1, \kappa_{2,1} = b + ia$$

$$\kappa_{2,2} = -b + ia$$

$$\beta_{2,1,1} = \beta_{2,1,2} = 1/2i$$

c. Response Solutions—We consider uniform, isotropic plates of dimensions L_1, L_2 along the cartesian axes x_1, x_2 which are simply supported along all edges. The response of such a plate at position (x_1, x_2) and at time t to any load of the class given by equations 57, 58, 59 and 34 is given by:

For $t < t_1$:

$$w_p(x_1,x_2,t) = \sum_n \sum_m A_{nm}\, \psi_{nm}(x_1,x_2)\, D_n(y_1,z_1)\, D_m(y_2,z_2)\, C_{nm}(t) \quad (63)$$

For $t > t_1$:

As in equation 63 with $C_{nm}(t)$ replaced by $E_{nm}(t,t_1)$ where n,m = mode numbers along x_1, x_2 directions, respectively.

$$\psi_{nm}(x_1,x_2) = \frac{2}{\sqrt{L_1 L_2}} \sin k_n x_1 \sin k_m x_2$$

$$k_n = \frac{n\pi}{L_1} \; ; \; k_m = \frac{m\pi}{L_2} \; ; \; n = 1,2 \ldots , m = 1,2 \ldots$$

$$A_{nm} = - \frac{1}{2(2\pi)^5 \mu \omega_{nm}}$$

ω_{nm} = natural frequencies

μ = surface mass density.

$$D_n(y_1,z_1) = \begin{cases} D_n(b_1,0) & ; \quad L_1 > b_1 > 0 > a_1 \\ D_n(b_1,a_1) & ; \quad L_1 > b_1 > a_1 > 0 \\ D_n(L_1,0) & ; \quad b_1 > L_1 > 0 > a_1 \\ D_n(L_1,a_1) & ; \quad b_1 > L_1 > a_1 > 0 \end{cases} \tag{64}$$

$$D_m(y_2,z_2) = \begin{cases} D_m(b_2,0) & ; \quad L_2 > b_2 > 0 > a_2 \\ D_m(b_2,a_2) & ; \quad L_2 > b_2 > a_2 > 0 \\ D_m(L_2,0) & ; \quad b_2 > L_2 > 0 > a_2 \\ D_m(L_2,a_2) & ; \quad b_2 > L_2 > a_2 > 0 \end{cases} \tag{65}$$

The functions $D_n(y_1,z_1)$ are given by equations 36 and 37 with L, $\beta_{j,r}$, κ_j. $\hat{p}_1(\pm k_n)$ replaced by L_1, $\beta_{1,j,r}$, $\kappa_{1,j}$, $\hat{p}_{11}(\pm k_n)$, respectively.

The functions $D_m(y_2,z_2)$ are given by equations 36 and 37 with L, n, $\beta_{j,r}$, κ_j, $\hat{p}_1(\pm k_n)$ replaced by L_2, m, $\beta_{2,j,r}$, $\kappa_{2,j}$, $\hat{p}_{12}(k_m)$, respectively.

The functions $C_{nm}(t)$, $E_{nm}(t,t_1)$ are given by equations 35 and 38, respectively, with n replaced by nm.

1. Examples

(a) The response of a rectangular plate of dimensions L_1, L_2 at $t > t_1$ excited by a load whose spatial distribution is such that

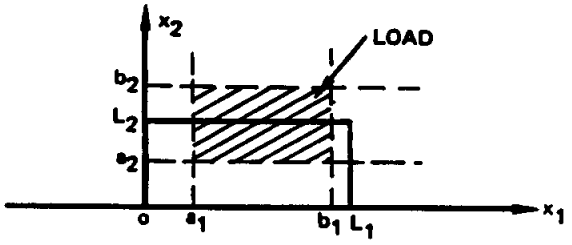
$$L_1 > b_1 > a_1 > 0$$

$$b_2 > L_2 > a_2 > 0$$

is given by equations 63, 64 and 65.

$$w_p(x_1,x_2,t) = \sum_n \sum_m A_{nm}\, \psi_{nm}(x_1,x_2)\, D_n(b_1,a_1)$$

$$D_m(b_2,a_2)\, E_{nm}(t,t_1)$$



(b) The response of the same plate at $t < t_1$, excited by a load whose spatial distribution is such that

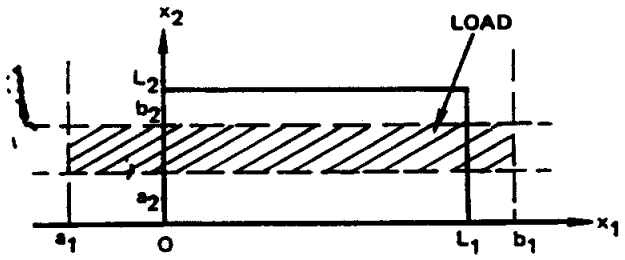
$$b_1 > L_1 > 0 > a_1$$

$$L_2 > b_2 > a_2 > 0$$

is given by:

$$w_p(x_1,x_2,t) = \sum_n \sum_m A_{nm}\, \psi_{nm}(x_1,x_2)\, D_n(L_1,0)$$

$$\cdot\ D_m(b_2,a_2)\, C_{nm}(t)$$



(c) Let the load be given by equations 37 and 60:

$$p(x_1,x_2,t) = P_o\, e^{-a(x_1+x_2)-0.5t}\left(x_1^2+1\right)\cos bx_2 \ ;$$

$$a,b = \text{constants}$$

The relevant load parameters ν_i, $\alpha_{i,s}$, κ_{ij}, $\beta_{1,j,r}$, $\beta_{2,j,r}$ were identified following equations 37 and 62. To obtain the response of a plate to this particular load, for all possible combinations of truncation, substitute the values of the above parameters in equations 45 through 48 under the change of notation indicated earlier. The response solutions are given by equation 63.

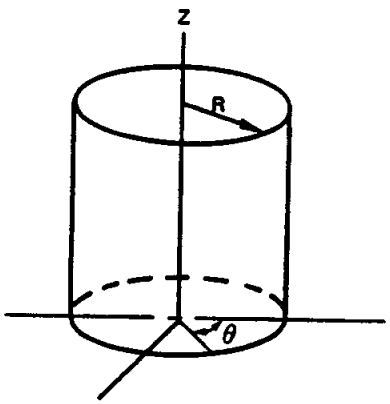
2. Remarks — The response solutions given in this section are applicable to orthotropic plates as well, with the appropriate values for the constants ω_{nm}, a_{nm}. The solutions are also applicable to any structure which can be approximated by an orthotropic plate; for instance, a uniform plate with closely spaced ribs attached to it, can be converted to an equivalent orthotropic plate.

5-1.6.2.3 Deterministic, Nonconvecting Pressure Fields — Response of Circular Cylindrical Shells

a. Load — For the coordinate system of the figure, we assume:

$$p(z,\theta,t) = p_1(z)\, p_3(\theta)\, p_2(t) \ ; \ a \le z \le b,\ 0 \le \theta \le \theta_o \le 2\pi,\ 0 < t \le t_1$$

$$= 0 \text{ otherwise.} \qquad (66)$$



The class of functions for which the response solutions are valid, are of the form:

$p_1(z)$—as in equation 33 with x replaced by z

$p_2(t)$—as in equation 34

$$p_3(\theta) = \sum_{\ell=1} \sum_{h=0} c_{\ell,h}\, e^{-a_\ell \theta}\, \theta^h \qquad (67)$$

$c_{\ell,h}$—constant coefficients

a_ℓ—complex constant

ℓ,h—integers

b. Example

1. Given:

$$p_3(\theta) = e^{-2\pi\theta} \cos \frac{3\pi}{2} \theta \qquad (68)$$

Comparing with equation 67,

$$h = 0$$

$$\ell = 1,2$$

$$c_{1,0} = c_{2,0} = \frac{1}{2}$$

$$a_1 = -\left(2\pi + \frac{3\pi i}{2}\right)$$

$$a_2 = 2\pi + \frac{3\pi i}{2}$$

2. The example of Paragraph 5-1.5.2 with x replaced by z.

c. Response Solutions — The response of a cylindrical shell of length L at a Point (z,θ) and at a time t, is given by:

For $t < t_1$

$$w_c(z,\theta,t) = \sum_n \sum_m A_{nm}\, \psi_n(z)\, H_m(\theta)\, C_{nm}(t) \begin{cases} D_n(b,0);\ L > b > 0 > a \\ D_n(b,a);\ L > b > a > 0 \\ D_n(L,0);\ b > L > 0 > a \\ D_n(L,a);\ b > L > a > 0 \end{cases} \quad (69)$$

where:

n,m — mode numbers in the axial and circumferential directions, respectively

$$A_{nm} = \frac{\varepsilon_m}{32\pi^4 i\, \mu\omega_{nm}}$$

(ω_{nm} = natural frequencies

μ = surface mass density)

$$\varepsilon_m = \begin{cases} 1, & m = 0 \\ 2, & \text{all } m > 0 \end{cases}$$

$$\psi_n(z) = \sqrt{\frac{2}{L}} \sin k_n z \; ; \; k_n = \frac{n\pi}{L}$$

$$H_m(\theta) = \frac{1}{2} e^{im\theta} \sum_{\ell=1} \sum_{h=0} c_{\ell,h} \left[\frac{h!}{(a_\ell + im)^{h+1}} \right.$$

$$\left. - e^{-(a_\ell+im)\theta_o} \sum_{q=0}^{h} \frac{h!}{q!} \frac{(\theta_o)^q}{(a_\ell + im)^{h-q+1}} \right]$$

$$+ \frac{1}{2} e^{-im\theta} \sum_{\ell=1} \sum_{h=0} c_{\ell,h} \left[\frac{h!}{(a_\ell - im)^{h+1}} \right.$$

$$\left. - e^{-(a_\ell-im)\theta_o} \sum_{q=0}^{h} \frac{h!}{q!} \frac{(\theta_o)^q}{(a_\ell - im)^{h-q+1}} \right] \qquad (70)$$

For $t > t_1$ —— as in equation 69 with $C_{nm}(t)$ replaced by $E_{nm}(t,t_1)$

The D_n functions in equation 69 are given by equations 46 and 47; $C_{nm}(t)$, $E_{nm}(t,t_1)$ are given by equations 45 and 48 with n replaced by nm.

d. **Example**—Suppose the load of equation 68 is applied on a shell for $0 \le \theta \le \pi$. For this case, the angular contribution to the response solution is given by equation 70 with $\theta_o = \pi$ and the constants $c_{\ell,h}$, a_ℓ given following equation 68:

$$H_m(\theta) = \frac{1}{2} e^{im\theta} \sum_{\ell=1}^{2} c_{\ell,0} \left[\frac{1}{a_\ell + im} - \frac{e^{-(a_\ell+im)\pi}}{a_\ell + im} \right]$$

$$+ \frac{1}{2} e^{-im\theta} \sum_{\ell=1}^{2} c_{\ell,0} \left[\frac{1}{a_\ell - im} - \frac{e^{-(a_\ell-im)\pi}}{a_\ell - im} \right]$$

e. Remark — The response solutions given in this section are equally applicable to orthotropic cylindrical shells with the proper values for the constants ω_{nm}, a_{nm}. This class of shells also includes structures which can be transformed to equivalent orthotropic shells, e.g., ring-stiffened shells where the stiffeners are closely spaced. The response of conical isotropic/orthotropic shells is also given by equation 69, if one transforms the conical shell to either a single equivalent cylindrical-shell or to a sequence of cylindrical shells (Reference 3).

5-1.6.2.4 Random, Nonconvecting Pressure Fields — Response of Beams

a. Load — Random loads are defined in terms of the cross-correlation function which supplies information with regard to the correlation of pressure at a point x at time t with the pressure at another point x′ at a different time t′. We assume that the loads are homogeneous (stationary) in both space and time. Under this assumption, the cross-correlation function depends on the separation distance $X = x - x'$ and the time delay $\tau = t - t'$ rather than the coordinates, x, x', t, t' themselves. That is,

$$R(x, x', t, t') = R(X, \tau) \qquad (71)$$

(R = cross-correlation function)

Three special cases of equation 71 are of practical concern:

$R(X = 0, \tau)$ — autocorrelation function

$R(X, \tau = 0)$ — spatial correlation function

$R(X = 0, \tau = 0)$ — mean square value

In all that follows, it is further assumed that:

$$R(X, \tau) = R_1(X)\, R_2(\tau) \qquad (72)$$

That is, the cross-correlation function is separable into the product of the spatial and temporal correlations. The class of functions for which the response solutions given in this section are valid are of the form:

$$R_1(X) = \begin{cases} i \sum_{\eta=1} \sum_{\sigma=1} \frac{C_{\eta,\sigma}}{(\sigma-1)!} e^{i\kappa_\eta X} (iX)^{\sigma-1} ; X > 0 \\ -i \sum_{\lambda=1} \sum_{\xi=1} \frac{D_{\lambda,\xi}}{(\xi-1)!} e^{i\kappa_\lambda X} (iX)^{\xi-1} ; X < 0 \end{cases} \quad (73)$$

$$R_2(\tau) = \begin{cases} i \sum_{\alpha=1} \sum_{\beta=1} \frac{A_{\alpha,\beta}}{(\beta-1)!} e^{i\nu_\alpha \tau} (i\tau)^{\beta-1} ; \tau > 0 \\ -i \sum_{\gamma=1} \sum_{\varepsilon=1} \frac{B_{\gamma,\varepsilon}}{(\varepsilon-1)!} e^{i\nu_\gamma \tau} (i\tau)^{\varepsilon-1} ; \tau < 0 \end{cases} \quad (74)$$

($A_{\alpha,\beta}$, $B_{\gamma,\varepsilon}$, $C_{\eta,\sigma}$, $D_{\lambda,\xi}$ = constant coefficients—in general, complex;

ν_α, κ_η = complex constants with positive imaginary parts—$\nu_\alpha = \nu'_\alpha + i\nu''_\alpha$, $\kappa_\eta = \kappa'_\eta + i\kappa''_\eta$,

$\nu''_\alpha > 0$, $\kappa''_\eta > 0$; ν_γ, κ_λ = complex constants with negative imaginary parts—$\nu_\gamma = \nu'_\gamma - i\nu''_\gamma$, $\kappa_\lambda = \kappa'_\lambda - i\kappa''_\lambda$, $\nu''_\gamma > 0$, $\kappa''_\lambda > 0$).

Note that equations 73 and 74 are of the precise form of equations 33 and 34, the only difference being that x, t are replaced by $|X|$, $|\tau|$. Since the latter variables are differences between two space-time points ($X = x - x'$, $\tau = t - t'$), they may be positive or negative.

The Fourier transforms of equations 73 and 74 are given by:

$$\hat{R}_1(k) = \sum_{\eta=1} \sum_{\sigma=1} C_{\eta,\sigma} (k - \kappa_\eta)^{-\sigma} + \sum_{\lambda=1} \sum_{\xi=1} D_{\lambda,\xi} (k - \kappa_\lambda)^{-\xi}$$

$$\equiv R_1^+(k) + R_1^-(k) \tag{75}$$

$$\hat{R}_2(\omega) = \sum_{\alpha=1} \sum_{\beta=1} A_{\alpha,\beta} (\omega - \nu_\alpha)^{-\beta} + \sum_{\gamma=1} \sum_{\varepsilon=1} B_{\gamma,\varepsilon} (\omega - \nu_\gamma)^{-\varepsilon}$$

$$\equiv R_2^+(\omega) + R_2^-(\omega) \tag{76}$$

Given any particular random load whose cross-correlation function falls in the category of equations 73 and 74, the first task is to identify the parameters involved.

b. Remark—Equation 71 holds only if the cross correlation is defined from $-\infty$ to $+\infty$. If the random pressure field is truncated in space and/or time, the cross correlation is not a homogeneous function of the variables. For instance, if a random field is applied to only part of the structure, the response solutions given in this section are not applicable since, in this case, $R(x,x', t,t') \neq R(X,\tau)$. Note that experimental definition of nonhomogeneous pressure-correlation functions are scarce if at all available.

c. **Example**

Given:

$$R_1(X) = e^{-a|X|} \cos bX \quad ; \quad a,b = \text{constants}$$

Write:

$$R_1(X) = \begin{cases} \frac{1}{2}\left[e^{i(b+ia)X} + e^{i(-b+ia)X}\right] ; & X > 0 \\ \frac{1}{2}\left[e^{i(b-ia)X} + e^{i(-b-ia)X}\right] ; & X < 0 \end{cases} \tag{77}$$

Comparing with equation 75:

$\sigma = \xi = 1$

$\eta = 1,2$

$\tau = 1,2$

$C_{1,1} = C_{2,1} = \frac{1}{2i}$

$D_{1,1} = D_{2,1} = -\frac{1}{2i}$

$\kappa_{\eta=1} = b + ia$

$\kappa_{\eta=2} = b + ia$

$\kappa_{\lambda=1} = b - ia$

$\kappa_{\lambda=2} = -b - ia$

Try the examples of Paragraph 5-1.5.2 with t replaced by $|\tau|$.

d. Response Solutions — The structure is a simply-supported, uniform beam of length L excited by any of the random loads given in equations 72, 73, and 74. The cross-correlation function for the displacement response at (x,t) and (x´,t´) is given by:

$$W(x,x',\tau) = \sum_n \sum_m \frac{\psi_n(x)\,\psi_m^*(x')}{(2\pi)^4\,\mu^2\,\omega_n\omega_m} A_{n,m}(L)\,B_{n,m}(\tau) \tag{78}$$

where:

$W(x,x',\tau)$ = response cross correlation function

$$\psi_n(x) = \psi_n^*(x) = \sqrt{\frac{2}{L}} \sin k_n x, \quad k_n = \frac{n\pi}{L} \qquad \text{(normalized mode shape)}$$

$$\psi_m(x) = \sqrt{\frac{2}{L}} \sin k_m x, \quad k_m = \frac{m\pi}{L}$$

n,m = mode numbers (1,2,3 ...)

μ = linear mass density

ω_n = natural frequencies

$$A_{n,m}(L) = \frac{2\pi^2}{L}\left[D_{n,m}(k_m,k_n,L) + D_{n,m}(-k_m,-k_n,L) - D_{n,m}(-k_m,k_n,L) - D_{n,m}(k_m,-k_n,L)\right] \tag{79}$$

$$D_{n,m}(k_m,k_n,L) = \frac{2\pi i}{k_n - k_m}\left\{ e^{i(k_m-k_n)L}\left[R_1^+(k_m) + R_1^-(k_n)\right] - \left[R_1^+(k_n) + R_1^-(k_m)\right] + \sum_{\lambda=1}\sum_{\xi=1}\sum_{h=0}^{\xi-1} \frac{D_{\lambda,\xi}}{h!} e^{i(k_m-\kappa_\lambda)L}(-iL)^h \cdot \left[(k_m-\kappa_\lambda)^{h-\xi} - (k_n-\kappa_\lambda)^{h-\xi}\right] - \sum_{\eta=1}\sum_{\sigma=1}\sum_{h=0}^{\sigma-1} \frac{C_{\eta,\sigma}}{h!} e^{-i(\kappa_n-\kappa_\eta)L}(iL)^h \left[(k_m-\kappa_\eta)^{h-\sigma} - (k_n-\kappa_\eta)^{h-\sigma}\right]\right\} \tag{80}$$

$$D_{n,m}(k_m,k_n,L)\Big|_{m=n} = 2\pi i \left\{ (-iL)\,\hat{R}_1(k_n) \right.$$

$$- \sum_{\lambda=1} \sum_{\xi=1} D_{\lambda,\xi} \frac{\xi}{(k_n - \kappa_\lambda)^{\xi+1}}$$

$$+ \sum_{\eta=1} \sum_{\sigma=1} C_{\eta,\sigma} \frac{\sigma}{(k_n - \kappa_\eta)^{\sigma+1}}$$

$$+ \sum_{\lambda=1} \sum_{\xi=1} \sum_{h=0}^{\xi-1} \frac{D_{\lambda,\xi}}{h!} e^{i(k_n-\kappa_\lambda)L} \frac{(\xi - h)(-iL)^h}{(k_n - \kappa_\lambda)^{\xi-h+1}}$$

$$\left. - \sum_{\eta=1} \sum_{\sigma=1} \sum_{h=0}^{\sigma-1} \frac{C_{\eta,\sigma}}{h!} e^{-i(k_n-\kappa_\eta)L} \frac{(\sigma - h)(iL)^h}{(k_n - \kappa_\eta)^{\sigma-h+1}} \right\} \quad (81)$$

$\hat{R}_1(k_n)$, $R_1^{\pm}(k_m)$, $R_1^{\pm}(k_n)$ — given by equation 75 with k replaced by k_m or k_n, e.g.,

$$R_1^{-}(k_m) = \sum_{\lambda=1} \sum_{\xi=1} D_{\lambda,\xi} (k_m - \kappa_\lambda)^{-\xi}.$$

$$B_{n,m}(\tau > 0) = \pi i \left\{ e^{i\Omega_{2,n}\tau} \hat{R}_2(\Omega_{2,n})\, \hat{g}_m(-\Omega_{2,n}) \right.$$

$$- e^{i\Omega_{1,n}\tau} \hat{R}_2(\Omega_{1,n})\, \hat{g}_m(-\Omega_{1,n})$$

$$- \frac{1}{2} \sum_{\alpha=1} \sum_{\beta=1} \sum_{h=0}^{\beta-1} \sum_{\ell=0}^{\beta-h-1} \frac{(-1)^{\beta-h}}{h!} A_{\alpha,\beta}\, e^{i\nu_\alpha \tau} (i\tau)^h$$

$$\cdot \left[(\nu_\alpha - \Omega_{2,n})^{-\ell-1} - (\nu_\alpha - \Omega_{1,n})^{-\ell-1} \right]$$

$$\left. \cdot \left[(\nu_\alpha + \Omega_{1,m})^{\ell+h-\beta} - (\nu_\alpha + \Omega_{2,m})^{\ell+h-\beta} \right] \right\} \qquad (82)$$

$$B_{n,m}(\tau < 0) = \pi i \left\{ e^{-i\Omega_{2,m}\tau} \hat{R}_2(-\Omega_{2,m})\, \hat{g}_n(-\Omega_{2,m}) \right.$$

$$- e^{-i\Omega_{1,m}\tau} \hat{R}_2(-\Omega_{1,m})\, \hat{g}_n(-\Omega_{1,m})$$

$$+ \frac{1}{2} \sum_{\gamma=1} \sum_{\varepsilon=1} \sum_{h=0}^{\varepsilon-1} \sum_{\ell=0}^{\varepsilon-h-1} \frac{(-1)^{\varepsilon-h}}{h!} B_{\gamma,\varepsilon}\, e^{i\nu_\gamma \tau} (i\tau)^h$$

$$\cdot \left[(\nu_\gamma - \Omega_{2,n})^{-\ell-1} - (\nu_\gamma - \Omega_{1,n})^{-\ell-1} \right]$$

$$\left. \cdot \left[(\nu_\gamma + \Omega_{1,m})^{\ell+h-\varepsilon} - (\nu_\gamma + \Omega_{2,m})^{\ell+h-\varepsilon} \right] \right\} \qquad (83)$$

$$B_{n,m}(\tau = 0) = \pi i \left[R_2^- (\Omega_{2,n})\, \hat{g}_m(-\Omega_{2,n}) - R_2^- (\Omega_{1,n})\, \hat{g}_m(-\Omega_{1,n}) \right.$$

$$\left. - R_2^+(-\Omega_{1,m})\, \hat{g}_n(-\Omega_{1,m}) + R_2^+(-\Omega_{2,m})\, \hat{g}_n(-\Omega_{2,m}) \right] \qquad (84)$$

$$\left.\begin{matrix} \Omega_{1,n} \\ \Omega_{2,n} \end{matrix}\right\} = \pm\omega_n + i a_n \quad ; \quad \left.\begin{matrix} \Omega_{1,m} \\ \Omega_{2,m} \end{matrix}\right\} = \pm\omega_m + i a_m \qquad (85)$$

$$\hat{g}_n(\omega) = \frac{1}{2}\left[\frac{1}{\omega - \Omega_{2,n}} - \frac{1}{\omega - \Omega_{1,n}} \right] \qquad (86)$$

$\hat{g}_m(\omega)$ —given by equation 86 with n replaced by m.

$\hat{g}_m(-\Omega_{2,n})$, $\hat{g}_m(-\Omega_{1,n})$, $\hat{g}_n(-\Omega_{2,m})$, and $\hat{g}_n(-\Omega_{1,n})$

are given by equation 86 with ω replaced by the appropriate Ω; e.g.,

$$\hat{g}_m(-\Omega_{2,n}) = \frac{1}{2}\left[\frac{1}{\omega - \Omega_{2,m}} - \frac{1}{\omega - \Omega_{1,m}} \right]\Bigg|_{\omega = -\Omega_{2,n}}$$

$$= \frac{1}{2}\left[\frac{1}{-\Omega_{2,n} - \Omega_{2,m}} - \frac{1}{-\Omega_{2,n} - \Omega_{1,m}} \right]$$

$$\hat{g}_n(-\Omega_{1,m}) = \frac{1}{2}\left[\frac{1}{-\Omega_{1,m} - \Omega_{2,n}} - \frac{1}{-\Omega_{1,m} - \Omega_{1,n}} \right]$$

$\hat{R}_2(\Omega_{2,n})$, $\hat{R}_2(\Omega_{1,n})$, $\hat{R}_2(-\Omega_{2,m})$, $\hat{R}_2(-\Omega_{1,m})$, $R_2^-(\Omega_{2,n})$, $R_2^-(\Omega_{1,n})$, $R_2^+(-\Omega_{1,m})$, $R_2^+(-\Omega_{2,m})$, are given by equation 76 with ω replaced by the appropriate Ω; e.g.,

$$R_2^-(\Omega_{1,n}) = \sum_{\gamma=1} \sum_{\epsilon=1} B_{\gamma,\epsilon}\, (\Omega_{1,n} - \nu_\gamma)^{-\epsilon}$$

$$R_2^+(-\Omega_{2,m}) = \sum_{\alpha=1} \sum_{\beta=1} A_{\alpha,\beta} \quad (-\Omega_{2,m} - \nu_\alpha)^{-\beta}$$

$$\hat{R}_2(-\Omega_{2,m}) = \sum_{\alpha=1} \sum_{\beta=1} A_{\alpha,\beta} \quad (-\Omega_{2,m} - \nu_\alpha)^{-\beta}$$

$$+ \sum_{\gamma=1} \sum_{\varepsilon=1} B_{\gamma,\varepsilon} \quad (-\Omega_{2,m} - \nu_\gamma)^{-\varepsilon}$$

Note — The terms in equation 78 for which $m \neq n$ are the so-called cross-terms.

e. Special Cases — Response autocorrelation function:

set $x = x'$ in equation 78

$$W(x,x,\tau) = \sum_n \sum_m \frac{\psi_n(x)\, \psi_m^*(x)}{(2\pi)^4\, \mu^2\, \omega_n \omega_m} A_{n,m}(L)\, B_{n,m}(\tau) \qquad (87)$$

Response to spatial correlation:

set $\tau = 0$ in equation 78, using equation 84

$$W(x,x',0) = \sum_n \sum_m \frac{\psi_n(x)\, \psi_m^*(x')}{(2\pi)^4\, \mu^2\, \omega_n \omega_m} A_{n,m}(L)\, B_{n,m}(0) \qquad (88)$$

Response mean square value at position x:

set $x = x'$, $\tau = 0$ in equation 78 and use equation 84

$$W(x,x,0) = \sum_n \sum_m \frac{\psi_n(x)\, \psi_m^*(x)}{(2\pi)^4\, \mu^2\, \omega_n \omega_m} A_{n,m}(L)\, B_{n,m}(0) \qquad (89)$$

Response mean square value at position x in the nth mode:

set $x = x'$, $\tau = 0$, $n = m$ in equation 78, using equation 79 with equations 81 and 84 with $n = m$; the individual terms in the nth sum are of the form:

$$W_n(x,x,0) = \frac{|\psi_n(x)|^2}{(2\pi)^4 \mu^2 \omega_n^2} A_{n,n}(L) B_{n,n}(0) \qquad (90)$$

To calculate any of the above response averages (given a particular random load) first identify the load parameters as in the example of the previous section and substitute the particular values in equations 79, and 82 through 84. The response averages are given by equations 78, and 87 through 90.

f. Examples

1. Find the mean square response in the nth mode of a beam excited by a random field whose cross-correlation function is:

$$R(X,\tau) = e^{-a|X|} \cos bX \, e^{-c|\tau|} \quad ; \quad a,b,c = \text{constants} \qquad (91)$$

Using the results of the example of the previous section (equation 77):

$$\sigma = \xi = 1$$

$$\eta = 1,2$$

$$\lambda = 1,2$$

$$C_{1,1} = C_{2,1} = \frac{1}{2i}$$

$$D_{1,1} = D_{2,1} = -\frac{1}{2i}$$

$$\kappa_{\eta=1} = b + ia$$

$$\kappa_{\eta=2} = -b + ia$$

$$\kappa_\lambda = \kappa_\eta^*$$

Also, in this case:

$$R_2(\tau) = \begin{cases} e^{-a\tau} , & \tau > 0 \\ e^{a\tau} , & \tau < 0 \end{cases}$$

so that, by comparison with equation 74:

$$\alpha = \beta = \gamma = \varepsilon = 1$$

$$A_{1,1} = \frac{1}{i}$$

$$B_{1,1} = -\frac{1}{i}$$

$$\nu_{\alpha=1} = ia$$

$$\nu_{\gamma=1} = -ia$$

The mean square response is given by equation 90 with equations 79, 81, and 84 with n = m under the substitution of the above constants.

2. Find the response cross-correlation function at $\tau > 0$ for the random load describable by equation 91.

The response is given by equation 78. For the terms $n \neq m$, substitute the constants of the previous example in equations 79, 80 and 82; the n = m terms are given by equations 79 and 82 with n = m using equation 81.

In substituting the above constants in equations 80 through 82 we note that, since $\sigma = \xi = 1$, $\beta = 1$, only the terms $h = 0$, $\ell = 0$ contribute. For instance, equation 80 now reads:

$$D_{n,m}(k_m,k_n,L) = \frac{2\pi i}{k_n - k_m}\left\{ e^{i(k_m-k_n)L}\left[R_1^+(k_m) + R_1^-(k_n)\right]\right.$$

$$- \left[R_1^+(k_n) + R_1^-(k_m)\right]$$

$$+ \sum_{\lambda=1}^{2} D_{\lambda,1}\, e^{i(k_m-\kappa_\lambda)L}\left(\frac{1}{k_m - \kappa_\lambda} - \frac{1}{k_n - \kappa_\lambda}\right)$$

$$\left. - \sum_{\eta=1}^{2} C_{\eta,1}\, e^{-i(k_n-\kappa_\eta)L}\left(\frac{1}{k_m - \kappa_\eta} - \frac{1}{k_n - \kappa_\eta}\right)\right\}$$

$$R_1^+(k_m) = \sum_{\eta=1}^{2} C_{\eta,1}\,(k_m - \kappa_\eta)^{-1}$$

$$R_1^+(k_n) = \sum_{\eta=1}^{2} C_{\eta,1}\,(k_n - \kappa_\eta)^{-1}$$

$$R_1^-(k_m) = \sum_{\lambda=1}^{2} D_{\lambda,1}\,(k_m - \kappa_\lambda)^{-1}$$

$$R_1^-(k_n) = \sum_{\lambda=1}^{2} D_{\lambda,1}\,(k_n - \kappa_\lambda)^{-1}$$

where we used equation 75. Now, expand the η and sums above and substitute the values for the constants $D_{1,\lambda,\xi}$, $C_{\eta,1}$, κ_λ, κ_η. Similarly, the remaining terms in the solution can be written down immediately.

5-1.6.2.5 Random, Nonconvecting Pressure Fields - Response of Plates

a. Load—Let x_1, x_2 be the two cartesian axes. The pressure-field cross correlation function is assumed to have the form:

$$R(x_1,x_1',x_2,x_2',t,t') = R_{11}(X_1)\, R_{12}(X_2)\, R_2(\tau) \tag{92}$$

$$X_1 = x_1 - x_1'$$

$$X_2 = x_2 - x_2'$$

$$\tau = t - t'$$

$$R_{11}(X_1) = \begin{cases} i \sum_{\eta=1} \sum_{\sigma=1} \frac{C_{1,\eta,\sigma}}{(\sigma-1)!}\, e^{i\kappa_{1,\eta}X_1}\, (iX)^{\sigma-1}, & X > 0 \\ -i \sum_{\eta=1} \sum_{\sigma=1} \frac{D_{1,\lambda,\xi}}{(\xi-1)!}\, e^{i\kappa_{1,\lambda}X_1}\, (iX_1)^{\sigma-1}, & X < 0 \end{cases} \tag{93}$$

$R_{1,2}(X_2)$ — as in equation 93 with subscript 1 replaced by subscript 2

$R_2(\tau)$ — as in equation 74

C,D — constants, in general, complex

$\kappa_{1,\eta}$ — complex constant with positive imaginary part

$\kappa_{1,\lambda}$ — complex constant with negative imaginary part

Note that the two-dimensional, spatial, pressure-correlation is simply the product of two one-dimensional correlation functions $R_{11}(X_1)\ R_{12}(X_2)$, each being of the precise form considered in the previous section. The corresponding Fourier transforms are given by equation 75 with the subscript "1" or "2" inserted, respectively. For instance, the Fourier transform of $R_{12}(X_2)$ is given by:

$$\hat{R}_{1,2}(k_2) = \sum_{\eta=1} \sum_{\sigma=1} C_{2,\eta,\sigma}\ (k_2 - \kappa_{2,\eta})^{-\sigma}$$

$$+ \sum_{\lambda=1} \sum_{\xi=1} D_{2,\lambda,\xi}\ (k_2 - \kappa_{2,\lambda})^{-\xi} \qquad (94)$$

The Fourier transform of $R_2(\tau)$ is given by equation 76.

Note — As was the case in the previous section, the assumption of homogeneity (stationarity) in both space and time implies that the pressure-cross-correlation function is defined in the whole range of the variables; truncated correlation functions can not, by definition, be homogeneous functions of the variables. The response solutions which follow are not applicable to truncated random fields.

b. Response Solutions — The cross-correlation function for the displacement response at (x_1,x_2,t), (x_1',x_2',t') of a simply supported, uniform plate is given by:

$$W(x_1,x_1',x_2,x_2',\tau) = \sum_{n} \sum_{m} \sum_{p} \sum_{q} \frac{\psi_{nm}(x_1,x_2)\ \psi^{*}_{pq}(x_1',x_2')}{(2\pi)^7\ \mu^2\ \omega_{nm}\ \omega_{pq}}$$

$$\cdot\ A_{n,p}(L_1)\ A_{m,q}(L_2)\ B_{nm,pq}(\tau) \qquad (95)$$

$W(x_1,x_1',x_2,x_2',\tau)$ = displacement-response cross correlation function,

n,p = mode numbers in the x_1 - direction (=1, 2, 3, ...)

m,q = mode numbers in the x_2 - direction (=1, 2, 3, ...)

ω_{nm}, ω_{pq} = natural frequencies

μ = surface mass density

$\tau = t - t'$

$$\psi_{nm}(x_1x_2) = \frac{2}{\sqrt{L_1L_2}} \sin k_n x_1 \sin k_m(x_2) \qquad \text{(normalized mode shapes)}$$

$$= \psi^*_{nm}(x_1,x_2) \ ;$$

$$k_n = \frac{n\pi}{L_1}$$

$$k_m = \frac{M\pi}{L_2}$$

$A_{n,p}(L_1)$ - given by equations 79 through 81 with m replaced by p and L replaced by L_1

$A_{m,q}(L_2)$ - given by equations 79 through 81 with n replaced by m, m replaced by q and L replaced by L_2

$B_{nm,pq}(\tau)$ - given by equations 82 through 84 with n replaced by nm and m replaced by pq.

c. Special Cases

Response autocorrelation function:

set $x_1 = x_1'$, $x_2 = x_2'$ in equation 95

Response to spatial correlation:

set $\tau = 0$ in equation 78

Response mean square value at position x_1, x_2:

set $x_1 = x_1'$, $x_2 = x_2'$, $\tau = 0$ in equation 95

Response mean-square-value at position x_1, x_2 in the nmth mode:

set $x_1 = x_1'$, $x_2 = x_2'$, $\tau = 0$, $p = n$, $q = m$

in equation 95.

To calculate any of the above averages for a particular random load of interest, first identify the load parameters as in the examples of the one-dimensional case (Paragraph 5-1.5.2) and substitute these particular values in the above equations.

If the plate is orthotropic, the above solutions hold, with the appropriate values for the natural frequencies and damping constant.

5-1.6.2.6 Random, Nonconvecting Fields—Response of Cylindrical Shells

a. Load—For the coordinate system of the figure in Paragraph 5-1.5.2, we assume the pressure cross-correlation function in the form:

$$R(z,z',\theta,\theta',t,t') = R_1(Z)\, R_3(\theta)\, R_2(\tau) \qquad (96)$$

$$Z = z - z'$$

$$\theta = \theta - \theta'$$

$$\tau = t - t'$$

$R_1(Z)$ —as in equation 73 with X replaced by Z

$R_2(\tau)$ —as in equation 74

$$R_3(\theta) = \sum_{j=1} \sum_{r=0} d_{j,r} \; e^{-a_j|\theta|} \; |\theta|^r \qquad (97)$$

$d_{j,r}$ — constant coefficients

$$a_j = a_j' \pm i a_j'' \quad ; \quad a_j' > 0$$

b. Examples —Try example (a) of Paragraph 5-1.5.2 with θ replaced by $|\theta|$ and the example of equation 77 with X replaced by Z.

c. Response Solutions —The cross-correlation function for the displacement response at (z,θ,t), (z',θ',t') of a simply-supported, uniform, isotropic/orthotropic circular-cylindrical shell is given by:

$$W(z,z',\theta,\tau) = \sum_n \sum_m \sum_p \sum_q \varepsilon_m \, \varepsilon_q \, \frac{\psi_n(z) \, \psi_p^*(z')}{(2\pi)^6 \, \mu^2 \, \omega_{nm} \, \omega_{pq}}$$

$$A_{n,p}(L) \; K_{m,q}(\theta) \; B_{nm,pq}(\tau) \qquad (98)$$

$W(z,z',\theta,\tau)$ = displacement-response cross-correlation function

n,p = mode numbers in axial direction (=1, 2, 3 ...)

m,q = mode numbers in circumferential direction (=0, 1, 2 ...)

ω_{nm}, ω_{pq} = natural frequencies

μ = surface mass density

τ = $t - t'$

$$\psi_n(z) = \frac{2}{\sqrt{L}} \sin k_n z, \qquad k_n = \frac{n\pi}{L} \qquad \text{(normalized mode shapes)}$$

$$\epsilon_m = \begin{cases} 1, & m = 0 \\ 2, & \text{all } m > 0 \end{cases}, \qquad \epsilon_q = \begin{cases} 1, & q = 0 \\ 2, & \text{all } q > 0 \end{cases}$$

$A_{n,p}(L)$ —given by equations 79 through 81 with m replaced by p

$B_{nm,pq}(\tau)$ —given by equations 82 through 84 with n replaced by nm and m replaced by pq

$$K_{m,q}(\theta) = \pi \delta_{mq} \cos m\theta \sum_{j=1} \sum_{r=0} d_{j,r}\, r! \left\{ \left[(a_j + im)^{-r-1} + (a_j - im)^{-r-1} \right] - (-1)^m e^{-a_j \pi} \sum_{\ell=0}^{r} \frac{\pi^\ell}{\ell!} \left[(a_j + im)^{\ell-r-1} + (a_j - im)^{\ell-r-1} \right] \right\} \tag{99}$$

$$\delta_{mq} = \begin{cases} 1, & m = q \\ 0, & m \neq q \end{cases}$$

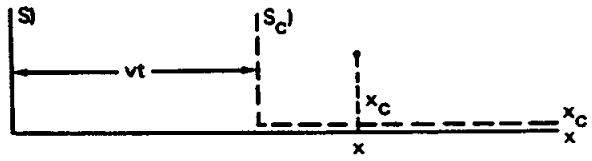
$$\theta = \theta - \theta'$$

d. Remark — The response solutions of this section are applicable to isotropic or orthotropic shells (with the proper values for the constants ω_{nm}, a_{nm}) and all other shells which can be approximated by same (e.g., ring-stiffened shells, truncated conical shells, etc.).

5-1.6.2.7 Random, Convecting Fields — Response of Beams, Plates Cylindrical Shells

a. Load — We let the random loads of the previous section move over the structure with constant velocity v. A consequence of the convecting nature of the field is the coupling of the space and time coordinates in equation 71. To see this, let us review briefly the Galilean transformation which connects two coordinate-systems in relative motion.

Let two frames of reference S and S_c coincide at some instant of time, say t = 0. We allow S_c to move with a velocity v to the right, so that, at a time t later it has travelled a distance vt.



A point x_c in S_c now occupies the position $x = x_c + vt_c$ in the S frame of reference. On the other hand, time is invariant, i.e., $t = t_c$, so that we have the transformation pairs:

$$x = x_c + vt \qquad x_c = x - vt$$

$$t = t_c \qquad t_c = t \qquad (100)$$

Let us associate the frame of reference S with a structure and the frame S_c with the pressure field (i.e., the pressure is at rest with respect to S_c but moves with velocity v with respect to S). We call S_c the "rest frame" of the pressure field. If in S_c the excitation is describable by some cross correlation $R_c(x_c - x_c', t - t')$, in the rest frame of the structure ", the excitation correlation is given by $R[(x - x') - v(t - t'), t - t']$ according to equation 100. It is this correlation function that the structure "sees," not R_c.

If we assume that in its rest frame the pressure correlation is separable into the product of functions of space and time -- equation 72 -- in the frame of reference of the structure:

$$R = R_1 \left[(x - x') - v(t - t')\right] R_2 (t - t') \tag{101}$$

again according to equation 100. We see, then, that the space and time variables are now coupled, even though in the rest frame of the excitation they are not.

Similarly, in the language of k and ω, even though the excitation spectrum is separable in k and ω in the S_c frame of reference, in the S frame it is not. In fact, it is a simple matter to show that the one-dimensional excitation spectrum in the frame of the structure is given by:

$$\hat{R}(k,\omega) = \hat{R}_1(k)\, \hat{R}_2(\omega + kv) \tag{102}$$

Thus, in the case of convecting random fields, the class of functions given by equation 73 is modified by replacing X by X - vt. In the case of a plate, we assume that the direction of the velocity v is along one of the principal axes, say x_1, so that equation 92 now reads:

$$R = R_{11}(X - vt)\, R_{12}(X_2)\, R_2(\tau) \tag{103}$$

where R_{11}, R_{12} are given by equation 93 with X_1 replaced by $X_1 - vt$.

Similarly, in the case of a cylindrical shell, we assume that the pressure field moves along the axial direction, so that equation 96 reads:

$$R(z,z',\theta,\theta',t,t') = R_1(Z - v\tau)\, R_3(\Theta)\, R_2(\tau) \tag{104}$$

Here, R_1 is given by equation 73 with X replaced by (Z - vt).

In all the foregoing cases, experimental definition of convecting random fields consists of taking measurements in the frame of reference of the structure (the "S-frame" of the figure); that is, one measures the cross-correlation R not R_c. More often than not, however, the data is reported in the frame of reference of the pressure field (the "S_c-frame" of the figure).

This is done by shifting the experimental points by increments ($v\tau$) along the direction of convection; that is, what is reported is R_c not the measured R. If this is the case, to use the response solutions of this section one need only identify the pressure parameters in precisely the way of the previous section. If, however, the raw data are given (that is, R) we must first transform the data to the rest frame of the pressure field and then identify the pressure parameters.

b. Example - A convecting field in the x-direction is defined in the frame of reference of the structure by:

$$R_1(X) = \exp\left(-a|X - v\tau|\right) \cos b\,(X - v\tau) \; ; \quad a,b = \text{constants}$$

First we transform to the rest frame of the pressure field by replacing (X - vt) by X:

$$R_{1c}(X) = \exp\left(-a|X|\right) \cos bX$$

We now identify the parameters in equation 73 as we did in the example of equation 77. It is these parameters that we substitute in the response solutions.

c. Response Solutions - The aforementioned coupling of the spatial and temporal variables (equation 100) leads to response solutions which are far too lengthy to be conventionally quoted in the handbook. The reader is referred to sections 3.0 and 4.2 of Reference 1 where these solutions are given. It will suffice to say here that to use them, one need only identify the pressure field parameters in the aforementioned manner and substitute them in the response solutions given in the above reference.

5-1.6.2.8 References

1. M. G. Cottis, P. Collas, *The Response of Linear Structures to Classes of Pressure Fields*, Douglas Report DAC-58686 (September 1968).

2. D. Kelly, *The Response of Linear Structures to Classes of Pressure Fields - Computer Program DA24*, Douglas Report DAC-61883 (April 1969).

5-1.6.3 Distributed Structure With Separable Elastic and Inelastic Solutions

Reference - *Response of Distributed Viscoelastic Structures to Arbitrary External Loads*, Professor K. C. Valanis

Dr. Valanis has formulated a general matrix method for the analysis of either elastic or viscoelastic structures subjected to arbitrary external loads. In this method, the elastic solution is established first and the viscous solution is based thereon. The viscosity of the material is obtained by either the relaxation or the creep function of a viscous model which adequately describes the material behavior. The method is founded on the three-dimensional theory of viscoelasticity, and is equally applicable to all viscoelastic structures (beams, plates, or shells). It includes initial displacements and velocities.

On the basis of the above theory, Dr. Valanis has considered a number of examples: one-dimensional applications include wave propagation in a finite rod and wave propagation in elastic beams, while a two-dimensional application is the wave propagation in a plane, reinforced, elastic sheet.

It should be noted that the above examples are not complete in their treatment. They merely clarify a number of points in the proposed theory. The theory, however, is complete in its formulation; for example, see Valanis' Reports 3 and 5 in Reference File 4.2 (39). To be used in an efficient manner, the method should be programmed for the computer.

The following procedural steps detail the method of solution proposed.

a. The given structure is divided into a number of elements. To every element and to every joint is assigned a set of coordinates.

b. The kinematic conditions between the joint and element coordinates are established from purely kinematic conditions.

c. A deflection or displacement distribution is assigned to every element.

d. The stress-strain relations for the elements are established.

e. The mass and stiffness matrices for the individual elements and the complete structure are computed.

f. The static solution is obtained in terms of the stiffness and input.

g. The mass matrix is diagonalized by means of an orthogonal transformation.

h. The real eigenvalue problem resulting from the application of the variational principle is solved.

i. The transient response is then obtained by means of the above information.

j. A viscous model is assumed and the typical Volterra equation is formulated and solved

k. The complete response including damping is finally formed.

For further details, see Memorandum A2-260-ABD1-244, dated 9-22-67.

5-2 COMPUTER ANALYSIS

In many cases the problem is sufficiently complex that it cannot be treated without using the computer. In general, computer analyses consist of four distinct operations:

a. Selecting a discrete element idealization of a structure.

b. Generating the matrices associated with the structural idealization.

c. Uncoupling the problem using an eigenvalue transformation.

d. Estimating the response of the structure.

The method used to predict the response depends on the nature of available forcing function data. If a force history is available, transient response can be calculated. If a shock spectrum is available, a peak response envelope can be estimated. If a probabilistic force description is available, a probabilistic response description can be calculated. If no force description is available, response may sometimes be estimated by extrapolation of existing data.

5-2.1 Structural Idealization

A structural idealization is a procedure by which a complicated structure is modeled as a set of structural elements (such as beams or panels) connected at points (or nodes). The deflection of these nodes is defined by a set of coordinates. In general, there may be six coordinates at a node; three rotations and three deflections. Deflections at points other than the nodes are assumed to be the static deflection associated with the deflections of the nodes. In this manner, the deflection at any point in the structure is represented by a finite set of coordinates. These coordinates must be independent. To reduce the problem size, coordinates may sometimes be eliminated if the forces acting on them are small. The procedure is discussed more fully in Hurty and Rubenstein, *Dynamics of Structures*.

The following practices are often useful:

a. The sign convention is important for coordinates. A sketch showing all the coordinates and their sign conventions should be made at the outset of an analysis. Sign convention errors are often found to be responsible for negative eigenvalues and anomalous mode shapes.

b. Coordinates should be distributed more-or-less uniformly throughout the structure to provide a thorough representation of the structure.

c. Coordinates should be located at the specific points where response is desired.

d. The number of coordinates should be several times the number of modes desired to ensure sufficient definition of the mode shape.

e. The coordinates should be distributed in such a manner as to avoid inordinately large or small mass or flexibility values.

5-2.2 Parameter Matrices

Once the coordinate system has been selected, the mass and stiffness matrices can be formed. The stiffness matrix can be generated from the elemental stiffness matrices. Computer program DA02 generates a stiffness matrix for a beam. The redundant force programs SA40-SA76 handle more complex structures. G750 "Dynamos III" analyzes cylindrical shells and SA80 "Sabor-Drastic" analyzes shells of revolution. For more complex systems, it is generally advisable to have the strength group formulate the stiffness matrix.

The mass matrix can be formulated in several ways, the most common of which is to arbitrarily partition the structure between the nodes. The mass matrix for the coordinates at a node is developed by assuming the mass in the partition surrounding the node acts as a rigid body with its position determined by the coordinates at the node. DA02 generates mass matrices for beams, G750 for cylindrical shells, and SA80 for shells of revolution.

A more accurate representation is the consistent mass matrix. Here, the structure is assumed to have a velocity profile proportional to the static displacements associated with the coordinates. The (i, j) element of the mass matrix is determined as the second partial derivative of the kinetic energy with respect to the ith and jth coordinates. This procedure is discussed in more detail in DAC-59062 (September 1966) *The Modal Analysis of Structure Idealized as an Assembly of Discrete Elements*.

The damping is usually assumed proportional (i.e., a linear combination of the mass and stiffness matrices) and is only used in computing response in modal coordinates. If the damping is not proportional, complex modes can be computed by doubling the size of the problem. This technique is explained in detail in Chapter 9 of Hurty and Rubenstein *Dynamics of Structures*.

The forcing function is treated in a manner similar to that used for the mass matrix. Either the force can be integrated over a partition surrounding the node, or for consistent force, the virtual work done by the static configuration due to a virtual displacement of a coordinate is equated to the virtual work done by the generalized force associated with that coordinate.

5-2.3 Eigenvalue Problem

In matrix notation, the equilibrium equations relating the mass (M), stiffness (K), damping (C), coordinates (X), and the generalized forces (F) is written:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\}$$

In general, the matrices are not diagonal so the response at any coordinate depends on the response at other coordinates. Such an equation is termed coupled. It is often more convenient to deal with uncoupled equations. A transformation based on the eigenvectors of the system will uncouple the equations, e.g., substituting $\{X\} = [\phi]\{\eta\}$ and premultiplying by $[\phi]^T$, where ϕ is the matrix whose columns are the eigenvectors of the system yields

$$[\overline{M}]\{\ddot{\eta}\} + [\overline{C}]\{\dot{\eta}\} + [\overline{K}]\{\eta\} = \{\overline{F}\}$$

where $\overline{M}$, $\overline{C}$, $\overline{K}$ are diagonal matrices. If the damping is proportional, the eigenvectors of the undamped system are also the eigenvectors of the damped system. The undamped eigenvalue problem can be stated as: find a set of λ's and associated Y's such that

$$\lambda[M]\{Y\} = [K]\{Y\}$$

The λ's are the natural frequencies (in radians/second) squared and the Y's are the mode shapes. There are several computer routines available that solve the equation:

$$[D]\{Z\} = \lambda'\{Z\} ,$$

so the problem must be converted to this form if a solution is to be obtained. There are three methods by which this is usually accomplished:

a. The first method is to premultiply by $[M]^{-1}$ to obtain

$$\lambda\{Y\} = [M]^{-1}[K]\{Y\} .$$

This is now the desired form (with $\lambda' = \lambda$, and $[D] = [M]^{-1}[K]$). Although M, K are symmetric, $M^{-1}K$ is not (in general). In such a case, an unsymmetric eigenvalue routine must be used. UNSEIG, EIGENP, and

EIG8 are available from the Univac 1108 fortran library. Of these, UNSEIG appears to be the best for general use. Program MW08 uses UNSEIG and writes and accepts tapes compatible with program SA40. Generally, the unsymmetric routine is less accurate and slower than the symmetric routine. In a similar manner to the above, premultiplying by K^{-1} yields the desired form (with $\lambda' = 1/\lambda$ and $D = K^{-1}M$).

b. The second method results in a symmetric dynamical matrix [D]. First, the eigenvectors of the mass matrix are obtained. The eigenvectors are then normalized such that the diagonalized mass matrix is an identity matrix. If the transform $\{Y\} = [\psi]\{Z\}$ is made, and the resulting equation is premultiplied by $[\psi]^T$, the following equation results:

$$\lambda[\psi]^T [M] [\psi] \{Z\} = [\psi]^T [K] [\psi] \{Z\}$$

This is in an acceptable form (with $\lambda' = \lambda$, and $D = \psi^T K \psi$). In this case D is symmetric. BIGMAT and JAKE are available subroutines on the Univac 1108 fortran library which obtain eigenvalues for symmetric matrices. Of these, BIGMAT appears to be the best suited for general use. BIGMAT is used in DA02, SA40, and SA48. This procedure (b.) is used in DA02. Similarly, if the eigenvectors of the stiffness matrix are normalized so $\psi^T K\psi = I$ then:

$$\lambda[\psi]^T [M] [\psi] \{Z\} = [\psi]^T [K] [\psi] \{Z\}$$

is in a compatible form with $\lambda' = 1/\lambda$ and $D = \psi^T M \psi$.

c. The third method is similar to the second, and also results in a symmetric dynamical matrix. The mass matrix is decomposed into the product of a lower triangular matrix (L) and its transpose using the Cholesky method. This matrix is inverted and the coordinate transform

$$\{Y\} = [L^T]^{-1} \{Z\}$$

is introduced. Premultiplying by $[L]^{-1}$ yields:

$$\lambda[L]^{-1} [M] [L^T]^{-1} \{Z\} = [L]^{-1} [K] [L^T]^{-1} \{Z\}$$

This is in a compatible form with $\lambda = \lambda'$ and

$$[D] = [L]^{-1} [K] [L^T]^{-1}$$

(Since $[M] = [L] [L^T]$). Similarly, if K is decomposed, ($[K] = [L][L^T]$)

$$\lambda[L]^{-1} [M] [L^T]^{-1} \{Z\} = [L]^{-1} [K] [L^T]^{-1} \{Z\}$$

is in a compatible form with $\lambda' = 1/\lambda$ and

$$D = [L]^{-1} [M] [L^T]^{-1}$$

This method offers several advantages, namely:

1. The decomposition and inversion procedure is algebraic.
2. It is quite fast and accurate.
3. The problem is in a symmetrical form, and the more efficient symmetric routines can be used.

More information on eigenvalue routines can be found in the manual *The Eigenvalue Problem and Available Routines for its Solution,* dated April 28, 1969.

Run time varies roughly as the cube of the order of the eigenvalue problem as illustrated in Figure 5-41. Thus, reduction in the number of coordinates can result in considerable time savings. There is a large difference in run times between problems which can be solved in core, and those requiring tape. An attempt should always be made to reduce the problem to core size. Round-off error is related to the log of the ratio of the highest to lowest frequency, as illustrated in Figure 5-42. Since round-off error can make the results meaningless, eigenvalues and vectors should always be checked.

In general, increasing the order of the problem increases the eigenvalue spread, providing an additional incentive for reducing the problem size. One way of effectively reducing problem size is modal coupling, discussed in paragraph 5-2.4. One other opportunity for increasing accuracy is worthy of mention. If a flexibility matrix is available instead of a stiffness matrix, the low frequency modes of the system are the highest eigenvalues (if the flexibility matrix is not

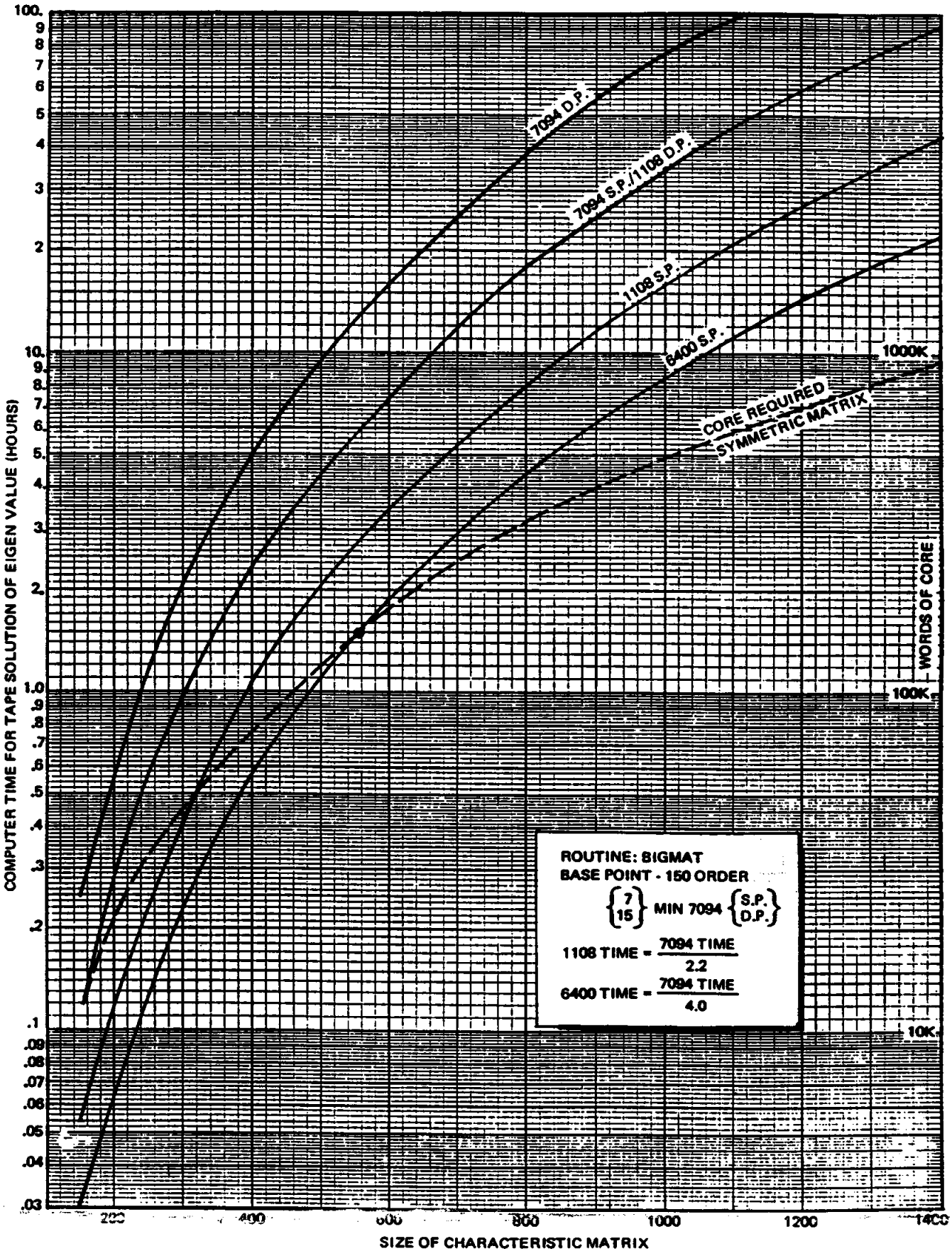


Figure 5-41. Computer Time and Storage Requirements for Eigen Solution of Characteristic Matrix.

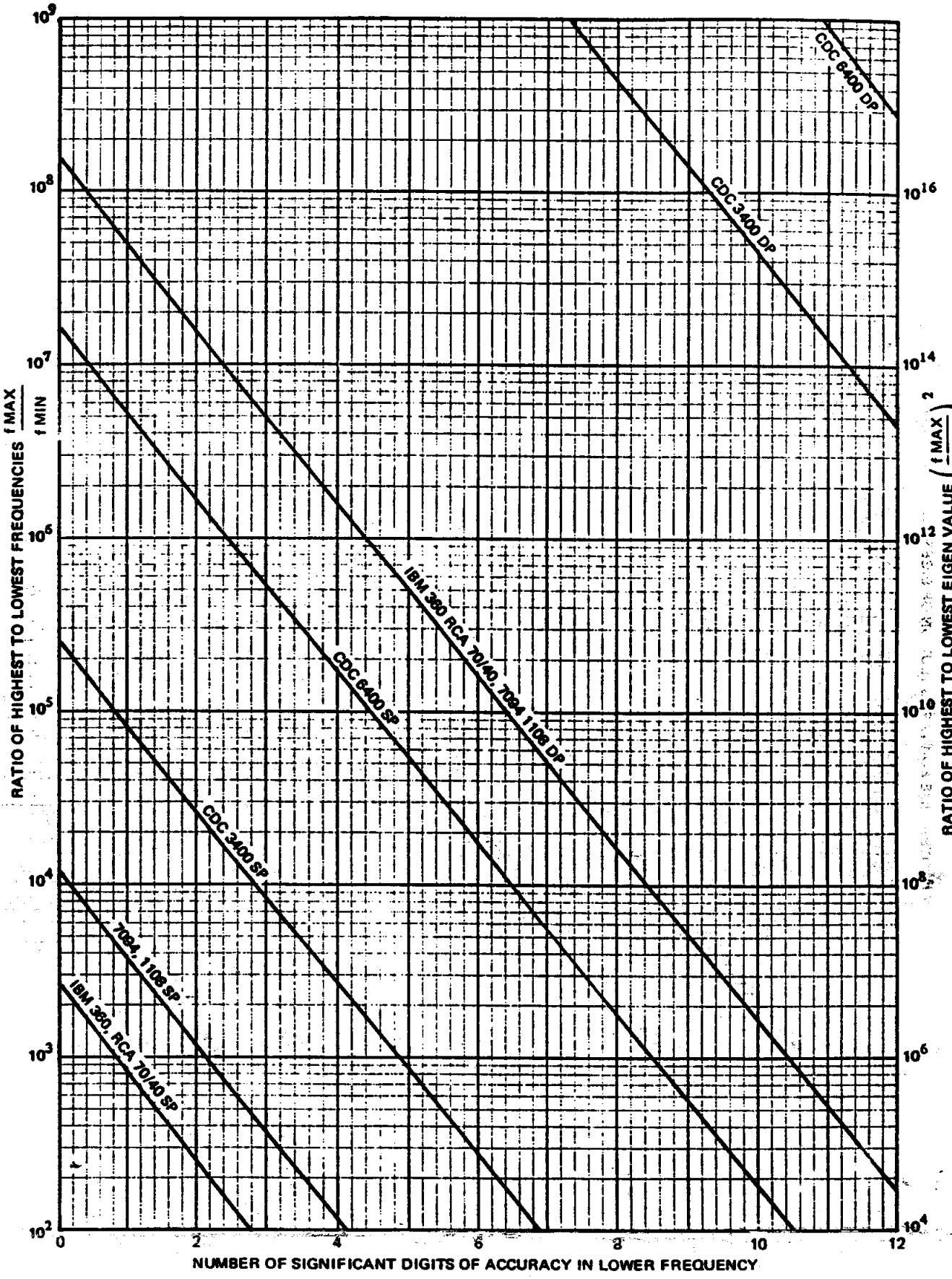


Figure 5-42. Frequency Ratio vs Accuracy for Eigenvalue Computation on Various Digital Computers.

inverted). Since the accuracy depends on the ratio of the highest frequency to the one of interest, these modes are the ones with the greatest accuracy.

5-2.4 Modal Coupling

One very simple method which may be used to reduce the size of an eigenvalue problem is to divide the problem into two parts, do each part separately, and then recombine them. In general, the dynamical matrix may be partitioned into an N1 order and an N2 order problem.

$$[D] = \left[\begin{array}{c|c} D_{11} & D_{12} \\ \hline D_{21} & D_{22} \end{array}\right] \begin{array}{l} \} \ N1 \\ \} \ N2 \end{array}$$

Let $[\psi_1]$ and $[\psi_2]$ be the eigenvectors of $[D_{11}]$ and $[D_{22}]$ and introduce the coordinate transform.

$$\{Y\} = \left\{\begin{array}{c} Y_1 \\ \hline Y_2 \end{array}\right\} = \left[\begin{array}{c|c} \psi_1 & 0 \\ \hline 0 & \psi_2 \end{array}\right] \left\{\begin{array}{c} M_1 \\ \hline M_2 \end{array}\right\} = [\psi] \{M\}$$

Since only the low frequency response is of interest, only the lowest modes of each partition need be retained. So ψ_1 and ψ_2 can be restricted to only M_1 and M_2 modes where M_1 and M_2 are much smaller than N_1 and N_2. [D] is transformed as follows:

$$[\psi^T][D][\psi] = \left[\begin{array}{ccc|ccc} \psi_1^T & D_{11} & \psi_1 & \psi_1^T & D_{12} & \psi_2 \\ \hline \psi_2^T & D_{21} & \psi_1 & \psi_2^T & D_{22} & \psi_2 \end{array}\right] \begin{array}{l} \} \ M_1 \\ \} \ M_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} \lambda_1 & & & \psi_1^T & D_{12} & \psi_2 \\ \hline \psi_2^T & D_{21} & \psi_1 & & \lambda_1 & \end{array}\right]$$

The eigenvalue problem is solved once more for the matrix $\psi^T D \psi$. The mode shapes ϕ diagonalize the matrix:

$$\left[\phi^T \psi^T D \psi \phi\right] = \left[\bar{\lambda}\right]$$

Thus the eigenvectors are $[\psi \phi]$ and the eigenvalues are $[\bar{\lambda}]$. In this way, the N-order eigenvalue problem has been reduced to solving an N1, an N2, and an M1 + M2 order problem. Each of these problems is approximately one half the order of the original problem.

This procedure can be extended in several directions. More than two partitions can be used, or the operations can be carried out on the mass and stiffness matrices instead of the dynamical matrix. Fixed modes can be used in place of the free modes of D. In many cases, the structure is composed of two or more well-defined substructures (i.e., a missile and a launch rail). In that case the division is obvious, and consists of running modes for the separate substructures (i.e., modes of the missile, and modes of the rail).

5-2.5 Transient Response

The matrix equations of motion are:

$$[M] \{\ddot{X}\} + [C] \{\dot{X}\} + [K] \{X\} = \{F\}$$

This is a set of N, coupled, second-order differential equations. When F is a given function of time, these equations can be solved by direct numerical integration using Mimic, CA28 (which also solves nonlinear equations), or using Sabor-Drastic (SA80).

The more usual technique is to uncouple the equations using the eigenvector transformation:

$$[\bar{M}] \{\ddot{\eta}\} + [\bar{C}] \{\dot{\eta}\} + [\bar{K}] \{\eta\} = [\phi]^T \{F\}$$

If the eigenvectors are normalized so that the modal mass is the identity matrix (or by premultiplying by $\bar{M}^{-1}$),

$$\{\ddot{\eta}\} + 2 [\omega] [\delta] \{\dot{\eta}\} + [\omega^2] \{\eta\} = [\phi]^T \{F\}$$

A set of uncoupled second-order differential equations are obtained. The ω's are the natural frequencies and the δ's are the fractions of critical damping. These can be solved using ordinary differential equation solution techniques. DA20 is designed to numerically solve these equations and then perform the modal summation. Dynamos III, G750, and DA16 do the same thing for certain closed-form forces on a cylindrical shell. SA48 and CA28 (Mimic) can be used for the smaller problems.

The relative merits of the two procedures are still a subject of discussion. The direct integration offers the advantage of eliminating the eigenvalue problem with the disadvantage of a more expensive integration scheme. The modal approach on the other hand,

a. Provides a rational method of introducing damping.

b. Is a more efficient method of integration for a set of uncoupled equations.

c. Provides results which can be refined by computing the response in more modes and simply adding this additional response to that already calculated (for fewer modes).

As a general rule of thumb, the forcing function should be defined for at least five points per cycle of the highest mode of interest. The output should be plotted with the same definition to fully describe the resultant response.

5-2.6 Response Based on Shock Spectra

In many cases, a detailed time history is not available, and the direct solution techniques of paragraph 5-2.5 cannot be applied. However, if a shock spectrum is available, it is possible to place an upper bound on the response. A shock spectrum (see Section 3, paragraph 3-3.8) is the maximum response of single degree-of-freedom systems to a given excitation as a function of the frequency of the oscillator. For base motion excitation, $X_o(t)$, the shock spectrum is the maximum of the solution to:

$$\ddot{X} + \omega^2 X = \omega^2 X_o$$

If a complex structure is subjected to a base motion excitation, the uncoupled equations can be written:

$$\{\ddot{\eta}\} + 2 [\omega] [\delta] \{\dot{\eta}\} + [\omega^2] \{\eta\} = \{S\}_{X_o} = [\omega^2] [\omega^2]^{-1} \{S\}_{X_o}$$

Comparing this with the shock spectrum, the maximum of n_i is s_i/ω_i^2 times the value of the shock spectrum. An upper bound on displacements can be obtained by summing the absolute values of the displacements in each mode. A similar procedure can be followed for obtaining upper bounds on loads or accelerations

The generalization of this procedure to several ground motions or other types of shock spectra (e.g., relative displacement shock spectra), is not completely trivial. The analyst should carefully develop the procedure for each case.

5-2.7 Random Response

Frequently, the forcing function is not deterministic in nature, and probabilistic techniques must be used to predict the response. The cross-power-spectral density (XPSD) of the response is computed from the XPSD of the forcing function using the relationship:

$$[S_x] = [\phi] \lceil Z \rfloor [\phi^T] [S_F] [\phi] \lceil \overline{Z} \rfloor [\phi^T]$$

Where S is the XPSD, ϕ is the matrix of mode shapes, and Z is the transfer function

$$Z_{ii} = \frac{1}{\left[\left(\omega_i^2 - \omega^2\right) + i\, 2\delta\, \omega\omega_i\right]}$$

and $\overline{Z}$ is the complex conjugate of Z. Note that S_x, Z, and S_F are functions of the frequency ω. The covariance matrix is given by:

$$[K] = \int_{-\infty}^{\infty} \left[S_x(\omega)\right] d\omega$$

The diagonal elements of the covariance matrix are the mean square displacements of the coordinates. This development is outlined in more detail in memos A2-260-ABD1-69-22, dated 23 January 1969, and A2-260-AED1-69-219, dated 18 August 1969.

Program MW11 generates a transfer function matrix at a set of frequencies including the natural frequencies, and interpolates a PSD for these frequencies. The input consists of natural frequencies and damping on an SA40 format tape. The appropriate matrix multiplications can then be done in SA40.

Computation of a full XPSD matrix requires large amounts of computer time. Care should be exercised in keeping these problems as small as possible.

5-2.8 Modal Scaling

In some cases, there is no information available about the forcing function. However, there may be flight data available from a similar vehicle. Under such circumstances, a crude estimate of the response can be obtained using the modal scaling technique.

The uncoupled equations of motion can be written:

$$\{\ddot{\eta}\} + [\omega^2]\{\eta\} = [MM]^{-1}[\phi]^T\{F\}$$

(where

$$[MM] = [\phi]^T[M][\phi]$$

is the modal mass).

If the forcing function can be associated with a particular position on the missile (e.g., engine vibrations, areas of high aerodynamically-induced pressures, etc.), the ith modal equation for forces at station j is given by

$$\ddot{\eta}^{(i)} + \omega_i^2 \eta^{(i)} = \frac{1}{MM_i} \phi_j^{(i)} F_j$$

The solution is given by:

$$\eta^{(i)} = \frac{\phi_j^{(i)}}{MM_i} G(F_j, \omega_i, \delta_i)$$

where G is a function of the forcing function, natural frequency, and system damping.

If the two missiles are sufficiently similar that the forcing function, natural frequency, and damping of the two missiles can be considered identical, then the solution for the second missile is related to the solution for the first missile by:

$$ {}_2\eta^{(i)} = \frac{{}_2\phi_j^{(i)}}{{}_2MM_i} \cdot \frac{{}_1MM_i}{{}_1\phi_j^{(i)}} \quad {}_1\eta^{(i)} $$

If the flight data on the first missile can be decomposed into the modal responses, the response at point K on the second missile is given by:

$$ {}_2Y_K = \sum_{i=1}^{N} {}_2Y_K^{(i)} = \sum_{i=1}^{N} {}_2\phi_K^{(i)} \quad {}_2\eta^{(i)} $$

For a single flight measurement at station ℓ with a modal decomposition:

${}_1Y_\ell^{(i)}$ the response ${}_1\eta^{(i)}$ is given by:

$$ \frac{{}_1Y_\ell^{(i)}}{{}_1\phi_\ell^{(i)}} \cdot $$

In this case, the predicted response in the ith mode is:

$$ {}_2Y_K^{(i)} = \frac{{}_1MM_i}{{}_2MM_i} \cdot \frac{{}_2\phi_j^{(i)} \; {}_2\phi_K^{(i)}}{{}_1\phi_j^{(i)} \; {}_1\phi_\ell^{(i)}} \quad {}_1Y_\ell^{(i)} $$

This method can be extended, in some cases, to include multiple forcing functions or multiple flight measurements. However, the problem of deriving modal amplitudes from flight data is not an easy one, and is subject to considerable error. It should be remembered that the modal scaling technique is a very approximate one, and should only be used in lieu of adequate forcing function data.

Section 7
TESTING

This section discusses the three major categories of tests (acoustical, shock, and vibration tests, in that order) and presents information which has proved to be useful in the performance of these tests at MDAC-WD. Information on test facility equipment is not emphasized, since the operation, maintenance, and scheduling of the equipment is the responsibility of the Engineering Laboratories and Services Department. However, for reference, a table summarizing the major characteristics of all types of facilities is given in Table 7-1. Comments on instrumentation which apply to all three types of tests are contained in paragraph 7-4.

7-1 ACOUSTICAL TESTS

Several references are available to augment the comments contained herein. These include accepted standards and manuals and are delineated in Sections 13 and 15. The tests described in this section are limited to acoustic excitation used to excite structural response.

7-1.1 Test Setup

Sound fields with a preferred direction of propagation are associated with the following usual types of test setups:

a. Distant high intensity source in the free field (open air)

b. Progressive wave tube

c. Terminated shroud or ducts

Such a sound field resembles (at least in some respects) the launch acoustic field and the in-flight, boundary-layer-induced, pressure fluctuations. Hence, these types of sound sources are often used for fatigue tests on primary structural panels and qualification or acceptance testing of sections, stages or the total vehicle.

The sound levels should always be surveyed close to and over the surface of the specimen. Usually, a broad band random signal is utilized to simulate the environment and induce the desired multi-mode response. In some special instances, however, the energy is band limited to simulate a high concentration of energy in a narrow band of frequencies.

The problem in the latter case is inducing a response representative of the actual in-flight situation when the boundary conditions are unique to the test setup. This latter problem may necessitate appropriately shifting the test band of energy in the frequency domain. Test spectrum levels may require correction if the criteria are based on a different pressure field than the one produced in the test facility (e.g., see Section 3, paragraph 3-2.3).

The other major acoustical test facility is the reverberation chamber, which produces a diffuse sound field (equal energy in all directions) throughout a large portion of the frequency domain, and simulates that sound field which is found within a reverberant enclosed structural volume excited externally (e.g., a primary vehicular shell).

For tests performed in the reverberation chamber, the test specimen is usually mounted so as to be completely immersed in a broad band, random, diffuse, sound field and should be isolated from all mechanical ties with the chamber. The suspension system should have a natural frequency of less than 5 Hz. In most cases, every surface of the test specimen should be exposed to the sound field. This is normally achieved by centrally locating the specimen in the test chamber. If the test chamber is rectangular, no major surface of the test specimen should be installed parallel to a chamber wall.

The sound pressure field is measured with the test item installed in the test chamber. Measurements are made by using one or more microphones to measure the sound field around the test item in proximity to each major dissimilar surface. The sound pressure spectrum is usually equalized at 6 dB or more below the test levels. The time required to conduct the survey should be much less than the final test time.

The small (approximately 1,000 cu. ft.) reverberation room at location A of MDAC-WD uses a transition section between the air horns and the room. This section can be converted to a small reverberant chamber by sealing it off with an existing concrete door. Small test specimens (ratio of chamber volume to specimen volume ≥ 10) can be tested to very high acoustic levels in this chamber.

Table 7-1. Facilities Comparisons

CAPABILITIES (PRESENT) \ FACILITY	RANDOM ACOUSTICAL TESTING																RANDOM VIBRATION TESTING								
	FIELD TESTS			BROAD BAND				SPLIT BROAD BAND (1)				SWEPT NARROW BAND				BROAD BAND			SPLIT NARROW BAND(11)			SWEPT NARROW BAND		
	"BATTLESHIP" FIRST FLIGHT (OBTAIN DYNAMIC DATA ALSO)	TEST ADJACENT TO ROCKET ENGINES	WIND TUNNEL TESTS	REVERBERATION CHAMBER	PROGRESSIVE WAVE TUBE	SHROUD CONTOURED TO SPECIMEN (3" GAP)	REVERBERATION CHAMBER WITH PROGRESSIVE WAVE SOURCE CONCENTRATED ON SMALL AREA	REVERBERATION CHAMBER	PROGRESSIVE WAVE TUBE	SHROUD, CONTOURED TO SPECIMEN (3" GAP)	REVERBERATION CHAMBER WITH PROGRESSIVE WAVE SOURCE CONCENTRATED ON SMALL AREA	REVERBERATION CHAMBER	PROGRESSIVE WAVE TUBE	SHROUD, CONTOURED TO SPECIMEN (3" GAP)	REVERBERATION CHAMBER WITH PROGRESSIVE WAVE SOURCE CONCENTRATED ON SMALL AREA	FIXTURE (10) MOUNTED, SINGLE, LARGE SHAKER, ONE LOCATION CONTROL ON FIXTURE	ADAPTER (12) MOUNTED, SINGLE, LARGE SHAKER, ONE (B) LOCATION CONTROL ON ADAPTER OR ON SPECIMEN FRAME	FRAME (13) MOUNTED, MULTI-SHAKERS IN RANDOM PHASE, PEAK OR AVG OF MULTIPLE CONTROLS ON SPECIMEN FRAMES (9) (10)	FIXTURE MOUNTED, SINGLE OR MULTI- IN-PHASE) SHAKERS ONE LOCATION CONTROL ON FIXTURE	ADAPTER MOUNTED, SINGLE OR MULTI-(IN-PHASE) SHAKERS ONE LOCATION CONTROL ON ADAPTER OR SPECIMEN FRAME	FRAME MOUNTED, MULTI- SHAKERS IN RANDOM PHASE, PEAK OR AVE OF MULTI- CONTROLS ON SPECIMEN FRAMES	FIXTURE MOUNTED, SINGLE OR MULTI-(IN-PHASE) SHAKERS, ONE LOCATION CONTROL ON FIXTURE	ADAPTER MOUNTED, SINGLE OR MULTI-(IN PHASE) SHAKERS ONE LOCATION CONTROL ON ADAPTER OR SPECIMEN FRAME	FRAME MOUNTED, MULTI- SHAKERS IN RANDOM PHASE, PEAK OR AVE. OF MULTI- CONTROLS ON SPECIMEN FRAMES
NOTES: (1) 0.8"S 1-5 THEN 0.8"S 5-7 (2) 1.0 GRMS OR 175 dB OA (3) e.g. VIBRATION & ACOUSTICAL (4) e.g. ACOUSTICAL & TEMPERATURE (5) SAMPLE EACH INPUT FOR SHORT DURATION (6) FOR 80% CORR AT 1000 Hz (7) WIND TUNNEL 1% CROSS SECTION & REVERB CHAMBER 10% OF VOLUME (8) ASSUMES 50% FLOOR ABSORBED BY ADAPTER (9) 6 SHAKERS OF 3000 Hz CAPABILITY (10) MODIFY TEST OR SPECIMEN DURING TEST (11) FIXTURE IS DEFINED HERE AS SUPPORT STRUCTURE BETWEEN THE SHAKER AND THE TEST SPECIMEN (12) ADAPTER IS DEFINED HERE AS A MODIFICATION TO THE TEST SPECIMEN TO PERMIT A SHAKER ATTACHMENT (13) FRAME IS DEFINED AS A STRUCTURAL MEMBER OF THE TEST SPECIMEN TO WHICH THE SHAKER IS ATTACHED																								
MAX ALLOWABLE WT OF SPECIMEN & FIXTURE	FLIGHT WEIGHT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	300#	150 #	30 #	400 #	200 #	40 #	600 #	300 #	60 #
MAX ALLOWABLE SPECIMEN SIZE (7)	NO LIMIT	NO LIMIT	(7)	1000ft³	NO BLOCKAGE	NO LIMIT	1000ft³	1000ft³	NO BLOCKAGE	NO LIMIT	1000ft³	1000ft²	NO BLOCKAGE	NO LIMIT	1000ft³	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT	NO LIMIT
THREE DIMENSIONAL FORCING FUNCTION	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	NO	NO	YES	NO	NO	YES	NO	NO	YES
SUFFICIENT POWER FOR FORCING FUNCTION (2)	YES	YES	YES	164 dB	166 dB	166 dB	164 dB	LOSS OF POWER ABOVE 6TH O B								YES	YES	YES	YES	YES	YES	YES	YES	YES
APPROX. SPATIAL CORRELATION OF FORCING FUNCTION (6)	.03 FT	1 FT	60 FT	.2 FT	5 FT	.5 FT	3 FT	.2 FT	.5 FT	.8 FT	.3 FT	.2 FT	5 FT	5 FT	3 FT	DIA	DIA	2 FT	DIA	DIA	2 FT	DIA	DIA	2 FT
TEMPORAL CORRELATION OF FORCING FUNCTION	RAPID DECAY	UNKNOWN	RAPID DECAY	NONE	NO DECAY	SOME DECAY	NO TIME DEPENDENCE		NO DECAY	SOME DECAY	NO TIME DEPENDENCE		NO DECAY	SOME DECAY	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
EXCITATION ABOVE 3000 Hz	YES	YES	YES	POOR	POOR	POOR	POOR	POOR	POOR	POOR	POOR	POOR	POOR	POOR	POOR	NO	NO	SOME	NO	NO	SOME	NO	NO	SOME
REALISTIC MOUNTING POINT IMPEDANCE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	NO	FAIR	FAIR	NO	FAIR	FAIR	NO	FAIR	FAIR
TYPICAL TOLERANCES ON FORCING FUNCTION	NONE	± 6dB	± 3dB	± 3dB	± 3dB	± 3dB	± 3dB	± 6dB	± 6dB	± 6dB	± 6dB	± 6dB	± 6dB	± 6dB	± 6dB	± 3dB	± 3dB	± 3dB	± 3dB	± 3dB	± 3dB	± 3dB	± 3dB	± 3dB	± 3dB
ASSUMPTIONS IN DEFINING FORCING FUNCTION	FLIGHT CONSTRAINT	FPL TO SPL	DEFINED BY EXP	FPL TO SPL	FPL TO SPL	FPL TO SPL	FPL TO SPL	FPL TO SPL	FPL TO SPL	FPL TO SPL	FPL TO SPL	FPL TO SPL	FPL TO SPL	FPL TO SPL	FPL TO SPL	SPL TO VIB	SPL TO VIB	SPL TO VIB	SPL TO VIB	SPL TO VIB	SPL TO VIB	SPL TO VIB	SPL TO VIB	SPL TO VIB
FLEXIBILITY OF TEST FACILITY (11)	NO	NO	FAIR	YES	FAIR	POOR	YES	YES	FAIR	POOR	YES	YES	FAIR	POOR	YES	YES	YES	FAIR	YES	YES	FAIR	YES	YES	FAIR
PRACTICALITY OF COMBINED DYNAMIC TESTS (3)	YES	NO	NO	YES	NO	NO	NO	YES	NO	NO	NO	YES	NO	NO	NO	YES	FAIR	NO	YES	FAIR	NO	FAIR	NO	NO
PRACTICALITY OF COMBINED ENVIRONMENTAL TESTS (4)	YES	ENFORCED	NO	FAIR	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	YES	FAIR	YES	YES	FAIR	YES	FAIR	YES	YES
RELATIVE COST OF ENTIRE TEST	HIGH	MODERATE	FAIRLY HIGH	VERY LOW	FAIRLY LOW	MOD LOW	LOW	VERY LOW	FAIRLY LOW	MOD LOW	LOW	VERY LOW	FAIRLY LOW	MOD LOW	LOW	FAIRLY LOW	FAIRLY LOW	MOD LOW	FAIRLY LOW	FAIRLY LOW	MOD LOW	FAIRLY LOW	FAIRLY LOW	MOD LOW

7-1.2 Test Procedures

a. After performing sonic fatigue tests to establish the design adequacy of a structural test specimen, testing should be continued to destruction of the specimen to establish ultimate fatigue strength. If the specimen fails soon after establishing its design adequacy, a second specimen should be tested to gain confidence that the first specimen was not at the high end of the fatigue scatter band.

b. The length of each fatigue crack should be marked at the conclusion of each acoustical-test run to establish its rate of propagation.

c. Components intended for attachment to mounting panels should be tested on the panels so that the acoustically-induced vibration of the panel will be input to the component as well as the acoustic field itself.

d. Whenever possible, the acoustic field which excites the specimen should be defined completely. This might necessitate calibration of the facility if a new source of geometry is used, and/or recording sound pressure measurements on tape for subsequent analysis.

7-1.3 Test Fixtures

a. If an acoustical test specimen is also tested on a mechanical shaker, the vibration test fixture should be considered for use for the acoustical test. This would require identical boundary conditions during dynamic testing, but eliminates additional test fixture fabrication costs.

b. All efforts should be made to ensure that a specimen exposed to an acoustic field is isolated from all mechanical vibrations induced artificially by structural ties between the specimen and facility structure. The exception to this is combined vibration and acoustical testing where both types of forcing functions are driving the structure. In the case of ducts, they should be isolated from the specimen and should not change the response of the specimen (adding damping or stiffness), or induce vibration through structural ties. In the case of the progressive wave tube, adjacent wall structure should have vibratory response, that is, at least an order of magnitude below that of the specimen.

7-2 SHOCK TESTS

There are many ways to impose a shock on a test specimen such as (in descending order of laboratory usage): mechanical shock test machines and shakers, dropping the specimen, explosion of pyrotechnic devices, air-sustained blast waves induced by a nearby explosion, and by X-Rays. Each method provides some unique features such as high amplitude, short or long duration, large or small spatial, correlation, propagating or nonpropagating sources (the response always propagating in the specimen), etc.

The intent in any laboratory shock test is to adequately envelope the real forcing function to verify operational and structural integrity. Sometimes, this requires excessive conservatism in some portion of the frequency domain unless fairly expensive testing techniques are used. Most shock test techniques are compared on the basis of shock spectra.

7-2.1 Test Set-Up

If the actual shock source to be simulated is a local, nonpropagating shock, a mechanical shaker, shock machine, or drop test is usually employed. For these test set-ups, the shock is localized and correlated at the point(s) of impact. The shaker is usually harder to program for its input, but can achieve long time durations (many milliseconds) whereas the shock machines or drop tests are usually used for higher amplitude (hundreds of g's), shorter duration (a few milliseconds) tests.

If a propagating shock source is required, a barrel test (pyrotechnic-type shock in a structure that can be repeatedly cut) or a blast-tube approach is used. For the pyrotechnic device, the explosion is used to cut metal and acts along a predetermined line of explosives. In a blast tube, an explosion causes an air blast that is correlated over the wave front which passes over the specimen at supersonic speeds. For the blast tube, the source can either be far away, resulting in a plane wave front at the specimen, or very close to the specimen causing a local curved wave front on the specimen.

Each of these devices can be shaped to provide amplitude variations and time variations, but each is normally employed to simulate short duration ($\simeq$ 1 millisecond) high amplitude (thousands of g's) shock.

Finally, for very short duration (microseconds) shocks, a blast tube shock wave or X-Ray impingement is used. These would be considered unusual (exotic) tests requiring special set-ups and considerable expense.

7-2.2 Test Procedures

The shock source should always be calibrated prior to exposing the test specimen to it. This may require a dummy mass or structure to simulate the specimen during the calibration. The purpose of this initial calibration step is to ensure a valid test for the right number of shocks without over- or under-testing the specimen.

Subsequent to the calibration runs, sufficient data must be gathered during the actual shock test to verify that the specimen was exposed to the required environment for the appropriate number of shocks. This requires care in setting instrumentation ranges so that the critical time history of the shock and the response to it can be evaluated.

7-2.3 Test Fixtures

As in all dynamic tests, specimen-supporting members (fixtures) should provide the physical support required without appreciably introducing excitation to the specimen, or causing a marked change in its response (boundary conditions which are not representative of the real situation). Thus, most shock tests require very flexible supports and systems for catching the specimen after exposure to the shock (to avoid additional shocks due to collisions of parts). Where possible, identical fixtures and supports should be utilized in multiple dynamic tests (shock, acoustics and vibration).

7-3 VIBRATION TESTS

As opposed to acoustical testing in which the excitation is airborne, this section deals with structurally-borne energy transmitted by direct mechanical ties. Hence, these tests utilize mechanical shakers attached to the specimen by means of structural members (fixtures).

7-3.1 Test Set-Up

a. Sufficient backup structure for the test item should be used (intercostals, frames, stringers, etc.) to ensure proper specimen response frequencies, and to allow structural loads to be distributed.

b. All tubing, wire bundles, or other items which influence the dynamic response of the test item should be included or simulated in the test specimen. This is particularly important when testing an item mounted on vibration isolators. The tubing and wire bundles should be installed in a production configuration (proper spacing of tie-down points, etc.). Simulation should be accurate to the second tie-down point adjacent to the test item, if possible. Where clearance problems exist in implementing this practice, the tubing/cable installation should be simulated to at least the first tie-down point adjacent to the test item, with a second tie-down clip added as near as possible to the production location for this tie-down.

c. A representative of the cognizant design section should always inspect the specimen prior to test for conformance of the test specimen and its installation to the latest production configuration.

d. When vibration testing several items in a common assembly, the use of "dummy" items in place of an unavailable item may be necessary to simulate the common assembly. Also, in the event of a test item failure during test, the substitution of a dummy item can allow the continuation of the assembly test. A dummy item may be a used test specimen or a mass having the same dynamic properties as the test item.

e. Very long lines can be tested in segments, provided that the individual segments are defined with adequate end-condition simulation at the segment parting plane. This usually requires extending the specimen to the next support point by using a dummy section of line.

f. Relatively massive items should be vibration tested in flight vehicle orientation whenever possible, providing test fixture cost and complexity are not prohibitive. Ducts not filled with liquid and relatively light items do not usually require vehicle orientation.

g. Vibration tests should be performed at operating temperature whenever possible, to simulate the effects of simultaneously applied thermal and dynamic loads. This is particularly important because it is usually impossible to analyze for the effects of these combined environments. The fatigue properties and notch sensitivity of structural materials may change significantly with temperature. Unequal thermal expansion can affect specimen "tightness," which in turn affects specimen equivalent damping and dynamic response.

h. Components which are internally pressurized during vibration environments should be pressurized during vibration tests. Internal pressure effects the mechanical force balane in valves, and effects the structural response of pressurized electronic boxes.

i. When a vibration test specimen includes a section of fuel tank, the tank section should be pressurized if the objective of the test is to prove structural adequacy and resistance to leakage. Pressurization is not required if the tank section is not undergoing structural test and is being used for dynamic simulation only.

j. Whenever technically justifiable, from a service sequence standpoint, vibration tests should be performed prior to life cycle tests. The large number of cycles at high loads which are specified for life cycle tests tend to precipitate fatigue failures in vibration tests. It should be noted that in service, life cycles often occur before the vibration environment. In these cases, the best procedure is to subject the specimen to sufficient life cycles to simulate the service conditions, then perform the vibration test, and finally perform additional life cycles as required by the test plan.

k. The location of the control accelerometer must be specified by the engineering group responsible for the generation and interpretation of the dynamic test specification (to ensure the correct input to the test item), and this location must not be changed at the test site without notification to this group.

7-3.2 Test Procedures

7-3.2.1 Sine Tests

a. Particular attention should be given to the type of equipment being used to control the double amplitude (d.a.) portion of the sine test specification. A laboratory may use an older shaker which employs a velocity coil or inductance-type pick-up mounted to the shaker armature. A good signal-to-noise ratio is obtained in the low frequency range (< 20 Hz) using this method, however, its major disadvantage is that d.a. control is monitored at a location remote from the control accelerometer location.

   For specimens using backup structure, an uncontrolled test can result, due to dynamic decoupling if that d.a. control is used at frequencies greater than about 50 percent of the system's first resonance frequency. At lower frequencies, where resonance amplification effects are not significant, this method of control is satisfactory, provided that the velocity coil is properly calibrated.

   A method that has been successfully used to avoid this decoupling effect is as follows: control on the basis of the control accelerometer signal at all frequencies, but manually control the acceleration through d.a. regions so that it approximates the specified d.a. slope. This technique is improved if a table of frequency and corresponding g level is prepared before the run, with an assistant to the console operator calling out d.a. frequencies during the run so that the operator need only concentrate on adjusting the proper g reading.

   Most labs use equipment which performs a double integration on the control accelerometer signal, making it proportional to displacement. The advantage of this method is that the control accelerometer is used at all times, thus avoiding the decoupling problem noted above. The disadvantage of this method is that the signal-to-noise ratio is often poor at low frequencies.

b. A sine specification crossover point is often verified prior to the test run by dwelling near the crossover frequency with specification level at the control. Care should be taken to ensure that the time spent in this procedure is minimized. If the crossover point occurs near a resonance frequency, this procedure can result in significant fatigue damage to the specimen.

c. When a sine specification is based on environmental data which have been filtered, it is technically correct to also filter the control accelerometer signal which is used to attain specification levels during test. A tracking filter is usually used to filter out harmonic distortion and feedback to the control signal due to specimen response.

   When tracking filters are not available, but schedule requires performance of the test, a choice must be made between the control modes: peak, average, or rms. Rms control results in the closest approximation to the filtered control, because, of the three methods, it goes furthest toward "smoothing out" the individual control signal peaks that can result from harmonic distortion and rattling.

d. When a tracking filter is used on the control signal, a very serious overtest can occur when a node point occurs at the control location (when the unfiltered signal is used for control, the overtest is not as serious because "hash" tends to "fill in" the spec.).

   A method which has been successfully used to alleviate this problem consists of selecting an alternate control location and constraining the filtered signal from this location to not exceed specification levels. Experience has shown that this constraint cannot be performed manually for structure-mounted test specimens at frequencies above 200 Hz, and if an automatic peak-limiting device is not available, practice has been to switch control back to the unfiltered signal of a single control accelerometer above 200 Hz.

e. With a single control accelerometer, it is likely that at some frequencies the acceleration at other similar locations will exceed specification levels. This is undesirable when the specification levels represent the maximum expected vibration levels for that type of structure. By using multiple control accelerometers and averaging or peak-limiting the outputs, the possibility of overtest is diminished in proportion to the number of accelerometers used.

7-3.2.2 Random Vibration Tests

a. A power spectral density (psd) analyzer (independent of the visual display on the automatic equalizer-analyzer) should be required for on-site verification that random vibration specification levels are achieved. A record of this verification should be made using an x-y recorder in conjunction with the independent analyzer. The filter bandwidths used in generating this psd should be noted on the plot.

b. Specification level tolerance bands apply only to peaks and notches in the spectrum that are difficult to eliminate using the equalization equipment. Deliberate control to the lower edge of the tolerance band is not acceptable.

c. For large complex structural tests, the power amplifier master gain control can often be used to optimize the control spectrum. The equalizer unit has a finite attenuation/amplification capability, and the master gain can be used to increase this finite capability. If a notch in the spectrum persists after the first equalization attempt, an increase in the master gain may raise it (care must be taken that the additional energy made available does not cause spectrum peaks by exceeding equalizer attenuation capabilities at other frequencies).

   Conversely, a spectrum peak might be lowered by "starving" the power amplifier to lower the available energy at all frequencies, bringing the peak within the dynamic range of the equalizer (in this case, care should be taken to avoid an undertest at other frequencies). It has been noted that when an undesirable spectrum peak occurs at a low master gain setting, it can sometimes be attenuated by cautiously increasing the master gain. This phenomenon apparently results from boosting a weak overall signal into a more optimum dynamic range for the equalizer.

d. When a severe peak or notch occurs on the spectrum, and it has been established that the fixture and specimen are physically sound, the following techniques can be helpful in alleviating the problem:

   1. Adjust equalizer filters immediately adjacent to the filter containing the problem frequency. Energy is absorbed at resonance over a fairly wide frequency band for all but very lightly damped resonances. Because most equalizers have constant filter bandwidths, the number of adjacent filters adjusted should increase at higher frequencies.

   2. Some spectrum peaks are due to harmonic response. These peaks can sometimes be lowered by decreasing the spectrum level at submultiples of the problem frequency. It may also be possible to obtain attenuation by decreasing the spectrum level at multiples of the problem frequency.

e. Particular caution should be exercised when using the low frequency filters of the Ling 80-channel equalizer-analyzer (channels 2 to 5, 20-70 Hz). Analyses of test data indicate that these filters have a different sensitivity than indicated by the visual display. These channels are also extremely sensitive to small manual adjustments to the equalizer sliders, and the visual display meters respond very slowly to manual changes. On-site psd verification plots should be used to determine the actual level at the low frequencies, rather than using the equalizer analyzer visual displays. The verification plot should have a logarithmic frequency scale to obtain adequate low frequency resolution.

f. Even though most random vibration specifications cut off at 20 Hz, the 10 to 20 Hz filter (channel 1) of the equalizer must not be set to zero, or the spectrum will roll off too rapidly and will be below specification from 20 to 30 Hz.

g. About 10 dB more dynamic range can be obtained when the equalizer filters are switched from the "automatic" mode to the "manual" mode. The "manual" mode can also be used on any filter to allow continuation of testing in event of a malfunction of the "automatic" mode on that filter.

h. At least one test laboratory has reported successfully increasing dynamic range by connecting two equalizer units in series.

7-3.2.3 General Test-Site Practice

a. It is important to establish the point in a vibration test sequence when specimen fatigue failure initiates. The test specimen should be inspected visually after each vibration and shock test in each axis, with particular attention paid to areas of stress concentration.

b. Test specimen conditioning and monitoring equipment, such as test facility flex hoses and checkout equipment cabling, should be tied down or otherwise restrained so that they do not influence the dynamic response of the test specimen. Any external insulation attached to the test specimen (to maintain a cold temperature environment) must be loose enough to allow the specimen to vibrate freely. The ratio of external insulation weight to specimen weight should be minimized. The clearance between specimen and test fixture should be checked to preclude unscheduled impact loads.

c. Test experience has shown that the repeatability of fatigue failures is much higher than that predicted by statistical analysis. That is, if a fatigue failure has been induced in a test specimen, a retest using an "identical" specimen is very likely to show a similar fatigue failure unless one of the following test parameters is changed: test setup, vibration levels, duration, or test sequence. In a failure analysis retest, only one of these parameters is usually changed (the one suspected of causing the failure). A change of more than one parameter precludes isolation of the cause of failure.

d. Special checkout of test facility equipment, in addition to standard setup procedures, is sometimes desirable. The control accelerometer calibration can be checked out in a crude but effective way by using a displacement wedge.

A method for checking the performance of the random vibration equalization system is: hard-mount the control accelerometer and a response accelerometer on the shaker head. Set up a white noise spectrum on the automatic equalizer-analyzer system. Generate psd plots from the output of the control and response accelerometers. Any significant deviations between the two psd plots should be resolved by a calibration check of the accelerometers. Any significant deviation of the psd plots from the desired white noise spectrum should be traced to either the automatic equalizer-analyzer or to the independent spectral density analysis equipment, and corrected. Finally, a tape loop of the equalized white noise spectrum should be made, to be used for future calibration checkouts of the equalizer-analyzer and independent spectral density analysis equipment.

e. A prime function of the engineering test director is to determine whether an out-of-tolerance test level is acceptable. A general statement cannot be made without approval of such a test condition, and the evaluation of each must consider contractual requirements, the test specimen itself, the test set-up, etc. However, the following factors must always be considered in the process of evaluating an out-of-tolerance condition:

1. It must be determined that the test equipment is operating satisfactorily. Approval of an out-of-tolerance condition caused by malfunctioning test equipment is unjustifiable.*

2. It must be determined that the test fixture is structurally adequate. A wide peak or notch in the frequency spectrum is often an indication that the fixture (or backup structure, if used) has failed* or that mounting bolts, control accelerometer block, or control accelerometer cable are loose.

3. If equipment and fixture are satisfactory, the criteria group may determine that the condition is acceptable in view of measured service environment data, the manner in which the specification was derived, and/or the critical resonances of the test item.

It should be noted that the inability to meet a required test specification is not in itself a justification for relocating a control accelerometer. Consider the case where a major peak occurs in the spectrum plot of a control accelerometer located on a backup structure. If the control accelerometer is moved closer to the rigid test fixture, an acceptable spectrum may be obtained; however, this would result in an uncontrolled overtest at the problem frequency, where previously some degree of attenuation of the resonance was obtained.

A case might occur in which a notch in the control spectrum coincides with a notch observed in the spectrum obtained from measured service data, indicating that the specimen is responding as it would in actual service, and that the test control accelerometer should not be moved.

f. The test director should not hesitate to halt a vibration test for control equipment checkout if the observed displacement of the shaker head appears to be grossly incompatible with the known test specification requirements. This is also true if an unusually "quiet" or "loud" shaker noise level is noted during random vibration equalization or test which is not reconcilable with the required rms level.

Numerous undertests and overtests have been averted by experienced personnel who were alert to these gross "test level indicators." A "Panic button" is used near the A-249 shaker at location "A" to alert the console operator of test level problems.

g. Engineering personnel should observe the test specimen vibrating at least once during each test mode, particularly during the low frequency portions of sine sweep tests, where specimen resonances generally occur and mode shapes can be observed.

7-3.2.4 Multiple Shaker Tests

a. The practice of attaching more than one shaker to a rigid test fixture should be avoided. In one instance this practice resulted in such adverse coupling effects that the control accelerometer immediately adjacent to one shaker was completely dominated at some frequencies by the other shaker; i.e., excessive, uncontrolled vibration levels were measured even when the adjacent shaker was turned off. A separate fixture should be provided for each shaker, with the only structural coupling between shakers occurring through the test specimen.

b. When a large, relatively flexible duct section is tested in a multiple shaker setup, it is not mandatory that the shaker axes have a parallel alignment.

Methods for determining that equipment and fixtures are performing adequately are presented in paragraph 7-3.3.2b

c. The output of each control accelerometer should be displayed at each control console so that a dump of any shaker will be immediately known. When multiple consoles are used, an interconsole alarm system should be set up to alert all console operators of the shut-down of any one shaker, to allow immediate cut-off of all shakers.

Extra care should be taken to avoid the occurrence of transient shocks. Extraneous transients that sometimes occur when multiple shakers are rapidly brought up to sine specification level (at the start of a sweep) can be avoided by mounting the control accelerometers on the bare shaker heads prior to test and optimizing servo and compressor settings at the initiating frequency of the sweep.

d. Random vibration test control should be accomplished using an individual equalizer-analyzer unit for each shaker of a multiple shaker test set-up. When equalizer-analyzer units are available for all but one shaker, but schedule requires performance of the test, an interactive tape-loop control method can be tried. This method consists of simultaneously alternating tape loop and equalizer control among the exciters, while producing new control tape loops for the exciter being controlled by the equalizer.

This method has been used with some success when structural coupling provided by the specimen between shakers is weak; that is, when the control accelerometer level at one shaker is not grossly influenced by the other shaker(s). The tape-loop recording quality (particularly the tape-splice quality) is especially critical for successful tape-loop control.

e. Sine-vibration phase control between multiple shakers can be maintained at frequencies below 500 Hz by electronic means. The question is whether phase control is desirable. While protecting the specimen from relative displacement strains at low frequencies, it provides an overly conservative simulation of a vehicle vibration environment which, in general, has random phase relationships.

7-3.3 Test Fixtures

7-3.3.1 Design Practice

a. The vibration test fixture designer should strive to obtain a one-to-one fixture dynamic response throughout the test frequency range. It is recognized that, since most test specifications require testing to at least 2,000 Hz, this goal is usually unattainable; however, any attempts to optimize fixture stiffness while maintaining a reasonable fixture weight are beneficial.

b. The line of action of applied shaker force should come as close as possible to intersecting the static cg of the combined test fixture and specimen. In cases where a serious attempt to fulfill this criterion still results in a significant mismatch (6 inches for a heavy fixture), team blocks or equivalent can be used to react the resulting undesirable dynamic moments. Team blocks should not be used as a panacea to correct a poorly-designed fixture.

c. Any test fixture or backup structure "overhangs" or cantilevers should be eliminated or reinforced, to minimize undesirable dynamic crosstalk effects. Long unsupported fixture-plate members should be stiffened using generous gussets, and the span length of tubular members should generally be kept below 20 inches for 4-inch diameter fixture tubing. (Formulas for determining the natural frequency of tubular fixture members are presented in Douglas Engineering Paper No. 3096.) The ends of channels, angles, etc., at the edges of backup structures should be stabilized using special clips that simulate actual vehicle restraint.

d. In general, maximum design efficiency is realized by the use of plate material for small vibration test fixtures, and tubular stock for large fixtures.

e. As noted in paragraph 7-3.2.4a, only one shaker should be attached to a vibration test fixture.

f. Available vibration test fixtures should be considered for use in new tests. Sometimes a relatively minor rework to an existing fixture can save the time and expense of fabricating a new one.

g. The Testing Manual (BN1) contains useful charts for use in selecting a shaker for a given test application. Given the specimen weight (including fixture), the charts show the "maximum sine capability" ($g_{o\text{-}pk}$), for certain shakers. It is very important to note that ". . . . in all cases, the curves refer to levels at the shaker head or on a 'rigid' fixture." Being thus based on "dead mass" theory, these curves are quite conservative in applications where test backup structure is used, for high-frequency structural resonances help to achieve the required excitation levels at the control accelerometer location.

Experience has shown that for test specimens employing backup structure, it is generally possible to use the "sine" curve to determine the random vibration (g_{rms}) capability of the shaker. This random vibration capability is the design consideration that usually limits the allowable fixture weight.

Figure 7-1 presents actual test datum points that illustrate the conservatism of a typical "shaker capability" chart. The solid datum point is particularly interesting because it represents a datum from a control accelerometer that was rigidly attached to the test fixture. It should be noted that the g_{rms} values shown are those obtained when an acceptable control spectrum was achieved; they are not necessarily the maximum g_{rms} values that could have been obtained.

h. Magnesium stock is usually used to fabricate vibration test fixtures. Magnesium has better internal damping properties and a better strength/weight ratio than aluminum, and earlier problems associated with magnesium welding have been solved by improved techniques. A disadvantage of magnesium is its low bearing strength. Evidence of this property is often found where the heads of shaker attachment bolts bear on the fixture base plate. The low bearing strength also often results in galling of tapped threads when bolts are tightened down. Inserts are therefore used in lieu of tapping threads in the parent material, but even these inserts can work loose with extended test use.

   Because of these disadvantages, magnesium fixtures should be thoroughly inspected before reuse, and bolt-bearing surfaces should be remachined and inserts replaced as required. When it is known that a fixture will be reused many times, the designer should consider making the fixture out of aluminum.

i. Experience has shown that 1 1/2 inches is the optimum thickness for a 30-inch diameter fixture base plate made of aluminum or magnesium.

j. Fixtures should be welded whenever possible to avoid possible dynamic decoupling that can occur at bolted connections. When bolts are used, lengths should be minimized so that the bolt axial frequencies (decoupling frequencies) are maximized. Bolts used to attach backup structure to a fixture should have at least a 1/4-inch diameter.

k. Fixture tubes are often filled with polyeurythane foam to improve damping and lower fixture response at fixture resonance frequencies. One disadvantage of this method is that whenever the fixture is rewelded, gases are released by the heated foam, making rewelding difficult and result in "dirty" welds.

l. As in all structures, stress concentrations in test fixtures greatly accelerate fatigue crack initiation. Whenever possible, large test fixtures should be stress relieved after welding. Areas of stress concentration should be avoided or minimized in the fixture design.

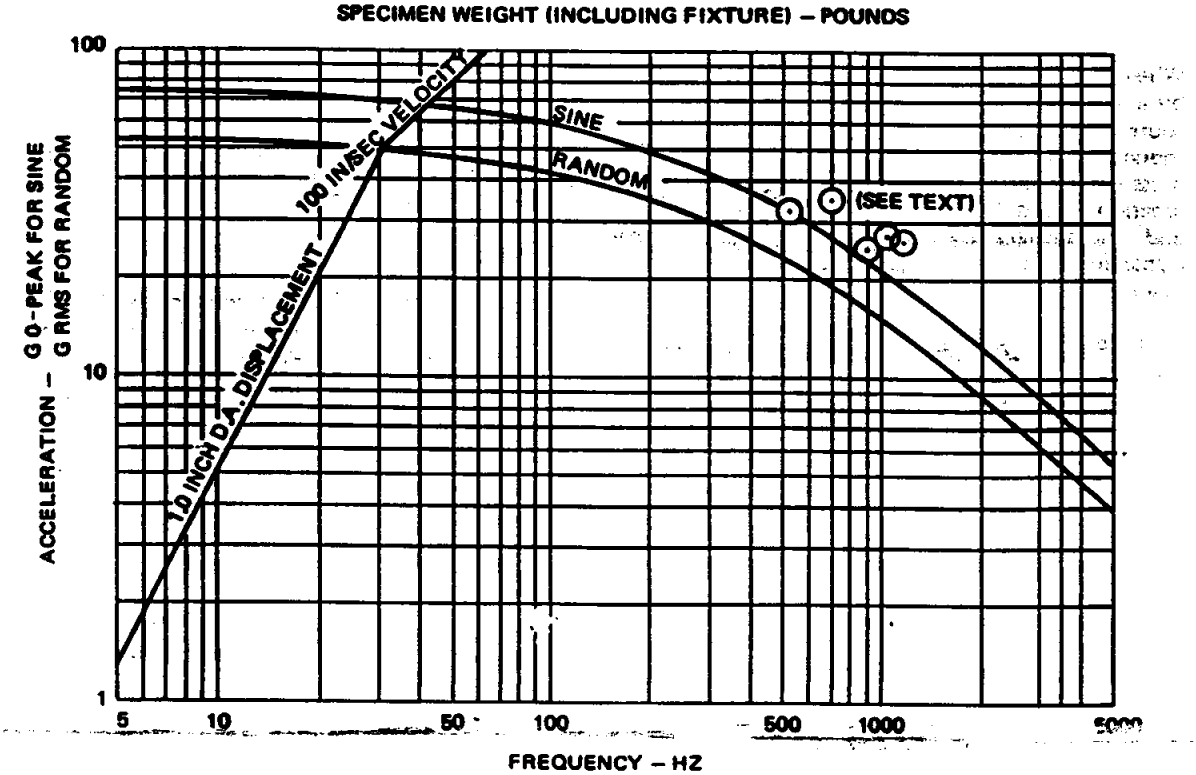


Figure 7-1. Datum Points Showing Achieved Random Vibration G_{rms} Levels for Typical Test Specimens

7-3.3.2 On-Site Checkout of Fixtures

a. A test engineer should visually inspect the fixture prior to test to make sure that all bolts are in place and tight, and that there are no fixture weld cracks. Small hairline cracks (approximately 1/8 inch long) are sometimes found where weldment has bubbled, but these "cooling cracks" do not propagate, and are of no significance.

b. It is good practice to mount an accelerometer directly to either the fixture base plate or the shaker head for use in trobleshooting when test specifications cannot be attained at the control accelerometer. The following table indicates how data from this accelerometer can be used to determine the source of a problem.

Control Spectrum	Shaker Head Spectrum	Probable Source or Problem
Wide Peak	Wide Peak at same frequency	Shaker or shaker control equipment
Wide Peak	Wide notch at same frequency	Fixture or backup structure resonance
Wide notch	Wide notch at same frequency	Shaker or shaker control equipment
Wide notch	Wide peak at same frequency	Energy not getting to control location. Possible: node point at control; structural failure in fixture or backup structure; loose bolts; loose control accelerometer block or cable.

c. When a wide control spectrum notch is accompanied by a wide shaker head peak (last entry in table), the source of the problem can often be pinpointed by mounting an accelerometer on each major bolted mass in the "drive train" between shaker and control location, inputting a low level white noise spectrum, and establishing by on-site data analysis that dynamic decoupling exists across one of the bolt planes.

d. A crude but fast laboratory method for determining fixture resonance frequencies consists of instrumenting the fixture (using an accelerometer and memoscope), and observing its response to a shock input supplied by a mallet.

7-4 TEST INSTRUMENTATION REQUIREMENTS

7-4.1 Test Instrumentation Planning

a. The location of accelerometers during a mechanically- or acoustically-induced vibration test is usually determined by one or more of the following considerations:

1. Exact duplication of flight measurements for comparison purposes.
2. Determination of the response of major spring-mass systems of the specimen.
3. Acquisition of information on specimen failure modes.
4. Acquisition of information for verification of structural dynamic analyses (these locations must be specified by the analyst).
5. Acquisition of information for use in later hardmount vibration tests of portions of the specimen.
6. Determination of the degree of correlation between tests when two "identical" vibration tests are performed. (Photographs, rather than sketches, of accelerometer locations on the previous test should be used to locate accelerometers for the second test.)

b. The use of strain gages is often the most direct way to obtain data to verify structural dynamic analyses.

c. It is good practice to have at least one cross-talk accelerometer on the specimen for each axis of testing.

d. As noted in paragraph 7-3.3.2b, it is good practice to mount an accelerometer on the shaker head or the fixture base plate.

e. Special care is required when attaching accelerometer mounting blocks to the test specimen. Most blocks are mounted using commercially available adhesives which have low shearing strengths. The flexibility of thin sheet structure often results in shear loads which cause block debonding.

In duct application, hose-clamp type straps have been used successfully to react this shear loading and preclude block debonding. The block is first tapped through its base and attached to the strap using a flat-head screw. The block and strap are bonded to the duct (in the vicinity of the block only) and the strap is clamped around the duct.

The use of mechanical fasteners in addition to adhesives becomes less important when blocks are mounted to heavier structure.

The control accelerometer block should always be fastened to structure using both adhesive and bolts or screws.

Temperature and surface preparation considerations are very important in the selection and use of adhesives. This is illustrated by two materials

currently in use, 3M Company EC 3515 and 3M Company EC 2216. Utilizing British Etch, an optimum cleaning agent, the shear strength of EC 3515 is approximately three times that of the EC 2216 (7,400 psi vs. 2,360 psi) at liquid nitrogen and colder temperatures.

However, British Etch is very difficult to use in a vibration laboratory. Much more practical for lab use is "solvent wipe" Methyl Ethyl Ketone (MEK). Use of this cleaning agent will lower the shear strength of the EC 3515 to about 16 percent of that attainable with the British Etch. or about 1,200 psi. However, use of MEK with the EC 2216 will lower its shear strength to only about 80 percent of that attainable with the British Etch or about 1,850 psi – stronger than EC 35.5. Cure time for EC 3515 is two days, half of which is at + 160°F, but a one-day ambient cure will suffice for the EC 2216.

In summary, EC 3515 can provide much more strength than EC 2216, but requires a two-day cure and a difficult cleaning operation. On a vibration laboratory situation where an MEK wipe is a reasonable cleaning operation, EC 2216 is strongest.

f. Care should be taken that the weight of an accelerometer and its mounting block does not mass-load a member to the point that its natural frequency is altered. Micro-miniature accelerometers or strain gages should be used when measuring the response of very light structure.

g. Slip-sync motion pictures should be considered for sine tests of items that have resonance frequencies below 200 Hz. Slip-sync movies can also be taken during random vibration tests with a reasonable degree of success. For random vibration use, the camera pulse rate is set at a frequency near the specimen resonance frequency, or is slowly swept through the resonance frequency range.

h. Microphones should never be large enough to cause a significant change in the acoustic field at the excitation frequencies of interest, or be near enough to the specimen to collide during exposure to the field.

i. Test instrumentation should provide electrical signals which are proportional to only one input and hence, must either be insensitive to other inputs, or isolated from them.

j. Dynamic range settings should be determined on the basis of calibration or equalization runs so that no loss of data occurs due to data acquisition systems being overdriven, or the use of unacceptable signal to noise ratios (< 10 dB). If this requires gain changes during the test, particular care must be exercised to properly identify gain settings before and after the change, and to accurately identify at what point in the test sequence the change was made.

k. If correlation is to be studied, particular care must be used in selecting data acquisition systems with minimal phase differences and recording channels which are on the same stock of record gaps (on standard multiple track recording machines all odd-numbered channels are on one record head and all even-numbered channels are on another which is physically displaced from the first, thus introducing an artificial time lag in the records between even and odd numbered channels). In addition, the stack of recording gaps can be skewed, introducing considerable error between channels assigned to gaps at opposite ends of the stack.

7-4.2 On-Site Checkout

a. Accelerometers should be tap-checked and microphones "whistled at" to ensure proper hookup and identification of test data channels.

b. When tests are performed at cryogenic temperatures, the following items should be considered:

   1. Accelerometer temperature ratings should be checked.

   2. Hard plastic accelerometer mounting blocks can be used to help thermally insulate accelerometers.

   3. Accelerometer connectors can be protected from moisture using silicone jelly.

   4. Some instances of erratic accelerometer outputs have been traced to thermal shock transients caused by cryogenic liquid dripping on the accelerometer casing.

   5. Accelerometer connectors should be retightened after cooling, when possible. If accelerometers are not accessible after cooling, connectors should be screwed down tighter at ambient temperature.

c. All accelerometer connectors should be checked for tightness at the beginning and end of each run. Any looseness found after a test should be documented to assist the data reduction group in determining the source of possible data anomalies.

d. For very important vibration tests, an on-site dynamic calibration of all accelerometers may be justified.

e. The following guidelines may be helpful to determine the proper dynamic range for response accelerometers:

   1. As a first estimate, allow for a dynamic amplification factor of 10 for loaded structure and 30 for unloaded structure.

2. When a sine test requires an up-sweep only, it is often useful to perform a low-level sweep (at 1/4 spec. levels or 1/2 g, whichever is lower) to determine specimen Q's. In adjusting accelerometer dynamic ranges, it is usually safe to assume that these Q's will not change when increasing the level because of limiting effects at high vibration levels. (The low-level sweep also serves to check out the shaker system.)

3. When a sine sweep requires both an upsweep and a downsweep, the test should be stopped between sweeps and the accelerometer ranges adjusted as required to eliminate any saturation and poor signal/noise ratios recorded on the oscillograms.

f. Sine test oscillograph data should be checked in the lab and any data anamolies documented for the information of the data reduction group. It is often useful to run the oscillograph during random vibration tests to check for data anamolies. A noticable g_{rms} variation during a random vibration test will indicate that the spectrum is not stationary and that the test specimen and set-up should be checked. Psd's should be made during test periods having different g_{rms} levels to establish the stationarity of the test.

g. An excellent quick check of accelerometer amplifier settings consist of establishing that all sine test response channels are indicating approximately the same g level at low test frequencies (< 30 Hz), where specimen resonances rarely occur.

h. Additional confidence in strain gage calibrations can often be obtained by the on-site application of a known static load to the specimen, which effectively provides an end-to-end calibration of the data system.

7-4.3 Checkout of Reduced Data

a. As noted in paragraph 7-4.2g, an excellent quick check of accelerometer amplifier settings consists of ensuring that all response channels are indicating approximately the same g level at the low test frequencies. This check also applies to reduced data from both sine and random vibration tests. Data that are "out" by factors of 3 or 10 in this frequency range should be considered suspect.

b. A quick but effective check on the overall level of random vibration data is to determine that the rms level of the psd is approximately one-third of the g (o-p) value read from an oscillogram record of the same data.

c. Sine vibration data which will be used in structural dynamic analysis work should be filtered with a narrow band-pass filter. In general, random vibration psd's used in structural dynamics analysis work should be reduced using filter bandwidths approximating one-third octave wide, since these bandwidths approximate the resonance bandwidths of most aerospace structure (one-third octave band being the effective or contributing band of energy for a system whose resonance frequency is the geometrical mean frequency of the band and whose Q is 6.8).

7-5 FLIGHT TESTS

Early planning for flight test programs is usually required. Typically, at the time that commitments have to be made as to type of instrument, location, dynamic environment amplitude, other environments, etc., little is known about the structural details, design problems or vibration sensitivity of components. Because of this "cart before the horse" sequence, considerable care should be exercised to clarify the goals where possible and provide for some flexibility in the plan.

For example, some means of resetting gain should be included so that late design changes can be considered. Also, if other than vibration measurements are to be made, this should be stated; otherwise Electronics will assume it to be a vibration measurement and estimate the cost on that basis.

The above are typical examples of why careful study is required early in the program (sometimes even before the contract is won) to properly plan a flight test program.

7-5.1 Location of Transducers

Flight measurements should be located, and identified as to type, on the basis of the major purpose of the measurement. These may be: measure the forcing function, measure the primary shell response, measure the transmission of energy, or measure specific internal vibration shock or acoustic environments. Preferably, a flight program would (in order of importance):

a. Measure dynamic environments which were significant for all flight critical components (Guidance and Control equipment, Telemetry, experiment critical equipment, etc.).

b. Measure environments for which no flight data previously existed.

c. Measure environments which required extrapolations (as opposed to interpolations) on existing data to predict the levels.

d. Measure environments which are representative (for reference) of the system.

e. Measure environments redundantly to provide statistical confidence (on the same flight or on subsequent flights).

The sensor should be placed as close as possible to the position at which a laboratory measurement or prediction of amplitude was made, and its orientation should be such as to provide comparable results.

7-5.2 Transducer Mounting Requirements

Some basic rules apply to mounting the transducers. They are:

a. Whenever possible, place a vibration or shock measuring sensor directly on the structure at the location at which response is of concern (to avoid recording sensor-mounting-bracket resonances).

b. Avoid measuring typical quantities at a position where local effects cause nontypical response.

c. Do not permit other environments to markedly effect the measurement by inducing a significant signal into the sensing system.

d. The presence of the sensing system should not appreciably effect the measured quantity.

7-5.3 Selection of Transducers

Vehicle instrumentation, including transducer selection, is not generally a function of the Acoustics/Vibration Engineer. His primary responsibility is the detailed description, by MRD (see Drafting Manual AA1, Section 11.801) of the following:

Parameter/location to be monitored.

Frequency range required for measurement.

Full scale calibration range and accuracy required.

Complete description of vibration environment at the point of measurement.

Time periods during which data should be acquired.

In addition to these five items, other information which is largely self explanatory will be required on the measurement request drawing. The five items above are listed because they contain certain pitfalls which must be avoided if useful data are to be obtained.

a. The engineer requesting a measurement should have a definite point in mind for the transducer location. (See 7-4.1 and 7-4.2) A little coordination with the installation designers at this point in time will save a great deal of rush work at a later date when it is discovered that the transducer cannot be mounted per the requests, or that the component to be monitored has been relocated, etc.

b. The frequency range required should take into account such items as available telemetry bandwidth and transducer characteristics, as well as the desired data frequencies. It does no good to request 100 measurements, each with a response from 5-10,000 Hz, if the available telemetry connot get the data back to the ground; or request one measurement with a range from DC-100,000 Hz if no transducers are available to do the job.

Also, a requested range of 5-2,000 Hz will, in the absence of clarifying notes, be interpreted by an instrumentation engineer to mean *at least* 5-2,000 Hz. The resulting measurement system may have uniform response from 5-10,000 Hz and if significant vibration exists between 2,000 Hz and 10,000 Hz, the electrical output from the transducer will probably overdrive the amplifier which is calibrated for a 5-2,000 Hz range.

Thus, it is very important to not just list the minimum frequency requirement. If any significant amplitudes exist outside this range, use notes on the request to require filters and other signal conditioning necessary to eliminate the unwanted frequencies.

The telemetry systems most often used, SSB-FM and FM-FM, provide a dynamic range of about 20-30 dB. This means that the ratio of the highest measurable data signal (full scale calibration) and the lowest measurable data signal is about 10-31. Larger signals will be clipped and smaller signals will be buried in system noise. The point: use care in choosing a calibration range; leaving a pad over the predicted level to avoid clipping but not so high that the data will be in the noise floor.

Also, if prediction methods are highly inaccurate, range the measurement on the high side. Acoustical/vibration data in the noise floor are still useful for two reasons.

1. Since the data are in the noise, the levels are low.

2. The noise spectrum puts an upper limit on the actual measurement spectrum.

For all measurements, request calibrations, accuracies, etc., in terms of the engineering units of the telemetered data, rather than units derived during data analysis. The best example is acoustical measurements.

A microphone is a dynamic pressure transducer which outputs a voltage which is linearly related to the pressure variation at the diaphragm. As such, the signal waveform is totally analogous to the output of an accelerometer. Therefore, these measurements should be calibrated as DPSI, or $\pm N/cm^2$, or $\pm N/m^2$ rather than decibels, which is a logarithmic value computed from a root-mean-square average during analysis.

d. A complete description of the environment includes all vibration amplitudes and frequencies that the transducer must withstand. This may mean estimating an environment from 5 Hz to 100,000 Hz, but it will eliminate possible transducer problems later.

e. Finally, note all time periods for which data are to be acquired. Due to test range requirements, possibly world-wide tracking station requirements, etc., telemetry systems are generally determined and *fixed* early in the program, making later changes difficult, if not impossible, to make.

7-5.4 Telemetry Systems

7-5.4.1 Single Sideband-FM

Single sideband-FM (SSB-FM)* will provide twenty-two continuous data channels acquired via one FM link, and all channels will have a frequency response up to 4,000 Hz. Single sideband systems were required by NASA for the Saturn vehicles. Reference (*) indicates a frequency response within ± 1dB from 30 Hz to 3,000 Hz is possible. The systems used on the Saturn S-IVB stage were specified to be within ± 2dB from 50 Hz to 3,000 Hz; however, in practice only seven of the 105 channels monitored during seven flight tests met this requirement. In general the Saturn systems were within ± 2dB only from *150* Hz to 3,000 Hz, with a sharp rolloff below 150 Hz (up to 11dB at 50 Hz).

When selecting a telemetry system the advantages of the higher frequency response of SSB-FM systems must be weighed against the following disadvantages:

a. To obtain valid data down to 50 Hz, the data spectra must be corrected for channel rolloff which requires a frequency response measurement for each channel.

b. The system band pass of 50-3,000 Hz is not compatible with normal vibration requirements, i.e., 5-2,000 Hz.

c. No phase or time correlation is possible. In fact, if a data tape is played back five times, it is probable that five *different* waveforms will be obtained.

d. Signal distortion by the system is quite high, so that periodic signals are not accurately transmitted. This makes calibration more difficult, as calibration signals are normally D.C. or sine.

e. Data quality obtained from processing is highly dependent on the setup of the ground equipment used, thus, increasing the chances for error.

7-5.4.2 FM-FM

Two types of FM-FM telemetry (proportional-band-width and constant-bandwidth) are now in use. Table 7-2 presents the IRIG standard for porportional bandwidth subcarriers and Tables 7-3 and 7-4 present AIA and IRIG standards for constant-bandwidth subcarriers. Both systems provide a frequency response from D.C. to some f_{max} with an accuracy of 1-5 percent (less than ±0.5 dB). Also, phase and time correlation are possible, and distortion is low.

Note that the frequency response for proportional bandwidth varies from channel to channel shile constant-bandwidth provides a uniform frequency response on all channels. Also, it is possible to intersperse proportional- and constant-bandwidth subcarriers in one system.

7-5.4.3 Pulse Modulation

Pulse modulation or time-sharing sampling is a method of modulation employing commutation; each channel is sequentially sampled. The samples may be transmitted directly in amplitude form by FM modulation of the transmitter or by a voltage-controlled subcarrier oscillator (VCO) or by encoding.

In the time-sharing category, the most frequently used systems are Pulse Duration Modulation (PDM), Pulse Amplutude Modulation (PAM), Pulse Code Modulation (PCM). PCM is further characterized by the fact that each sample is first quarterized, or forced to assume the closest value of certain allowable values. It is then encoded, usually in serial binary form, and transmitted either through a VCO or directly through the transmitter. All time-sharing sampling devices inherently require the transmission of synchronizing signals to provide channel discrimination at the receiving end.

a. Pulse Modulation Systems – The simplest of all time-sharing systems is PAM. It consists of an electronic or mechanical commutator and occasionally, a unity gain amplifier for the purpose of lowering the output impedance. The amplifier may also be provided with nominal gain for low level sources. The PDM system consists of a PAM commutator with a conversion device, often referred

*Handbook of Telemetry and Remote Control; Elliott L. Gruenberg, Editor in Chief; McGraw Hill, 1967; TK-399-G78-C.1 DAC Library

Table 7-2. Proportional-Bandwidth Subcarrier Channels (IRIG)

CHANNEL	CENTER FREQUENCY KHZ	SUBCARRIER DEVIATION %	FREQUENCY RESPONSE Hz **M = 5	FREQUENCY RESPONSE Hz **M = 2.5
1	.400	± 7.5%	6.0	11
2	.560		8.4	14
3	.730		11.0	20
4	.960		14.0	25
5	1.300		20	35
6	1.700		25	45
7	2.300		35	59
8	3.000		45	81
9	3.900		59	110
10	5.400		81	160
11	7.350		110	220
12	10.5		160	330
13	14.5		220	450
14	22.0		330	660
15	30.0		450	900
16	40.0		600	1200
17	52.5		790	1600
18	70.0		1050	2100
19	93.0		1400	2800
20	124.0		1900	3700
21	165.0	± 7.5%	2500	5000
A*	22.0	± 15%	660	
B	30.0		900	
C	40.0		1200	
D	52.5		1600	
E	70.0		2100	
F	93.0		2800	
G	124.0		3700	
H	165.0	± 15%	5000	

*TO USE A LETTERED CHANNEL, THE ADJACENT NUMBERED AND LETTERED CHANNELS MUST BE OMITTED. EXAMPLE: USE OF CHANNEL C WOULD PROHIBIT THE USE OF CHANNELS 15, 16, 17, B, AND D.

**THE DYNAMIC RANGE (MAXIMUM SIGNAL TO NOISE RATIO) DECREASES WITH DECREASING MODULATION INDEX (M).

Table 7-3. Constant-Bandwidth Subcarrier Channels (AIA)

CHANNEL	CENTER FREQUENCY KHZ	DEVIATION KHZ	FREQUENCY RESPONSE - KHZ	
			M = 2	M = 5
1C	12.5	± 2.0	1.0	0.4
2C	20.8			
3C	29.2			
4C	37.5			
5C	45.8			
6C	54.2			
7C	62.5			
8C	70.8			
9C	79.2			
10C	87.5			
11C	95.8			
12C	104.2			
13C	112.5			
14C	120.8			
15C	129.2			
16C	137.5			
17C	145.8			
18C	154.2			
19C	162.5			
20C	170.8			
21C	179.2			
22C	187.5	± 2.0	1.0	0.4
2CW*	20.8	± 4.0	2.0	0.8
4CW	37.5			
6CW	54.2			
8CW	70.8			
10CW	87.5			
12CW	104.2			
14CW	120.8			
16CW	137.5			
18CW	154.2			
20CW	170.8			
22CW	187.5	± 4.0	2.0	0.8

*USE OF A CW CHANNEL (± 4.0 KHZ DEVIATION) PROHIBITS THE USE OF THE ADJACENT ODD NUMBERED C CHANNELS

Table 7-4. Constant-Bandwidth Subcarrier Channels (IRIG)

DEVIATION ± 2 KHZ NOMINAL RESPONSE DC - 400 Hz *MAXIMUM RESPONSE DC - 2000 Hz		DEVIATION ± 4 KHZ NOMINAL RESPONSE DC - 800 Hz *MAXIMUM RESPONSE DC - 4000 Hz		DEVIATION ± 8 KHZ NOMINAL RESPONSE DC - 1600 Hz *MAXIMUM RESPONSE DC - 8000 Hz	
CHANNEL	CENTER FREQUENCY KHZ	CHANNEL	CENTER FREQUENCY KHZ	CHANNEL	CENTER FREQUENCY KHZ
1A	16				
2A	24				
3A	32	3B	32		
4A	40				
5A	48	5B	48		
6A	56				
7A	64	7B	64	7C	64
8A	72				
9A	80	9B	80		
10A	88				
11A	96	11B	96	11C	96
12A	104				
13A	112	13B	112		
14A	120				
15A	128	15B	128	15C	128
16A	136				
17A	144	17B	144		
18A	152				
19A	160	19B	160	19C	160
20A	168				
21A	176	21B	176		

*MAXIMUM RESPONSE IS THE MAXIMUM THEORETICAL RESPONSE OBTAINABLE WITHIN THE DEVIATION LIMITS.

to as a "keyer," which converts the data-related amplitude of the pulse to a constant amplitude pulse with data-related time duration.

A PCM system accepts analog voltages and commutates them in the same manner as the PAM multicoder. The serial analog samples are then encoded into a digital word, usually binary, since such a code permits transmission of the data by means of a series of "on" or "off" signals. The amplitude of these signals is noncritical provided that "ons" and "offs" can be distinguished from each other. These data, as transmitted, can be fed directly into a digital computor for analog presentation or further analysis with no degradation of accuracy caused by further conversions.

b. Comparative Features

1. PDM systems are useful for providing many channels of low frequency response information. They are capable of medium to high accuracy and are comparatively efficient.

2. PAM systems can provide many channels of low frequency information. They are very efficient and have medium accuracy.

3. PCM systems can provide many channels of low and medium frequency information or, with super commutation, high frequency channels. These systems are very efficient and have high accuracy. They have rather an abrupt threshold which promotes user confidence due to the fact that signals can usually be determined to be either good or bad. PCM is the only readily available system that can maintain transmission system error below one percent. Combinations of PCM, PDM, PAM and FM can be used in conjunction with a single transmitter.

7-6 DATA ACQUISITION AND PROCESSING SYSTEMS

7-6.1 Digital Systems (as of 1968)

Acoustical/vibration data acquisition systems are primarily analog, as discussed in 7-4.4 and 7-5.2, whereas, data reduction and analysis are often performed digitally. The main function of data processing has therefore become the analog-to-digital conversion of analog-acquired data. The analyses are then performed by the programs discussed in Section 9, paragraph 9-3 (see Figures 7-2, 7-3, and 7-4).

The equipment in use at A3 Data Processing is capable of a real-time (overall) conversion rate (A to D) of approximately 15,000 samples per second (sps) while A location Data Processing has 30,000 sps capability. This real-time conversion can be for a single channel at the maximum rate, or two or more channels simultaneously at rates representative of equal sharing between the channels (up to 10 channels at A3 and 6 at A). Higher sampling rates may be obtained by lowering the tape playback speed (by factors of 2 or more) to obtain sampling rates up to 320,000 sps. It should be noted that no significant data degradation is introduced by this process (up to a tape speed reduction of 1/8th) unless the data are direct recorded; in the latter case, one octave of low frequency information is lost for each factor of two in tape speed reduction. A sample and hold amplifier may be required if the tape speed is reduced to analyze high frequency data.

The method whereby two or more channels are simultaneously converted is very valuable if data are to be cross analyzed, as no time/phase lag is introduced between the various channels. If the data are not to be cross analyzed, tapes should be specified as single channel because tape read time for multichannel tapes is slightly higher than that for single channel tapes.

The digital tape format now used for most acoustical/vibration analysis is CAT 74. These tapes may contain from one to ten channels of data at any conceivable sampling rate, stored in as many files as the physical limits of the tape permit (tape reel capacity is approximately 3.2 million data words). No restrictions are placed on data sources, so that each file, which represents one continuously sampled time interval, may contain data from different types of measurements, acquired by different methods on different tests. The one requirement is that the sampling frequency remain constant throughout the tape.

If the data are not available in CAT-74 format, programs such as UT13 are available to reformat data for input to the analysis programs (see Section 9, paragraph 9-3).

7-6.2 Analog Systems (as of 1968)

Analog data acquisition systems include SSB-FM and FM-FM telemetry, discussed in 7-4.4, as well as numerous hardwire/record methods used for ground tests. The most often used method for ground test recording is hardwire/FM. Using a center frequency of 54 KHZ ± 40 percent deviation, a frequency response from DC to 2,500 Hz, 5,000 Hz, 10,000 Hz, or 20,000 Hz may be obtained (as desired), at a tape speed of 30 IPS. A response of DC to 2,500 Hz will yield a dynamic range of 45 dB. Other combinations of center frequency and tape speed are also available, and a frequency response of DC to approximately 40 KHZ can be obtained.

For high frequency data, such as produced by pyrotechnic shock, direct recording will provide a frequency response from 300 Hz to approximately 120,000 Hz at a tape speed of 60 IPS, with a dynamic range of about 30 dB.

The foregoing methods are the most commonly used. For descriptions of other methods, the Data Processing Section or Instrumentation Design Sections should be contacted.

Analog data processing available includes the following:

a. Oscillograms — Instantaneous level vs. time.

b. Sanborn graphic level recorder — RMS level vs. time.

c. Bruel and Kjaer one-third octave spectrum analyzer — output of one-third octave filters recorded on a c b.

d. Data filtering — low pass, high pass or band pass.

e. Sonic analyzer — Instantaneous peak spectrum levels.

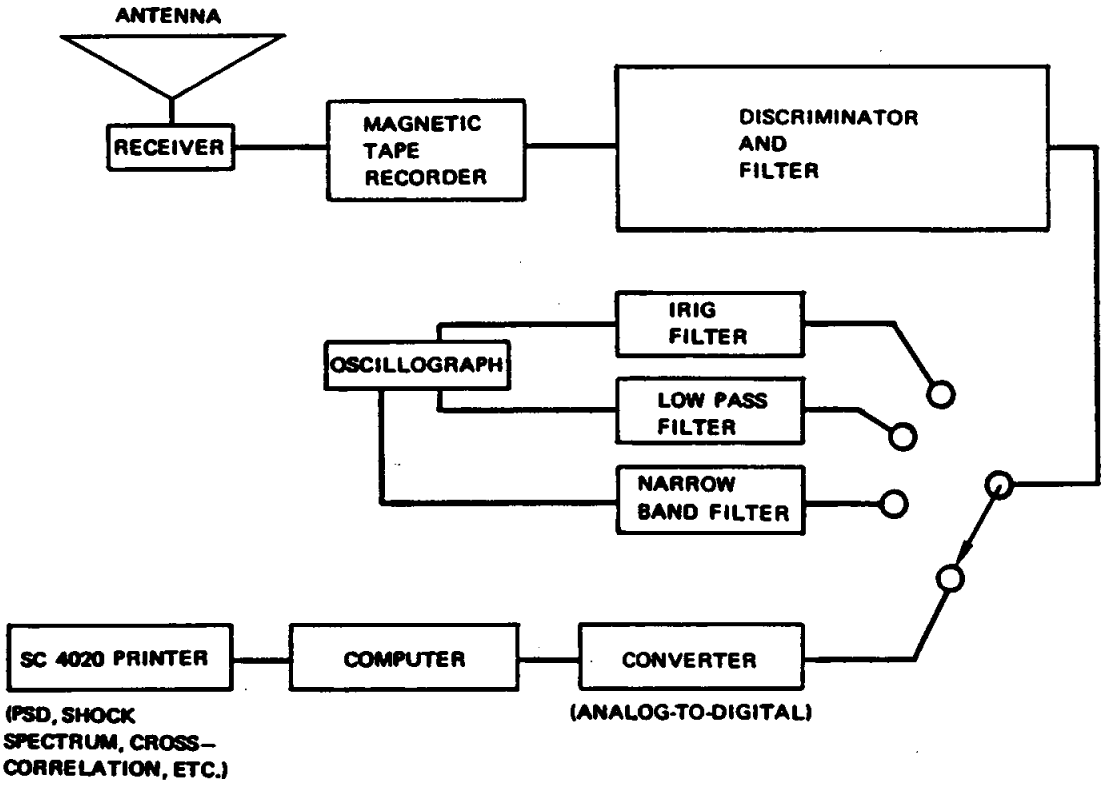


Figure 7-2. Vibration and Shock Data Reduction System

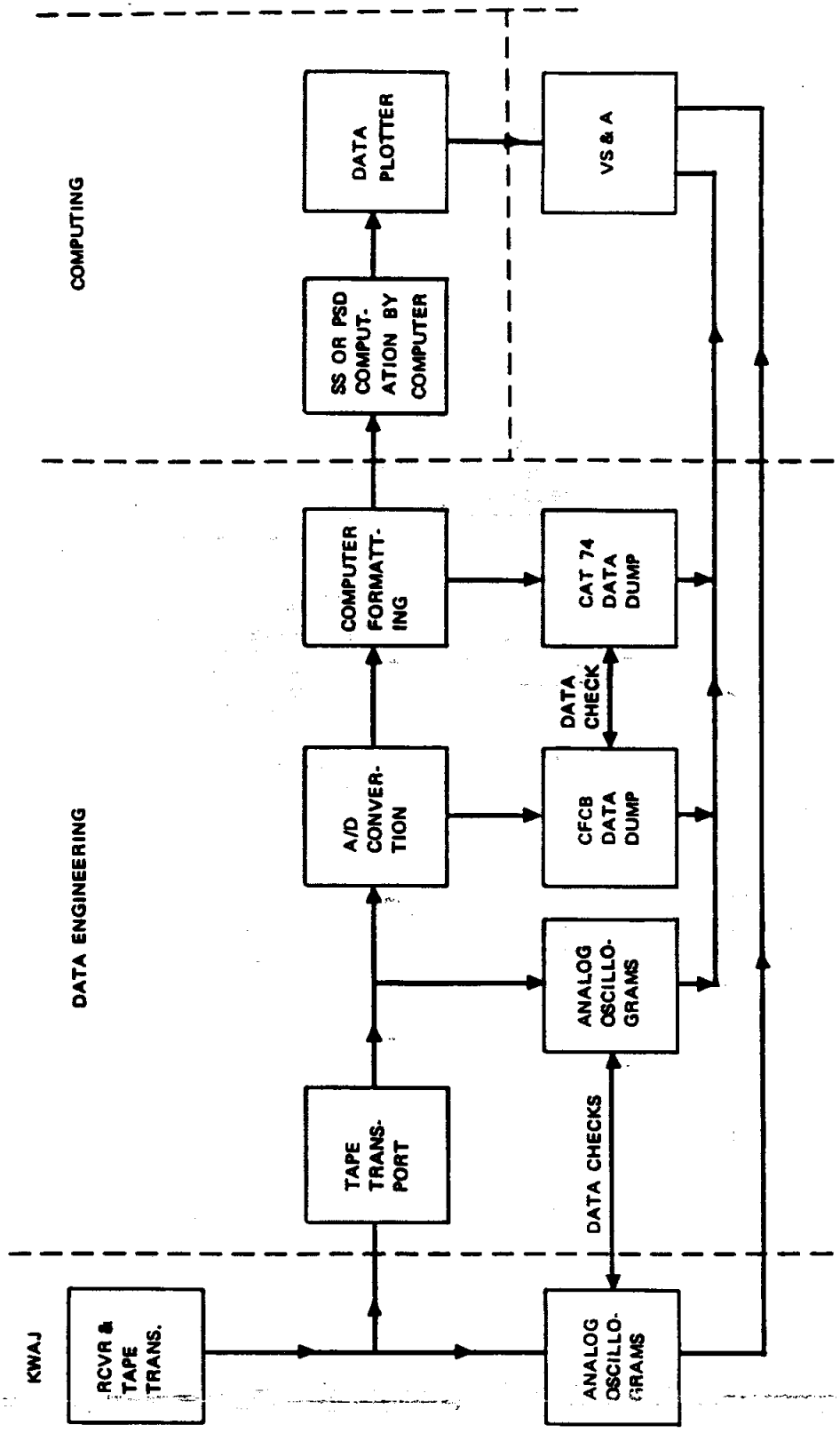


Figure 7-3. Flight Data Flow Diagram

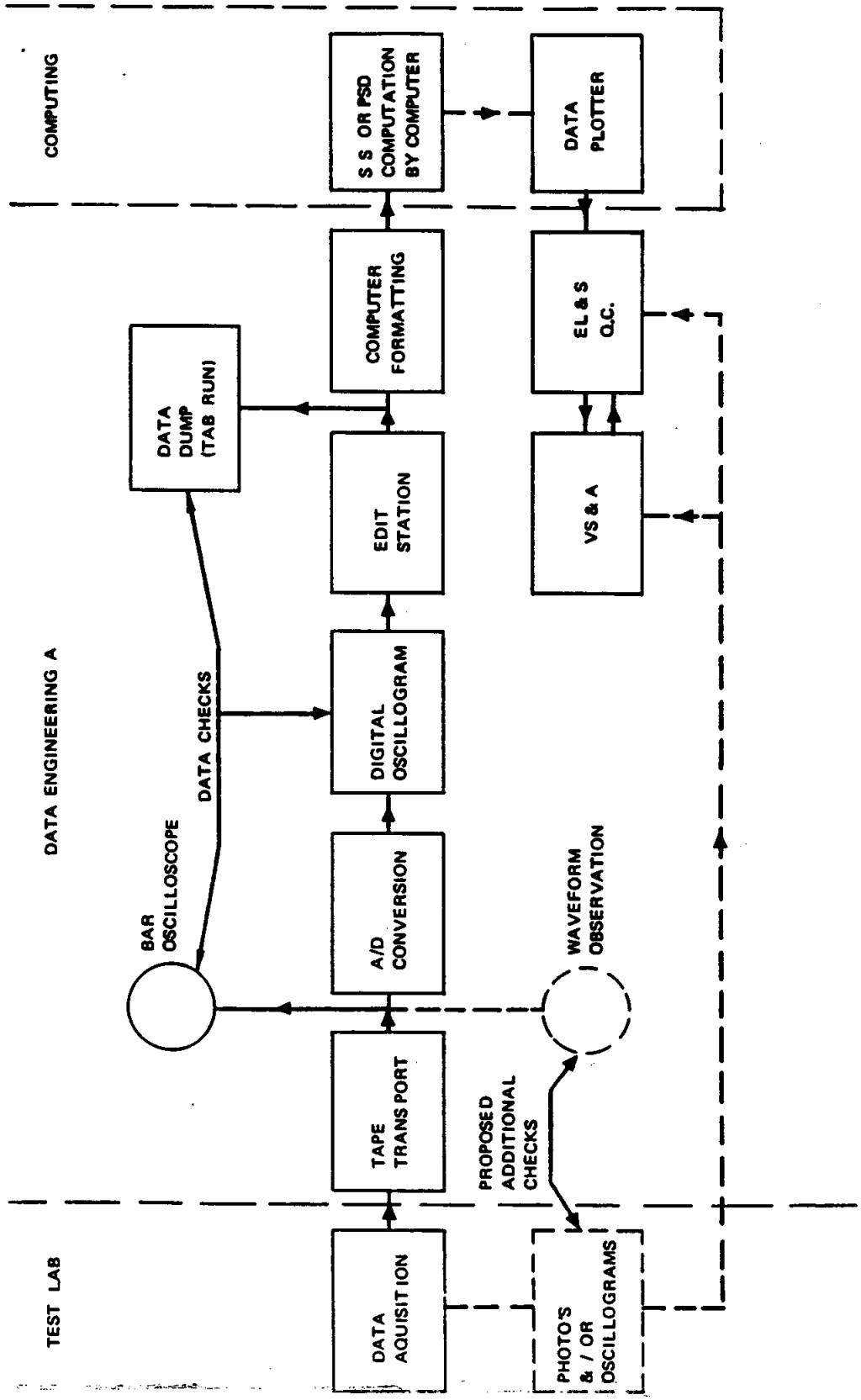
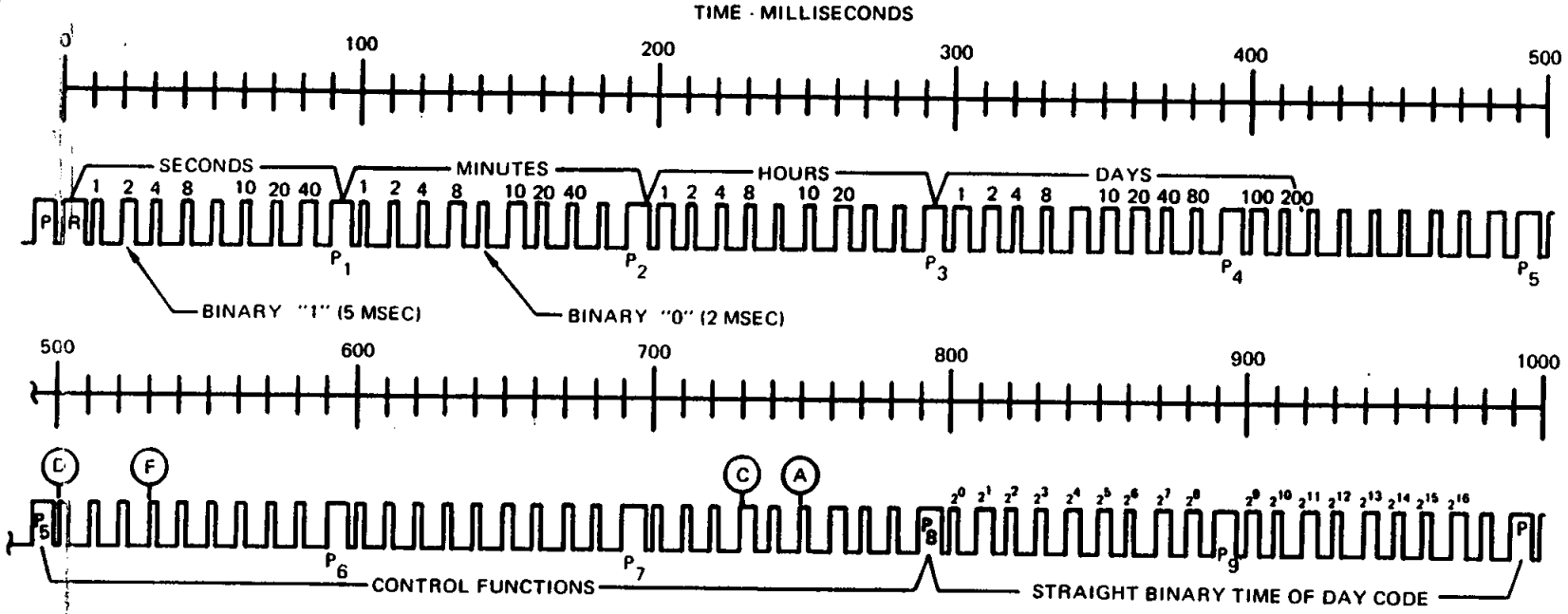


Figure 7-4. Ground Test Data Flow Diagram

7-6.3 Timing

Timing signals are of importance in relating specific events with data anomalies. For flight measurements, the most commonly used timing code is IRIG-B. Figure 7-5 can be used to read that real time code. Verbal identification is always helpful if labels are lost or forgotten, but sometimes quite useless as a means of timing because of insufficient playback electronics. Hence, augmentation of voice cueing with some type of timing signal is recommended.

When performing ground tests, the usual timing signal available is merely a steady-state frequency of pulses or cycles. When utilizing such a signal, it is important to plan for tape speed changes (reduction or increase) so that the pulses are not lost due to an inadequate frequency response. However, the preferable timing signal is amplitude or frequency modulated so that specific pulses can be indisputably identified (in preference to a steady-state signal which is only good for determining relative elapsed time).



1. 100 PPS MODULATING 1-KC. CARRIER IN ONE SECOND TIME FRAME, COMBINATION TIME DISPLAY CONSISTING OF 30-BIT BINARY CODED DECIMAL AND 17-BIT STRAIGHT BINARY.
2. TIME CODE READOUT IS REFERENCED TO LEADING EDGE OF REFERENCE MARKER R WHICH ALSO DENOTES BEGINNING OF ONE SECOND TIME FRAME.
3. POINT P DENOTES POSITION IDENTIFIERS, 8 MS DURATION.
4. POINT (C) DENOTES CONTROL ELEMENTS, 5 MS DURATION.
5. POINT (D) DENOTES 0.1 SECONDS INDEX MARKER; POINT (F) DENOTES 0.01 INDEX MARKER.
6. TIME AT POINT (A) IS 21 HRS., 18 MIN., 42 SEC. +0.7 + .05 = 2118:42.75 ON 133 RD DAY.
7. WEIGHTED CODE DIGITS HAVE 5 MS DURATION, UNWEIGHTED DIGITS HAVE 2 MS DURATION.

Figure 7-5. IRIG-B Time Code

Section 8
STATISTICS

Statistics provides a particularly useful method to describe processes in which the exact behavior in time cannot be predicted for any instant of time, but which behave in a certain describable manner from a probabilistic point of view. Stationary or weakly-stationary random time histories typify such processes. The technologies of vibration and acoustics are often concerned with dynamic environments which are such random processes. Hence, it is appropriate to discuss the meaning, interpretation, and use of statistical quantities as descriptors for these dynamic environments. For concise definitions of the terms utilized in the following subsections, refer to Section 14, Terminology.

8-1 STATISTICAL QUANTITIES

Several basic, interrelated statistical quantities are utilized to advantage in describing random vibration or random fluctuating-pressure fields. Theoretically, these quantities are determined from time histories that extend from minus infinity to plus infinity, and from enumerable sets of these. Since it is not practical to obtain such unlimited information, the mathematical definitions involving integration with infinite limits (found in numerous textbooks) for the statistical quantities, are replaced in practice with finite summations which are carried far enough to indicate convergence. Even so, it can be shown that sufficient accuracy is maintained to provide valid results (e.g., see *Measurement and Analysis of Random Data* by J. Bendat and A. Piersol, J. Wiley & Sons).

From the time history of a random signal (assuming stationarity so that a finite time interval of this time history may be examined as representative of any other time interval of the same duration which could occur anywhere between minus infinity and plus infinity, and assuming ergodicity so that this same time history may be assumed to represent any of the set of time histories of the random processes under consideration) the probability density functions (e.g., peak, amplitude, etc.) or an autocorrelation can be obtained.

A spectral density (distribution throughout the frequency domain of a quantity that is proportional to the power in a one hertz band) can be determined from the time history itself or can be calculated as the Fourier Transform of the autocorrelation. If two or more time histories are available for analysis, joint probability functions, cross-correlations and their Fourier Transforms cross-spectral-densities, and coherence functions can all be obtained. (See Section 14, Terminology for mathematical definitions.)

8-2 INTERPRETATION OF STATISTICAL QUANTITIES

8-2.1 Autocorrelation

Autocorrelation is a comparison of a time history with itself at an earlier or later time. Hence, it defines the degree that a time history signature remains intact (recognizable) in time.

Since most correlation work deals with correlation coefficients (the correlation normalized to itself at zero time delay, the latter being always equal to the maximum possible value of correlation), the coefficient ranges from a value of 1 to -1. If the autocorrelation coefficient is equal to 1, the signature is identical to the original signature; if equal to any positive value between 0 and 1, it is partially recognizable and in phase; if 0, the time-displaced signature is completely unlike the original signature; if some negative value between 0 and -1, then the time displaced signature is partially recognizable as the original signature but out of phase; and if -1, the time-displaced signature is exactly like the original signature but 180° out of phase with it.

A sine wave has an autocorrelation coefficient that is a cosine. A random signal has an autocorrelation coefficient which usually resembles a decaying trigonometric function (the rate of decay depending on whether sine components are present or whether the random signal is a broad band – rapid-decay, or narrow band – slow-decay signal).

Since sine waves continue to show correlation (even when low in amplitude) while a truly random signal quickly (in time) loses all resemblance to its previous self, the autocorrelation analysis is an excellent way to identify the presence of unwanted sinusoidal components in a random signal. The reverse is also true,

for if a significant contribution is made to a sinusoidal signal by random "hash," then the perfect cosine correlation function will be distorted (i.e., it will not reach coefficient values of plus and minus one).

8-2.2 Power Spectral Density

Power Spectral Density is the Fourier Transform of the autocorrelation function, and reflects the frequency distribution of the energy. As is typical with Fourier Transforms, when the signature shows good correlation in time, it is reflected as concentrated energy in the frequency domain, and conversely, poorly time-correlated signals have energy distributed throughout much of the frequency domain.

By definition, the spectral density (of whatever the variable of concern) is proportional to the power (the variable squared) in a one Hertz bandwidth at a frequency. If the word spectrum or spectral density is *mis-applied* to a series of amplitudes which are proportional to the energy in successive bands of frequencies larger than one hertz (e.g., fractions of, or whole octave bands) then an appropriate adjective modifier should be added to the word spectrum (e.g., 1/3-octave band spectrum, etc.).

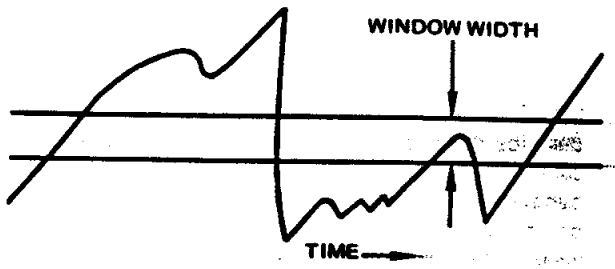
The reason for this additional identification is obvious, since the shape and amplitude of the curve in the frequency domain depends on the bandwidth of the energy. (e.g., A spectral density with a constant amplitude (white noise) has an octave band "spectrum" that increases 3 dB per octave. The octave band level for the band whose geometrical mean frequency is 500 Hz is 25.5 dB above the spectrum level.)

8-2.3 Probability Density Function

Probability Density Function indicates the probability that the function will assume a certain magnitude. By examining the density function for amplitudes, the presence of a bias (non-zero position of the maximum of the density function) can be detected, the mean square and root mean square values can be calculated, and the standard deviation (variance) is determined (and equal to the mean square if the mean value is zero and the random process is stationary). Also, the distribution function can be compared to known functions by goodness of fit tests for similarity.

The integral of the probability density function with minus and plus infinity integration limits is equal to 1. The presence of a sine wave with a constant amplitude is detectable in a random signal amplitude density function by the obvious positive and negative maxima on either side of the mean.

Other characteristics of a signal can be detected by proper interpretation of the distribution function (e.g., a clipped signal does not have a normal amplitude distribution function). A peak distribution function is different from an amplitude distribution function in that the amplitude must change sign (±) of slope within the window or it is not a peak. For example in the sketch following, there are 4 counts of amplitude but only one peak count within the window.



Where the autocorrelation and spectral density are interrelated (being a Fourier Transform pair) the probability density function is not related to either of them (unless the data are Gaussian and the mean value is zero – no bias). The normal or Gaussian distribution is the easiest distribution function with which to work, and in many respects the most useful (e.g., if Gaussian, all the higher order moments are determinable). However, a stationary non-Gaussian signal looks Gaussian when viewed through narrow bands.

8-2.4 Stationarity

Stationarity is not one of the major functions to be discussed, but deserves a word of explanation. The existence of stationarity (whether by assumption or by actual fact) permits statistical analysis techniques to be used on the data.

In actuality, no data (of the sort under consideration) are stationary since they have a finite beginning and end in time. Weakly-stationary signals are indicated in the practical way by the autocorrelation and the mean value being invariant with time (within the sampling error). True stationarity requires that higher moments are invariant. However, if weak stationarity can be shown for vibration, shock and acoustical data, then statistical analyses of the random processes is warranted.

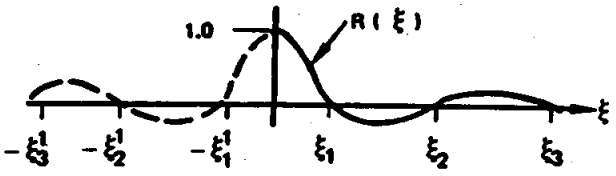
8-2.5 Cross-Correlation

Cross-Correlation is computationally similar to the auto-correlation function except that two signatures taken at different locations in space are compared in the time domain. Much information can be gleaned from this type of investigation. It provides space-time data which defines convection velocity (speed and direction), distribution in space of the in-phase pressures either in the frame of reference of the forcing function, or the structure, the latter being determined by examining a

particular "instant" of time, $\tau = 0$, when the two frames of reference are identical – the "instant" being actually a short finite time in which averages are taken), and the retarded-time definition of the total space-time correlation function if a constant convection velocity is in evidence.

Working in the space-time domain has advantages from the aspect of measured data, but for analysis, this same information is best expressed in the wave number-frequency domain which is the double Fourier Transform of the cross-correlation. For much of the analysis work, a hybrid system of space-frequency is utilized, since the added sophistication of either acquiring data in sufficient quantities and over sufficient space to permit wave number filtering or performing a Fourier Transform on the spatial results is not usually required.

Correlation lengths (or areas) can be calculated in a variety of ways from the $\tau = 0$ spatial correlation. Since each method provides a different answer, care should be exercised in comparing quantitative results from different sources unless the method is specified. Specifically, a correlation length (area) gives an estimate of the distance (area) over which the pressures are in phase. Since a spatial correlation coefficient function often has positive and negative values between +1 and -1, the following methods have been used to describe a correlation length (see sketch).



Length 1 $= (\xi_1 - \xi_1^1) = 2\,\xi_1$

Length 2 $= \int_{-\infty}^{\infty} R(\xi)\, d\,\xi = 2 \int_0^{\infty} R\,(\xi)\, d\,\xi$

Length 3 $= \int_{-\infty}^{\infty} |\, R(\xi)\,|\, d\,\xi = 2 \int_0^{\infty} |\, R(\xi)\,|\, d\,\xi$

Length 4 $= \int_{\xi_1^1}^{\xi_1} R\,(\xi)\, d\,\xi = 2 \int_0^{\xi_1} R(\xi)\, d\,\xi$

Length 1 is strictly a crossover point to crossover point distance, between which the pressures are in phase but not everywhere perfectly correlated. Lengths 2 and 3 take into account the whole curve, either letting pressure cancellation occur (length 2), or adding all portions of the curve (length 3), while length 4 determines a length over which the pressure is perfectly correlated that is the equivalent of the correlated length 1, over which the pressures are only partially correlated.

If the structural lengths (areas) are large compared to $2\xi_n$ (the last significant crossover point) then length 2 (or an area based on a similar consideration extended to two dimensions) is the correct correlation length (area) definition.

If, however, the structural lengths (areas) are small compared to $2\,\xi_1$, then the length (area) 4 would be a preferred definition. For structural lengths (areas) between ξ_1, and ξ_n, good judgment must be applied as to whether the additional factor of pressure cancellation should be included.

8-2.6 Cross-Spectral Density

Cross-Spectral Density is a complex quantity with a real part (co-spectral density) and an imaginary part (quadrature or quad-spectral density). This function is the Fourier time transformation of the cross-correlation. As such, it describes the relationship in the frequency domain of the time dependence. As in the case of the autocorrelation-power spectral density relationship, a long time dependence of the cross-correlation means a narrow frequency band in which there is similarity in the cross-spectral density, and for a short time dependence, there is a broad frequency identification.

Much information can be extracted from plots of the spectral density (whether plotted as real and imaginary parts or as the amplitude and phase of the function, the amplitude being the square root of the sum of the squares of the co and quad-spectra, and the phase being the arctan of the ratio of the quad to the co-spectra).

For example:

a. Narrow band (band-limited) crossover points for determining correlation lengths for various frequency components can be ascertained from broad-band cross-spectral densities taken from various separation distances because the crossover point occurs at a distance for which the phase angle equals 90° for the frequency of interest $\xi_1 = U_c\pi/2\omega$.

b. A frequency-dependent convection velocity can be determined by dividing the phase angle by the frequency (in radians/sec) (convection velocity can also be obtained by multiplying $2\omega/\pi$ times the distance between zero and the first zero crossing of the co-spectrum).

c. Time delays versus frequency can be obtained from the cross-spectral densities (which are not directly obtainable from broadband cross-correlation analyses).

8-2.7 Coherence Functions

Coherence functions are a measure of the linearity of one function with respect to the other. Hence, they can be used to detect structural transmission paths for a response due to single or multiple inputs. They can also be used to predict errors in frequency-response-function estimates. If there is no linear relationship between two functions the coherence is zero. Since the function is one of frequency, the amount of coherency is determined at each frequency, with the maximum amount of coherency being 1.

A shift in the frequency of maximum coherency versus spatial separation suggests that as distance increases, the predominant contributors to the recognizable similarity of the two signatures are changing (coming from different frequencies). Plots of coherence versus frequency also quickly inform the viewer as to what portions of the spectrum contributes significantly to the signature similarity (correlation). The lack of coherence between signatures, which are known to be identical, could result from extraneous noise or the presence of other signatures which were not previously being considered.

8-3 EXPLANATION OF CONFIDENCE BANDS

The purpose of a confidence band is to convey the accuracy with which a number is embued. A wide confidence band obviously suggests much less accuracy in predicting the value for a given number than a narrow one.

If a Gaussian distribution of values is assumed to exist around the mean, then a variance of one standard deviation σ, as given by

$$\sigma = \sqrt{\frac{\sum_{i=0}^{n} d_i^2}{n}}$$

(where d_i is the ith deviation from the mean and n is the number of data points available for analysis) provides a 68 percent confidence band (68 percent of the time the value will be the mean $\pm\sigma$). A 2σ confidence band includes 95 percent of the data, and a 3σ confidence band includes 99.7 percent of the data.

8-4 CONFIDENCE LIMITS ON PSD'S DUE TO SAMPLE DURATION

When data are digitally reduced to spectral densities, the error in decibels is approximately equal to $20/\sqrt{2BT-1}$ when the factor 2BT is greater than 16. The constant (20) changes if 2BT is less than 16. Here B is the bandwidth of resolution and T is the sample time interval. This error or confidence band is the width of the 90 percent band (90 percent of the time, the value is within the error band around the given value).

Obviously, the longer the time interval of the data, or the less accurate the frequency determination (wider bandwidths) the higher the confidence (smaller the error band) that the quoted value is correct. Unfortunately, it is usually desirable to have narrow bandwidths to resolve signatures into frequency components, and core storage limits the time interval for digital analysis. With these factors tending to make the "BT" error large, good judgment must be applied when deciding resolution details.

Section 9
COMPUTER APPLICATIONS

9-1 NON-PRODUCTION PROGRAMS FOR DYNAMICS

This paragraph presents brief descriptions of non-production computer programs written for the vibration, shock, and acoustical technologies, as follows:

F147 Prediction of internal SPL's due to external FPL's.

F949 Vibratory response of a multiple-degree-of-freedom system.

G300 Acoustic power levels from measured SPL's.

G356 Response of a simply-supported flat panel to a moving pressure field with an assumed correlation.

9-1.1 Program F147 – Prediction of Internal SPL's Due to External FPL's (1962)

Program F147 is a study of internal acoustic levels as produced by the turbulent boundary layer exciting the shell, resulting in several comparative techniques of prediction. This computer program calculates the internal SPL's based on a modified Ribner[1] calculation which assumes most of the energy is generated due to coincidence effects. In addition, an empirically deduced equation is also included based on data from Zeus, Transit Courier, and Nike Hercules flights. These last named data gave the following result:

$$\text{Internal SPL} = 10 \log_{10} \frac{AF\, \rho_e\, V_{oo}\, \rho_i\, \delta^* K}{m^2\, P_o^2}$$

where:

AF = Non-dimensional amplification factor due to reverberation

ρ_e = External air density

ρ_i = Internal air density

V_{oo} = Free stream velocity

δ^* = Boundary layer displacement thickness

K = Power spectral density of FPL's (fluctuating pressures at the external surface

m = Surface mass of the wall

P_o = Standard sound pressure reference level squared

F147 is a working program (non-production status) using FORTRAN II language with standard input 1 loadsheets on an IBM 7094 Computer.

For information relative to programming and program usage, contact C. M. Ailman.

9-1.2 Program F949 Vibratory Response of a Multiple-Degree-of-Freedom System (1964)

Program F949 is a digital computer program for a vibration analysis of multiple-degree-of-freedom systems. The steady-state response to a sinusoidal forcing function or the transient response to some arbitrary shock pulse is obtained by normal mode theory. The superposition of natural modes is used to obtain the total response at any mass point of the system.

The essence of the F949 program is found in the following equation for a multiple-degree-of-freedom system in a steady-state response to a sinusoidal forcing function:

$$\frac{R}{A} = \sum_{j=1}^{m} -\frac{r_j\, \mathrm{Re}(\bar{p}_j)}{\omega^2 M_j} + \sum_{j=1}^{n} \frac{r_j\, \mathrm{Re}(\bar{p}_j)(K_j - \omega^2 M_j) + \mathrm{Im}(\bar{p}_j)\, B_j}{(K_j - \omega^2 M_j)^2 + B_j^2}$$

$$+ i \left[\sum_{j=1}^{m} -\frac{r_j \mathrm{Im}(\bar{p}_j)}{\omega^2 M_j} \right] + \sum_{j=1}^{n} \frac{r_j \mathrm{Im}(\bar{p}_j)(K_j - \omega^2 M_j) - \mathrm{Re}(\bar{p}_j)\, B_j}{(K_j - \omega^2 M_j)^2 + B_j^2}$$

[1] *Ribner, H. S., Boundary-Layer-Induced Noise in the Interior of Aircraft, Institute of Aerophysics, University of Toronto, Report No. 37, April 1956*

where:

- A = Forcing function amplitude
- R = Total response of the summed r_j modes
- j = Modal subscript
- r_j = Value of a specific response due to unit amplitude of the j^{th} mode
- Re = Real part of complex numbers
- Im = Imaginary part of complex numbers
- $\bar{p}_j$ = Complex amplitude of the forcing function
- ω = Forcing function frequency in radians/sec
- M_j = Modal mass
- K_j = Modal stiffness
- B_j = Modal damping

This general analysis assumes that the rigid body modes and the flexible modes must, when combined, conform to the notation of orthogonality that had been introduced to uncouple the equations (the space coordinate system is orthogonal to the mass weighting function).

Any output of the F949 program is normalized to an input of one g excitation, whether acceleration, displacement, bending moment, etc. For example, the output of the F949 program can be bending moment per g versus frequency for some position on the multiple-degree-of-freedom-system. Taking this answer and multiplying it by the g level of the excitation at a particular frequency results in a value for the bending moment.

F949 is a working program (non-production status) using FORTRAN II language with standard Input 1 loadsheets.

Publication references are as follows:

Bisplinghoff, R. L., et al, *Aeroelasticity,* Addison-Wesley Pub. Co., Inc., 1957

Bisplinghoff, R. L. and Ashley, H., *Principles of Aeroelasticity,* John Wiley and Sons, Inc., 1952

Myklestad, N. O., *Fundamental of Vibration Analysis,* McGraw-Hill Book Co., 1956

Gardner, M. F. and Barnes, J. L., *Transients in Linear Systems,* John Wiley and Sons, Inc., 1956

Aseltine, J. A., *Transform Methods in Linear System Analysis,* McGraw-Hill Book Co., Inc., 1958

For information relative to programming contact C. M. Fuller.

9-1.3 Program G300 – Acoustic Power Levels From Measured SPL's (1965)

Program G300 is a digital computer program to calculate total acoustic power of a sound source. The program uses measured far field sound pressure levels with corrections being added or subtracted if desired to correct for system responses, spherical divergence losses, etc. These measured pressures are assumed constant over a hemispherical surface area and, hence, total power radiated over the surface can be calculated. The program also permits the power to be calculated in octave bands while the overall acoustic power is summed over the entire range of frequencies.

G300 is a working program (non-production status) using FORTRAN IV language with standard Input 1 loadsheets.

For information relative to programming, contact A. H. Marsh of Douglas Aircraft Company. For usage information contact C. M. Ailman of McDonnell Douglas Astronautics Company.

9-1.4 Program G356 – Response of a Simply-Supported Flat Panel to Moving Pressure Field With an Assumed Correlation (1965)

The closed form solution of the response (displacement) of a simply-supported flat plate was calculated using Green's functions after Dyer[1]. A different correlation function was assumed based on broadband cross-correlation calculations of the acoustic field approximately 110 feet from an upward-firing nuclear engine.

Program G356 calculates frequencies of assumed modes and calculates displacement and acceleration for each at any point on the plate. The program also sums the response to give a total mean-squared displacement and zero to peak acceleration at the position of interest. Reference-run capability permits examination of different positions on the same plate.

G356 is a working program (non-production status) using FORTRAN IV language and standard Input 1 loadsheets on an IBM 7094 Computer.

A report which may be helpful as a reference is SM-48903 dated May 1965. For information relative to programming and usage contact C. M. Ailman.

[1] *Dyer, I., Response of Plates to a Decaying and Convecting Random Pressure Field – Journal of Acoustical Society of America, Vol 31, Number 7, July 1959*

9-2 PRODUCTION PROGRAMS FOR DYNAMICS

This section briefly describes production (unless status otherwise stated) programs written to predict dynamic response of structures. For current detailed information such as cognizant programmer and user, refer to the Production Programs Abstract Catalog. The programs included are:

A16 Redundant force analysis

DA02 Determination of vibration modes by K-Matrix displacement method

DA16 The dynamic response of a simply-supported, finite, thin, elastic, cylindrical shell of circular cross-section

DA20 Transient response and dynamic loads

F690 Manual of Auxiliary FORTRAN programs for redundant force analysis

G612 Redundant force structure plotter

G750 Dynamos III – A FORTRAN Program for determining dynamic response of multi-layered orthotropic cylindrical shells

H026 Matrix abstraction program

H195 Dynamic elastic-plastic response of simple structures (DEPROSS)

H195B Dynamic elastic-plastic response of a hard-bonded double-layered free ring

H195C Dynamic elastic-plastic response of a soft-bonded double-layered free ring

H195G Dynamic, elastic, perfectly-plastic response of an orthotropic cylindrical shell

H243 Orthotropic analysis of composites, laminates, and elastic structures (ORACLES)

H258 Dynamic response analysis of thin orthotropic shells of revolution

H417 Free vibration of preloaded orthotropic stiffened cylinders

H523 Shock propagation and decay by equivalent depth technique (SPADEDT)

MW10 Redundant force analysis master table duplication

MW11 Transfer function matrix generation

SA 40 Matrix analysis: FORMAT II

SA 42 Automated structural design (formerly principal force)

SA 48 Matrix abstraction and eigenvalve, eigenvector capability

SA 74 Output identification for SA 40 (FORMAT II)

SA 80 The linear, elastic, static and dynamic analysis of multilayered orthotropic shells of revolution by the matrix displacement method

9-2.1 Program A16 – Redundant Force Analysis (1958)

Program A16 provides an analysis of statically indeterminate structures. The method of analysis used by this program is matrix formulation of the equilibrium and the Maxwell-Mohr equations. In the Maxwell-Mohr Method, the structure is cut to create a statically determinate structure of basic system.

The actual structure to be analyzed is simulated by an idealized discrete model consisting of bars and panels. In general, the bars can carry tension, torque, four components of bending moment, and associated sheers. The panels can carry shear. In the idealization, bars are considered straight between joints.

The Redundant Force Method is applicable to any linear discrete structure. The term "discrete" denotes a finite number of members connected at a finite number of joints; "linear" refers to the assumed linear relationships between external loads, support displacements, internal forces, and deflections.

The program consists of six phases and is made up of the following programs:

Phase I and II

A05.0 MATRIX GENERATOR – This program reads in the input tables and generates the matrices required for the redundant force method. The input consists of structural geometry and properties.

Phase III

A05.1 STRUCTURE CUTTER – This program determines which of the forces are redundant forces and solves the equations of static equilibrium.

Phase IV

A17.0 MATRIX TAPE EDITOR – This program is a matrix input routine.

Phase IV

A09.0 TAPE MATRIX COMPILER – This program provides a simple and efficient method of performing algebra on large order matrices.

A54.0 MATRIX ABSTRACTION – Like A09.0, this program also provides a method of performing matrix algebra. It is usually used in place of A09.0. It is more difficult to code, but takes less machine time.

Phase V

A05.2 REDUNDANT FORCE MATRIX OUTPUT – This program provides a final printout of the Redundant Force Method matrices. Output consists of joint and bar external reactions, bar forces, panel forces, and deflections.

Phase VI

A05.3 STRESS ANALYSIS – This program computes the individual and combined bar stresses and the panel stresses.

A16 is a production program using DEMON language with an IBM 7094 Computer. Loadsheets used are Forms 60-994 Series, 60-983 Series, 60-1013 Series, 60-1030, 60-1087 and 60-1157.

Refer to Report SM-30470 dated 12-20-60 and DAC Engineering Paper No. 834 dated 9-17-59 for additional reference material.

For information relative to programming contact K. W. Springer and for usage information contact R. J. Soukup.

9-2.2 Program DA02 – Determination of Vibration Modes by K-Matrix Displacement Method (1967)

Program DA02 uses the displacement method to compute the vibration modes in a flexible beam or coupled system of beams. The program is designed to be as unspecialized as possible and to allow for a variety of systems and subsystems with the addition of masses, branch beams, springs, pins, and clamps.

Boundary conditions in the form of grounded springs, pins and clamps may be applied at desired reference stations on the main beam. A total of forty reference stations are allowed to yield a maximum of eighty degrees of freedom for bending problems. Compression (or torsion) modes may also be computed. In addition to modes, computed output includes internal forces (i.e., moments and shears), the mass and stiffness matrices, the modal mass matrix, and the largest and smallest non-zero eigenvalues obtained from the dynamical matrix.

DA02 is a production program using FORTRAN IV language and Forms 60-1741 Series loadsheets and a Univac 1108 Computer.

Refer to Douglas Report Number DAC-58633, dated 6-20-67 for additional data. For information relative to programming contact D. N. Kelly and for usage information contact A. S. Hopkins.

9-2.3 Program DA16 – Dynamic Response of a Simply-Supported, Finite, Thin, Elastic, Cylindrical Shell of Circular Cross-Section (1969)

DA16 is a digital computer program, written in FORTRAN IV language, for the dynamic response of a simply-supported, finite, thin, elastic, cylindrical shell of circular cross-section. This shell is assumed to obey the linearized Reissner shallow-shell theory, modified to include viscous-damping terms, which provides two coupled equations of motion in the radial displacement and the Airy stress function.

The Green's function method is used to solve the Reissner equations for a moving shock wave and viscous damping. This shock wave is a step function discontinuity incident at an arbitrary angle upon a shell whose interior is at atmospheric pressure.

DA16 will print out radial displacements at the intersection of each circumferential angle and each plane section through the shell, and, at user option, none or one or more of the following plots will be generated on the S-D 4060: (1) maximum displacement versus each plane section through the shell; (2) maximum displacement versus each circumferential angle; (3) maximum displacement versus velocity; (4) maximum displacement versus time; (5) maximum displacement versus the angle of incidence of the shock wave; and (6) displacement versus each plane section through the shell for a given circumferential angle.

DA16 is a production program using FORTRAN IV language, Forms 60-2287 Series loadsheets and a Univac 1108 Computer. For information relative to programming and usage contact P. R. Lindsey. The following reference documents may be useful in understanding the theory presented:

a. PDL-36737, *Green's Function Technique in the Dynamics of a Finite Cylindrical Shell*, dated January, 1965

b. PDL-70745, *On the Dynamic Response of an Orthotropic Finite Cylindrical Shell to an Arbitrary Pressure Field*, dated 1968

c. Douglas Report No. DAC-61873, *The Dynamic Response of a Simply-Supported, Finite, Thin, Elastic, Cylindrical Shell of Circular Cross-Section,* dated May 31, 1969

9-2.4 Program DA20 – Transient Response and Dynamic Loads (1969)

Program DA20 computes the response of a general structural system, using the normal mode method. In this method, the equations of motion for each mode are uncoupled. Input and output is in modal coordinates. Second order modal differential equations are solved by a closed form method.

Program input includes initial conditions for the modal displacement and velocity; and arbitrary forcing function; and the damping, natural frequency, and modal mass for each mode. Input matrices may be from cards or tape.

Program output consists of modal displacement, velocities, and accelerations (optional) at the indicated time points. The output may be printed, written on tape, or both, and there is tape compatibility between DA20, SA40 and SA48.

DA20 is a production program using FORTRAN IV language and a Univac 1108 Computer.

For information relative to programming contact Jean Smith. For user information contact A. S. Hopkins.

Refer to Memorandum A2-773-LCAO-DA20-1, dated 3-15-68, for additional data.

9-2.5 Program F690 – Manual of Auxiliary FORTRAN Programs for Redundant Force Analysis (1963)

Program F690 determines mechanical properties and bar deformation at elevated temperatures for use in A16.0 Redundant Force Analysis Program.

F690 is a production program using FORTRAN II (version 3) language and standard Input 1 loadsheets with IBM 7094 and IBM 360/65 Computers.

For information relative to programming contact H. I. Kerr. For user information contact R. J. Soukup.

Refer to Douglas Report SM-42958, *Manual of Auxiliary FORTRAN Programs for Redundant Force Analysis* for additional reference data.

9-2.6 Program G612 – Redundant Force Structure Plotter (1964)

Program G612 plots several pictures of the structure represented by A050 card input, A160 tape input or by input 1 FORTRAN cords. Plotting is done on the SC 4020 plotter using the 9-inch camera or the 35 mm camera, or both.

Standard output consists of 10 views of the structure recorded on both 9-inch and 35-mm cameras.

Output options include 1-10 views of the structure, any orientation of any of the views, any or both cameras to record the series.

Input options include A050 cards, A160 tape, or Input 1 FORTRAN cards, or certain combinations of the former.

G612 is a working program (non-production status) using FORTRAN IV language and either DEMON or standard Input 1 loadsheets on an IBM 7094 computer. For information relative to programming and usage contact W. Laughlin.

For reference data consult Douglas Paper No. 1555, *Computer Method for Perspective Drawing* by Harvey R. Puckett

9-2.7 Program G750 – FORTRAN Program for Determining Dynamic Response of Multi-Layered Orthotropic Cylindrical Shells (DYNAMOS III) (1968)

Program G750 determines the linear, elastic, dynamic response of multi-layer, orthotropic, cylindrical shells subjected to blast or impluse loading. Analysis is done for several shapes of side-on pressure loading which is constant along the length of the cylinder but which decays either linearly or exponentially with time. Both inplane and radial inertia terms are used in calculating frequencies and response.

Input consists of material properties, geometry, yield stresses for each layer, computing time step, and various control flags. Displacements, strains and stresses are computed versus time for each layer, and the output can either be printed or plotted. Suitable yield criteria for each layer are input and critical pressures and impulses are then computed based on the yield criteria and stress histories of each layer.

G750 is a production program using FORTRAN IV language with standard Input 1 loadsheets on a Univac 1108 Computer.

Refer to DAC Report 58784 dated March 1968 and DAC Report 61298 (User's Manual) dated May 1968 for additional data.

For information relative to programming contact J. J. Dietrich and for usage information contact J. J. Dietrich, MW. Wolfson or W. H. Schneider.

9-2.8 Program H026 — Matrix Abstraction Program (1965)

Program H026 executes various matrix operations using card input. These operations include element by element, standard matrix operations, and matrix scalar operations. The program uses scratch tapes, and handles up to eight matrices each with as many as 2500 elements. Any one or several of the eight matrices may be replaced with computed matrices, thus maintaining a maximum of eight.

H026 is a working program (non-production status) using FORTRAN II language with standard Input 1 loadsheets on an IBM 7094 computer. For reference literature and programming or usage information contact H. R. Puckett.

9-2.9 Program H195 — Dynamic Elastic-Plastic Response of Simple Structures (1960)

The DEPROSS series of programs are designed to compute the dynamic elastic-plastic response of simple structures subjected to impulsive loading. Strain hardening and strain-rate effects can be included for analysis of rings, beams, plates and shells.

H195 is a working program (non-production status) using FORTRAN IV language on a Univac 1108 computer.

For information relative to programming and usage contact H. L. Chane.

Refer to Douglas Report DAC-58420, *Description of DEPROSS Code for Rings and Beams,* October 1960 for additional data.

9-2.10 Program H195B — Dynamic Elastic-Plastic Response of a Hard-Bonded, Double-Layered Free Ring (DARTS) 1967)

Program H195B calculates the dynamic elastic-plastic response of a hard-bonded double-layered free ring due to blast, impulse and rapidly-applied thermal loads. Static thermal prestresses may be applied prior to dynamic loading. Strain hardening, strain rate and large deflection effects may be accounted for.

H195B is a working program (non-production status) using FORTRAN IV language on a Univac 1108 computer.

Refer to Douglas Report, DAC-60633, *The Effects of Thermal Stresses on the Dynamic Elastic-Plastic Response of Single and Bonded Double-Layered Ring Structures* for additional data.

9-2.11 Program H195C — Dynamic Elastic-Plastic Response of a Soft-Bonded Double-Layered Free Ring (BOND)(1968)

Program H195C calculates the dynamic elastic-plastic response of a soft-bonded double-layered free ring due to impulse loads. Strain hardening, strain rate and large deflection effects may be accounted for.

H195C is in checkout status, using FORTRAN IV language on a Univac 1108 computer.

For information relative to programming and usage contact H. L. Chane.

9-2.12 Program H195C — Dynamic-Elastic Perfectly-Plastic Response of an Orthotropic Cylindrical Shell (CYLINDER) (1968)

Program H195G calculates the dyamic, elastic, perfectly-plastic response of an orthotropic cylindrical shell due to impulsive and transient surface loading. Static thermal prestresses may be applied prior to dynamic loading.

H195G is in checkout status, using FORTRAN IV language on a Univac 1108 computer.

For information relative to programming and usage contact H. L. Chane.

Refer to Kaman Nuclear-Report KN-67-533, *A Computer Program for the Inelastic Dynamic Response of Orthotropic Cylindrical Shells* for additional data.

9-2.13 Program H243 — Orthotropic Analysis of Composites, Laminates, and Elastic Structures (ORACLES)(1965)

Program H243 computes the elastic constants and stiffness properties of orthotropic laminates. It will also compute the shear stresses, bending stresses, membrane stresses, and buckling loads for orthotropic rib-stiffened or unstiffened cylindrical shells and buckling loads for orthotropic plates. Input consists of the basic properties of the filament and matrix from which the structures are constructed.

H243 is a production program using FORTRAN IV language with standard Input 1 loadsheets on a Univac 1108 computer.

For information relative to programming and usage contact R. J. Miller.

Refer to Miller, R. J., Oracles: *Computer Program and Analysis Methods for Orthotropic Cylinders,* Douglas Report SM-47852, April 1965, for additional data.

9-2.14 Program H258 – Dynamic Response Analysis of Thin Orthotropic Shells of Revolution (1968)

Program H258 has two basic capabilities – a dynamic response capability, and a static buckling capability. The dynamic response program can handle single-layer thin orthotropic-shells of revolution under asymmetric time-dependent loading. The static buckling part can handle thin orthotropic shells of revolution under axisymmetric loading.

H258 is in checkout status, using FORTRAN IV language with standard Input 1 loadsheets on a Univac 1108 computer.

For information relative to programming and usage contact M. H. Schneider.

Refer to DAC Report 60671, dated June 1967, for additional data.

9-2.15 Program H417 – Free Vibration of Preloaded Orthotropic-Stiffened Cylinders (1967)

Program H417 determines the natural frequencies and normalized eigenvectors (i.e., displacement components between modal lines) for preloaded orthotropic-stiffened cylindrical-shell-structure. Input consists of the shell dimensions, the stiffening rigidities and the hoop and axial loads per unit length. The stiffening rigidities may be calculated or obtained from Program H312.

H417 is a production program using FORTRAN IV language with standard Input 1 loadsheets on a Univac 1108 computer.

For information relative to programming or usage contact R. R. Meyer.

For additional data refer to:

1. DAC-60562, *Free Vibration of Preloaded Orthotropic Stiffened Cylinders,* R. R. Meyer, January 1967
2. SM-49354, *Buckling of Generalized Orthotropic Stiffened Circular Cylinders,* R. R. Meyer, February 1966
3. SM-47915, *Static and Dynamic Response of the Stiffened Cylinder,* R. R. Meyer, July 1965

9-2.16 Program H523 – Shock Propagation and Decay by Equivalent Depth Technique (SPADEDT)(1968)

Program H523 calculates the pressure attenuation through a multi-layered composite subjected to an impulse loading. The program can handle a single material or composites of two or three material layers. Basic assumptions and equations for the pressure decay and pulse travel calculations are based on weak shock theory.

H523 is in checkout status, using FORTRAN IV language with Form 60-615 and standard Input 1 loadsheets on a Univac 1108 computer.

For information relative to programming or usage contact Jay L. Carter.

For additional data refer to Memorandum A-830-BRG1-49, May 23, 1968, *Equivalent Depth Techniques.*

9-2.17 Program MW10 – Redundant Force Analysis Master Table Duplicator (1967)

Program MW10 is designed to copy loadsheet requested cases from an RFA Master Table tape and generate a new Master Table tape to be used as the initial input to the RFA programs. All tables from each case are copied, with the option provided to change the analysis and case numbers.

Input required is an RFA Master-Table tape and associated load sheet control parameters. In addition to the new RFA Master Table copy, a list indicating which cases are on the new tape is produced as output.

MW10 is a production program using SAMSON language with standard Input 1 loadsheets on an IBM 7094 computer.

For information relative to programming or usage contact R. W. Duesing or W. Won respectively.

For additional data refer to memorandum A3-773-LCBO-MW10-1.

9-2.18 Program MW11 – Transfer Function Matrix Generator (1968)

Program MW11 defines a set of frequency points using a given set of resonance frequencies (from an SA40 formatted tape) and a given number of frequencies to be computed between each two resonance frequencies. The totality of these frequencies (including the resonance frequencies) will be written in ascending order as a column matrix on an SA40 formatted tape.

Under an option, the transfer function is calculated for each resonance frequency and computed frequency. These values are placed on the SA40-formatted output tape as a matrix whose columns vary according to resonance frequency and whose rows vary according to computed frequency.

Under another option, PSD curves in the form of column matrices of values (which were evaluated at equally-spaced frequency points) and the frequency increment are input from an SA40 formatted tape. These curves are then interpolated (by linearly interpolating the logarithms of input values) at each computed frequency. Interpolated values are then placed on the SA40-formatted output tape as a matrix consisting of one column for each PSD curve.

MW11 is a production program using FORTRAN IV language with 806-066 loadsheets on an IBM 7094 computer.

For information relative to programming or usage contact C. H. Lenhart or C. M. Fuller respectively.

For additional data refer to memorandum A3-773-LCBO-MW11-1.

9-2.19 Program SA40 – Matrix Analysis (FORMAT II) (1968)

Program SA40 is a digital computer program for matrix analysis of complex structures. The basic program provides for abstraction of matrix and pseudo-matrix operations on matrices of up to 2000 order. Available operations include all standard operations useful in structural analysis, e.g., the "structure-cutter" for automatic selection of optimum redundants in the matrix force method.

Outstanding features of the basic program are:

a. The engineer programs his problem directly in matrix notation using simple FORTRAN-like statements with mnemonic names for matrices and without concern for the bookkeeping required for storage of data on external devices (e.g., tapes).

b. The program automatically allocates utilization of external storage devices for each job.

c. Up to nine user-coded subroutines can be added to temporarily function as integral parts of the program.

d. Where profitable and practical, data are stored and processed in compressed mode (zero-valued elements omitted) to take advantage of sparsity in a general and fundamental manner.

e. The program is written entirely in FORTRAN IV for maximum universality.

Two special user-coded subroutines are included for automatic generation of matrices required for analysis of structures by the Redundant Force Method and the Direct Stiffness Method. The force-method matrix generator provides for bars that carry axial load, bending and shear in two planes, and torsion and panels that carry shear only. The displacement-method matrix generator provides for bars that carry axial load, bending and shear in two planes, and torsion, and rectangular and triangular panels that carry inplane forces only. Both generators provide for thermal load. Applied loads and reactions can be either translational or rotational forces.

The program is documented in four volumes. Volume I is primarily a user's manual, Volume II is a programmer's manual, and Volumes III and IV describe the force-method and displacement-method matrix generators, respectively.

SA40 is a production program using FORTRAN IV language, with loadsheets as outlined in the following reference, on a Univac 1108 computer.

For information relative to programming or usage contact J. Rice or W. L. Laughlin respectively.

For additional data refer to Technical Report, AFFDL-TR-66-207, *Format II Second Version of Fortran Matrix Abstraction Technique,* dated December 1966 (Volumes I through IV described in abstract).

9-2.20 Program SA42 – Automated Structural Design (Formerly Principle Force)(1967)

Program SA42 provides an analysis of statically indeterminate structures. The method of analysis used by this program is matrix formulation of the equilibrium and the Maxwell-Mohr equations. In the Maxwell-Mohr Method, the structure is cut to create a statically determinate structure or basic system.

The actual structure to be analyzed is simulated by an idealized discrete model consisting of bars and panels. In general, the bars can carry tension, and the panels can carry shear. In the idealization, bars are considered straight between joints.

SA42 is applicable to any linear discrete structure. The term *discrete* denotes a finite number of members connected at a finite number of joints; *linear* refers to the assumed linear realationships between external loads, support displacements, internal forces, and deflections.

An input option allows the user to submit run data cards in 36AD or A16.0 format, 36AD being the C1 version of SA42.

SA42 also contains dynamics solution capability. Harmonic frequencies and mode shapes can be computed.

SA42 is a production program using FORTRAN IV language with standard Input 1 loadsheets on an IBM 7094 or 360/65 computer.

For information relative to programming or usage contact A. L. Eshleman or R. J. Soukup respectively.

For additional data refer to

1. 36AD Users Manual, *Preliminary Proof Copy,* dated 10-4-66.
2. SM-30470, *Redundant Force Analysis Manual,* dated 12-20-60.
3. Computing Memorandum A2-260-AKE1-SA42, dated 5-5-67.

9-2.21 Program SA48 – Matrix Abstraction With Eigenvalue, Eigenvector Capability

Program SA48 is a FORTRAN IV/1108 version of Program ZC10 but with significant additional capability. Program SA48 can be used for doing many matrix operations including computation of eigenvectors of a symmetric matrix. Operation cards coded by the user specify when data are to be input and how they are to be manipulated.

The allowed matrix operations include, add, subtract, multiply, input, compute the determinant, solve a system of simultaneous equations, print, punch and compute eigenvalues and eigenvectors of a symmetric matrix. In addition, operations for plotting, duplicating a matrix, creating an identity matrix, making a diagonal matrix from a row matrix, generating a null matrix, manipulating partitions of a matrix, taking reciprocals, absolute values and square roots of matrix elements, taking an envelope of values of a matrix and reading and writing matrices on tape are available.

Arithmetic can be performed only on matrices which are completely in core, though partitions of larger matrices can, of course, be manipulated and saved temporarily on tape. Up to 40,000 matrix elements may be stored in core at one time.

The program contains an integration scheme for the solution of sets of ordinary second-order differential equations. Forcing functions must be input and all initial conditions are assumed zero.

SA48 is a production program using FORTRAN IV language with Form 60-1974 loadsheets on a Univac 1108 computer.

For information relative to programming or usage contact D. N. Kelly or A. S. Hopkins respectively.

For additional data refer to memorandum DAC-58746-13 (11-30-67; rev. 6-3-68).

9-2.22 Program SA74 – Output Identification for SA40 (FORMAT II) (1967)

Program SA74 processes a magnetic tape containing matrix results which were computed and readout during a run of Program SA40, *Structural Analysis Using the Matrix Force Method.*

Objective of Program SA74 is to provide specialized printout relating to the structure being analyzed. There are four types of printout:

a. The DEFLECTIONS at the joints at which external loads are applied.

b. The reactive forces and moments (REACTIONS) which sustain static equilibrium.

c. The loads acting on each structure BAR. Each bar may carry axial loads, bending shears and moments, and torsional moments.

d. The SHEAR stresses within each structure PANEL.

SA74 is a production program using FORTRAN language on a Univac 1108 or IBM 7094 computer. Input to Program SA74 consists of the tape output from SA40 and the structural description portion of the SA40 run data cards.

For information relative to programming or usage contact J. C. Rice or H. R. Puckett respectively. For additional data refer to Douglas Report No. DAC-58685, dated 8-31-67.

9-2.23 Program SA80 – Linear Elastic, Static and Dynamic Analysis of Multilayered Orthotropic Shells of Revolution by the Matrix Displacement Method (1968)

Program SA80 integrates the functions of the set of programs known as SABOR III, DRASTIC and DRASTIC OUTPUT into a single program with the additional capability of treating multilayered orthotropic shells. Additional SC-4020 plot features will also be incorporated. The current triad of programs operate under the open shop program H400.

SABOR III utilizes the matrix displacement method for the linear elastic analysis of shells of revolution. The shell is idealized as a series of conical frusta, joined at nodal circles. The program uses an assumed displacement field in terms of harmonic coefficients and computes displacements, reactive forces and shell stress resultants for each harmonic in turn, for given applied generalized forces. Total stresses and strains are computed by summing over the various harmonics. Both axisymmetric and asymmetric loadings may be treated.

SABOR III also computes mass and stiffness matrices for the shell which are utilized by the DRASTIC program to determine transient response to some given force system. DRASTIC OUTPUT provides printout and SC-4020 plots of any computed parameters displayed as a function of axial or circumferential location.

SA80 is in checkout status, using FORTRAN language on a Univac 1108 computer. Loadsheets are currently in preparation.

For information on either programming or usage contact J. J. Dietrich or M. H. Schneider.

For additional data refer to Percy, J. H.; Pian, T. H. H.; Navaratna, P. R., and Klein, S.; *Application of the Matrix Displacement Method to the Linear Elastic Analysis of Shells of Revolution,* M.I.T. A.S.R.L., ASRL121-7, January 1967.

Computing Engineering document on SA80 (in preparation).

9-3 DATA REDUCTION – DIGITAL

This paragraph contains descriptions of the following programs:

DA07	Data Spectrum Storage and Retrieval
DA-11	Analysis of Time Varying Signals
DA11M	Time Series Analysis
DA18	Random Vibration Analysis
G664	Statistical Analysis Which Determines the Arithmetic Mean, Standard Deviation, and Correlation Coefficient
MW02	Correlation and Power Spectrum
UT03	Curve Fit and Plot
UT13	Data Edit, Filter and Merge
UT15	SC 4020 Print/Plot Tape Formatter

For current detailed information refer to the Production Programs Abstract Catalog.

9-3.1 Program DA07 – Data Spectrum Storage and Retrieval

All spectra computed by DA11 may be stored on magnetic tape. DA07 has the capability of inputting the stored spectra from the DA11 tapes, updating a master storage tape, performing operations on selected spectra and plotting the results. At present, all output from the program options (listed below) is plotted. It is planned to lengthen the list of options in the future and to provide for tape storage of intermediate results. The present list of options is:

a. Plot the difference of two spectra

b. Plot the ratio of two spectra

c. Plot the maximum and minimum envelope of N spectra

d. Plot the arithmetic average of N spectra

e. Options 3 and 4

f. Option 5 and plot a loadsheet entered curve

g. Plot N spectra on one graph

h. Plot N spectra on separate graphs

i. Option 8 and a printout of the spectrum levels

Further details on the use of this program are available in memorandum A3-773-LCCO-DA07-1 dated 4/16/69.

9-3.2 Program DA11 – Analysis of Line-Varying Signals

DA11 (see memo's A3-773-LCOO-DA11-2,3) is a general analysis program written for the reduction of shock, vibration and acoustical data. Program capabilities include spectrum analysis, data plotting, instantaneous peak/amplitude distribution analysis, and storage of all program output (see DA07). The spectrum analysis methods are applicable to the following classes of data:

a. Quasi-Stationary Random - Random data with less than 1-2 dB variation of the RMS composite during the period of interest, where the RMS is averaged over at least five cycles of the lowest data frequency.

b. Periodic and Aperiodic - Data with a discrete (or line) spectrum.

c. Transient

Note that data from other types of measurements, e.g., dynamic strain, ocean wave, earthquake, fluctuating pressure, etc., will also fall into one of these categories.

It is expected that the most common problem will consist of power spectrum and/or one-third octave band spectrum analysis of random data; or shock spectrum analysis of transient data. Thus, a sample problem is presented at the end of this paragraph. The following discussion concerns data input to the program and analysis methods applicable to the three classes of data.

The input format of the program is very general due to the large class of error signals (low-frequency trends, exponential trends, DC shifts, etc.,) that may be introduced into data signals by the acquisition system without complete loss of the data. The acquisition system is here meant to include everything from the transducer through the playback recorder. Error signals are removed in the program by least-squares fitting a polynomial (up to 3rd degree) and/or a Fourier series to the input signal and then subtracting the fitted curve from the input. The resulting data sequence is then analyzed.

If necessary, up to five separate segments (time slices) of data may be input. Each segment can then be independently curve fitted, and the resulting segments butted end-to-end, or summed, or averaged to obtain a data sequence for analysis. The spectrum analysis performed will depend on the type of input data as indicated in the following discussion:

a. Quasi-Stationary Random Data - The program will generate a power spectrum and/or a one-third octave band spectrum. The power spectrum may be computed with a constant filter bandwidth or a cascade filter bandwidth. The latter method varies the filter bandwidth with frequency in a range from 1/4 to 1/3 of an octave and tends to smooth the high frequency portion of the spectrum. The constant bandwidth analysis produces the autocorrelation function as additional output and generates confidence bands which tend to be independent of frequency. If two functions are input, the cross power spectrum is computed with a constant filter bandwidth, and the output includes absolute spectrum amplitude, phase, coherence spectrum and frequency response.

   The one-third octave-band spectra are computed by filtering the input data through successive one-third-octave bandpass filters and computing the RMS level of the filter output. The RMS levels are then plotted versus the filter center frequencies to obtain spectra; or, if the input data are scaled in PSID, the RMS levels may be coverted to dB ref. 0.0002 dynes/cm^2 and then plotted.

   It should be noted that the digital filter design is based on stationarity of the input data. If this condition is not satisfied (approximately), the output may be invalid.

b. Periodic and Aperiodic Data - The applicable analysis is a Fourier series. Spectrum values are computed at integral multiples of a loadsheet-entered frequency. Output includes absolute spectrum amplitude and phase; and if two functions are input, the frequency response function $F_x(\omega)/F_y(\omega)$ is computed. Note that the computed spectrum will be periodic regardless of the input data, with a period determined by the loadsheet frequency.

c. Transient Data - The applicable analyses are Fourier series and/or shock spectrum. The shock spectrum is computed as the maximum responses (positive negative and absolute) of a set of damped single-degree-of-freedom systems to the data sequence considered as the input excitation.

The program computes the spectrum at logrithmically spaced frequencies $f_j \equiv F_s\,(2^{1/N})\,j$, $j = 0, 1, 2 - - - - j_{max}$; where F_s, N and j_{max} are loadsheet entries. Values of N (points per octave) greater than $1/\delta$, where $1/\delta$ is the damping ratio entered on the loadsheet, should not be used, as no additional spectrum resolution will be obtained and computer run time is directly proportional to N.

9-3.2.1 Program DA11 – Sample Problem

The most common analyses performed are:

a. Power spectrum.

b. One-third octave band specturm.

c. Shock spectrum.

Where the input consists of a single time slice of data from one data channel.

The following sample case illustrates a and b, while the entries necessary for c are noted. Plotted output is shown in Figure 9-1 and a sample loadsheet is shown in Figure 9-2.

The first three columns of each IBM card (see loadsheet) contain the case number. As many cases as necessary may be analyzed during one computer run. The second three columns (4, 5, and 6) contain the card number. Four IBM cards (010, 020, 031, and 050) are required to analyze one time slice from one data channel. If two functions are to be cross analyzed, one additional card (041) is required.

Card 010 contains the 54 character line that appears as the first title line above the two spectrum plots, card 020 contains the second line of the title. These two lines appear on all output plots except the cross correlation function and instantaneous peak/amplitude histograms.

Card 031 contains all information concerning input of function (X) to the program, card 041 controls input of function (Y). The format of the two cards is the same, which is as follows:

Columns 7-12

230240 – Identification number for data tape (CAT-74 tape) obtained from A3 data processing.

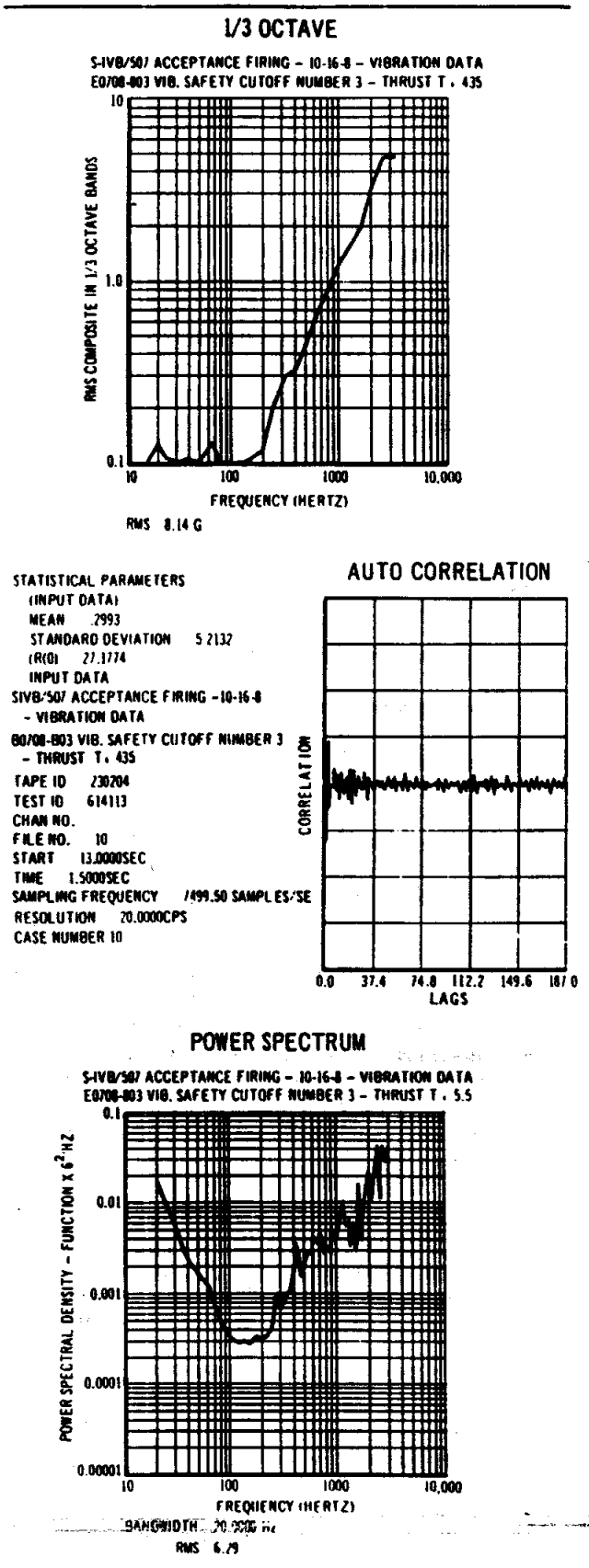


Figure 9-1. Sample Output Program DA11

Columns 13-18

614113 – Identification number of test involved. (In this sample case, S-IVB/507 Acceptance Firing).

Columns 19-21

10 – File number where data are stored on the CAT-74 tape. The CAT-74 tape format places an identification file at the beginning of the tape (File 000) followed by from 1 to 99 data files. The data files are independent in that each may contain data from different sources. The one requirement is that the sampling frequency, which is stored in the ID file, remains constant throughout the tape.

Columns 22-29

13.0 – Start time in seconds from the beginning of the data file. Note that CAT-74 tapes contain no time words, and the program assumes that the beginning of EACH data file is 00.00 seconds.

Columns 30-37

1.5 – Length of data time slice. For a discussion of analysis confidence levels versus analysis bandwidth and time slice length, see Section 8, paragraph 8-4.

Columns 38-41

1 – *Tape Channel* number of required data. Acoustical and vibration tapes are generally single-channel, due to equipment limitations, but a CAT-74 tape may contain up to 10 channels.

Columns 46-48

1 – This entry (if nonzero) causes data points to be skipped on input. (In this sample, the 1 causes every other point to be read. A 2 would cause every third point to be read and so on). The effect of skipping points is to lower the effective sampling frequency of the data. This option is very useful where data are obtained from several channels with different frequency responses (e.g. IRIG FM/FM). These data may all be digitized on one tape at the highest sampling frequency required, and read in with points skipped to save analysis time.

Columns 49-56

.283 – Calibration Factor. CAT-74 data tapes are not calibrated in engineering units as this would be wasteful of computing time. The digitizer used is an 11 bit machine so that the range of numbers for data amplitudes is ± 1023. For S-IVB data we have requested data to set the full-scale channel calibrations to ± 750. This provides a pad of 36 percent against clipping in the digitizers and a dynamic range of approximately 56 dB

FORM NO. 66-3657-2 (5/68)

DA11 ACOUSTIC AND VIBRATION ANALYSIS

LOADSHEET A

PREPARED BY G. CARSON

CASE NO. 0.1.0

DUPLICATE ON ALL CARDS

	TITLE LINE ONE	CROSS PSD ID	SA40 ID
0.1.0	S-IVB/SG2 ACCEPTANCE FIRING - 1.0-16-B - VIBRATION DATA		

	TITLE LINE TWO	REPORT NO.
0.2.0	E-670B-B03 VIB. SAFETY CUTOFF NUMBER 3 - THRUST T=+3.5	

FUNCTION X

	TAPE NO.	TEST NO.	FILE NO.	ST. TIME (LAT 27 - UTO7) ST. POINT (BCD)	INTERVAL LENGTH (LAT 27) NO. OF POINTS (UTO7 - BCD)	FUNCTION CODE (UTO7) CHANNEL (LAT 27)	FUNCTION CODE (UTO7) FIELD DES. (BCD)	PTS TO SKIP	CONVERSION CONSTANT	L E N	FOURIER SEC	T Y P E	SAMPLE RATE (UTO7 - BCD)	SPARE	DATA FORM	DATA TYPE	ID FUNC X
0.3.1	2.30.24.06.14.11.3	1.0	1.3		1.5	1		1	.393							1	99.1.07.709.1
0.3.2																	
0.3.3																	
0.3.4																	
0.3.5																	

FUNCTION Y

	TAPE NO.	TEST NO.	FILE NO.	ST. TIME (LAT 27 - UTO7) ST. POINT (BCD)	INTERVAL LENGTH (LAT 27) NO. OF POINTS (UTO7 - BCD)	FUNCTION CODE (UTO7) CHANNEL (LAT 27)	FUNCTION CODE (UTO7) FIELD DES. (BCD)	PTS TO SKIP	CONVERSION CONSTANT	L E N	FOURIER SEC	T Y P E	SAMPLE RATE (UTO7 - BCD)	SPARE	DATA FORM	DATA TYPE	ID FUNC Y
0.4.1																	
0.4.2																	
0.4.3																	
0.4.4																	
0.4.5																	

	PSD							1/3 OCTAVE			FOURIER TRANSFORM			SHOCK SPECTRUM				TIME PLOTS						
	UNITS	SET	DOF/OCT	Δf	f_{BW}	f_c	FILTER	MAX AMP	SET	f_{START}	# BANDS	DATA	SET	Δf	SET	f_{START}	PTS/OCT	TOTAL POINTS	DAMPING RATIO	DENSITY	PTS TO SKIP	SPARE	CUMULATIVE	SPECIAL	SPECIAL	
0.5.0	G	3		.20	3.000	.2500	1		2	1.0		A.A.V														

KEYPUNCH: PUNCH AS WRITTEN

Figure 9-2. Sample Loadsheet – Program DA11

for data. The calibration factor is then computed by the analyst as the measurement range divided by 750.

Column 69

1 – The number of time slices input. Up to five slices may be input using cards 031-035 and 041-045. A date sequence is generated from the time slices using one of three options determined by Column 70.

Column 70

Blank – In this case, only one time slice is input, so no option is specified. The three options, Butt, Summ and Average are discussed in the referenced memorandum.

Columns 71-80

0971077081 – Spectrum identification number. The middle eight digits are for the analysis use. The first digit and the last digit are controlled by the program to identify the type of spectrum (PSD, shock, etc.,) stored.

Card 050 contains all information determining output from the program, as follows:

Columns 7-9

G – Abbreviation of the engineering units for the input data. It appears in the ordinate labels of the output.

Column 10

3 – Power spectrum option. 3 requests a constant filter bandwidth analysis, plotted and stored on tape. 5 requests both a constant filter bandwidth and a cascade filter bandwidth analysis, with the output plotted and stored on tape. 1 and 4 request the same analyses without tape storage of the output.

Column 11

If two data channels (X and Y are input AND a 1 is placed in this column, a cross-power spectrum analysis will be generated.

Columns 12-16

20 – Filter bandwidth for constant bandwidth power spectrum.

Columns 17-22

3000 – Maximum frequency for the power spectrum plot. The program always computes the spectrum to $0.5f_s$ where f_s is the sampling frequency. However, random data should be sampled at a frequency at least 2.5 times the highest data frequency, in which case $0.5f_s$ is higher than the data frequencies of interest. Thus, this entry is used to suppress plotting of the spectrum above the data frequencies.

Columns 23-27

2500 – Two normalized filter response curves are stored in the program, covering IRIG standard FM/FM, and FM/hardwire discriminator filters. By entering the filter's cutoff frequency in this field, and the option number in Columns 28 and 29, the computed spectra may be corrected for the filter's response.

Columns 28-29

1 – This option requests spectrum correction for an FM/hardwire filter. 2 would correct for an IRIG FM/FM filter. The filter type is constant delay (Gaussian). If another filter is required, the normalized response may be input by loadsheet (see referenced memorandum) and requested in the program by loading 10 in this field.

Column 33

2 – This is the one-third octave band spectrum option. 2 requests both plotting and tape storage of the output. 1 requests plotting of the output without tape storage.

Columns 34-40

10 – This is the center frequency of the first one-third octave band filter.

Columns 41-42

26 – The number of one-third octave band filters to be generated. In this case, the first filter is centered at 10 Hz and the remaining 25 filters are generated above 10 Hz by successive multiplication by $2^{1/3}$.

Column 43

V – This option plots the RMS value in each one-third octave band on a log-log grid. If the input data are scaled in PSID, and the A option is requested, the RMS values are converted to dB Ref 0.0002 dynes/cm^2 and plotted on a semi-log grid.

Column 77

1 – This entry indicates that one data function, X, has been input. A 2 indicates that both functions, X and Y, have been input. This entry must be loaded.

This discussion has indicated how a power spectrum analysis and a one-third octave band spectrum analysis may be obtained. If one of these analyses is not desired, its corresponding option number should be left blank.

If a shock spectrum analysis is required, cards 010, 020, and 031 should be loaded as above. The only differences occur on card 050, as follows:

Columns 7-9

Physical units, as above.

Columns 10-43

Blank

Column 51

Shock Spectrum Option as discussed on Page 3-9 of memorandum A3-773-LCCO-DA11-3, dated 5/17/68.

Columns 52-58

The lowest frequency at which the spectrum is computed.

Columns 59-61

The number of logrithmically-spaced frequency points to be computed in one octave bandwidth. This number should not be greater than the reciprocal of the damping ratio, as entered in Columns 65-69.

Columns 62-64

The total number of frequency points to be computed.

Columns 65-69

The system damping ratio.

9-3.2.2 Restrictions

Several restrictions should be remembered when running the program.

a. The maximum number of data points that may be input to the program is 15000. This number is determined as the product of the sampling frequency and the time-slice length. Further, if (a) two functions are input, or (b) the input data are curved fitted, or (c) a cascade PSD is requested, the maximum number of points is 7500.

b. The maximum number of frequency points that may be computed for a constant bandwidth power spectrum analysis is 450. Thus the ratio $f_s/2\Delta f \leq 450$, where f_s is the sampling frequency and Δf is the bandwidth of the analysis.

c. The highest degree polynomial that may be fitted to the input data is cubic.

d. When many cases are run from one data (CAT-74) tape, the cases should be ordered to read data sequentially from the tape.

e. Data may be input from other types of data tapes, as discussed in memorandum A3-773-LCCO-DA11-3, dated 5-17-68.

f. If a plot of the input data is requested (card 050) the maximum number of data points per SD 4060 frame is 501.

9-3.3 Program DA11M – Time Series Analysis

Program DA11M (A3-773-LCBO-DA11m-1,2) contains the same analysis methods as DA11. The output (plot) format has been modified to include an "MDAC CORPORATION" heading and a report number.

In addition, the program contains a routine which integrates one-third octave band spectra to obtain octave band spectra.

9-3.4 Program DA18 – Random Data Analysis (1968)

Program DA18 analyzes random data. If the data are steady state (stationary or weakly stationary), one or two signals can be analyzed for autocorrelation and power spectral density or cross-correlation and cross-power spectral density respectively. The spectral densities can be determined for constant bandwidths or two-stage cascade bandwidths. Some of the optional features of the program are: data smoothing, trend removal and numerical filtering, one-dimensional frequency distribution, pulse height and width determination, probability density function for pulse heights and widths, coherency function and transfer function if two signals are compared, and plots of the various analyses (either SC-4020 plots for 1 or 2 series, or Calcomp plots for 1 series data). In the analysis of steady state data, DA18 replaces the analysis programs A076, K049, K051 and plot programs K067 and K068. It also eliminated the need for program K087 by accepting input data in a different form than A076, K049 and K051.

DA18 analyzes transient data identically to the procedures and rules of DA11 for shock spectrum analysis.

DA18 is a production program using FORTRAN IV language on a Univac 1108 computer. Input data are prepared on Forms 60-2454-1 and 2.

For information relative to programming or usage contact Diane Conte or Bill Bare respectively.

For additional data refer to Douglas Report No. DAC-58630A, dated 10-31-67 and revised 10-31-69.

9-3.5 Program G664 – Statistical Analysis Which Determines the Arithmetic Mean, Standard Deviation, and Correlation Coefficient (1967)

Program G664 determines the arithmetic mean, the standard deviation, and the correlation coefficient of two hypothetically related sets of data.

G664 is in research/checkout status and uses FORTRAN IV language with standard Input 1 loadsheets on a Univac 1108 computer.

For information relative to programming or usage contact M. E. Mortenson.

For additional data refer to DAC 58719, *A Compilation of Design Programs*.

9-3.6 Program MW02 – Correlation and Power Spectrum

MW02 (see Memo A3-610-EABA-MW02-1) is a specialized analysis program, capable of computing up to 4000 lag products of the autocorrelation function using a data time slice of unlimited extent. This compares to 450 lag products and 15000 data points for DA11, where the number of lag products computed determines the spectrum resolution obtained, and the number of data points analyzed determines the width of the spectrum confidence bands.

The power spectrum is computed with a constant filter bandwidth by the same method as DA11, and for most acoustical and vibration analysis, DA11 will be sufficient. MW02 is advantageous when very high spectrum resolution is required, or when the autocorrelation is of primary interest.

9-3.7 Program UT03 – Curve Fit and Plot (1965)

Program UT03 provides the capability of curve fitting, plotting, calculating various statistical quantities, coefficients and other related items from input data sets. All curve fitting methods employed utilize, where necessary, double or triple precision logic to ensure greater accuracy. Each technique allows a maximum of 250 data points as input. Residuals are calculated after each fit to verify the validity of the coefficients and these may be tabulated on request. Some of the program options are:

a. Least-square fit polynomials of degree 1-9, normalized and unnormalized.

b. Orthogonal polynomial fits.

c. Fourier fit.

d. Multi-variable fit.

e. Selected coefficient fit (fractional powers allowable).

f. Plot over a specified range for a given set of coordinates.

g. Exponential fit of Ae^{bx}.

UT03 is a production program using FORTRAN IV language with Forms 60-1628 and 60-1628-1 loadsheets on a Univac 1108 computer.

For information relative to programming or usage contact L. W. Carroll or J. J. Dietrich respectively.

For additional data refer to memorandum A3-860-KECC-UT03-1.

9-3.8 Program UT13 – Data Edit, Filter and Merge

UT13 (see memo's A3-773-LCCO-UT13-1,2) performs two independent operations. The first is extrapolation of the input data sequence to obtain a new sequence either shifted in time from the input or at a different sampling frequency. This operation will probably not be applied to acoustical and vibration data due to the high sampling frequencies required. It will usually be cheaper to have the data digitized in the appropriate format.

The second operation is digital filtering of the input data. This may be done with one of the two (low pass) filters built into the program or most likely with a filter designed by the analyst. The latter type filter may be entered into the program in two different ways. The analyst may load the appropriate filter weights[1], or load the required gain and phase characteristics and use the program to compute the weights. If the weights are entered, the filter may be either recursive or non-recursive (with feedback or without). If the program computes the weights, the filter will be non-recursive. The following points should be noted in regard to filtering:

a. The computer time required is directly proportional to the number of weights.

b. The number of weights required will increase as the sharpness of the filter cutoff is increased.

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[1] *The following reports, available in the Douglas library, present a discussion of digital filtering techniques.*

Rader, C. M. et. al., Digital Filter Design Techniques, PDL 61905, SL-2, Lincoln Laboratory, Lexington, Massachussetts, 23 December 1965

Martin, M. A., Digital Filters for Data Processing, PDL 53294, General Electric Company Missiles and Space Division, October 1962

c. The number of weights required will increase as the passband ripple is decreased.

d. For a given gain characteristic, a recursive filter will require fewer weights than a non-recursive filter.

9-3.9 Program UT15 – S-C 4020 Print/Plot Tape Formatter (1966)

Program UT15 is a generalized post-processing program designed to accept and process information contained on magnetic tapes of specific format. This information is plotted and printed on the S-C 4020 plotter.

UT15 is a production program using FORTRAN IV language with 860-77 and 16-1834-1 loadsheets on a UNIVAC 1108 computer.

For information relative to programming or usage contact H. L. Montgomery or E. S. Rand respectively.

For additional data refer to memorandum A3-830-KBCA-UT15-1, dated August 10,1966.

Section 10
GEOMETRIC PROPERTIES

This section is a collection of standard formulas for determining section properties of common geometric shapes. Ways to find area, centroid, radius of gyration squared, and product of inertia are described and illustrated. Inertias are described in terms of their radii of gyration for the following reasons:

a. Radius of gyration k^2 is a direct imput to our computer programs.

b. Radius of gyration lends itself to an "eye-ball" accuracy check more readily than moment of inertia.

c. Radius of gyration squared is a term in the equation for moment of inertia, $I = Wk^2$.

The products of inertia equations were taken from Santa Monica Report No. SM-19671, entitled, "Manual of Moments of Inertia Group" and from "Applied Mechanics – Dynamics" by Housner and Hudson.

The symbols used in the geometric properties tables are in agreement with "American Standard Letter Symbols for Aeronautical Sciences" (ASA-Y10.7-1954), published by the American Society of Mechanical Engineers.

10-1 THIN PLATES

Thin plate is defined as having a thickness of less than 1/30 of the length or width.

The various geometric shapes are illustrated in Figure 10-1.

10-2 THIN RODS

Thin rod is defined as having a length equal to 30 diameters.

The various shapes are illustrated in Figure 10-2.

10-3 THIN SHELLS

Thin shell is defined as having a thickness of 1/30 of the length or width.

The various shapes are illustrated in Figure 10-3.

10-4 SOLIDS

Solids are illustrated in Figure 10-4.

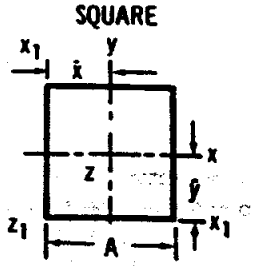
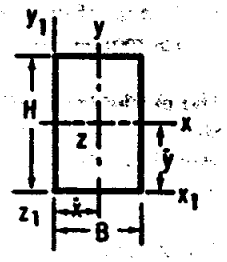
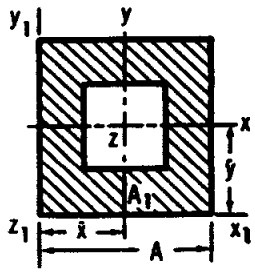
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
A^2	$\bar{x} = \bar{y} = \frac{1}{2} A$	$K_x^2 = K_y^2 = \frac{1}{12} A^2$ $K_z^2 = K_x^2 + K_y^2 = \frac{1}{6} A^2$ $K_{x_1}^2 = K_{y_1}^2 = \frac{1}{3} A^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2 = \frac{2}{3} A^2$	$I_{xy} = 0$ $I_{x_1y_1} = \frac{A^4}{4}$	SQUARE 
BH	$\bar{x} = \frac{1}{2} B$ $\bar{y} = \frac{1}{2} H$	$K_x^2 = \frac{1}{12} H^2$ $K_y^2 = \frac{1}{12} B^2$ $K_z^2 = \frac{1}{12} (H^2 + B^2)$ $K_{x_1}^2 = \frac{1}{3} H^2$ $K_{y_1}^2 = \frac{1}{3} B^2$ $K_{z_1}^2 = \frac{1}{3} (H^2 + B^2)$	$I_{xy} = 0$ $I_{x_1y_1} = \frac{B^2H^2}{4}$	RECTANGLE 
$A^2 - A_1^2$	$\bar{x} = \bar{y} = \frac{1}{2} A$	$K_x^2 = K_y^2 = \frac{1}{12} (A^2 + A_1^2)$ $K_z^2 = \frac{1}{6} (A^2 + A_1^2)$ $K_{x_1}^2 = K_{y_1}^2 = K_x^2 + \frac{1}{4} A^2$ $K_{z_1}^2 = 2K_{x_1}^2$	$I_{xy} = 0$ $I_{x_1y_1} = \frac{1}{4} A^2 (A^2 - A_1^2)$	HOLLOW SQUARE 

Figure 10-1. Thin Plates (Sheet 1 of 11)

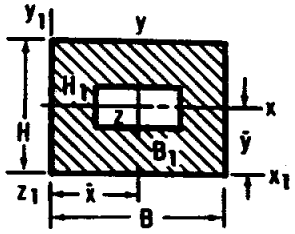
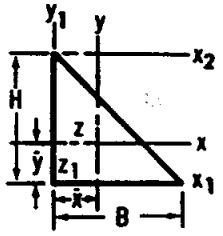
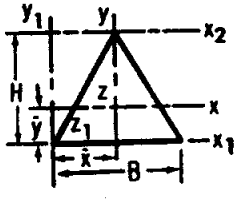
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$BH - B_1H_1$	$\bar{x} = \frac{1}{2} B$ $\bar{y} = \frac{1}{2} H$	$K_x^2 = \frac{1}{12}\left[\frac{BH^3 - B_1H_1^3}{BH - B_1H_1}\right]$ $K_y^2 = \frac{1}{12}\left[\frac{B^3H - B_1^3H_1}{BH - B_1H_1}\right]$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = K_x^2 + \frac{1}{4} H^2$ $K_{y_1}^2 = K_y^2 + \frac{1}{4} B^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = 0$ $I_{x_1y_1} = \frac{BH}{4}(BH - B_1H_1)$	HOLLOW RECTANGLE 
$\frac{1}{2} BH$	$\bar{x} = \frac{1}{3} B$ $\bar{y} = \frac{1}{3} H$	$K_x^2 = \frac{1}{18} H^2$ $K_y^2 = \frac{1}{18} B^2$ $K_z^2 = \frac{1}{18}(H^2 + B^2)$ $K_{x_1}^2 = \frac{1}{6} H^2$ $K_{y_1}^2 = \frac{1}{6} B^2$ $K_{z_1}^2 = \frac{1}{6}(H^2 + B^2)$ $K_{x_2}^2 = \frac{1}{2} H^2$	$I_{xy} = -\frac{H^2B^2}{72}$ $I_{x_1y_1} = \frac{H^2B^2}{24}$	RIGHT TRIANGLE 
$\frac{1}{2} BH$	$\bar{x} = \frac{1}{2} B$ $\bar{y} = \frac{1}{3} H$	$K_x^2 = K_y^2 = \frac{1}{24} B^2$ $K_z^2 = \frac{1}{12} B^2$ $K_{x_1}^2 = \frac{1}{8} B^2$ $K_{y_1}^2 = \frac{7}{24} B^2$ $K_{z_1}^2 = \frac{5}{12} B^2$ $K_{x_2}^2 = \frac{1}{2} H^2$	$I_{xy} = 0$ $I_{x_1y_1} = \frac{1}{12} B^2H^2$	EQUILATERAL TRIANGLE 

Figure 10-1. Thin Plates (Sheet 2 of 11)

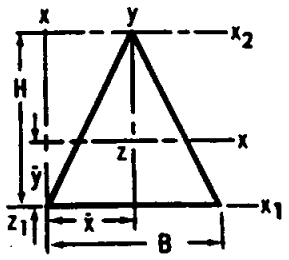
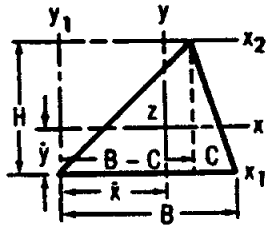
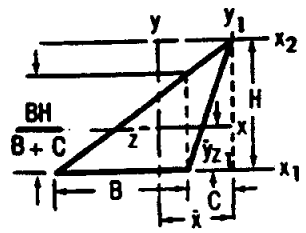
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\frac{1}{2}$ BH	$\bar{x} = \frac{1}{2}$ B $\bar{y} = \frac{1}{3}$ H	$K_x^2 = \frac{1}{18} H^2$ $K_y^2 = \frac{1}{24} B^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{1}{6} H^2$ $K_{y_1}^2 = \frac{7}{24} B^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$ $K_{x_2}^2 = \frac{1}{2} H^2$	$I_{xy} = 0$ $I_{x_1y_1} = \frac{1}{12} B^2 H^2$	ISOSCELES TRIANGLE 
$\frac{1}{2}$ BH	$\bar{x} = \frac{2}{3}(B - C)$ $\bar{y} = \frac{1}{3}$ H	$K_x^2 = \frac{1}{18} H^2$ $K_y^2 = \frac{1}{18}(B^2 - BC + C^2)$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{1}{6} H^2$ $K_{x_2}^2 = \frac{1}{2} H^2$	$I_{xy} = \frac{BH^2}{72}[2(B - C) - B]$ $I_{x_1y_1} = \frac{BH^2}{72}[2(B - C) + B]$	OBLIQUE TRIANGLE 
$\frac{1}{2}$ BH	$\bar{x} = \frac{1}{3}(B + 2C)$ $\bar{y} = \frac{1}{3}$ H	$K_x^2 = \frac{1}{18} H^2$ $K_y^2 = \frac{1}{18}(B^2 + BC + C^2)$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{1}{6} H^2$ $K_{y_1}^2 = K_y^2 + \bar{x}^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$ $K_{x_2}^2 = \frac{1}{2} H^2$	———	OBTUSE ANGLED TRIANGLE 

Figure 10-1. Thin Plates (Sheet 3 of 11)

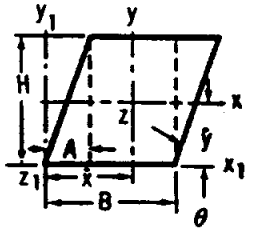
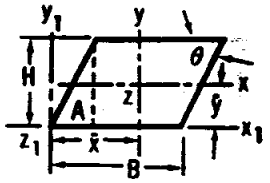
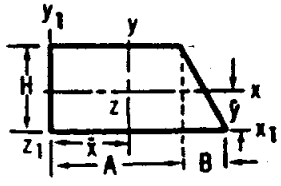
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
BH	$\bar{x} = \frac{1}{2}(A + B)$ $\bar{y} = \frac{1}{2} H$	$K_x^2 = \frac{1}{12} H^2$ $K_y^2 = \frac{1}{12} (A^2 + B^2)$ $K_z^2 = \frac{1}{12} (A^2 + B^2 + H^2)$ $K_{x_1}^2 = \frac{1}{3} H^2$ $K_{y_1}^2 = \frac{1}{6} (2A^2 + 2B^2 + 3AB)$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = \frac{A^4}{12} \sin^2\theta \cos\theta$ $I_{x_1y_1} = \frac{A^4}{12} \sin^2\theta$ $(3 + 4\cos\theta)$	RHOMBUS 
BH	$\bar{x} = \frac{1}{2}(A + B)$ $\bar{y} = \frac{1}{2} H$	$K_x^2 = \frac{1}{12} H^2$ $K_y^2 = \frac{1}{12} (A^2 + B^2)$ $K_z^2 = \frac{1}{12} (A^2 + B^2 + H^2)$ $K_{x_1}^2 = \frac{1}{3} H^2$ $K_{y_1}^2 = \frac{1}{6} (2A^2 + 2B^2 + 3AB)$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = \frac{H^3B}{12 \sin\theta} (\cos\theta)$ $I_{x_1y_1} = \frac{H^2B}{12} (3B + 4\cos\theta)$	PARALLELOGRAM 
$\frac{1}{2}H(2A + B)$	$\bar{x} = \frac{3A^2 + 3AB + 3B^2}{3(2A + B)}$ $\bar{y} = \frac{H(3A + B)}{3(2A + B)}$	$K_x^2 = K_{x_1}^2 - \bar{y}^2$ $K_y^2 = K_{y_1}^2 - \bar{x}^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{H^2(4A + B)}{6(2A + B)}$ $K_{y_1}^2 =$ $\frac{2A^2(2A + 3B) + B^2(4A + B)}{6(2A + B)}$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = \frac{H^2}{24} [\, 2A^2 + (2A + B)^2$ $- 4(2A + B)\bar{x}\,]$ $I_{x_1y_1} = \frac{H^2}{24}$ $[\, 2A^2 + (2A + B)^2\,]$	RIGHT ANGLED TRAPEZOID 

Figure 10-1. Thin Plates (Sheet 4 of 11)

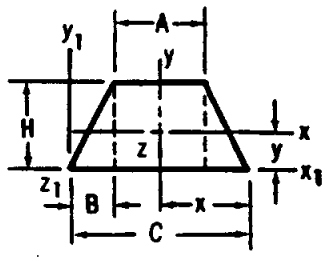
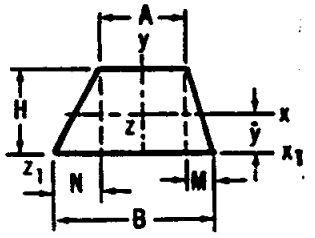
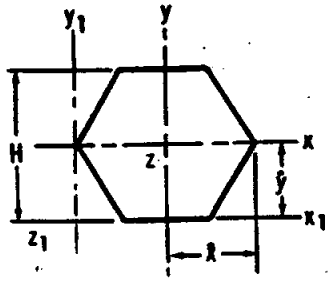
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$H(A + B)$	$\bar{x} = \frac{A}{2} + B$ $\bar{y} = \frac{H(3A + 2B)}{6(A + B)}$	$K_x^2 = K_{x_1}^2 - \bar{y}^2$ $K_y^2 = \frac{A^2(A + 3B) + 2B^2(2A + B)}{12\,(A + B)}$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{H^2(2A + B)}{6(A + B)}$ $K_{y_1}^2 = K_y^2 + \bar{x}^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = 0$ $I_{x_1y_1} = \frac{H^2}{12}(c^2 + 2AC)$	ISOSCELES TRAPEZOID 
$\frac{1}{2} H(A + B)$	$\bar{x} = \frac{1}{3(A + B)}$ $[\,3A(A + M)$ $+ 2N(3A + N)$ $+ M(3N + M)\,]$ $\bar{y} = \frac{H}{3}\left(\frac{2A + B}{A + B}\right)$	$K_x^2 = \frac{H^2}{18}\left[\,1 + \frac{2AB}{(A + B)^2}\,\right]$ $K_{x_1}^2 = \frac{H^2(3A + B)}{6(A + B)}$	$I_{xy} = I_{x_1y_1} -$ $\frac{1}{2} H(A + B)\,\bar{x}\,\bar{y}$ $I_{x_1y_1} = \frac{H^2}{24}[\,3N(N + 4A)$ $+ 4M(N + A) + M^2 + 6A^2\,]$	OBLIQUE TRAPEZOID 
$.866H^2$	$\bar{x} = .577H$ $\bar{y} = \frac{1}{2} H$	$K_x^2 = K_y^2 = .06944H^2$ $K_z^2 = 2K_x^2$ $K_{x_1}^2 = K_x^2 + \bar{y}^2$ $K_{y_1}^2 = K_y^2 + \bar{x}^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = 0$ $I_{x_1y_1} = \frac{1}{4} H^4$	REGULAR HEXAGON 

Figure 10-1. Thin Plates (Sheet 5 of 11)

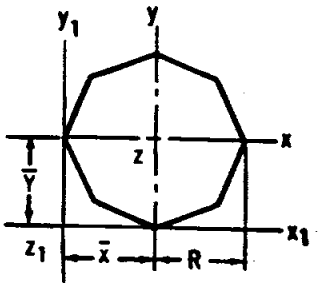
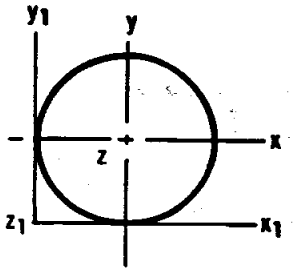
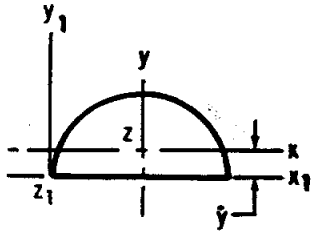
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$2.8286R^2$	$\bar{x} = \bar{y} = R$	$K_x^2 = K_y^2 = .2256R^2$ $K_z^2 = 2K_x^2$ $K_{x_1}^2 = K_{y_1}^2 = 1.2256R^2$ $K_{z_1}^2 = 2K_{x_1}^2$	———	REGULAR OCTAGON 
πR^2	———	$K_x^2 = K_y^2 = \frac{1}{4}R^2$ $K_z^2 = \frac{1}{2}R^2$ $K_{x_1}^2 = K_{y_1}^2 = 1.25R^2$ $K_{z_1}^2 = 2.50R^2$	$I_{xy} = 0$ $I_{x_1y_1} = \pi R^4$	CIRCLE 
$\frac{1}{2}\pi R^2$	$\bar{y} = .424R$	$K_x^2 = .07R^2$ $K_y^2 = .25R^2$ $K_z^2 = .32R^2$ $K_{x_1}^2 = K_x^2 + \bar{y}^2$ $K_{y_1}^2 = 1.25R^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2 = 1.5R^2$.	$I_{xy} = 0$ $I_{x_1y_1} = \frac{2}{3}R^4$	SEMI-CIRCLE 

Figure 10-1. Thin Plates (Sheet 6 of 11)

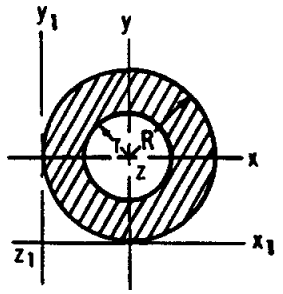
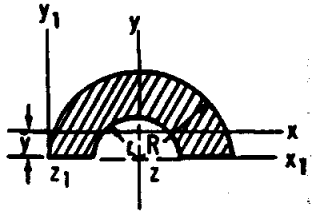
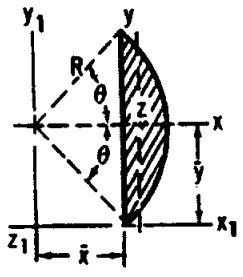
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\pi(R^2 - r^2)$	———	$K_x^2 = K_y^2 = \frac{1}{4}(R^2 + r^2)$ $K_z^2 = \frac{1}{2}(R^2 + r^2)$ $K_{x_1}^2 = K_{y_1}^2 = \frac{1}{4}(5R^2 + r^2)$ $K_{z_1}^2 = \frac{1}{2}(5R^2 + r^2)$	$I_{xy} = 0$ $I_{x_1 y_1} = \pi R^2 (R^2 - r^2)$	HOLLOW CIRCLE 
$\frac{1}{2}\pi(R^2 - r^2)$	$\bar{y} = .424\left(R + r - \frac{Rr}{R+r}\right)$	$K_x^2 = K_{x_1}^2 - \bar{y}^2$ $K_y^2 = \frac{1}{4}(R^2 + r^2)$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{1}{4}(R^2 + r^2)$ $K_{y_1}^2 = \frac{1}{4}(5R^2 + r^2)$ $K_{z_1}^2 = \frac{1}{2}(3R^2 + r^2)$	$I_{xy} = 0$ $I_{x_1 y_1} = \frac{2}{3} R(R^3 - r^3)$	HOLLOW SEMI-CIRCLE 
$R^2(\theta - \sin\theta\cos\theta)$ IN RADIANS	$\bar{x} = \frac{2}{3}\left[\frac{A\sin^3\theta}{\theta - \sin\theta\cos\theta}\right]$ $\bar{y} = A\sin\theta$	$K_x^2 = \frac{R^2}{4}\left[1 - \frac{2\sin^3\theta\cos\theta}{3(\theta - \sin\theta\cos\theta)}\right]$ $K_y^2 = K_{y_1}^2 - \bar{x}^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = K_x^2 + \bar{y}^2$ $K_{y_1}^2 = \frac{R^2}{4}\left[1 + \frac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = 0$ $I_{x_1 y_1} = 0$	CIRCULAR SEGMENT 

Figure 10-1. Thin Plates (Sheet 7 of 11)

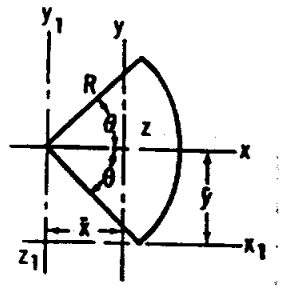
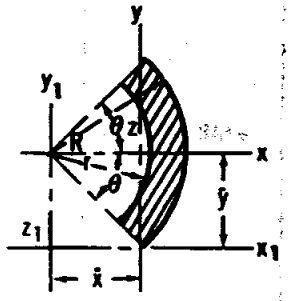
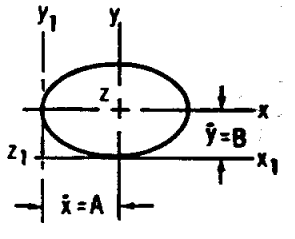
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$R^2\theta$ θ IN RADIANS	$\bar{x} = \frac{2}{3}\frac{R\sin\theta}{\theta}$ $\bar{y} = R\sin\theta$	$K_x^2 = \frac{R^2}{4}\left(1 - \frac{\sin\theta\cos\theta}{\theta}\right)$ $K_y^2 = K_{y_1}^2 - \bar{x}^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = K_x^2 + \bar{y}^2$ $K_{y_1}^2 = \frac{R^2}{4}\left(1 + \frac{\sin\theta\cos\theta}{\theta}\right)$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = 0$ $I_{x_1y_1} = 0$	CIRCULAR SECTOR 
$\theta(R^2 - r^2)$ θ IN RADIANS	$\bar{x} = \frac{2\sin\theta(R^3 - r^3)}{3\theta(R^2 - r^2)}$ $\bar{y} = R\sin\theta$	$K_x^2 = \frac{1}{4}(R^2 + r^2)\left[1 - \frac{\sin\theta\cos\theta}{\theta}\right]$ $K_y^2 = K_{y_1}^2 - \bar{x}^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = K_x^2 + \bar{y}^2$ $K_{y_1}^2 = \frac{1}{4}(R^2 + r^2)\left[1 + \frac{\sin\theta\cos\theta}{\theta}\right]$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	———	HOLLOW CIRCULAR SECTOR 
πAB	$\bar{x} = A$ $\bar{y} = B$	$K_x^2 = \frac{1}{4}B^2$ $K_y^2 = \frac{1}{4}A^2$ $K_z^2 = \frac{1}{4}(A^2 + B^2)$ $K_{x_1}^2 = 1.25B^2$ $K_{y_1}^2 = 1.25A^2$ $K_{z_1}^2 = 1.25(A^2 + B^2)$	$I_{xy} = 0$ $I_{x_1y_1} = \pi A^2B^2$	ELLIPSE 

Figure 10-1. Thin Plates (Sheet 8 of 11)

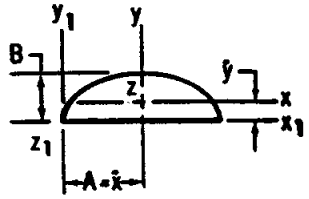
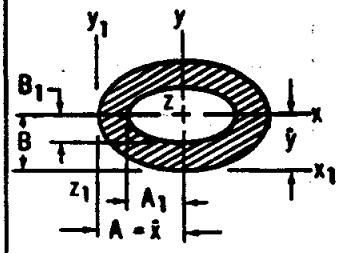
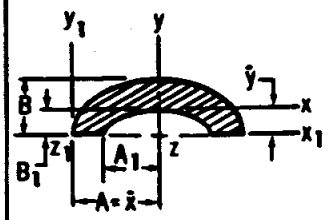
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\frac{1}{2}\pi AB$	$\bar{x} = A$ $\bar{y} = .424B$	$K_x^2 = K_{x_1}^2 - \bar{y}^2$ $K_y^2 = \frac{1}{4}A^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{1}{4}B^2$ $K_{y_1}^2 = 1.25A^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = 0$ $I_{x_1y_1} = \pi A^2B^2(\frac{1}{2} - \frac{2}{3\pi})$	SEMI-ELLIPSE 
$\pi(AB - A_1B_1)$	$\bar{x} = A$ $\bar{y} = B$	$K_x^2 = \frac{1}{4}\frac{AB^3 - A_1B_1^3}{AB - A_1B_1}$ $K_y^2 = \frac{1}{4}\frac{A^3B - A_1^3B_1}{AB - A_1B_1}$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = K_x^2 + \bar{y}^2$ $K_{y_1}^2 = K_y^2 + \bar{x}^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = 0$ $I_{x_1y_1} = \pi AB(AB - A_1B_1)$	HOLLOW ELLIPSE 
$\frac{1}{2}\pi(AB - A_1B_1)$	$\bar{x} = A$ $\bar{y} = .424\frac{AB^2 - A_1B_1^2}{AB - A_1B_1}$	$K_x^2 = K_{x_1}^2 - \bar{y}^2$ $K_y^2 = \frac{1}{4}\frac{A^3B - A_1^3B_1}{AB - A_1B_1}$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{1}{4}\frac{AB^3 - A_1B_1^3}{AB - A_1B_1}$ $K_{y_1}^2 = K_y^2 + \bar{x}^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = 0$ $I_{x_1y_1} = \frac{2A}{3}(AB^2 - A_1B_1^2)$	HOLLOW SEMI-ELLIPSE 

Figure 10-1. Thin Plates (Sheet 9 of 11)

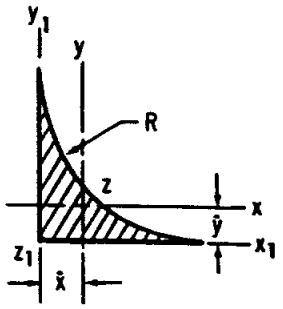
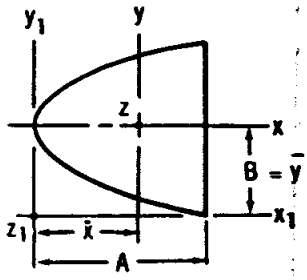
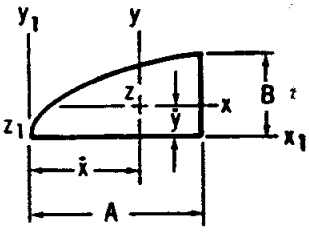
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$.2146R^2$	$\bar{x} = \bar{y} = .2233R$	$K_x^2 = K_y^2 = .035R^2$ $K_z^2 = .07R^2$ $K_{x_1}^2 = K_x^2 + \bar{y}^2$ $K_{y_1}^2 = K_y^2 + \bar{x}^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = .00444R^4$ $I_{x_1y_1} = .00627R^4$	CIRCULAR COMPLEMENT 
$\frac{4}{3}AB$	$\bar{x} = .6A$ $\bar{y} = B$	$K_x^2 = \frac{1}{5}B^2$ $K_y^2 = .0685A^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = 1.2B^2$ $K_{y_1}^2 = .4286A^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = 0$ $I_{x_1y_1} = 0$	PARABOLIC SEGMENT 
$\frac{2}{3}AB$	$\bar{x} = \frac{3}{5}A$ $\bar{y} = \frac{3}{8}B$	$K_x^2 = .0594B^2$ $K_y^2 = .0686A^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{1}{5}B^2$ $K_{y_1}^2 = .4285A^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	$I_{xy} = \frac{1}{60}A^2B^2$ $I_{x_1y_1} = \frac{1}{6}A^2B^2$	PARABOLIC HALF SEGMENT 

Figure 10-1. Thin Plates (Sheet 10 of 11)

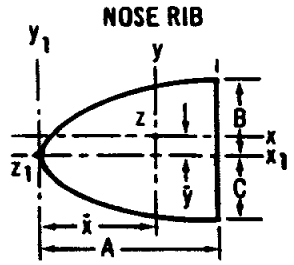
AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\frac{2}{3}A(B+C)$	$\bar{x} = \frac{3}{5}A$ $\bar{y} = \frac{3}{8}(B-C)$	$K_x^2 = \frac{19B^2 + 26BC + 19C^2}{320}$ $K_y^2 = .0686A^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{1}{5}(B^2 - BC + C^2)$ $K_{y_1}^2 = .4286A^2$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	———	NOSE RIB 

Figure 10-1. Thin Plates (Sheet 11 of 11)

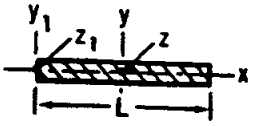
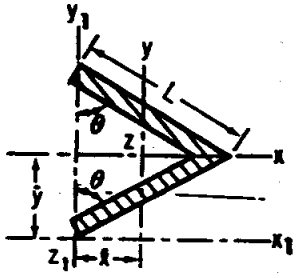
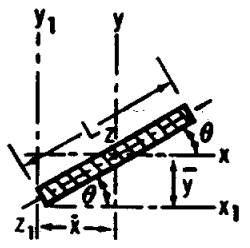
LENGTH	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
L	$\bar{x} = \frac{L}{2}$	K_x^2 IS NEGLIGIBLE $K_y^2 = K_z^2 = \frac{1}{12} L^2$ $K_{y_1}^2 = K_{z_1}^2 = \frac{1}{3} L^2$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1z_1} = I_{y_1z_1} = 0$	STRAIGHT ROD 
2L	$\bar{x} = \frac{L}{2} \sin\theta$ $\bar{y} = L\cos\theta$	$K_x^2 = \frac{1}{3} L^2 \cos^2\theta$ $K_y^2 = \frac{1}{12} L^2 \sin^2\theta$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = \frac{4}{3} L^2 \cos^2\theta$ $K_{y_1}^2 = \frac{1}{3} L^2 \sin^2\theta$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	———	BROKEN ROD 
L	$\bar{x} = \frac{L}{2}\cos\theta$ $\bar{y} = \frac{L}{2}\sin\theta$	$K_x^2 = \frac{1}{12} L^2 \sin^2\theta$ $K_y^2 = \frac{1}{12} L^2 \cos^2\theta$ $K_z^2 = \frac{1}{12} L^2$ $K_{x_1}^2 = \frac{1}{3} L^2 \sin^2\theta$ $K_{y_1}^2 = \frac{1}{3} L^2 \cos^2\theta$ $K_{z_1}^2 = \frac{1}{3} L^2$	———	INCLINED ROD 

Figure 10-2. Thin Rods (Sheet 1 of 4)

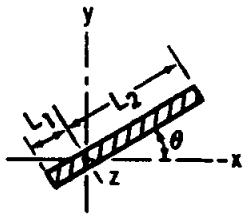
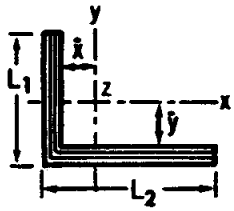
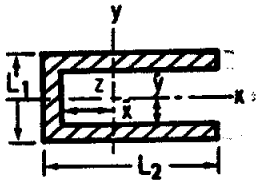
LENGTH	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$L_1 + L_2$	———	$K_x^2 = \frac{1}{3}\sin^2\theta\,(L_1^2 - L_1L_2 + L_2^2)$ $K_y^2 = \frac{1}{3}\cos^2\theta\,(L_1^2 - L_1L_2 + L_2^2)$ $K_z^2 = K_x^2 + K_y^2$	———	INCLINED ROD NOT THROUGH C.G. AXIS 
$L_1 + L_2$	$\bar{x} = \frac{L_2^2}{2(L_1+L_2)}$ $\bar{y} = \frac{L_1^2}{2(L_1+L_2)}$	$K_x^2 = \frac{1}{12}\left[\frac{L_1^3(L_1+4L_2)}{(L_1+L_2)^2}\right]$ $K_y^2 = \frac{1}{12}\left[\frac{L_2^3(4L_1+L_2)}{(L_1+L_2)^2}\right]$ $K_z^2 = L_1 + L_2$	———	"L" ROD 
$L_1 + 2L_2$	$\bar{x} = \frac{L_2^2}{L_1 + 2L_2}$ $\bar{y} = \frac{L_1}{2}$	$K_x^2 = \frac{1}{12}\left[\frac{L_1 + 6L_2}{L_1 + 2L_2}\right]$ $K_y^2 = \frac{1}{3}L_2^3\left[\frac{2L_1 + L_2}{(L_1 + 2L_2)^2}\right]$ $K_z^2 = K_x^2 + K_y^2$	———	"U" ROD 

Figure 10-2. Thin Rods (Sheet 2 of 4)

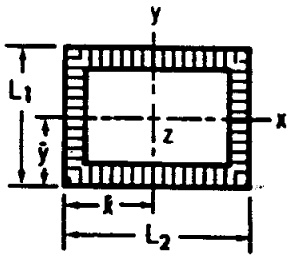
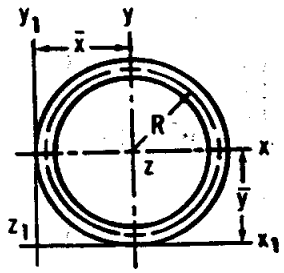
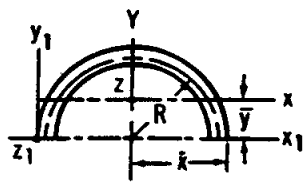
LENGTH	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$2(L_1 + L_2)$	$\bar{x} = \frac{L_2}{2}$ $\bar{y} = \frac{L_1}{2}$	$K_x^2 = \frac{L_1^2}{12}\left[\frac{L_1 + 3L_2}{L_1 + L_2}\right]$ $K_y^2 = \frac{L_2^2}{12}\left[\frac{3L_1 + L_2}{L_1 + L_2}\right]$ $K_z^2 = K_x^2 + K_y^2$	—	RECTANGULAR ROD 
$2\pi R$	$\bar{x} = \bar{y} = R$	$K_x^2 = K_y^2 = \frac{1}{2}R^2$ $K_z^2 = R^2$ $K_{x_1}^2 = K_{y_1}^2 = \frac{3}{2}R^2$ $K_{z_1}^2 = 3R^2$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = WR^2$ $I_{xz} = I_{yz} = 0$	CIRCULAR ROD 
πR	$\bar{x} = R$ $\bar{y} = .6366R$	$K_x^2 = .09474R^2$ $K_y^2 = \frac{1}{2}R^2$ $K_z^2 = .59474R^2$ $K_{x_1}^2 = 0.5R^2$ $K_{y_1}^2 = 1.5R^2$ $K_{z_1}^2 = 2R^2$	—	SEMI-CIRCULAR ROD 

Figure 10-2. Thin Rods (Sheet 3 of 4)

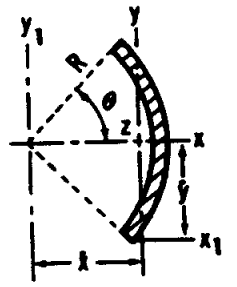
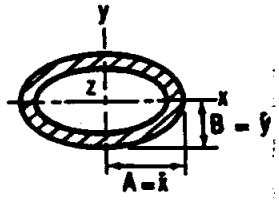
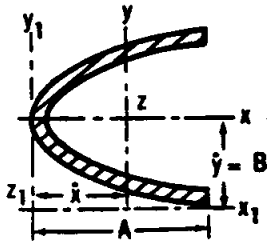
LENGTH	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$2R\theta$ θ IN RADIANS	$\bar{x} = \frac{R \sin\theta}{\theta}$ $\bar{y} = R \sin\theta$	$K_x^2 = R^2(\frac{1}{2} - \frac{\sin\theta\cos\theta}{2\theta})$ $K_y^2 = R^2(\frac{1}{2} + \frac{\sin\theta\cos\theta}{2\theta} - \frac{\sin^2\theta}{\theta^2})$ $K_z^2 = R^2(1 - \frac{\sin^2\theta}{\theta^2})$ $K_{x_1}^2 = R^2(\frac{1}{2} - \frac{\sin\theta\cos\theta}{2\theta} + \sin^2\theta)$ $K_{y_1}^2 = R^2(\frac{1}{2} + \frac{\sin\theta\cos\theta}{2\theta})$ $K_{z_1}^2 = R^2(1 + \sin^2\theta)$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1z_1} = I_{y_1z_1} = 0$	SEGMENT OF A CIRCULAR ROD 
$\frac{\pi}{2A}(3A^2 + B^2)$	$\bar{x} = A$ $\bar{y} = B$	$K_x^2 = \frac{B^2}{2}\left[\frac{55A^4 + 10A^2B^2 - B^4}{45A^4 + 22A^2B^2 - 3B^4}\right]$ $K_y^2 = \frac{A^2}{2}\left[\frac{35A^4 + 34A^2B^2 + 5B^4}{45A^4 + 22A^2B^2 - 3B^4}\right]$ $K_z^2 = \frac{(A^2 + B^2)}{2}\left[\frac{90A^4 + 44A^2B^2 - 6B^4}{45A^4 + 22A^2B^2 - 3B^4}\right]$	—	ELLIPTIC ROD 
$L = \sqrt{4A^2 + B^2} + \frac{B^2}{2A}\text{LOG}\,C$ $C = \frac{2A + \sqrt{4A^2 + B^2}}{B}$	$\bar{x} = \frac{(4A^2 + B^2)^{3/2}}{8AL} - \frac{B^2}{16A}$ $\bar{y} = B$	$K_x^2 = \frac{B^2}{8A^2}\left[\frac{(4A^2 + B^2)^{3/2}}{L} - \frac{B^2}{2}\right]$ $K_y^2 = K_{y_1}^2 - \bar{x}^2$ $K_z^2 = K_x^2 + K_y^2$ $K_{x_1}^2 = K_x^2 + \bar{y}^2$ $K_{y_1}^2 = \frac{1}{12}\left[\frac{4A^2 + B^2)^{3/2}}{L}\right] - \frac{K_x^2}{8}$ $K_{z_1}^2 = K_{x_1}^2 + K_{y_1}^2$	—	PARABOLIC ROD 

Figure 10-2. Thin Rods (Sheet 4 of 4)

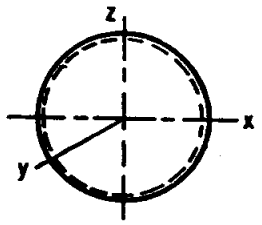
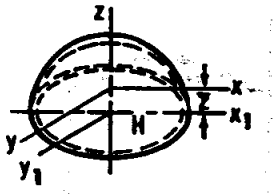
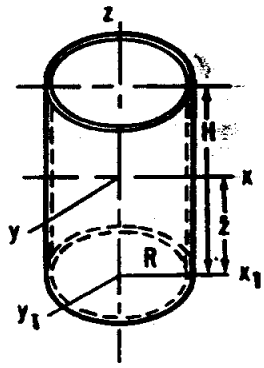
SURFACE AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$4\pi R^2$	———	$K_x^2 = K_y^2 = K_z^2 = \frac{2}{3} R^2$	———	SPHERICAL SHELL 
$2\pi R^2$	$\bar{z} = \frac{1}{2} R$	$K_x^2 = K_y^2 = \frac{5}{12} R^2$ $K_z^2 = K_{x_1}^2 = K_{y_1}^2 = \frac{2}{3} R^2$	———	HEMISPHERICAL SHELL 
$2\pi RH$	$\bar{z} = \frac{H}{2}$	$K_x^2 = K_y^2 = \frac{1}{2}(R^2 + \frac{H^2}{6})$ $K_z^2 = R^2$ $K_{x_1}^2 = K_{y_1}^2 = \frac{1}{6}(3R^2 + 2H^2)$	———	LATERAL CYLIN. SHELL (CIRCULAR) 

Figure 10-3. Thin Shells (Sheet 1 of 4)

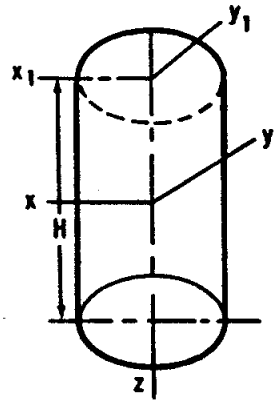
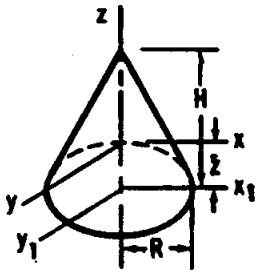
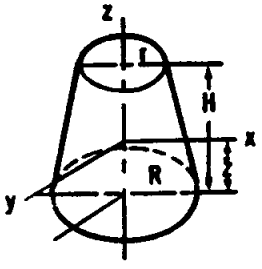
SURFACE AREA	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$2\pi R(R+H)$	$\bar{z} = \frac{H}{2}$	$K_x^2 = K_y^2 = \frac{1}{12(R+H)}$ $[\,3R^2(R+2H) + H^2(3R+H)\,]$ $K_z^2 = \frac{R^2}{2}\left[\frac{R+2H}{R+H}\right]$ $K_{x_1}^2 = K_{y_1}^2 = \frac{1}{12(R+H)}$ $[\,3R^2(R+2H) + 2H^2(3R+2H)\,]$	—	TOTAL CYLIN. SHELL (CIRCULAR) 
$\pi R\sqrt{R^2+H^2}$	$\bar{z} = \frac{1}{3}H$	$K_x^2 = K_y^2 = \frac{1}{4}(R^2 + \frac{2}{9}H^2)$ $K_z^2 = \frac{1}{2}R^2$ $K_{x_1}^2 = K_{y_1}^2 = \frac{1}{12}(3R^2 + 2H^2)$	—	LATERAL SURFACE OF A CIRC. CONE 
$\pi(R+r)\sqrt{H^2+(R-r)^2}$	$\bar{z} = \frac{H}{3}\left[\frac{2r+R}{r+R}\right]$	$K_x^2 = K_y^2 = \frac{1}{4}(R^2+r^2) + \frac{H^2}{18}\left[1 + \frac{2Rr}{(R+r)^2}\right]$ $K_z^2 = \frac{1}{2}(R^2+r^2)$	—	LATERAL SURF. OF FRUSTUM OF CIRC. CONE 

Figure 10-3. Thin Shells (Sheet 2 of 4)

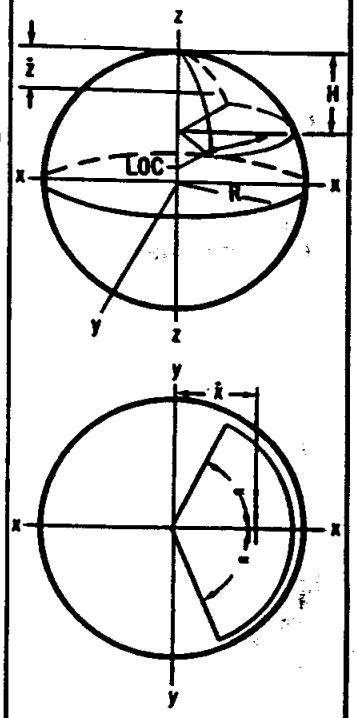
VOLUME	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$2RH\alpha t$ α = HALF ANGLE IN RADIANS t = SHELL THICKNESS	$\bar{X} = \frac{\sin\alpha}{2\alpha H}\left[R^2\cos^{-1}\left(\frac{R-H}{R}\right)\right.$ $\left.-(R-H)\sqrt{H(2R-H)}\right]$ $\bar{Y} = 0$ $\bar{Z} = \frac{H}{2}$ (FROM TOP)	$K_x^2 = \frac{H}{6}\left[\frac{6R-H}{2}\right.$ $\left.-(3R-H)\frac{\sin\alpha\cos\alpha}{\alpha}\right]$ $K_y^2 = \frac{H}{6}\left[\frac{6R-H}{2}\right.$ $\left.+(3R-H)\frac{\sin\alpha\cdot\cos\alpha}{\alpha}\right]$ $-\frac{\sin^2\alpha}{4H^2\alpha^2}\left[R^2\cos^{-1}\left(\frac{R-H}{R}\right)\right.$ $\left.-(R-H)\sqrt{H(2R-H)}\right]^2$ $K_z^2 = \frac{H(3R-H)}{3}$ $-\frac{\sin^2\alpha}{4H^2\alpha^2}\left[R^2\cos^{-1}\left(\frac{R-H}{R}\right)\right.$ $\left.-(R-H)\sqrt{H(2R-H)}\right]^2$	$I_{xy} = I_{yz} = 0$ $I_{xz} = \frac{W\sin\alpha}{4H\alpha}(2R-H)$ $\left[\sqrt{H(2R-H)}\left(\frac{H+3R}{3}\right)\right.$ $\left.-R^2\cos^{-1}\left(\frac{R-H}{R}\right)\right]$	SPHERICAL LUNE SEGMENT 

Figure 10-3. Thin Shells (Sheet 3 of 4)

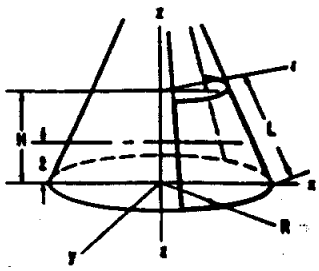
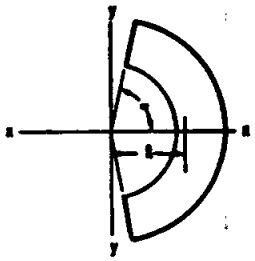
VOLUME	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$(R + r)\, L\alpha\, t$ α = HALF ANGLE IN RADIANS t = SHELL THICKNESS	$\bar{X} = \frac{2 \sin\alpha}{3\alpha}\left[(R+r) - \frac{Rr}{(R+r)}\right]$ $\bar{Y} = 0$ $\bar{Z} = \frac{H}{3}\left[\frac{2r+R}{R+r}\right]$	$K_x^2 = \frac{R^2+r^2}{4}\left(1 - \frac{\sin 2\alpha}{2\alpha}\right) + \frac{H^2}{18}\left[1 + \frac{2Rr}{(R+r)^2}\right]$ $K_y^2 = \frac{R^2+r^2}{4}\left(1 + \frac{\sin 2\alpha}{2\alpha}\right) + \frac{H^2}{18}\left[1 + \frac{2Rr}{(R+r)^2}\right] - \frac{4\sin^2\alpha}{9\alpha^2}\left[(R^2+r^2) + \frac{R^2r^2}{(R+r)^2}\right]$ $K_z^2 = \frac{R^2+r^2}{2} - \frac{4\sin^2\alpha}{9\alpha^2}\left[(R^2+r^2) + \frac{R^2r^2}{(R+r)^2}\right]$	$I_{xy} = I_{yz} = 0$	SEGMENT OF A CONE FRUSTUM  

Figure 10-3. Thin Shells (Sheet 4 of 4)

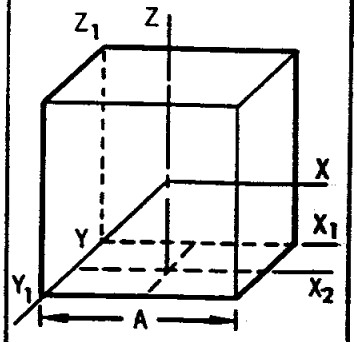
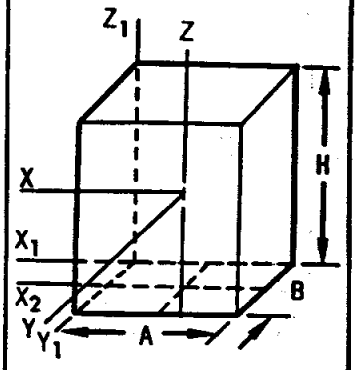
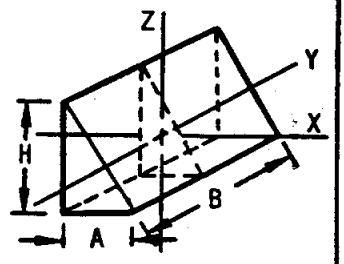
VOLUME	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
A^3	$\bar{X} = \bar{Y} = \bar{Z} = \frac{A}{2}$	$K_x^2 = K_y^2 = K_z^2 = 0.167A^2$ $K_{x_1}^2 = K_{y_1}^2 = K_{z_1}^2 = 0.666A^2$ $K_{x_2}^2 = 0.417A^2$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1y_1} = I_{y_1z_1} = \frac{1}{4}WA^2$	CUBE 
ABH	$\bar{X} = \frac{1}{2}A$ $\bar{Y} = \frac{1}{2}B$ $\bar{Z} = \frac{1}{2}H$	$K_x^2 = \frac{1}{12}(B^2 + H^2)$ $K_y^2 = \frac{1}{12}(A^2 + H^2)$ $K_z^2 = \frac{1}{12}(A^2 + B^2)$ $K_{x_1}^2 = 4K_x^2$ $K_{y_1}^2 = 4K_y^2$ $K_{z_1}^2 = 4K_z^2$ $K_{x_2}^2 = \frac{1}{12}(B^2 + 4H^2)$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = \frac{1}{4}WAB$ $I_{x_1z_1} = \frac{1}{4}WAH$ $I_{y_1z_1} = \frac{1}{4}WHB$	RECTANGULAR PRISM 
$\frac{1}{2}$ ABH	$\bar{X} = \frac{1}{3}A$ $\bar{Y} = \frac{1}{2}B$ $\bar{Z} = \frac{1}{3}H$	$K_x^2 = \frac{1}{36}(2H^2 + 3B^2)$ $K_y^2 = \frac{1}{18}(H^2 + A^2)$ $K_z^2 = \frac{1}{36}(2A^2 + 3B^2)$	———	RIGHT ANGLED WEDGE 

Figure 10-4. Solids (Sheet 1 of 7)

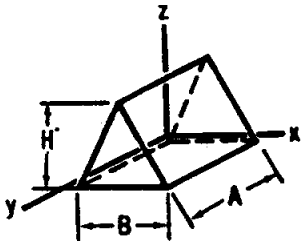
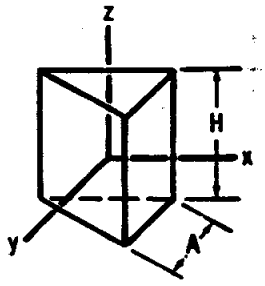
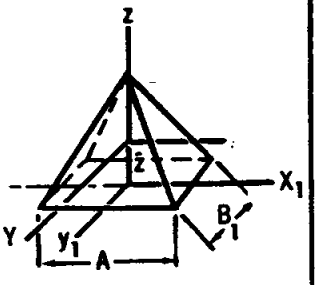
VOLUME	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\frac{1}{2}ABH$	$\bar{z} = \frac{1}{3}H$	$K_x^2 = \frac{1}{36}(2H^2 + 3A^2)$ $K_y^2 = \frac{1}{72}(4H^2 + 3B^2)$ $K_z^2 = \frac{1}{24}(2A^2 + B^2)$	—	ISOSCELES WEDGE 
$\frac{\sqrt{3}}{4}A^2H$	$\bar{z} = \frac{1}{2}H$	$K_x^2 = K_y^2 = \frac{1}{24}(A^2 + 2H^2)$ $K_z^2 = \frac{1}{12}A^2$	—	REGULAR TRIANGULAR PRISM 
$\frac{1}{3}ABH$	$\bar{z} = \frac{1}{4}H$	$K_x^2 = \frac{1}{20}(B^2 + \frac{3}{4}H^2)$ $K_y^2 = \frac{1}{20}(A^2 + \frac{3}{4}H^2)$ $K_z^2 = \frac{1}{20}(A^2 + B^2)$ $K_{x_1}^2 = \frac{1}{20}(B^2 + 2H^2)$ $K_{y_1}^2 = \frac{1}{20}(A^2 + 2H^2)$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1z_1} = I_{y_1z_1} = 0$	RIGHT RECTANGULAR PYRAMID 

Figure 10-4. Solids (Sheet 2 of 7)

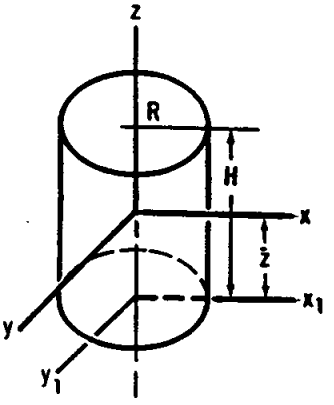
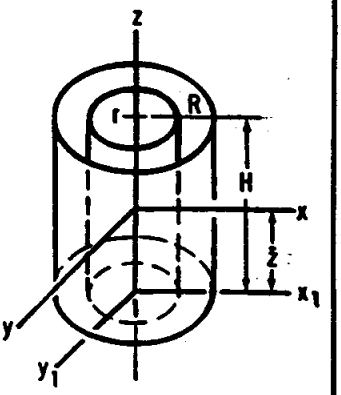
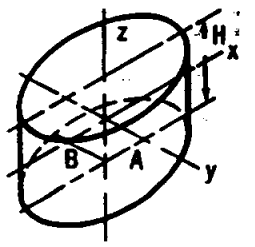
VOLUME	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\pi R^2 H$	$\bar{z} = \frac{H}{2}$	$K_x^2 = K_y^2 = \frac{1}{12}(3R^2 + H^2)$ $K_z^2 = \frac{1}{2}R^2$ $K_{x_1}^2 = K_{y_1}^2 = \frac{1}{12}(3R^2 + 4H^2)$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1z_1} = I_{x_1z_1} = 0$	RIGHT CIRCULAR CYLINDER 
$\pi H(R^2 - r^2)$	$\bar{z} = \frac{H}{2}$	$K_x^2 = K_y^2 = \frac{1}{12}\left[3(R^2 + r^2) + H^2\right]$ $K_z^2 = \frac{1}{12}(R^2 + r^2)$ $K_{x_1}^2 = K_{y_1}^2 = \frac{R^2 + r^2}{4} + \frac{H}{3}$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1z_1} = I_{y_1z_1} = 0$	HOLLOW RIGHT CIRCULAR CYLINDER 
πABH	$\bar{z} = \frac{H}{2}$	$K_x^2 = \frac{1}{12}(3B^2 + H^2)$ $K_y^2 = \frac{1}{12}(3A^2 + H^2)$ $K_z^2 = \frac{1}{4}(A^2 + B^2)$	———	ELLIPTICAL CYLINDER 

Figure 10-4. Solids (Sheet 3 of 7)

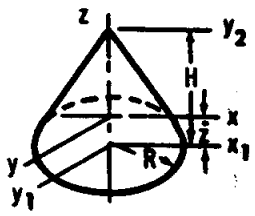
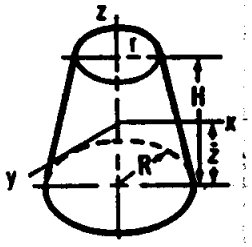
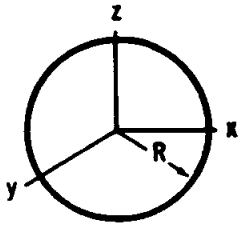
VOLUME	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\frac{1}{3}\pi R^2 H$	$\bar{z} = \frac{1}{4} H$	$K_x^2 = K_y^2 = \frac{3}{20}(R^2 + \frac{1}{4}H^2)$ $K_z^2 = \frac{3}{10} R^2$ $K_{x_1}^2 = K_{y_1}^2 = \frac{1}{20}(3R^2 + 2H^2)$ $K_{x_2}^2 = \frac{3}{20}(R^2 + 4H^2)$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1z_1} = I_{y_1z_1} = 0$	RIGHT CIRCULAR CONE 
$\frac{1}{3}\pi H(R^2 + Rr + r^2)$	$\bar{z} = \frac{H}{4}\left[\frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}\right]$	$K_z^2 = \frac{3}{10}\left[\frac{R^5 - r^5}{R^3 - r^3}\right]$	—	FRUSTUM OF A CONE 
$\frac{4}{3}\pi R^3$	—	$K_x^2 = K_y^2 = K_z^2 = \frac{2}{5} R^2$	$I_{xy} = I_{xz} = I_{yz} = 0$	SPHERE 

Figure 10-4. Solids (Sheet 4 of 7)

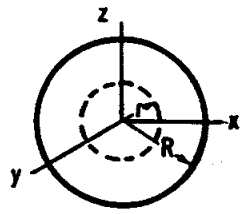
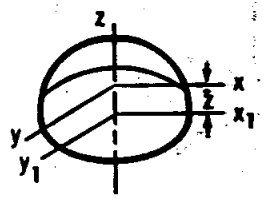
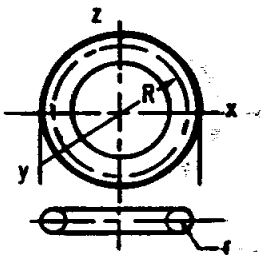
VOLUME	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\frac{4}{3}\pi(R^3 - r^3)$	———	$K_x^2 = K_y^2 = K_z^2 = \frac{2}{5}\left[\frac{R^5 - r^5}{R^3 - r^3}\right]$	$I_{x_1y_1} = I_{x_1z_1} = I_{y_1z_1} = 0$	HOLLOW SPHERE 
$\frac{2}{3}\pi R^3$	$\bar{z} = \frac{3}{8}R$	$K_x^2 = K_y^2 = .26R^2$ $K_{x_1}^2 = K_{y_1}^2 = K_z^2 = .4R^2$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1z_1} = I_{y_1z_1} = 0$	HEMISPHERE 
$2\pi^2 r^2 R$	———	$K_x^2 = K_z^2 = \frac{1}{8}(4R^2 + 5r^2)$ $K_y^2 = \frac{1}{4}(4R^2 + 3r^2)$	———	TORUS 

Figure 10-4. Solids (Sheet 5 of 7)

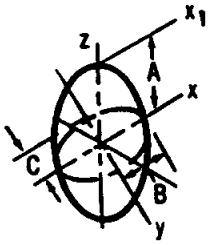
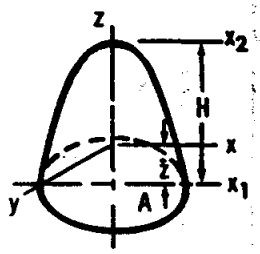
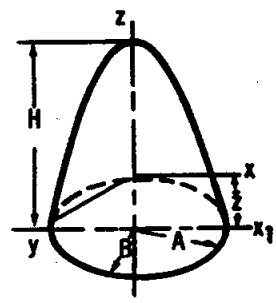
VOLUME	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\frac{4}{3}\pi ABC$	———	$K_x^2 = \frac{1}{5}(A^2 + C^2)$ $K_y^2 = \frac{1}{5}(A^2 + B^2)$ $K_z^2 = \frac{1}{5}(B^2 + C^2)$ $K_{x_1}^2 = \frac{1}{5}(6A^2 + C^2)$ $K_{y_1}^2 = \frac{1}{5}(6A^2 + B^2)$	$I_{xy} = I_{xz} = I_{yz} = 0$	ELLIPSOID 
$\frac{1}{2}\pi R^2 H$	$\bar{z} = \frac{1}{3}H$	$K_x^2 = K_y^2 = \frac{1}{18}(3R^2 + H^2)$ $K_z^2 = \frac{1}{3}R^2$ $K_{x_1}^2 = \frac{1}{6}(R^2 + H^2)$ $K_{x_2}^2 = \frac{1}{6}(R^2 + 3H^2)$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1z_1} = I_{y_1z_1} = 0$	PARABOLOID OF REVOLUTION 
$\frac{1}{2}\pi ABH$	$\bar{z} = \frac{1}{3}H$	$K_x^2 = \frac{1}{18}(3B^2 + H^2)$ $K_y^2 = \frac{1}{18}(3A^2 + H^2)$ $K_z^2 = \frac{1}{6}(A^2 + B^2)$ $K_{x_1}^2 = \frac{1}{6}(B^2 + H^2)$ $K^2 \quad \frac{1}{6}(A^2 + H^2)$	$I_{xy} = I_{xz} = I_{yz} = 0$ $I_{x_1y_1} = I_{x_1z_1} = I_{y_1z_1} = 0$	ELLIPTIC PARABOLOID 

Figure 10-4. Solids (Sheet 6 of 7)

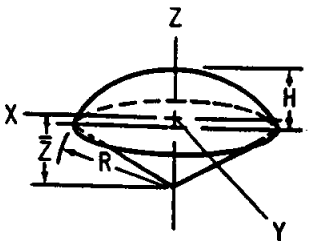
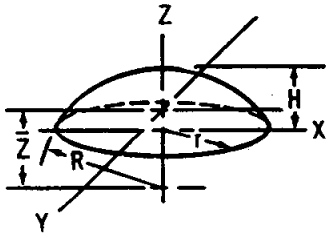
VOLUME	CENTROID	RADIUS OF GYRATION SQUARED	PRODUCT OF INERTIA	
$\frac{2}{3}\pi R^2 H$	$\bar{z} = \frac{3}{8}(2R - H)$	$K_z^2 = \frac{H}{5}(3R - H)$	———	SPHERICAL SECTOR 
$\frac{1}{6}\pi H(3r^2 + H^2)$	$\bar{z} = \frac{3}{4}\frac{(2R + H)^2}{(3R - H)}$	$K_z^2 = \frac{2H}{3R - H}$ $(R^2 - \frac{3}{4}RH + \frac{3}{20}H^2)$	———	SPHERICAL SEGMENT 

Figure 10-4. Solids (Sheet 7 of 7)

Section 11
MATERIALS

11-1 PROPERTIES OF STRUCTURAL MATERIALS

Properties of structural materials are listed in Tables 11-1 through 11-13.

11-2 PROPERTIES OF PROPELLANTS

Properties of propellants are listed in Tables 11-14 through 11-16.

The tables have been reproduced from the current edition of the SAWE weight handbook.

Table 11-1. Physical Properties

Name	Sym	Atomic Weight Approx.	Specific Gravity	Specific Heat cal/g/°C (20°C)	Therm Cond cal/cm²/cm/°C/sec (20°C)	Coef of Lin Therm Expan /°C x 10⁻⁶ (20°C)	Electrical Conductivity % IACS
Actinium	Ac	227.	....	....	.....	.....	.....
Aluminum	Al	26.98	2.704*	0.215	0.53	23.9	64.9c
Americium	Am	243.	11.7*	....	.....	.....	.....
Antimony	Sb	121.8	6.63*	0.049	0.045	8.5–10.8	4.22a
Argon	A	39.94	1.357#	0.125	0.406 x 10⁻⁴	.....	.....
Arsenic	As	74.91	5.74*	0.082	.....	4.7	4.7a
Astatine	At	211.	.....	....	.....	.....	.....
Barium	Ba	137.4	3.5*	0.068	.....	.....	.....
Berkelium	Bk	245.	....	....	.....	.....	.....
Beryllium	Be	9.013	1.82*	0.52	0.38	12.4	28.0a
Bismuth	Bi	209.0	9.82*	0.034	0.020	13.3	1.54a
Boron	B	10.82	2.3*	0.309	.....	8.3	9.1×10^{-11}a
Bromine	Br	79.92	3.13*	0.070	.....	.....	.....
Cadmium	Cd	112.4	8.67*	0.055	0.22	29.8	24.1a
Calcium	Ca	40.08	1.55*	0.149	0.3	22.	48.0a
Californium	Cf	246.	.....	....	.....	.....	.....
Carbon	C	12.01	2.22*	0.165	0.057	0.6–4.3	0.120a
Cerium	Ce	140.1	6.9*	0.042	.....	.....	2.2c
Cesium	Cs	132.9	1.9*	0.052	.....	97.	8.74a
Chlorine	Cl	35.46	1.51*	0.116	0.172 x 10⁻⁴	.....	.....
Chromium	Cr	52.01	7.20*	0.11	0.16	6.2	14.d
Cobalt	Co	58.94	8.9*	0.099	0.165	12.3	27.6c
Copper	Cu	63.54	8.98*	0.092	0.94	16.5	103.c
Curium	Cm	243.	....	....	.....	.....	.....
Dysprosium	Dy	162.5	8.58*	...	.....	.....	.....
Einsteinium	E	253.	....	....	.....	.....	.....
Erbium	Er	167.3	9.18*	....	.....	.....	.....
Europium	Eu	152.0	5.25*	....	.....	.....	.....
Fermium	Fm	255.	....	....	.....	.....	.....
Fluorine	F	19.00	1.14*	0.18	.....	.....	.....
Francium	Fr	223.	....	....	.....	.....	.....
Gadolinium	Gd	157.3	7.96*	....	.....	.....	.....
Gallium	Ga	69.72	5.92*	0.079	.....	18.	3.08a
Germanium	Ge	72.60	5.37*	0.073	0.14	.....	1.8×10^{-3}a
Gold	Au	197.0	19.35*	0.031	0.71	14.2	75.1a
Hafnium	Hf	178.5	11.4*	....	.....	.....	.....
Helium	He	4.003	0.1358#	1.25	3.32×10^{-4}	.....	.....
Holmium	Ho	164.9	10.14*	....	.....	.....	.....
Hydrogen	H	1.008	0.0684#	3.45	4.06×10^{-4}	.....	.....
Indium	In	114.8	7.32*	0.057	0.057	33.	19.7a
Iodine	I	126.9	4.94*	0.052	10.4×10^{-4}	93.	1.3×10^{-13}c
Iridium	Ir	192.2	22.5*	0.031	0.14	6.8	32.c
Iron	Fe	55.85	7.88*	0.11	0.18	11.7	17.c
Krypton	Kr	83.80	2.847#	....	0.21×10^{-4}	.....	.....
Lanthanum	La	138.9	6.16*	0.045	.....	.....	2.9c
Lead	Pb	207.2	11.36*	0.031	0.083	29.3	8.35c
Lithium	Li	6.940	0.53*	0.79	0.17	56.	19.2a
Lutetium	Lu	175.0	9.76*	....	.....	.....	.....
Magnesium	Mg	24.32	1.74*	0.25	0.38	26.	38.7c
Manganese	Mn	54.94	7.44*	0.115	.....	22.	0.932c
Mendelevium	Mv	256.	....	....	.....	.....	.....
Mercury	Hg	200.6	13.57*	0.033	0.0201	.....	1.75a
Molybdenum	Mo	95.95	10.2*	0.061	0.35	4.9	31.8a
Neodymium	Nd	144.3	7.06*	0.045	.....	.....	2.2c
Neon	Ne	20.18	0.6846#	....	1.1×10^{-4}	.....	.....
Neptunium	Np	237.	19.5*	....	.....	.....	.....
Nickel	Ni	58.71	8.9*	0.105	0.22	13.3	25.2c
Niobium	Nb	92.91	8.59*	0.065		7.1	13.2c
Nitrogen	N	14.01	0.9509#	0.247	0.600×10^{-4}	.....	.....
Nobelium	No	253.	....	.....	.....	....	.....
Osmium	Os	190.2	22.5*	0.031	.....	4.6	18.c
Oxygen	O	16.00	1.087#	0.218	0.590×10^{-4}	.....	.....

Table 11-1. Physical Properties (Continued)

Name	Sym	Atomic Weight Approx.	Specific Gravity	Specific Heat cal/g °C (20°C)	Therm Cond cal cm²/cm °C. sec (20°C)	Coef of Lin Therm Expan °C x 10⁻⁶ (20°C)	Electrical Conductivity % IACS
Palladium	Pd	106.4	12.0*	0.058	0.17	11.8	16.0c
Phosphorus	P	30.98	1.82*	0.177	.....	125.	2. x 10⁻¹⁵b
Platinum	Pt	195.1	21.49*	0.032	0.17	8.9	16.7a
Plutonium	Pu	242.	....	....	.....	.....	.....
Polonium	Po	210.	9.26*	....	.....	.....	4.1c
Potassium	K	39.10	0.86*	0.177	0.24	83.	26.8a
Praseodymium	Pr	140.9	6.64*	0.048	.....	.....	2.0c
Promethium	Pm	145.	....	....	.....	.....	.....
Protactinium	Pa	231.	....	....	.....	.....	.....
Radium	Ra	226.0	5.0*	....	.....	.....	.....
Radon	Rn	222.	....	....	.....	.....	.....
Rhenium	Re	186.2	20.5*	0.033	.....	.....	.....
Rhodium	Rh	102.9	12.46*	0.059	0.21	8.3	38.c
Rubidium	Rb	85.48	1.53*	0.080	.....	90.	13.8c
Ruthenium	Ru	101.1	12.2*	0.057	.....	9.1	22.a
Samarium	Sm	150.4	7.7*	....	.....	.....	.....
Scandium	Sc	44.96	2.5*	....	.....	.....	.....
Selenium	Se	78.96	4.82*	0.084	.....	37.	1.4 x 10⁻¹¹c
Silicon	Si	28.09	2.33*	0.162	0.20	2.8–7.3	2. x 10⁻³a
Silver	Ag	107.9	10.51*	0.056	0.975	19.7	108.c
Sodium	Na	22.99	0.97*	0.295	0.32	71.	39.a
Strontium	Sr	87.63	2.6*	0.176	.....	.....	7.5c
Sulphur	S	32.07	2.07*	0.175	6.31x10⁻⁴	64.	9. x 10⁻²²c
Tantalum	Ta	181.0	16.6*	0.036	0.13	6.5	13.9c
Technetium	Tc	99.	11.48*	....	.....	.....	.....
Tellurium	Te	127.6	6.25*	0.047	0.014	16.8	9 x 10⁻⁴c
Terbium	Tb	158.9	8.34*	....	.....	.....	.....
Thallium	Tl	204.4	11.87*	0.031	0.093	28.	9.1a
Thorium	Th	232.0	11.5*	0.034	.....	11.1	9.1c
Thulium	Tm	168.9	9.37*	....	.....	.....	.....
Tin	Sn	118.7	7.311*	0.054	0.16	23.	15.0c
Titanium	Ti	47.90	4.55*	0.126	.....	8.5	2.1a
Tungsten	W	183.9	19.3*	0.032	0.48	4.3	31.c
Uranium	U	238.1	18.7*	0.028	0.064	.....	2.9c
Vanadium	V	50.95	6.0*	0.120	.....	7.8	6.6c
Xenon	Xe	131.3	4.486#	....	1.25 x 10⁻⁴	.....	.....
Ytterbium	Yb	173.0	7.02*	....	.....	.....	.....
Yttrium	Y	88.92	5.52*	....	.....	.....	.....
Zinc	Zn	65.38	7.146*	0.092	0.27	15–16	29.1c
Zirconium	Zr	91.22	6.5*	0.066	.....	5.	4.01a

#For gases, $P_o = 1.2250 \times 10^{-3}$ g/cm³ (density of air at standard sea level).
*For liquids and solids, $P_o = 9.98203 \times 10^{-1}$ g/cm³ (density of water at 20°C)

a At 0°C
b At 11°C
c At 20°C
d At 28°C

Atomic Number – The number of positive charges on the nucleus of an atom.

Atomic Weight – by definition – the weight of an atom relative to that of an atom of the isotope of carbon with a mass number of 12 (C^{12}). (Note: Prior to 1961 the standard was the element oxygen. The chemical atomic weight was the average weight of atoms of a particular element relative to the average weight of an oxygen atom which was defined as 16.000 atomic mass units (a.m.u.).

Atom – Weight. (not atomic weight) The weight of a single atom in grams is the atomic weight divided by Avogadro's Number (6.023×10^{23}). Thus, the weight of one atom of oxygen (O^{16}) is

$$16.000 \div 6.023 \times 10^{23} = 2.66 \times 10^{-23} \text{ grams.}$$

Table 11-1A. Specific Stiffness*

$(E/\gamma) \times 10^{-6}$ Inches

Material	Ratio
Beryllium	657
Silicon Carbide	607
Boron Carbide	483
Alumina Ceramics	420
Mica, Natural	337
Mica, Synthetic	281
Titanium Carbide Cermet	248
Polycrystalline Glass	219
Alumina Cermets	195
Tungsten-Titanium Carbide Cermet	192
Zircon	188
Tungsten Carbide Cermet	185
Boron Nitride	163
Standard Electrical Ceramics	143
Glass, Alumino-Silicate	140
Ruthenium	136
Glass, Fused Silica	129
Mica, Glass-Bonded	129
Molybdenum and Its Alloys	127
Glass, 96% Silica	124
Glass, Borosilicate	123
Stealite	120
Cobalt-Base Superalloys	120
Nickel-Base Superalloys	112
Titanium and Its Alloys	111
Glass, Soda Lime	111
High Temperature Steels	111
Ultra High Strength Steels	107
Ingot Irons	106
Carbon Steels (cast)	106
Wrought Iron	106
Pearlitic Malleable Irons	106
Carbon Steels	106
Alloy Steels	106
Nitriding Steels	106
Stainless Steels (cast)	106
Aluminum and Its Alloys	105
Age Hardenable Stainless Steels	104
Heat Resistant Alloys (cast)	104
Ferritic Stainless Steels	104
Martensitic Stainless Steels	104
Iron-Base Superalloys (Cr-Ni-Co)	103
Free-Cutting Steels	102
Iron-Base Superalloys (Cr-Ni)	102
Magnesium Alloys	102
Austenitic Stainless Steels	100
Nodular Irons (cast)	100
Osmium	98
Reinforced Plastics, Phenolic	96
Malleable Irons (cast)	96
Rhodium	94
Cobalt	94
Nickel and Its Alloys	93
Cordierite	93
Iridium	91
Vanadium	87
Nickel and Its Alloys (cast)	86
Phenolics, Electrical	85
Tungsten	84
Glass, Lead Silicate	82
Monel	82
Low Expansion Nickel Alloys	81
Gray Irons (cast)	77
Columbium Alloys	[illegible]
Cupro-Nickels	68

Material	Ratio
Aluminum Bronzes (cast)	66
Beryllium Copper	64
Zirconium and Its Alloys	59
Yellow Brasses (cast)	58
Nickel Silvers	57
Phenolics, Shock Resistant	56
Silicon Bronzes	56
Red Brass, 85%	54
Commercial Bronze, 90%	54
Leaded Brasses	53
Gilding, 95%	53
Phosphor Bronzes	53
Copper	53
Tin and Aluminum Brasses	53
Cartridge Brass, 70%	52
Low Brass, 80%	51
Leaded Tin Bronzes (cast)	51
Tellurium Copper	50
Muntz Metal	50
Yellow Brass	49
Columbium	48
Leaded Red Brass (cast)	47
Tantalum	45
Uranium	43
Hafnium	43
Melamines	42
Carbon	41
Paliadium	39
Ureas	34
Polystyrenes, Glass-Filled	33
Reinforced Plastics, Polyester	33
Graphite	31
Pewter	29
Silver	29
Tin-Lead-Antimony Alloys	29
Platinum	27
Diallyl Phthalate	25
Grade A Tin	25
Thorium	24
Phenolics, GP	23
Polyvinyl Formal	19
Modified Polystyrenes	18
Gold	17
Polyesters (cast)	16
Polystyrenes, GP	16
Acrylics, GP	14
Phenolics (cast)	12
Polyvinyl Butyral	12
Nylon 66	12
Ethyl Cellulose, GP	10
Nylon 6	10
Acrylics, High Impact	9
Acetal	9
ABS Resins	9
Ethyl Cellulose, High Impact	9
Polycarbonate	9
Nylon 610	8
Polypropylene	6
Nylon 11	6
Cellulose Nitrate	5
Lead and Its Alloys	5
CFE Fluorocarbons	5
Polyethylenes, Low Density	1
TFE Fluorocarbons	1
Polyvinyl Chloride, Nonrigid	0.07

*Specific stiffness, or stiffness/weight ratios, were obtained by dividing modulus of elasticity in tension (psi) by weight density (lb/cu in.). Specifically, the highest value of a range of typical values was divided by that material's weight density.

Table 11-2. Properties of Metals

Alloy Designation	Specification	Lb/Cu. Inch	Room Temp KSI FTU	Room Temp KSI FTY	Coeff. of Expans. In/In °F(10-6)	Form
ALUMINUM (Wrought)						
2EC-T6		.098	43	34		
1100-0		.098	11	3.5	12.1	S
1100-H14		.098	16	14	12.1	S
2014-T6	QQ-A-367	.101	65	55	11.9	F
2017-T4	QQ-A-367	.101	62	40		F
2020	MIL-A-8882A					S
2024-T4	QQ-A-355	.100	64	40	11.9	P
2024-T6	QQ-A-355	.100	64	50	11.9	P
2024-TB1	QQ-A-355	.100	67	58	11.9	S
2117-T4	MIL-R-5674	.099	43	24		
2219-TB1		.103	61	42		
3003-0		.099	16	6		S
5052-0	QQ-A-318	.097	25	6	12.2	S
5486-0	MIL-A-21170	.096	42	19	12.3	B
5486-H321	MIL-A-21170	.096	42	25	12.3	B
6061-T6	QQ-A-327	.098	42	35	12.0	S
7075-T6	QQ-A-283	.101	76	65	12.0	S
7075-T73		.101	66	56	12.0	F
7079-T6	QQ-A-367	.101	72	62	12.0	F
7178-T6	MIL-A-9180	.101	83	72	12.0	S
ALUMINUM (CAST)						
218	QQ-A-591	.093	41		13.3	
B195-T6	QQ-A-596	.101	35		12.7	
356-T6	AMS4260	.097	33	22	11.9	
195-T6	QQ-A-601	.101	32	20	12.7	
220-T4	QQ-A-601	.093	42	22	14.0	
A356-T61	MIL-A-21180	.097	45	34		
C355-T61	MIL-A-21180	.098	50	40		
AL-MAG	MIL-A-10935	.095	35	18	12.0	
BERYLLIUM						
2% BeO Max	AMS7901	.067	40	30		B
CHROMIUM STEELS						
Haynes Alloy 6	AMS5387	.303	110	65		
Stellite No. 21	AMS5385C	.300	101	82		
Stellite No. 30	AMS5380B	.300	96	81		
Stellite No. 31	AMS5382B	.311	113	79		
Multimet	AMS5372B	.296	101	54		
COBALT						
Pure		.321				
L-605	AMS5759	.330	125	45	6.83	B
COLUMBIUM						
Unalloyed		.310	40		4.06	
Cb-1Zr		.311	72	60		
Cb-5Zr		.310				
Cb-5V		.306				
Cb-10Hf-1Ti-.5Zr		.320				
Cb-10Ta-10W		.347				
Cb-28Ta-10W-1Zr		.383				
Cb-33Ta-1Zr		.343	95	80	4.10	
Cb-10Ti-5Zr		.286				
Cb-10Ti-10Mo-.1C		.292				
Cb-5W-1Zr		.317				
Cb-10W-1Zr-.1C		.326				
Cb-10W-2.5Zr		.326				
Cb-10W-10Hf		.343				
Cb-15W-5Mo-1Zr		.340				
Cb-15W-5Mo-5Ti-1Zr		.330				
Cb-20W-1Zr		.347				
Cb-20W-10Ti-6Mo		.320				
Cb-5V-5Mo-1Zr		.305				

Alloy Designation	Specification	Lb/Cu. Inch	Room Temp KSI FTU	Room Temp KSI FTY	Coeff. of Expans. In/In °F(10-6)	Form
COPPER ALLOYS						
110 (Electrolytic)	QQ-C-502	.322	32	10	9.4	
122	WW-T-775	.323	32	10	9.4	S,P
360 Free Cutting	QQ-B-626	.307	49	18	11.4	E,R
464 Naval Br.	QQ-B-637	.304	60	27	11.8	S,R
510 Phosper Bronze	WW-T-401	.320	47	19	9.9	S
637 Al Bronze	AMS4631	.274	90	45	9.9	
Beryllium Cu	AMS4650	.298	150	120	9.3	
Al Bronze	AMS4640	.274	100	52	9.9	B
Beryllium Cu	AMS4890	.298	150	120	9.3	
Beryllium Cu	QQ-B-530	.298	180	130	9.3	P
Al Bronze	QQ-B-671	.274	65	25	9.6	C
Naval Brass	QQ-N-35	.274	54	25		
Cu Soft	QQ-C-576	.323	30	10	9.9	
Cu Hard	QQ-C-576	.323	43	38	9.9	
Cartridge Br.	QQ-B-613	.308	45	15	11.1	
614 Al Bronze	QQ-C-450	.274	72	32	9.9	B
Phosper Br.	QQ-P-330	.320	91	65	9.9	S
Al Bronze	QQ-A-679	.274	85	42	9.9	
Beryllium Cu	QQ-C-533	.297	190	165	9.3	S
Cu Tube Soft	WW-T-775	.323				
IRON - DUCTILE CASTING	Grade					
ASTM-A339-55	69-45-10	.257	60	45	6.4	C
ASTM-A339-55	80-60-3	.257	80	60	6.4	C
MIL-I-11466	Class 2	.257	100	75	6.4	C
Steel Alloy	QQ-S-681	.283	150	125	6.3	C
LEAD						
Antimony 4-6%	Rolled	.395			15.1	S
Antimony 4-6%	Cast	.395			15.1	C
MAGNESIUM						
AZ31B	QQ-M-31	.064	35	21	14.5	E
AZ80A	QQ-M-31	.066	43	28	14.5	E
ZK60A	QQ-M-31	.066	45	36	14.5	E
HM31A-T5	MIL-M-8916	.065	37	26	14.8	E
AZ91C	QQ-M-56	.066	34	16	14.6	C
AZ63A	QQ-M-56	.066	34	16	14.6	C
EZ33A	QQ-M-56	.066	20	14	14.5	C
HK31A	QQ-M-56	.065	27	13	14.5	C
AZ91	QQ-M-55	.066	34	16	14.5	C
HM21XA-T5		.064	31	20	14.5	F
AZ31B-H24	QQ-M-44	.064	39	29	14.5	S
NICKEL						
Monel	QQ-N-281	.319	110	85	7.6	B
Inconel	MIL-N-6710	.304	120	90	6.4	B
Inconel-X	AMS5667	.300	165	105	7.7	B
Inconel 718		.293	175	145	7.1	B
Inconel 718		.297	180	150	7.1	F
Rene 41	AMS5545	.298	170	130	7.5	S
Hastelloy X	AMS2262	.297	100	40	7.7	S
Udimet 500		.290	190	125	8.3	S
Udimet 600		.286	190	130	6.4	S
T.D. Nickel		.322	80	59	8.5	S
MOLYBDENUM						
Unalloyed		.369	90	75		B
Mo-1.5Cb		.368	155	120		B
Mo-.05Zr		.369	126	115		S
Mo-.5Zr		.368	128	94		S
Mo-.5Ti		.367	75	65	3.06	S
Mo-.5Ti-.08Zr-.03C		.367	130	105		S
Mo-1.2Ti-.25Zr-.15C		.363	143	104		B
Mo-25W-.1Zr-.03C		.418	145	126		B
Mo-30W		.430	121	106		B
Mo-50W		.483	144	133		B

Table 11-2. Properties of Metals (Continued)

Alloy Designation	Specification	Lb/ Cu. Inch	Room Temp KSI FTU	Room Temp KSI FTY	Coeff. of Expans. In/ In/ °F(10⁻⁶)	Form
STAINLESS STEELS						
302 Cond A	QQ-S-763	.286	90	45	9.5	B
302 Cond B	QQ-S-763	.286	125	100	9.5	B
303 Cond B	QQ-S-763	.286	125	100	9.5	B
316	MIL-S-867	.290	70	30	9.3	C
321	QQ-S-763	.290	75	30	9.3	F
19-9DL	AMS5526	.287	95	45	8.52	S
17-4PH	AMS5343	.282	150	130	6.0	C
A286	AMS5735	.286	130	85	9.17	F
304 Annealed	MIL-T-8506	.290	115	35	9.5	T
347	MIL-S-6721	.290	80	30	9.3	S
301 1/4 Hard	MIL-S-5059	.285	125	75	9.4	S
301 Full Hard	MIL-S-5059	.285	185	140	9.4	S
PH15-7Mo	AMS5520	.277	190	170	6.1	S
17-7PH	MIL-S-25043	.276	180	150	6.2	S
17-4PH	AMS5643	.282	190	170	6.0	F
N-155	AMS5532	.295	100	58	8.7	S
STEEL ALLOYS						
SAE 1018		.283	60	33	7.0	B
SAE 1095	MIL-S-8559	.283			7.2	B
SAE 4130	MIL-S-6758	.283	180	165	6.5	F,S
SAE 4140	MIL-S-5626	.283	200	180	6.3	B,F
SAE 4330	AMS6427	.283	220	190	6.3	B,F
SAE 4340	MIL-S-5000	.283	180	165	6.3	B,F
SAE 8620	MIL-S-8690	.283			6.3	B
SAE 9310	AMS6260	.283				B
17-22AS	AMS6302	.283	150	130	7.01	B,C
H-11		.281	280	230	6.8	B
SAE 4130	MIL-S-18729	.283	180	165	6.3	S,T
SAE 4340	AMS6359	.283	200	180	6.3	S,T
T1		.283	105	90	7.01	F
TANTALUM						
Unalloyed	AMS7840	.600	60	52	3.6	S,P
Ta-10W		.608	150	140	2.96	
Ta-12.5W		.610	102	85		S
Ta-30Cb-7.5V		.425	155	139		S
Ta-5W-2.5Mo		.595	96	88		S
Ta-8W-2Hf		.604	112	96	3.1	S
Ta-10W-2.5Hf		.604	110	105		S
Ta-10W-2.5Mo		.599	110	102		S
TITANIUM						
99.5		.163	38	27		S
99.2	AMS4902	.163	60	45		S
99.0	AMS4900	.163	75	60		S
99.0	AMS4901	.163	90	75		S
98.9	AMS4921	.164	100	85		S
.15-.20 Pd		.163	62	46		S
5Al-2.5Sn	AMS4926	.161	125	117		S
5Al-2.5Sn low O		.162	110	95		S
5Al-5Sn-5Zr		.166	125	120		S
7Al-12Zr		.165	165	159		B
7Al-2Cb-1Ta		.159	126	120		B
8Al-1Mo-1V		.158	160	150		S
8Mn	AMS4908A	.171	137	125		S
2Fe-2Cr-2Mo	AMS4923	.171	179	71		B
2.5Al-16V		.165	180	165		S
3Al-2.5V		.162	100	85		S
4Al-4Mn	AMS4925	.163	148	135		B
4Al-3Mo-1V	AMS4913	.163	195	167		S
5Al-1.25Fe-2.75Cr		.162	190	175		B
5Al-1.5Fe-1.4Cr-1.2Mo	AMS4969	.162	195	184		B
6Al-4V	AMS4935	.160	170	155		S
6Al-4V low O		.160	135	127		S
6Al-6V-2Sn-1FeCu		.164	190	180		B
7Al-4Mo		.162	185	175		B

Alloy Designation	Specification	Lb/ Cu. Inch	Room Temp KSI FTU	Room Temp KSI FTY	Coeff. of Expans. In/ In/ °F(10⁻⁶)	Form
TITANIUM (Cont'd.)						
1Al-8V-5Fe		.168	221	215		B
3Al-13V-11Cr		.175	260	245		S
TUNGSTENS						
Unalloyed	AMS7899	.697	95			S
Doped		.697	400			W
W-.38ToC		.696				B
W-1ThO_2		.690				
W-2ThO_2		.683				
W-.6Cb		.692				
W-2Mo		.685	40			C
W-15Mo		.615	45			C
W-3Re		.699				
W-5Re		.701				
W-26Re		.714				
	AMS7725	.614				
Kennertium W-2		.670	100	80		
URANIUM (Depleted)						
U-2		.664	110	65		C
U-8	MIL-U-46045A	.629	120	115		S
MISC. METALS						
Bismuth		.354			7.4	
Cadmium		.313			16.6	
Cerrobend		.350				
Chromium		.258			3.4	
Constanten		.322				
Gold		.700	19		14.2	
Kirksite "A"		.250				
Lead - Pure		.409	1.7	.6	16.3	
Manganese		.268			12.2	
Nickel - Pure		.322			7.4	
Platinum		.774	20		8.8	
Silver		.380	24		19.7	
Tin		.264			13.0	
Mallory 1000		.606				
Vanadium		.205	31	16	4.3	
White Metal (Babitt)		.263				
Zinc - Pure		.258			22.1	
Zirconium		.234	49		3.0	

Note: Since the properties of any material may differ with alloy, temperature, form, heat treatments, etc., the above values should be used only as a rough comparison. For exact values refer to applicable specifications or materials and processes data.

Form: B - Bar, S - Sheet, P - Plate, E - Extrusions, C - Cast, F - Forgings, W - Wire.

Table 11-3. Tubing

Steel Tubing (Lb/in.)

O.D. Inches	Wall Thickness – Inches															
	.016	.020	.022	.028	.035	.042	.049	.058	.065	.083	.095	.109	.120	.156	.188	.250
3.16	.0024	.0030	.0032	.0040	.0048	.0055	.0061									
1.4	.0034	.0041	.0045	.0055	.0067	.0078	.0088	.0099	.0107							
5.16	.0042	.0052	.0057	.0071	.0087	.0101	.0115	.0132	.0143	.0170	.0184	.0198				
3.8	.0051	.0063	.0069	.0087	.0106	.0125	.0142	.0164	.0180	.0216	.0237	.0259	.0273	.0305		
7.16	.0060	.0074	.0082	.0102	0126	0148	0170	.0196	.0216	.0263	.0290	.0320	.0340	.0392	.0418	
1/2	.0069	.0085	.0094	.0118	.0145	.0172	.0197	.0229	.0252	.0309	.0343	.0380	.0407	.0479	.0523	
9.16	.0077	.0097	.0106	.0133	.0165	.0195	.0225	.0261	.0288	.0355	.0396	.0441	.0474	.0566	.0628	
5 8	.0087	.0108	.0118	.0149	.0184	.0218	.0252	.0293	.0325	.0401	.0449	.0502	.0541	.0653	.0733	
3/4	.0105	.0130	.0143	.0180	.0223	.0265	.0306	.0358	.0397	.0494	.0555	.0623	.0674	.0827	.0943	.1115
7 8	.0123	.0153	.0167	.0212	.0262	.0312	.0361	.0423	.0470	.0586	.0661	.0745	.0808	.1001	.1152	.1394
1	.0141	.0175	.0192	.0243	.0301	.0359	.0416	.0487	.0542	.0679	.0767	.0866	.0942	.1175	.1362	.1673
1-1 8	.0159	.0197	.0217	.0275	.0341	.0407	.0471	.0554	.0595	.0772	.0873	.0988	.1076	.1349	.1572	.1952
1-1 4	.0176	.0219	.0242	.0304	.0378	.0458	.0523	.0614	.0687	.0864	.0980	.1110	.1210	.1523	.1781	.2231
1-3.8	.0196	.0243	.0267	.0338	.0418	.0500	.0580	.0682	.0760	.0957	.1085	.1231	.1344	.1697	.1991	.2509
1-1 2	.0212	.0264	.0297	.0361	.0457	.0546	.0634	.0746	.0832	.1049	.1191	.1353	.1477	.1871	.2201	.2788
1-5.8	.0228	.0285	.0314	.0399	.0497	.0593	.0689	.0811	.0905	.1142	.1297	.1474	.1611	.2045	.2410	.3067
1-3.4	.0246	.0308	.0338	.0430	.0536	.0640	.0744	.0876	.0977	.1235	.1403	.1596	.1745	.2219	.2620	.3346
1-7 8	.0265	.0331	.0364	.0460	.0575	.0687	.0798	.0940	.1050	.1327	.1509	.1718	.1879	.2393	.2830	.3625
2	.0283	.0354	.0389	.0493	.0613	.0734	.0853	.1005	.1122	.1420	.1615	.1839	.2013	.2567	.3039	.3903
2-1.4	.0308	.0398	.0438	.0554	.0690	.0831	.0962	.1134	.1267	.1605	.1827	.2082	.2281	.2915	.3459	.4461
2-1 2	.0354	.0442	.0486	.0618	.0769	.0920	.1072	.1264	.1412	.1790	.2039	.2325	.2548	.3263	.3878	.5019
2-3.4	.0390	.0487	.0535	.0680	.0848	.1015	.1181	.1393	.1557	.1975	.2250	.2568	.2816	.3610	.4297	.5576

Alluminum Alloy Tubing (Lb/In.)

O.D.	Wall Thickness															
	.016	.020	.022	.028	.035	.042	.049	.058	.065	.083	.095	.109	.120	.156	.188	.250
3.16		.0011	.0011	.0014	.0017	.0019	.0021	.0024	.0025	.0027						
1 4		.0014	.0016	.0020	.0024	.0027	.0031	.0035	.0038	.0044	.0046	.0048	.0049			
5.16		.0018	.0020	.0025	.0031	.0036	.0041	.0046	.0051	.0060	.0065	.0070	.0073			
3 8		.0022	.0024	.0031	.0037	.0044	.0050	.0058	.0063	.0076	.0084	.0091	.0096			
7 16		.0026	.0029	.0036	.0044	.0052	.0060	.0069	.0076	.0092	.0102	.0113	.0120	.0138		
1 2		.0030	.0033	.0042	.0051	.0060	.0069	.0081	.0089	.0109	.0121	.0134	.0143	.0169		
9.16		.0034	.0037	.0047	.0058	.0069	.0079	.0092	.0102	.0125	.0140	.0155	.0167	.0199	.0221	
5.8		.0038	.0042	.0053	.0065	.0077	.0089	.0103	.0114	.0141	.0158	.0177	.0190	.0230	.0258	
3 4		.0046	.0050	.0064	.0079	.0093	.0108	.0126	.0140	.0174	.0196	.0220	.0238	.0291	.0332	.0393
7.8		.0054	.0059	.0075	.0092	.0110	.0127	.0149	.0165	.0207	.0233	.0262	.0285	.0352	.0406	.0491
1		.0062	.0068	.0086	.0106	.0126	.0146	.0172	.0191	.0239	.0270	.0305	.0332	.0414	.0480	.0589
1-1.8		.0069	.0076	.0097	.0120	.0143	.0166	.0194	.0216	.0272	.0307	.0348	.0379	.0475	.0553	.0687
1-1.4		.0077	.0085	.0108	.0134	.0159	.0185	.0217	.0242	.0304	.0345	.0391	.0426	.0536	.0627	.0785
1-3.8		.0085	.0094	.0118	.0147	.0176	.0204	.0240	.0268	.0337	.0382	.0434	.0473	.0597	.0701	.0884
1-1.2		.0094	.0102	.0130	.0161	.0192	.0223	.0263	.0293	.0370	.0419	.0476	.0520	.0659	.0775	.0982
1-5 8		.0102	.0111	.0141	.0175	.0209	.0243	.0286	.0319	.0402	.0457	.0519	.0567	.0720	.0849	.1080
1-3.4		.0110	.0119	.0152	.0189	.0225	.0262	.0308	.0344	.0435	.0494	.0562	.0615	.0781	.0923	.1178
1-7.8		.0118	.0128	.0163	.0202	.0242	.0281	.0331	.0370	.0467	.0531	.0605	.0662	.0843	.0996	.1276
2		.0125	.0137	.0174	.0216	.0258	.0300	.0354	.0395	.0500	.0569	.0648	.0709	.0904	.1070	.1375
2-1.4		.0141	.0155	.0196	.0244	.0291	.0339	.0399	.0446	.0565	.0643	.0733	.0803	.1026	.1218	.1571
2-1 2		.0157	.0173	.0217	.0271	.0324	.0377	.0445	.0497	.0630	.0718	.0819	.0897	.1149	.1366	.1767
2-3.4		.0173	.0192	.0239	.0299	.0357	.0416	.0491	.0548	.0695	.0792	.0904	.0992	.1271	.1513	.1964

Capacity of Tubing Cubic Inches Per Inch

O.D.	Wall Thickness															
	.016	.020	.022	.028	.035	.042	.049	.058	.065	.083	.095	.109	.120	.156	.188	.250
3.16	.0190	.0171	.0162	.0137	.0108	.0084	.0063	.0040	.0026	.0004						
1 4	.0373	.0346	.0333	.0296	.0254	.0216	.0182	.0141	.0113	.0055	.0028	.0008				
5 16	.0618	.0583	.0566	.0517	.0462	.0410	.0361	.0303	.0262	.0169	.0118	.0070	.0041			
3.8	.0924	.0881	.0861	.0799	.0731	.0665	.0603	.0527	.0471	.0343	.0269	.0194	.0143	.0031		
7 16	.1292	.1241	.1216	.1143	.1061	.0981	.0905	.0812	.0743	.0579	.0481	.0378	.0306	.0124	.0030	
1 2	.1720	.1662	.1633	.1548	.1452	.1359	.1269	.1158	.1075	.0876	.0755	.0625	.0531	.0278	.0121	
9 16	.2211	.2144	.2115	.2015	.1905	.1798	.1695	.1566	.1469	.1235	.1090	.0933	.0817	.0493	.0273	.0031
5 8	.2762	.2688	.2651	.2543	.2419	.2299	.2181	.2035	.1924	.1655	.1486	.1301	.1164	.0769	.0487	.0123
3.4	.4049	.3959	.3915	.3783	.3632	.3484	.3339	.3157	.3019	.2679	.2463	.2223	.2043	.1507	.1099	.0491
7 8	.5581	.5476	.5424	.5268	.5090	.4914	.4742	.4525	.4359	.3949	.3685	.3390	.3167	.2489	.1956	.1104
1	.7359	.7238	.7178	.6999	.6793	.6590	.6390	.6138	.5945	.5463	.5153	.4803	.4537	.3718	.3058	.1963
1-1.8	.9381	.9245	.9177	.8973	.8742	.8511	.8282	.7996	.7776	.7223	.6866	.6461	.6151	.5191	.4406	.3068
1-1.4	1.165	1.150	1.142	1.120	1.094	1.068	1.042	1.010	.9852	.9228	.8824	.8364	.8012	.6912	.5999	.4418
1-3.8	1.416	1.400	1.391	1.366	1.338	1.309	1.281	1.245	1.217	1.148	1.103	1.051	1.012	.8875	.7838	.6013
1-1 2	1.692	1.674	1.665	1.638	1.606	1.575	1.544	1.504	1.474	1.377	1.348	1.291	1.247	1.108	9924	.7854
1-5 8	1.994	1.974	1.964	1.934	1.899	1.865	1.832	1.789	1.756	1.672	1.618	1.554	1.507	1.354	1 225	.9941
1-3 4	2.319	2.297	2.286	2.254	2.217	2.180	2.143	2.097	2.061	1.971	1.911	1.844	1.791	1.624	1 483	1.227
1-7 8	2.668	2.645	2.633	2.599	2.559	2.519	2.480	2.430	2.392	2.294	2.230	2.156	2.100	1.919	1.765	1.485
2	3.042	3.017	3.005	2.968	2.926	2.883	2.841	2.788	2.746	2.642	2.573	2.494	2.433	2.238	2.071	1.767
2-1.4	3.868	3.836	3.827	3.761	3.733	3.684	3.636	3.577	3.530	3.411	3.333	3.243	3.173	2.950	2.758	2.405
2-1 2	4.784	4.753	4.737	4.691	4.638	4.585	4.532	4.464	4.412	4.279	4.191	4.090	4.011	3.760	3.543	3.142
2-3 4		5.768	5.751	5.700	5.641	5.582	5.523	5.449	5.391	5.244	5.147	5.035	4 948	4 668	4.427	3.976

Table 11-4. Sandwich Materials

Typical Properties of Hexcel Aluminum Honeycomb at Room Temperature

Type MIL	Type HEXCEL	Core Density (Tolerance +8% – 6%) Lb./Cu.Ft.	Nominal Weight Lbs. per Inch/Foot²	Average Physical Properties – PSI: Bare Compressive Strength 5/8 Inch Core	Flat Compressive Strength 5/8 Inch Core with Facings	Flexual Shear Strength 6 Inch Span L**	Flexual Shear Strength 6 Inch Span W**	Shear Modulus from Flexure Test 6 Inch Span L**	Shear Modulus from Flexure Test 6 Inch Span W**	Facing Thickness for Flexure Specimens, Inch
	Alloy 3003–H19									
I–A	AL 1/8–3003 .0007 P	3.1	.26	210	240	160	90	40,000	20,000	.020
I–B	AL 1/8–3003 .001 P	4.5	.37	400	470	285	160	64,000	32,000	.032
I–C	AL 1/8–3003 .0015 P	6.1	.51	640	760	465	260	96,000	46,000	.051
I–D	AL 1/8–3003 .002 P	8.1	.67	970	1200	720	415	140,000	63,000	.091
II–A	AL 3/16–3003 .001 P	3.1	.26	210	240	160	90	40,000	20,000	.020
II–B	AL 3/16–3003 .0015 P	4.4	.37	365	450	275	155	63,000	31,000	.032
II–C	AL 3/16–3003 .002 P	5.7	.47	575	680	415	230	87,000	42,000	.051
II–D	AL 3/16–3003 .003 P	8.1	.67	970	1200	720	415	140,000	63,000	.091
III–A	AL 1/4–3003 .001 P	2.3	.19	115	135	100	55	27,000	14,000	.020
	AL 1/4–3003 .0015 P	3.4	.28	250	285	180	105	45,000	23,000	.020
III–B	AL 1/4–3003 .002 P	4.3	.36	370	435	265	150	61,000	30,000	.032
III–C	AL 1/4–3003 .003 P	6.0	.50	625	745	450	250	94,000	45,000	.051
III–D	AL 1/4–3003 .004 P	7.9	.66	930	1150	690	395	137,000	61,000	.091
IV–A	AL 3/8–3003 .001 P	1.6	.13	45	60	60	30	16,000	8,000	.020
IV–B	AL 3/8–3003 .002 P	3.0	.25	195	225	150	85	39,000	19,000	.020
IV–C	AL 3/8–3003 .003 P	4.2	.35	360	415	250	145	60,000	30,000	.032
IV–D	AL 3/8–3003 .004 P	5.4	.45	530	630	380	210	82,000	40,000	.051
IV–E	AL 3/8–3003 .005 P	6.5	.54	700	850	510	290	110,000	49,000	.072
	Alloy 5052–H39									
	AL 1/8–5052 .0007 P	3.1	.26	250	285	180	110	40,000	20,000	.032
	AL 1/8–5052 .001 P	4.5	.37	480	560	340	210	64,000	32,000	.032
	AL 1/8–5052 .0015 P	6.1	.51	760	910	580	350	96,000	46,000	.050
	AL 1/8–5052 .002 P	8.1	.67	1160	1440	1000	630	140,000	63,000	.091
	AL 3/16–5052 .001 P	3.1	.26	250	285	180	110	40,000	20,000	.032
	AL 3/16–5052 .0015 P	4.4	.37	440	540	330	200	63,000	31,000	.032
	AL 3/16–5052 .002 P	5.7	.47	690	815	510	305	87,000	42,000	.050
	AL 3/16–5052 .003 P	8.1	.67	1160	1440	1000	630	140,000	63,000	.091
	AL 1/4–5052 .0007 P	1.6	.13	50	75	70	40	16,000	8,000	.032
	AL 1/4–5052 .001 P	2.3	.19	140	160	110	65	27,000	14,000	.032
	AL 1/4–5052 .0015 P	3.4	.28	300	340	210	130	45,000	23,000	.032
	AL 1/4–5052 .002 P	4.3	.36	445	520	310	190	61,000	30,000	.032
	AL 1/4–5052 .003 P	6.0	.50	750	895	565	340	94,000	45,000	.050
	AL 1/4–5052 .004 P	7.9	.66	1115	1380	970	600	137,000	61,000	.091

Typical Properties of Hexcel Glass Fabric-Plastic Honeycomb at Room Temperature

		Tolerance Unknown		(No Set Dim. on Core)	(No Set Dim. on Core)	(No Set Dim. on Span)		(No Set Dim. on Span)		
Polyester										
I–A	PE 3/16–112–4.5	4.5	.38	340	510	270	150	17,000	7,500	
I–B	PE 3/16–112–6.5	6.5	.54	840	1250	470	240	20,500	9,000	
I–C	PE 3/16–112–9.0	9.0	.75	1500	1900	660	420	34,000	18,000	
II–A	PE 1/4–112–4.0	4.0	.33	440	620	250	150	13,000	6,700	
II–B	PE 1/4–112–6.0	6.0	.50	910	1140	460	280	21,000	10,000	
II–C	PE 1/4–112–8.0	8.0	.67	1320	1700	520	400	22,000	16,000	
III–A	3/8–112–2.5	2.5	.21	150	220	160	70	6,000	2,800	
III–B	3/8–112–4.5	4.5	.38	330	490	330	190	15,000	6,200	
Nylon Phenolic										
I–A	NP 3/16–112–4.5	4.5	.38	430	580	270	140	14,500	5,900	
I–B	NP 3/16–112–6.5	6.5	.54	970	1150	490	250	27,500	11,000	
I–C	NP 3/16–112–9.0	9.0	.75	1700	1800	700	430	34,000	19,000	
II–A	NP 1/4–21–4.0	4.0	.33	380	440	280	140	15,500	6,500	
II–B	NP 1/4–21–6.0	6.0	.50	900	930	420	290	20,500	11,000	
II–C	NP 1/4–21–8.0	8.0	.67	1300	1350	560	390	25,000	17,000	
III–A	NP 3/8–21–2.5	2.5	.21	180	200	150	70	10,000	4,500	
III–B	NP 3/8–21–4.5	4.5	.38	340	380	320	170	15,000	6,600	
Nylon Phenolic Forming										
I–B	NPF 3/16–112–6.5	6.5	.54	1020	1200	510	270	30,000	11,500	
I–C	NPF 3/16–112–9.0	9.0	.75	1800	1900	670	410	32,500	17,000	
Over Expanded										
II–A	NPOX 1/4–21–4.0	4.0	.33	350		160	190	5,000	12,000	
II–A	NPOX 1/4–21–6.0	6.0	.50	700		275	325	7,500	19,500	
III–B	NPOX 3/8–21–4.5	4.5	.38	350		150	280	5,000	15,500	
Heat Resistant Types										
I–A	HRL 3/16 GF 5.5	5.5	.46	725	500	270		25,000	13,500	
I–B	HRL 3/16 GF 7.0	7.0	.58	1200	600	360		30,000	17,000	
I–C	HRL 3/16 GF 9.0	9.0	.75	1750	700	480		36,000	22,000	
I–A	HRP 3/16 GF 5.5	5.5	.46	575	500	300		26,000	16,000	
I–B	HRP 3/16 GF 7.0	7.0	.58	1050	700	430		32,000	20,000	
I–C	HRP 3/16 GF 9.0	9.0	.75	1600	950	600		45,000	30,000	

Maximum Weight of Adhesive (Lbs./Ft.² Bonded Surface) } .120 .110 .105

Cell Size } 1/8 3/16 1/4

Specific gravity of plastic insert compound is 1.05 (.0389 lb./in.³)
Specific gravity of plastic edging compound is 1.10 (.0397 lb./in.³)

**Span parallel to W or L Direction, as applicable.

Table 11-5. Glass, Ceramics, Insulation

Glass

Corning Glass Code	Type	Color	Forms Usually Available	Thermal Expansion 10^{-7} in./in./°C. 0–300°C	Thermal Expansion 10^{-7} in./in./°C. Room Temp.-Setting Point	Upper working Temperatures (Mechanical Considerations Only) Annealed Normal Service °C.	Annealed Extreme Limit °C.	Tempered Normal Service °C.	Tempered Extreme Limit °C.	Density grams Per C.C.	Young's Modulus (10^6 lb./sq. in.)	Refractive Index Sod. D Line (.5893) Microns)
0010	Potash soda lead	Clear	T	93	100	110	380			2.86	8.9	1.539
0080	Soda lime	Clear	BMT	92	103	110	460	220	250	2.47	10.0	1.512
0120	Potash Soda Lead	Clear	TM	89	98	110	380			3.05	8.6	1.560
1720	Aluminosilicate	Clear	BT	42	52	200	650	400	450	2.52	12.7	1.530
1723	Aluminosilicate	Clear	ST	46	54	200	650	400	450	2.64	12.5	1.547
1990	Potash Soda Lead	Clear	–	124	136	100	310	–	–	3.47	8.4	–
2405	Borosilicate	Red	BPU	43	51	200	480	–	–	2.50	9.9	1.507
2475	Soda Zinc	Red	T	93		110	440	–	–	2.59	10.0	1.511
3320	Borosilicate	Canary	–	40	43	200	480	–	–	2.27	9.4	1.481
6720	Soda Zinc	Opal	P	80	92	110	480	220	275	2.58	10.2	1.507
6750	Soda barium	Opal	BPR	88	–	110	420	220	220	2.59	–	1.513
6810	Soda zinc	Opal	BPR	69	–	120	470	240	270	2.65	–	1.508
7040	Borosilicate	Clear	BT	48	54	200	430			2.24	8.6	1.480
7050	Borosilicate	Clear	T	46	51	200	440	235	235	2.24	8.7	1.479
7052	Borosilicate	Clear	BMPT	46	53	200	420	210	210	2.28	8.2	1.484
7056	Borosilicate	Clear	BTP	51	57	200	460	–	–	2.29	9.2	1.487
7070	Borosilicate	Clear	BMPT	32	39	230	430	230	230	2.13	7.4	1.469
7250	Borosilicate	Clear	P	36	38	230	460	260	260	2.24	9.2	1.475
7570	High Lead	Clear	–	84	92	100	300	–	–	5.42	8.0	–
7720	Borosilicate	Clear	BPT	36	43	230	460	260	260	2.35	9.1	1.487
7740	Borosilicate	Clear	BPSTU	33	35	230	490	260	290	2.23	9.1	1.474
7760	Borosilicate	Clear	BP	34	37	230	450	250	250	2.23	9.1	1.473
7900[1]	96% Silica	Clear	BPTUM	8	7	800	1100	–	–	2.18	10.0	1.458
7913	96% Silica	Clear	BPRST	8	7	900	1200	–	–	2.18	9.6	1.458
7940	Fused Silica	Clear	U	5.5	7	900	1100	–	–	2.20	10.5	1.459
8160	Potash Soda Lead	Clear	PT	91	100	110	380	–	–	2.98	–	1.553
8161	Potash Lead	Clear	PT	90	97	110	390	–	–	4.00	7.8	1.659
8363	High Lead	Clear	LC	104	112	100	200	–	–	6.22	7.4	1.97
8871	Potash Lead	Clear	–	102	113	125	300	–	–	3.84	8.4	–
9010	Potash soda barium	Grey	P	89	102	110	380	–	–	2.64	9.8	1.507
9700	Borosilicate	Clear	TU	37	39	220	500	–	–	2.26	9.6	1.478
9741	Borosilicate	Clear	BUT	39	49	200	390	–	–	2.16	7.2	1.468

B – Blown ware P – Pressed Ware U – Panels LC – Large Castings
M – Multiform R – Rolled Sheet T – Tubing and Rod S – Plate Glass

Insulation

Designation	Specification	Lb./Cu.In.	K*	Max. Temp. °F
Fiberglass Batt, Phenolic Bonded "AA" Fiber Dia.	MIL–B–5924 Type I	.000347	.019	350
Fiberglass Batt, Unbonded, "AA" Fiber Dia.		.000289 .001736	.025 .022	1000 1000
Fiberglass Batt, Phenolic Bonded "B" Fiber Dia.		.000289 .000579 .001157	.022 .020 .019	350 350 350
Fiberglass Batt Unbonded, "B" Fiber Dia.		.000289 .000579 .001736	.026 .025 .022	1000 1000 1000
Refrasil, A–100 H.I. Thompson Fiber Glass Co.		.001736	.028	1600
MIN–K, 1301 Johns-Manville Co.		.011570	.015	1300
Micro Quartz (Q-Felt), LOF Glass Fibers Co.		.001736	.023	2000
Thermoflex Johns-Manville Co.		.001736 .003472 .004629 .006944	.024 .021 .028 .027	2000 2000 2000 2000
Fiberfrax XSW (Blanket)		.003472	.020	2300
Fiberfrax, Ceramic Fiberboard		.014470	.020	2000

*K = Btu-ft/hr/sq.ft./°F

Ceramic Materials

Material	Lb./Cu.In.	Melting Point Temp. °F	Coef. of Expan. °C x 10^{-6}
Alumina	.107	3660	8.1
Beryllium Oxide	.104		9.3
Boron Carbide	.091	4440	4.5
Chromic Oxide	.188	3614	7.0–12.0
Graphite	.058	6332	1.5
Magnesia	.090	5072	14.3
Niafrax A	.100		6.7
Niafrax (Lt.wt.)	.083		
Silicon Carbide	.102	3990	4.5
Thoria	.265	5522	9.4
Titanium Carbide	.154	5748	
Zircon	.120	4532	5.1
Zirconia	.159	4892	11.0
Zirconium Boride	.220		5.5
Zirconium Oxide	.199		8.3

Table 11-6. Plastics

Type	Lb. Cu.In.	Hardness Rockwell	Tens. Str. Psi (10^3)	Flexural Yield Str. Psi (10^3)	Impact, Izod Ft-Lb In. 73°	Elastic Modulus (10^5)	Water Absorpt %	Coeff. of Therm. Exp. per Degree F (10^{-5})	Heat Dist. Temp. °F	Refract. Index
Thermosets										
Alkyds										
Putty	.076	Barcol 60–70	4–5	8–11	3–4	20–27	.1	1–3	350	
Rope			4–6	12–20	2–4	20–25				
Granular	.080	Barcol 60–70	3–5	6–10	3–.4	16–26	.1	1–3	350	
Allylics										
Diallyl phthalate	.046	M114–116		7–9	.2–.3	5	0–.2			1.57
Diallyl isophthalate	.046	M119–121	4 3	7.4–8.3	.2–.3	5	1			1.57
Amines										
Urea										
Alpha Cellulose	.054	E94–97	5.5–7	11–18	2–.3	14–15	.4–.8	4–6.5		
Melamine										
Alpha cellulose	.054	E110	7–8	10–15	3	13	.1–.4	3.6–10.3		
Rag	.054	E100	8–10	12–15	.6–.9	14	.3–.6	4.5–5.4		
Asbestos	.064	E90	5.5–6.5	7.4–10	.3–.4	18	.1–.2	3.8–7.7		
Glass Fiber	.071		5.9	13–24	.5–6.0	24	.1–.3	2.2–4.5		
Unfilled	.053	E110		9 5–13	.1	10	.2–.4	7.7–8.5		
Epoxies										
Molding compounds										
Glass fiber filler	.068	M100–108	14–30	20–26	8–15		.1	400–500		
Mineral filler	.066	M101	5–7	10–15	3–.5		.1	250–450		
Casting resins										
Unfilled	.045	M80–110	4–13	13.3–21	2–1		.1–.2	115–550		
Silica filler	.065	M85–120	7–13	8–14	3–.5		.04–.1	160–550		
Phenolic										
General purpose	.050	M110–116	7	10	.3		.6		300	
Nonbleeding	.050	M107–110	7.3	9.8	.3		.9		325	
Heat resistant	.058	M112	6	9	.3		.3		300–360	
Impact	.050	M110–113	6.7	10.7	.7		.9		300	
Electrical	.056	M100–115	5.6	9.6	.4		.1–.7		240–300	
Special purpose	.050	M107–114	6	9.1	.4		.4–1.0		300	
Polyesters										
Type: Rigid	.045	Barcol 45–55	7 5–11	10–17	.2–.5	6–7		8.8–10.8	185–392	
Resilient	.041	Barcol 20–45	6–12	15–24	.4–.6	4 5–6		8.8–18	158–185	
Flexible	.040	Barcol 0–20	5–2	0–2	6–2.5					
Silicones										
Molding compound										
Fibrous, glass filler	.068	M87	6.5	19	10	25	2			
Granular, silica filler	.067	M90	3.5	8	3	16	.1			
1/8 in. glass laminates										
Hi pressure general	.061	Barcol 58	20	43			.1			
Low pressure 30–200 psi	.070	Barcol 73	41	50			1			
Solventless rigid										
Cured – low viscosity	.042		3.5	7.6	.2			12.5		
Cured – interm. viscosity	.043		2.8	5.9	2			15		
Flexible type										
Dielectric gel	.035						1			1.40
Clear	.038	Shore A 40					.1			1.43
Filled	.045	Shore A 45					1			
Foams										
Premixed powders	.0086						2.3			
Expanded and cured	.0020						.3			
Urethanes										
Flexible foam										
Polyether	.0010									
Polyester	.0013									
Rigid foam										
Polyether	.0010						3			
Polyester	.0010									

Table 11-6. Plastics (Continued)

Type	Lb. Cu.In.	Hardness Rockwell	Tens. Str. Psi (10^3)	Flexural Yield Str. Psi (10^3)	Impact, Izod Ft-Lb./In. 73°	Elastic Modulus (10^5)	Water Absorpt. %	Coeff. of Therm. Exp. per Degree F (10^{-5})	Heat Dist. Temp. °F	Refract. Index
Thermoplastics										
ABS Resins										
Molding Gr. general	.038	R104	6.4	10	3–5	3.2	.3–.4	17.1	192	
Sheet extr. grade	.037	R88	5	7.5	5–7	2.4	.3–.4	18.7	186	
Acetal Resins										
Homopolymer	.051	M94	10	14.1	1.4	4.1	.25	4.5	255	
Copolymer	.051	M80	8	13	1.2	3.75	.25	4.7	230	
Acrylic Resins										
Implex A (Mold.)	.040	M20	5.6	8.5	2.3	2.25	.3	5.6	169	
Plexiglass G (Cast sheet)	.043	M93	10.5	16	.4	4.5	.2	3.9	205	1.49
Plexiglass H (Mold)	.042	M100	11	13	.3	4.7	.2		228	1.52
Cellulose Resins										
Cellulose acetate	.046	R40–120	1.9–7.8	2.3–11.6	.4–5.8	.86–4	1.9–4.5	8–16	111–165	1.48
Cellulose propionate	.044	R10–116	1.6–7.8	3.2–11.4	.5–11.5	1.5–3.4	1.2–2.3	11–16	120–161	1.47
Ethyl cellulose	.040	R106	4.8	6.8	3	2.5	1.0–1.7	5		
Butyrate	.043	R31–116	2.6–6.9	1.8–9.2	.8–6.3	.55–1.85	.9–2.2	6.1–9.4		1.48
Chlorinated Polyether	.051	R100	10	5	.4	1.3	Negl.	5–8.2	210	
Fluorocarbons										
CTFE (Mod. cryst.)	.077	S85	5.4	10.7	3.1	2.54			350	
FEP	.077	R25	1.8		No break	.95	<.01	3.2–5.2	158	
TFE	.078	R58	1.3		3	.90	<.01	4.7–5.5	250	
Polyamides (Nylon)										
Type 6	.041	R119	11.8		1		1.6	4.6	365	
Type 6/6	.041	R118	11.8		.9		1.5	4.5	360	
Type 6/10	.039	R111	8.2		1.2		.4	5	300	
Type 11	.038	A55	8.5		3.3		.4	10	154	
Glass reinforc. 6/6 (40%)	.055	E75–80	30		3.7		.6	.9	509	
MoS_2 filled (2½%)	.042	R110–125	10–14				.5–1.4	3.5	400	
Polymerized	.042	R112–120	11–14		.9			5	400	
Polycarbonates	.043	M70, R118	9–10.5		12–16		.35	3.9	283–290	1.59
Polyethylenes										
Type I	.033		1.7				<.1	9.5		
Type II	.034		1.5				<.1	8		
Type III	.035		2.2		.7–3.0		<.1	7		
Polyimides										
Polymer SP–1	.051		13.5			4.5			~473	
H film	.051		25							
Mylar	.051		23							
Polypropylenes	.033	R90–95	5	6	.7	1.8	.01	6	240	
Polystyrenes	.039	M65–80	4–7	10–17	.2–.4	4–5	.2			1.59
Vinyls										
Elastomeric	.048	Shore A 70	1.6							
Semirigid	.048	Shore D 48	3.4							
Rigid	.051	Shore D 80	7	12.5	.7	4.3	.04	3.5		

Table 11-7. Rubber Compounds

			Common Name	Natural Rubber	Synthetic Rubber	SBR	Butyl	Poly-butadiene	EP Rubber	Neoprene	NBR	Poly-sulfide	Poly-urethane	Silicone	Hypalon	Acrylics	Fluoro-elastomer
			Weight – lb/cu.in.	.033	.033	.034	.033	.034	.031	.044	.036	.048	.039	.036	.040	.040	.05–.07
Physical Properties			Durometer, range	30–100	40–80	40–100	30–100	45–80	30–90	40–95	20–90	20–80	55–100	25–80	50–95	40–90	65–90
			Resilience	O	O	G	F	O	G	E	G	F	G to E	P to E	G	G	F
			Tensile strength, psi (reinforc.)	4000+	2–3000	2000+	2000+	2500	2–3000	3000+	1–3500	500–1500	4–8000	600–1500	15–2500	1700	15–3000
			Elongation, % (reinforced)	500	3–700	450	3–800	450	500	650–850	4–600	2–550	250–800	90–800	250–500	450	1–450
			Drift, room temperature	E	G	E	F	G	F	F to G	G	P	G to E	F to E	F	F	G
			Compression set	G	F	G	F	F	F	F to G	G	P to F	E	G to E	F to G	F to G	G to E
			Electrical resistivity	E	E	E	E	E	E	F	P	F	G	E	G	F	G
			Impermeability, gas	G	G	F	O	G	G	G	G	E	G	F	E	G	E
Resistance Properties	Mechanical	Resistance to:	Impact	E	E	E	G	G	G	G	F	P	E	P to F	F to G	P	P to G
			Abrasion	E	E	E	G	E	G	G to E	E	P to F	E	P to G	G	F to G	G
			Tear	E	G	F	G	G	P	G	G	P to F	E	P to G	F to G	F	P to G
			Cut growth	E	E	G	E	F	G	G	G	P	F to E	P to G	F to G	G	P to G
	Temperature	Tens. str. psi, at:	250°F	1800	1800	1200	1000	1200	2000	1500	700	700	1800	850	500	1300	300–800
			400°F	125	125	170	350	170	400	180	130	<25	200	400	200	225	150–300
		Elongation, %, at:	250°F	500	500	250	250	250	3–500	350	120	140	300	350	60	400	100–350
			400°F	80	80	60	80	60	0–120	0–100	20	<25	140	200	20	150	50–160
		Drift at 212°F		G	G	G	F	G	F	F to G	E	P	E	E	F	F	G to E
		Heat aging at 212°F		G	F	G	E	F	E	G	G	G	F to G	E	E	E	O
		Flame resistance		P	P	P	P	P	P	G	P to F	P	P to F	F to G	G	P to F	E
		Low temperature	Stiffening, F	–20 to –50	–20 to –50	0 to –50	–10 to –40	–30 to –60	–20 to –50	+10 to –20	+30 to –20	–10 to –45	–10 to –30	–60 to –160	–30 to –50	+35 to +10	+20 to –30
			Brittle point, F	–80	–80	–80	–80	–100	–90	–45	–65	–60	–60	–90 to –180	–60	–10	+10 to –60
	Environmental	Weather		F	F	F	E	F	E	E	G	E	E	E	E	E	E
		Oxidation		G	E	G	E	E	G	G	F to G	E	E	E	E	E	O
		Ozone		P	P	P	E	P	E	E	P	E	E	E	O	E	O
		Radiation		F to G	F to G	G	P	P	P	F to G	F to G	F to G	G	F to E	F to G	P to G	F to G
		Water		E	E	E	E	E	G to E	G	E	G	G	G	G	F	G
		Acid		F to G	F to G	F to G	E	F to G	G to E	G	G	F	P to F	P to G	E	F	G to E
		Alkali		F to G	F to G	F to G	E	F to G	G to E	G	F to G	G	P to F	P to F	E	P	P to G
		Gasoline, kerosens, etc.		P	P	P	P	P	P	G	E	E	E	P to F	F	E	E
		Benzol, toluol, etc.		P	P	P	F to G	P	F	P	G	E	F to G	P	P to F	P	E
		Degreaser solvents		P	P	P	P	P	P	P	P	F to G	F to G	P to G	P to F	P	G
		Alcohol		G	G	F	E	F to G	P	F	E	G	G	G	G	P	E
		Synthetic lubricants		P to F	P to F	P	F	P to F	P to F	P	F to G	G	P	P to F	P	G	F to G
		Hydraulic fluids:	Silicates	P	P	P to F	F	P	F to G	G	F	P to F	–	P	G	G	G
			Phosphates	P to F	P to F	P	G	P to F	G to E	P	P	P to F	P	G	P to F	P	P

Ratings are in this decreasing order: O = Outstanding, E = Excellent, G = Good, F = Fair, P = Poor.

Table 11-8. Fluids Gases

Fluids

Description	Std. Density		Description	Std. Density		Description	Std. Density	
	Lb./Cu.In.	Lb./Gal		Lb./Cu.In.	Lb./Gal		Lb./Cu.In.	Lb./Gal.
Acetic acid	.03830	8.85	Hydraulic fluid MIL-O-5606	.03030	7.00	Oil – MIL-O-6082 Gr. 1120	.03190	7.37
Alcohol – ethyl	.02857	6.60	Hydraulic fluid – skydrol			Oil – MIL-O-6082 Gr. 1055	.03158	7.30
Alcohol – isopropyl	.02861	6.61	Jet fuel – JP-1	.02944	6.80	Oil – MIL-O-6082 Gr. 1010	.03129	7.23
Alcohol – methyl	.02922	6.75	Jet fuel – JP-3	.02814	6.50			
Ammonia	.03215	7.43	Jet fuel – JP-4	.02814	6.50	Olive oil	.03324	7.68
Benzine	.02493	5.76	Jet fuel – JP-5	.02944	6.80	Palm oil	.03505	8.10
Benzol	.03117	7.20	Jet fuel – kerosene	.03030	7.00	Petroleum oil	.02963	6.84
Carbolic acid	.03468	8.01	Jet oil – MIL-L-7808	.03333	7.70	Phosphoric acid	.06431	14.86
Carbon disulphate	.04552	10.52	Jet oil – MIL-L-23699	.03333	7.70	Rape oil	.03324	7.68
Carbon tetrocloride	.05758	13.30	Linseed oil	.03396	7.84	Sulphuric acid	.06648	15.36
Castor oil	.03723	8.60	Mineral oil	.03324	7.68	Tar	.03613	8.33
Cotton seed oil	.03360	7.76	Muriatic acid	.04335	10.01	Turpentine oil	.03143	7.26
Ether	.02601	6.00	Naptha	.02746	6.34	Vinegar	.03902	9.01
Ethylene glycol	.04043	9.34	Nitric acid	.04408	10.18	Water – pure	.03613	8.33
Fluoric acid	.05419	12.52	Oil – recir Eng.	.03247	7.50	Water – sea	.03701	8.55
Gasoline –all grades	.02597	6.00						
Glycerine	.04502	10.40						

Gases

Gas	Symbol	Approx. Mol. Wt.	Weight in Lb. of 1 Cu.Ft. at Std. Atmos. Pressure and 68°F	Density Relative to Air	Gas Constant, R, Ft-Lb Per Lb, Deg. R	Specific Heat Per Lb. at R.T. Btu Per Lb, Deg. F		Heat Capacity Per Cu.Ft. at Std. Atmos. Pressure and 68°F, Btu Per Cu.Ft., Deg. F		$k = C_p/C_v$
						C_p	C_v	C_p	C_v	
Helium	He	4.0	0.01039	0.138	386.30	1.250	0.7540	0.0130	0.0078	1.66
Argon	A	40.0	0.10370	1.377	38.70	0.124	0.0743	0.0129	0.0077	1.67
Air		29.0	0.07528	1.000	53.30	0.241	0.1725	0.0181	0.0130	1.40
Oxygen	O_2	32.0	0.08305	1.103	48.31	0.217	0.1549	0.0180	0.0129	1.40
Nitrogen	N_2	28.0	0.07274	0.966	55.16	0.247	0.1761	0.0179	0.0128	1.40
Hydrogen	H_2	2.0	0.005234	0.0695	766.80	3.420	2.4350	0.0179	0.0127	1.40
Nitric oxide	NO	30.0	0.07788	1.034	51.52	0.231	0.1648	0.0180	0.0128	1.40
Carbon monoxide	CO	28.0	0.07269	0.965	55.19	0.243	0.1721	0.0177	0.0125	1.41
Hydrochloric acid	HCl	36.5	0.09460	1.256	42.41	0.191	0.1365	0.0181	0.0129	1.40
Steam	H_2O	18.0		0.623	85.81	0.460	0.3600			1.28
Carbon dioxide	CO_2	44.0	0.11420	1.516	35.13	0.205	0.1599	0.0234	0.0183	1.28
Nitrous oxide	N_2O	44.0	0.11430	1.518	35.12	0.221	0.1759	0.0253	0.0201	1.26
Sulphur dioxide	SO_2	64.0	0.16630	2.208	24.13	0.154	0.1230	0.0256	0.0204	1.25
Ammonia	NH_3	17.0	0.04420	0.587	90.77	0.523	0.4064	0.0231	0.0179	1.29
Acetylene	C_2H_2	26.0	0.06754	0.897	59.40	0.350	0.2737	0.0236	0.0185	1.28
Methyl chloride	CH_3Cl	50.5	0.13090	1.738	30.62	0.240	0.2006	0.0314	0.0263	1.20
Methane	CH_4	16.0	0.04163	0.553	96.37	0.593	0.4692	0.0247	0.0195	1.26
Ethylene	C_2H_4	28.0	0.07280	0.967	55.11	0.400	0.3292	0.0291	0.0240	1.22

Cryogenic Properties of Liquefied Gases

Gas	O_2 Oxygen	N_2 Nitrogen	A_2 Argon	CH_4 Methane	F_2 Fluorine	H_2 Hydrogen	He Helium	C_2H_4 Ethylene
Density 32°F lb./cu.ft. 1 atm.	.0892	.0781	.1113	.0448	.1069	.00561	.01114	.07868
Boiling point °F 1 atm.	–297.4	–320.4	–302.4	–258.7	–306.5	–423.0	–452.0 26 atm	–154.68
Melting point °F 1 atm.	–361.1	–345.8	–308.7	–299.2	–363.3	–434.6	–458.0	–272.47
Vapor density at N.B.P. lb./cu.ft.	.296	.288	.368	.1124	—	.0830	.999	.130
Liquid density at N.B.P. lb. cu.ft.	71.29	50.19	86.77	26.47	94.4	4.37	7.803	35.49
Heat of vaporization at N.B.P. BTU/lb.	91.6	85.7	70.0	248.4	73.7	194.4	10.3	207.9
Heat of vaporization at N.B.P. BTU/cu.ft.	6530	4301	6074	6575	6957	849.5	80.37	7378
Heat of fusion at N.M.P. BTU/lb.	5.9	11.0	12.1	26.1	5.8	25.2	1.8	—
Critical temperature °F	–181.1	–232.8	–188.5	–116.5	–200.2	–399.8	–450.2	49.82
Critical pressure psia	730.3	492.3	705.2	672.0	808.3	188.1	33.2	747.7

N.B.P. Normal boiling point
Temperature at which boiling occurs when pressure is one atmosphere.

N.M.P. Normal melting point
Temperature at which melting occurs when pressure is one atmosphere.

Heat of vaporization BTU/lb
Amount of heat added to change one pound of liquid to vapor at normal boiling point.

Heat of fusion BTU/lb
Amount of heat added to change one pound of solid to liquid at normal melting point.

Critical temperature
The temperature above which the substance cannot exist in the liquid state.

Critical pressure
The pressure of the saturated vapor at the critical temperature.

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Table 11-9. Wood

Strength Properties of Commercial Woods in the Air-Dry Condition (12 % Moisture Content)

Species	Specific Gravity (Oven-Dry Volume)	Static Bending: Modulus of Rupture, PSI	Static Bending: Modulus of Elasticity, 1,000 PSI	Static Bending: Work to Maximum Load, In.-Lb/Cu.In.	Compression Parallel to Grain: Max. Crushing Strength, PSI	Compression Perpendicular to Grain: Stress at Proportional Limit, PSI	Shear, PSI	Tension Perpendicular to Grain: Max. Tensile Strength, PSI	Hardness, lb.: End	Hardness, lb.: Side
Hardwoods:										
Alder, red	0.43	9,800	1,380	8.4	5,820	540	1,080	420	980	590
Ash, commercial white	0.62	14,600	1,680	15.6	7,280	1,510	1,920	850	1,680	1,260
Aspen, quaking	0.40	8,400	1,180	7.6	4,250	460	850	260	510	350
Balsa	0.13	2,100	420	–	1,250	70	245	115	170	80
Basswood, American	0.40	8,700	1,460	7.2	4,730	450	990	350	520	410
Beech	0.67	14,900	1,720	15.1	7,300	1,250	2,010	1,010	1,590	1,300
Birch, sweet and yellow	0.68	16,700	2,070	19.8	8,310	1,250	2,020	930	1,660	1,340
Birch, paper	0.60	12,300	1,590	16.0	5,690	740	1,210	–	890	910
Cherry, black	0.53	12,300	1,490	11.4	7,110	850	1,700	560	1,470	950
Cottonwood, eastern	0.43	8,500	1,370	7.4	4,910	470	930	580	580	430
Elm, American	0.55	11,800	1,340	13.0	5,520	850	1,510	660	1,110	830
Elm, rock	0.66	14,800	1,540	19.2	7,050	1,520	1,920	–	1,510	1,320
Greenheart	1.06	25,500	3,700	22.0	12,920	1,970	1,830	1,020	2,140	2,630
Hickory, pecan	0.69	16,300	1,780	18.8	8,280	2,040	2,080	–	1,930	1,820
Hickory, true	0.78	19,700	2,100	27.2	8,970	2,310	2,130			
Mahogany, American	0.51	11,460	1,500	7.5	6,780	1,090	1,230	740	970	800
Maple, red	0.55	13,400	1,640	12.5	6,540	1,240	1,850		1,430	950
Maple, sugar	0.68	15,800	1,830	16.5	7,830	1,810	2,330		1,840	1,450
Oak, red	0.66	14,400	1,810	15.0	6,920	1,260	1,830	820	1,490	1,300
Poplar, yellow	0.43	9,200	1,500	6.8	5,290	580	1,100	520	560	450
Sweetgum	0.53	11,900	1,490	11.3	5,800	860	1,610	800	950	690
Sycamore, American	0.54	10,000	1,420	8.5	5,380	860	1,470	720	920	770
Teak	0.62	13,770	1,670	9.3	7,520	1,190	1,360	980	1,010	1,100
Tupelo, black	0.55	9,600	1,200	6.2	5,520	1,150	1,340	500	1,240	810
Walnut, black	0.56	14,600	1,680	10.7	7,580	1,250	1,370	690	1,050	1,010
Softwoods:										
Cedar, incense	0.37	8,000	1,040	5.4	5,200	730	880	270	830	470
Cedar, Port Orford	0.44	11,300	1,730	9.1	6,470	760	1,080	400	730	560
Cedar, northern white	0.32	6,500	800	4.8	3,960	380	850	240	450	320
Cedar, eastern red	0.49	8,800	880	8.3	6,020	1,140	–	–	900	900
Cedar, western red	0.34	7,700	1,120	5.8	5,020	610	860	220	660	350
Cypress, southern	0.48	10,600	1,440	8.2	6,360	900	1,000	270	660	510
Douglas fir, coast type	0.51	12,200	1,950	9.8	7,430	870	1,160	340	900	710
Fir, balsam	0.41	7,600	1,230	5.1	4,530	380	710	180	510	400
Hemlock, western	0.44	10,100	1,490	7.5	6,210	680	1,170	310	940	580
Larch, western	0.61	13,900	1,960	12.6	8,110	980	1,410	430	1,120	830
Pine, eastern white	0.36	8,600	1,240	6.8	4,800	510	900	310	480	380
Pine, lodgepole	0.43	9,400	1,340	6.8	5,370	750	880	290	530	480
Pine, longleaf	0.62	14,700	1,990	11.8	8,440	1,190	1,500	470	920	870
Pine, shortleaf	0.54	12,800	1,760	11.0	7,070	1,000	1,310	470	750	690
Pine, western white	0.42	9,500	1,510	8.8	5,620	540	850	–	440	370
Redwood	0.42	10,000	1,340	6.9	6,150	860	940	240	790	480
Spruce, eastern	0.43	10,100	1,450	8.5	5,620	590	1,070	350	640	490
Tamarack	0.56	11,600	1,640	7.1	7,160	990	1,280	400	670	590

Plywood

Thickness	Number of Plies	Species of Face & Back: / Other Plies:	Weights – Lbs. Per Square Inch: Birch / Birch	Birch / Poplar	Mahogany / Poplar	Poplar / Poplar	Walnut / Walnut
3/64	3	The weights given for Poplar Plywood are for all practical purposes the same as for Spruce Plywood.	.00146	.00146	.00125	.00118	
1/16	3		.00208	.00174	.00146	.00139	
3/32	3		.00278	.00243	.00193	.00195	
1/8	3		.00361	.00299	.00241	.00243	.00330
5/32	3		.00417	.00347	.00299	.00278	
3/16	3		.00493	.00403	.00347	.00331	.00492
3/16	5			.00403	.00368	.00347	
7/32	5			.00514	.00451	.00431	
1/4	5			.00556	.00505	.00527	.00604
5/16	5			.00660	.00583	.00632	.00763
3/8	5			.00729	.00682	.00646	
7/16	7			.00903	.00826	.00799	
1/2	7			.01021	.00944	.00917	
9/16	7			.01125	.01021	.01015	.01503
5/8	9			.01361	.01144	.01104	.01701
3/4	9			.01632	.01396	.01306	.01854

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Table 11-10. Electrical Wiring

Wire Size – Weight Pounds Per 1000 Inches																			
30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	1	0	00	000	0000
MS 17331 – Asbestos Reinforced TFE – 200°C – MIL–W–22759																			
				.458	.642	.883	1.08	1.71	2.40	3.67	5.83								
MS 17332 – Asbestos Reinforced TFE – 260°C – MIL–W–22759																			
				.458	.642	.883	1.08	1.71	2.40	3.67	5.83								
MS 17410 – Fibrous Glass TFE – 200°C – MIL–W–22759																			
				.667	.833	1.17	1.50	2.08	2.83	4.29	6.83	9.83	15.0	23.3	30.25	36.3	45.7		
MS 17411 – Type II Extruded TFE – 200°C – MIL–W–22759																			
			.500	.667	.833	1.17	1.50	2.08	2.83	4.33									
MS 17412 – Extruded TFE – 260°C – MIL–W–22759																			
			.500	.667	.833	1.17	1.54	2.08	2.83	4.33	6.50	9.83	15.0						
MS 18000 – Extruded TFE – 200°C – MIL–W–22759																			
			.358	.500	.675	.917	1.15	1.55	2.33	3.63	6.00	9.00	13.8						
MS 18001 – Extruded TFE – 260°C – MIL–W–22759																			
			.358	.500	.675	.917	1.15	1.55	2.33	3.63	6.00	9.00	13.8						
MS 18032 – Fused Laminate TFE – 200°C – MIL–W–22759																			
.067	.088	.129	.188	.275	.400	.650	.792	1.19	1.88	3.00	4.88	7.76	12.4	19.1					
MS 18033 – Fused Laminate TFE – 260°C – MIL–W–22759																			
.067	.088	.129	.188	.275	.400	.650	.792	1.19	1.88	3.00	4.88	7.76	12.4	19.1					
MS 21985 – Extruded TFE, 600 Volt – 200°C – MIL–W–22759																			
	.107	.152	.210	.294	.435	.641	.785												
MS 21986 – Extruded TFE, 600 Volt – 260°C – MIL–W–22759																			
	.107	.152	.210	.294	.435	.641	.785												
MS 24284 – 600 Volt Copper – – MIL–W–27300																			
		.166	.228	.321	.469	.687	.709	1.46	2.31										
MS 25190 – 600 Volt Copper – 105°C – MIL–W–5086 Type I																			
				.367	.525	.717	.891	1.37	2.06										
MS 25190 – 600 Volt Copper – 105°C – MIL–W–5086 Type II																			
				.392	.567	.791	.991	1.52	2.16	3.67	5.83	9.17	13.7	20.8	25.4	33.3	41.6	50.8	65.4
MS 25190 – 600 Volt Copper – 105°C – MIL–W–5086 Type III																			
				.475	.633	.916	1.17	1.67	2.36	3.67	5.83	9.43	14.2	20.8	25.8	33.3	41.6	50.8	65.4
MS 25191 – 600 Volt Aluminum – – MIL–W–7072																			
											2.67	4.58	6.16	8.92	11.1	13.9	16.9	20.2	24.7
MS 25471 – 600 Volt Copper – 200°C – MIL–W–8777																			
				.484	.650	.900	1.12	1.67	2.42	3.75	6.00	8.92	13.7	21.8	26.4	32.5	41.6		
MS 27110 – 600 Volt Copper – 200°C – MIL–W–8777																			
				.591	.750	1.01	1.19	1.79	2.54	4.00	6.25	9.5	14.4						
MS 27125 – High Temperature & Fire Resistant – MIL–W–25038																			
				.833	1.00	1.25	1.58	2.08	2.92	4.13	7.07	10.6	16.0	24.2	28.9	34.6	43.3	54.0	66.1
MIL–W–7139B Class I – Glass Reinforced TFE – 200°C																			
				.575	.729	1.09	1.47	1.74	2.72	3.86	5.82	9.85							
MIL–W–7139B Class II – Glass Reinforced TFE – 260°C																			
				.575	.729	1.09	1.47	1.74	2.72	3.86	5.82	9.85							
MIL–W–16878 Type B – 105°C – 600 Volts																			
.067	.094	.125	.183	.258	.392	.592	.760	.925											
MIL–W–16878 Type C – 105°C – 1000 Volts																			
		.221	.258	342	.484	.700	.863	1.21	1.92										
MIL–W–16878 Type D – 105°C – 3000 Volts																			
			.416	.508	.675	.750	1.12	1.55	2.42	3.40	5.95	8.92	13.8	21.4	26.1	30.8			
MIL–W–16878 Type E and Type K – 200°C – 600 Volts																			
.084	.108	.141	.202	.282	.421	.628	.808	1.25	1.88	2.78									
MIL–W–16878 Type EE and Type KK – 200°C – 1000 Volts																			
.125	.158	.153	.267	.354	.501	.715	.916	1.37	2.02	2.96	5.39								
MIL–C–27500 Type V Shielded and Jacketed – 200°C – 600 Volt – Single Conductor																			
				1.47	1.67	2.08	2.65	3.44	4.42										
MIL–C–27500 Type V Shielded and Jacketed – 200°C – 600 Volt – Two Conductor																			
				3.02	3.62	4.47	5.52	6.84	9.34										
MIL–C–27500 Type V Shielded and Jacketed – 200°C – 600 Volt – Three Conductor																			
				3 86	4 46	5.78	7 05	9 56	12 5										
MIL–C–27500 Type V Shielded and Jacketed – 200°C – 600 Volt – Four Conductor																			
				4.58	5.45	7.15	9.38	12.1	15.8										
MIL–C–27500 Type V Shielded and Jacketed – 200°C – 600 Volt – Seven Conductor																			
				6.97	8.33	11.5	15.6	21.1	28.0										

Table 11-11. Cable

Coaxial Cable

Weight in Pounds Per Inch

RG-	Wt.	RG-	Wt.	RG-	Wt.	RG-	Wt.	RG-	Wt.	RG-	Wt.	RG-	Wt.
-1/U		-27/U	.0253	-59B/U	.00292	-94/U	.0156	-138/U	.006	-180/U	.00148	-228/U	.0500
-2/U		-27A/U	.0253	-60/U		-94A/U	.0206	-139/U	.007	-180A/U	.00155	-229/U	Cancel
-3/U		-28/U	.0308	-61/U		-95/U	.037	-140/U	.00375	-181/U	.00089	-230/U	
-4/U	.00208	-28A/U	.0308	-62/U	.00300	-96/U	.015	-141/U	.00292	-182/U		-231/U	.0129
-5/U	.00717	-28B/U	.0308	-62A/U	.00292	-97/U	.013	-141A/U	.00292	-183/U		-232/U	.0483
-5A/U	.00692	-29/U	.00183	-62/3	.00250	-98/U	.009	-142/U	.00375	-184/U		-233/U	.0900
-5B/U	.00692	-30/U	Obsole	-62C/U	.00333	-99/U	.008	-142A/U	.00375	-185/U		-234/U	.2592
-6/U	.00667	-31/U	.00883	-63/U	.00733	-100/U		-143/U	.00850	-186/U		-235/U	.0130
-6A/U	.00667	-32/U	Obsole	-63A/U	Obsole	-101/U		-143A/U	.00850	-187/U	.00100	-236/U	.0138
-7/U	.00667	-33/U	.0325	-63B/U	.00700	-102/U		-144/U	.0100	-188/U	.00093	-237/U	.0167
-8/U	.00875	-34/U	.0191	-64/U	.0171	-103/U	.159	-145/U		-189/U		-238/U	
-8A/U	.00858	-34A/U	.0187	-64A/U	.0171	-104/U	.341	-146/U		-190/U	.0294	-239/U	
-9/U	.0107	-34B/U	.0187	-65/U	.00800	-105/U	.106	-147/U		-191/U	.122	-240/U	.0775
-9A/U	.0107	-35/U	.0382	-65A/U	.00800	-106/U	.027	-148/U		-192/U		-241/U	
-9B/U	.0106	-35A/U	.0378	-66/U	.021	-107/U	.008	-149/U	.00767	-193/U		-242/U	.2250
-10/U	.0108	-35B/U	.0378	-67/U	.014	-108/U	.00258	-150/U	.00967	-194/U		-243/U	
-10A/U	.0108	-36/U	.0671	-68/U	.022	-108A/U	.00242	-151/U		-195/U	.00174	-244/U	.00983
-11/U	.00767	-37/U	.00333	-69/U	.511	-109/U	.207	-152/U		-196/U	.00050	-245/U	.0128
-11A/U	.00750	-38/U	.00917	-70/U		-110/U	.0821	-153/U		-197/U	.0417	-246/U	.0286
-12/U	.00967	-39/U	.00833	-71/U	.00358	-111/U	.0121	-154/U		-198/U	.0725	-247/U	.0353
-12A/U	.00950	-40/U	.0125	-71A/U	.00358	-111A/U	.0121	-155/U		-199/U	.0371	-248/U	.0790
-13/U	.0101	-41/U	.0125	-71B/U	.00358	-112/U	.271	-156/U	.0176	-200/U	.0775	-249/U	.0915
-13A/U	.0101	-42/U	.00417	-72/U		-113/U	.084	-157/U	.0264	-201/U		-250/U	.1996
-14/U	.0172	-43/U	Obsole	-73/U		-114/U	.00725	-158/U	.0317	-202/U		-251/U	.2338
-14A/U	.0167	-44/U		-74/U	.0195	-114A/U	.00725	-159/U		-203/U		-252/U	.0146
-15/U	.0163	-45/U		-74A/U	.0197	-115/U	.0116	-160/U		-204/U		-253/U	.0158
-16/U	.0212	-46/U		-75/U	.069	-115A/U	.0133	-160A/U		-205/U		-254/U	.0575
-17/U	.0386	-47/U		-76/U		-116/U	.0187	-161/U	.00058	-206/U		-255/U	.0463
-17A/U	.0383	-48/U	.223	-77A/U		-117/U	.0375	-162/U		-207/U		-256/U	.0463
-17B/U	Cancel	-49/U	.118	-78A/U		-118/U	.0500	-163/U		-208/U		-257/U	.1000
-18/U	.0433	-50/U	.088	-79/U	.00950	-119/U	.0188	-164/U	.0408	-209/U	.0338	-258/U	.1117
-18A/U	.0429	-51/U	.072	-79A/U	Obsole	-120/U	.0235	-165/U	.0101	-210/U	.00333	-259/U	.00833
-19/U	.0621	-52/U	.045	-79B/U	.00925	-121/U	.005	-166/U	.0120	-211/U	.0375	-260/U	.00917
-19A/U	.0617	-53/U	.017	-80/U		-122/U	.00167	-167/U	.044	-212/U	.00692	-261/U	
-20/U	.0672	-54/U	Obsole	-81/U	.0143	-123/U		-168/U	.023	-213/U	.00858	-262/U	
-20A/U	.0667	-54A/U	.00333	-82/U	.0582	-124/U		-169/U	.017	-214/U	.0106	-263/U	.1833
-21/U	.00692	-55/U	.00300	-83/U	.0102	-125/U	.0150	-170/U	014	-215/U	.0106	-264/U	
-21A/U	.00667	-55A/U	.00300	-84A/U	.1104	-126/U	.00633	-171/U	.009	-216/U	.0101	-265/U	
-22/U	.00858	-55B/U	.00283	-85A/U	.2425	-127/U		-172/U	.005	-217/U	.0167	-266/U	
-22A/U	.00992	-56/U		-86/U		-128/U		-173/U	.003	-218/U	.0389	-267/U	
-22B/U	.0100	-57/U	.0207	-87/U	Obsole	-129/U		-174/U	.00006	-219/U	.0429	-268/U	
-23/U	.0317	-57A/U	.0201	-87A/U	.0145	-130/U	.0183	-175/U		-220/U	.0621	-269/U	
-23A/U	.0308	-58/U	.00225	-88A/U		-131/U	.0246	-176/U		-221/U	.0771	-270/U	
-24/U	.0358	-58A/U	.00225	-89/U	.0167	-132/U		-177/U	.0392	-222/U	.00700		
-24A/U	.0342	-58B/U	.00208	-90/U		-133/U	.00783	-178/U	.00049	-223/U	.00300		
-25/U	.0171	-58C/U	.00208	-91/U	.026	-134/U		-178A/U	.00045	-224/U	.0197		
-25A/U	.0171	-59/U	.00308	-92/U		-135/U	.007	-179/U	.00078	-225/U	.0145		
-26/U	.0158	-59A/U	.00292	-93/U	.0396	-136/U	.007	-179A/U	.00085	-226/U	.0206		
-26A/U	.0158.					-137/U	.007			-227/U	.0187	-316/U	.00100

Preformed Aircraft Strand and Cable Galvanized and Stainless Steel

Constructions, Breaking Strengths, Weights

6 x 19 1 WRC

Cable Diam. Inches	Galvanized MIL-W-1511A Breaking Strength Pounds	Stainless MIL-C-5424 Breaking Strength Pounds	Non-Mag. Stainless MIL-C-18735 Breaking Strength Pounds	Weight per 100 Feet in Pounds
7/16	17,600	16,300	14,900	35.60
1/2	22,800	22,800	19,300	45.80
9/16	28,500	28,500	24,300	59.00
5/8	35,000	35,000	30,100	71.50
3/4	49,600	49,600	42,900	105.20
7/8	66,500	66,500	58,000	143.00
1	85,400	85,400	75,200	187.00
1-1/8	106,400	106,400		240.00
1-1/4	129,400	129,400		290.00
3/8	153,600	153,600		330.00
1-1/2	180,500	180,500		420.00

7 x 19

Cable Diam. Inches	Galvanized MIL-W-1511A Breaking Strength Pounds	Stainless MIL-C-5424 Breaking Strength Pounds	Non-Mag. Stainless MIL-C-18735 Breaking Strength Pounds	Weight per 100 Feet in Pounds
1/8	2,000	1,760	1,300	2.90
5/32	2,800	2,400	2,000	4.50
3/16	4,200	3,700	2,900	6.50
7/32	5,600	5,000	3,800	8.60
1/4	7,000	6,400	4,900	11.00
9/32	8,000	7,800	6,100	13.90
5/16	9,800	9,000	7,600	17.30
11/32	12,500			20.70
3/8	14,400	12,000	11,000	24.30

Table 11-12. Fabrics

Fabric-Webbing

Specification Designation	Width Inches	Thickness In Inches	Break Strength In Lb.	Weight Oz. Per Linear Yd.
MIL-W-4088 D Webbing, Cotton				
Type I	.56	.03-.04	500	.28
II	1.00	.03-.04	600	.42
III	1.25	.03-.04	800	.52
IV	3.00	.03-.04	1,800	1.20
VI	1.72	.05-.06	2,200	1.20
VII	1.72	.08-.11	5,000	2.60
VIII	1.72	.065-.085	3,600	1.70
IX	3.00	.100-.125	8,200	4.20
X	1.72	.125-.145	8,700	3.70
XII	1.72	.03-.04	1,200	.85
XIII	1.72	.100-.120	6,000	2.90
XIV	.50	.08-.10	1,200	.80
XV	2.00	.04-.05	1,500	1.25
XVI	1.72	.070-.095	4,500	2.20
XVII	1.00	.055-.070	2,400	1.15
XVIII	1.00	.11-.16	6,000	2.05
XIX	1.75	.11-.13	10,000	3.40
XX	1.00	.190-.235	9,000	3.25
XXI	1.25	.065-.085	3,600	1.70
XXII	1.72	.090-.12	7,300	3.50
XXIII	1.125	.20-.30	12,000	3.70
XXIV	1.94	.055-.075	5,500	2.25
XXV	1.00	.070-.095	4,500	1.50
MIL-W-27265 Webbing, Nylon				
Type I	.56	.026-.04	500	.31
II	1.00	.026-.04	600	.46
III	1.25	.026-.04	800	.57
IV	3.00	.026-.04	1,800	1.32
VI	1.72	.044-.06	2,200	1.32
VII	1.72	.070-.11	5,000	2.86
VIII	1.72	.057-.085	3,600	1.87
IX	3.00	.088-.125	8,200	4.62
X	1.72	.110-.145	8,700	4.07
XII	1.72	.026-.04	1,200	.935
XIII	1.72	.088-.120	6,000	3.190
XIV	.50	.070-.10	1,200	.88
XV	2.00	.035-.05	1,500	1.38
XVI	1.72	.062-.095	4,500	2.42
XVII	1.00	.048-.070	2,400	1.27
XVIII	1.00	.097-.16	6,000	2.26
XIX	1.75	.097-.13	10,000	3.74
XX	1.00	.167-.235	9,000	3.58
XXI	1.25	.057-.085	3,600	1.87
XXII	1.72	.079-.12	7,300	3.85
XXIII	1.125	.176-.30	12,000	4.07
XXIV	1.94	.048-.075	5,500	2.48
XXV	1.00	.062-.095	4,500	1.65
MIL-W-25361 Webbing, Dacron				
Type I	1.72	.046-.065	3,400	1.65
II	1.72	.065-.085	5,500	2.10
III	1.72	.080-0.10	6,300	2.50
IV	3.0	.070-.095	8,200	3.50
MIL-W-5665 Webbing, Cotton				
Type I	.56	.040-.050	350	.40
II	1.00	.040-.050	575	.75
III	1.25	.040-.050	750	.90
IV	3.00	.050-.100	1,900	2.50
V	5.00	.050-.100	3,100	4.30
VI	1.75	.070-.090	1,800	2.10
VII	1.75	.140-.170	2,600	3.00
VIII	1.75	.070-.085	2,900	3.00
IX	3.00	.090-.115	4,500	4.65
X	1.75	.125-.135	5,000	3.35
XI	1.00	.090-.110	1,000	1.35
XII	1.75	.040-.050	1,000	1.25
XIII	1.75	.100-.120	3,400	3.40
XIV	3.00	.090-.120	5,200	4.95
XV	1.75	.125-.140	4,500	3.40
XVI	1.75	.095-.115	2,700	2.60
XVII	1.00	.075-.095	1,000	1.25
XVIII	2.50	.050-.060	1,250	1.40
MIL-W-530 Webbing, Cotton				
Type I	Light Weight	.375	50	.15
		.500	70	.20
		.625	85	.25
		.750	100	.30
		.875	115	.35
		1.000	130	.40
		1.125	145	.45
		1.25	165	.50
		1.50	190	.60
		2.00	250	.80

Fabric - Cotton Cloth

Commercial Designation	Specification	Breaking Strength in Lb. Warp	Breaking Strength in Lb. Filling	Weight Oz. Per Sq. Yd.
Cloth, cotton duck	CCC-C-419 Type I Hard texture ◆ Duck no. 12/0	800	430	48.00
	2/0	465	435	31.90
	1/0	450	405	30.31
	1	440	370	28.71
	2	420	345	27.12
	3	390	330	25.53
	4	375	300	23.93
	5	345	285	22.33
	6	335	250	20.74
	8	285	210	17.55
	10	245	160	14.35
	11	240	140	12.75
	12	195	120	11.16
	Medium texture ◆ Duck no. 1	425	435	28.71
	2	410	320	27.12
	3	370	315	25.53
	4	350	290	23.93
	5	315	285	22.33
	6	305	250	20.74
(Army duck)	CCC-C-419 Type III (Specify weight)	125	120	8.25
		160	110	9.85
		210	130	12.29
		235	175	14.77
		245	200	15.90
		315	200	18.48
Cloth, cotton airplane	MIL-C-5646	80	80	
Cloth, cotton duck (Fire, water & weather resistant)	CCC-D-746 Type I Duck no. 2	420	345	43.32
	4	375	300	38.28
	5	345	285	37.93
	6	335	250	35.24
	8	285	210	29.83
	10	245	160	24.40
	12	195	120	19.96
	CCC-D-746 Type II 8 oz.	125	120	14.40
	10 oz.	160	110	17.00
	12 oz.	210	130	20.40
	15 oz.	235	175	23.63

Fabric-Cotton, Coated

Commercial Designation	Specification	Thickness Inches	Breaking Strength Pounds/Inch Grip Warp	Breaking Strength Pounds/Inch Grip Filling	Weight Oz. Per Sq. Yd.
Coated cloth (Vinyl coated cotton)	MIL-C-10799C Type I Class 1		80	80	7.75
	Class 2		80	80	10.0
	Type II Class 1		160	110	14.5
	Class 2		130	85	14.5
	Class 3		125	120	12.0
	Class 4		210	130	18.0
	Class 5		235	175	22.0
Coated cloth (Vinyl coated cotton)	MIL-C-7642 Type I		130	90	14.0
	Type II		130	90	14.0
Coated cloth (Neoprene coated cotton)	AMS 3270 (Specify thickness)	.008	40	40	8.0
		.010	37	19	10.0
		.025	90	90	27.0
		.035	250	170	35.0
		.050	250	170	54.0

Felt

Drawing Callout: Commercial Designation	Drawing Callout: Specification	S.A.E. Felt Number	Properties: T.S. (PSI)	Properties: Splitting Resistance (Lbs/2 In. Width)	Properties: Lb. Per Cu.In.
Wool felt, (White back check)	CF-206 Type I Class 16RI	F-1	500	33	.012

Table 11-13. Miscellaneous Materials

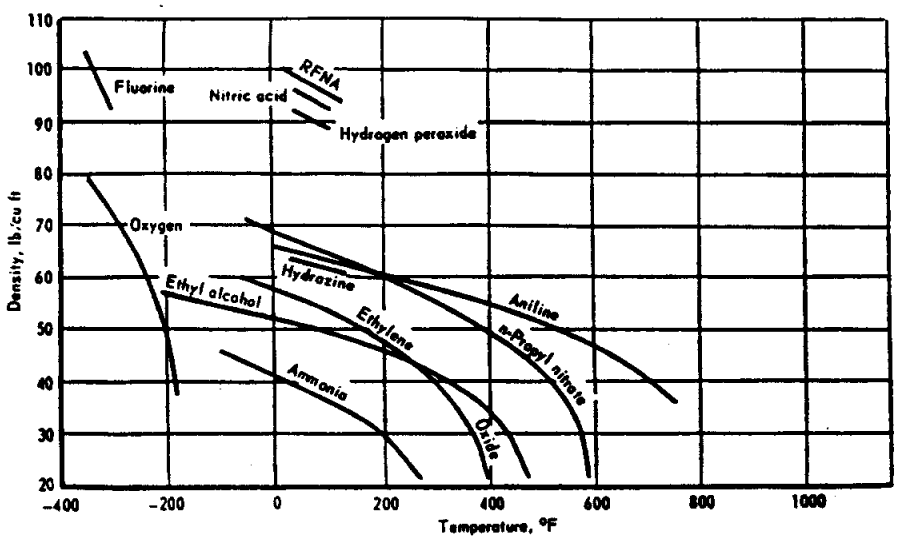
Substance	Weight, Pounds per Cu. Ft.
Metals, Alloys, Ores	
Aluminum, cast-hammered	165
Aluminum, bronze	481
Brass, cast-rolled	534
Bronze, 7.9 to 14% Sn	509
Copper, cast-rolled	556
Copper ore, pyrites	262
Gold, cast-hammered	1205
Iron, cast, pig	450
Iron, wrought	485
Iron, steel	490
Iron, spiegel-eisen	468
Iron, ferro-silicon	437
Iron ore, hematite	325
Iron ore, hematite in bank	160–180
Iron ore, hematite loose	130–160
Iron ore, limonite	237
Iron ore, magnetite	315
Iron slag	172
Lead	710
Lead ore, galena	465
Manganese	475
Manganese ore, pyrolusite	259
Mercury	849
Nickel	565
Nickel, monel metal	556
Platinum, cast-hammered	1330
Silver, cast-hammered	656
Tin, cast-hammered	459
Tin ore, cassiterite	418
Zinc, cast-rolled	440
Zinc ore, blende	253
Various Solids	
Cereals, oats, bulk	32
Cereals, barley, bulk	39
Cereals, corn, rye bulk	48
Cereals, wheat, bulk	48
Hay and straw, bales	20
Cotton flax, hemp	93
Fats	58
Flour, loose	28
Flour, pressed	47
Glass, common	156
Glass, plate or crown	161
Glass, crystal	184
Leather	59
Paper	58
Potatoes, piled	42
Rubber, caoutchouc	59
Rubber goods	94
Salt, granulated, piled	48
Saltpeter	67
Starch	96
Sulphur	125
Wool	82
Various Liquids	
Water, 4°C, max. density	62.428
Water, 100°C	59.830
Water, ice	56
Water, snow, fresh fallen	8
Water, sea water	64

Substance	Weight, Pounds per Cu. Ft.
Ashlar Masonry	
Granite, syenite, gneiss	165
Limestone, marble	160
Sandstone, bluestone	140
Mortar Rubble Masonry	
Granite, syenite, gneiss	155
Limestone, marble	150
Sandstone, bluestone	130
Dry Rubble Masonry	
Granite, syenite, gneiss	130
Limestone, marble	125
Sandstone, bluestone	110
Brick Masonry	
Pressed brick	140
Common brick	120
Soft brick	100
Concrete Masonry	
Cement, stone, sand	144
Cement, slag, etc.	130
Cement, cinder, etc.	100
Various Building Material	
Ashes, cinders	40–45
Cement, portland, loose	90
Cement, portland, set	183
Lime, gypsum, loose	53–64
Mortar, set	103
Slags, bank slag	67–72
Slags, bank screenings	98–117
Slags, machine slag	96
Slags, slag sand	49–55
Earth, etc., Excavated	
Clay, dry	63
Clay, damp, plastic	110
Clay and gravel, dry	100
Earth, dry, loose	76
Earth, dry, packed	95
Earth, moist, loose	78
Earth, moist, packed	96
Earth, mud, flowing	108
Earth, mud, packed	115
Riprap, limestone	80–85
Riprap, sandstone	90
Riprap, shale	105
Sand, gravel, dry, loose	90–105
Sand, gravel, dry, packed	100–120
Sand, gravel, dry, wet	118–120

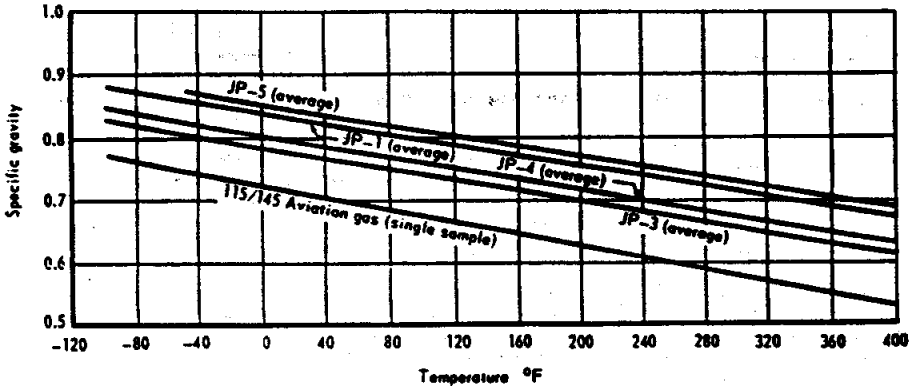
Substance	Weight, Pounds per Cu. Ft.
Excavation in Water	
Sand or gravel	60
Sand or gravel and clay	65
Clay	80
River mud	90
Soil	70
Stone riprap	65
Minerals	
Asbestos	153
Barytes	281
Basalt	184
Bauxite	159
Boron	109
Chalk	137
Clay, marl	137
Dolomite	181
Feldspar, orthoclase	159
Gneiss, serpentine	159
Granite, syenite	175
Greenstone, trap	187
Gypsum, alabaster	159
Hornblende	187
Limestone, marble	165
Magnesite	187
Phosphate rock, apatite	200
Porphyry	172
Pumice, natural	40
Quartz, flint	165
Sandstone, bluestone	147
Shale, slate	175
Soapstone, talc	169
Stone, Quarried, Piled	
Basalt, granite, gneiss	96
Limestone, marble, quartz	95
Sandstone	82
Shale	92
Greenstone, hornblende	107
Bituminous Substances	
Asphaltum	81
Coal, anthracite	97
Coal, bituminous	84
Coal, lignite	78
Coal, peat, turf, dry	47
Coal, charcoal, pine	23
Coal, charcoal, oak	33
Coal, coke	75
Graphite	131
Paraffine	56
Petroleum	54
Petroleum, refined	50
Petroleum, benzine	46
Petroleum, gasoline	42
Pitch	69
Tar, bituminous	75
Coal and Coke, Piled	
Coal, anthracite	47–58
Coal, bituminous, lignite	40–54
Coal, peat, turf	20–26
Coal, charcoal	10–14
Coal, coke	23–32

Table 11-14. Densities and Specific Gravities

Density of Some Rocket Propellants



Variation of Specific Gravity of Some Aircraft Fuels with Temperature



Variation of Specific Gravity of Some Aircraft Oils with Temperature

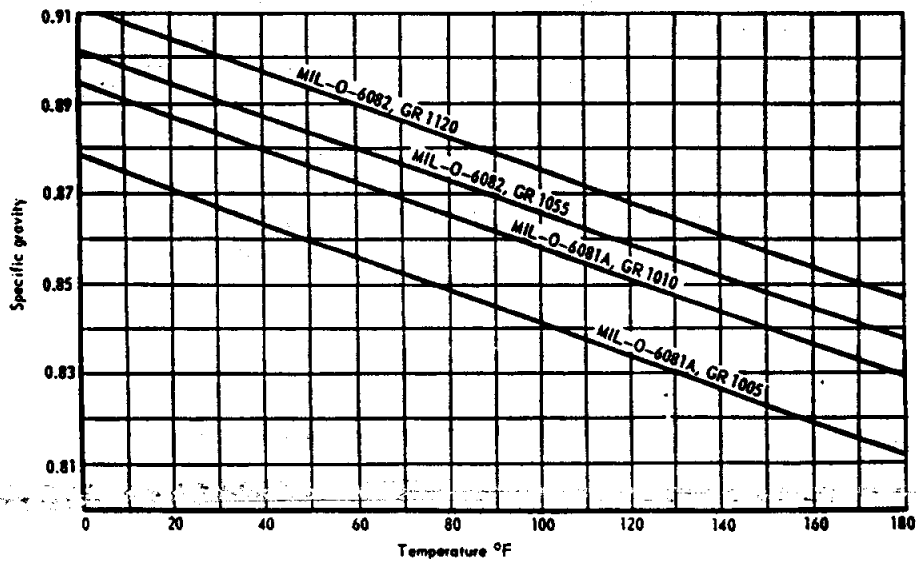


Table 11-15. Rocket Propellants

Name	Formula	Fr. Pt. °C	B. Pt. °C	Critical Temp. °C	Crit. Press. Atmosphere	Specific Gravity ($t^\circ/4^\circ C$) ③	Specific Heat g. cal/g °C ③	Thermal Conductivity g. cal/(sec)(cm²)(°C/cm) ③	Viscosity Millipoises ③
FUELS									
Acetylene	C_2H_2	− 81.5 ①	− 84	36	62	0.613 (−80)			
Aluminum Borohydride	$Al(BH_4)_3$	− 65	45			0.61 Approx.			
Ammonia	NH_3	− 77.7	− 33.4	132.9	112.3	0.817 (−79)	1.05 (−60)	1.188×10^{-8} (−15 to 30)	2.66 (−33.5)
Aniline	$C_6H_5NH_2$	− 6.2	184.4	426	52.4	1.022 (20)	0.512 (8–82)	0.408×10^{-8} (12)	29.3 (32)
Benzene	C_6H_6	5.5	80.1	288.5	47.7	0.879 (20)	0.419 (6–60)	0.333×10^{-8} (12)	5.61 (30)
Butyl Mercaptan	C_4H_9SH	−116	98			0.837 (25)			
Ethane	C_2H_6	−172	− 88.6	32.1	48.8	0.546 (−88)			3.01 (−130)
Ethyl Alcohol	C_2H_5OH	−112	78.4	243.1	63.1	0.789 (20)	0.680 (0 − 98)	3.369×10^{-8} (51) 0.369	11.94 (20)
Ethyl Alcohol − Water (75 − 25)	$CH_3OH + H_2O$	− 60	82			0.854 (20)	0.785 (41)	0.54×10^{-8} (25)	1.9 (25)
Ethyl Amine	$C_2H_5NH_2$	− 80.6	16.6	183.2	55.5	0.69 (15)			
Ethylene	C_2H_4	−169	−103.9	9.7	50.9	0.566 (−102)			
Ethylene Oxide	C_2H_4O	−111.3	10.5	192	71	0.887(7)	0.47(7)		2.8(10)
Ethyl Alc − Methyl Alc (95–5)	$C_2H_5OH + CH_3OH$					0.790(20)			
Ethyl Nitrate	$C_2H_5NO_3$	−102	88			1.100(20)			
Furfuryl Alcohol	$C_4H_3OCH_2OH$	− 32	170			1.129(25)	0.50(25)		46.2(25)
Gasoline (MIL−F−5572)	Mixture	− 60 max.	104 ev			0.71 ev.	0.53 approx.	0.32×10^{-8}(30)	3 to 6(25)
Heptane	C_7H_{16}	− 90.6	98.4	266.8		0.684(20)	0.507(0–50)	0.377×10^{-8}(4)	4.16(20)
Hydrazine	N_2H_4	1.4	113.5	380	145	1.01(15)	0.74(25)		9.8(25)
68% Hydrazine	$N_2H_4 + H_2O$	− 53	121			1.000(20)			
Unsymmetrical dimethyl hydrazine	$(CH_3)_2N-NH_2$	− 58	63	249	60	0.784(25)	0.653(25)	0.48×10^{-8}	5.1(25)
Hydrogen	H_2	−259.1	−252.7	−239.9	12.8	0.071(−253)	3.39(0)		0.130(−252.57)
Isopropyl Alcohol	C_3H_7OH	− 85.8	82.5	235	53	0.789 (20)		0.368×10^{-8} (0)	23.7 (20)
JP − 3 (Mil−F−5624A)	Mixture	− 60	315 ②			0.76 Av.			
JP − 4 (Mil−F−5624A)	Mixture	− 60 max.	121–315			0.78 Av.			
Kerosene	Mixture	− 37	202–279			0.82 Av.	0.50 (0–100)	0.335×10^{-8} (75)	16 (43)
Lithium	Li	186	1317			0.51 (200)	1.0 (200–1000)	0.09 (220)	5.6 (200)
Lithium Hydride	LiH	680	1375			0.820 (20)	1.07 (50)		
Methane	CH_4	−182.6	−161.4	− 82.5	45.8	0.38 (−164)	0.992 (−113)		1.42 (−170)
Methylal	$CH_2(OCH_3)_2$	−105	42	224		0.854 (20)	0.521 (15–41)		3.4 (15)
98% Methylal	$CH_2(OCH_3)_2 + H_2O$	−105	42+			0.866 (20)			
Methyl Alcohol	CH_3OH	− 97.8	64.7	240.0	78.7	0.792 (20)	0.601 (15–20)	0.51×10^{-8} (20)	5.93 (20)
80% Methyl Alcohol	$CH_3OH + H_2O$	− 51	65.5			0.829 (20)	0.709 (20)	0.64×19^{-8} (20)	9.9 Approx. (25)
Methyl Alc. − Hydrazine Hydrate − Water (57–32–11)	$CH_3OH + N_2H_4 \cdot H_2O + H_2O$	− 50	80			0.873 (20)			
Methyl Amine	CH_3NH_2	− 92.5	− 6.5	156.9	73.6	0.699 (−11)	0.77 (−50)		2.364 (0)
Nitro Methane	CH_3NO_2	−28.5	101			1.4 (20)	0.412 (17)	0.516×10^{-8} (30)	6.31 (25)

Table 11-15. Rocket Propellants (Continued)

Name	Formula	Fr. Pt. °C	B. Pt. °C	Critical Temp. °C	Crit. Press. Atmosphere	Specific Gravity (t°/4°C) ③	Specific Heat g. cal/g °C ③	Thermal Conductivity g. cal/(sec)(cm²)(°C/cm) ③	Viscosity Millipoises ③
n-Octane	C_8H_{18}	- 56.5	125.7	296.0	24.6	0.703 (20)	0.578 (20–123)	0.375×10^{-8} (4)	4.83 (30)
Propane	C_3H_8	-187.1	- 42.2	96.8	42.0	0.585 (-44.5)	0.576 (0)		4.33 (-100)
n-Propyl Nitrate	$C_3H_7NO_3$	<-100	110	307	40	1.06 (20)	0.42 (25)		6.9 (20)
o-Toluidine	$CH_3C_6H_4NH_2$	- 16.3	199.7			0.999 (20)	0.598 (22–195)		43.9 (20)
Triethylamine	$(C_2H_5)_3N$	-114.8	89.4	262.0	30.0	0.728 (20)			3.63 (25)
Trimethyl Trithiophosphite	$(CH_3S)_3P$	- 57	74			1.227 (20)			
Turpentine (α-Pinene)	$C_{10}H_{16}$	- 80	156–176			0.845 Av.	0.42 (18)	(-15)	12.7 (30)
2, 3-Xylidine	$(CH_3)_2C_6H_3NH_2$	>- 50	220			0.991 (15)			
OXIDIZERS									
Chlorine Trifluoride	ClF_3	- 76.3	11.3	153.5	32.3	1.77 (20)			
Fluorine	F_2	-220	-188	-129	55	1.51 (-188)	0.29 (-205.5)		2.6 (188)
Oxygen Bifluoride	OF_2	-224	-145	- 81	70	1.71 (-195)	0.21 (-145)		
Hydrogen Peroxide	H_2O_2	- 0.89	151	457	214	1.44 (20)	0.578 (0)		12.5 (20)
Nitric & Sulfuric Acid	88%HNO_3 + 12%H_2SO_4	- 50	189			1.62 (25)			59.3 (25)
Nitrogen Tetroxide	N_2O_4	- 9.3	21.3	158	99	1.45 (20)	0.36 (-10 to 20)		4.2 (20)
Nitrogen Trifluoride	NF_3	-208.5	-129	- 39	44.7	1.535 (-129)			
Nitrogen Pentoxide	N_2O_5	30	47			1.63 (18)	0.239(s)(-80 to -5)		
Oxygen (AN-O-1)	O_2	-218	-183	-118.8	49.7	1.14 (-183)	0.35 (-200 to -183)	0.150 (0)	8.7 (-219)
Ozone	O_3	-251	-112	- 5.0	67	1.71 (-183)			
RFNA (1 NO_2)	$HNO_3 + NO_2 + H_2O$	- 43	- NO_2			1.54 (25)			
RFNA (16 NO_2)	$HNO_3 + NO_2 + H_2O$	- 58	- NO_2			1.58 (25)	0.42 (25)	0.15 (25)	12.5 (25)
WFNA (100%)	HNO_3	- 43	86			1.50 (20)	0.42 (25)	0.16 (25)	9.6 (20)
Bromine Pentafluoride	BrF_5	- 61.3	40.5	197	53.4	2.45 (25)			
MONOPROPELLANTS									
Ethyl Nitrate	$C_2H_5NO_3$	-102	88			1.10 (25)			
Ethylene Oxide	C_2H_4O	-111.3	10.5	192	71	0.89 (7)	0.47 (7)		2.8 (10)
Hydrazine	N_2H_4	1.4	113.5	380	145	1.01 (15)	0.74 (25)		9.8 (10)
Nitromethane	CH_3NO_2	- 28.5	101.5			1.13 (25)	0.412 (17)	0.516×10^{-8} (30)	6.31 (25)
n-Propyl Nitrate	$C_3H_7NO_3$	<-100	110	307	40	1.06 (20)	0.42 (25)		6.9 (20
Hydrogen Peroxide	H_2O_2	- 0.89	151	457	214	1.44 (20)	0.578 (0)		13.07 (15.6)
Hydrogen Peroxide 90%	$H_2O_2 + H_2O$	- 6 .	141			1.40	0.62 (15.5)		13.00 (15.6)
H_2O_2 80%	$H_2O_2 + H_2O$	- 22				1.35 (15.6)			12.98 (15.6)
H_2O_2 70%	$H_2O_2 + H_2O$	- 37				1.29 (15.6)			12.90 (15.6)

① 891 mm. ② end point ③ temperatures in parenthesis in °C.

Table 11-16. Possible Propellant Combinations

Oxidizer	Fuel	O/F Ratio	$T_{combust}$ °F	I_{sp} 300 — 14.7 psi	Bulk Sp Gr.			
Chlorine Trifluoride	Ammonia*	3.93	2619	240	1.43			
	Hydrazine*	0.677	5840	251	1.22			
	Methyl Alcohol*	2.88	5150	230 (S.E.)	1.35			
Fluorine	Ammonia*	2.90	7512	295	1.017			
	Hydrazine*	1.98	7692	300	1.070			
	Hydrogen	9.42	8072	371	0.457			
		3.77	4469	356	0.269			
	Lithium	1.83	8319	309	0.785			
	Methyl Alcohol*	2.37	7472	298	0.991			
70% Fluorine 30% Oxygen	JP-4	3.78	7370	278	1.21			
90% Hydrogen Peroxide:	JP-4	7.75	4508	234.5 (S.E.)	1.26			
	uns-Dimethyl-hydrazine*	4.40	4490	239.5 (S.E.)	1.21			
	Hydrazine*	2.04	4350	242.6 (S.E.)	1.235			
Hydrogen Peroxide	Aluminum Boronhydride*	3.0	5772	200	0.82			
Mixed Oxides of Nitrogen (24% NO)	uns-Dimethyl-hydrazine*	2.5	5642	255 (S.E.)	1.17			
	Cyclohexane		4150	234				
	Hydrazine*	1.69	4200	240	1.22			
	Methyl Alc-Hydrazine Hyd.* -H_2O(57-32-11)	2.54	3934	224	1.196			
	Nitromethane	0.42	4719	232	1.200			
	n-Octane	5.10	4095	230	1.205			
Nitrogen Tetroxide	Ammonia	2.029	4627	237 (S.E.)	1.155			
	Aniline*	3.87	5742	221	1.34			
	Benzene	4.418	5598	214	1.201			
	Carbon	3.833	5852	210	1.533			
	Ethylamine*	4.096	5538	229	1.205			
	Hydrazine*	1.00	4905	249	1.193			
	Hydrogen	11.5	5610	279	0.565			
	Isopropyl Alc.	3.06	4773	224	1.204			
	JP-3	3.00	5472	229	1.19			
	n-Octane	3.00	4870	228	1.19			
	Propane	4.15	5121	240	1.128			
	Turpentine	4.7	5542	221	1.305			
	UDMH	2.25	5288	249.5 (S.E.)	1.143			
	Xylidine*	3.00	5470	223	1.302			
Nitrogen Trifluoride: Oxygen	Ammonia	4.2	6128	277 (S.E.)	1.24			
	Acetylene	1.23	6012	266	0.828			
	Aluminum Borohydride	1.32	6000	276	0.634			
	Ammonia	1.25	4834	250	0.969			
	Ethane	2.30	5308	254	0.867			
	Ethyl Alcohol	1.50	5297	242	0.970			
	75% Ethyl Alc.	1.31	4887	234	0.984			
	Ethyl Alc. Methyl Alc (95-5)	0.84		240	0.922			
	Ethylene	1.86	5538	264	0.773			
	Gasoline	2.26	5660	252	0.955			
	Hydrazine	0.83	5382	263	1.065			
	68% Hydrazine	1.00	4572	242	1.07			
	Hydrogen (Z)	2.89	3886	345	0.23			
	Isopropyl Alc.	1.85	5553	241	0.985			
	Kerosene	2.28	5702	249	1.02			
	Lithium*	1.15	13000	318	0.747			
	JP-4	2.4	5737	263 (S.E.)	1.005			
	Lithium Borohydride*	1.47	8300	306				
	Lithium hydride*	1.34	6400	268	0.975			
	Methane	2.33	4874	263	0.747			
	Methylal	1.052	5103	239	0.985			

Table 11-16. Possible Propellant Combinations (Continued)

Oxidizer	Fuel	O/F Ratio	$T_{combust}$ °F	I_{sp} 300 – 14.7 psi	Bulk Sp Gr
Oxygen	Methyl Alcohol	1.15	5076	237	0.943
	80% Methyl Alc.	1.50	4881	228	0.99
	Methylamine	2.06	5600	252	1.07
	Nitromethane	0.076	4703	226	1.13
	n-Octane	2.20	5498	248	0.975
Oxygen	Ammonia	2.08	6042	292 (S.E.)	1.08
Difluoride:	Hydrazine	1.35	6285	299 (S.E.)	1.24
Oxygen	Hydrazine	1.33	6290	298 (S.E.)	1.28
Bifluoride	Ammonia	2.13	6065	292 (S.E.)	1.12
Ozone	Ammonia	1.13	5175	267	1.13
	Hydrazine	0.63	5418	277	1.20
	Hydrogen	2.65	4280	373	0.23
	JP–4	2.2	6327	286 (S.E.)	1.193
Perchloryl-	Hydrazine*	1.30	5516	262 (S.E.)	1.30
fluoride:	JP–4	3.85	6092	249 (S.E.)	1.34
	uns-Dimethyl-hydrazine	2.45		254.5 (S.E.)	1.27
Red Fuming	Aniline-Furfuryl				
Nitric Acid	Alc* (80–20)	3.35		228	1.39
(6.5% NO_2)					
	Hydrazine*	1.00	4437	240	1.22
	68% Hydrazine*	0.494	3519	211	1.136
	Isopriopyl Alc.	3.33	4797	218.1	1.252
	n-Octane	4.56	5005	222	1.265
RFNA (14% NO_2)	Ammonia	2.20	4202	231.5 (S.E.)	1.11
	Diethylenetriamine*	3.61	4908	234 (S.E.)	1.37
	Hydrazine*	1.40	4796	246 (S.E.)	1.267
	JP–4	4.65	5012	231.5 (S.E.)	1.31
	Turpentine (α pinene)*	4.4	5113	231 (S.E.)	1.353
	Toluene	4.1	5130	227 (S.E.)	1.345
	uns-Dimethyl-hydrazine	2.7	4920	239 (S.E.)	1.23
RFNA	Aniline*	3.00	5067	221	1.39
(16% NO_2)	Ethyl Alcohol	2.50	4555	219	1.23
	Hydrazine*	1.16	4728	242	1.25
	o-Toluidine*	3.00	4905	219	1.38
WFNA	Aniline*	3.00	4942	222	1.355
	Butyl Mercaptan*	4.20	5980	234	1.30
	Cyclohexane	3.22	4113	217	1.23
	Furfuryl Alcohol*	2.65	4885	210	1.38
	Gasoline	4.60	4941	223	1.25
	Hydrazine*	1.22	4681	246 (S.E.)	1.23
	Hydrogen	12.6	5360	298	0.60
	JP–4	4.7	5000	229	1.30
	Methyl Alcohol	2.36	4480	219	1.19
	Methyl Alc. Furfuryl Alc* (50–50)	2.52	4699	213	1.30
	Octane	4.00	4744	229	1.225

NOTES: 1. Mixture ratio given is for the data presented and is not necessarily optimum or stoichiometric value.
2. Specific impulse figures are theoretical values calculated, assuming frozen composition.
3. Asterisk denotes that propellant combination is hypergolic.
4. S.E. (performance calculated on basis of shifting equilibrium).

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Section 12
CONVERSION TECHNIQUES

It is frequently necessary to convert between various acoustical and dynamical relationships. This section contains common and useful conversions and conversion techniques including Sound Pressure Level conversion for varied pressure units; decibel ratio conversions for units of power, voltage, and current; decibel additions; calculation of rms g's from power spectral density spectra; velocity amplitude relationships for vibration; equivalent sinusoidal signal and loading for random excitations; the relationships between sinusoidal sweep, and resonance dwell and amplification factors; and wave length versus frequency for air.

Where pertinent, the theory related to the specific conversion technique is given. Specific applications for various conversions are not included.

12-1 CONVERSION OF SOUND PRESSURE LEVELS

See Figure 12-1.

12-2 TABULATED DATA CONVERSIONS

See Table 12-1.

The conversion listing of some useful acoustics constants is as follows:

1 in.	= 2.54 cm	1 slug	= 14.6188 Kgm
1 cm	= 0.3937 in.	1 lb	= $4.448\ (10)^5$ dynes
1 ft	= 30.48 cm	1 newton	= 10^5 dynes
1 ft	= 0.3048 meter	1 dyne/cm^2	= 1 μ bar
1 cm	= $3.281\ (10)^{-2}$ ft	1 dyne/cm^2	= $14.5\ (10)^{-6}$ lb/in.^{2}
1 meter	= 100 cm	1 lb/in.^{2}	= $6.895\ (10)^4$ dynes/cm^2
1 dyne	= $2.248\ (10)^{-6}$ lb		

12-3 DECIBEL/VOLTAGE CONVERSION

$$N_{dB} = 10 \text{ LOG}_{10} \frac{P_1}{P_2} = 20 \text{ LOG } \frac{E_1}{E_2}$$

where:

N_{dB} is the number of decibels corresponding to the ratio between two amounts of power, P_1 or P_2 or two voltages, E_1 or E_2.

Values outside the range of either Table 12-2 or 12-3 can be found by following these simple rules:

a. Reference Table 12-2

Number of decibels positive (+): Subtract +20 decibels successively from the given number of decibels until the remainder falls within range of Table 12-2. To find the *voltage ratio*, multiply the corresponding value from the right-hand voltage-ratio column by 10 for each 20 dB subtracted. To find the *power ratio*, multiply the corresponding value from the right-hand power-ratio column by 100 for each 20 dB subtracted.

Example:

Given: 49.2 dB

49.2 dB − 20 dB − 20 dB = 9.2 dB

Voltage ratio: 9.2 dB → 2.884

2.884 x 10 x 10 = 288.4 → 49.2 dB

Power ratio: 9.2 dB → 8.318

8.318 x 100 x 100 = 83180 → 49.2 dB

b. Reference Table 12-3

For ratios smaller than those in tables: Multiply the given ratio by 10 successively until the product can be found in the table. From the number of decibels thus found, subtract +20 decibels for each multiplication by 10.

Example:

Given: Voltage ratio = 0.0131

0.0131 x 10 x 10 = 1.31

From Table 12-3, 1.31 → 2.345 dB

2.345 dB − 20 dB − 20 dB = −37.655 dB

Number of decibels negative (−): Add +20 decibels successively to the given number of decibels until the sum falls within the range of Table 12-2. For the *voltage*

ratio, divide the value from the left-hand voltage-ratio column by 10 for each 20 dB added. For the *power ratio*, divide the value from the lefthand power-ratio column by 100 for each 20 dB added.

Example:

Given: –49.2 dB

–49.2 dB + 20 dB + 20 dB = –9.2 dB

Voltage ratio: –9.2 dB → 0.3467

0.3467 x 1/10 x 1/10 = 0.003467 → –49.2 dB

Power ratio: –9.2 dB → 0.1202

0.1202 x 1/100 x 1/100 = 0.00001202 → –49.2 dB

For ratios greater than those in table: Divide the given ratio by 10 successively until the remainder can be found in the table. To the number of decibels thus found, add +20 dB for each division by 10.

Example:

Given: Voltage ratio = 712

712 x 1/10 x 1/10 = 7.12

From Table 12-3, 7.12 → 17.050 dB

17.050 dB + 20 dB + 20 dB = 57.050 dB

12-4 DECIBEL ADDITION TECHNIQUE

See Figure 12-2 for a simple technique for decibel addition. If only a few numbers of nearly equal magnitude are to be added, Figure 12-3 gives fast results.

12-5 RMS OF SPECTRAL DENSITY

To obtain the rms value for any arbitrary spectral density (composed of straight line segments on log-log paper), the relationships shown in Figure 12-4 are valid.

12-6 RANDOM TO SINE TEST EQUIVALENCE

The following derivation is based on Miner's hypothesis that fatigue damage accumulates according to:

$$F = \sum_{j=1}^{k} \frac{n_j}{N_j} \qquad (1)$$

where:

F = fatigue damage, $F = 1$ indicates fatigue

S_j = peak stress

n_j = number of cycles at S_j

N_j = number of cycles of fatigue ($F = 1$)

These definitions are valid for a stress of the form:

$S = S_j \sin \omega t$

Test data show that a plot of S vs N on log-log paper gives a straight line with a slope of $-1/\alpha$.

Therefore:

$N = S^{-\alpha}C$ or $S^{\alpha}N = C$ = constant, and

$$F = \sum_{j=1}^{k} \frac{n_j S_j^{\alpha}}{C} = \int_{n_i}^{n} S(n)^{\alpha} dn \qquad (2)$$

For a vibrating system: $S \propto g$ and $t \propto n$.

Substituting these values into equation 2 gives:

$$F \propto \int_{t_i}^{t_k} S(t)^{\alpha} dt \qquad (3)$$

The response of interest is random in nature and it is assumed that a Rayleigh distribution describes the distribution of the peaks or:

$$dt \propto dy = \frac{S}{\sigma^2} e^{-g^2/2\sigma^2} dg$$

Substituting this value for dt into equation 3 yields:

$$F \propto \int_{0}^{\infty} (g)^{\alpha} \frac{g}{\sigma^2} e^{-g^2/2\sigma^2} dg \qquad (4)$$

let $x = g^2/2\sigma^2$

$$F \propto (2\sigma^2)^{\alpha/2} \int_{0}^{\infty} \left(x^{\alpha/2} e^{-x} dx \right) = (2\sigma^2)^{\alpha/2} \Gamma\left(1 + \frac{\alpha}{2}\right)$$

An equivalent amount of fatigue would result from a response of the form $g = g_e \sin \omega t$ where:

$F \propto g_e$ and

$$g_e = \sigma \sqrt{2} \left[\Gamma\left(1 + \frac{\alpha}{2}\right) \right]^{1/\alpha}$$

See Figure 12-5 for the values of

$$\sigma\sqrt{2}\left[\Gamma\left(1+\frac{\alpha}{2}\right)\right]^{1/\alpha}$$

The value of σ can be expressed as:

$$\sigma = \left[\omega_o f(\omega_o)\left(\frac{\pi}{4\delta}\right)\frac{S_o^2\,\omega_o^4}{F_o^2}\right]^{1/2}$$

where:

$$\frac{S_o^2}{F_o^2} = \frac{\text{stress static}}{\text{force static}}$$

ω_o = 2π x natural frequency

δ = damping/critical damping

$f(\omega_o)$ = power spectral density in g^2/radian for the exciting force if $f(\omega)$ is reasonably flat in the region of resonance.

This gives:

$$g_e = \left[\omega_o f(\omega_o)\,\frac{\pi}{4\delta}\,\frac{S_o^2\,\omega_o^4}{F_o^2}\right]^{1/2}\sqrt{2}\left(\Gamma\left(1+\frac{\alpha}{2}\right)\right)^{\frac{1}{\alpha}} \quad (5)$$

The g level (g_s) for a sinusoidal exciting force that would give a response of g_e at resonance would be:

$$g_e = \frac{g_s\ S_o}{2\delta\ F_o}\,\omega_o^2 \quad (6)$$

Equating (5) and (6) and letting $Q = 1/2\delta$ and using the fact that $[\omega_o f(\omega_o)] = \text{freq}_o\ f(\text{freq}_o)$ gives:

$$g_s = \left(\pi\frac{\text{freq}_o}{Q}\right)^{1/2}\left(\Gamma\left(1+\frac{\alpha}{2}\right)\right)^{\frac{1}{\alpha}}\left(f(\text{freq}_o)\right)^{1/2} \quad (7)$$

The above expression for g_s, an equivalent sinusoidal input to produce the same fatigue results as the random input g_e, is comparable but more general than the expression 24.41, page 24-28 of Harris and Crede *Shock and Vibration Handbook*. The reason for its greater generality lies, of course, in the presence of the gamma term. If, as in the case of the Harris and Crede equation, a particular value is assumed for the gamma function (some value on Figure 12-5 for example) then a quantized coefficient results.

If the forcing function g_s were expressed in one-third octave band levels $\sigma_{1/3}$ (in g_{rms}) instead of spectral densities like $f(\text{freq})^{1/2}$ in g^2/Hz then

$$g_s = 3.68\sigma_{1/3}\ \left[\Gamma\left(1+\frac{\alpha}{2}\right)\right]^{1/\alpha}\ Q^{-1/2} \quad (8)$$

When values for Q and α are assumed, quantitative values for g_s may be obtained. A typical sample problem is worth discussing to illustrate the benefits of accelerated testing (note that one-third octave band levels are utilized in this particular example)

Assume that Q = 15 and α = 6 which yields:

$$g_s = \frac{3.68}{15^{1/2}}\ (3!)^{1/6}\ \sigma_{1/3} = 1.28\ \sigma_{1/3}$$

The required test duration per resonance at this equivalent level is 7.1 hours.

The time required to run such a test as a sweep test, sweeping from 5 to 2,000 Hz (equal time in each third octave), can be computed as:

$$7.1 \times \frac{620}{6.67} = 660 \text{ hours.}$$

This time can be reduced to 6.0 hours by increasing the excitation 2.19 times. This gives:

$$g_s = 2.8\ \sigma_{1/3}$$

This acceleration not only reduces the test time required but also reduces the dependence of the test level on α. Figure 12-5 represents this dependence for a test that is not accelerated.

An acceleration factor of 110 results in a test which is conservative as a function of α. This time acceleration yields a test time of 6.0 hours at a level g_s.

The natural scatter (in stress level) in an SN curve is taken into account by increasing $2.8\ \sigma_{1/3}$ to $3.0\ \sigma_{1/3}$. The final resulting test levels and durations are:

g_s = 3.0 σ (based on third-octave levels for the random excitation)

t = 6.0 hours

The sweep will be from 5 Hz to 2,000 Hz so that the sweep rate is proportional to the frequency. The sweep testing to be used is from 5 to 2,000 and back to 5 Hz in 1.5 hours.

12-6.1 References

1. Miner, M. A., *Cumulative Damage in Fatigue*, J. Appl. Mech Vol. 12. pp 159-164.1, 1945

2. Harris & Crede, *Shock and Vibration Hand Book*, Chapt. 24, McGraw-Hill

3. Miles, J. W., *One Structural Fatigue Under Random Loading*, Journal of Aeronautical Sciences, dated November 1954, pp 753-762.

12-7 VIBRATION NOMOGRAPH

See Figure 12-6.

12-8 SWEEP RATES TO DURATIONS AND EFFECT ON "Q"

See Figures 12-7 and 12-8.

12-9 WAVELENGTH IN AIR VERSUS FREQUENCY

See Figure 12-9.

12-10 DECIMAL EQUIVALENTS

See Table 12-4.

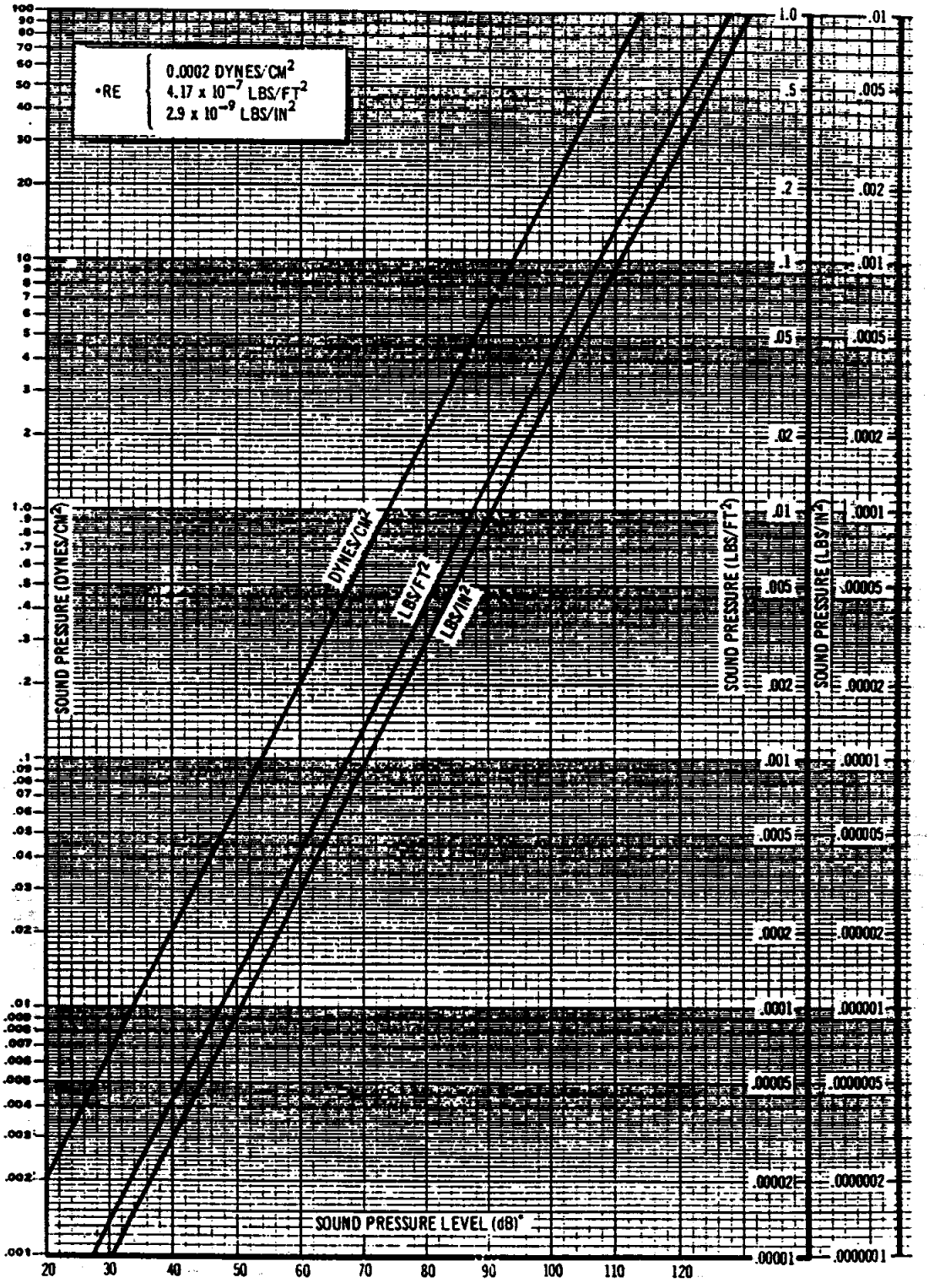


Figure 12-1. Conversion of SPL's in Sound Pressure in Dynes/CM^2, Lb/Ft^2, and Lbs/In^2 (Sheet 1 of 2)

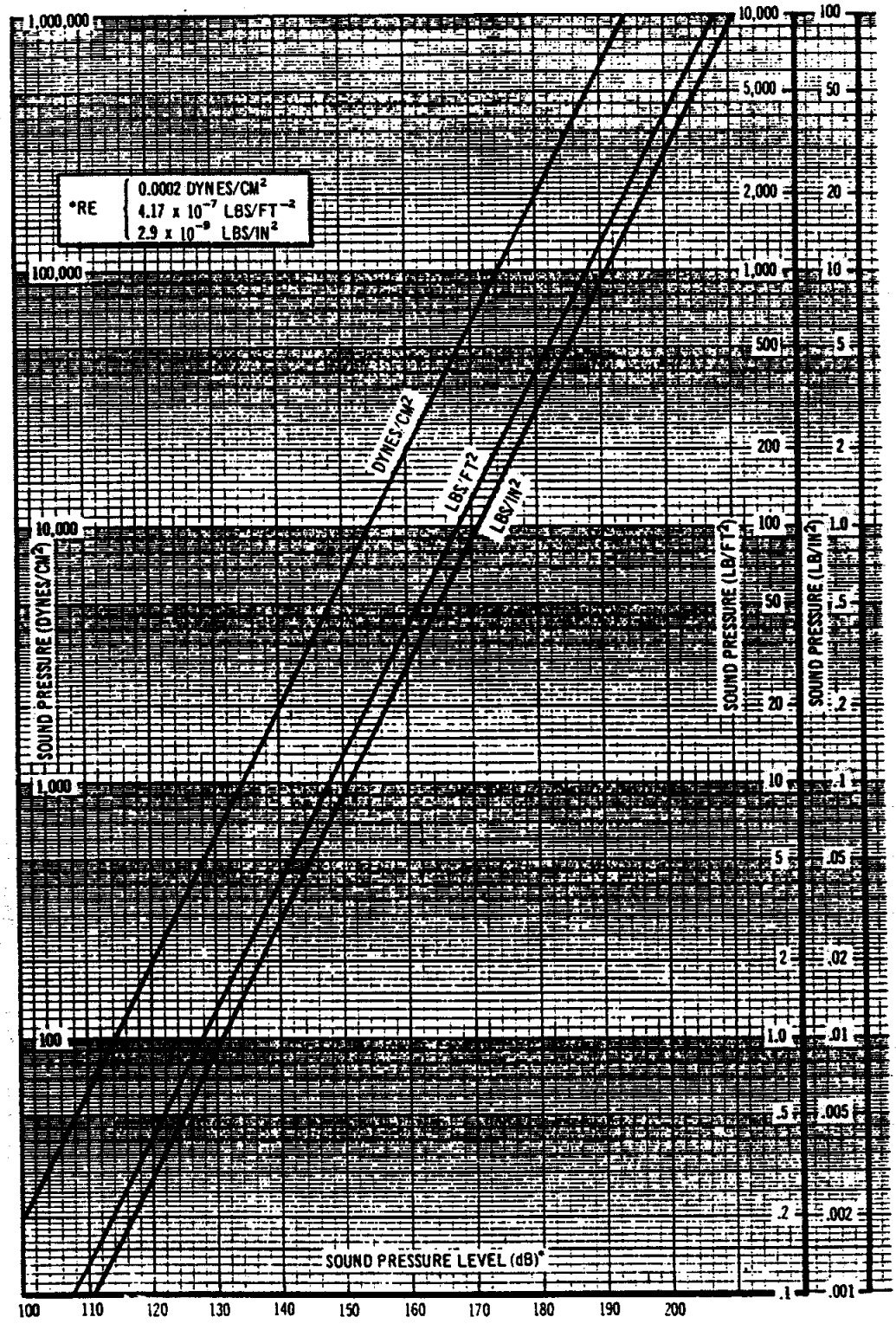


Figure 12-1. Conversion of SPL's in Sound Pressure in Dynes/CM², Lb/Ft², and Lbs/In² (Sheet 2 of 2)

1. MAKE 3 COLUMNS AND LABEL WITH THREE HIGHEST DECADES
2. USE ONLY LAST DIGIT OF dB TO OBTAIN "SUM" FROM TABLE
3. ADD "SUMS AND USE LEADING DIGIT PAIR OF ADDITION TO OBTAIN LAST DIGIT OF dB FROM TABLE, AND COLUMN HEADING LEADING DIGIT TO OBTAIN DECADE OF dB

EXAMPLE 1. FIND dB SUM OF 135, 137, 140, 145, 150, 152 dB'S

5	4	3
		3 2
		5 0
	1	0
	3	2
1	0	
1	6	
3	1	0 2

A. NEAREST dB FOR 3.1 IS 5
B. COLUMN HEADING OF 3 IS 5
C. ANSWER: 155 dB

EXAMPLE 2. FIND dB SUM OF 139, 149, 150½, 152½, 158, 159 dB'S

5	4	3
		8 0
	8	0
1	1	
1	8	
6	3	
8	0	
1 8	0	8 0

A. NEAREST dB FOR 1.8 is 2½
B. COLUMN HEADING OF 1 IS 6 (5 + 1)
C. ANSWER: 162½ dB

dB	SUM
	8.9
9	8.0
	7.1
8	6.3
	5.6
7	5.0
	4.5
6	4.0
	3.6
5	3.2
	2.8
4	2.5
	2.2
3	2.0
	1.8
2	1.6
	1.4
1	1.3
	1.1
0	1.0

Figure 12-2. Decibel Addition Technique

LIMITATIONS: USE OF THIS TABLE LIMITED TO THE TWO FOLLOWING CONDITIONS:

A. COMBINATIONS OF SINE WAVES NO TWO OF WHICH HAVE THE SAME FREQUENCY.

B. ANY COMBINATION OF RANDOM NOISE SOURCES, WITH OR WITHOUT COMBINATIONS OF SINE WAVES.

EXPLANATION OF TABLE: THE GROUP OF NUMBERS BENEATH THE BOLD NUMERAL AT THE TOP OF EACH BOX, REPRESENTS THE DIFFERENCE IN dB, BETWEEN ANY TWO SOUND PRESSURE LEVELS, L_1 & L_2. ($L_1 \geq L_2$)

THE VALUES IN THE RIGHT HAND COLUMN OF EACH BOX ARE THE NUMBER OF dB TO BE ADDED TO L_1 TO OBTAIN THE RESULTANT OF L_1 & L_2 IN dB.

0	
0.0	3.0
0.1	
0.2	2.9
0.3	
0.4	2.8
0.5	
0.6	2.7
0.7	
0.8	2.6
0.9	

1	
1.0	2.5
1.1	
1.2	2.4
1.3	
1.4	
1.5	2.3
1.6	
1.7	2.2
1.8	
1.9	

2	
2.0	2.1
2.1	
2.2	
2.3	2.0
2.4	
2.5	1.9
2.6	
2.7	
2.8	1.8
2.9	

3	
3.0	1.8
3.1	
3.2	1.7
3.3	
3.4	
3.5	1.6
3.6	
3.7	
3.8	1.5
3.9	

4	
4.0	1.5
4.1	
4.2	1.4
4.3	
4.4	
4.5	1.3
4.6	
4.7	
4.8	1.2
4.9	

5	
5.0	1.2
5.1	
5.2	
5.3	
5.4	1.1
5.5	
5.6	
5.7	
5.8	1.0
5.9	

6	
6.0	1.0
6.1	
6.2	
6.3	
6.4	0.9
6.5	
6.6	
6.7	
6.8	0.8
6.9	

7	
7.0	0.8
7.1	
7.2	
7.3	
7.4	
7.5	
7.6	0.7
7.7	
7.8	
7.9	

8	
8.0	0.6
8.1	
8.2	
8.3	
8.4	
8.5	
8.6	
8.7	0.5
8.8	
8.9	

9	
9.0	0.5
9.1	
9.2	
9.3	
9.4	
9.5	
9.6	0.4
9.7	
9.8	
9.9	

10	
10.0	0.4
10.1	
10.2	
10.3	
10.4	
10.5	
10.6	
10.7	
10.8	0.3
10.9	

11	
11.0	0.3
11.1	
11.2	
11.3	
11.4	
11.5	
11.6	
11.7	
11.8	
11.9	

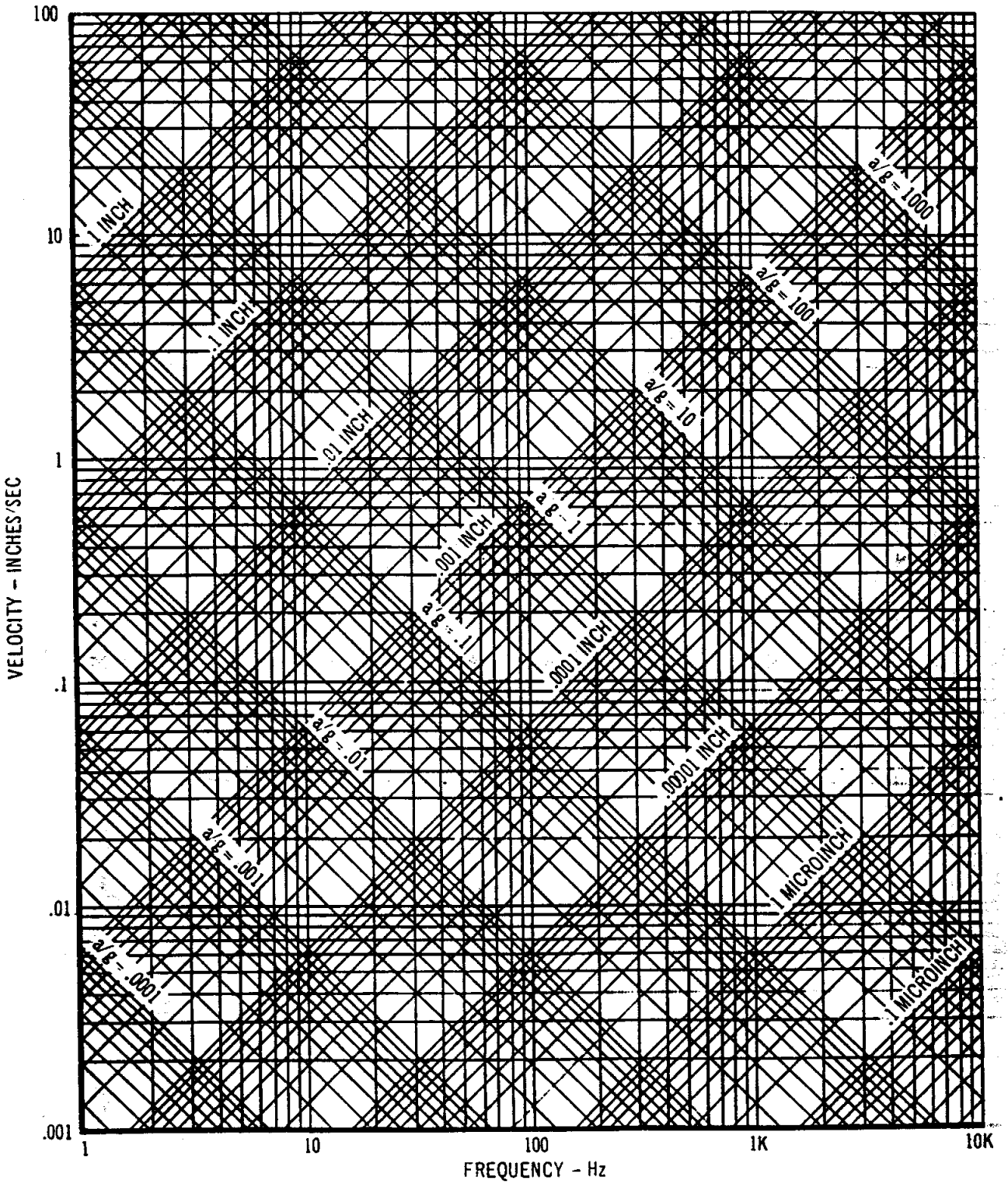
12	
12.0	0.3
12.1	
12.2	
12.3	
12.4	0.2
12.5	
12.6	
12.7	
12.8	
12.9	

13	
13.0	0.2
13.1	
13.2	
13.3	
13.4	
13.5	
13.6	
13.7	
13.8	
13.9	

14	
14.0	0.2
14.1	
14.2	
14.3	
14.4	
14.5	
14.6	0.1
14.7	
14.8	
14.9	

15	
15.0	0.1
15.1	
15.2	
15.3	
15.4	
15.5	
15.6	
15.7	
15.8	
15.9	

Figure 12-3. Combining Sound Pressure Levels in Decibels



NOTE: EITHER VECTOR (SINGLE AMPLITUDE) OR RMS QUANTITIES MUST BE USED CONSISTENTLY

EXAMPLE: GIVEN FREQUENCY = 100 Hz
AND DISPLACEMENT = 0.01 INCH (VECTOR)
THEN ACCELERATION = 10g (VECTOR)
AND, VELOCITY 6.2 IN. SEC (VECTOR)

Figure 12-6. Vibration Nomograph – Displacement to Velocity to Acceleration Relationships

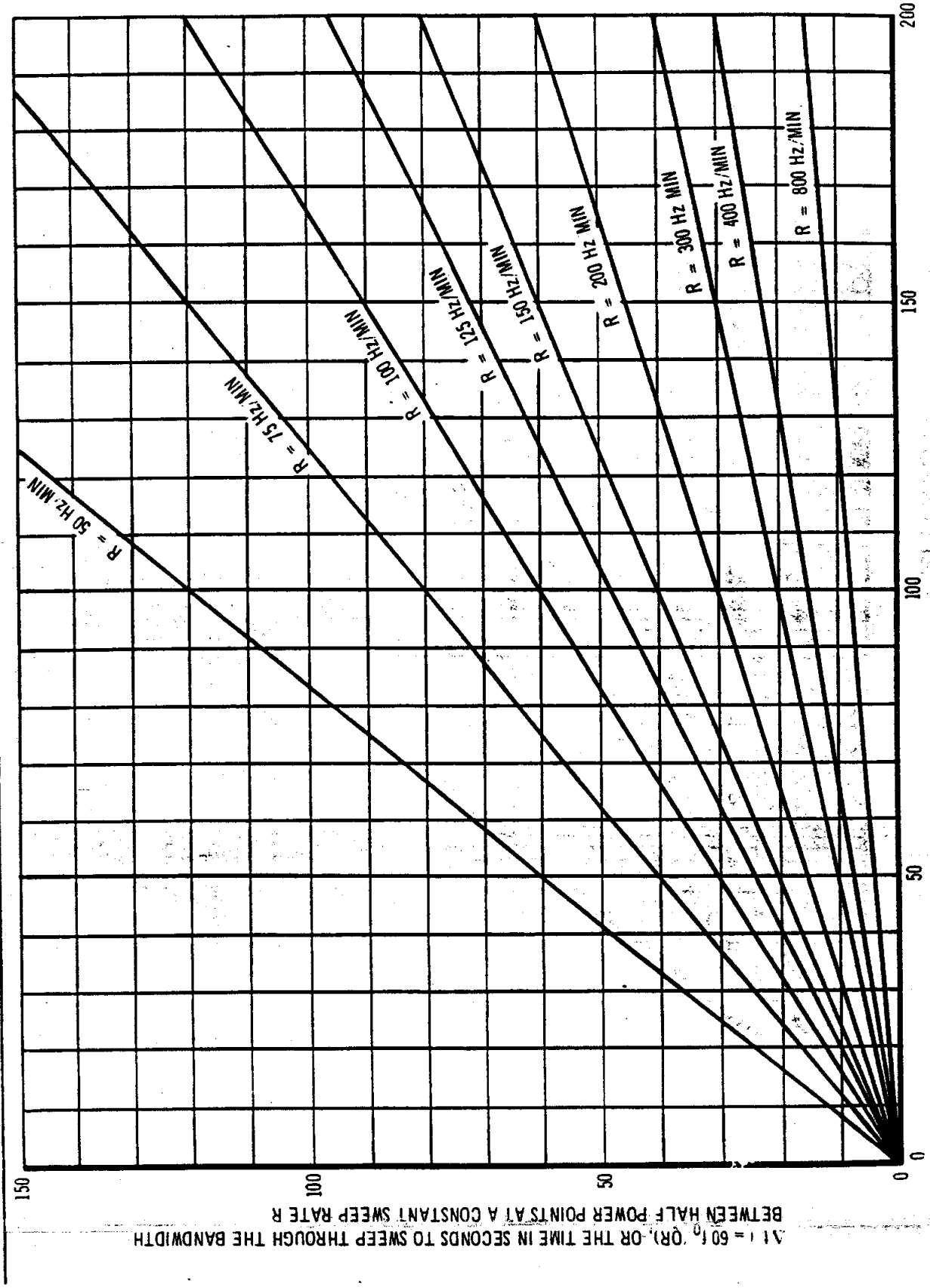
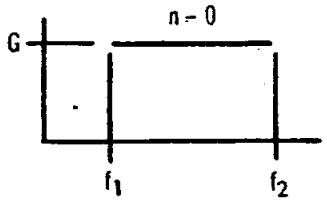
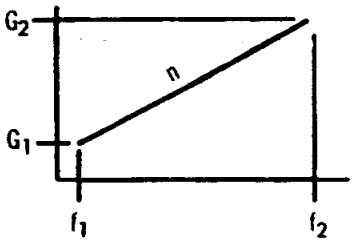
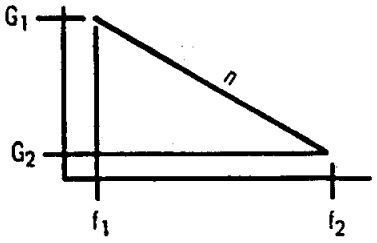


Figure 12-7. Time Spent In Passing Through a Resonance When Using a Constant Sweep Rate



$$(g\,rms)^2 = G\,(f_2 - f_1)$$





$$n = \frac{\log G_2/G_1}{\log f_2/f_1}$$

$$n = \frac{dB/oct}{3} \quad ; \; n = -1 \text{ when slope} = -3\,dB/oct.$$

when $n \neq -1$,

$$(g\,rms)^2 = \frac{G_2 f_2 - G_1 f_1}{n+1}$$

when $n = -1$,

$$(g\,rms)^2 = G_1 f_1 \ell n \frac{f_2}{f_1} = 2.3\, G_1 f_1 \log \frac{f_2}{f_1}$$

$$G\,rms = \sqrt{\Sigma\,(g\,rms)^2}$$

GENERAL SOLUTIONS

let f_p = frequency at G_p, etc.

and $f_1 \leq f_p \leq f_2$;

then $(g\,rms)^2 = \dfrac{G_p}{(n+1) f_p^n} (f_2^{n+1} - f_1^{n+1})$ for $n \neq -1$

$= G_p f_p \ell n \dfrac{f_2}{f_1}$ for $n = -1$

and

$$n = \frac{\log G_2/G_1}{\log f_2/f_1} = \frac{\ell n\, G_2/G_1}{\ell n\, f_2/f_1} = \frac{\log G_1/G_2}{\log f_1/f_2} = \frac{\ell n\, G_1/G_2}{\ell n\, f_1/f_2} = \frac{dB/oct}{3.01}$$

Figure 12-4. RMS of Spectral Density

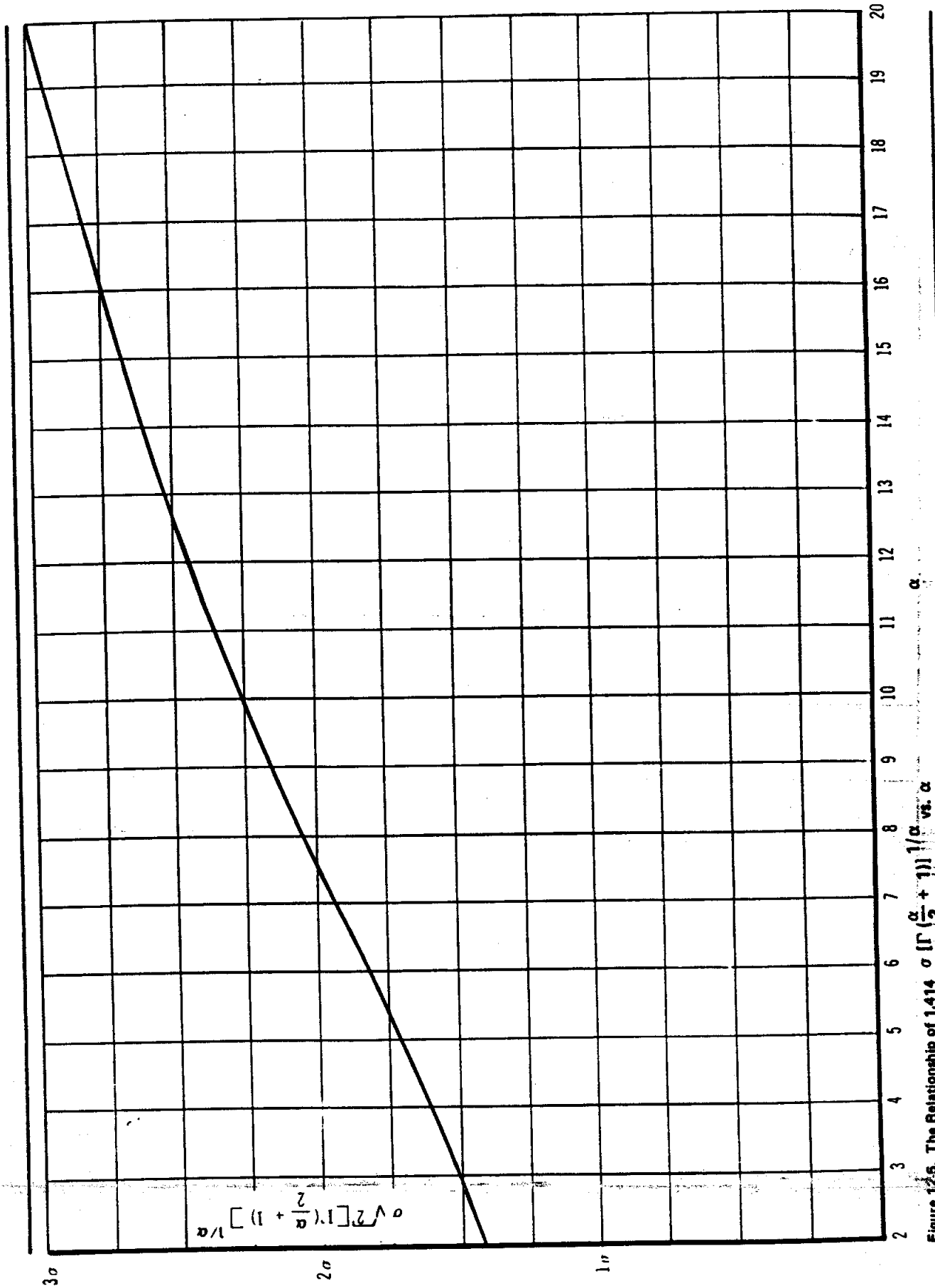


Figure 12.5. The Relationship of $1.414\ \sigma\left[\Gamma\left(\frac{\alpha}{2}+1\right)\right]^{1/\alpha}$ vs. α

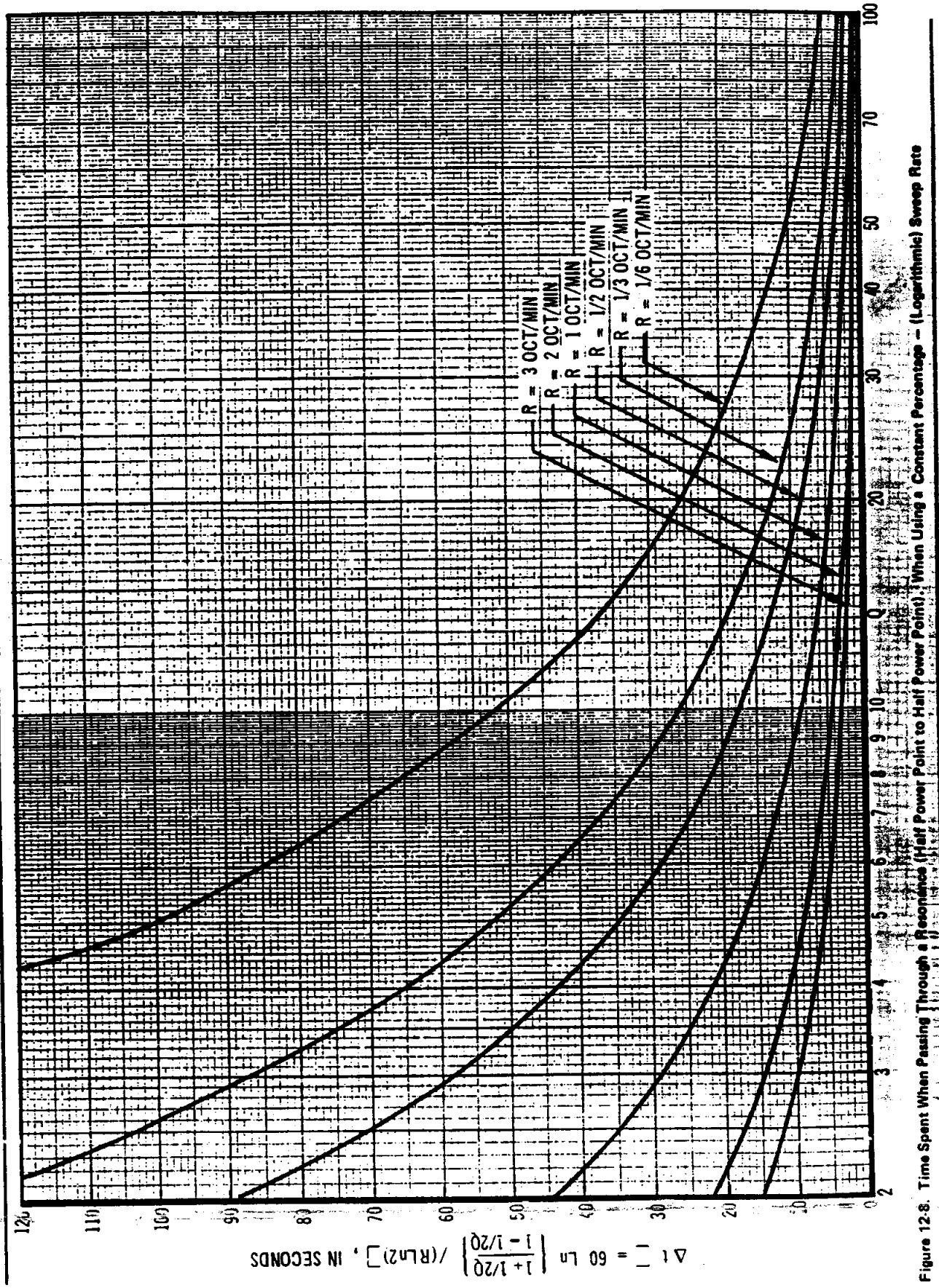
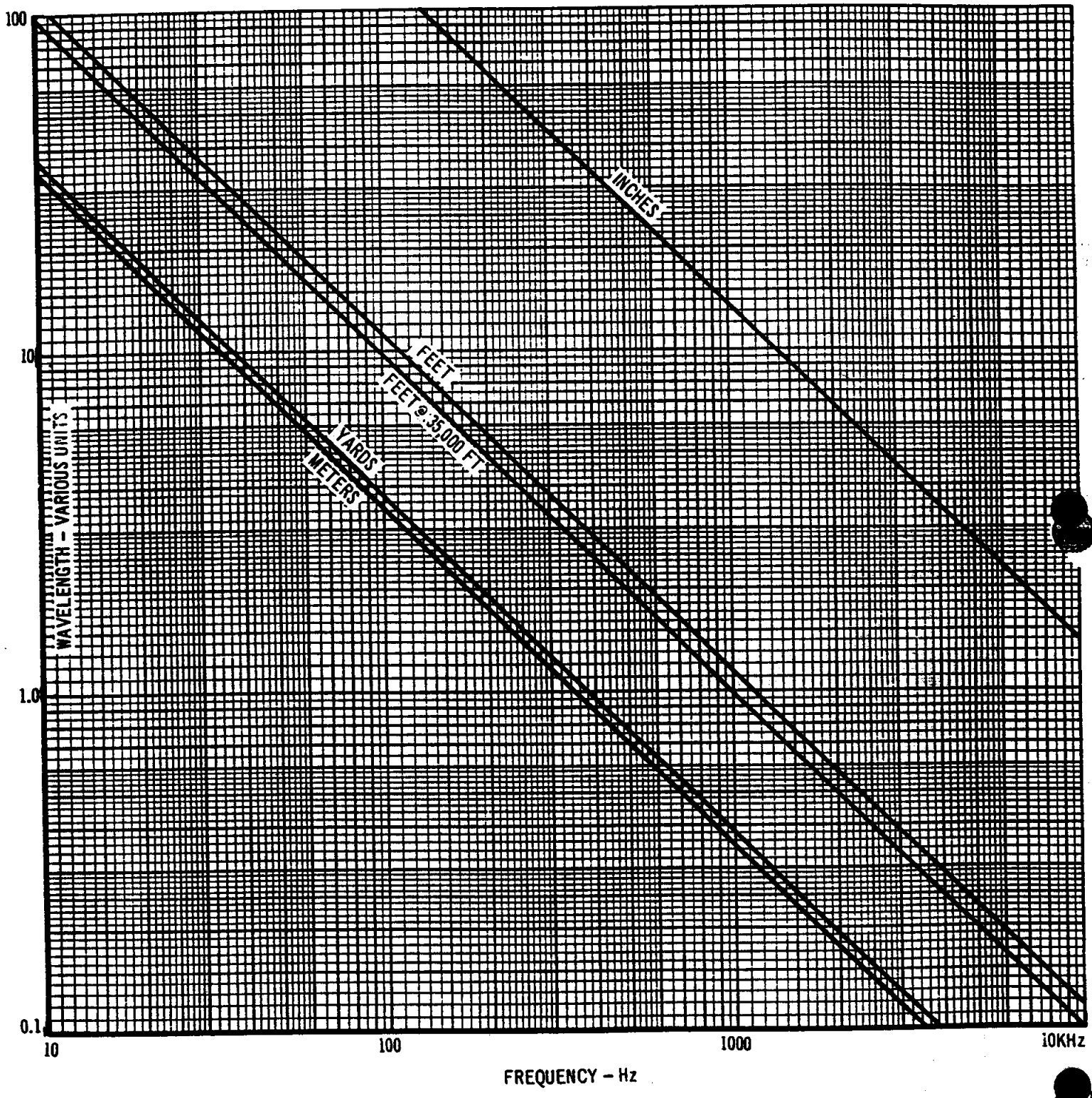


Figure 12-8. Time Spent When Passing Through a Resonance (Half Power Point to Half Power Point). When Using a Constant Percentage – (Logarithmic) Sweep Rate



NOTE: DATA IS FOR SEA LEVEL UNLESS OTHERWISE STATED.

Figure 12-9. Wavelength in Air versus Frequency $\lambda = c/f$ at Sea Level c = 1125 Ft/Sec: at 35000 Ft,c = 973 Ft/Sec

Table 12-1. Tabulated Data Conversions

SYSTEM ELEMENT	C.G.S.	M.K.S.	ENGLISH	
			FT-LB-SEC	IN-LB-SEC
P_α* STANDARD ATMOS. PRESSURE	$1.013 (10)^6$ DYNES/CM^2	$1.013 (10)^5$ NEWTONS/M^2	$2.116 (10)^3$ LBS/FT^2	14.70 LBS/IN.^{2}
ρ* AIR DENSITY	$1.2250 (10)^{-3}$ GRAMS/ CM^3	1.2250 KGM M^3	$2.3760 (10)^{-3}$ SLUGS FT^3	$1.1453 (10)^{-7}$ LB-SEC^2/IN^4 (MASS/IN.^{3})
c* SPEED OF SOUND IN AIR	$3.4340 (10)^4$ CM SEC	$3.4340 (10)^2$ METERS SEC	$1.1266 (10)^3$ FT/SEC	$1.3520 (10)^4$ IN- SEC
ρc (RAYLS) IMPEDANCE	42.067 GRAMS/CM^2–SEC	420.67 KGM M^2–SEC	2.6734 SLUGS FT^2–SEC	$1.5471 (10)^{-3}$ LB-SEC/FT^3 (MASS IN.^{2}-SEC)
P_o "0" dB REFERENCE PRESSURE LEVEL	$2.00 (10)^{-4}$ DYNES CM^2	$2.00 (10)^{-5}$ NEWTONS M^2	$4.18 (10)^{-7}$ LB/FT^2	$2.90 (10)^{-9}$ LB/IN.^{2}
I_o "0" dB REFERENCE INTENSITY LEVEL	$1.000 (10)^{-11}$ DYNES CM –SEC	$1.000 (10)^{-12}$ WATTS M^2	$6.8470 (10)^{-14}$ LB/FT-SEC	$5.7058 (10)^{-15}$ LB/IN.–SEC
W_o "0" dB REFERENCE ENERGY LEVEL	$1.00 (10)^{-6}$ DYNES – CM SEC	$1.00 (10)^{-13}$ WATTS	$7.37 (10)^{-14}$ FT-LB SEC	$8.84 (10)^{-13}$ IN.–LB/SEC
1.000 HORSEPOWER		746 WATTS	550 FT-LB/SEC	
$1.341 (10)^{-3}$ HORSEPOWER		1.000 WATTS	0.737 FT-LB/SEC	
$1.818 (10)^{-3}$ HORSEPOWER		1.356 WATTS	1.000 FT-LB SEC	
NOTE: ALL VALUES OF P_α, ρ, AND c ARE GIVEN FOR "STANDARD" ATMOSPHERIC CONDITIONS.				

Table 12-2. Decibels — Power Ratios

GIVEN: DECIBELS

TO FIND: POWER AND | VOLTAGE / CURRENT | RATIOS

TO ACCOUNT FOR THE SIGN OF THE DECIBEL

FOR POSITIVE (+) VALUES OF THE DECIBEL – BOTH VOLTAGE AND POWER RATIOS ARE GREATER THAN UNITY. USE THE TWO RIGHT-HAND COLUMNS.

FOR NEGATIVE (–) VALUES OF THE DECIBEL – BOTH VOLTAGE AND POWER RATIOS ARE LESS THAN UNITY. USE THE TWO LEFT-HAND COLUMNS

EXAMPLE – GIVEN: +9.1 dB. FIND:

	POWER RATIO	VOLTAGE RATIO
+9.1 dB	8.128	2.851
−9.1 dB	0.1230	0.3508

← −dB → VOLTAGE RATIO	← −dB → POWER RATIO	dB	+ dB → VOLTAGE RATIO	+ dB → POWER RATIO	← −dB → VOLTAGE RATIO	← −dB → POWER RATIO	dB	+ dB → VOLTAGE RATIO	+ dB → POWER RATIO
1.0000	1.0000	0	1.000	1.000	.5623	.3162	5.0	1.778	3.162
.9886	.9772	.1	1.012	1.023	.5559	.3090	5.1	1.799	3.236
.9772	.9550	.2	1.023	1.047	.5495	.3020	5.2	1.820	3.311
.9661	.9333	.3	1.035	1.072	.5433	.2951	5.3	1.841	3.388
.9550	.9120	.4	1.047	1.096	.5370	.2884	5.4	1.862	3.467
.9441	.8913	.5	1.059	1.122	.5309	.2818	5.5	1.884	3.548
.9333	.8710	.6	1.072	1.148	.5248	.2754	5.6	1.905	3.081
.9226	.8511	.7	1.084	1.175	.5188	.2692	5.7	1.928	3.715
.9120	.8318	.8	1.096	1.202	.5129	.2630	5.8	1.950	3.802
.9016	.8128	.9	1.109	1.230	.5070	.2570	5.9	1.972	3.890
.8913	.7943	1.0	1.122	1.259	.5012	.2512	6.0	1.995	3.981
.8810	.7762	1.1	1.135	1.288	.4955	.2455	6.1	2.018	4.074
.8710	.7586	1.2	1.148	1.318	.4898	.2399	6.2	2.042	4.169
.8610	.7413	1.3	1.161	1.349	.4842	.2344	6.3	2.065	4.266
.8511	.7244	1.4	1.175	1.380	.4786	.2291	6.4	2.089	4.365
.8414	.7079	1.5	1.189	1.413	.4732	.2239	6.5	2.113	4.467
.8318	.6918	1.6	1.202	1.445	.4677	.2188	6.6	2.138	4.571
.8222	.6761	1.7	1.216	1.479	.4624	.2138	6.7	2.163	4.677
.8128	.6607	1.8	1.230	1.514	.4571	.2080	6.8	2.188	4.786
.8035	.6457	1.9	1.245	1.549	.4519	.2042	6.9	2.213	4.898
.7943	.6310	2.0	1.259	1.585	.4467	.1995	7.0	2.239	5.012
.7852	.6166	2.1	1.274	1.622	.4416	.1950	7.1	2.265	5.129
.7762	.6026	2.2	1.288	1.660	.4365	.1905	7.2	2.291	5.248
.7674	.5888	2.3	1.303	1.698	.4315	.1862	7.3	2.317	5.370
.7586	.5754	2.4	1.318	1.738	.4266	.1820	7.4	2.344	5.495
.7499	.5623	2.5	1.334	1.778	.4217	.1778	7.5	2.371	5.623
.7413	.5495	2.6	1.349	1.820	.4169	.1738	7.6	2.399	5.754
.7328	.5370	2.7	1.365	1.862	.4121	.1698	7.7	2.427	5.888
.7244	.5248	2.8	1.380	1.905	.4074	.1660	7.8	2.455	6.026
.7161	.5129	2.9	1.396	1.950	.4027	.1622	7.9	2.483	6.166
.7079	.5012	3.0	1.413	1.995	.3981	.1585	8.0	2.512	6.310
.6998	.4898	3.1	1.429	2.042	.3936	.1549	8.1	2.541	6.457
.6918	.4786	3.2	1.445	2.089	.3890	.1514	8.2	2.570	6.607
.6839	.4677	3.3	1.462	2.138	.3846	.1479	8.3	2.600	6.761
.6761	.4571	3.4	1.479	2.188	.3802	.1445	8.4	2.630	6.918
.6683	.4467	3.5	1.496	2.239	.3758	.1413	8.5	2.661	7.079
.6607	.4365	3.6	1.514	2.291	.3715	.1380	8.6	2.692	7.244
.6531	.4266	3.7	1.531	2.344	.3673	.1349	8.7	2.723	7.413
.6457	.4169	3.8	1.549	2.399	.3631	.1318	8.8	2.754	7.586
.6383	.4074	3.9	1.567	2.455	.3589	.1288	8.9	2.786	7.762
.6310	.3981	4.0	1.585	2.512	.3548	.1259	9.0	2.818	7.943
.6237	.3800	4.1	1.603	2.570	.3508	.1230	9.1	2.851	8.128
.6166	.3802	4.2	1.622	2.630	.3467	.1202	9.2	2.884	8.318
.6095	.3715	4.3	1.641	2.692	.3428	.1175	9.3	2.917	8.511
.6026	.3631	4.4	1.660	2.754	.3388	.1148	9.4	2.951	8.710
.5957	.3548	4.5	1.679	2.818	.3350	.1122	9.5	2.985	8.913
.5888	.3467	4.6	1.698	2.884	.3311	.1096	9.6	3.020	9.120
.5821	.3388	4.7	1.718	2.951	.3273	.1072	9.7	3.055	9.333
.5754	.3311	4.8	1.738	3.020	.3236	.1047	9.8	3.090	9.550
.5689	.3236	4.9	1.758	3.090	.3199	.1023	9.9	3.126	9.772

Table 12-2. Decibels — Power Ratios (Continued)

← −dB+ → VOLTAGE RATIO	POWER RATIO	dB	VOLTAGE RATIO	POWER RATIO
.3162	.1000	10.0	3.162	10.000
.3126	.09772	10.1	3.199	10.23
.3090	.09550	10.2	3.236	10.47
.3055	.09333	10.3	3.273	10.72
.3020	.09120	10.4	3.311	10.96
.2985	.08913	10.5	3.350	11.22
.2951	.08710	10.6	3.388	11.48
.2917	.08511	10.7	3.428	11.75
.2884	.08318	10.8	3.467	12.02
.2851	.08128	10.9	3.508	12.30
.2818	.07943	11.0	3.548	12.59
.2786	.07762	11.1	3.589	12.88
.2754	.07586	11.2	3.631	13.18
.2723	.07413	11.3	3.673	13.49
.2692	.07244	11.4	3.715	13.80
.2661	.07079	11.5	3.758	14.13
.2630	.06918	11.6	3.802	14.45
.2600	.06761	11.7	3.846	14.79
.2570	.06607	11.8	3.890	15.44
.2541	.06457	11.9	3.936	15.49
.2512	.06310	12.0	3.981	15.85
.2483	.06166	12.1	4.027	16.22
.2455	.06026	12.2	4.074	16.60
.2427	.05888	12.3	4.121	16.98
.2399	.05754	12.4	4.169	17.38
.2371	.05623	12.5	4.217	17.78
.2344	.05495	12.6	4.266	18.20
.2317	.05370	12.7	4.315	18.62
.2291	.05248	12.8	4.365	19.05
.2265	.05129	12.9	4.416	19.50
.2239	.05012	13.0	4.467	19.95
.2213	.04898	13.1	4.519	20.42
.2188	.04786	13.2	4.571	20.89
.2163	.04677	13.3	4.624	21.38
.2138	.04571	13.4	4.677	21.88
.2113	.04467	13.5	4.732	22.39
.2089	.04365	13.6	4.786	22.91
.2065	.04266	13.7	4.842	23.44
.2042	.04169	13.8	4.898	23.99
.2018	.04074	13.9	4.955	24.55
.1995	.03981	14.0	5.012	25.12
.1972	.03890	14.1	5.070	25.70
.1950	.03802	14.2	5.129	26.30
.1928	.03715	14.3	5.188	26.92
.1905	.03631	14.4	5.248	27.54
.1884	.03548	14.5	5.309	28.18
.1862	.03467	14.6	5.370	28.84
.1841	.03388	14.7	5.433	29.51
.1820	.03311	14.8	5.495	30.20
.1799	.03236	14.9	5.559	30.90
.1778	.03162	15.0	5.623	31.62
.1758	.03090	15.0	5.689	32.36
.1738	.03020	15.2	5.754	33.11
.1718	.02951	15.3	5.821	33.88
.1698	.02884	15.4	5.888	34.67
.1679	.02818	15.5	5.957	35.48
.1660	.02754	15.5	6.026	36.31
.1641	.02692	15.7	6.095	37.15
.1622	.02630	15.8	6.166	38.02
.1603	.02570	15.9	6.237	38.90

← −dB+ → VOLTAGE RATIO	POWER RATIO	dB	VOLTAGE RATIO	POWER RATIO
.1585	.02512	16.0	6.310	39.81
.1567	.02455	16.1	6.383	40.74
.1549	.02399	16.2	6.457	41.69
.1531	.02344	16.3	6.531	42.66
.1514	.02291	16.4	6.607	43.65
.1496	.02239	16.5	6.683	44.67
.1479	.02188	16.6	6.761	45.71
.1462	.02138	16.7	6.839	46.77
.1445	.02089	16.8	6.918	47.86
.1429	.02042	16.9	6.998	48.98
.1413	.01995	17.0	7.079	50.12
.1396	.01950	17.1	7.161	51.29
.1380	.01905	17.2	7.244	52.48
.1365	.01862	17.3	7.328	53.70
.1349	.01820	17.4	7.413	54.95
.1334	.01778	17.5	7.499	56.23
.1318	.01738	17.6	7.586	57.54
.1303	.01698	17.7	7.674	58.88
.1288	.01660	17.8	7.762	60.26
.1274	.01622	17.9	7.852	61.66
.1259	.01585	18.0	7.943	63.10
.1245	.01549	18.1	8.035	64.57
.1230	.01514	18.2	8.128	66.07
.1216	.01479	18.3	8.222	67.61
.1202	.01445	18.4	8.318	69.18
.1189	.01413	18.5	8.414	70.79
.1175	.01380	18.6	8.511	72.44
.1161	.01349	18.7	8.610	74.13
.1148	.01318	18.8	8.710	75.86
.1135	.01288	18.9	8.811	77.62
.1122	.01259	19.0	8.913	79.43
.1109	.01230	19.1	9.016	81.28
.1096	.01202	19.2	9.120	83.18
.1084	.01175	19.3	9.226	85.11
.1072	.01148	19.4	9.333	87.10
.1059	.01122	19.5	9.441	89.13
.1047	.01096	19.6	9.550	91.20
.1035	.01072	19.7	9.661	93.33
.1023	.01047	19.8	9.772	95.50
.1012	.01023	19.9	9.886	97.72
.1000	.01000	20.0	10.000	100.00

−dB+

VOLTAGE RATIO	POWER RATIO	dB	VOLTAGE RATIO	POWER RATIO
3.162×10^{-1}	10^{-1}	10	3.162	10
$\times 10^{-1}$	10^{-2}	20	10	10^2
3.162×10^{-2}	10^{-3}	30	3.162×10	10^3
$\times 10^{-2}$	10^{-4}	40	$\times 10^2$	10^4
3.162×10^{-3}	10^{-5}	50	3.162×10^2	10^5
10^{-3}	10^{-6}	60	10^3	10^6
3.162×10^{-4}	10^{-7}	70	3.162×10^3	10^7
10^{-4}	10^{-8}	80	10^4	10^8
3.162×10^{-5}	10^{-9}	90	3.162×10^4	10^9
10^{-5}	10^{-10}	100	10^5	10^{10}

Table 12-3. Voltage/Current Ratio – Decibels

GIVEN: | VOLTAGE / CURRENT | RATIO

TO FIND: DECIBELS

POWER RATIOS

TO FIND THE NUMBER OF DECIBELS CORRESPONDING TO A GIVEN POWER RATIO – Assume the given power ratio to be a voltage ratio and find the corresponding number of decibels from the table. The desired result is exactly one-half of the number of decibels thus found.

EXAMPLE – Given: a power ratio of 3.41.
Find: 3.41 in the table:
3.41 → 10.655 dB (voltage)
10.655 dB x 1/2 = 5.328 dB (power)

VOLTAGE RATIO	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.0	.000	.086	.172	.257	.341	.424	.506	.588	.668	.749
1.1	.828	.906	.984	1.062	1.138	1.214	1.289	1.364	1.438	1.511
1.2	1.584	1.656	1.727	1.798	1.868	1.938	2.007	2.076	2.144	2.212
1.3	2.279	2.345	2.411	2.477	2.542	2.607	2.671	2.734	2.798	2.860
1.4	2.923	2.984	3.046	3.107	3.167	3.227	3.287	3.346	3.405	3.464
1.5	3.522	3.580	3.637	3.694	3.750	3.807	3.862	3.918	3.973	4.028
1.6	4.082	4.137	4.190	4.244	4.297	4.350	4.402	4.454	4.506	4.558
1.7	4.609	4.660	4.711	4.761	4.811	4.861	4.910	4.959	5.008	5.057
1.8	5.105	5.154	5.201	5.249	5.296	5.343	5.390	5.437	5.483	5.529
1.9	5.575	5.621	5.666	5.711	5.756	5.801	5.845	5.889	5.933	5.977
2.0	6.021	6.064	6.107	6.150	6.193	6.235	6.277	6.319	6.361	6.403
2.1	6.444	6.486	6.527	6.568	6.608	6.649	6.689	6.729	6.769	6.809
2.2	6.848	6.888	6.927	6.966	7.008	7.044	7.082	7.121	7.159	7.197
2.3	7.235	7.272	7.310	7.347	7.384	7.421	7.458	7.495	7.532	7.568
2.4	7.604	7.640	7.676	7.712	7.748	7.783	7.819	7.854	7.889	7.924
2.5	7.959	7.993	8.028	8.062	8.097	8.131	8.165	8.199	8.232	8.266
2.6	8.299	8.333	8.366	8.399	8.132	8.465	8.498	8.530	8.563	8.595
2.7	8.627	8.659	8.691	8.723	8.755	8.787	8.818	8.850	8.881	8.912
2.8	8.943	8.974	9.005	9.036	9.068	9.097	9.127	9.158	9.188	9.218
2.9	9.248	9.278	9.308	9.337	9.367	9.396	9.426	9.455	9.484	9.513
3.0	9.542	9.571	9.600	9.629	9.657	9.686	9.714	9.743	9.771	9.799
3.1	9.827	9.855	9.883	9.911	9.939	9.966	9.994	10.021	10.049	10.076
3.2	10.103	10.130	10.157	10.184	10.211	10.238	10.264	10.291	10.317	10.344
3.3	10.370	10.397	10.423	10.449	10.475	10.501	10.527	10.553	10.578	10.604
3.4	10.630	10.655	10.681	10.706	10.731	10.756	10.782	10.807	10.832	10.857
3.5	10.881	10.906	10.931	10.955	10.980	11.005	11.029	11.053	11.078	11.102
3.6	11.126	11.150	11.174	11.198	11.222	11.246	11.270	11.293	11.317	11.341
3.7	11.364	11.387	11.411	11.434	11.457	11.481	11.504	11.527	11.550	11.573
3.8	11.596	11.618	11.641	11.664	11.687	11.709	11.732	11.754	11.777	11.799
3.9	11.821	11.844	11.860	11.888	11.910	11.932	11.954	11.976	11.998	12.019
4.0	12.041	12.063	12.085	12.106	12.128	12.149	12.171	12.192	12.213	12.234
4.1	12.256	12.277	12.298	12.319	12.340	12.361	12.382	12.403	12.424	12.444
4.2	12.465	12.486	12.506	12.527	12.547	12.568	12.588	12.609	12.629	12.649
4.3	12.669	12.690	12.710	12.730	12.750	12.770	12.790	12.810	12.829	12.849
4.4	12.869	12.889	12.908	12.928	12.948	12.967	12.987	13.006	13.026	13.045
4.5	13.064	13.084	13.103	13.122	13.141	13.160	13.179	13.198	13.217	13.230
4.6	13.255	13.274	13.293	13.312	13.330	13.349	13.308	13.386	13.405	13.423
4.7	13.442	13.460	13.479	13.497	13.516	13.534	13.552	13.570	13.589	13.607
4.8	13.625	13.643	13.661	13.679	13.697	13.715	13.733	13.751	13.768	13.786
4.9	13.804	13.822	13.839	13.857	13.875	13.892	13.910	13.927	13.945	13.962
5.0	13.979	13.997	14.014	14.031	14.049	14.100	14.083	14.100	14.117	14.134
5.1	14.151	14.168	14.185	14.202	14.219	14.270	14.253	14.270	14.287	14.303
5.2	14.320	14.337	14.353	14.370	14.387	14.436	14.420	14.436	14.453	14.469
5.3	14.486	14.502	14.518	14.535	14.551	14.599	14.583	14.599	14.616	14.632
5.4	14.618	14.664	14.680	14.696	14.712	14.760	14.744	14.760	14.776	14.791

Table 12-3. Voltage/Current Ratio — Decibels (Continued)

VOLTAGE RATIO	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
5.5	14.807	14.823	14.839	14.855	14.870	14.886	14.902	14.917	14.933	14.948
5.6	14.964	14.979	14.995	15.010	15.026	15.041	15.056	15.072	15.087	15.102
5.7	15.117	15.133	15.148	15.163	15.178	15.193	15.208	15.224	15.239	15.254
5.8	15.269	15.284	15.298	15.313	15.328	15.343	15.358	15.373	15.388	15.402
5.9	15.417	15.432	15.446	15.461	15.476	15.490	15.505	15.519	15.534	15.549
6.0	15.563	15.577	15.592	15.606	15.621	15.635	15.649	15.664	15.678	15.692
6.1	15.707	15.721	15.735	15.749	15.763	15.778	15.792	15.806	15.820	15.834
6.2	15.848	15.862	15.876	15.890	15.904	15.918	15.931	15.945	15.959	15.973
6.3	15.987	16.001	16.014	16.028	16.042	16.055	16.069	16.083	16.096	16.110
6.4	16.124	16.137	16.151	16.164	16.178	16.191	16.205	16.218	16.232	16.245
6.5	16.258	16.272	16.285	16.298	16.312	16.325	16.338	16.351	16.365	16.378
6.6	16.391	16.404	16.417	16.430	16.443	16.456	16.469	16.483	16.496	16.509
6.7	16.521	16.534	16.547	16.560	16.573	16.586	16.599	16.642	16.625	16.637
6.8	16.650	16.663	16.676	16.688	16.701	16.714	16.726	16.739	16.752	16.764
6.9	16.777	16.790	16.802	16.815	16.827	16.840	16.852	16.865	16.877	16.890
7.0	16.902	16.914	16.927	16.939	16.951	16.964	16.976	16.988	17.001	17.013
7.1	17.025	17.037	17.050	17.062	17.074	17.086	17.098	17.110	17.122	17.135
7.2	17.147	17.159	17.171	17.183	17.195	17.207	17.219	17.231	17.243	17.255
7.3	17.266	17.278	17.290	17.302	17.314	17.326	17.338	17.349	17.361	17.373
7.4	17.385	17.396	17.408	17.420	17.431	17.443	17.455	17.466	17.478	17.490
7.5	17.501	17.513	17.524	17.536	17.547	17.559	17.570	17.582	17.593	17.605
7.6	17.616	17.628	17.639	17.650	17.662	17.673	17.685	17.696	17.707	17.719
7.7	17.730	17.741	17.752	17.764	17.775	17.786	17.797	17.808	17.820	17.831
7.8	17.842	17.853	17.864	17.875	17.886	17.897	17.908	17.919	17.931	17.942
7.9	17.953	17.964	17.975	17.985	17.996	18.007	18.018	18.029	18.040	18.051
8.0	18.062	18.073	18.083	18.094	18.105	18.116	18.127	18.137	18.148	18.159
8.1	18.170	18.180	18.191	18.202	18.212	18.223	18.234	18.244	18.255	18.266
8.2	18.276	18.287	18.297	18.308	18.319	18.329	18.340	18.350	18.361	18.371
8.3	18.382	18.392	18.402	18.413	18.423	18.434	18.444	18.455	18.465	18.475
8.4	18.486	18.496	18.506	18.517	18.527	18.537	18.547	18.558	18.568	18.578
8.5	18.588	18.599	18.609	18.619	18.629	18.639	18.649	18.660	18.670	18.680
8.6	18.690	18.700	18.710	18.720	18.730	18.740	18.750	18.760	18.770	18.780
8.7	18.790	18.800	18.810	18.820	18.830	18.840	18.850	18.860	18.870	18.880
8.8	18.890	18.900	18.909	18.919	18.929	18.939	18.949	18.958	18.968	18.978
8.9	18.988	18.998	19.007	19.017	19.027	19.036	19.046	19.056	19.066	19.075
9.0	19.085	19.094	19.104	19.114	19.123	19.133	19.143	19.152	19.162	19.171
9.1	19.181	19.190	19.200	19.209	19.219	19.228	19.238	19.247	19.257	19.226
9.2	19.276	19.285	19.295	19.304	19.313	19.323	19.332	19.342	19.351	19.360
9.3	19.370	19.379	19.388	19.398	19.407	19.416	19.426	19.435	19.444	19.453
9.4	19.463	19.472	19.481	19.490	19.499	19.509	19.518	19.527	19.536	19.545
9.5	19.554	19.564	19.573	19.582	19.591	19.600	19.609	19.618	19.627	19.636
9.6	19.645	19.654	19.664	19.673	19.682	19.691	19.700	19.709	19.718	19.726
9.7	19.735	19.744	19.753	19.762	19.771	19.780	19.789	19.798	19.807	19.816
9.8	19.825	19.833	19.842	19.851	19.860	19.869	19.878	19.886	19.895	19.904
9.9	19.913	19.921	19.930	19.939	19.948	19.956	19.965	19.974	19.983	19.991

VOLTAGE RATIO	0	1	2	3	4	5	6	7	8	9
10	20.000	20.328	21.584	22.279	22.923	23.522	24.082	24.609	25.105	25.575
20	26.021	26.444	26.848	27.235	27.604	27.959	28.299	28.627	28.943	29.248
30	29.542	29.827	30.103	30.370	30.630	30.881	31.126	31.364	31.596	31.821
40	32.041	32.256	32.465	32.669	32.869	33.064	33.255	33.442	33.625	33.804
50	33.979	34.151	34.320	34.486	34.648	34.807	34.964	35.117	35.269	35.417
60	35.563	35.707	35.848	35.987	36.124	36.258	36.391	36.521	36.650	36.777
70	36.902	37.025	37.147	37.266	37.385	37.501	37.616	37.730	37.842	37.953
80	38.062	38.170	38.276	38.382	38.486	38.588	38.690	38.790	38.890	38.988
90	39.085	39.181	39.276	39.370	39.463	39.554	39.645	39.735	39.825	39.913
100	40.000	—	—	—	—	—	—	—	—	—

Table 12-4. Decimal Equivalents

1/64	.015625	33/64	.515625
1/32	.03125	17/32	.53125
3/64	.046875	35/64	.546875
1/16	.0625	9/16	.5625
5/64	.078125	37/64	.578125
3/32	.09375	19/32	.59375
7/64	.109375	39/64	.609375
1/8	.125	5/8	.625
9/64	.140625	41/64	.640625
5/32	.15625	21/32	.65625
11/64	.171875	43/64	.671875
3/16	.1875	11/16	.6875
13/64	.203125	45/64	.703125
7/32	.21875	23/32	.71875
15/64	.234375	47/64	.734375
1/4	.25	3/4	.75
17/64	.265625	49/64	.765625
9/32	.28125	25/32	.78125
19/64	.296875	51/64	.796875
5/16	.3125	13/16	.8125
21/64	.328125	53/64	.828125
11/32	.34375	27/32	.84375
23/64	.359375	55/64	.859375
3/8	.375	7/8	.875
25/64	.390625	57/64	.890625
13/32	.40625	29/32	.90625
27/64	.421875	59/64	.921875
7/16	.4375	15/16	.9375
29/64	.453125	61/64	.953125
15/32	.46875	31/32	.96875
31/64	.484375	63/64	.984375
1/2	.5	1	1

Section 13
STANDARDS

13-1 ATMOSPHERIC PARAMETERS

See Figure 13-1

13-2 PROPERTIES OF STANDARD ATMOSPHERE

See Table 13-1

13-3 RELATIVE HUMIDITY VERSUS ABSOLUTE HUMIDITY

See Figure 13-2

13-4 STANDARD BANDWIDTHS

See Table 13-2

NOTE

To 1 percent accuracy, the bandwidth of some fraction n of an octave band, is given by:

$$BW = 0.7\, n\, f_o$$

where:

BW = Bandwidth in Hz.

n = Fraction of an octave band, $0 < n \leq 1$.

f_o = Geometrical mean frequency.

13-5 CALCULATION OF PNdB, SONES, NOYS, AND LOUDNESS LEVELS

See Figures 13-3, 13-4, 13-5, and 13-6 and Tables 13-3, 13-4, and 13-5.

13-6 SUMMARY OF REFERENCE STANDARDS

Contents of several of the more applicable standards are briefly summarized in this section. If applied to some problem, the engineer is urged to refer directly to the standard for definitive information.

a. MIL-STD-810B, *Military Standard — Environmental Test Methods* — Supersedes previous versions (810 and 810A) and is sometimes referred to by name as a mandatory standard when working on military contracts. It attempts to establish "uniform environmental test methods for determining the resistance of equipment to the affects of natural and induced environments" such as conditions of climate (humidity, temperature, rain, sunshine, salt, fog, dust, etc.), induced dynamics (vibration, shock, and acoustics), pressure, vacuum (space) etc.

When referring to a specific test method in the standard, the engineer should distinguish whether he is referring to just the method (the Standard being a test methods document) or to the method and the levels described therein.

b. MIL-HDBK-5A, *Metallic Materials and Elements for Aerospace Vehicle Structures* — This handbook supersedes MIL-HDBK-5 and presents data which are uniformly accepted, and thereby eliminates the necessity for innumerable handbooks and bulletins. It extensively covers the characteristics of commonly-used materials, but does not include new or exotic materials. It's main intent is to give the strengths of the materials under various conditions (e.g. at room or elevated temperatures), but is also includes the more commonly used methods and formulas by which the strengths are calculated.

c. *Standard Methods for Analysis and Presentation of Shock and Vibration Data* — Proposed by the Writing Group S2-W-30 of the U.S. Standards Institute (dated July 1968), it is intended to explain the general principles of the analysis and presentation of shock and vibration data, and to describe concisely several methods of reducing and formatting these data. It deals with periodic, aperiodic, random and transient time-histories of a variable associated with shock and vibration (acceleration, velocity, force, etc.) assuming all system error has been eliminated. It also gives definitions which have been, for the most part, included in Terminology, Section 15 of this manual.

d. S1.1 — 1960, *Acoustical Terminology* — An American Standard (including mechanical, shock and vibration) prepared by the Acoustical Society of

America in 1960 (superceding Z24.1-1951 and Z24.1a). Contents are confined to a definition of terms, the most common of which have been included in Terminology, Section 15 of this manual.

e. S3.4, *Computation of Loudness of Noise* – A proposed American Standard Procedure sponsored by the Acoustical Society of America. It specifies a simplified procedure for calculating the loudness of a complex sound so that sounds of diverse levels and spectra may be compared subjectively.

f. S1.2 – 1962, *Physical Measurement of Sound* – An Acoustical Society of America sponsored American Standard and supersedes Z24.7-1950. It establishes the accepted methods by which sound pressure levels are measured and reported, and by which sound powers are calculated therefrom. It primarily deals with airborne sound generated by equipment which operates in air and radiates sound as a by-product of its primary function.

g. Z24.19 – 1957, *Laboratory Measurement of Air-Borne Sound Transmission Loss of Building Floors and Walls* – An American Standard prepared by the Acoustical Society of America. It covers only the recommended practice for measuring air-borne sound reduction through building floors and walls as determined by a random incidence or reverberant sound method. From these measurements the sound transmission loss of the test specimen can be calculated if the appropriate descriptors of the test facility are known (absorptivity, geometry, diffusivity of the sound field, etc.).

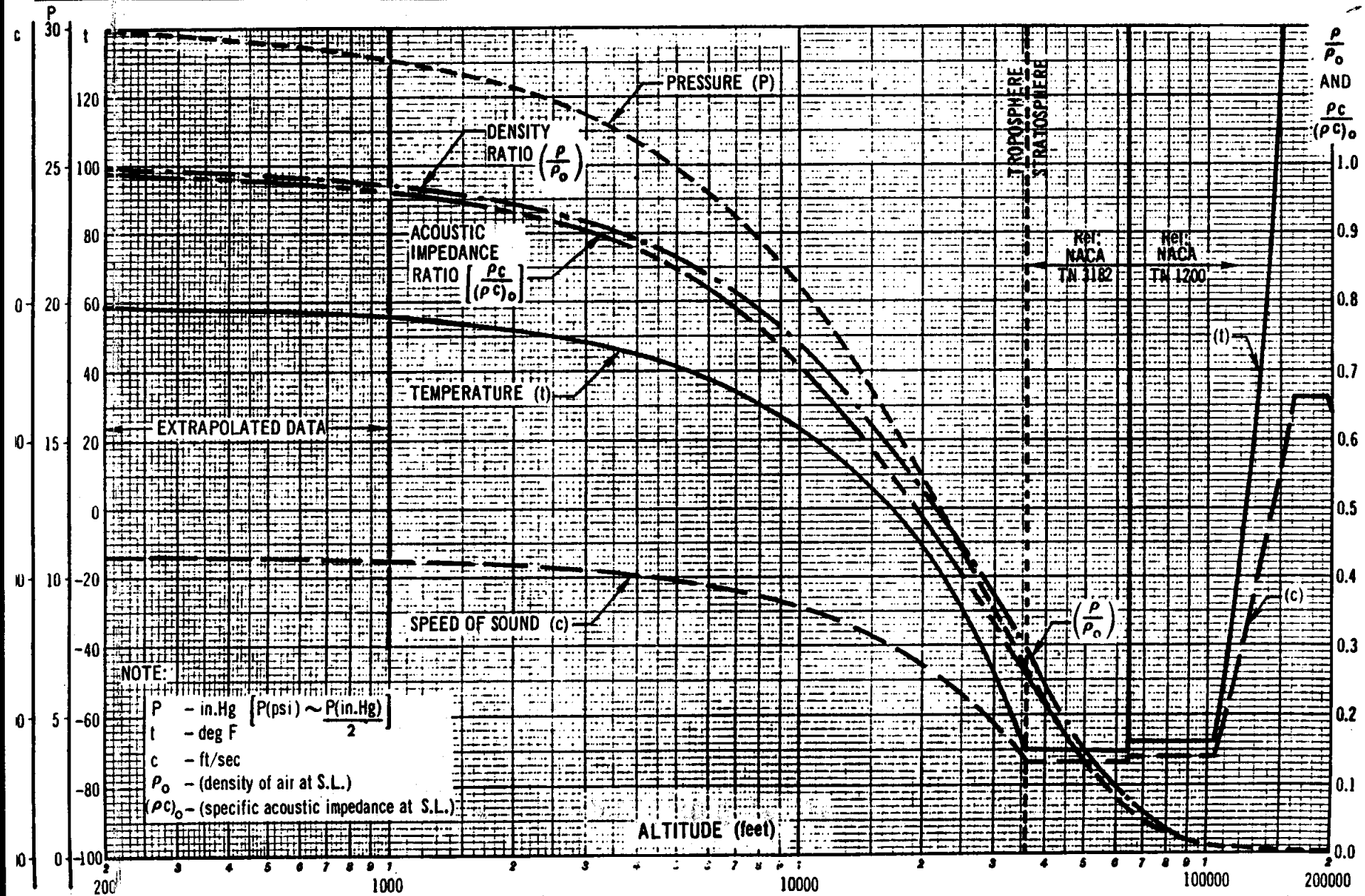


Figure 13-1. Atmospheric Parameters vs Altitude

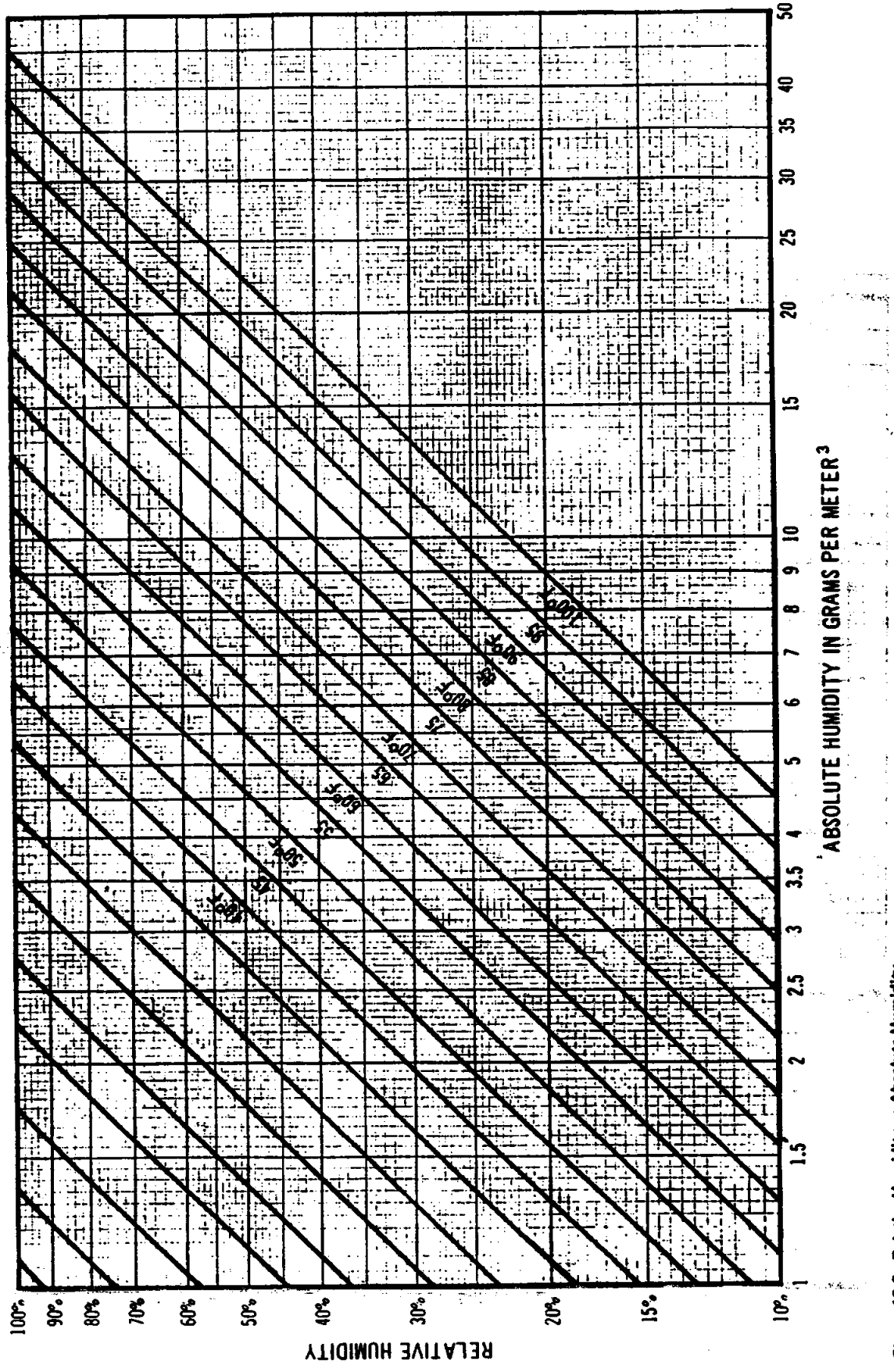


Figure 13-2. Relative Humidity vs Absolute Humidity

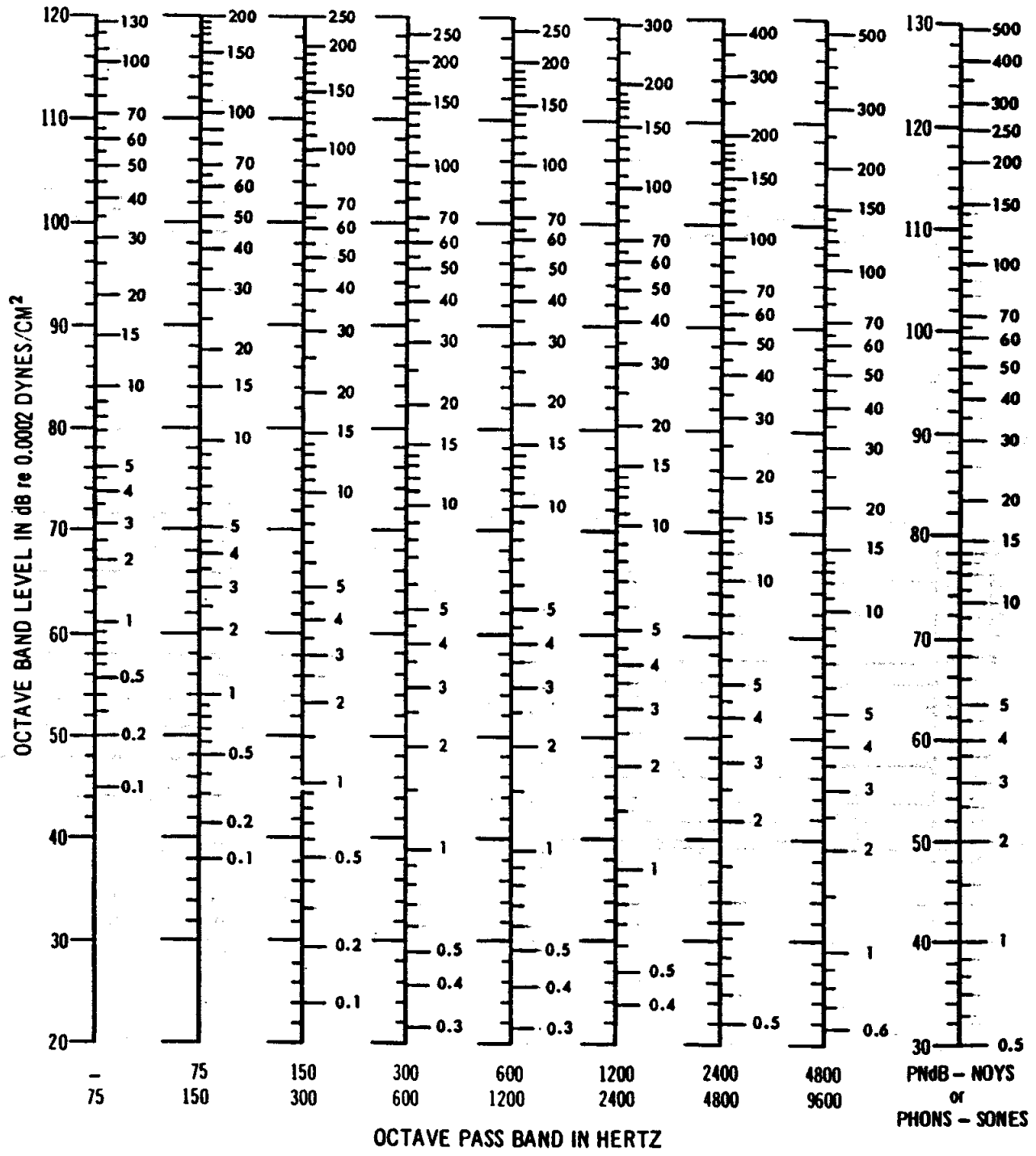


Figure 13-3. Nomograms Relating SPL in Octave Bands to Loudness in Sones

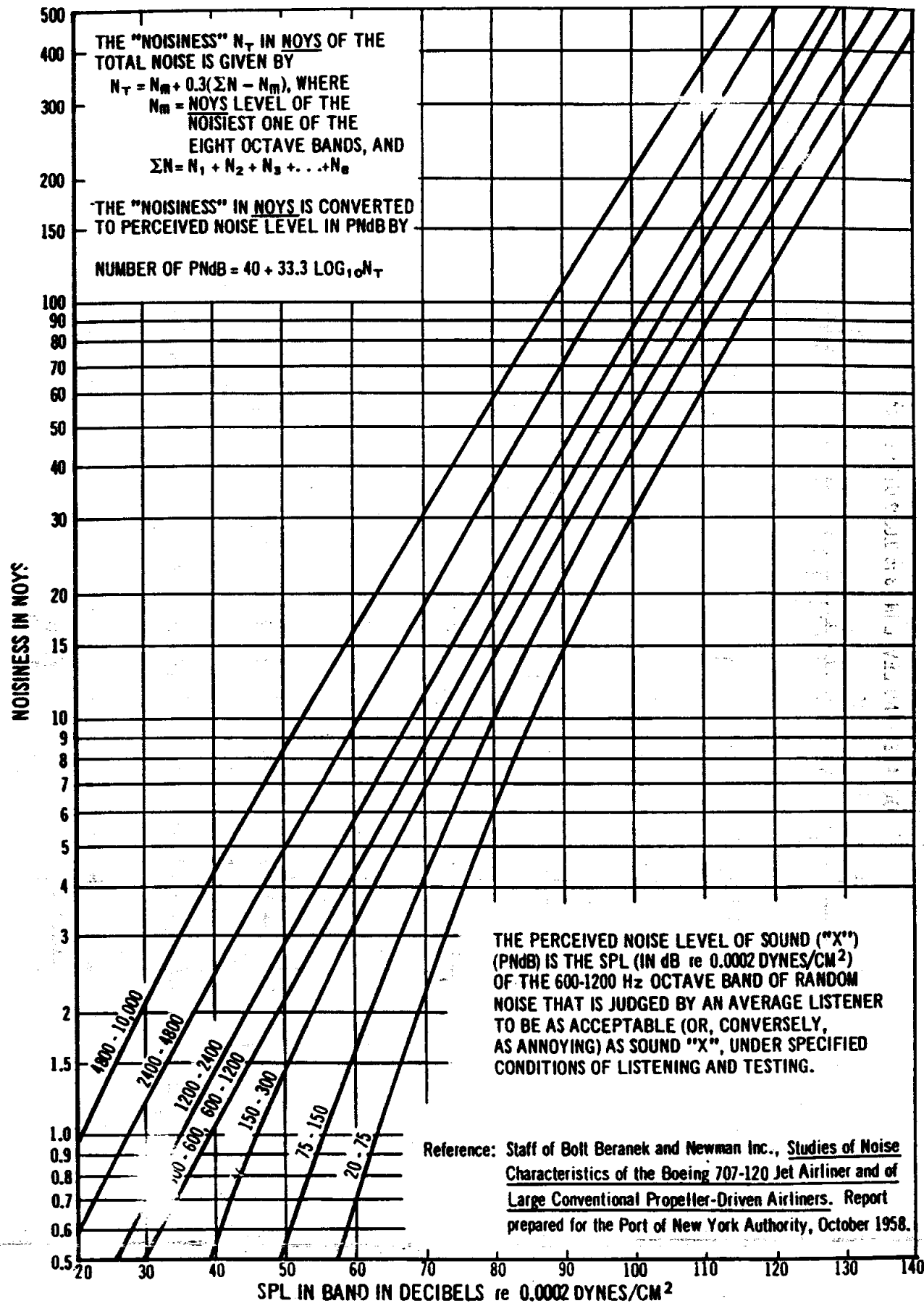


Figure 13-4. Relationship Between SPL in Frequency Bands and Noisiness in NOYS

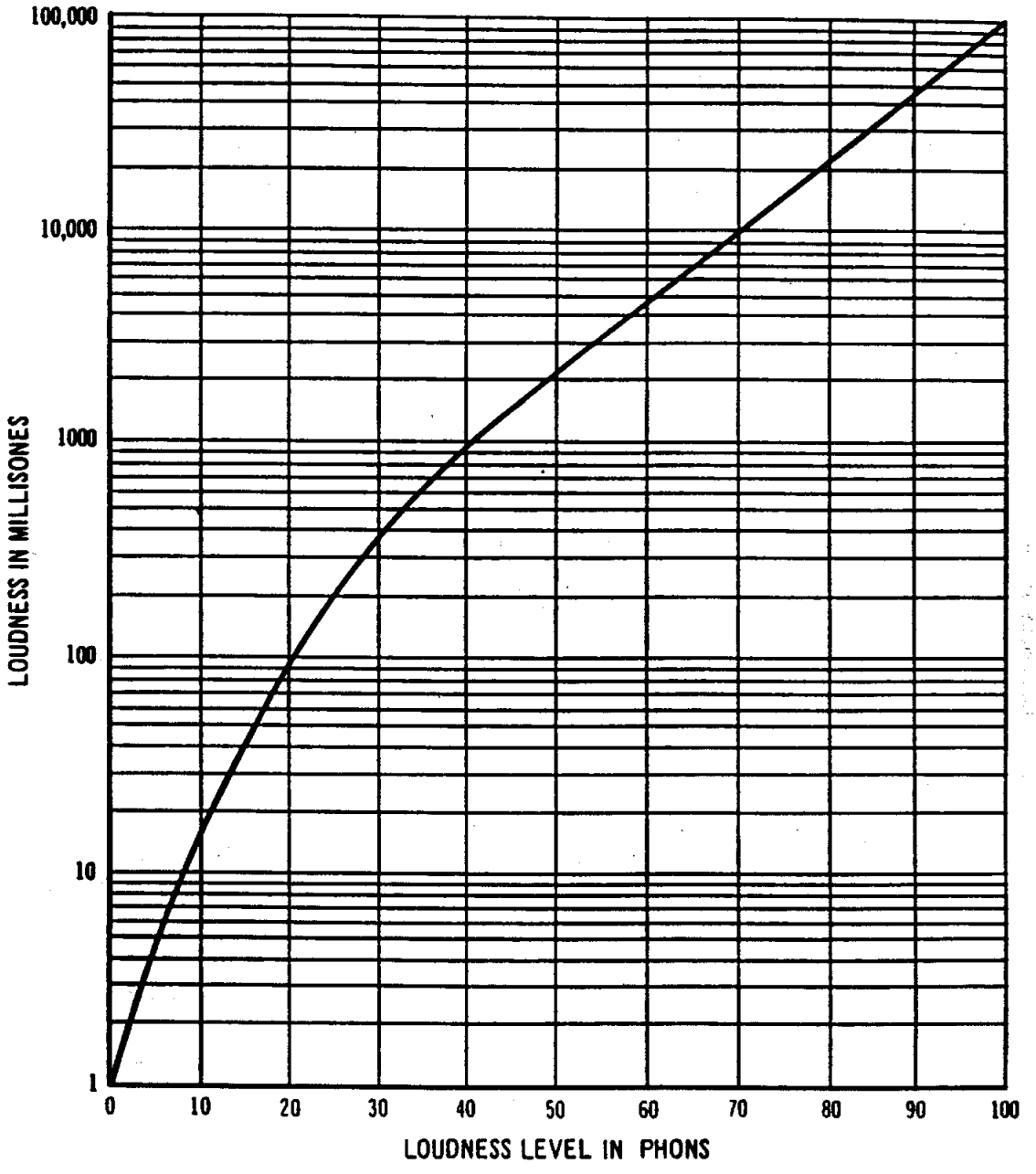


Figure 13-5. Loudness vs Loudness Level for Pure Tones

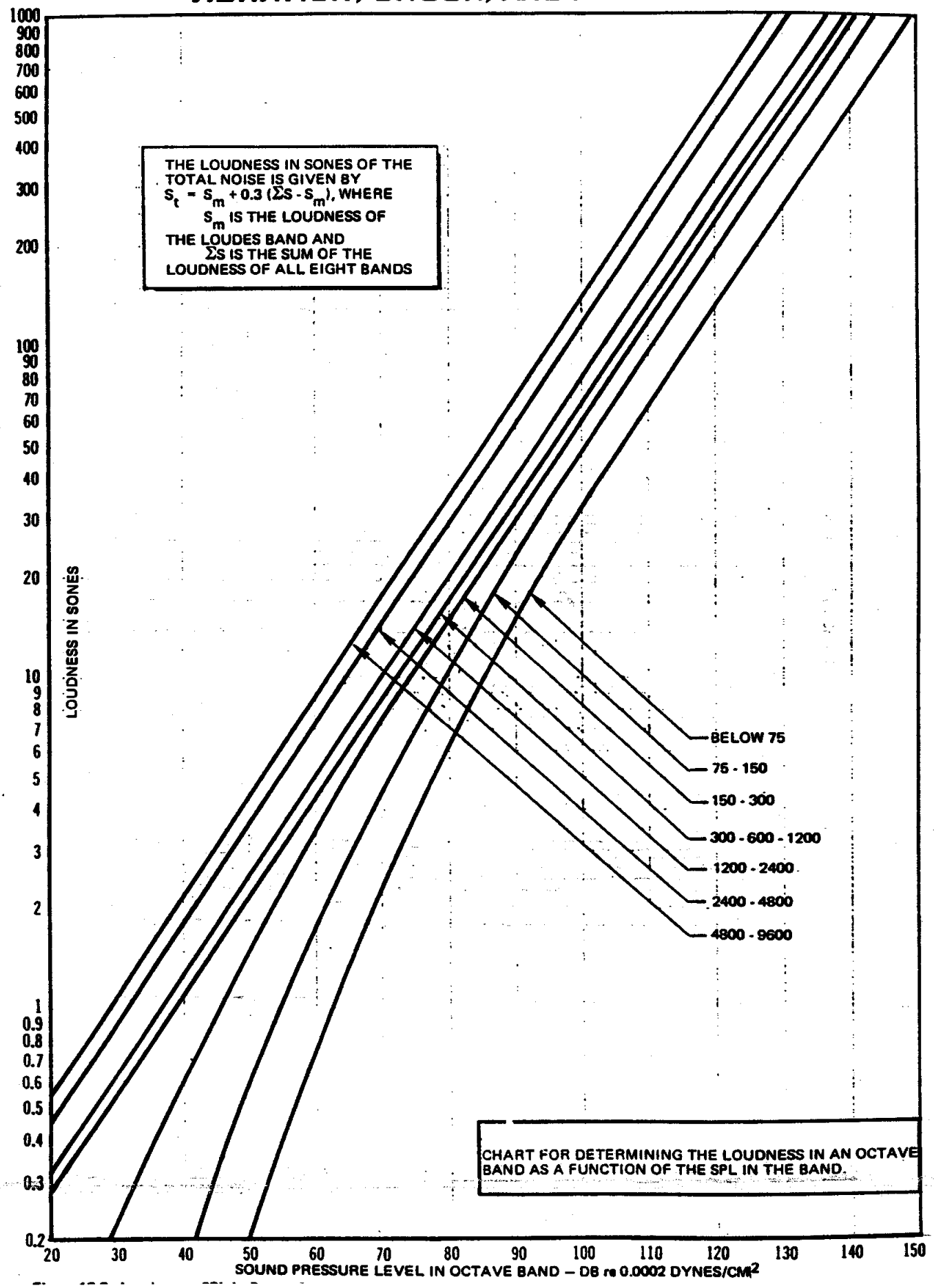


CHART FOR DETERMINING THE LOUDNESS IN AN OCTAVE BAND AS A FUNCTION OF THE SPL IN THE BAND.

Table 13-1. Properties of the Standard Atmosphere

h (ft)	t (°F)	t (°C)	a/aSL	a (ft/sec)	a (mph)	p/pSL	p (lb/sq ft)	p millibars	$\sigma = \rho/\rho_{SL}$	ρ (slug/cu ft)	$\sigma^{\frac{1}{2}}$	$\mu \times 10^7$ (slug/ft sec)	$\nu \times 10^6$ (ft²/sec)	q/M² (lb/ft²)
0	59.00	15.00	1.0000	1117	761.6	1.0000	2116.2	1013.2	1.0000	0.002378	1.0000	3.719	1.564	1481.3
1,000	57.44	13.02	.9966	1113	759.0	.9644	2040.9	977.1	.9711	.002310	.9854	3.699	1.602	1428.6
2,000	51.87	11.04	.9931	1109	756.3	.9298	1967.7	942.1	.9428	.002242	.9710	3.679	1.641	1377.4
3,000	48.31	9.06	.9896	1105	753.7	.8963	1896.7	908.1	.9152	.002177	.9566	3.659	1.681	1327.7
4,000	44.74	7.08	.9862	1102	751.0	.8637	1827.7	875.1	.8881	.002112	.9424	3.639	1.723	1279.4
5,000	41.18	5.10	.9827	1098	748.4	.8321	1760.8	843.0	.8617	.002049	.9283	3.618	1.766	1232.5
6,000	37.62	3.12	.9792	1094	745.7	.8014	1696.0	812.0	.8359	.001988	.9143	3.598	1.810	1187.2
7,000	34.05	1.14	.9756	1090	743.0	.7717	1633.0	781.8	.8107	.001928	.9004	3.577	1.855	1143.1
8,000	30.49	- 0.84	.9721	1086	740.4	.7428	1571.9	752.6	.7860	.001869	.8866	3.557	1.903	1100.3
9,000	26.92	- 2.82	.9686	1082	737.7	.7148	1512.8	724.3	.7620	.001812	.8729	3.536	1.951	1059.0
10,000	23.36	- 4.80	.9650	1078	734.9	.6877	1455.4	696.8	.7385	.001756	.8594	3.515	2.002	1018.8
11,000	19.80	- 6.78	.9614	1074	732.2	.6614	1399.8	670.2	.7156	.001702	.8459	3.495	2.054	979.9
12,000	16.23	- 8.76	.9579	1070	729.5	.6360	1345.9	644.4	.6932	.001649	.8328	3.474	2.107	942.1
13,000	12.67	-10.74	.9543	1066	726.8	.6113	1293.7	619.4	.6714	.001597	.8194	3.453	2.163	905.6
14,000	9.10	-12.72	.9507	1062	724.0	.5875	1243.2	595.2	.6500	.001546	.8063	3.432	2.220	870.2
15,000	5.54	-14.70	.9470	1058	721.2	.5644	1194.3	571.8	.6292	.001497	.7933	3.411	2.280	836.0
16,000	1.98	-16.68	.9434	1054	718.5	.5420	1147.0	549.1	.6090	.001448	.7804	3.390	2.341	802.9
17,000	- 1.59	-18.66	.9397	1050	715.7	.5203	1101.1	527.2	.5892	.001401	.7676	3.369	2.404	770.8
18,000	- 5.15	-20.64	.9361	1046	712.9	.4994	1056.9	506.0	.5699	.001355	.7549	3.347	2.470	739.8
19,000	- 8.72	-22.62	.9324	1041	710.1	.4792	1014.0	485.5	.5511	.001311	.7424	3.326	2.538	709.8
20,000	-12.28	-24.60	.9287	1037	707.3	.4596	972.6	465.6	.5328	.001267	.7299	3.305	2.608	680.8
21,000	-15.84	-26.58	.9250	1033	704.5	.4406	932.5	446.4	.5150	.001225	.7176	3.283	2.681	652.8
22,000	-19.41	-28.56	.9213	1029	701.6	.4223	893.8	427.9	.4976	.001183	.7054	3.262	2.757	625.7
23,000	-22.97	-30.54	.9175	1025	698.8	.4047	856.4	410.0	.4807	.001143	.6933	3.240	2.834	599.5
24,000	-26.54	-32.52	.9138	1021	695.9	.3876	820.3	392.7	.4642	.001104	.6813	3.218	2.915	574.2
25,000	-30.10	-34.50	.9100	1017	693.1	.3711	785.3	376.0	.4481	.001066	.6694	3.196	2.999	549.7
26,000	-33.66	-36.48	.9062	1012	690.2	.3552	751.7	359.9	.4325	.001029	.6576	3.174	3.087	526.2
27,000	-37.23	-38.46	.9024	1008	687.3	.3399	719.2	344.3	.4173	.000993	.6460	3.153	3.177	503.4
28,000	-40.79	-40.44	.8986	1004	684.4	.3251	687.9	329.3	.4025	.000957	.6345	3.130	3.270	481.5
29,000	-44.36	-42.42	.8948	999	681.5	.3108	657.6	314.9	.3882	.000923	.6230	3.108	3.367	460.3
30,000	-47.92	-44.40	.8909	995	678.5	.2970	628.5	300.9	.3741	.000890	.6116	3.086	3.469	440.0
31,000	-51.48	-46.38	.8871	991	675.6	.2837	600.4	287.5	.3606	.000858	.6005	3.064	3.573	420.2
32,000	-55.05	-48.36	.8832	987	672.6	.2709	573.3	274.5	.3473	.000826	.5894	3.041	3.682	401.3
33,000	-58.61	-50.34	.8793	982	669.7	.2586	547.3	262.0	.3345	.000796	.5784	3.019	3.795	383.1
34,000	-62.18	-52.32	.8754	978	666.7	.2467	522.2	250.0	.3220	.000766	.5675	2.997	3.913	365.6
35,000	-65.74	-54.30	.8714	973	663.7	.2353	498.0	238.4	.3099	.000737	.5567	2.974	4.036	348.8
35,332	-67.6	-55.33	.8693	971	662.1	.2314	489.8	234.5	.3058	.000727	.5530	2.961	4.073	342.9
36,000	-67.6	-55.33	.8693	971	662.1	.2244	474.6	227.3	.2981	.000709	.5460	2.951	4.176	332.4
37,000	-67.6	-55.33	.8693	971	662.1	.2138	452.5	216.7	.2845	.0006766	.5334	2.961	4.376	316.8
38,000	-67.6	-55.33	.8693	971	662.1	.2038	431.2	206.5	.2711	.0006448	.5207	2.961	4.592	301.8
39,000	-67.6	-55.33	.8693	971	662.1	.1942	411.0	196.8	.2584	.0006145	.5083	2.961	4.819	287.7
40,000	-67.6	-55.33	.8693	971	662.1	.1851	391.8	187.6	.2463	.0005857	.4963	2.961	5.055	274.3
41,000	-67.6	-55.33	.8693	971	662.1	.1764	373.4	178.8	.2347	.0005582	.4845	2.961	5.305	261.4
42,000	-67.6	-55.33	.8693	971	662.1	.1681	355.8	170.4	.2237	.0005320	.4730	2.961	5.566	249.1
43,000	-67.6	-55.33	.8693	971	662.1	.1603	339.1	162.4	.2132	.0005071	.4617	2.961	5.839	237.4
44,000	-67.6	-55.33	.8693	971	662.1	.1527	323.2	154.8	.2032	.0004833	.4508	2.961	6.127	226.2
45,000	-67.6	-55.33	.8693	971	662.1	.1456	308.0	147.5	.1936	.0004605	.4400	2.961	6.430	215.6
46,000	-67.6	-55.33	.8693	971	662.1	.1387	293.6	140.6	.1846	.0004390	.4296	2.961	6.745	205.5
47,000	-67.6	-55.33	.8693	971	662.1	.1322	279.8	134.0	.1759	.0004184	.4194	2.961	7.077	195.9
48,000	-67.6	-55.33	.8693	971	662.1	.1260	266.6	127.7	.1676	.0003987	.4094	2.961	7.427	186.6
49,000	-67.6	-55.33	.8693	971	662.1	.1201	254.1	121.7	.1598	.0003800	.3997	2.961	7.792	177.9
50,000	-67.6	-55.33	.8693	971	662.1	.1145	242.2	116.0	.1523	.0003622	.3902	2.961	8.175	169.5
60,000	-67.6	-55.33	.8693	971	662.1	.0713	150.9	72.2	.0942	.0002240	.3069	2.961	13.219	105.6
70,000	-67.6	-55.33	.8693	971	662.1	.0442	93.5	44.8	.0584	.0001389	.2417	2.961	21.317	65.5
80,000	-67.6	-55.33	.8693	971	662.1	.0274	58.0	27.8	.0362	.0000861	.1903	2.961	34.390	40.6
90,000	-67.6	-55.33	.8693	971	662.1	.0170	36.0	17.2	.0225	.0000535	.1500	2.961	55.346	25.2
100,000	-67.6	-55.33	.8693	971	662.1	.0106	22.4	10.7	.0140	.0000331	.1183	2.961	89.456	15.7
104,987	-67.6	-55.33	.8693	971	662.1	.00831	17.59	8.42	.0110	.0000261	.1048	2.961	113.4	12.31
110,000	-47.4	-44.11	.8917	996	679.0	.00655	13.92	6.66	.00827	.0000197	.09093	3.090	157.2	9.744
120,000	- 7.2	-21.78	.9346	1043	711.1	.00426	9.026	4.32	.00448	.0000116	.06988	3.339	287.6	6.318
130,000	33.0	.56	.9749	1089	742.5	.00287	6.071	2.91	.00302	.0000071?	.05493	3.579	498.9	4.250
140,000	73.3	22.94	1.0143	1132	771.8	.00199	4.213	2.02	.00193	.00000460	.04399	3.809	827.9	2.949
150,000	113.5	45.28	1.0510	1174	800.5	.00142	3.003	1.45	.00128	.00000305	.03582	4.032	1322	2.102
160,000	153.7	67.61	1.0877	1215	828.4	.00103	2.190	1.05	.00087	.00000208	.02957	4.247	2043	1.533
164,042	170.0	76.67	1.1021	1231	839.3	.000916	1.938	.928	.00075	.00000179	.02746	4.332	2417	1.357
170,000	170.0	76.67	1.1021	1231	839.3	.000767	1.624	.777	.00063	.00000150	.02513	4.332	2886	1.137
180,000	170.0	76.67	1.1021	1231	839.3	.000570	1.206	.577	.00047	.00000111	.02165	4.332	3885	.8442
190,000	170.0	76.67	1.1021	1231	839.3	.000423	.8956	.429	.00035	.00000083	.01866	4.332	5232	.6269
196,850	170.0	76.67	1.1021	1231	839.3	.000345	.7305	.350	.00028	.00000068	.01685	4.332	6412	.5114
200,000	159.4	70.78	1.0931	1220	831.8	.000314	.6646	.318	.00026	.00000062	.01621	4.277	6844	.4652
210,000	125.9	52.17	1.0627	1187	809.3	.000230	.4869	.233	.00020	.00000048	.01427	4.099	8467	.3408
220,000	92.4	33.56	1.0322	1152	785.5	.000166	.3504	.168	.00015	.00000037	.01247	3.916	10600	.2453
230,000	58.9	14.94	1.0000	1117	761.6	.000117	.2470	.118	.00012	.00000028	.01080	3.727	13400	.1729
240,000	25.3	-3.72	.9669	1080	736.4	.000080	.1699	.081	.00009	.00000020	.00926	3.533	17330	.1189
250,000	- 8.2	-22.33	.9329	1042	710.5	.000054	.1139	.054	.00006	.00000015	.00786	3.333	22700	.0797
255,905	-28.0	-33.33	.9123	1019	694.8	.000042	.0886	.042	.00005	.00000012	.00709	3.212	26680	.0621
260,000	-28.0	-33.33	.9123	1019	694.8	.000035	.0742	.035	.00004	.00000010	.00649	3.212	32090	.0520

List of symbols: h = height; t = temperature; a = velocity of sound; p = pressure; ρ = density; σ = density relative to sea level; μ = coefficient of viscosity; ν = coefficient of kinematic viscosity

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The subscript SL refers to sea level (h = 0)

Table 13-2. A.S.A. Recommended Octave Designations

OLD[1]		NEW[2]			
FREQUENCY BAND	CENTER FREQUENCY (GEOMETRIC MEAN)	FREQUENCY BAND		CENTER FREQUENCY (GEOMETRIC MEAN)	
(Hz)	(Hz)	⅓ OCTAVE (Hz)	OCTAVE (Hz)	⅓ OCTAVE (Hz)	OCTAVE (Hz)
4.7-9.4	6.6	5.7- 7.1 7.1- 8.8 8.8-11.3	5.7-11.3	6.3 8.0 10.0	8.0
9.4-18.7	13.2	11.3-14.2 14.2-17.7 17.7-22.4	11.3-22.4	12.5 16.0 20.0	16.0
18.7-37.5	26.5	22.4-28.3 28.3-35.4 35.4-44.5	22.4-44.5	25.0 31.5 40.0	31.5
37.5-75	53.0	44.5-56.5 56.5-70.7 70.7-88.7	44.5-88.7	50.0 63.0 80.0	63.0
75-150	106	88.7-112.2 112.2 -141 141-177	88.7-177	100 125 160	125
150-300	212	177-224 224-283 283-354	177-354	200 250 315	250
300-600	424	354-445 445-566 566-707	354-707	400 500 630	500
600-1200	848	707- 887 887-1130 1130-1414	707-1414	800 1000 1250	1000
1200-2400	1696	1414-1770 1770-2240 2240-2830	1414-2830	1600 2000 2500	2000
2400-4800	3392	2830-3540 3540-4450 4450-5660	2830-5660	3150 4000 5000	4000
4800-9600	6784	5660- 7070 7070- 8870 8870-11300	5660-11300	6300 8000 10000	8000
9600-19200	13568	11300-14140 14140-17700 17700-22400	11300-22400	12500 16000 20000	16000
19200-38400	27136	22400-28300 28300-35400 35400-44500	22400-44500	25000 31500 40000	31500

For a constant spectrum level within the frequency band of interest, add 0.7 dB to each of the "old-octave-band" SPL's to determine a comparable "new-octave-band" SPL.

[1] "Octave-Band Filter Set for the Analysis of Noise and Other Sounds," American Standard Association, Spec Z24.10-1953

[2] "Preferred Frequencies of Accoustical Measurements," American Standards Association, Spec S1.6-1960

Table 13-3. Relation Between SPL in Octave Bands to Loudness in Sones

OCTAVE BAND LEVEL IN dB	OCTAVE BANDS								
	1	2	3	4	5	6	7	8	9
88	13	21	27	30	30	34	49	60	
87	12	19	25	28	28	32	45	56	
86	11	18	24	26	26	30	42	52	
85	10	16	22	25	25	28	39	48	
84	9	15	21	23	23	26	36	45	
83	8	14	20	21	21	25	34	42	
82	7.5	13	18	20	20	23	32	39	
81	7	12	17	18	18	21	30	36	
80	6	11	16	17	17	20	28	34	
79	5.5	10	15	16	16	19	26	32	
78	5	9.5	14	15	15	18	25	30	
77	4.5	9.0	13	14	14	16	23	28	
76	4.0	8.0	12	13	13	15	21	26	
75	4.0	7.5	11	12	12	14	19	24	
74	3.5	7.0	10	11	11	13	18	22	
73	3.0	6.0	10	10	10	12	17	21	
72	3.0	6.0	9	10	10	11	16	20	
71	2.5	5.5	8	9	9	11	15	18	
70	2.5	5.0	8	9	9	10	14	17	
69	2.0	4.5	7	8	8	9	13	16	
68	2.0	4.0	6.5	7.5	7.5	9	12	15	
67	1.5	4.0	6	7	7	8	11	14	
66	1.5	3.5	5.5	7	7	7.5	10.5	13	
65	1.5	3.0	5	6	6	7	10	12	
64	1.0	3.0	5	6	6	7	9	11	
63	1.0	3.0	4.5	5	5	6	9	11	
62	1.0	2.5	4	5	5	6	8	10	
61	1.0	2.5	4	5	5	5	8	9	
60	1.0	2.0	3.5	4.5	4.5	5	7	8.5	
59						5	7	8	

NOTE: dB re: 0.0002 dynes/cm^2

Table 13-3. (Continued)

OCTAVE BAND LEVEL IN dB	OCTAVE BANDS								
	1	2	3	4	5	6	7	8	9
120	139	195	250	272	276	310	450	550	
119	129	181	228	254	258	290			
118	119	168	212	238	241	272			
117	112	159	200	222	225	254			
116	105	148	188	208	209	237			
115	98	138	176	194	194	221			
114	92	129	164	181	181	206			
113	86	120	154	169	170	194			
112	80	111	144	157	159	182			
111	75	103	134	146	148	171			
110	70	95	125	136	138	160	224	280	
109	65	88	116	127	128	149	208	260	
108	60	82	107	119	119	138	194	240	
107	56	77	100	111	111	129	180	225	
106	52	72	93	104	104	120	168	210	
105	48	67	87	97	97	112	156	195	
104	45	63	81	91	91	104	145	180	
103	42	59	76	85	85	98	135	167	
102	39	55	71	79	79	92	126	155	
101	36	51	67	73	74	86	119	143	
100	33	48	63	68	69	80	112	133	
99	31	45	59	64	64	74	105	124	
98	29	42	55	60	60	69	98	116	
97	27	39	51	56	56	64	91	109	
96	25	36	47	52	52	60	84	102	
95	23	34	44	49	48	56	78	96	
94	21	31	41	46	45	52	72	90	
93	19	29	39	43	42	49	68	84	
92	18	27	36	40	39	46	64	78	
91	16	26	33	37	37	43	60	73	
90	15	24	31	34	35	40	56	68	
89	14	22	29	32	33	37	53	64	

Table 13-4. Relation Between SPL and Noisiness in Noys — Both in Octave Bands

SPL - dB re 0.0002 DYNES/CM²	FREQUENCY BAND							
	20–75	75–150	150–300	300–600	600–1200	1200–2400	2400–4800	4800–10,000
	NOYS	NOYS	NOYS	NOYS	NOYS	NOYS	NOYS	NOYS
20							0.6	1.0
30				0.5	0.5	0.7	1.2	2.1
40			0.6	1.0	1.0	1.4	2.4	4.3
45			0.9	1.5	1.5	2.0	3.3	6.0
50		0.6	1.4	2.1	2.1	2.9	4.7	8.5
51		0.6	1.6	2.2	2.2	3.0	5.1	9.0
52		0.7	1.7	2.4	2.4	3.2	5.4	9.7
53		0.8	1.8	2.6	2.6	3.5	5.8	10.5
54		0.9	2.0	2.8	2.8	3.7	6.2	11.0
55		1.0	2.2	3.0	3.0	4.0	6.7	11.8
56		1.1	2.4	3.2	3.2	4.3	7.2	12.5
57	0.5	1.2	2.6	3.4	3.4	4.6	7.7	13.4
58	0.6	1.3	2.7	3.6	3.6	5.0	8.2	14.4
59	0.6	1.5	3.0	3.9	3.9	5.3	8.7	15.4
60	0.7	1.6	3.2	4.2	4.2	5.7	9.3	16.5
61	0.8	1.8	3.4	4.5	4.5	6.1	10.0	17.6
62	0.9	2.0	3.7	4.8	4.8	6.5	10.7	18.8
63	1.0	2.2	4.0	5.2	5.2	7.0	11.5	20.0
64	1.1	2.4	4.3	5.6	5.6	7.5	12.3	21.3
65	1.2	2.6	4.7	6.0	6.0	8.0	13.1	22.7
66	1.4	2.9	5.1	6.5	6.5	8.6	14.0	24.0
67	1.6	3.2	5.5	7.0	7.0	9.2	15.0	25.6
68	1.8	3.5	5.9	7.5	7.5	9.9	16.0	27.4
69	2.0	3.9	6.3	7.9	7.9	10.7	17.2	29.3
70	2.2	4.3	6.8	8.4	8.4	11.5	18.5	31.3
71	2.5	4.7	7.3	9.0	9.0	12.2	19.8	33.4
72	2.7	5.1	7.8	9.6	9.6	13.0	21.0	35.5
73	3.0	5.6	8.4	10.3	10.3	14.0	22.5	38.0
74	3.4	6.1	9.0	11.0	11.0	15.0	24.0	40.5
75	3.8	6.6	9.7	11.8	11.8	16.0	25.7	43
76	4.2	7.3	10.5	12.7	12.7	17.0	27.5	46
77	4.6	7.9	11.2	13.7	13.7	18.3	29.5	49
78	5.0	8.5	12.0	14.7	14.7	19.5	31.5	51
79	5.5	9.2	13.0	15.8	15.8	21.0	33.7	55
80	6.2	9.9	14.0	17.0	17.0	22.5	36	59
81	6.7	10.8	15.0	18.3	18.3	24.0	38	63
82	7.3	11.7	16.0	19.5	19.5	25.5	41	67
83	8.0	12.7	17.1	21.0	21.0	27.5	44	71
84	8.7	13.8	18.3	22.5	22.5	29.5	47	76

SPL – dB re 0.0002 DYNES/CM²	FREQUENCY BAND							
	20–75	75–150	150–300	300–600	600–1200	1200–2400	2400–4800	4800–10,000
	NOYS	NOYS	NOYS	NOYS	NOYS	NOYS	NOYS	NOYS
85	9.4	14.8	19.5	24.5	24.5	31	50	81
86	10.2	15.9	21.0	26	26	34	53	86
87	11.1	17.0	22.5	28	28	36	57	92
88	12.1	18.3	24	30	30	38	61	97
89	13.1	19.5	26	32	32	41	65	103
90	14.2	21	28	34	34	44	70	110
91	15.3	22	30	37	37	47	75	117
92	16.5	24	32	40	40	50	80	124
93	17.0	26	34	43	43	53	85	132
94	19.5	28	36	46	46	57	91	141
95	21.0	30	39	49	49	61	97	150
96	22.5	32	42	53	53	66	104	160
97	24.0	34	45	57	57	71	111	170
98	26.5	37	48	61	61	75	119	180
99	28.0	40	51	65	65	80	127	191
100	30.5	42	55	70	70	86	135	203
101	32.5	45	59	75	75	92	144	215
102	35	49	63	80	80	98	159	228
103	38	53	67	86	86	105	165	242
104	41	56	71	92	92	113	175	256
105	43	60	76	98	98	120	187	273
106	46	64	82	106	106	128	200	290
107	50	68	88	113	113	136	213	310
108	53	73	94	120	120	145	226	328
109	57	78	100	129	129	155	240	348
110	61	84	106	139	139	165	260	370
111	66	90	114	150	150	177	275	395
112	71	96	122	166	166	190	293	420
113	76	103	130	170	170	203	315	445
114	82	111	139	181	181	215	335	470
115	87	119	148	192	192	230	360	500
116	93	125	158	205	205	245	380	
117	100	134	169	220	220	260	405	
118	106	145	180	237	237	280	435	
119	113	155	192	255	255	300	465	
120	120	165	204	275	275	320	500	
125	170	225	280	385	385	440		
130	235	315	390		510			
135	325	425						
140	450							

Reference: L. L. Beranek, K.D. Kryter and L. N. Miller, "The Reaction of People to Exterior Aircraft Noise," ASME Publication, Paper Number 59-AV-50, Table I, (Presented at the Aviation Conference, March 9-12, 1959, of The American Society of Mechanical Engineers).

Table 13-5. Relation Between -- Loudness Level in Noys and Loudness in Sones: Perceived Noise Level in PNdB and Noisiness in Noys

PNdB L. L.	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.062	.063	.063	.064	.064	.065	.065	.066	.066	.067
1	.067	.067	.068	.068	.069	.069	.070	.070	.071	.071
2	.072	.072	.073	.073	.074	.074	.075	.075	.076	.076
3	.077	.077	.078	.079	.079	.080	.080	.081	.081	.082
4	.082	.083	.084	.084	.085	.085	.086	.087	.087	.088
5	.088	.089	.090	.090	.091	.092	.092	.093	.093	.094
6	.095	.095	.096	.097	.097	.098	.099	.099	.100	.101
7	.102	.102	.103	.104	.104	.105	.106	.107	.107	.108
8	.109	.110	.110	.111	.112	.113	.113	.114	.115	.116
9	.117	.117	.118	.119	.120	.121	.122	.122	.123	.124
10	.125	.126	.127	.128	.129	.129	.130	.131	.132	.133
11	.134	.135	.136	.137	.138	.139	.140	.141	.142	.143
12	.144	.145	.146	.147	.148	.149	.150	.151	.152	.153
13	.154	.155	.156	.157	.158	.159	.160	.162	.163	.164
14	.165	.166	.167	.168	.170	.171	.172	.173	.174	.174
15	.177	.178	.179	.180	.182	.183	.184	.186	.187	.188
16	.189	.191	.192	.193	.195	.196	.198	.199	.200	.202
17	.203	.204	.206	.207	.209	.210	.212	.213	.215	.216
18	.218	.219	.221	.222	.224	.225	.227	.228	.230	.232
19	.233	.235	.237	.238	.240	.241	.243	.245	.247	.248
20	.250	.251	.253	.255	.257	.259	.261	.262	.264	.266
21	.268	.270	.272	.274	.275	.277	.279	.281	.283	.285
22	.287	.289	.291	.293	.295	.297	.299	.301	.304	.306
23	.308	.310	.312	.314	.316	.319	.321	.323	.325	.328
24	.330	.332	.334	.337	.339	.342	.344	.346	.349	.351
25	.354	.356	.358	.361	.363	.366	.369	.371	.374	.376
26	.379	.382	.384	.387	.390	.392	.395	.398	.401	.403
27	.406	.409	.412	.415	.418	.420	.423	.426	.429	.432
28	.435	.438	.441	.444	.448	.451	.454	.457	.460	.463
29	.467	.470	.473	.476	.480	.483	.486	.490	.493	.497
30	.500	.503	.507	.511	.514	.518	.521	.525	.529	.532
31	.536	.540	.543	.547	.551	.555	.559	.563	.566	.570
32	.574	.578	.582	.586	.590	.595	.599	.603	.607	.611
33	.616	.620	.624	.629	.633	.637	.642	.646	.651	.655
34	.660	.664	.669	.674	.678	.683	.688	.693	.697	.702
35	.707	.712	.717	.722	.727	.732	.737	.742	.747	.753
36	.758	.763	.768	.774	.779	.785	.790	.796	.801	.807
37	.812	.818	.824	.829	.835	.841	.847	.853	.859	.865
38	.871	.877	.883	.889	.895	.901	.908	.914	.920	.927
39	.933	.940	.946	.953	.959	.966	.973	.979	.986	.993
40	1.00	1.01	1.01	1.02	1.03	1.04	1.04	1.05	1.06	1.06
41	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.13	1.13	1.14
42	1.15	1.16	1.16	1.17	1.18	1.19	1.20	1.21	1.21	1.22
43	1.23	1.24	1.25	1.26	1.27	1.27	1.28	1.29	1.30	1.31
44	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39	1.39	1.40
45	1.41	1.42	1.43	1.44	1.45	1.46	1.47	1.48	1.49	1.51
46	1.52	1.53	1.54	1.55	1.56	1.57	1.58	1.59	1.60	1.61
47	1.62	1.64	1.65	1.66	1.67	1.68	1.69	1.71	1.72	1.73
48	1.74	1.75	1.77	1.78	1.79	1.80	1.82	1.83	1.84	1.85
49	1.87	1.88	1.89	1.91	1.92	1.93	1.95	1.96	1.97	1.99

Example: $1.17 \frac{\text{sones}}{\text{noys}} = 42.3 \frac{\text{Noys}}{\text{PNdB}}$

Table 13-5. Relation Between — Loudness Level in Noys and Loudness in Sones: Perceived Noise Level in PNdB and Noisiness in Noys (Continued)

PNdB L. L.	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50	2.00	2.01	2.03	2.04	2.06	2.07	2.08	2.10	2.11	2.13
51	2.14	2.16	2.17	2.19	2.20	2.22	2.23	2.25	2.27	2.28
52	2.30	2.31	2.33	2.35	2.36	2.38	2.39	2.41	2.43	2.45
53	2.46	2.48	2.50	2.51	2.53	2.55	2.57	2.58	2.60	2.62
54	2.64	2.66	2.68	2.69	2.71	2.73	2.75	2.77	2.79	2.81
55	2.83	2.85	2.87	2.89	2.91	2.93	2.95	2.97	2.99	3.01
56	3.03	3.05	3.07	3.10	3.12	3.14	3.16	3.18	3.10	3.23
57	3.25	3.27	3.29	3.32	3.34	3.36	3.39	3.41	3.43	3.46
58	3.48	3.51	3.53	3.56	3.58	3.60	3.63	3.66	3.68	3.71
59	3.73	3.76	3.78	3.81	3.84	3.86	3.89	3.92	3.94	3.97
60	4.00	4.03	4.06	4.08	4.11	4.14	4.17	4.20	4.23	4.26
61	4.29	4.32	4.35	4.38	4.41	4.44	4.47	4.50	4.53	4.56
62	4.59	4.63	4.66	4.69	4.72	4.76	4.79	4.82	4.86	4.89
63	4.92	4.96	4.99	5.03	5.06	5.10	5.13	5.17	5.21	5.24
64	5.28	5.31	5.35	5.39	5.43	5.46	5.50	5.54	5.58	5.62
65	5.66	5.70	5.73	5.78	5.82	5.86	5.90	5.94	5.98	6.02
66	6.06	6.11	6.15	6.19	6.23	6.28	6.32	6.36	6.41	6.45
67	6.50	6.54	6.59	6.63	6.68	6.73	6.77	6.82	6.87	6.92
68	6.96	7.01	7.06	7.11	7.16	7.21	7.26	7.31	7.36	7.41
69	7.46	7.52	7.57	7.62	7.67	7.73	7.78	7.84	7.89	7.94
70	8.00	8.06	8.11	8.17	8.22	8.28	8.34	8.40	8.46	8.51
71	8.57	8.63	8.69	8.75	8.82	8.88	8.94	9.00	9.06	9.13
72	9.19	9.25	9.32	9.38	9.45	9.51	9.58	9.65	9.71	9.78
73	9.85	9.92	9.98	10.1	10.1	10.2	10.3	10.3	10.4	10.5
74	10.6	10.6	10.7	10.8	10.9	10.9	11.0	11.1	11.2	11.2
75	11.3	11.4	11.5	11.6	11.6	11.7	11.8	11.9	12.0	12.0
76	12.1	12.2	12.3	12.4	12.5	12.6	12.6	12.7	12.8	12.9
77	13.0	13.1	13.2	13.3	13.4	13.5	13.5	13.6	13.7	13.8
78	13.9	14.0	14.1	14.2	14.3	14.4	14.5	14.6	14.7	14.8
79	14.9	15.0	15.1	15.2	15.3	15.5	15.6	15.7	15.8	15.9
80	16.0	16.1	16.2	16.3	16.4	16.6	16.7	16.8	16.9	17.0
81	17.1	17.3	17.4	17.5	17.6	17.8	17.9	18.0	18.1	18.3
82	18.4	18.5	18.6	18.8	18.9	19.0	19.2	19.3	19.4	19.6
83	19.7	19.8	20.0	20.1	20.3	20.4	20.5	20.7	20.8	21.0
84	21.1	21.3	21.4	21.6	21.7	21.9	22.0	22.2	22.3	22.5
85	22.6	22.8	22.9	23.1	23.3	23.4	23.6	23.8	23.9	24.1
86	24.3	24.4	24.6	24.8	24.9	25.1	25.3	25.5	25.6	25.8
87	26.0	26.2	26.4	26.5	26.7	26.9	27.1	27.3	27.5	27.7
88	27.9	28.1	28.2	28.4	28.6	28.8	29.0	29.2	29.4	29.7
89	29.9	30.1	30.3	30.5	30.7	30.9	31.1	31.3	31.6	31.8
90	32.0	32.2	32.4	32.7	32.9	33.1	33.4	33.6	33.8	34.1
91	34.3	34.5	34.8	35.0	35.3	35.5	35.8	36.0	36.3	36.5
92	36.8	37.0	37.3	37.5	37.8	38.1	38.3	38.6	38.9	39.1
93	39.4	39.7	39.9	40.2	40.5	40.8	41.1	41.4	41.6	41.9
94	42.2	42.5	42.8	43.1	43.4	43.7	44.0	44.3	44.6	44.9
95	45.3	45.6	45.9	46.2	46.5	46.9	47.2	47.5	47.8	48.2
96	48.5	48.8	49.2	49.5	49.9	50.2	50.6	50.9	51.3	51.6
97	52.0	52.3	52.7	53.1	53.4	53.8	54.2	54.6	54.9	55.3
98	55.7	56.1	56.5	56.9	57.3	57.7	58.1	58.5	58.9	59.3
99	59.7	60.1	60.5	61.0	61.4	61.8	62.2	62.7	63.1	63.6

Table 13-5. Relation Between — Loudness Level in Noys and Loudness in Sones: Perceived Noise Level in PNdB and Noisiness in Noys (Continued)

PNdB L. L.	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
100	64.0	64.4	64.9	65.3	65.8	66.3	66.7	67.2	67.6	68.1
101	68.6	69.1	69.6	70.0	70.5	71.0	71.5	72.0	72.5	73.0
102	73.5	74.0	74.5	75.1	75.6	76.1	76.6	77.2	77.7	78.2
103	78.8	79.3	79.9	80.4	81.0	81.6	82.1	82.7	83.3	83.9
104	84.4	85.0	85.6	86.2	86.8	87.4	88.0	88.6	89.3	89.9
105	90.5	91.1	91.8	92.4	93.1	93.7	94.4	95.0	95.7	96.3
106	97.0	97.7	98.4	99.0	99.7	100	101	102	103	103
107	104	105	105	106	107	108	108	109	110	111
108	111	112	113	114	115	115	116	117	118	119
109	119	120	121	122	123	124	124	125	126	127
110	128	129	130	131	132	133	133	134	135	136
111	137	138	139	140	141	142	143	144	145	146
112	147	148	149	150	151	152	153	154	155	156
113	158	159	160	161	162	163	164	165	167	168
114	169	170	171	172	174	175	176	177	179	180
115	181	182	184	185	186	187	188	190	191	193
116	194	195	197	198	199	201	202	204	205	206
117	208	209	211	212	214	215	217	218	220	221
118	223	224	226	228	229	231	232	234	236	237
119	239	241	242	244	246	247	249	251	252	254
120	256	258	260	261	263	265	267	269	271	272
121	274	276	278	280	282	284	286	288	290	292
122	294	296	298	300	302	304	307	309	311	313
123	315	317	319	322	324	326	329	331	333	335
124	338	340	343	345	347	350	352	355	357	360
125	362	365	367	370	372	375	377	380	383	385
126	388	391	393	396	399	402	405	407	410	413
127	416	419	422	425	428	431	434	437	440	443
128	446	449	452	455	458	461	465	468	471	474
129	478	481	484	488	491	495	498	501	505	508
130	512	516	519	523	526	530	534	537	541	545
131	549	553	556	560	564	568	572	576	580	584
132	588	592	596	600	605	609	613	617	622	626
133	630	635	639	644	648	653	657	662	666	671
134	676	680	685	690	695	699	704	709	714	719
135	724	729	734	739	744	750	755	760	765	771
136	776	781	787	792	798	803	809	815	820	826
137	832	838	843	849	855	861	867	873	879	885
138	891	898	904	910	917	923	929	936	942	949
139	955	962	969	976	982	989	996			

Reference: S. S. Stevens, "Calculation of the Loudness of Complex Noise", The Journal of the Acoustical Society of America, Vol. 28, NO. 5, 807-832, September 1956, Table VII, pp 830-832.

Section 14
TERMINOLOGY

14-1 DEFINITIONS

ACCELERATION is a vector quantity that specifies the time rate of change of velocity.

ACOUSTIC, ACOUSTICAL. The qualifying adjectives "acoustic" and "acoustical" mean containing, producing, arising from, actuated by, related to, or associated with sound. Acoustic is used when the term being qualified designates something that has the properties, dimensions, or physical characteristics associated with sound waves; acoustical is used when the term being qualified does not designate explicitly something that has such properties, dimensions, or physical characteristics.

NOTE 1

The following examples qualify as having the "properties or physical characteristics associated with sound waves" and hence would take acoustic: impedance, inertance, load (radiation field), output (sound power), energy, wave, environment, signal, conduit, absorptivity, transducer.

NOTE 2

The following examples do not have the requisite physical characteristics and therefore take acoustical: society, method, engineer, school, glossary, symbol, problem, measurement, point of view, end-use, device.

NOTE 3

As illustrated in the preceding notes, usually the generic term is modified by acoustical, whereas the specific technical implication calls for acoustic.

ACOUSTIC IMPEDANCE of a fluid medium on a given surface lying in a wave front is the complex ratio of the sound pressure (force per unit area) on that surface to the flux (volume velocity, or particle velocity multiplied by the area) through the surface. When concentrated rather than distributed impedances are considered, the impedance of a portion of the medium is based on the pressure difference effective in driving that portion and the flux (volume velocity). The acoustic impedance may be expressed in terms of mechanical impedance divided by the square of the area of the surface considered.

NOTE 1

Velocities in the direction along which the impedance is to be specified are considered positive.

NOTE 2

The real part of an acoustic impedance is *acoustic resistance* and the imaginary part is *acoustic reactance.*

ALTITUDE is the perpendicular distance from the surafce of the earth at sea level.

AMBIENT NOISE is the all-encompassing noise associated with a given environment, being usually a composite of sounds from many sources near and far.

AMPLITUDE is the maximum value of a sinusoidal quantity.

NOTE 1

The maximum value of a sinusoidal quantity is the amplitude; it is sometimes called (for emphasis) the *single* amplitude to distinguish from the *double* amplitude, the latter being the total peak-to-peak excursion of the sinusoidal quantity.

NOTE 2

For quantities that are not sinusoidal, the instantaneous or probabilistic values for the quantities are designated magnitudes.

ANECHOIC ROOM (Free-Field Room) is one whose boundaries absorb effectively all the sound incident thereon, thereby affording essentially free-field conditions.

ANGLE OF ATTACK is the angle made by the line of zero lift of a structure with the free stream velocity.

ANGULAR FREQUENCY (Circular Frequency) of a periodic quantity, in radians per unit time, is the frequency multiplied by 2π The usual symbol is ω.

ANTINODE is a point, line, or surface in a standing wave (structure) where some characteristic of the wave field (vibration) has maximum amplitude.

NOTE

The appropriate modifier should be used before the word *antinode* to signify the type that is intended; e.g., displacement antinode, velocity antinode, pressure antinode.

APERIODIC VIBRATION is a continuing deterministic vibration which is not periodic.

AUDIO FREQUENCY is any frequency corresponding to a normally audible sound wave.

NOTE 1

Audio frequencies range roughly from 15 to 20,000 cycles per second.

NOTE 2

The word *audio* may be used as a modifier to indicate a device or system intended to operate at audio frequencies, e.g., *audio amplifier*.

AUTOCORRELATION COEFFICIENT $\rho(\tau)$ of a quantity $\xi(t)$ is the ratio of the correlation function $R(\tau)$ to the mean-square value $R(0)$ of the quantity:

$$\rho(\tau) = \frac{R(\tau)}{R(0)}$$

At any delay, τ, the autocorrelation coefficient satisfies $-1 \leq \rho(\tau) \leq 1$.

AUTOCORRELATION FUNCTION $R(\tau)$ of a quantity $\xi(t)$ is the average of the product of the quantity at time t with the quantity at time $(t + \tau)$ for an appropriate averaging time T:

$$R(\tau) = \frac{1}{T}\int_0^T \xi(t)\,\xi(t+\tau)\,dt$$

NOTE

For a stationary random quantity $\xi(t)$ which persists for all time, T should approach infinity. In practice, T must be finite, and then the formula gives an estimate only with a certain statistical uncertainty which increases as T decreases.

AVERAGE VALUE. (Same as *Mean Value*.)

AVERAGE ABSOLUTE VALUE $|\overline{\xi}|$ of a quantity $\xi(t)$ is the time average of the absolute value of the quantity for an appropriate averaging time T:

$$|\overline{\xi}| = \frac{1}{T}\int_0^T |\xi(t)|\,dt$$

See NOTE under *Autocorrelation Function*.

BACKGROUND NOISE is the total of all sources of interference in a system used for the production, detection, measurement, or recording of a signal, independent of the presence of the signal.

NOTE 1

Ambient noise detected, measured, or recorded with the signal becomes part of the background noise.

NOTE 2

Included in this definition is the interference resulting from primary power supplies, that separately is commonly described as hum.

BAND-PASS FILTER (ideal) is a filter that effectively passes (without distortion) only those frequency components in the band between a lower cutoff frequency (greater than zero) and a finite upper cutoff frequency. Practical filters involve phase shifting as well as attenuation of frequency components both within and without the filter band.

BASE PRESSURES (both the static and fluctuating components thereof) are those pressures measured on the base (aft end) of a blunt-ended structure which has a relative velocity between itself and the surrounding medium.

BEATS are periodic variations that result from the superposition of two simple harmonic quantities of different frequencies f_1 and f_2. They involve the periodic increase and decrease of amplitude at the beat frequency $(|f_1 - f_2|)$.

BOUNDARY LAYER (Laminar and Turbulent) is a thin layer of the flow about a solid body in which the external velocity changes relative to the solid surface. In this layer, viscosity plays an essential part, the large velocity gradients across this flow giving rise to shear stresses and surface (skin) friction.

A turbulent boundary layer is one in which local random fluctuations in velocity, pressure, and other physical quantities exist. It is formed from a laminar boundary layer which has become dynamically unstable, and undergone transition at some streamwise location along the body (depending primarily on the value of the local Reynolds number).

BOUNDARY LAYER THICKNESS is usually defined as that distance from the wall where the velocity differs by 1 percent from the external velocity.

BUFFET was formerly defined as the phenomenon of rapidly alternating static pressure between two different, but fixed levels due to unstable flow. The term has since been generalized to include any alternating pressure (regardless of the source of characteristics of the pressure field), and has even been applied to structural response which results from an alternating pressure (buffet response).

CAVITATION (Sonically-induced) in a liquid is the formation, growth, and collapse of gaseous and vapor bubbles due to the action of intense sound waves.

CAVITATION NOISE is the noise produced in a liquid by gaseous or vaporous cavitation.

COHERENCE FUNCTION $\gamma^2_{1,2}$ (f) of two quantities $\xi_1(t)$ and $\xi_2(t)$ is the ratio of the square of the absolute value of the cross-spectral density function to the product of the spectral density functions of the two quantities:

$$\gamma^2_{1,2}(f) = \frac{|G_{1,2}(f)|^2}{G_1(f)G_2(f)}$$

At any frequency, f, the coherence function satisfies $0 \leq \gamma^2_{1,2}(f) \leq 1$.

COMPRESSIBLE – INCOMPRESSIBLE FLOWS. Compressible flows are those in which the volume (or density) of the medium changes under the action of external forces. Contrarily, an incompressible flow is one in which the volume (density) can be considered unchanged under the action of external forces. Liquids are considered practically incompressible. Gases or air can be considered incompressible (to a good approximation if the dynamic pressure (q) is small compared to the modulus of elasticity of the gas (air). It can be shown that this is equivalent to saying that compressibility can be neglected if $M_\infty^2 << 2$ where M_∞ is the Mach number of the free stream.

COMPRESSIONAL WAVE in an elastic medium is one which causes an element of the medium to change its volume without undergoing rotation.

NOTE 1

Mathematically, a compressional wave is one whose velocity field has zero curl.

NOTE 2

A compressional plane wave is a longitudinal wave.

CONFIDENCE INTERVAL (Band) is the range within which the true value of a measured quantity will lie with a given value of uncertainty.

CONVECTION VELOCITY is the velocity at which some particle or phenomenon, such as a pressure peak, is traveling relative to a fixed reference system. It may not be a constant with distance or frequency, and may not necessarily be parallel with the free stream velocity vector.

CONTINUOUS SYSTEM (Distributed System) is one that is considered to have an infinite number of possible independent displacements. Its configuration is specified by a function of a continuous spatial variable or variables in constrast to a discrete or lumped parameter system which requires only a finite number of coordinates to specify its configuration.

CO-SPECTRAL DENSITY. (See *Cross-Spectral Density*).

COULOMB DAMPING (Dry Friction Damping) is the dissipation of energy that occurs when a particle in a vibrating system is resisted by a force whose magnitude is a constant independent of displacement and velocity, and whose direction is opposite to the direction of the velocity of the particle.

COUPLED MODES are modes of vibration that are not independent, but which influence one another because of energy transfer from one mode to the other.

NOTE 1

Although all modes predicted analytically are independent, modes may be coupled in the actual structure due to structural characteristics such as non-linearities or some types of damping.

NOTE 2

When two parts of a structure for which modes have been determined are analytically connected, the modes for each part are then coupled since excitation of a mode in one part results in excitation of the modes of the other part. If modes of a structure are determined by this process of modal coupling (see Section 5, paragraph 5-2.4) these modes are sometimes designated coupled modes.

CRITICAL DAMPING is the least amount of viscous damping which allows a displaced elastic system to return to its equilibrium position without overshoot. In a single degree-of-freedom system, the coefficient of critical damping is $c_c = 2\sqrt{km}$.

CROSS-CORRELATION COEFFICIENT $\rho_{1,2}(\tau)$ of two quantities $\xi_1(t)$ and $\xi_2(t)$ is the ratio of the cross-correlation function $R_{1,2}(\tau)$ to the square root of the product of the mean-square values of the quantities:

$$\rho_{1,2}(\tau) = \frac{R_{1,2}(\tau)}{\sqrt{R_1(0)\,R_2(0)}}$$

where:

$$R_i(0) = \overline{\xi_i^2(t)} \text{ for } i = 1,2.$$

At any delay, τ, the cross-correlation coefficient satisfies $-1 \leq \rho_{1,2}(\tau) \leq 1$.

CROSS-CORRELATION FUNCTION $R_{1,2}(\tau)$ of two quantities $\xi_1(t)$ and $\xi_2(t)$ is the average of the product

of ξ_1 at the time t and ξ_2 at time $(t + \tau)$ for an appropriate averaging time T:

$$R_{1,2}(\tau) = \frac{1}{T}\int_0^T \xi_1(t)\,\xi_2(t+\tau)\,dt.$$

See NOTE UNDER *Autocorrelation Function*.

CROSS-SPECTRAL DENSITY $G_{1,2}(f)$ of two quantities $\xi_1(t)$ and $\xi_2(t)$ is a complex-valued function of frequency f defined by:

$$G_{1,2}(f) = C_{1,2}(f) - j\,Q_{1,2}(f)$$

The real part $C_{1,2}(f)$, called the co-spectral density function, is the average product of $\xi_1(t)$ and $\xi_2(t)$ at frequency f per unit bandwidth, in the limit as the bandwidth approaches zero for an appropriate averaging time T:

$$C_{1,2}(f) = \lim_{B\to 0}\frac{1}{BT}\int_0^T \xi_1(f,t,B)\,\xi_2(f,t,B)\,dt$$

where $\xi_1(f,t,B)$ and $\xi_2(f,t,B)$ are the results of passing $\xi_1(t)$ and $\xi_2(t)$, respectively, through identical narrow band-pass filters of bandwidth B centered at f.

The imaginary part $Q_{1,2}(f)$, called the *quadrature* (or quad-) spectral density function, is also an average product of $\xi_1(t)$ and $\xi_2(t)$ at frequency f per unit bandwidth, except the $\xi_2(t)$ is shifted in phase to produce a 90-degree phase lag:

$$Q_{1,2}(f) = \lim_{B\to 0}\frac{1}{BT}\int_0^T \xi_1(f,t,B)\;\xi_2^{\circ}(f,t,B)\,dt$$

where $\xi_1(f,t,B)$ is the result of passing $\xi_1(t)$ through a narrow band-pass filter of bandwidth B centered at f and $\xi_2^{\circ}(f,t,B)$ denoted a 90-degree phase lag from $\xi_2(f,t,B)$ which is the result of passing $\xi_2(t)$ through a narrow band-pass filter of bandwidth B centered at f.

NOTE 1

In practice, B must be nonzero and T must be finite so that the formulas only give estimates (which contain both a bias error and a statistical uncertainty error).

NOTE 2

The cross-spectral density may be defined in complex polar notation as

$$G_{1,2}(f) = |\,G_{1,2}(f)\,|\,e^{-j\theta_{1,2}(f)}$$

where the magnitude $|\,G_{1,2}(f)\,|$ and the angle $\theta_{1,2}$ are related to $C_{1,2}(f)$ and $Q_{1,2}(f)$ by

$$|\,G_{1,2}(f)\,| = \sqrt{C_{1,2}^2(f) + Q_{1,2}^2(f)}$$

$$\theta_{1,2}(f) = \tan^{-1}\left[\frac{Q_{1,2}(f)}{C_{1,2}(f)}\right]$$

NOTE 3

For stationary processes, the cross-spectral density is twice the Fourier Transform (see *Fourier Spectrum*) of the cross-correlation function:

$$G_{1,2}(f) = 2\int_{-\infty}^{\infty} R_{1,2}(\tau)\,e^{-j2\pi f\tau}\,d\tau \quad (f \geq 0)$$

$$= 2\int_0^{\infty} [R_{1,2}(\tau) + R_{2,1}(\tau)]\cos 2\pi f\tau\,d\tau$$

$$-2j\int_0^{\infty} [R_{1,2}\tau - R_{2,1}(\tau)]\sin 2\pi f\tau\,d\tau$$

CYCLE is the complete sequence (continuum) of values of a periodic quantity that occur during one period.

CYLINDRICAL WAVES have wave fronts which are coaxial cylinders.

DAMPED NATURAL FREQUENCY is the frequency of free vibration of a damped linear system. The free vibration of a damped system may be considered periodic in the limited sense that the time interval between zero crossings in the same direction is constant, even though successive amplitudes decrease progressively. The frequency of the vibration is the reciprocal of this time interval.

DAMPING is the dissipation of energy with time or distance.

DAMPING RATIO ζ for a system with viscous damping is the ratio of actual damping coefficient c to critical damping coefficient c_c:

$$\zeta = c/c_c = 1/2\,Q$$

where Q is the quality (amplification) factor. Damping ratio is sometimes known as fraction of critical damping or as damping factor.

DATA are the continuous or sampled measures of a quantity.

DECIBEL (One tenth of a bel) is a unit of level (where the level of a quantity is the logarithm of the ratio of that quantity to a reference quantity of the same kind) when the base of the logarithm is the tenth root of ten and the quantities concerned are proportional to power.

NOTE 1

Examples of quantities that qualify are pow sound pressure squared, particle velo squared, sound intensity, and sound-ener density. Thus the decibel is a unit of sound pressure squared level; it is common practice, however, to shorten this to sound pressure level because ordinarily no ambiguity results.

NOTE 2

The logarithm to the base the tenth root of 10 is the same as ten times the logarithm to the base 10.

NOTE 3

The decibel always expresses a RATIO, not an absolute quantity. We may have a sound pressure (SP) measured in dynes/cm^2. If we wish to express it in decibels, we then call it a sound pressure level (SPL) and compare it (i.e., form a ratio) to some arbitrarily chosen reference sound pressure, usually 0.0002 dynes/cm^2. Thus

$$\text{SPL (dB)} = 10 \log_{10} \left(\frac{SP}{0.0002}\right)^2$$

$$= 20 \log_{10} \left(\frac{SP}{0.0002}\right)$$

Meticulous usage demands that the reference pressure be specified in each case: e.g., ". . . . . where we measured an SPL of 83 dB re 0.0002 dynes/cm^2."

DEGREES OF FREEDOM (Number of) of a mechanical system is equal to the minimum number of independent generalized coordinates required to define completely the positions of all parts of the system at any instant of time. In general, it is equal to the number of independent generalized displacements that are possible.

NOTE 1

It is also possible to talk about the statistical number of degrees of freedom which is the number of independent variables in an estimate of some quantity. In this case, the number of degrees of freedom determines the statistical accuracy of an estimate. For spectral density estimates of random data, the effective number of degrees of freedom n = 2 BT where B is the effective filter bandwidth and T is the effective averaging time.

DETERMINISTIC VIBRATION is one whose value can be calculated (or determined) from knowledge of its behavior at previous times.

DIFFUSE SOUND FIELD is one in which the time average of the mean-square sound pressure is everywhere the same and the flow of energy in all directions is equally probable.

DISPLACEMENT is a vector quantity that specifies the change of position of a body or particle and is usually measured from the mean position or position of rest. In general, it can be represented as a rotation vector or a translation vector, or both.

DISPLACEMENT THICKNESS (of the Boundary Layer) δ^* indicates the distance by which the external streamlines are shifted owing to the formation of the boundary layer near the wall. It is defined mathematically by

$$U_\infty \delta^* = \int_{y=0}^{\infty} \frac{\rho(y)}{\rho_\infty} (U_\infty - u) dy$$

where U_∞ is the free stream velocity, and u(y) is the local flow velocity at some perpendicular distance y from the surface.

NOTE

For incompressible flow, $\frac{\rho(y)}{\rho_\infty} = 1$

DISTORTION is an undesired change in waveform. Noise and certain desired changes in waveform, such as those resulting from modulation or detection, are not usually classed as distortion.

DOPPLER EFFECT (Shift) is the phenomenon evidenced by the change in the observed frequency of a wave in a transmission system caused by a time rate of change in the effective length of the path of travel between the source and the point of observation.

NOTE

The effect is described quantitatively by:

$$f_r = \frac{(1 + V_r/c)}{(1 - V_s/c)} f_s$$

where:

- f_r = observed frequency
- f_s = frequency at source
- V_r = component of velocity (relative to the medium) of observation point toward source
- V_s = component of velocity (relative to the medium) of source toward observation point
- c = speed of sound in a stationary medium

DYNAMIC PRESSURE of a system is the kinetic energy of that system and equal to 1/2 ρU^2. In aerodynamics, dynamic pressure plus the static pressure is defined as a constant (and equal to the total pressure) along a streamline in steady incompressible flow so long as large changes in potential energy and viscous action do not occur.

EFFECTIVE BANDWIDTH of a specified transmission system is the bandwidth of an equivalent ideal system which (1) has uniform transmission in its pass band equal to the maximum transmission of the specified system and (2) transmits the same power as the specified system when the two systems are receiving equal input signals having a uniform distribution of energy at all frequencies.

NOTE 1

The effective bandwidth is equal to the area under the square of the transmission system characteristic curve divided by the square of the maximum transmission. It may be measured by dividing the mean-square response of the system to white noise excitation by the product of the excitation spectral density and the square of the maximum transmission.

NOTE 2

The term effective bandwidth is also used to describe the bandwidth of excitational energy which contributes to the response of a system. For a single degree-of-freedom system this effective bandwidth is π^2 times f_o divided by Q where f_o is the frequency of resonance of the system and Q is the quality factor (amplification).

EIGENVALUE, EIGENVECTOR: If A is an NxN matrix, the eigenvalues of A are defined to be those complex or real constants λ for which the characteristic equation $A\underline{C} = \lambda\underline{C}$ has a non-trivial solution ($C \neq 0$). The solution vector $\underline{C}$ is called the eigenvector belonging to the eigenvalue λ.

NOTE

In vector algebra, the eigenvector, $\underline{C}$, is the only vector that remains unchanged in direction under the co-ordinate transformation A.

ELECTROACOUSTIC TRANSDUCER is a transducer for receiving waves from an electric system and delivering waves to an acoustic system, or vice versa.

ELECTROMECHANICAL TRANSDUCER is a transducer for receiving waves from an electric system and delivering waves to a mechanical system, or vice versa.

EQUIVALENT STATIC ACCELERATION for an applied dynamic excitation is the steadily-applied (inertial) acceleration required to produce the same maximum relative displacement in a single degree-of-freedom system as does the dynamic excitation. The equivalent static acceleration is given by $(2\pi f_n)^2 A_m$, where A_m is the maximum relative displacement and f_n is the undamped natural frequency of the SDF system. (See Shock Spectrum for a definition of A_m.) The equivalent static acceleration is equal to the maximum absolute acceleration in an undamped system; the equality is approximate in a lightly-damped system.

EQUIVALENT VISCOUS DAMPING is a value of viscous damping assumed for the purpose of analysis of a vibratory motion, such that the dissipation of energy per cycle at resonance is the same for either the assumed or actual damping force.

ERGODIC PROCESS is a stationary process involving an ensemble of time-histories where time averages are the same for every time-history. It follows that these time averages from any time-history will then be equal to corresponding statistical averages over the ensemble.

FILTER is a device for separating the frequency components of an oscillation by amplitude suppression of components outside of one or more selected bands of frequencies.

FINENESS RATIO of a structure is the length (in the direction of the flow) divided by the height from the solid boundary. It is often applied to steps which interrupt aerodynamic flow.

FLUCTUATING PRESSURE LEVEL (FPL) in decibels, is 20 times the logarithm to the base 10 of the ratio of the fluctuating pressure to a reference pressure. The reference pressure shall be explicitly stated. This term is more general than Sound Pressure Level in that it includes non-propagating (though often convected) fluctuating pressures as well as sound waves. Both types of fluctuating pressures can induce structural response.

NOTE 1

The following reference pressure is in common use; 2×10^{-4} dynes/cm^2 (2×10^{-5} newtons/m^2).

NOTE 2

Unless otherwise explicitly stated, it is to be understood that the fluctuating pressure is the effective (rms) fluctuating pressure.

NOTE 3

The many and diverse sources of fluctuating pressures requires that some source definition be included in the term's use unless the source is understood (e.g. boundary-layer-induced FPL etc.)

FLUTTER is a self-induced vibration resulting from an interchange of energy between a flow field and its boundary causing either a constrained or divergent (catastrophic) motion.

FLUTTER PARAMETER is defined for panel flutter as $t/L\ (\sqrt{M_\infty^2 - 1}\ E/q_\infty)^{1/3}$ where M_∞ is the free stream Mach number, E is the modulus of elasticity of the panel material, q_∞ is free stream dynamic pressure, t is the panel thickness and L is the panel length in the direction of the flow. By comparison with experimental data, it is used to indicate regions of instability for a panel in which the structural parameters and aerodynamic flow are suitably valued to permit the interchange of energy from the flow to vibration of the panel.

VIBRATION, SHOCK, AND ACOUSTICS

FORCED VIBRATION is the oscillatory motion that results from an applied, oscillatory, external, forcing function. If the excitation is periodic and continuing, the oscillation is steady state.

FOURIER COEFFICIENTS are the coefficients of a Fourier series. If a quantity ξ (t) is periodic with period T, the mathematical expressions for the Fourier coefficients are as follows:

$$a_o = \frac{1}{T}\int_{-T/2}^{T/2} \xi(t)\, dt$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} \xi(t) \cos 2\pi\, nft\, dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} \xi(t) \sin 2\pi\, nft\, dt$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} \xi(t) e^{-j2\pi nft} dt$$

where f = 1/T is the fundamental frequency and n is the order of the harmonic. The integration interval (limits) may be replaced by any interval of length T. General expressions exist for the coefficients if the quantity is not periodic.

FOURIER SERIES can be used to express a periodic quantity ξ (t) in terms of the individual frequency components:

$$\xi(t) = \xi(t+T) = a_o + \sum_{n=1}^{\infty} (a_n \cos 2\pi nft + b_n \sin 2\pi nft)$$

or in complex form:

$$\xi(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nft}$$

where T represents the period of the quantity and a_n, b_n, and c_n are the Fourier coefficients. Fourier series can also be used to describe other than periodic quantities if an artificial frequency composition (the weighted frequency components which, when combined, reproduce a deterministic time history) is desired.

FOURIER SPECTRUM (Fourier Transform) F(f) for a general quantity ξ(t) is a continuous function of frequency f given by

$$F(f) = \int_{-\infty}^{\infty} \xi(t)\, e^{-j2\pi ft}\, dt$$

assuming ξ (t) is suitably defined so that F(f) exists. The time function ξ(t) is obtained by the following integration:

$$\xi(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(f)\, e^{j2\pi ft}\, df$$

where F(f) and ξ (t) are known as the direct and inverse Fourier transforms respectively.

NOTE 1

Since F(f) represents a frequency distribution in complex form, it may be written in terms of the real and imaginary parts:

$$F(f) = \mathrm{Re}\,[F(f)] - j\,\mathrm{Im}\,[F(f)]$$

where

$$\mathrm{Re}\,[F(f)] = \int_{-\infty}^{\infty} \xi(t) \cos 2\pi ft\, dt$$

$$\mathrm{Im}\,[F(f)] = \int_{-\infty}^{\infty} \xi(t) \sin 2\pi ft\, dt$$

Alternatively, the Fourier spectrum may be defined in terms of its absolute value and phase angle, | F(f) | and θ (f), respectively:

$$F(f) = |F(f)|\, e^{-j\theta(f)}$$

The function | F(f) | is called the Fourier Amplitude Spectrum; θ (f) is called the Fourier Phase Spectrum. The absolute value and the phase angle are related to the real and imaginary parts by

$$F(f) = \sqrt{\mathrm{Re}^2\,[F(f)] + \mathrm{Im}^2\,[F(f)]}$$

$$\theta(f) = \tan^{-1}\left\{\mathrm{Im}\,[F(f)]\,/\mathrm{Re}\,[F(f)]\right\}$$

NOTE 2

For engineering use it is desirable to consider positive frequencies only. The inverse transform for even functions then takes the form:

$$\xi(t) = \frac{1}{\pi}\mathrm{Re}\left[\int_0^{\infty} F(f)\, e^{j2\pi ft} df\right]$$

when ξ (t) = 0 for all t < 0

$$\xi(t) = \frac{2}{\pi}\int_0^{\infty} \mathrm{Re}\,[F(f)] \cos 2\pi ft\, df$$

NOTE 3

Other formulations of this pair of transforms appear in the literature and employ the factor of $1/2\pi$ in different ways. Care should be exercised in comparing results directly.

FREE SOUND FIELD (Free Field) is a field in a homogeneous, isotropic medium free from boundaries. In practice it is a field in which the effects of the boundaries are negligible over the region of interest.

NOTE

The actual pressure impinging on an object (e.g., electroacoustic transducer) placed in an otherwise free sound field will differ from the pressure which would exist at that point with the object removed, unless the acoustic impedance of the object matches the acoustic impedance of the medium.

FREE STREAM means that region of space in which no appreciable effects are observed due to a relative motion between an object and the medium in which it is immersed.

NOTE

As an example of the correct usage, the relative velocity between an object flying in the earth's atmosphere and the air particles unaffected by the said object would be called the free stream velocity U_∞, whereas the relative velocity between those air particles given a velocity disturbance due to the presence of the object and the object itself would be a local velocity (U_ℓ) and be different from U_∞.

FREE VIBRATION is oscillatory motion that occurs in the absence of an external force after disturbance from the equilibrium position.

FREQUENCY of a function that is periodic in time is the reciprocal of the period. The unit is cycle per unit time and must be specified.

NOTE

The cycles per second unit is called hertz (Hz).

FUNDAMENTAL is that sinusoidal component of a periodic vibration which has the lowest non-zero frequency.

g is the acceleration caused by the force resulting from the phenomenon of gravity, which varies with the latitude and elevation of the point of observation. By international agreement, the value 980.665 cm/sec^2 = 386.087 in./sec^2 = 32.1739 ft/sec^2 has been chosen as the standard acceleration due to gravity.

GAUSSIAN DISTRIBUTION (Normal Distribution) See *Amplitude Density Function*.

HARMONIC is a sinusoidal quantity having a frequency that is an integral multiple of the frequency of a periodic quantity to which it is related.

HIGH-PASS FILTER is a wave filter having a single transmission band extending from some critical or cutoff frequency, not zero, up to infinite frequency.

HYPERSONIC refers to that speed domain in which gas molecules begin to disassociate in appreciable numbers in the high temperature regions of the gas surrounding a body. For air, Mach numbers greater than 5 are usually considered hypersonic.

IMPEDANCE is the ratio of two complex quantities whose arguments increase linearly with time and whose real (or imaginary) parts represent a force-like and velocity-like quantity respectively.

NOTE 1

Examples of force-like quantities are: force, sound pressure, voltage, temperature, electric field strength. Examples of velocity-like quantities are: velocity, volume velocity, current, heat flow, magnetic flux.

NOTE 2

The terms and definitions related to impedance pertain to single-frequency quantities in the steady state, and to systems whose properties are independent of the magnitudes of these quantities. These quantities can be represented mathematically by complex exponential functions of time. Under these conditions the factors involving time cancel out in the ratios called for, leaving complex numbers independent of time. Solutions based on complex exponential functions under these conditions give the solution for real sinusoidal oscillations.

Because of the similarity of electrical, mechanical, and acoustical transmission theory, the same terminology is used in the three cases. Where confusion is likely to occur, the proper term should be prefixed to the general term; e.g., "acoustic transfer impedance," but unless otherwise specified the definitions apply not only in acoustics but in mechanics as well. While acoustics is a branch of mechanics, it is found convenient to distinguish an acoustical system from a mechanical one whenever elastic wave motion is an essential feature.

INFLUENCE COEFFICIENT, α, is defined as the static deflection at coordinate i due to a unit load applied at coordinate j.

INFRASONICS is the study of sound waves with frequencies which are below audibility (about 15 Hz).

NOTE 1

The word "infrasonic" may be used as a modifier to indicate a device or system intended to operate at an infrasonic frequency.

NOTE 2

The term "subsonic" was once used in acoustics synonymously with infrasonic; such usage is now deprecated.

INSERTION LOSS in decibels (resulting from insertion of a transducer in a transmission system) is 10 times the logarithm to the base 10 of the ratio of the power delivered to that part of the system that will follow the transducer, before the insertion of the transducer, to the power delivered to that same part of the system after insertion of the transducer.

JERK is a vector that specifies the time rate of change of acceleration (Jerk is the third derivative of displacement with respect to time).

JOINT ACCEPTANCE of a two-dimensional system is defined as

$$J^2_{nm} = \frac{1}{S^2} \iint \bar{c}(\vec{x},\vec{y})\, s_n(\vec{x})\, s_m(\vec{y})\, dS(\vec{x})\, dS(\vec{y})$$

where S is the total surface area, $\bar{c}$ is the correlation coefficient (the pressure field correlation normalized by the mean squared pressure), $s(\vec{x})$ and $s(\vec{y})$ are the (assumed) separable mode shapes in the x and y directions. The joint acceptance is a measure of the effectiveness of the pressure field in loading the mode of vibration in question since the generalized force $<F^2> = <p^2>_{ref.}\ S^2\ j^2_{nm}$ where $<p^2>_{ref.}$ is the mean, squared reference pressure.

JOINT PROBABILITY DENSITY FUNCTION $P_{1,2}(\xi_1,\xi_2)$ for two quantities $\xi_1(t)$ and $\xi_2(t)$ is the joint probability that $\xi_1(t)$ will assume the magnitude ξ_1 while $\xi_2(t)$ *simultaneously* assumes the magnitude ξ_2, per unit magnitude windows, in the limit as the magnitude windows approach zero.

NOTE 1

The magnitude window refers to an ideal magnitude gate width W centered at the magnitudes ξ_1 and ξ_2, respectively.

NOTE 2

The volume under the entire surface of joint probability density is unity.

JOINT PROBABILITY DISTRIBUTION FUNCTION (Joint Cumulative Distribution Function) $P_{1,2}(\xi_1,\xi_2)$ defines the joint probability that $\xi_1(t)$ will not exceed ξ_1 and that $\xi_2(t)$ will not exceed ξ_2. From the definition of the joint probability density function,

$$P_{1,2}(\xi_1,\xi_2) = \int_{-\infty}^{\xi_1} \int_{-\infty}^{\xi_2} P_{1,2}(u,v)\, du\, dv$$

where u and v are dummy variables of integration for ξ_1 and ξ_2, respectively.

KINEMATIC VISCOSITY is the ratio of the absolute viscosity to the density (where the absolute viscosity is the proportionality function between the shear stress and the rate of strain, and is a function of both pressure and temperature).

LINEAR SYSTEM is defined as one for which the response of every element in the system is proportional to the excitation.

NOTE

This definition implies that the dynamic properties of each element in the system can be represented by a set of linear differential equations, and that for the system as a whole, superposition holds.

LINE SPECTRUM one whose components occur only at a number of discrete frequencies. (See *Fourier Series*.)

LOGARITHMIC DECREMENT is the natural logarithm of the ratio of any two successive amplitudes of like sign, in the decay of a single-frequency oscillation.

LONGITUDINAL WAVE is one in which the direction of displacement at each point of the medium is normal to the wave front.

LOW-PASS FILTER is a filter having a single transmission band extending from zero frequency up to some critical or cutoff frequency which is not infinite. (See *Band-pass Filter*.)

MACH ANGLE the Mach angle $\mu = \sin^{-1}(1/M)$ and is the angle between the velocity vector and an infinitesimally week wave front (sound wave) propagating from a disturbance in a flow with a Mach number (M) greater than 1.

MACH NUMBER is the amplitude of the velocity divided by the speed of sound. Since the velocity could either be local or free stream, the Mach number must possess a subscript to designate local (M_ℓ) or free stream (M_∞) conditions (unless the conditions are understood).

MACROSONICS is the technology of sound at signal amplitudes so large that linear approximations are not valid.

MEAN SQUARE VALUE $\overline{\xi^2}$ of a quantity $\xi(t)$ is the time average of the square of the quantity for an appropriate averaging time T:

$$\overline{\xi^2} = \frac{1}{T}\int_0^T \xi^2(t)\, dt$$

See NOTE under *Autocorrelation Function*.

MEAN VALUE $\overline{\xi}$ of a quantity $\xi(t)$ is the time average of the quantity for an approriate averaging time T:

$$\overline{\xi} = \frac{1}{T}\int_0^T \xi(t)\, dt$$

See NOTE under *Autocorrelation Function*.

MECHANICAL IMPEDANCE is the ratio of force to velocity during simple harmonic motion. (See *Impedance*)

NOTE 1

The ratio of force to displacement is sometimes also called mechanical impedance; this usage is deprecated.

NOTE 2

If the force and velocity are measured at the same point, the ratio is designated driving point impedance; if they are measured at different points, the ratio is designated transfer impedance.

VIBRATION, SHOCK, AND ACOUSTICS

MECHANICAL SHOCK is a non-periodic excitation (e.g. a motion of the foundation, or an applied force) of a mechanical system. It is characterized by a suddenness and severity, and usually develops significant relative displacements and internal forces in the system.

MICROPHONE is an electroacoustic transducer that responds to sound waves and delivers essentially equivalent electric signals.

MODE OF VIBRATION in a system undergoing vibration, is a characteristic pattern assumed by the system, in which the motion of every particle is simple harmonic with the same frequency. Two or more modes may exist concurrently in a multiple degree-of-freedom system.

MOMENTUM THICKNESS θ (of the Boundary Layer) is defined mathematically as

$$U_\infty^2 \theta = \int_{y=0}^{\infty} \frac{\rho(y)}{\rho_\infty} u\,(U_\infty - u)\,dy$$

where the U, u, and y symbols have the same definition as in displacement thickness. A physical interpretation of the integral in the above equation is that it represents the loss of momentum in the boundary layer due to friction.

NOTE

For incompressible flow, $\frac{\rho(y)}{\rho_\infty} = 1$

NATURAL FREQUENCY is a frequency of free vibration of a system. For a multiple degree-of-freedom system, the natural frequencies are the frequencies of the normal modes of vibration.

NODE is a point, line, or surface in a standing wave where some characteristic of the wave field has essentially zero amplitude.

NOTE

The appropriate modifier should be used before the word "node" to signify the type that is intended; e.g., displacement node, velocity node, pressure node.

NOISE (1) Noise is any undesired sound. By extension, noise is any unwanted disturbance within a useful frequency band, such as undesired electric waves in a transmission channel or device. (2) Noise is an erratic, intermittent, or statistically random oscillation.

NOTE1

If ambiguity exists as to the nature of the noise, a phrase such as "Acoustic noise" or "electric noise" should be used.

NOTE 2

Since the above definitions are not mutually exclusive, it is usually necessary to depend upon context for the distinction.

NOISE REDUCTION (NR) is defined as the difference between the measured sound power level (or its approximate equivalent the average sound pressure level) in an enclosed volume containing all the sound sources, and the levels measured in an adjacent enclosure. NR therefore inherently includes a dependence on room volumes, absorptivity of exposed surfaces, leakage, etc., but is a measurable quantity. Because of its dependence on variables that are not fixed, it is never used to describe the transmissibility of structures (see *Transmission Loss*.)

NOMINAL BANDWIDTH of a filter is the difference between the nominal upper and lower cutoff frequencies. The difference may be expressed (1) in Hz (cycles per second); (2) as a percentage of the pass-band center frequency; or (3) as the interval between the upper and lower nominal cutoff frequencies in octaves.

NOMINAL PASS-BAND CENTER FREQUENCY is the geometric mean of the nominal cutoff frequencies.

NOMINAL UPPER AND LOWER CUTOFF FREQUENCIES of a filter pass-band are those frequencies above and below the frequency of maximum response of a filter at which power due to the response to a sinusoidal signal is 3 dB below the maximum response.

NONLINEAR DAMPING is damping that is not proportional to velocity or displacement.

NONSTATIONARY PROCESS is any process which is not a stationary process.

NOTE

For nonstationary processes, statistical quantities are functions of the time at which the process is being measured and are not invariant with respect to translations in time. Definitions given herein for terms such as mean value, mean square value, probability density, correlation function, and spectral density function do not apply to nonstationary processes (though in special instances they may be applied to processes which are considered weakly stationary because their mean values and covariance functions are independent of time, while their other distributions are not).

NORMAL MODE OF VIBRATION is a mode of free vibration on an undamped system. In general, any composite motion of the system is analyzable into a summation of its normal modes.

NOTE 1

The characteristic pattern of motion typically consists of a space distribution, one part of which is negative in relation to the other part.

Thus, at the same time that the particles in one part are moving outward in the positive direction from their positions of equilibrium, the particles in the other part are moving outward in the negative direction, and conversely.

NOTE 2

Vibration in a normal mode occurs at a natural frequency of the undamped system.

NOTE 3

The terms "natural mode," "characteristic mode," and "eigen mode" are synonymous with normal mode.

OCTAVE is a frequency interval for which $f_2 = 2f_1$.

OCTAVE BAND is a band of frequencies whose upper and lower limits constitute an interval of one octave.

ORTHOGONALITY is that property meaning perpendicularity. In vector notation, the condition of orthogonality requires that the scalar product of the vectors vanishes (=0).

OSCILLATION is the variations, usually with time, of the instantaneous value of a quantity with respect to a specified reference when the value is alternately greater and smaller than the reference.

PEAK VALUE of a quantity ξ (t) is the value of ξ (t) at a maximum or minimum value. (Note that a quantity ξ (t) may have many peak values during a finite time interval).

PERIOD of a periodic quantity is the smallest increment of the independent variable (e.g., time) for which the function repeats itself.

PERIODIC VIBRATION is vibration whose instantaneous value ξ (t) recurs for integer multiples of the period T:

$$\xi(t) = \xi(t \pm nT)$$

for $n = 1, 2, 3 \ldots$.

NOTE

In general, a periodic function can be expanded into a series of the form

$$y = f(t) = A_0 + A_1 \sin(\omega t + a_1) + A_2 \sin(2\omega t + a_2) + \ldots$$

where ω, a positive constant, equals 2π divided by the period T, and the A's and a's are constants which may be positive, negative, or zero.

PHASE OF A PERIODIC QUANTITY for a particular value of the independent variable, is the fractional part of a period through which the independent variable has advanced, measured from an arbitrary reference.

PIEZOELECTRIC TRANSDUCER (Crystal or Ceramic) is a transducer that converts a change in pressure to a voltage. It depends for its operation on the interaction between the electric charge and the deformation of certain asymmetric crystals having piezoelectric properties.

NOTE

Some ceramics can have induced piezoelectric properties due to the application of an electric field during manufacturing.

PINK NOISE by definition, has a spectral intensity that is inversely proportional to frequency over a specified range, and therefore dissipates, in a constant resistance, equal power in any octave bandwidth in that range.

PLANE WAVE has wave fronts which are everywhere parallel planes normal to the direction of propagation.

POWER LEVEL in decibels, is 10 times the logarithm to the base 10 of the ratio of a given power to a reference power. The reference power must be indicated.

NOTE

In sound recording, for example, a reference electric power often used is the milliwatt, and the symbol dbm is employed to indicate both the unit of power level, the decibel, and the reference power, the milliwatt.

POWER SPECTRAL DENSITY (Autospectral Density, Spctral Density) G(f) of a quantity ξ (t) is the mean square value at frequency f per unit bandwidth, in the limit as the bandwidth approaches zero for an appropriate averaging time T:

$$G(f) = \lim_{B \to 0} \frac{1}{BT} \int_0^T \xi^2(f, t, B)\, dt$$

where ξ^2 (f, t, B) is the result of passing ξ (t) through a narrow band-pass filter of bandwidth B centered at f and then squaring the output. In other words, G(f) is the rate of change of mean square value with frequency.

NOTE 1

In practice B must be non-zero and T must be finite so that the formula gives only an estimate (which contains both a bias error and a statistical uncertainty error).

NOTE 2

Power spectral density is a generic term used regardless of the physical quantity represented by the time-history. The word "power" merely

implies a squaring and averaging operation, and has no necessary relation to a physical power. It is desirable that the physical quantity involved be indicated in referring to particular data. For example, the term "acceleration spectral density" may be used instead of power spectral density when $\xi(t)$ is an acceleration time-history. If desired, the terms "autospectral density" or "spectral density" may be used instead of "power spectral density".

NOTE 3

For stationary processes, the power spectral density is twice the Fourier Transform (see Fourier Spectrum) of the autocorrelation function:

$$G(f) = 2\int_{-\infty}^{\infty} R(\tau)\, e^{-j2\pi f\tau}\, d\tau$$

$$= 4\int_{0}^{\infty} R(\tau)\cos 2\pi f\tau d\tau (f \geq 0)$$

PRESSURE COEFFICIENT c_p of a pressure p_1 is defined to be equal to $(p_1 - p_\infty)/q_\infty$, where p_∞ and q_∞ are the free stream static and dynamic pressures respectively. It is a dimensionless term, and often used to describe the local static pressure (being zero when p_1 equals the free stream static pressure).

NOTE

A term sometimes called fluctuating pressure coefficient has also been used, and is defined as $\Delta c_p = (\sqrt{\overline{p^2}} - p_\infty)/q_\infty$ where Δc_p designates a fluctuating rather than a static level whose average value is given by the root mean squared pressure $\sqrt{\overline{p^2}}$ in the equation rather than a static pressure quantity.

PROBABILITY DENSITY FUNCTION $p(\xi)$ of a quantity is the probability that the quantity will assume the magnitude ξ per unit magnitude window, in the limit as the magnitude window approaches zero.

$$p(\xi) = \lim_{W \to 0} \frac{P(\xi, W)}{W}$$

where $P(\xi W)$ is the probability that the quantity lies in the magnitude window of width W centered at the magnitude ξ. In other words, $p(\xi)$ is the rate of change of probability with magnitude.

NOTE 1

The magnitude window refers to an ideal magnitude gate width W centered at the magnitude.

NOTE 2

The area under the entire curve of probability density is unity.

NOTE 3

It is sometimes convenient to normalize the probability density by computing $P(\xi/\sigma)$, where σ is the standard deviation of $\xi(t)$.

PROBABILITY DISTRIBUTION FUNCTION (Cumulative Distribution Function) $P(\xi)$ defines the probability that ξ will not be exceeded. From the definition of the probability density function, $P(\xi)$ is equal to the area under the curve of $p(\xi)$ between minus infinity and ξ.

$$P(\xi) = \int_{-\infty}^{\xi} p(u)\, du$$

where u is a dummy variable of integration for ξ.

QUADRATURE SPECTRAL DENSITY (See *Cross-spectral Density*).

Q (Quality Factor or Amplification) is a measure of the sharpness of resonance or frequency selectivity of a resonant vibratory system having a single degree-of-freedom, either mechanical or electrical.

NOTE

In a viscously-damped mechanical system, this quantity is equal to one-half the reciprocal of the damping ratio (c/c_c). It is commonly used only with reference to a lightly-damped single degree-of-freedom system, and is then approximately equal to the following:

(1) Transmissibility at resonance,

(2) π/logarithmic decrement,

(3) $2\pi W/\Delta W$ where W is the stored energy and ΔW the energy dissipation per cycle, and

(4) $f_r/\Delta f$ where f_r is the resonance frequency and Δf is the bandwidth between the half-power points.

RANDOM INCIDENCE. If an object is in a diffuse sound field the sound is said to strike the object at random incidence.

RANDOM NOISE (or Vibration) is an oscillation whose instantaneous magnitude is not predictable with certainty for any given instant of time. The instantaneous magnitudes of a random noise (vibration) are specified only by probability distribution functions giving the fraction of the total time that the magnitude, or some sequence of magnitudes, lies within a specified range. Such random oscillations do not contain any periodic constituents.

NOTE

If the instantaneous values occur according to a Gaussian distribution, it is called Gaussian random noise (vibration).

VIBRATION, SHOCK, AND ACOUSTICS

RAYLEIGH WAVE is a surface wave associated with the free boundary of a solid, such that a surface particle describes an ellipse whose major axis is normal to the surface, and whose center is at the undisturbed surface. At maximum particle displacement away from the solid surface the motion of the particle is opposite to that of the wave.

NOTE

The propagation velocity of a Rayleigh wave is slightly less than that of a shear wave in the solid; the wave amplitude of the Rayleigh wave diminishes exponentially with depth.

RESONANCE of a system in forced oscillation exists when any change, however small, in the frequency of excitation causes a decrease in the response of the system.

NOTE

Velocity resonance, for example, can occur at a frequency different from that of displacement resonance.

RESONANCE FREQUENCY is one at which resonance exists.

NOTE

In case of possible confusion, the type of resonance must be indicated; e.g. velocity, resonance frequency.

REVERBERATION is the persistence of sound in an enclosed space, as a result of multiple reflections after the sound source has stopped emitting.

NOTE

The repeated reflections of residual sound in an enclosure can alternatively be described in terms of the transient behavior of the modes of vibration of the medium bounded by the enclosure.

REVERBERATION TIME of a room is the time that would be required for the mean-square sound pressure level therein, orginally in a steady state, to decrease 60 dB after the source is stopped.

REYNOLDS NUMBER is defined as $Re = U_\infty\ d/\nu$ (dimensionless), where U_∞ is free stream velocity, d is a characteristic linear dimension (usually the distance from the leading edge of the solid boundary) and ν is the kinematic viscosity. In wind tunnel testing, a unit Reynolds Number is often used for convenience in which case the definition is $Re_U = U_\infty/\nu$ (length^{-1}) or Reynolds Number per unit length. The Reynolds Number arises from considerations of the dynamic similarity of different flows, and signifies (physically) the ratio of the inertial forces to the viscous forces. (See *Boundary Layer* and *Transition*.)

ROOM CONSTANT is equal to the product of the average absorption coefficient of the room and the total internal surface area of the room divided by the quantity one minus the average absorption coefficient.

ROOT-MEAN-SQUARE VALUE (rms) is the square root of the mean-square value. For the case of a quantity with a mean value of zero, the rms value is equal to the standard deviation.

SCHLIEREN METHOD is a technique by which light, refracted by density variations, is used to produce a visible image of a sound or flow field.

SHEAR WAVE (Rotational Wave) in an elastic medium causes an element of the medium to change its shape without a change of volume.

NOTE 1

Mathematically, a shear wave is one whose velocity field has zero divergence.

NOTE 2

A shear plane wave in an isotropic medium is a transverse wave.

NOTE 3

When shear waves combine to produce standing waves, linear displacements may result.

SHOCK is a transient excitation (acceleration, force, etc.) which produces a sudden change in the motion of the responding system.

SHOCK PULSE is a transient excitation characterized by a single rise of the variable (acceleration, force, etc.) from a constant value and return of the variable to the previous constant value.

SHOCK SPECTRUM (Shock Response Spectrum) is an expression of the maximum response (displacement, velocity, or acceleration) to an applied shock of an array of linear single-degree-of-freedom systems, as a function of their natural frequencies for a specified damping ratio of the systems. The three forms of Shock Spectra are the *Displacement, Velocity,* and *Acceleration Shock Spectra* which are presentations of the maximum relative displacement A_m, the pseudo velocity $2\pi f_n A_m$, and the equivalent static acceleration $(2\pi f_n)^2 \cdot A_m$, respectively. (See *Equivalent Static Acceleration*.)

NOTE 1

A broader definition of a shock spectrum is any spectrum which is determined from the time-history of a shock.

NOTE 2

For a single degree-of-freedom system responding to a motion at the base of the spring

defined by u(t), the relative displacement time history δ(t) of the mass relative to the base is (for zero initial conditions):

$$\delta(t) = \frac{1}{2\pi f_d}\int_0^t \ddot{u}(t')e^{-2\pi\zeta f_n(t-t')} \sin 2\pi f_d(t-t')\, dt'$$

where $f_n = (k/m)^{1/2}/2\pi$ is the undamped natural frequency, ζ is the damping ratio (c/c_c) or fraction-of-critical damping, and $f_d = f_n(1-\zeta^2)^{1/2}$ is the damped natural frequency, and t' is the dummy time variable. If, instead of acceleration, a force q(t) is applied to the mass, the relative displacement is given by the above expression with ü(t) replaced by q(t)/m. Spectra are based on maximum values of δ(t) denoted by A_m. When consideration is given to the sign of the response, the spectra are labeled positive or negative.

NOTE 3

The "primary" or "initial" shock spectrum refers to responses which occur during the time of the shock excitation. The "residual" shock spectrum refers to responses which occur after the time of termination of the shock excitation. When necessary to distinguish a spectrum for which the time of the response maximum is not considered, the designation "overall" shock spectrum is employed.

SHOCK STRENGTH is the ratio of the plateau static pressure (the pressure immediately downstream of the shock which has a zero gradient with respect to distance) to the static pressure just before the shock. This is a measure of the strength of the shock in terms of a static pressure increase.

SIMPLE HARMONIC MOTION (Harmonic Motion) is motion such that the displacement is a sinusoidal function of time.

SINGLE DEGREE-OF-FREEDOM SYSTEM is one for which only one coordinate is required to define completely the configuration of the system at any instant.

SKIN FRICTION COEFFICIENT is defined as the viscous shear stress at the surface divided by the dynamic pressure. It can be related to other aerodynamic parameters such as Reynolds Number, total drag, boundary layer thicknesses, etc., through mathematical relationships which differ between laminar and turbulent boundary layers.

SOUND (1) Sound is an oscillation in pressure, stress, particle displacement, particle velocity, etc. in a medium with internal forces (e.g., elastic, viscous), or the superposition of such propagated oscillations. (2) Sound is an auditory sensation evoked by the oscillation described above.

NOTE 1

In case of possible confusion, the term "sound wave" or "elastic wave" may be used for concept (1) and the term "sound sensation" for concept (2). Not all sound waves can evoke an auditory sensation; e.g., ultrasound.

NOTE 2

The medium in which the sound exists is often indicated by an appropriate adjective; e.g., air-borne, water-borne, structure-borne.

SOUND-ABSORPTION COEFFICIENT of a surface is the fraction of incident sound energy absorbed or otherwise not reflected by the surface. Unless otherwise specified, a diffuse sound field is assumed.

SOUND PRESSURE at a point is the total instantaneous pressure at that point in the presence of a sound wave minus the static pressure at that point.

SOUND PRESSURE LEVEL in decibels, is 20 times the logarithm to the base 10 of the ratio of the sound pressure to a reference pressure. The reference pressure shall be explicitly stated.

NOTE 1

The following reference pressure is in common use:

2×10^{-4} dynes/cm^2 or

2×10^{-5} Newtons/m^2

NOTE 2

Unless otherwise explicitly stated, it is to be understood that the sound pressure is the effective (rms) sound pressure.

NOTE 3

It is to be noted that in many sound fields the sound pressure ratios are not the square roots of the corresponding power ratios.

SPECIFIC ACOUSTIC IMPEDANCE (Unit Area Acoustic Impedance) at a point in the medium is the complex ratio of sound pressure to particle velocity.

SPECIFIC HEAT RATIO of a gas is the ratio of the specific heat at constant pressure to the specific heat at constant volume for that gas.

SPECTRUM (1) The spectrum of a function of time is a description of its resolution into components, each of different frequency and (usually) different amplitude

VIBRATION, SHOCK, AND ACOUSTICS

- 2.4 Effect of Noise on Animals
  - 2.4.1 Physiological
  - 2.4.2 Psychological
  - 2.4.3 Speed Interference
  - 2.4.4 Headgear
- 2.5 Noise Control
  - 2.5.1 Absorption
  - 2.5.2 Noise Cancellation Techniques
  - 2.5.3 Refraction
  - 2.5.4 Noise Reduction and Transmission Loss
- 2.6 Testing
  - 2.6.1 Equipment
  - 2.6.2 Facilities
- 2.7 Noise Propagation
  - 2.7.1 Focusing Effects
  - 2.7.2 Air Attenuation
  - 2.7.3 Ground Attenuation
- 2.8 Ultrasonics

3. Shock
- 3.1 Sources of Shock
  - 3.1.1 Combustion (Unstable Burning)
  - 3.1.2 Explosives (Explosive Bolts, EBW's)
  - 3.1.3 Impact
  - 3.1.4 Shock Waves (Blast, Overpressure)
- 3.2 Effect of Shock on Structure
- 3.3 Effect of Shock on Equipment
- 3.4 Effect of Shock on Animals
  - 3.4.1 Physiological
  - 3.4.2 Psychological
- 3.5 Control of Shock
  - 3.5.1 Isolators
  - 3.5.2 Damping
  - 3.5.3 Stiffness
  - 3.5.4 Closure Devices
- 3.6 Testing
  - 3.6.1 Equipment
  - 3.6.2 Facilities

4. Vibration
- 4.1 Sources of Vibration
  - 4.1.1 Machinery (Rotation and Oscillation)
  - 4.1.2 Rockets and Jets (Direct Mechanical)
  - 4.1.3 Acoustically and Aerodynamically Induced Vibration
  - 4.1.4 Transportation and Handling
- 4.2 Effects of Vibration on Structure
  - 4.2.1 Beams
  - 4.2.2 Plates
  - 4.2.3 Shells
  - 4.2.4 Membranes
- 4.3 Effects of Vibration on Equipment
- 4.4 Effect of Vibration on Animals
  - 4.4.1 Physiological
  - 4.4.2 Psychological
- 4.5 Control of Vibration
  - 4.5.1 Isolators
  - 4.5.2 Damping
  - 4.5.3 Stiffness
- 4.6 Testing
  - 4.6.1 Equipment
  - 4.6.2 Facilities

5. Flutter
- 5.1 Panel
- 5.2 Vehicle Stability

6. Instrumentation and Data Reduction
- 6.1 Transducers
- 6.2 Recording Equipment
  - 6.2.1 Magnetic Tape
  - 6.2.2 Oscillographs
  - 6.2.3 Strip-Chart Recorders
- 6.3 Data Reduction Techniques

7. Computer Programs
- 7.1 Acoustics
- 7.2 Shock
- 7.3 Vibration

a. "Dynamics Training Course Lectures," Vols. I and II SM-42981 May 1963

b. "Structures Training Course Study Guide – Vibration, Shock and Acoustics" MDAC-WD. November 1968.

This is a compilation of the notes for a lecture series dealing (in a cursory fashion) with the why, how, and what of vibration, shock and acoustical criteria for design and test. It was intended for those individuals in peripheral (touching) technologies requiring a working knowledge with criteria, design impact, and testing in these areas.

c. "Testing Manual" MDAC-WD SPME-6000.3.27 Sept. 1968. This manual contains descriptions of test operating methods and procedures for the environmental laboratories. It contains good background material on the available MDAC-WD facilities. Any engineer preparing a test control drawing should be cognizant of its contents.

15-2 REFERENCE FILE INDEX

The Vibration, Shock and Acoustics (VS&A) reference file contains a good selection of specific technology-oriented information (not all of which are contained in the Library). These references pertain to various major technology subjects (vibration, shock, bio-acoustics, acoustics, etc.), and some closely-related subtopics (e.g., instrumentation and data reduction).

Each document is assigned a basic number on the basis of its subject matter, and in addition, a parenthetical number to uniquely identify its position in the files. In addition to being card-indexed and filed by contents, documents are also card-indexed by author (providing a dual approach to a document search).

A complete listing of the subject matter categories and associated number for each is given in the following reference file index. For specific problems beyond the immediate capability of an engineer, it is strongly recommended that this reference file be consulted as it is an excellent source of pertinent additional information.

Vibration, Shock & Acoustics Reference File Subject Index

1. General
   - 1.1 Theory
     - 1.1.1 Wave Theory in Gases
     - 1.1.2 Wave Theory in Liquids
     - 1.1.3 Wave Theory in Solids
     - 1.1.4 Correlation
   - 1.2 Factual Information
     - 1.2.1 Douglas Brochures
     - 1.2.2 Books
     - 1.2.3 Bibliographies
     - 1.2.4 Symposia
     - 1.2.5 J.A.S.A.'s
     - 1.2.6 Shock and Vibration Bulletins
     - 1.2.7 NASA Design Criteria Monographs
     - 1.2.8 Shock and Vibration Monographs
   - 1.3 Systems and General Documents Pertaining Thereto
     - 1.3.1 General
     - 1.3.2 Genie
     - 1.3.3 Thor
     - 1.3.4 Skybolt
     - 1.3.5 Zeus
     - 1.3.6 Saturn
     - 1.3.7 Nike Hercules
     - 1.3.8 Airplanes
     - 1.3.9 Voyager
     - 1.3.10 Delta
     - 1.3.11 MOL
     - 1.3.12 Spartan
     - 1.3.13 Special Space
     - 1.3.14 Titan
     - 1.3.15 UpSTAGE
   - 1.4 Properties of Material
     - 1.4.1 Effects of Temperature
     - 1.4.2 Effects of Pressure
     - 1.4.3 Effects of Radiation
   - 1.5 Specifications, Definitions, and Tables
     - 1.5.1 Standards
   - 1.6 Mathematics
2. Acoustics
   - 2.1 Noise Sources
     - 2.1.1 Jet and Rocket Noise
     - 2.1.2 Impeller Noise
     - 2.1.3 Equipment Noise
     - 2.1.4 Aerodynamic Sources
       - 2.1.4.1 Boundary Layer (Includes Ducts)
       - 2.1.4.2 Turbulence
       - 2.1.4.3 Cavities
       - 2.1.4.4 Wakes
       - 2.1.4.5 Oscillating Shocks
     - 2.1.5 Sonic Boom
   - 2.2 Effect of Noise on Structure (Sonic Fatigue)
   - 2.3 Effect of Noise on Equipment

Section 15
BIBLIOGRAPHY

15-1 STANDARD BIBLIOGRAPHY OF BACKGROUND MATERIAL

15-1.1 Books (Recommended)

Beranek, L. L., *Noise Reduction,* McGraw Hill Co. Inc., New York, 1960

Harris, C. M., and C. E. Crede, *Shock and Vibration Handbook,* McGraw Hill Co., Inc., New York, 1961

Harris, C. M., *Handbook of Noise Control,* McGraw Hill Co. Inc., New York, 1957

Bendat, J. S., and A. G. Piersol, *Measurement and Analysis of Random Data,* J. Wiley & Sons, New York, 1966

Morse, P. M., *Vibration and Sound,* McGraw Hill Co. Inc., New York, 1948

Hurty and Rubenstein, *Dynamics of Structures*

Crandall, *Random Vibrations*

Crede, C. E., *Vibration and Shock Isolation,* J. Wiley & Sons, New York 1951

Housner and Hudson, *Applied Mechanics — Statics*

15-1.2 Publications

a. Journal of the Acoustical Society of America — This monthly American Journal includes papers on the following general topics: architectural acoustics (including structural design for acoustics), physiological and psychological acoustics, acoustical instrumentation, musical acoustics, noise and noise control, standards for acoustics, speech acoustics, ultrasonics, radiation and scattering, mechanical vibrations and shock, underwater sound, aeroacoustics and macrasonics, bioacoustics, and acoustic signal processing. December issues of the journal index the previous years papers.

b. Shock and Vibration Bulletin — This biannual publication documents papers presented at the Shock and Vibration Symposia sponsored by the Naval Research Laboratory, Washington, D.C. The bulletins include classified as well as unclassified presentations on all manner of material relating to the source of, response to, measurement of, analysis of, and isolation from vibration and shock. A special index issue is periodically released which lists all the published papers in previous issues.

c. Sound and Vibration — This journal is an English publication and tends to bridge the somewhat ill-defined boundary between the above two journals by including articles on any subject covered by either. A yearly index is published, listing all the papers printed during that reporting period.

d. NASA CR's — CR is the current abbreviation for a Contractor's Report (one which is prepared under a NASA contract but one for which they claim no technical responsibility with respect to its contents) as opposed to a TN which is a Technical Note prepared by NASA personnel (in house) and for which they assume total technical responsibility for its validity. MDAC-WD personnel have prepared several CR's in the areas of vibration shock and acoustics. No complete index is available, but any literature search prepared by the MSSD library can include these NASA publications and many of them are on file in the library and/or the VS&A reference file.

e. AF Technical Document Reports — Similar to NASA CR's and TN's, these TDR's represent technical effort funded by the Air Force (whether performed by in-house or outside personnel). Comments similar to the ones under NASA CR's apply as to indexing, searching for, and availability of these documents.

15-1.3 Engineering Papers and DAC Reports

MDAC reports and engineering papers on the subjects of vibration, shock and acoustics are too numerous to include in a listing such as this. The reader interested in specific problem areas is referred to the reference file kept by the VS&A branches (see index in paragraph 15-2). Therefore, only those comprehensive, summary-type documents, compiled for training purposes, are included herein.

VIBRATION, SHOCK, AND ACOUSTICS

VET	vehicle elapsed time
VFO	variable frequency oscillator
VHF	very high frequency
VIB	Vertical Integration Building
VIP	Value in Performance Very Important People
VSWR	voltage standing-wave ratio
VTC	Vandenberg Test Center
VTS	Vandenberg Tracking Station
VTVM	vacuum-tube voltmeter

W

WBS	work breakdown structure
WGA	weighted guidelines analysis
WO	work order
WOA	work order authorization
WRO	work release order
WS	weapon system (not weapons system)
WSWG	work statement working group
WT	Wind Tunnel
WTR	Western Test Range

X Y Z

XSM	experimental strategic missile
ZI	zone of interior

SMAB	Solid Motor Assembly Building
SMS	Safety Manual Supplement
	strategic missile squadron
S/N	signal-to-noise ratio
SO	sales order
	shop order
	stop order
SOR	specific operational requirement
SOW	statement of work
SP	Standard Practice
SPADATS	space detection and tracking system
SPB	Standard Practice Bulletin
SPC	stored program command
SPD	System Program Director
SP/DR	System Performance/Design Requirements, General Specification for
SPL	sound pressure level
SPM	Standard Practice Memorandum
SPO	System Program Office
S-RD	Secret- -Restricted Data (security class.)
SRM	solid rocket motor
SS	source selection
	stainless steel
	system segment
S-SAR	Secret- -Special Access Required (security class.)
SSB	Source Selection Board
	single side-band
SSC	Space Systems Center
SSEP	System Safety Engineering Plan
SSG	system safety group
SSL	Space Simulation Laboratory
SSLV	standard space launch vehicle
SSM	system support manager
SSP	System Safety Plan
STC	Sacramento Test Center
	Satellite Test Center
STE	special test equipment
STINFO	scientific and technical information
STO	system test objectives
STOL	Saturn test-oriented language
ST&TE	special tools and test equipment

T

t	time
TAC	Tactical Air Command
TAD	thrust-augmented Delta
TAN	task authorization notice
TAP	unit of impulse
TAT	Technical Approval Team
	thrust-augmented Thor
TBD	to be determined
TCD	Test Control Drawing
TCO	termination contracting officer
T-count	terminal count (flight readiness count)
TCTO	time compliance technical order
TD&GS	Technical Documentation & Graphic Services
TDP	Technical Development Plan
TDY	temporary duty
TEPI	training equipment planning information
TF	Transfer Function
TGSE	test ground support equipment
TICO	Titusville — Cocoa
Tiros	Television Infrared Observation Satellite
TL	Transmission Loss (acoustic)
TLS	time line sheet
T&M	time and materials
TM	technical manual
TMS	type-model-series
TO	technical order
TOL	test-oriented language
TOP	technical operating procedure
TOR	technical operating report
TPD	Test Procedure Drawing
TP&E	Test Planning & Evaluation
TPS	thermal protection system
TPST	Terminal Peak Sawtooth
TPU	tape preparation unit
TR	technical report
	technical requirement
	time record
TRD	Test Requirements Drawing
TRS	technical requirements specification
TS	Top Secret (security class.)
TSE	test support equipment
TSR	trade study report
TTC	tracking, telemetry, and command
TTCV	tracking, telemetry, command, and voice
TVC	thrust vector control
TWT	traveling-wave tube
TWX	teletypewriter exchange

U

U	Unclassified (security class.)
UHF	ultrahigh frequency
UR	unsatisfactory report (AFTO Form 29)
USA	ullage simulation assembly
USAF	United States Air Force
USM	underwater-to-surface missile
USSG	United States standard gage
UT	umbilical tower

V

vac	volts alternating current
VAFB	Vandenberg Air Force Base
VCS	voltage-current-sequence
VE	value engineering
	vernier engine
VECO	vernier engine cutoff
VESR	value engineering study request

VIBRATION, SHOCK, AND ACOUSTICS

Q

Q	Amplification (Quality) Factor
QA	quality assurance
QC	quality control
QCM	Quality Control Manual
QDR	Quality Data & Reporting
QE	Quality Engineering
QQPRI	qualitative and quantitative personnel requirements information
QR	quality review
QS	quality standard

R

RA	Reliability Analysis (Branch)
	Reliability Assurance
RAS	reproduction assembly sheet
	requirements allocation sheet
R-count	readiness count
RCS	reentry control system
RCSO	Research Contract Support Office
R&D	research and development
RD	Restricted Data
RDT&E	Research, Development, Test, & Evaluation
Re	Reynolds Number
re.	with respect (or reference) to
REM	reentry module
RF	radio frequency
RFA	radio frequency allocation
	radio frequency attenuation
RFE	request for estimate
	request for expenditure
RFI	radio frequency interference
RFP	request for proposal
RFQ	request for quotation
RFW	request for waiver
RH	relative humidity
RIM	receiving, inspection, and maintenance (building)
R&M	Release & Material
rms	root mean square
ROC	Range Operations Center (WTR)
ROD	record of discussion
ROM	rough order of magnitude
ROTI	recording optical tracking instrument
RSC	range safety command
RSO	range safety officer
RSP	Recovery Support Plan
RTCC	real-time computer complex
RTD	Research & Technology Division
RTN	remote tracking network
RTS	remote tracking station
RTV	reentry test vehicle
RV	reentry vehicle

S

S	Secret
S/A	Saturn/Apollo
S&A	safing and arming
SAB	Scientific Advisory Board
SAC	Strategic Air Command
SACTO	Sacramento Test Operations
SAMSO	Space & Missiles Systems Organization
SAR	Special Access Required (security class.)
SATAF	Site Activation Task Force
SC	scope change
	significant characteristics
	Support Center (Employe Relations)
SCC	Satellite Control Center
	space chamber complex
	System Control Center
SCD	specification control drawing
SCF	Satellite Control Facility
SCN	specification change notice
SCSG	Signal Conditioning Subsystem Group
SDF	Single-degree of freedom
SDI	subcontractor data item
SD&I	System Development & Integration
SDRL	subcontract data requirements list
SE	System Engineering
SEA	Statistical Energy Analysis
SED	system engineering documentation
SEO	salvage engineering order
	serial engineering order
SEP	System Effectiveness Plan
SERO	system engineering release order
SFCW	search for critical weakness
SGLS	Space-Ground Link System
SI	International System of Units
SIA	Systems Integration Area
SIC	system integration contractor
SIL	schedules interface log
	Systems Integration Laboratory
SIR	subcontractor information request
SIRL	support item requirement list
SIRS	Skills Information Retrieval System
SIV	MDAC Saturn stage
SIVB	MDAC Saturn stage
SL	sea level
	section list
SLC	Space Launch Complex
SLN	section list number
SLS	Space Launch System
SLV	simulated launch vehicle
	standard launch vehicle
SM	strategic missile
SMA	squadron maintenance area

N

NA	next assembly not applicable
NACA	National Advisory Committee for Aeronautics (now NASA)
NAF	Naval Aircraft Factory (Standards)
NAS	National Aerospace Standard
NASA	National Aeronautics and Space Administration
NASC	National Aeronautics and Space Council
NBS	National Bureau of Standards
NEC	National Electrical Code
NEL	Navy Electronics Laboratory
NHA	next higher assembly
NOL	Navy Ordnance Laboratory
NORAD	North American Air Defense Command
NR	not required Noise Reduction (acoustic)
NRD	National Range Division
NSC	National Security Council

O

OAO	Orbiting Astronomical Observatory
OBC	on-board computer
OGE	operational ground equipment
OGO	Orbiting Geophysical Observatory
OL	Organization List
OM	Operating Memorandum
OMI	Operating Memorandum- -Information
OMIS	Operating Management Information System
OMP	Operating Memorandum- -Policy
OMPA	Operating Memorandum- -Personnel Appointment/Assignment Operating Memorandum- -Policy Administration
OP	Operating Policy
OPR	office of prime responsibility
O&R	overhaul and repair
ORD	Orbital Requirements Document
OSAF	Office of the Secretary of the Air Force
OSE	operational support equipment
OSO	Orbiting Solar Observatory
OSP	Orbital Support Plan
OSS	Orbital Space Station
OSTF	Operational Systems Test Facility
OTEF	Operational Training and Evaluation Facility
OV	orbiting vehicle
OVSB	Orbiting Vehicle Support Building (formerly Orbiting Vehicle Assembly Building)

P

PALC	Point Arguello Launch Complex
PAM	primary auxiliary memory (unit) pulse amplitude modulation
PAST	propulsion and associated systems test
PAT	Procedures Authorized Task
PBS	program breakdown structure
PC	prime contractor Program Control
PCM	pulse code modulation
PCO	procuring contracting officer
PCP	program change proposal
PCS	Precision Casting Standard
PDM	pulse duration modulation
PDP	program definition phase
PDR	preliminary design review
PECL	preliminary engineering configuration list
PED	personnel and equipment data
PEO	process engineering order
PERT	program evaluation review technique
PFRT	preflight rating test
PG	Position Guide
PIM	Pricing Instructions Memorandum
PLOT	probability of launch on time
PM	phase modulation photo master
PME	precision measuring equipment
PMEL	precision measuring equipment laboratory
PMI	program management instruction
PMN	program management network
PMR	Pacific Missile Range
P/N	part number pin number
PNdB	Perceived Noise Level in dB
PO	purchase order
P&P	Procurement & Production
PP	purchased part
PPB	provisioning parts breakdown
PPL	planning parts list
P&PM	Packing and Packaging Manual
PPS	piece part specification
PR	Public Relations
PRD	Program Requirements Document
PRF	pulse repetition frequency
PS	process solution
PSA	pressure suit assembly
PSCA	pressure suit conditioning assembly
PSD	procedural support data power spectral density
PSIA	Production System Integration Area
PSPP	proposed system package plan
PSQ	personnel security questionnaire
PST	Pacific standard time
PTDP	preliminary technical development plan
PTOP	Program Test and Operations Plan
PTV	propulsion test vehicle
PTVE	Propulsion & Vehicle Engineering (NASA/MSFC)
PWP	planning work package
PWS	program work statement

VIBRATION, SHOCK, AND ACOUSTICS

IV	intravehicular
IVA	intravehicular activity
IWS	integrated work statement

J K

JAN	Joint Army-Navy
JDL	job drawing list
JI	jet interaction
JOP	joint operating procedure
KSC	Kennedy Space Center
KTS	Kodiak Tracking Station
KWIC	key word in context

L

laser	light amplification by stimulated emission of radiation
LAX	Los Angeles International Airport
LCC	Launch Control Center
LCO	launch control officer
LCSO	launch control safety officer
LDC	labor data collection
LEP	lowest effective power
LeRC	Lewis Research Center (NASA)
LID	line item description
LM	laboratory module
LMSE	laboratory module simulation equipment
LO	Launch Operations (MDAC)
LOAP	List of Applicable Publications
LOCC	launch operations control center
LRC	Langley Research Center (NASA)
LRS	Laboratory Release System
LRV	lifting reentry vehicle
LSB	launch service building
LST	local sidereal time
LSWO	Logistics Support work order
LTD	Launch Test Directive
LV	laboratory vehicle
LVSS	Laboratory Vehicle System Segment

M

M&A	maintenance and assembly
MAB	missile assembly building
MAP	Manned Awareness Program
MAS	manufacture assembly specifications
maser	microwave amplification by stimulated emission of radiation
MATS	Military Air Transport Service
MBE	missileborne equipment
MCAIR	McDonnell Aircraft Company
MCAUTO	McDonnell Automation Company
MCC	Mission Control Center
MCP	military construction program
MCR	Master Control Room
MDAC	McDonnell Douglas Astronautics Company
MDAC-East	McDonnell Douglas Astronautics Company – East
MDAC-LO	McDonnell Douglas Astronautics Company – Launch Operations
MDAC-TICO	McDonnell Douglas Astronautics Company – TICO Division
MDAC-West	McDonnell Douglas Astronautics Company – West
MDC	McDonnell Douglas Corporation
MECO	main engine cutoff
MEL	master equipment list
MEMOS	Manufacturing Engineering Management Operations System
MEP	Manufacturing Engineering Planning
MGE	maintenance ground equipment
MIM	manufacturing information memorandum
MIS	Management Information Systems
MLB	manufacturing load boards
MLS	maintenance loading sheet
MLV	medium launch vehicle
MM-RE	Material & Methods – Research & Engineering
MOL	Manned Orbiting Laboratory
MORL	Manned Orbiting Research Laboratory
MP	maintenance procedure mass properties mission payload
MPL	Manufacturing parts list
MPSS	Mission Payload System Segment
MRB	Material Review Board modification review board
MRD	Mission Requirements Document Measurement Request Drawing
MR&PM	Materials Research & Production Methods
MRV	maneuvering reentry vehicle
MS	military standard mission simulator
MSA	monitor and switching assembly
MSB	Mission Simulator Building most significant bit
MSC	Maintenance Support Center
MSF	Mission Simulator Facility
MSFC	Marshall Space Flight Center (NASA)
MSK	major subcontract major subcontractor
MSR	Management Systems Representative missile site radar
MST	mobile service tower
MTBF	mean time between failures
MTBM	mean time between maintenance
MTSC	Magnetic Tape Selectric Composer
MTST	Magnetic Tape Selectric Typewriter
MTTF	mean-time-to-failure
MTTR	mean-time-to-repair
MTU	mobile training unit

EOS	emergency oxygen system
EPS	Electrical Power Subsystem
ERN	engineering reference number
ESD	Electronic Systems Division
ESM	Engineering Service Memorandum
ETC	estimate to completion
etc.	and so forth
ETR	Eastern Test Range (formerly Atlantic Missile Range)
EV	extravehicular
EVA	extravehicular activity
EVT	extravehicular transfer
EWO	engineering work order
EWR	engineering work request

F

f	frequency
FAA	Federal Aviation Agency
FACI	first article configuration inspection
FAO	fabrication assembly order
FARR	failure and rejection report
FCEI	facility contract end item
FCR	facility change request
FFP	firm fixed price
FFPI	fixed-fee-plus-incentive
FHS	flame hardness standard
FM	frequency modulation
FMEA	failure modes and effects analysis
FO	fabrication order fabrication outline
FORTRAN	formula translation
F.P.	flutter parameter
FP	fixed price
FPI	fixed price incentive
FPIF	fixed price incentive fee
FPL	fluctuating pressure level
FR	facilities request
FSN	Federal Stock Number
FTC	Florida Test Center
FTP&E	Flight Test Planning & Evaluation
FV	flight vehicle
FY	fiscal year

G

GB	Gemini B
GBPS	Gemini B Procedures Simulator
G&C	guidance and control
GEEIA	Ground Electronics Engineering Installation Agency
GEOS	geodetic observation satellite
GFE	Government-furnished equipment
GFP	Government-furnished property
GIRLS	General Information Retrieval List System
GMT	Greenwich mean time
GOPE	Government-owned plant equipment
g_{pk}	zero-to-peak acceleration in g's
$g_{pk\text{-}pk}$	peak-to-peak acceleration in g's
GPO	Government Printing Office
GSA	General Services Administration
GSE	ground support equipment
GSFC	Goddard Space Flight Center (NASA)
GTP	Ground Test Plan
GTR	ground test reactor
GTS	Guam Tracking Station

H

H&CR	handling and checkout requirements
HE	human engineering
HEOS	highly eccentric orbiting satellite
HFE	human factors engineering
HQ	Headquarters (MDAC)
HIBEX	high-g boost experiments
Hz	hertz

I

IA	interchangeable alternate
IAC	integration, assembly, and checkout
I&C	installation and checkout
ICBM	intercontinental ballistic missile
ICD	interface control document interface control drawing
ID	inside diameter
I/D&C	Instrumentation/Displays and Controls (Subsystem)
IDEA	interface and display electronics assembly
IDEP	Interservice Data Exchange Program
i.e.	that is
IFB	invitation for bid
IFS	interface specification
IGY	International Geophysical Year
IL	Insertion Loss (acoustic)
ILC	initial launch capability integrated launch complex
IOC	initial operational capability
IOS	Indian Ocean Station inspection operation sheet
IPB	illustrated parts breakdown
IPWO	interplant work order
IR	illustration request
IRAD	independent research and development
IRAN	inspection and repair as necessary
IRBM	intermediate range ballistic missile
IRIG	Interrange Instrumentation Group
IRS	Information Retrieval System
ISM	Industrial Security Manual
ISS	Information Systems (subdivision)
IST	integrated-systems test
I&T(P)	Inspection & Test (Planning)
ITL	integrate-transfer-launch (facility)
IUM	interim use material

VIBRATION, SHOCK, AND ACOUSTICS

COMSEC	communication security
ConAC	Continental Air Command
ConAD	Continental Air Defense
ConUS	continental United States
CP	Company Policy
	Control Procedure
Cp	pressure coefficient
CPE(R)	contractor performance evaluation (report)
CPFF	cost-plus-fixed-fee
CPI	change package identification
CPIF	cost-plus-incentive-fee
CPS	central power system
	Company Policy Statement
CRAD	contract research and development
CRB	Change Review Board
CRR	cost reduction reports
CRS	calibration requirements summary
CSAR	Configuration Status Accounting Report
CSF	contractor support facility
CSG	Command Subsystem Group
CSK	component subcontract
	component subcontractor
CST	combined-systems test
CVL	configuration verification list
CW	carrier wave
	continuous wave

D

D/A	digital to analog (conversion)
DA	double amplitude
DAC	Douglas Aircraft Company
DACAN	Douglas Aircraft Company of Canada, Ltd.
DARL	Douglas Advanced Research Laboratories
DASG	Data Acquisition Subsystem Group
dB	decibel
DCAS	Defense Contract Administrative Services Office
DCL	drawing change list
DCN	design change notice
	document change notice
	document control number
DCR	drawing change request
DCREO	drawing change request engineering order
DCRSEO	drawing change request serial engineering order
DCS	dimensional control standard
DCSG	Data Computation Subsystem Group
DCU	digital counting unit
DDC	Defense Documentation Center
DE	Development Engineering
DETD	Development Engineering Task Description
DF	development fixture
	direction finder
	direction-finding
DIS	Douglas inspection standard
DM	data management
	Douglas model
	Drafting Manual
DMS	Data Management Subsystem
	data management system
	Douglas material specification
DOD	Department of Defense
DP	data processing
	Douglas paper
	Douglas private
DPS	detail process standard
DQS	Douglas quality specification
	Douglas quality standard
DRA	Design Review Agreement
DRL	design review list
DRN	documentation revision notice
DSV	Douglas space vehicle

E

E	Young's Modulus
EAC	estimate at completion
EB	external burning
ECIPL	engineering change identity parts list
ECL	engineering configuration list
EC/LS	Environmental Control and Life Support (Subsystem)
ECN	engineering change notice
ECO	engineering change order
ECP	engineering change proposal
ECR	engineering change request
ECS	environmental control system
ECSG	Environmental Control Subsystem Group
EDCTU	Electronic Development and Compatibility Test Unit (not hot mockup)
E&DO	Experimental & Development Operations
EDP	electronic data processing
EEG	electroencephalogram
EET	event elapsed time
	equivalent exposure time
e.g.	for example
EKG	electrocardiogram
EL	Engineering Laboratories
EMC	electromagnetic compatibility
EMI	electromagnetic interference
EO	engineering order
EOB	Engineering and Operations Building
EOC	explosive ordnance components
EOMSF	Earth Orbital Military Space Force

14-2 ABBREVIATIONS AND ACRONYMS

A

AB	Administrative Bulletin
ACO	administrative contracting officer
A/D	analog to digital (conversion)
AD	Administrative Directive
ADC	Air Defense Command
ADPE	automated data processing equipment
AEC	Atomic Energy Commission
AF	Air Force
AFE	authority for expenditure
AFLC	Air Force Logistics Command
AFM	Air Force manual
AFPR(O)	Air Force Plant Representative (Office)
AFR	Air Force regulation
AFSC	Air Force specialty code Air Force Systems Command
AFTO	Air Force technical order
A/G	air to ground
AGE	aerospace ground equipment
A/I	accident/incident
AM	amplitude modulation
AMA	Air Materiel Area
AMD	Aerospace Medical Division
AMO	advance material order
AMS	aerospace material specification
ANA	Air Force-Navy Aeronautical (Bulletin)
AO	assembly outline
AOM	assembly outline master
AORL	Apollo Orbiting Research Laboratory
APL	approved parts list
AP&PO	Advance Programming & Proposal Operations
ARC	Ames Research Center (NASA)
ARPA	Advanced Research Projects Agency (DOD)
ASM	air-to-surface missile
ASPR	Armed Services Procurement Regulations
AS&T	Advanced Systems & Technology
ASTEG	All-Systems Test Equipment Group
ASV	aircraft-to-surface vessel aerothermodynamic structural vehicle all-systems vehicle
A&T	assembly and test
ATC	Air Training Command
ATOLL	acceptance, test, or launch language
ATP	authority to proceed
ATW	Aerospace Test Wing
avc	automatic volume control
AVE	aerospace vehicle equipment
AWG	American wire gage

B

BCD	binary-coded decimal
BECO	booster engine cutoff
BFO	beat-frequency oscillator
BIOS	biological orbiting satellite
bit	binary digit
B/M	bill of material
BMEWS	Ballistic Missile Early Warning System
BMRS	ballistic missile reentry system
BO	burnout
BOSU	Bioastronautics Operational Support Unit
BPS	bits per second
BSD	Ballistic Systems Division
Btu	British thermal unit

C

C	Confidential (security class.)
CAB	Change Analysis Board
CADC	Continental Air Defense Command
CAGE	computerized aerospace ground equipment
CC	Committee Charter
CCB	Configuration Control Board (Air Force)
CCBD	Configuration Control Board Directive
CCCB	Configuration Change Control Board
CCN	contract change notice
CCP	contract change proposal Corporate Control Procedure
CD	chief draftsman contract definition
CDA	command data acquisition
CDP	contract definition phase
CDR	critical design review
CDRL	Contract Data Requirements List
CDRS	Computer Data Recording System
CEI	contract end item
CFAE	contractor-furnished aeronautical equipment contractor-furnished aerospace equipment
CFE	contractor-furnished equipment
CFP	contractor-furnished property
CG	center of gravity
CIB	Change Implementation Board
CIP	central information processor
CM	Configuration Management
CMD	Contract Management Division
C/O	checkout
CO	change order contracting officer
COBOL	common business-oriented language
COMSAT	communications satellite

and phase. (2) "Spectrum" is also used to signify a continuous range of components, usually wide in extent, within which waves have some specified common characteristic; e.g., "audio-frequency spectrum."

NOTE 1

The term "spectrum" is also applied to functions of variables other than time, such as distance.

NOTE 2

A line spectrum has nonzero values only at discrete frequencies (for example see *Fourier Series*). A continuous spectrum is a continuous function of frequency (for example, see *Fourier Spectrum*, and *Power Spectral Density*).

SPHERICAL WAVE is one in which the wave fronts are concentric spheres.

STAGNATION a point of stagnation in aerodynamics is one in which a streamline either meets or leaves the body surface.

NOTE

In the absence of viscous and compressibility effects, the stagnation pressure is the sum of the static and dynamic pressures.

STANDARD DEVIATION is the positive square root of the variance; i.e., the square root of the mean of the squares of the deviation from the mean value of a quantity.

STANDING WAVE is a periodic wave having a fixed distribution in space, which is the result of interference of progressive waves of the same frequency and kind. Such waves are characterized by the existence of nodes, or partial nodes, and antinodes that are fixed in space.

STATIC PRESSURE is the hydrostatic pressure that exists at a point if viscous or dynamic effects, due to fluid motion or sound fields, are not included.

STATIONARY PROCESS is an ensemble of time histories having statistical properties which are invariant with respect to translations in time.

STEADY-STATE VIBRATION exists in a system if the velocity of each particle is a continuing periodic quantity.

NOTE

In the case of random response, the velocity of each particle is a continuing periodic quantity with random amplitude.

STIFFNESS is the ratio of change of force (or torque) to the corresponding change in translational (or rotational) displacement of an elastic element.

STING is a mounting device used to support models in a wind tunnel. It usually houses a balance for measuring forces and moments and provides the capability of controlling the roll, pitch, and yaw of the model. Also it often provides a means of protecting instrumentation leads from the flow.

SUBSONIC designates velocities with an amplitude less than the speed of sound. (The use of subsonic to describe frequencies below the audible range is deprecated. See *Infrasonic*.)

SUPERSONIC currently designates the range of velocity magnitudes above the speed of sound. Velocities that are several times the speed of sound may be called hypersonic (see *Hypersonic* and *Subsonic*).

TIME-HISTORY is the magnitude of a quantity expressed as a function of time.

TRANSIENT VIBRATION is temporarily sustained vibration of a mechanical system. It may consist of forced or free vibration, or both.

TRANSITION as used in boundary layer theory, refers to the change from a laminar to turbulent layer. (See *Boundary Layer*.)

TRANSMISSIBILITY is the nondimensional ratio of the response amplitude of a system in steady-state forced vibration to the excitation amplitude. The ratio may be one of forces, displacements, velocities, or accelerations.

TRANSMISSION LOSS is the reduction in the magnitude of some characteristic of a signal, between two stated points in a transmission system.

NOTE 1

The characteristic is often some kind of level, such as power level or voltage level; in acoustics the characteristic that is commonly measured is sound pressure level. Thus, if the levels are expressed in decibels the transmission loss level is likewise in decibels.

NOTE 2

It is imperative that the characteristic concerned (such as the sound pressure level) be clearly identified because, in all transmission systems, more than one characteristic is propagated.

NOTE 3

Sound transmission loss (TL) of a piece of structure is a measure of the insulation that the structure provides against airborne incident sound. TL is a characteristic of an object and is therefore independent of location or time unless some physical deformation or chemical change takes place. TL is dependent on both frequency

and angle of incidence of the sound wave. In general, TL is quoted for random incidence (diffuse sound field) and is thereby an "average" type of descriptor. TL, as such, is not a measurable quantity in the case of sound, as the sending and receiving sides of the structure modify the sound field. Hence, noise reduction is measured, while TL must be calculated.

TRANSVERSE WAVE is one in which the direction of displacement at each point of the medium is parallel to the wave front.

ULTRASONICS is the technology of sound at frequencies above the audible range.

NOTE

Supersonics is the general subject covering phenomena associated with speeds higher than the speed of sound (as in the case of aircraft and projectiles traveling faster than sound). This term was once used in acoustics synonymously with "ultrasonic"; such usage is now deprecated.

UNDAMPED NATURAL FREQUENCY of a mechanical system is the frequency of free vibration resulting from only elastic and inertial forces of the system.

VARIANCE is the mean of the squares of the deviations from the mean value of a quantity.

VELOCITY is a vector quantity that specifies the time rate of change of displacement with respect to a reference frame. If the reference frame is not standard, the velocity is often designated "relative velocity".

VELOCITY SHOCK is a mechanical shock resulting from a non-oscillatory change in velocity of an entire system.

VIBRATION is oscillation of a mechanical system. (See *Oscillation*.)

VISCOUS DAMPING causes the dissipation of energy, and occurs when a particle in a vibrating system is resisted by a force that has a magnitude proportional to the magnitude of the velocity of the particle and a direction opposite to the direction of the velocity of the particle.

WALL SHEAR STRESS τ_w in boundary layer theory is equal to the product of the free stream dynamic pressure times the local coefficient of friction. It is a measure of the frictional stress near the wall.

WAVE INTERFERENCE is the phenomenon which results when waves of the same or nearly the same frequency are superposed and is characterized by a spatial or temporal distribution of amplitude of some specified characteristic differing from that of the individual superposed waves.

WAVELENGTH of a periodic wave in an isotropic medium is the perpendicular distance between two wave fronts in which the displacements have a difference in phase of one complete period.

WAVE VELOCITY (Propagation Velocity) is a vector quantity that specifies the speed and direction with which a sound wave travels through a medium.

WHITE NOISE possesses a spectrum density (or spectrum level) that is substantially independent of frequency over a specified range.

NOTE

White noise need not be random.