A CIRCULAR PLATE SUBJECTED TO UNIFORM PRESSURE

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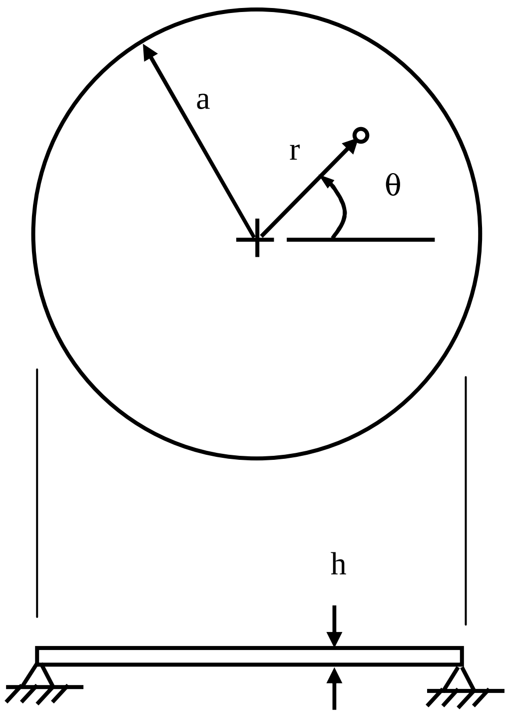

Figure 1.

Governing Equation

The governing equation for the lateral displacement w is

$$\nabla^2 \nabla^2 w = p / D \quad (1)$$

where

p is the pressure

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right) w = \frac{p}{D} \quad (2)$$

The displacement for a constant pressure p is

$$w = \frac{p\, r^4}{64 D} + A_1 + A_2 \ln r + B_1 r^2 + B_2 r^2 \ln r \quad (3)$$

where A_1, A_2, B_1, B_2 are constants that depend on the boundary conditions.

<u>Bending Moments per Unit Length</u>

The radial moment per unit length is

$$M_r = -D\left(\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right) \quad (4)$$

The tangential moment per unit length is

$$M_t = -D\left(\frac{1}{r}\frac{\partial w}{\partial r} + \nu \frac{\partial^2 w}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) \quad (5)$$

Clamped Circular Plate, Symmetrical Bending

The displacement equation is

$$w = \frac{pa^4}{64D}\left[1-\left(\frac{r}{a}\right)^2\right]^2 \tag{6}$$

The radial moment per unit length is

$$M_r = \frac{p}{16}\left[a^2(1+\nu)-r^2(3+\nu)\right] \tag{7}$$

The tangential moment per unit length is

$$M_t = \frac{p}{16}\left[a^2(1+\nu)-r^2(1+3\nu)\right] \tag{8}$$

The maximum radial stress at a given radius r is

$$\sigma_r = -\frac{6}{h^2}M_r \tag{10}$$

$$\sigma_r = -\frac{3p}{8h^2}\left[a^2(1+\nu)-r^2(3+\nu)\right] \tag{11}$$

The maximum tangential stress at a given radius r is

$$\sigma_t = -\frac{6}{h^2}M_t \tag{12}$$

$$\sigma_t = -\frac{3p}{8h^2}\left[a^2(1+\nu)-r^2(1+3\nu)\right] \tag{13}$$

Simply Supported Circular Plate, Symmetrical Bending

The displacement equation is

$$w = \frac{pa^4}{64D}\left[1-\left(\frac{r}{a}\right)^2\right]\left[\frac{5+\nu}{1+\nu}-\left(\frac{r}{a}\right)^2\right] \tag{14}$$

The radial moment per unit length is

$$M_r = \frac{pa^2}{16}(3+\nu)\left[1-\left(\frac{r}{a}\right)^2\right] \tag{15}$$

The tangential moment per unit length is

$$M_t = \frac{pa^2}{16}\left[3+\nu-(1+3\nu)\left(\frac{r}{a}\right)^2\right] \tag{16}$$

The maximum radial stress at a given radius r is

$$\sigma_r = -\frac{6}{h^2}M_r \tag{17}$$

$$\sigma_r = -\frac{3pa^2}{8h^2}(3+\nu)\left[1-\left(\frac{r}{a}\right)^2\right] \tag{18}$$

The maximum tangential stress at a given radius r is

$$\sigma_t = -\frac{6}{h^2}M_t \tag{19}$$

$$\sigma_t = -\frac{3pa^2}{8h^2}\left[3+\nu-(1+3\nu)\left(\frac{r}{a}\right)^2\right] \tag{20}$$

References

1. Boresi et al, Advanced Mechanics of Materials, Third Edition, Wiley, New York, 1978.
2. Timoshenko and Woinowsky-Krieger, Theory of Plates and Shells, McGraw-Hill, International Student Edition, 1970.