BIFILAR PENDULUM

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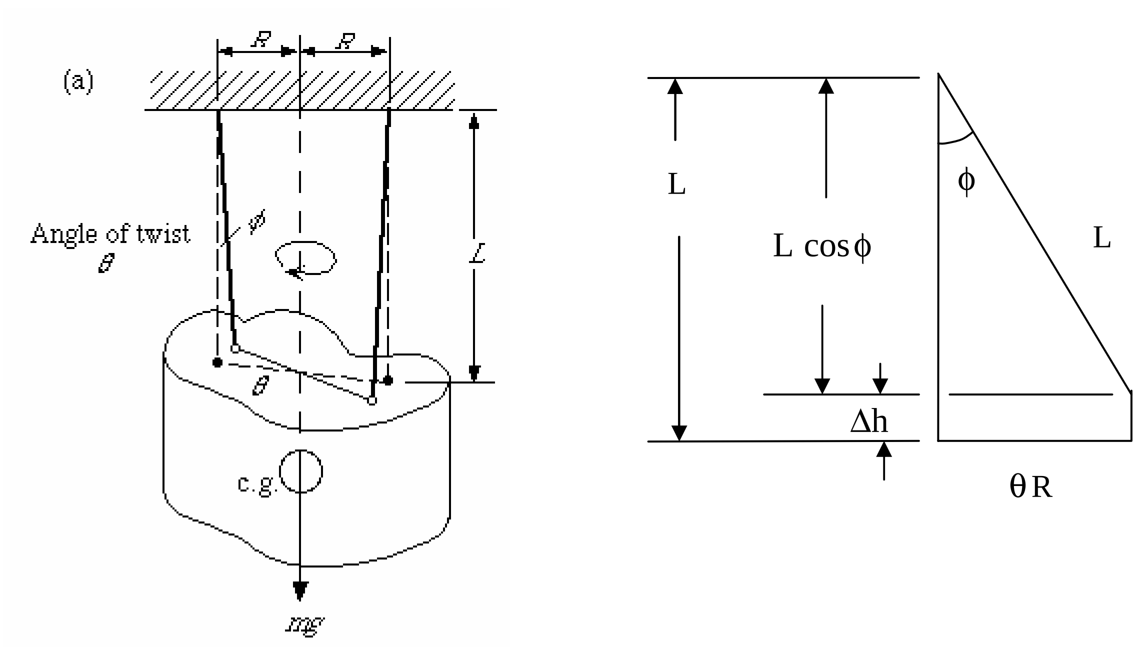

Figure 1. Bifilar Pendulum

The diagram on left is courtesy of University of Texas.

Let

$$d = 2R \tag{1}$$

The kinetic energy T is

$$T = \frac{1}{2} J\dot{\theta}^2 \qquad (2)$$

where J is the mass moment of inertia.

The potential energy V is

$$V = mg\,\Delta h \qquad (3)$$

$$\Delta h = L - \sqrt{L^2 - \frac{\theta^2 d^2}{4}} \qquad (4)$$

$$V = mg\left[L - \sqrt{L^2 - \frac{\theta^2 d^2}{4}} \right] \qquad (5)$$

The energy method is

$$\frac{d}{dt}[T + V] = 0 \qquad (6)$$

$$\frac{d}{dt}\left\{ \frac{1}{2} J\dot{\theta}^2 + mg\left[L - \sqrt{L^2 - \frac{\theta^2 d^2}{4}} \right] \right\} = 0 \qquad (7)$$

$$J\,\dot{\theta}\,\ddot{\theta} + \frac{1}{4} mg \frac{\theta\,\dot{\theta}\,d^2}{\sqrt{L^2 - \frac{\theta^2 d^2}{4}}} = 0 \qquad (8)$$

For small angular displacements,

$$\sqrt{L^2 - \frac{\theta^2 d^2}{4}} \approx L \tag{9}$$

Substitute equation (9) into (8).

$$J \dot{\theta} \ddot{\theta} + \frac{mg\, d^2}{4L} \theta \dot{\theta} = 0 \tag{10}$$

Divide through by $\dot{\theta}$.

$$J \ddot{\theta} + \frac{mg\, d^2}{4L} \theta = 0 \tag{11}$$

$$\ddot{\theta} + \frac{mg\, d^2}{4JL} \theta = 0 \tag{12}$$

Let

$$\omega_n^2 = \frac{mg\, d^2}{4JL} \tag{13}$$

The angular natural frequency is thus

$$\omega_n = \frac{d}{2}\sqrt{\frac{mg}{JL}} \tag{14}$$

The mass moment of inertia can be found experimentally as

$$J = \frac{m\,g\,d^2}{4\,\omega_n^2\,L} \tag{15}$$

$$\omega_n = 2\,\pi\,f_n \tag{16}$$

$$J = \frac{m\,g\,d^2}{16\,\pi^2\,f_n^2\,L} \tag{17}$$