Figure 1. Bifilar Pendulum

The diagram on left is courtesy of University of Texas.

Let

\[ d = 2R \]  \hspace{1cm} (1)
The kinetic energy $T$ is

$$T = \frac{1}{2} J \dot{\theta}^2 \tag{2}$$

where $J$ is the mass moment of inertia.

The potential energy $V$ is

$$V = mg \Delta h \tag{3}$$

$$\Delta h = L - \sqrt{L^2 - \frac{\theta^2 d^2}{4}} \tag{4}$$

$$V = mg \left[ L - \sqrt{L^2 - \frac{\theta^2 d^2}{4}} \right] \tag{5}$$

The energy method is

$$\frac{d}{dt} [T + V] = 0 \tag{6}$$

$$\frac{d}{dt} \left\{ \frac{1}{2} J \dot{\theta}^2 + mg \left[ L - \sqrt{L^2 - \frac{\theta^2 d^2}{4}} \right] \right\} = 0 \tag{7}$$

$$J \ddot{\theta} + \frac{1}{4} mg \frac{\theta \dot{\theta} d^2}{\sqrt{L^2 - \frac{\theta^2 d^2}{4}}} = 0 \tag{8}$$
For small angular displacements,

\[ \sqrt{L^2 - \frac{\theta^2 d^2}{4}} \approx L \]  

(9)

Substitute equation (9) into (8).

\[ J \ddot{\theta} + \frac{mgd^2}{4L} \dot{\theta} = 0 \]  

(10)

Divide through by \( \dot{\theta} \).

\[ J \ddot{\theta} + \frac{mgd^2}{4L} \theta = 0 \]  

(11)

\[ \ddot{\theta} + \frac{mgd^2}{4JL} \theta = 0 \]  

(12)

Let

\[ \omega_n^2 = \frac{mgd^2}{4JL} \]  

(13)

The angular natural frequency is thus

\[ \omega_n = \frac{d}{2} \sqrt{\frac{mg}{JL}} \]  

(14)
The mass moment of inertia can be found experimentally as

\[ J = \frac{m g d^2}{4 \omega_n^2 L} \]  \hspace{1cm} (15)

\[ \omega_n = 2 \pi f_n \]  \hspace{1cm} (16)

\[ J = \frac{m g d^2}{16 \pi^2 f_n^2 L} \]  \hspace{1cm} (17)