EQUATION OF MOTION OF A
SPRING-CYLINDER SYSTEM ON A MOVING FLOOR

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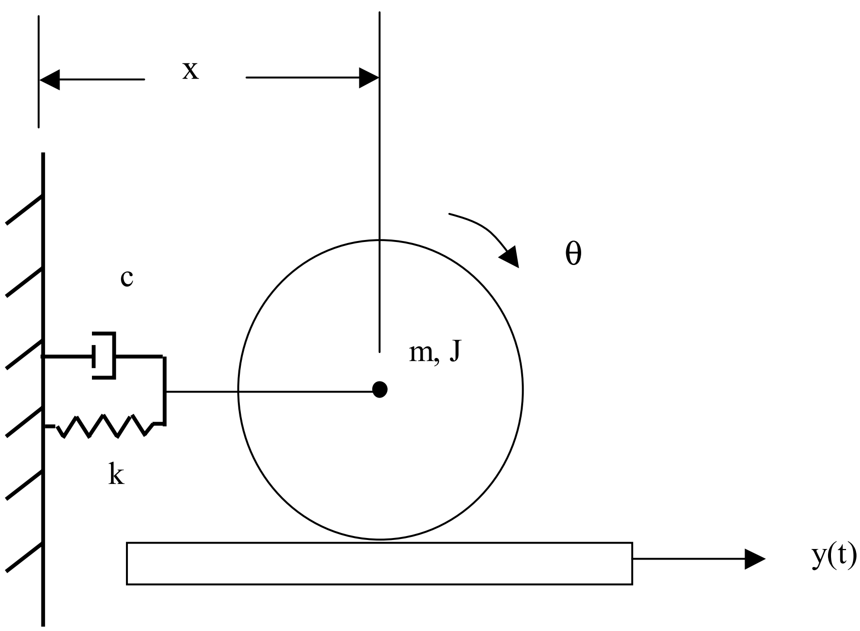
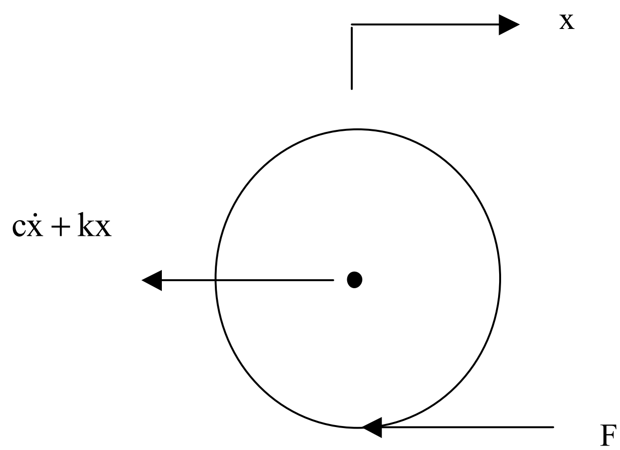
Assume no slipping.





$$\theta = \frac{1}{R}[x - y] \quad (1)$$

$$\dot{\theta} = \frac{1}{R}[\dot{x} - \dot{y}] \quad (2)$$

$$\ddot{\theta} = \frac{1}{R}[\ddot{x} - \ddot{y}] \quad (3)$$

The frictional force F results from the sum of the moments.

$$\sum M = J\ddot{\theta} \quad (4)$$

$$J\ddot{\theta} = FR \quad (5)$$

$$F = (J / R)\ddot{\theta} \quad (6)$$

Sum the forces.

$$\sum \hat{F} = m\ddot{x} \tag{7}$$

$$m\ddot{x} = -c\dot{x} - kx - (J/R)\ddot{\theta} \tag{8}$$

$$m\ddot{x} = -c\dot{x} - kx - \frac{J}{R^2}\left[\ddot{x} - \ddot{y}\right] \tag{9}$$

$$\left[m + \frac{J}{R^2}\right]\ddot{x} + c\,\dot{x} + kx = \frac{J}{R^2}\,\ddot{y} \tag{10}$$

The result agrees that in Reference 1, Chapter 1, problem 18; except that the problem in Reference 1 has a stationary floor.

Reference

1. W. Seto, Mechanical Vibrations, McGraw-Hill, New York, 1964.

APPENDIX A

Energy Method

Repeat the problem from the main text via the energy method.

$$KE = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} J\dot{\theta}^2 \quad \text{(A-1)}$$

$$PE = \frac{1}{2} k x^2 \quad \text{(A-2)}$$

$$\text{Work} = -c\,\dot{x} \quad \text{(A-3)}$$

$$\frac{d}{dt}\left\{\frac{1}{2} m\dot{x}^2 + \frac{1}{2} J\dot{\theta}^2 + c\,\dot{x} + \frac{1}{2} k x^2\right\} = 0 \quad \text{(A-4)}$$

$$m\dot{x}\ddot{x} + J\dot{\theta}\ddot{\theta} + c\,\dot{x}^2 + k x \dot{x} = 0 \quad \text{(A-5)}$$

$$\theta = \frac{1}{R}[x - y] \quad \text{(A-6)}$$

$$\dot{\theta} = \frac{1}{R}[\dot{x} - \dot{y}] \quad \text{(A-7)}$$

$$\ddot{\theta} = \frac{1}{R}[\ddot{x} - \ddot{y}] \quad \text{(A-8)}$$

$$m\dot{x}\ddot{x} + J\frac{1}{R^2}[\dot{x} - \dot{y}][\ddot{x} - \ddot{y}] + c\,\dot{x}^2 + k x \dot{x} = 0 \quad \text{(A-9)}$$

Equation (A-9) yields two equations.

$$\dot{x}\left[J\frac{1}{R^2}[\ddot{x}-\ddot{y}]+m\ddot{x}+c\dot{x}+kx\right]=0 \qquad \text{(A-10)}$$

$$-\dot{y}\left[J\frac{1}{R^2}[\ddot{x}-\ddot{y}]\right]=0 \qquad \text{(A-11)}$$

Rewrite equation (A-10).

$$\left[m+\frac{J}{R^2}\right]\ddot{x}+c\dot{x}+kx=\frac{J}{R^2}\ddot{y} \qquad \text{(A-12)}$$