V-BAND STRUCTURAL ANALYSIS
Revision A

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September 11, 2009

Introduction

A V-band is used to connect two segments of a rocket vehicle. The V-band is ejected via bolt-cutters so that the segments may separate at the proper time during flight. Consider a V-band which consists of two semicircular halves with two bolts on each V-band half.

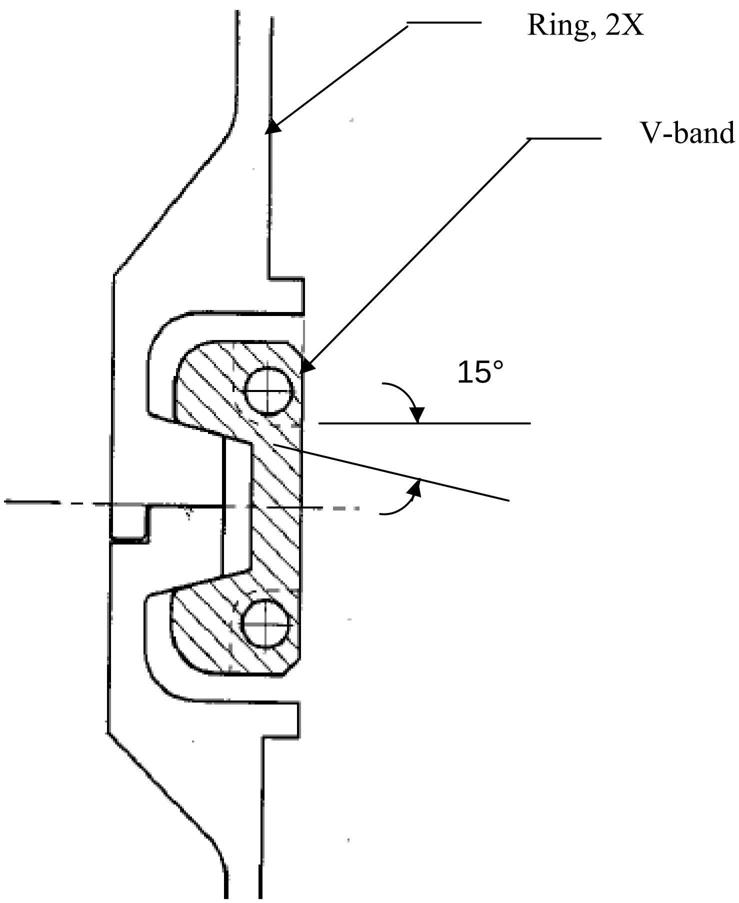


Figure 1. V-band Joint Cross Section

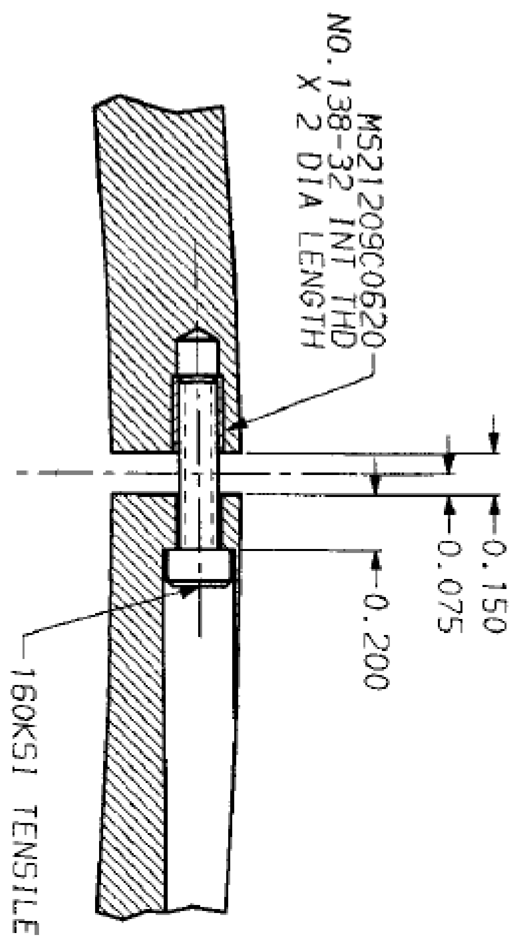


Figure 2. Top Looking Down View

There are two bolts for each half. Only the top bolt is shown in the diagram.

Loading

The V-band is subjected to an external bending moment.

The applied moment will first cause the initial preload to reduce. Once this preload is overcome by the moment, then the V-band tension increases as the band resists the force acting to separate the rings from each other.

Objective

Determine the force in each bolt. Also determine the maximum allowable bending moment based on the combined tensile strength in the bolts.

Assume that the mating surface between the V-bands and the rings is frictionless.

Variables

The variables are

M_A	=	Applied Bending moment
M	=	Maximum allowable bending moment
F_{AX}	=	Axial force due to drag, acceleration, etc.
F_{SP}	=	Spring force
F_{BM}	=	Force in each bolt (2 bolts on each half)
R	=	Radius of V-band
T	=	Combined tensile strength (force) of the two bolts on either side of the V-band.

Equations

The force in each bolt is

$$F_{BM} = \left[\frac{M_A}{4R} - \frac{1}{8}\left(F_{AX} - F_{SP}\right) \right] \tan 15^\circ \tag{1}$$

The maximum allowable bending moment M is

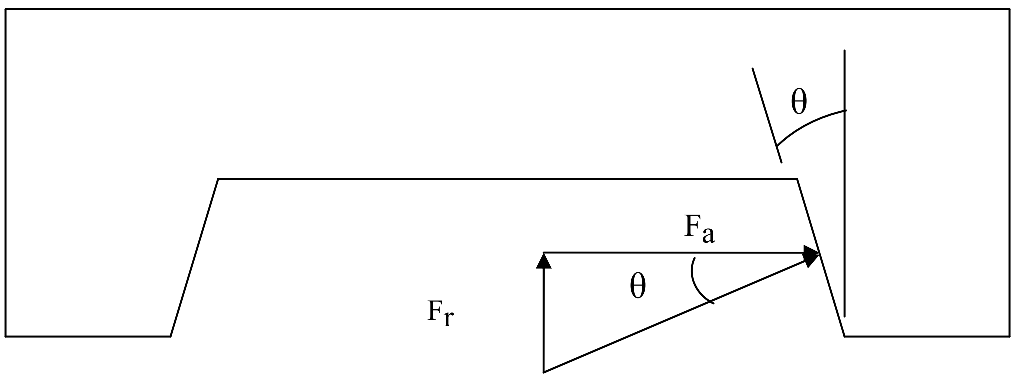
$$M = \frac{2\ R\ T}{\tan 15^\circ} + \frac{\left(F_{AX} - F_{SP}\right) R}{2} \tag{2}$$

A partial derivation of equation (2) is given in Appendix C via an approximate method. The axial and drag forces are neglected in the derivation.

A separate derivation using a more rigorous method is given in Appendix E.

APPENDIX A

Forces Against V-band Cross-section



$$\theta = 15^\circ \tag{A-1}$$

The factor 2 accounts for the forces on both sides of the V-band.

$$F_r = 2F_a \tan 15^\circ \tag{A-2}$$

$$F_a = \frac{1}{2} F_r \,/\, \tan 15^\circ \tag{A-3}$$

APPENDIX B

<u>Total Force Acting on V-band as a Function of Bending Moment</u>

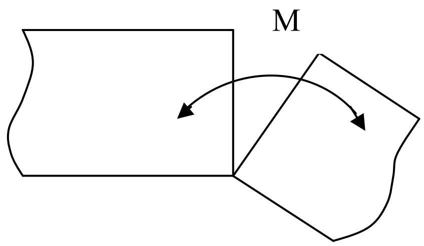


Figure B-1.

Assume that the rocket vehicle body structure is much stiffer than the V-band. The deflection of the V-band joint under a moment M will appear as shown in Figure B-1.

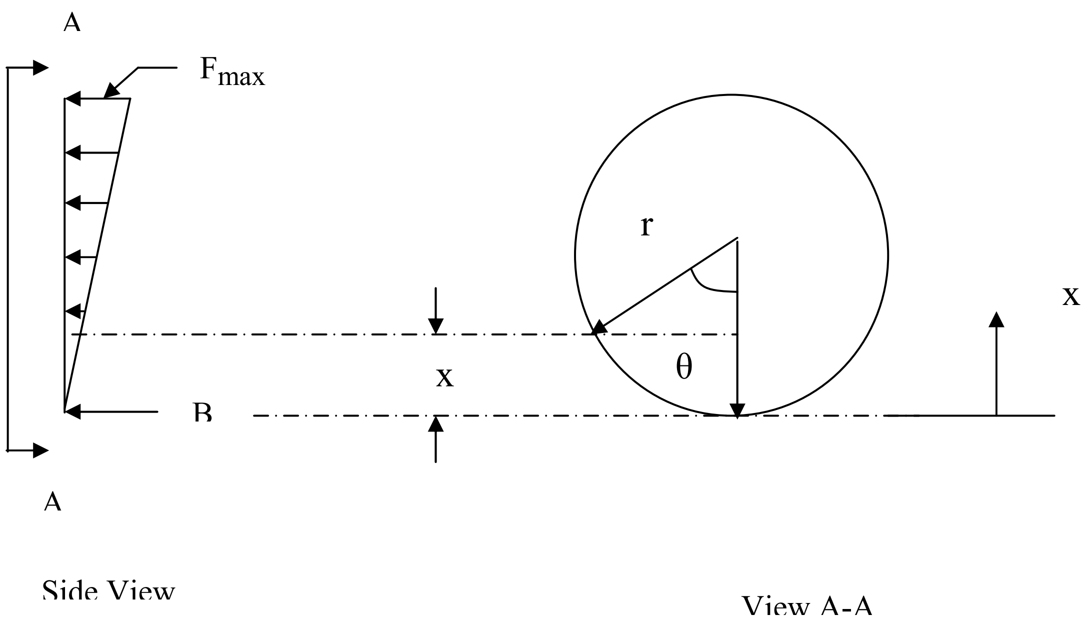


Figure B-2.

Variables:

F(x)	=	Line load at location x
C	=	Slope

$$F(x) = C\,x \tag{B-1}$$

$$C = \frac{F_{max}}{2R} \tag{B-2}$$

$$x = R\,(1 - \cos\theta) \tag{B-3}$$

$$F(\theta) = C\,R\;\;(1 - \cos\theta) \tag{B-4}$$

$$M = \int_0^{2\pi} F(\theta)\,x(\theta) R\,d\theta \tag{B-5}$$

$$M = \int_0^{2\pi} C\;\;(1 - \cos\theta)^2 R^3\,d\theta \tag{B-6}$$

$$M = C\,R^3 \int_0^{2\pi} (1 - \cos\theta)^2\,d\theta \tag{B-7}$$

$$M = C\,R^3 \int_0^{2\pi} \left[1 - 2\cos\theta + \cos^2\theta\right] d\theta \tag{B-8}$$

$$M = C\,R^3 \int_0^{2\pi} \left[1 - 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\right] d\theta \tag{B-9}$$

$$M = C\,R^3 \int_0^{2\pi} \left[\frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta \right] d\theta \tag{B-10}$$

$$M = C\,R^3 \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta \right]\Bigg|_0^{2\pi} \tag{B-11}$$

$$M = 3\pi C\,R^3 \tag{B-12}$$

$$C = \frac{M}{3\pi\ R^3} \tag{B-13}$$

$$F(\theta) = C\,R\ \ (1 - \cos\theta) \tag{B-14}$$

$$F(\theta) = \frac{M}{3\pi\ R^3} R\ \ (1 - \cos\theta) \tag{B-15}$$

$$F(\theta) = \frac{M}{3\pi\ R^2}\ \ (1 - \cos\theta) \tag{B-16}$$

$$F_{total} = \int_0^{2\pi} F(\theta)\,R\,d\theta \tag{B-17}$$

$$F_{total} = \int_0^{2\pi} \frac{M}{3\pi\ R^2}\ \ (1 - \cos\theta)\,R\,d\theta \tag{B-18}$$

$$F_{total} = \frac{M}{3\pi\ R} \int_0^{2\pi} (1 - \cos\theta)\ d\theta \tag{B-19}$$

$$F_{total} = \frac{M}{3\pi\ R} \left[\theta - \sin\theta\right]\Big|_0^{2\pi} \tag{B-20}$$

$$F_{total} = \frac{2M}{3R} \tag{B-21}$$

Reference: 006-007, page 14.

APPENDIX C

Allowable Bending Moment for a Given Bolt Tensile Strength

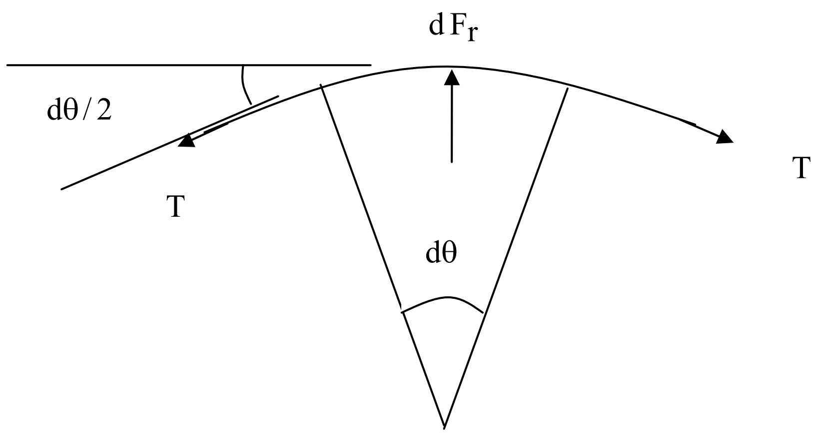


Figure C-1.

For incremental angles,

$$dF_r = 2T\sin\frac{d\theta}{2} \tag{C-1}$$

For small angles,

$$\sin\frac{d\theta}{2} \approx \frac{d\theta}{2} \tag{C-2}$$

$$dF_r = T\,d\theta \tag{C-3}$$

$$\int_0^{\pi} dF_r = T \int_0^{\pi} d\theta \tag{C-4}$$

$$F_r = \int_0^{\pi} dF_r \tag{C-5}$$

$$F_r = \pi T \tag{C-6}$$

From Appendix A,

$$F_r = F_a \tan 15^{\circ} \tag{C-7}$$

$$2\, F_a \tan 15^{\circ} = \pi T \tag{C-8}$$

$$F_a = \frac{\pi T}{2 \tan 15^{\circ}} \tag{C-9}$$

Recall equation (B-21).

$$F_{total} = \frac{2M}{3R} \tag{C-10}$$

$$F_{total} = F_a \tag{C-11}$$

$$\frac{2M}{3R} = \frac{\pi T}{2 \tan 15^{\circ}} \tag{C-12}$$

The maximum allowable bending moment M is

$$M = \left(\frac{3\pi}{4}\right)\frac{R\,T}{\tan 15^\circ} \approx 2.36\,\frac{R\,T}{\tan 15^\circ} \tag{C-13}$$

Reference: 006-007, page 16.

Again, T is combined tensile strength (force) of the two bolts on either side.

Equation (2) in the main text neglecting the axial and spring stiffness forces is

$$M = 2\,\frac{R\,T}{\tan 15^\circ} \tag{C-14}$$

APPENDIX D

Force on V-band due to Preload

This is a beam approach. Assume that the V-band only resists tension.

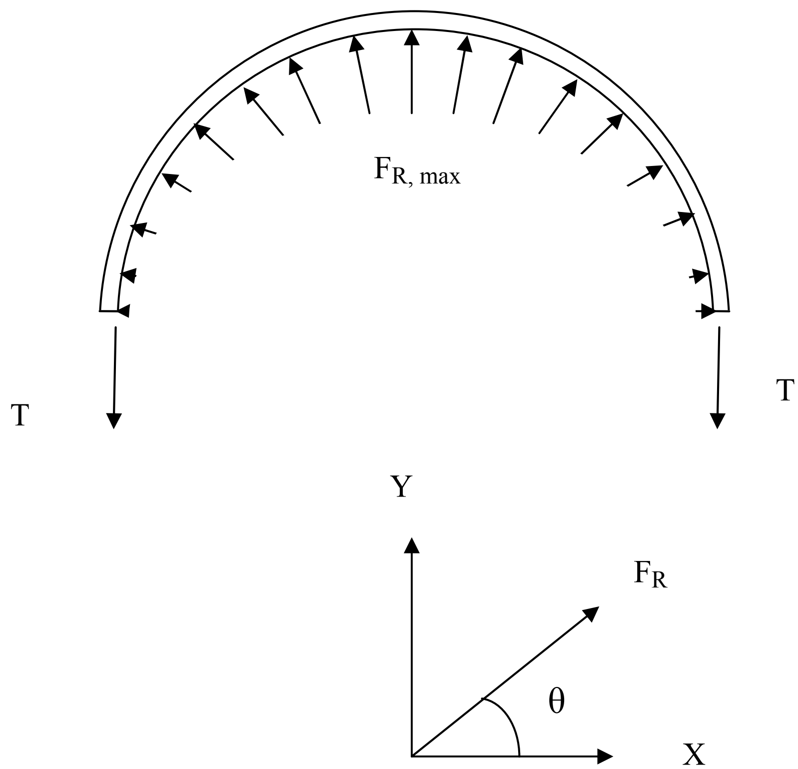


Figure D-1.

$$F_R = F_{R,max} \sin\theta \tag{D-1}$$

$$F_{R,y} = F_R \sin\theta \tag{D-2}$$

$$F_{R,y} = F_{R,max} \sin^2\theta \tag{D-3}$$

$$2T = \int_0^{\pi} F_{R,y}\, r\, d\theta \qquad \text{(D-4)}$$

$$2T = \int_0^{\pi} F_{R,max} \sin^2\theta\, r\, d\theta \qquad \text{(D-5)}$$

$$2T = F_{R,max}\, R \int_0^{\pi} \sin^2\theta\, d\theta \qquad \text{(D-6)}$$

$$2T = F_{R,max}\, R \int_0^{\pi} \left[\frac{1}{2} - \frac{1}{2}\cos 2\theta\right] d\theta \qquad \text{(D-7)}$$

$$2T = F_{R,max}\, R \left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]\Bigg|_0^{\pi} \qquad \text{(D-8)}$$

$$2T = \frac{1}{2}\pi F_{R,max}\, R \qquad \text{(D-9)}$$

$$F_{R,max} = \frac{4T}{\pi R} \qquad \text{(Line Load)} \qquad \text{(D-10)}$$

Reference: 006-007, page 36.

APPENDIX E

Line Load as a Function of Moment

Assume that the joint acts like a beam where V-band provides the joint tension and the structure provides the compression.

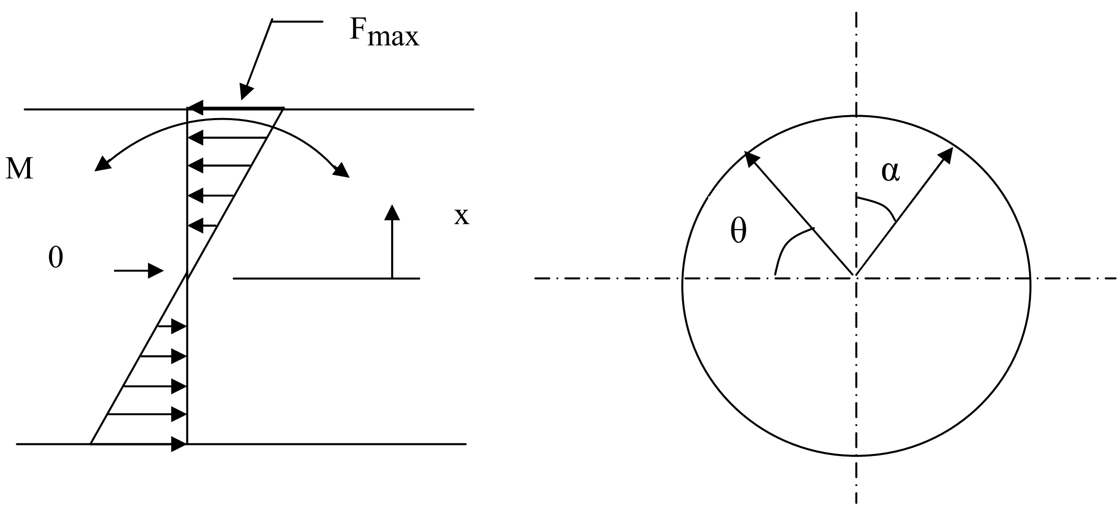


Figure E-1.

Assume that the centroid is in the center of the beam. The V-band will not take compression. It will be taken by the two structural rings of the joint.

$$x = R \sin\theta \tag{E-1}$$

Note that F is a line load with dimensions of (force/length).

$$F = a\,x \qquad \text{where } a \text{ is the slope} \tag{E-2}$$

$$F = F_{max} \qquad \text{at } x = R \tag{E-3}$$

Thus

$$a = \frac{F_{max}}{R} \tag{E-4}$$

$$F = \left(\frac{F_{max}}{R}\right) x \tag{E-5}$$

$$F = \left(\frac{F_{max}}{R}\right) R \ \sin\theta \tag{E-6}$$

$$F \ = \ F_{max} \sin\theta \tag{E-7}$$

The total moment M is

$$M = \int_0^{2\pi} F\, R \sin\theta\, R\, d\theta \tag{E-8}$$

$$M = R^2 \int_0^{2\pi} F \sin\theta \ d\theta \tag{E-9}$$

$$M = R^2 \ F_{max} \int_0^{2\pi} \sin^2\theta \ d\theta \tag{E-10}$$

$$M = R^2 \ F_{max} \int_0^{2\pi} \frac{1}{2}\left[1 - \cos 2\theta\right] d\theta \tag{E-11}$$

$$M = \frac{1}{2} R^2 \ F_{max} \left[\theta - \frac{1}{2}\sin 2\theta\right]\Bigg|_0^{2\pi} \tag{E-12}$$

$$M = \pi R^2 \ F_{max} \tag{E-13}$$

$$F_{max} = \frac{M}{\pi R^2} \tag{E-14}$$

Notational change:

$$F_{a,\,max} = F_{max} = \frac{M}{\pi R^2} \tag{E-15}$$

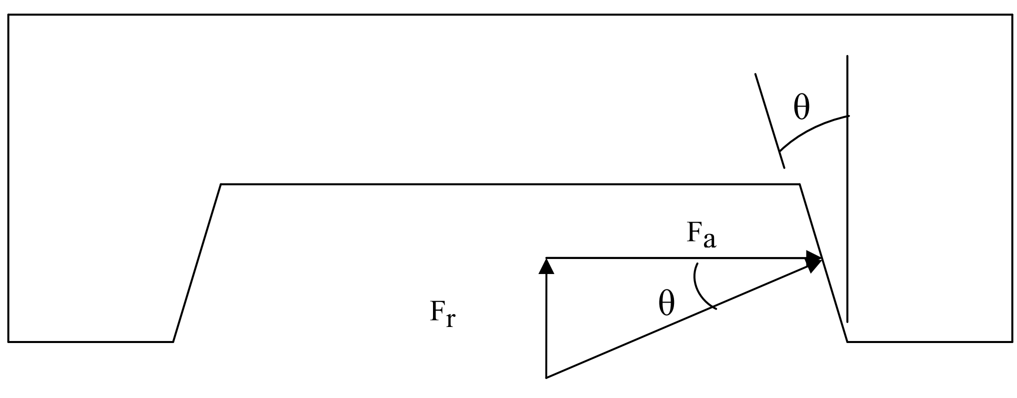


Figure E-2.

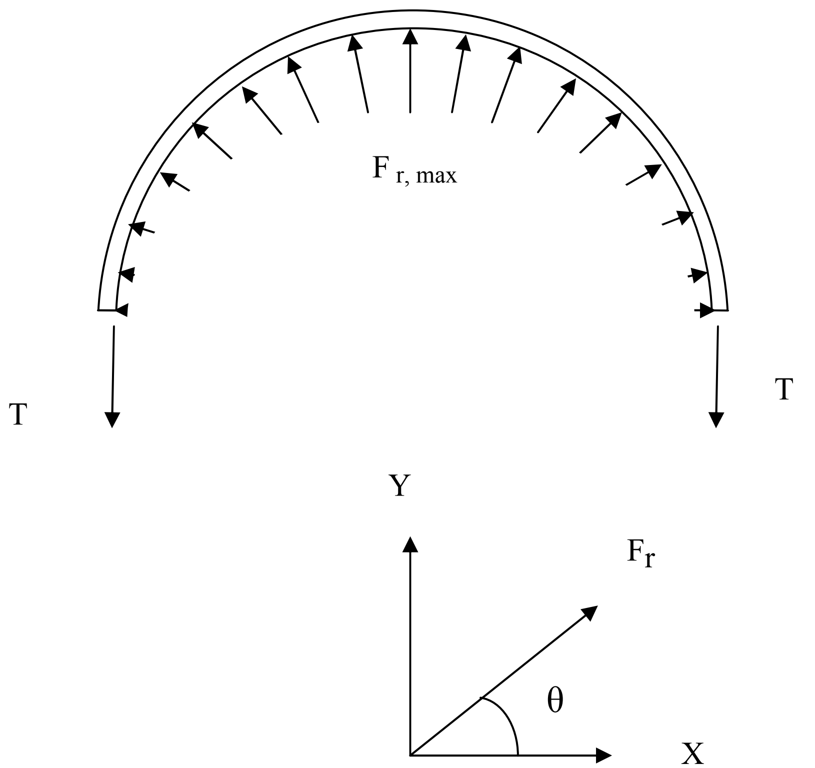


Figure E-3.

$$2T = \int_0^{\pi} F_Y \; R \, d\theta \tag{E-16}$$

$$F_Y = F_r \sin\theta \tag{E-17}$$

$$F_r = 2 F_a \tan 15^\circ \tag{E-18}$$

$$F_y = 2\, F_a \; \tan 15^\circ \sin\theta \tag{E-19}$$

$$2T = \int_0^{\pi} 2\, F_a \tan 15^\circ \sin\theta\, R\, d\theta \tag{E-20}$$

$$T = \int_0^{\pi} F_a \tan 15^\circ \sin\theta\, R\, d\theta \tag{E-21}$$

$$F_a = F_{a,max} \sin\theta \tag{E-22}$$

$$F_a = \left(\frac{M}{\pi R^2}\right)\sin\theta \tag{E-23}$$

$$T = \int_0^{\pi} \left(\frac{M \tan 15^\circ}{\pi R^2}\right) \sin^2\theta\, R\, d\theta \tag{E-24}$$

$$T = \left(\frac{M \tan 15^\circ}{\pi R}\right)\int_0^{\pi} \sin^2\theta\, d\theta \tag{E-25}$$

$$T = \left(\frac{M \tan 15^\circ}{\pi R}\right)\int_0^{\pi} \frac{1}{2}\left[1-\cos 2\theta\right] d\theta \tag{E-26}$$

$$T = \left(\frac{M \tan 15^\circ}{2\pi R}\right)\left[\theta - \frac{1}{2}\sin 2\theta\right]\Bigg|_0^{\pi} \tag{E-27}$$

$$T = \left(\frac{M}{2R} \right) \tan 15^\circ \tag{E-28}$$

The maximum allowable bending moment is thus

$$M = \frac{2\,R\,T}{\tan 15^\circ} \tag{E-29}$$

Reference: 006-007, page 22.

APPENDIX F

V-band Capability before Gapping

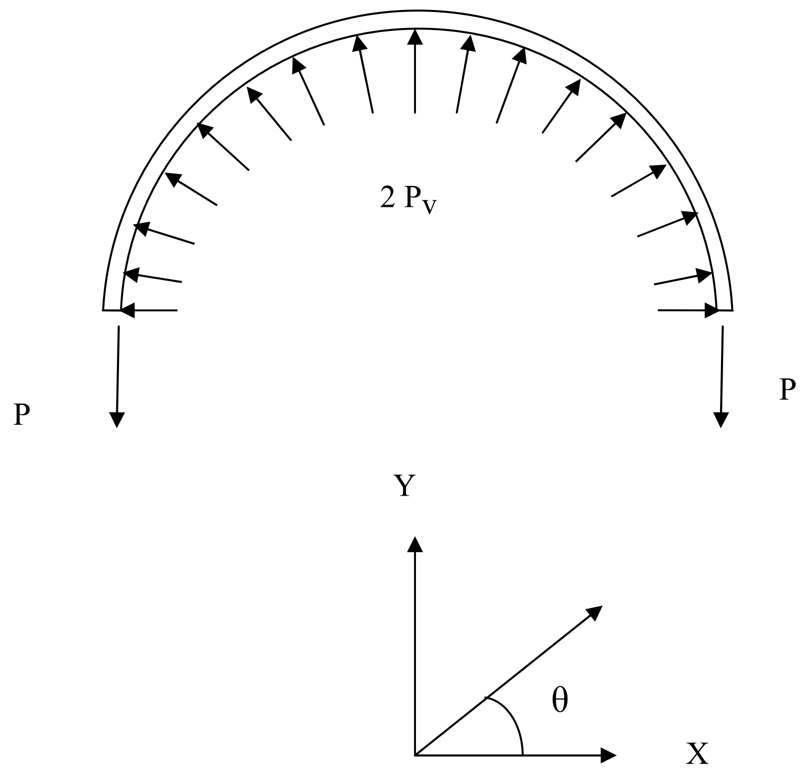


Figure F-1.

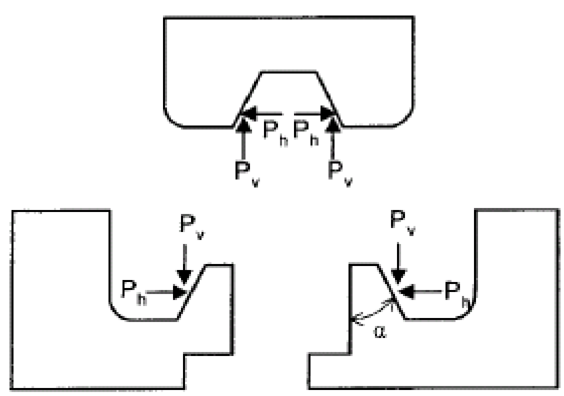


Figure F-2.

Preload Force Equation

The distributed force $F_y(\theta)$ in the Y-axis is

$$F_y(\theta) = 2\,P_v \sin(\theta) \tag{F-1}$$

The preload tension P is related to the distributed preload by

$$2P = \int_0^{\pi} F_y(\theta) R d\theta \tag{F-2}$$

$$2P = \int_0^{\pi} 2\,P_v \sin(\theta) R d\theta \tag{F-3}$$

$$P = P_v\, R \int_0^{\pi} \sin(\theta) d\theta \tag{F-4}$$

$$P = P_v\, R\left[-\cos(\theta)\right]\Big|_0^{\pi} \tag{F-5}$$

$$P = 2P_v\, R \tag{F-6}$$

$$P_v = \frac{P}{2\,R} \tag{F-7}$$

Gapping Threshold

The amount of bending moment that the V-band can be subjected to before the torque preload is overcome depends on the torques in the bolts.

Assume a torque friction coefficient of 0.20 because the condition (dry or lubricated) is unknown.

The preload P is

$$P = \frac{T}{0.20\,D} \tag{F-8}$$

where T is the bolt torque and D is the bolt diameter.

Reference 061-093, page 7.

The maximum allowable bending moment for no gapping is derived as follows.

Recall equation (E-14) where the force is now induced by the distributed preload P_V.

$$P_V = \frac{M}{\pi R^2} \tag{F-9}$$

$$M = \pi R^2 P_V \tag{F-10}$$

$$P_V = \frac{P}{2R} \qquad \text{(from bolt preload)} \tag{F-11}$$

The maximum allowable moment for no gapping is thus

$$M = \frac{\pi P R}{2 \tan 15^\circ} \tag{F-12}$$

This is also the moment that would cancel the preload.