A CIRCULAR PLATE SUBJECTED TO UNIFORM PRESSURE

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Figure 1.

Governing Equation

The governing equation for the lateral displacement w is

\[ \nabla^2 \nabla^2 w = \frac{p}{D} \]

(1)

where

p is the pressure
\[ D = \frac{Eh^3}{12(1-v^2)} \]

\[
\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) w = \frac{p}{D} \tag{2}
\]

The displacement for a constant pressure \( p \) is

\[ w = \frac{pr^4}{64D} + A_1 + A_2 \ln r + B_1 r^2 + B_2 r^2 \ln r \tag{3} \]

where \( A_1, A_2, B_1, B_2 \) are constants that depend on the boundary conditions.

**Bending Moments per Unit Length**

The radial moment per unit length is

\[ M_r = -D \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) \tag{4} \]

The tangential moment per unit length is

\[ M_t = -D \left( \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) \tag{5} \]
Clamped Circular Plate, Symmetrical Bending

The displacement equation is

\[ w = \frac{pa^4}{64D}\left[1 - \left(\frac{r}{a}\right)^2\right]^2 \]  \hspace{1cm} (6)

The radial moment per unit length is

\[ M_r = \frac{p}{16}\left[a^2(1 + \nu) - r^2(3 + \nu)\right] \]  \hspace{1cm} (7)

The tangential moment per unit length is

\[ M_t = \frac{p}{16}\left[a^2(1 + \nu) - r^2(1 + 3\nu)\right] \]  \hspace{1cm} (8)

The maximum radial stress at a given radius \( r \) is

\[ \sigma_r = -\frac{6}{h^2} M_r \]  \hspace{1cm} (10)

\[ \sigma_r = -\frac{3p}{8h^2}\left[a^2(1 + \nu) - r^2(3 + \nu)\right] \]  \hspace{1cm} (11)

The maximum tangential stress at a given radius \( r \) is

\[ \sigma_t = -\frac{6}{h^2} M_t \]  \hspace{1cm} (12)

\[ \sigma_t = -\frac{3p}{8h^2}\left[a^2(1 + \nu) - r^2(1 + 3\nu)\right] \]  \hspace{1cm} (13)
Simply Supported Circular Plate, Symmetrical Bending

The displacement equation is

\[
w = \frac{pa}{64D} \left[ 1 - \left( \frac{r}{a} \right)^2 \right] \left[ \frac{5 + v}{1 + v} - \left( \frac{r}{a} \right)^2 \right]
\]  
(14)

The radial moment per unit length is

\[
M_r = \frac{pa^2}{16} (3 + v) \left[ 1 - \left( \frac{r}{a} \right)^2 \right]
\]  
(15)

The tangential moment per unit length is

\[
M_t = \frac{pa^2}{16} \left[ 3 + v - (1 + 3v) \left( \frac{r}{a} \right)^2 \right]
\]  
(16)

The maximum radial stress at a given radius \( r \) is

\[
\sigma_r = -\frac{6}{h^2} M_r
\]  
(17)

\[
\sigma_r = -\frac{3pa^2}{8h^2} (3 + v) \left[ 1 - \left( \frac{r}{a} \right)^2 \right]
\]  
(18)

The maximum tangential stress at a given radius \( r \) is

\[
\sigma_t = -\frac{6}{h^2} M_t
\]  
(19)

\[
\sigma_t = -\frac{3pa^2}{8h^2} \left[ 3 + v - (1 + 3v) \left( \frac{r}{a} \right)^2 \right]
\]  
(20)
References