BUCKLING OF COLUMNS
Revision B

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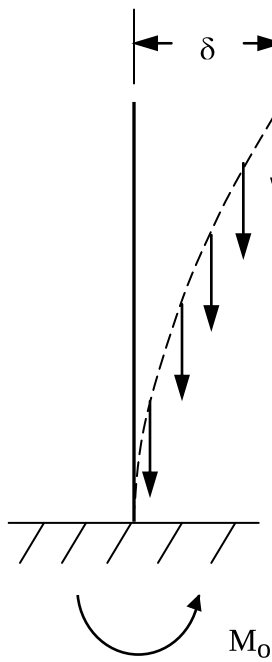
Introduction

The critical load is the maximum load that a structure can support prior to structural instability or collapse. The collapse of the structure is reached when the displacements become relatively large for a small load increment. Thus, the overall stiffness of the structure becomes very small.

Variables

L	=	length
E	=	modulus of elasticity
I	=	area moment of inertia
q	=	uniform distributed load (load per length)
P	=	concentrated load

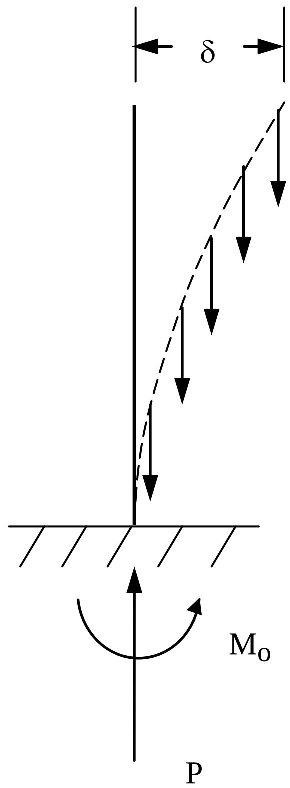
Lower End Fixed, Upper End Free Column with Distributed Body Weight Load



$$\left(q\,L\right)_{cr} = \frac{7.837\,EI}{L^2} \tag{1}$$

Equation (1) is taken from Reference 1, Chapter 2, p103.

Lower End Fixed, Upper End Free Column with Combined Distributed Body Weight Load and Applied Axial Load



$$P_{cr} = \frac{\pi^2 EI}{4L^2} - 0.3qL \tag{2}$$

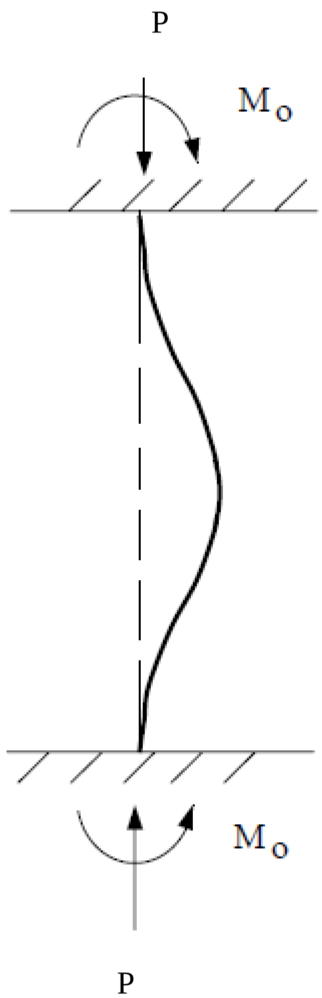
Note that if

$$(qL) \geq \frac{7.837\,EI}{L^2}$$

Then buckling will occur due to the distributed load alone. In this case a tension force would be needed to prevent buckling.

Equation (2) is taken from Reference 1, Chapter 2, p 104.

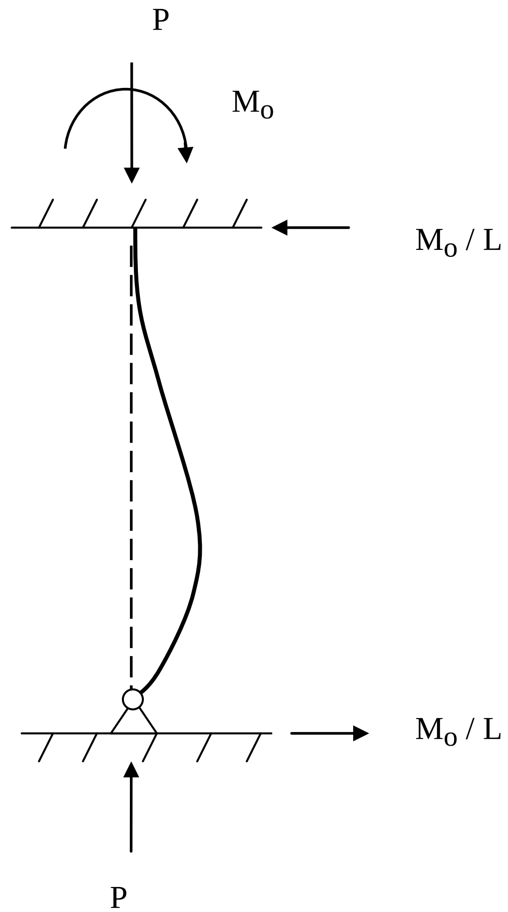
Both Ends Fixed Column with Applied Axial Load



$$P_{cr} = \frac{4\pi^2 EI}{L^2} \tag{3}$$

Equation (3) is taken from Reference 2, equation (1.14).

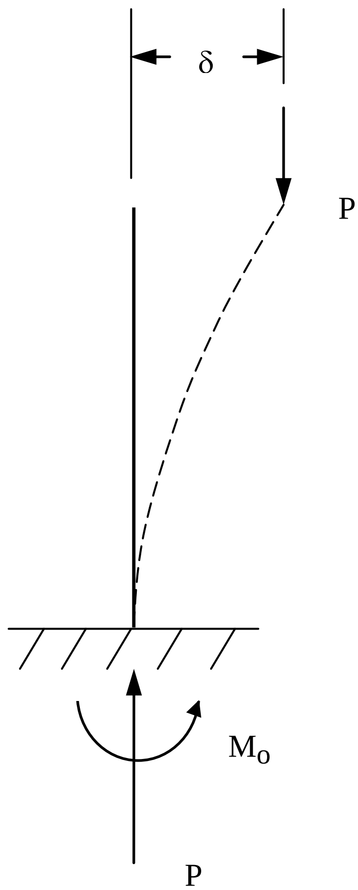
Lower End Hinged, Upper End Fixed Column with Applied Axial Load



$$P_{cr} = \frac{20.19\pi^2 EI}{L^2} \tag{4}$$

Equation (4) is taken from Reference 1, equation (2-12) and from Reference 2, equation (1.24). Note that the critical load is the same if the boundary conditions are switched.

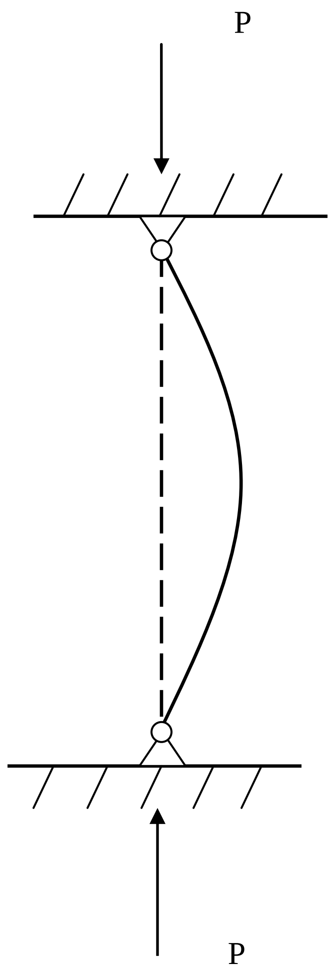
Lower End Fixed, Upper End Free Column with Applied Axial Load



$$P_{cr} = \frac{\pi^2 EI}{4L^2} \tag{5}$$

Equation (5) is taken from Reference 2, equation (1.19).

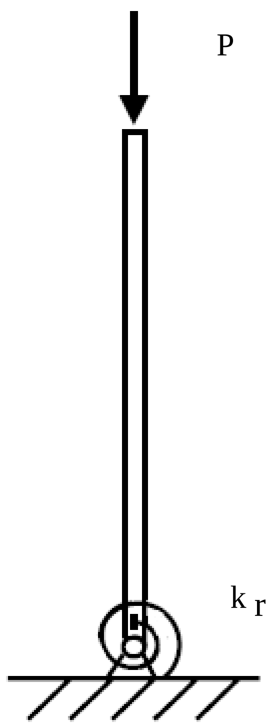
Both Ends Hinged Column with Applied Load



$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{6}$$

Equation (6) is taken from Reference 1, equation (2-5).

Rigid Column, Hinged at Base, with Rotational Spring



$$P_{cr} = \frac{k_r}{L} \tag{7}$$

Equation (7) is taken from Reference 3, page 81.

References

1. Timoshenko and Gere, Theory of Elastic Stability, International Student Edition, 2nd Edition, McGraw-Hill, New Delhi, 1963.

2. Alexander Chajes, Principles of Structural Stability Theory, Prentice-Hall, New Jersey, 1974.

3. Bathe, Finite Element Procedures in Engineering Analysis, Prentice-Hall, New Jersey, 1982. Section 12.3.1.