THE VIBRATION RESPONSE OF A PANEL TO AN ACOUSTIC PRESSURE FIELD
Revision B

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October 7, 2006

Introduction
Consider an acoustic pressure field which excites a homoegenous panel. Assume that the pressure field has random incidence upon the panel.

As an example, a satellite is mounted inside of a launch vehicle nosecone fairing. The satellite's solar panels must withstand the liftoff acoustic field. The fairing attenuates a great deal of this energy. Nevertheless, a portion of the acoustical energy is transmitted to the satellite.

The purpose of this report is to derive a relationship between the acoustic pressure and the panel vibration response.

Critical Frequency
The critical frequency is the frequency at which the airborne acoustic wavelength matches the panel bending wavelength. The critical frequency \( f_c \) for a homogeneous panel is

\[
f_c \approx \frac{c^2}{1.8 C_L h}
\]

(1)

where
- \( c \) is the speed of sound in the surrounding air medium
- \( C_L \) is the longitudinal wave velocity
- \( h \) is the panel thickness

Equation (1) is taken from Reference 1, chapter 6, equation (56a).
Panel Velocity Response to Acoustic Pressure

Let \( V_{\text{RMS}} \) be the velocity root-mean-square.
Let \( P_{\text{RMS}} \) be the pressure root-mean-square.

The velocity and pressure are related by the following semi-empirical equation.

\[
[ V_{\text{RMS}} ]^2 = \beta [ P_{\text{RMS}} ]^2
\]  
(2)

Equation (2) is taken from Reference 1, chapter 8.

Note that each term in equation (2) is a function of frequency.

The coefficient \( \beta \) is

\[
\beta = \begin{cases}
\frac{1}{2\pi^2 f^2 m^2} \left[ 1 - \frac{\rho c}{\pi f m} \arctan \left( \frac{\pi f m}{\rho c} \right) \right], & \text{for } f < f_c \\
\frac{c^2}{8\pi f^2 m^2 c_B^2 \eta}, & \text{for } f \geq f_c
\end{cases}
\]  
(3)

where

- \( f \) is the excitation frequency
- \( m \) is the mass per area
- \( c \) is the speed of sound in the surrounding air
- \( \rho \) is the mass per volume of the surrounding air
- \( c_B \) is the phase speed of the bending waves in the panel
- \( \eta \) is the loss factor

Equation (3) is taken from Reference 1, chapter 8, section c for frequencies below the critical frequency.

Equation (3) is derived from Reference 1, chapter 8, section d, equations (101) through (103) for frequencies above the critical frequency.

Note that the phase speed \( c_B \) and the loss factor \( \eta \) each vary with frequency.
Panel Phase Speed

Consider a panel with thickness $h$ and a longitudinal wave velocity of $C_L$. The longitudinal velocity is a constant for a given material. The phase speed is

$$C_B \approx \sqrt{1.8 \ C_L \ h \ f}$$  \hspace{1cm} (4)

Equation (4) is taken from Reference 1, chapter 3.

Mass Law

Equation (2) essentially follows the "mass law" below the critical frequency. A doubling of either the mass density or the frequency causes the velocity to decrease to one-half its value, which is equivalent to a 6 dB drop.

Equation (2) is complicated above the critical frequency due to the variation of the phase speed and loss factor with frequency.

Loss Factor

The loss factor $\eta$ is an empirical parameter. The loss factor is assumed to be due mainly to acoustic radiation above the critical frequency.

A mean loss factor curve for aluminum panels is shown in Figure 1. The frequency scale is normalized in terms of the critical frequency. The panels were suspended from a frame by elastic cords such that the boundary conditions were essentially free around the perimeter. The data was taken from two panels. One panel was 1/4 inch thick. The other was 1/2 inch thick. Each panel was 4.0 by 6.5 feet.

The data in Figure 1 is thus approximate data for a particular material, geometry, and boundary condition case. Loss factor measurements on a particular structure of interest must thus be taken in order to accurately use the calculation methods in this report.
Figure 1.
Power Spectrum Format

Acoustic pressure fields are typically represented in one-third octave band power spectra.

Let $V_{PS}$ be the velocity power spectrum.
Let $P_{PS}$ be the pressure power spectrum.

Equation (2) can be modified as follows

$$[V_{PS}] = \beta [P_{PS}]$$  \hspace{1cm} (5)

The acceleration is often of greater interest than velocity for structural vibration.

Let $A_{PS}$ be the acceleration power spectrum.

$$[A_{PS}] = \beta [2\pi f]^2 [P_{PS}]$$  \hspace{1cm} (6)

Reference Data

The parameter $\rho_c$ is called the "characteristic impedance." The values for air are shown in Table 1. The speed of sound for gases and solid is given in Tables 2 and 3, respectively.

<table>
<thead>
<tr>
<th>Gas and Temperature</th>
<th>Impedance</th>
<th>Impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Pa $\cdot$ s/m)</td>
<td>(lbf $\cdot$ s / ft$^3$)</td>
</tr>
<tr>
<td>Air at 0$^\circ$ C</td>
<td>428</td>
<td>2.73</td>
</tr>
<tr>
<td>Air at 20$^\circ$ C</td>
<td>415</td>
<td>2.64</td>
</tr>
</tbody>
</table>
Table 2. Gases at a pressure of 1 atmosphere

<table>
<thead>
<tr>
<th>Gases</th>
<th>Molecular Mass (kg/kgmole)</th>
<th>Temperature (°C)</th>
<th>Density (kg/m³)</th>
<th>Ratio of Specific Heats</th>
<th>Speed of Sound (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>28.97</td>
<td>0</td>
<td>1.293</td>
<td>1.402</td>
<td>332</td>
</tr>
<tr>
<td>Air</td>
<td>28.97</td>
<td>20</td>
<td>1.21</td>
<td>1.402</td>
<td>343</td>
</tr>
<tr>
<td>Oxygen (O₂)</td>
<td>32.00</td>
<td>0</td>
<td>1.43</td>
<td>1.40</td>
<td>317</td>
</tr>
<tr>
<td>Hydrogen (H₂)</td>
<td>2.016</td>
<td>0</td>
<td>0.09</td>
<td>1.41</td>
<td>1270</td>
</tr>
<tr>
<td>Steam</td>
<td>-</td>
<td>100</td>
<td>0.60</td>
<td>-</td>
<td>404.8</td>
</tr>
</tbody>
</table>

Table 3. Solids

<table>
<thead>
<tr>
<th>Solid</th>
<th>Density (kg/m³)</th>
<th>Elastic Modulus (Pa)</th>
<th>Shear Modulus (Pa)</th>
<th>Poisson’s Ratio</th>
<th>Speed of Sound (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bar</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2700</td>
<td>7.0 (10¹⁰)</td>
<td>2.4 (10¹⁰)</td>
<td>0.33</td>
<td>5100</td>
</tr>
<tr>
<td>Brass</td>
<td>8500</td>
<td>10.4 (10¹⁰)</td>
<td>3.8 (10¹⁰)</td>
<td>0.37</td>
<td>3500</td>
</tr>
<tr>
<td>Copper</td>
<td>8900</td>
<td>12.2 (10¹⁰)</td>
<td>4.4 (10¹⁰)</td>
<td>0.35</td>
<td>3700</td>
</tr>
<tr>
<td>Steel</td>
<td>7700</td>
<td>19.5 (10¹⁰)</td>
<td>8.3 (10¹⁰)</td>
<td>0.28</td>
<td>5050</td>
</tr>
<tr>
<td>Ice</td>
<td>920</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Glass (Pyrex)</td>
<td>2300</td>
<td>6.2 (10¹⁰)</td>
<td>2.5 (10¹⁰)</td>
<td>0.24</td>
<td>5200</td>
</tr>
</tbody>
</table>

Alternate Method

An alternate equation based on statistical energy analysis is given in Appendix A.
References


APPENDIX A

Diffuse Sound Field Driving a Freely Hung Panel

The mean square, space-time average velocity \( \langle v^2 \rangle \) of the panel is

\[
\langle v^2 \rangle = \langle p^2 \rangle \frac{\sqrt{12} \pi c_{\text{air}}^2}{2 (\rho c)_{\text{air}} h c_{L} \rho_s \omega^2} \left\{ \frac{1}{1 + \frac{\rho_s \omega \eta}{2 (\rho c)_{\text{air}} \sigma_{\text{rad}}}} \right\}
\]

(A-1)

where

\( \sigma_{\text{rad}} \) is the radiation efficiency

\( \rho_s \) is the mass per surface area of the panel

\( \eta \) is the loss factor of the panel

\( \langle p^2 \rangle \) is the mean square, space-time average pressure

Equation (A-1) is taken from Reference 2, section 9.8.
The radiation efficiency for supersonic bending waves in structures is

\[
\sigma_{\text{rad}} = \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad (A-2)
\]

Equation (A-2) is taken from Reference 3.

The radiation efficiency of a vibrating body generating sound energy in air is

\[
\sigma_{\text{rad}} = \frac{W_{\text{rad}}}{\left\langle v^2 \right\rangle (\rho c)_{\text{air}} S} \quad (A-3)
\]

Equation (A-3) is taken from Reference 2, section 9.6.