LAUNCH VEHICLE BODY-BENDING DYNAMICS

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Introduction

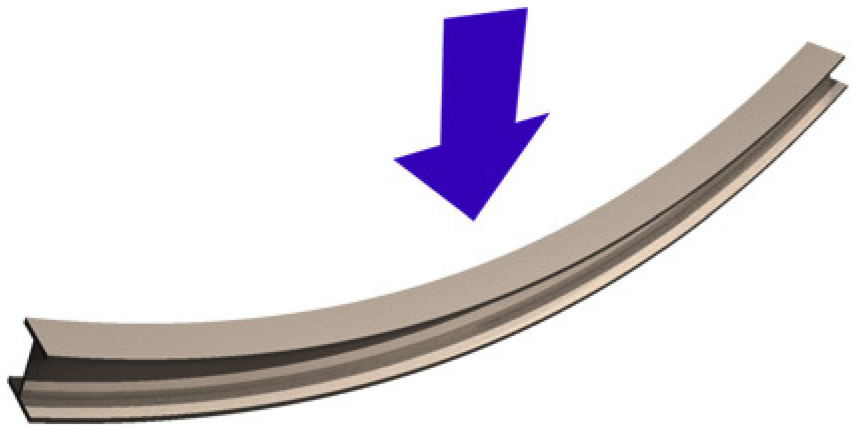

Figure 1. Free-Free Beam, Fundamental Bending Mode

A rocket vehicle in flight is conceptually similar to the beam in Figure 1 in terms of structural dynamics. The beam is unconstrained, with free boundary conditions.

Both the beam and the rocket vehicle have a fundamental body bending mode which oscillates back and forth. Both have higher bending modes as well. The vehicle's bending natural frequencies must be determined via analysis and test data prior to flight so that the autopilot control system can be designed accordingly.

Note that the vehicle's bending modes may be excited in flight by thrust offsets, maneuvers, wind gusts, etc.

Unlike the beam, the launch vehicle's bending frequencies changes with time. The vehicle's frequencies increase as the propellant mass is expelled.

Furthermore, the launch vehicle may have a gimbaled nozzle with a thrust vector control actuator. The autopilot system commands the nozzle to rotate in-flight as needed to steer the vehicle on its programmed trajectory and to correct for any wind gusts or perturbations.

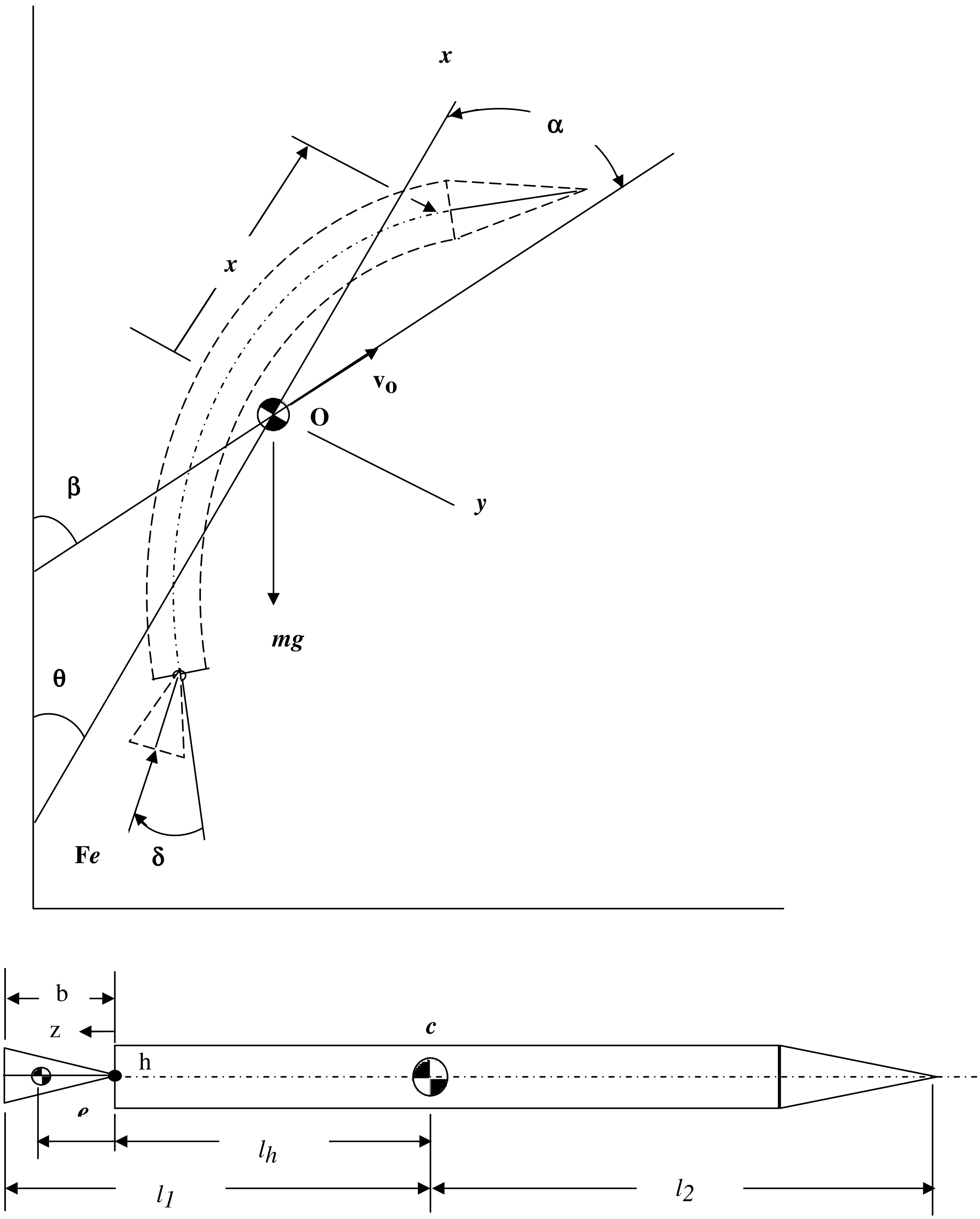

Figure 1. Launch Vehicle Geometry and Coordinates

$\mathbf{v_o}$ is the rigid body velocity vector.

Assumptions

1. The mass is constant. (The analysis could be for mass variation with time.)
2. The vehicle has a solid rocket motor, so that liquid fuel slosh is not a factor.
3. Restrict the motion to the x-y plane.
4. Neglect shear deformation and rotary inertia.
5. The nozzle is gimbaled.

Coordinate Definition

The coordinate system origin is at the center of mass. The coordinate system is moving. The x-axis is the longitudinal axis of the undeformed vehicle.

Variables

c	=	Center of mass of the entire vehicle
e	=	Nozzle center of mass
h	=	Nozzle hinge point
I_e	=	Mass moment of inertia of the nozzle about its hinge
I_o	=	Mass moment of inertia of the entire vehicle about the center of mass
m	=	Total mass of the vehicle
m_e	=	Mass of the nozzle
l_1	=	Length from nozzle exit plane to center of mass
l_2	=	Length from nose cone tip to center of mass
y_c	=	Rigid-body (or vehicle center of mass) displacement due to hinge rotation δ
z	=	Variable distance from hinge along nozzle centerline
z_e	=	Distance from hinge to nozzle center of mass
α	=	Angle-of-attack
δ	=	Nozzle angular displacement with respect to vehicle longitudinal axis
θ	=	Angle between global Y-axis and the vehicle's local X-axis
$\dot{\theta}_f$	=	Rigid-body angular velocity of the vehicle forward of the hinge due to hinge rotation

Variables (continued)

$\phi_i(x)$	=	Normal mode shapes for body-bending modes
$q_i(t)$	=	Generalized coordinate associate with the corresponding mode
ω	=	Rate of rotation of the coordinate axis. $\omega = \dot{\theta}$.

Note that nozzle system would be referred to as an engine if the vehicle had liquid propellant.

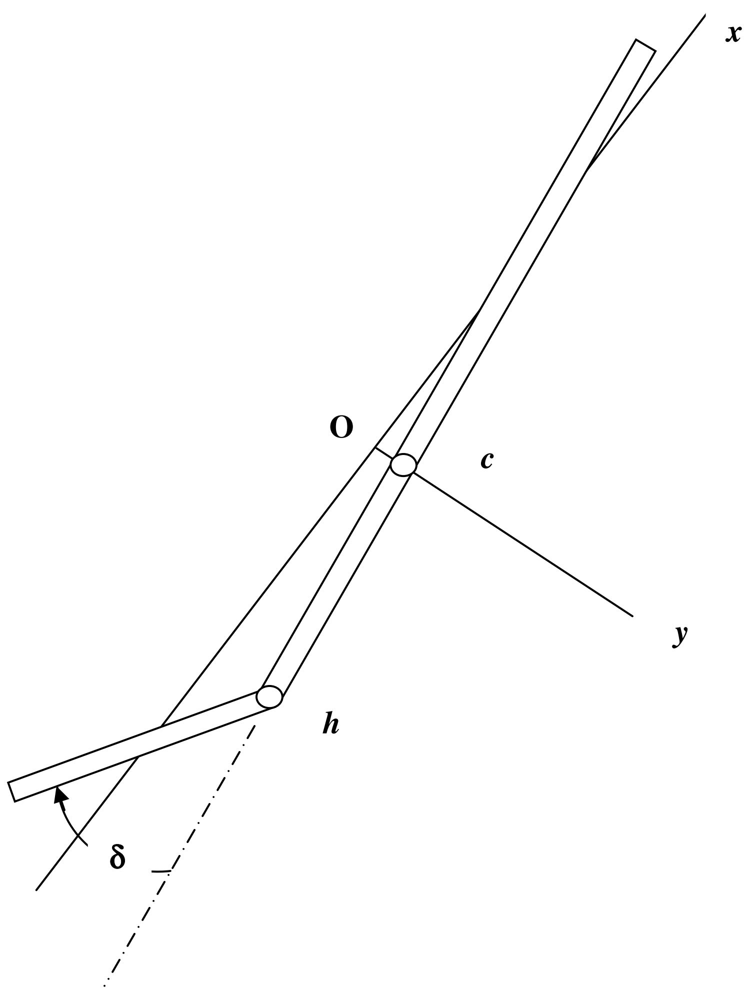

Figure 2. Rigid-Link Model for Momentum Analysis

Linear Momentum

Note that the bending modes are orthogonal and have zero linear momentum and zero angular momentum.

The vehicle undergoes a longitudinal displacement as the nozzle rotates.

There is no external force. Thus the condition of zero linear momentum within the moving coordinate frame requires that

$$\int_{-l_1}^{l_2} \dot{y}\,dm - \dot{\delta}\int_0^b z\,dm = 0 \tag{1}$$

The lateral velocity of the vehicle at the center of mass is thus

$$m\,\dot{y}_c = m_e\,z_e\,\dot{\delta} \tag{2}$$

$$\dot{y}_c = \frac{m_e\,z_e}{m}\,\dot{\delta} \tag{3}$$

The corresponding displacement equation is

$$y_c = \frac{m_e\,z_e}{m}\,\delta \tag{4}$$

Let

$$C_1 = \frac{m_e\,z_e}{m} \tag{5}$$

$$y_c = \frac{m_e\,z_e}{m}\,\delta = C_1\,\delta \tag{6}$$

Note that the displacement of the vehicle at c in the x-axis is omitted because it is negligibly small.

Angular Momentum

The condition of zero angular momentum within the moving coordinate frame requires that

$$\int_{-l_1}^{l_2} \dot{y}(x)\,x\,dm - \dot{\delta}\int_0^b z\,x\,dm = 0 \tag{7}$$

Note that

$$\dot{y}(x) = \dot{y}_c - x\,\dot{\theta}_f \tag{8}$$

$$x = -(l_h + z) \tag{9}$$

$$z = -x - l_h \tag{10}$$

$$\int_{-l_1}^{l_2} x\,dm = 0 \tag{11}$$

$$I_o = \int_{-l_1}^{l_2} x^2\,dm \tag{12}$$

$$I_h = \int_0^b x^2\,dm \tag{13}$$

By substitution,

$$\int_{-l_1}^{l_2} \left[\dot{y}_c - x\,\dot{\theta}_f\right] x\,dm + \dot{\delta}\int_0^b \left[x + l_h\right] x\,dm = 0 \tag{14}$$

$$\int_{-l_1}^{l_2} \dot{y}_c\, x\,dm - \int_{-l_1}^{l_2} x^2\,\dot{\theta}_f\,dm + \dot{\delta}\int_0^b x^2\,dm + \dot{\delta}\; l_h \int_0^b x\,dm = 0 \tag{15}$$

$$-I_o\,\dot{\theta}_f + I_h\,\dot{\delta} + l_h\, m_e\, z_e\;\dot{\delta} = 0 \tag{16}$$

$$I_o\,\dot{\theta}_f = \left[I_h + l_h\, m_e\, z_e\right]\dot{\delta} \tag{17}$$

$$\dot{\theta}_f = \frac{\left[I_h + l_h\, m_e\, z_e\right]}{I_o}\,\dot{\delta} \tag{18}$$

The corresponding angular displacement equation is

$$\theta_f = \frac{\left[I_h + l_h\, m_e\, z_e\right]}{I_o}\,\delta \tag{19}$$

Let

$$C_2 = \frac{[I_h + l_h m_e z_e]}{I_o} \tag{20}$$

$$\theta_f = \frac{[I_h + l_h m_e z_e]}{I_o} \delta = C_2 \delta \tag{21}$$

Equation (6) gives the rigid vehicle displacement y_c due to hinge rotation δ.

Equation (21) gives the rigid vehicle angular displacement θ_f due to hinge rotation δ.

The y-axis displacement due to hinge rotation and elastic bending modes is

$$y(x, t) = (C_1 - C_2 x)\delta(t) + \sum_i q_i(t)\phi_i(x) \tag{22}$$

Equation (22) is the displacement relative to the moving coordinate system.

Equations of Motion

To be added in the next revision.

Reference

1. W. Thomson, Introduction to Space Dynamics, Dover, New York, 1986.