

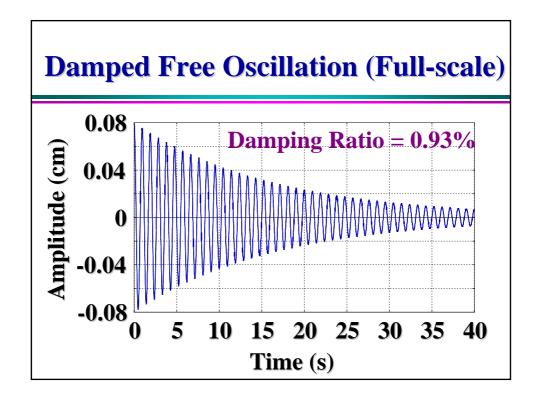
Lecture 10

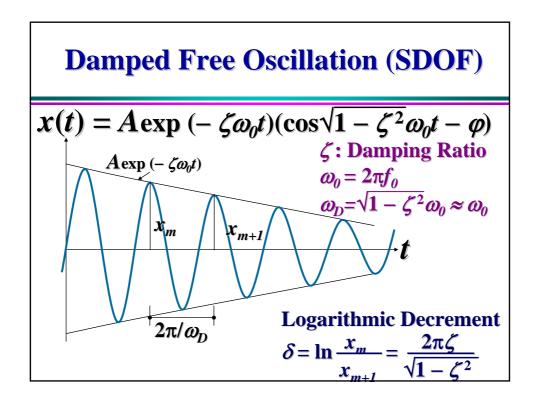
Damping in Buildings

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The 21st Century Center of Excellence Program
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Damping

- Reduction of intensity with time or spatial propagation
 - Vibration Energy Thermal Energy
 - Radiation to Outside
- **■** Cease of vibration with time
- Reduction of wind-induced/earthquake-induced vibration
- Increase of onset wind speed of aerodynamic instability
- etc.





Damping in Buildings

- Estimation of damping
 - no theoretical method
 - based on full-scale data significant scatter

Dispersion of Damping Data

- Structural Materials
- Soil & Foundations
- Architectural Finishing
- Joints
- Non-structural Members
- **Vibration Amplitude**
- Non-stationarity of Excitations
- **Vibration Measuring Methods**
- Damping Evaluation Techniques etc.

Uncertainty of Response Prediction Due to Uncertainty of Damping Ratio

Coefficient of variation of full-scale damping data

ex. Havilland (1976) C.O.V. 70%

If damping ratio was estimated at $\zeta = 2\%$ on average,

 ζ can generally take 0.6% ~ 3.4% (2% ± 1.4%)

Wind-induced acceleration response

$$A(\zeta = 0.6\%) / A(\zeta = 2\%) = 1.8$$

 $A(\zeta = 3.4\%) / A(\zeta = 2\%) = 0.8$ 2.3 times

provides significant reduction of reliability of structural design

Importance of Damping

■ Improvement of Reliability of Structural Design

Accurate Response Prediction Accurate Damping Predictor Reliable Damping Database

Physical Causes of Damping in Buildings

	Energy Dissipation Inside			Energy I	Dissipation Outside		
	Solid	Liquid	Gas	S-S	S–L	S–G	
Friction	Internal Friction Damping	-		External Friction Damping	_		
Viscosity	_	Internal Viscous Damping		_	External Viscous Damping		
Radiation	_		Radiation	n Damping –			
Interaction	_			_	Hydro- dynamic Damping	Aero- dynamic Damping	
Plasticity	Hysteretic – Damping				_		

Internal Friction Damping

Energy dissipation due to internal friction of solid materials

■ Deformation of Materials

Relative displacement between molecules Slip of micro-cracks in microscopic structures such as crystals

Macroscopic: Elastic

Microscopic:

Friction damping between microscopic structures

Elastic hysteretic loss

Very small in metals (Energy loss $\approx 0.5\%$)

<< Different from energy loss due to plastic hysteresis>>

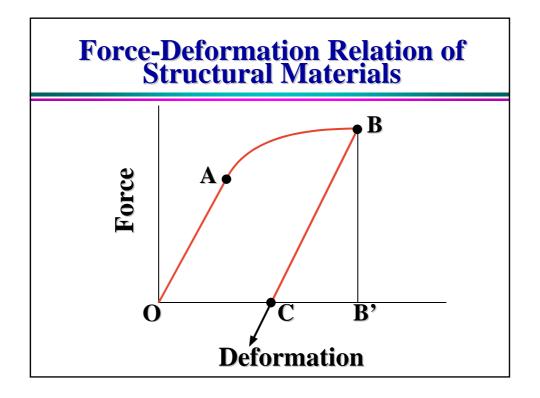
Plasticity Damping

Energy dissipation due to plasticity of solids

■ Hysteresis due to Plasticity

Change in microscopic structure of materials Hysteretic characteristics / Plasticity Rate

Significantly greater than the energy dissipation due to internal material friction



Internal Viscous Damping

Energy dissipation due to internal viscosity of liquids

- Molecular Viscosity
 Collisions of molecules
 Coefficient of Kinetic Viscosity v
 Conversion of kinetic energy to thermal energy
- Turbulence Viscosity

 Reynolds Stress (Virtual stress due to correlation of fluctuating velocity components of fluids)

 Coefficient of Kinetic Vortex Viscosity 14

Mixture and diffusion of kinetic energy and so on

External Friction Damping

Energy dissipation due to friction between solids

Mainly Sliding Friction Coefficient of Friction

Work done by friction force preventing relative motion between solid bodies

Conversion of vibration energy to thermal energy

- Sticking of molecules due to contact
- Damage and replacement of sticking due to relative motion
- Digging up by projections
- ex. Friction between joints, Friction between members, finishing etc.

Radiation Damping

Energy transfer between Solid - Solid, or Solid - Liquid

- Propagation and loss of a system's energy to outside
 - Necessary work for exciting a body contacting the system
 - Penetration of wave energy through boundary ex.
 - Radiation damping due to soil-structure interaction
 - Damping due to wave generation for a floating body

Reflection of ground motions from building surface: Input loss

External Viscous Damping

Energy dissipation due to viscosity of liquids or gas contacting the body

- Viscous resistance acting on a moving body in oil or water
 - Large velocity gradient near body surface
 - A function of relative velocity

ex.

Oil Damper, Viscous Wall Damper

Fluid-Dynamic Damping (Aerodynamic Damping)

Fluid-body interaction

- **■** Effects of relative velocity
- Effects of additional unsteady flow induced by body motion (Feedback system)

Ex.

Along-wind Vibrations (Buffeting) due to turbulence: Positive Damping

Across-wind Vibrations (Galloping, Vortex-resonance etc.):

Negative Damping

Damping and Building Vibration

Careful and precise observation of Vibration Phenomena

Analytical Model with high accuracy

Damping Evaluation appropriate for the model

Equivalent Model, Mathematical Formula

Treatment of Damping: Restriction in numerical analysis

Soil-Structure Dynamic Interaction Ground Domain / Boundary Treatment, Internal Damping of Ground

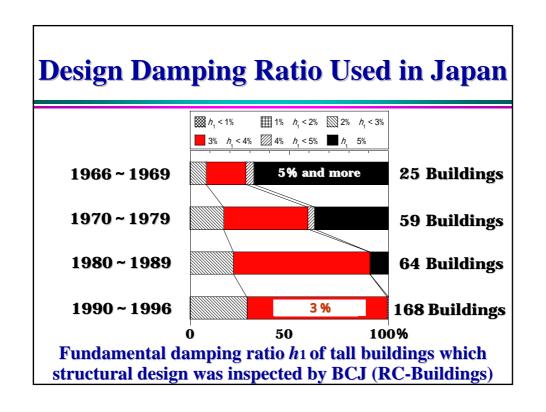
Evaluation of higher mode damping Damping Matrix, Value of Damping Ratio, Non-linear Range

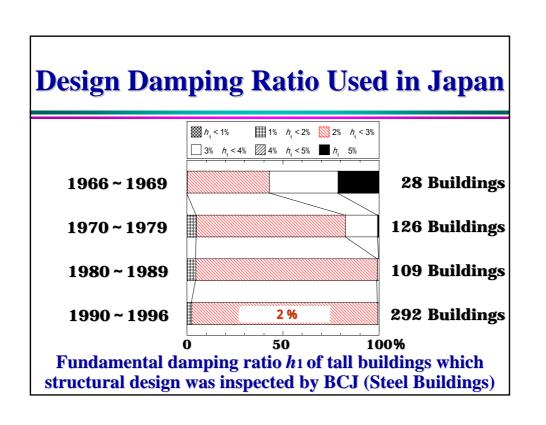
Damping Ratio of Buildings

- Damping Matrix Proportional to Stiffness Matrix
- Realistic Proportional Matrix Meeting Conditions
- Actual Damping Ratio
- Design Damping Ratio Closely Following Actual Phenomena
- Variation of Natural Frequency and Damping Ratio With Amplitude / Effects of Secondary Members
- Initial Stiffness / Instantaneous Stiffness
- *Q-∆* and Damping Characteristics in Inelastic Range During Extremely Strong Earthquake
- Damping in Above Ground Structure / Soil-Structure Interaction / Full-scale Values of Damping Ratio
- Damping for Vertical Vibrations ?

Currently Used Design Damping Values

- AS 1170.2 Part 2
- Chinese Standards
- DIN1055, Teil 4
- ESDU 83009
- EUROCODE 1
- ISO4354
- ISO/CD 3010
- ONORM B4014
- Swedish Code
- US Atomic Energy Commission etc.





Currently Used Damping Values (Steel Buildings)

Country	Actions/Stress Le	vels	Joints/Structures	Damping ratios ζ_1 (%)
Australia	Serviceability			0.5 – 1.0
(AS1170.2)	Ultimate & Pern	nissible	Frame Bolted Frame Welded	5 2
Austria	(ÖNORM B4014))		
China	(GB50191-93)		Steel (TV) Tower	2
France			Standard Bolt High Resistance Bol Welded	0.8 t 0.5 0.3
	Earthquake		Bolt Welded	4 2
Germany	Wind	(DIN 1055)		~
Italy	Wind Earthquake	(EUROCO	DE 1)	5
Japan	Habitability Earthquake			1 2
Singapore				1
Sweden	(Swedish Code of	Practice)		0.9
United King	dom Wind	(ESDU)		
USA (Penzio	en, US Atomic Energ	y Commissi	ion)	

Currently Used Damping Values (RC Buildings)

C .	A /C. T		G: .	D 1 11
Country	Actions/Stress L	evels	Structures	Damping ratios
				ζ_1 (%)
Australia	Serviceability		RC or Prestressed C	0.5 - 1.0
(AS1170.2)	Ultimate & Per	missible	RC or Prestressed C	5
Austria	(ÖNORM B4014	1)		
China	(GB50191-93)		RC Structures	5
	,		RC (TV) Towers Prestressed RC Towe	5 5 er 3
_				
France			Standard Reinforced	1.6 0.65
	Earthquake		Standard	0.03 3 - 4
	Lartiquake		Reinforced	2
Germany	Wind	(DIN 105		~
Italy	Wind	(EUROC	ODE 1)	
J	Earthquake	`	,	5
Japan	Habitability			1
•	Earthquakě			3
Singapore				2
Sweden	(Swedish Code o	f Practice)		1.4
United King	dom Wind	(ESDU)		
USA	(US Atomic En	ergy Commis	ssion)	

DIN 1055

Teil 4, The German Pre-Standard

Wind (Actual Wind Load Code)

Structures	Conditions Da	mping ratios ζ_l (%)
- Steel	Bolted Welded	0.5 - 0.8 0.3
- Reinforced C	Without cracks With cracks	0.6 1.6
- Prestressed C		0.6

ESDU

Damping of Structures – Part 1 Tall Buildings, 83009, 1983

Wind

■ 1st mode damping ratio ζ₁ (%)

$$\zeta_1 = \zeta_s + \zeta_a$$

$$\zeta_s : \text{Structural damping ratio}$$

$$\zeta_s = 100(\zeta_{s0} + \zeta^2 \frac{x_H}{H}) \le \frac{60}{H} + 1.3 \quad (\%)$$

 $\zeta_{s\theta} = f_I / 100$ (Most Probable), $f_I / 250$ (Lower Limit) $\zeta' = 10^{\sqrt{D}/2}$ (Most Probable), $10^{\sqrt{D}/2.5}$ (Lower Limit)

 ζ_a : Aerodynamic damping ratio

 x_H : Tip displacement (m), H: Building height (m)

 f_1 : 1st mode natural frequency (Hz)

EUROCODE

Wind Actions, ENV-1991, 1994

Wind

• 1st mode damping ratio ζ_I (%)

$$\zeta_1 = \zeta_s + \zeta_a + \zeta_d$$

 ζ_s : Structural damping ratio

$$\zeta_s = a f_1 + b \ge \zeta_{min}$$

 $f_1 = 46/H$ (1st mode natural frequency)

a = 0.72 (Steel), 0.72(RC)

b = 0 (Steel), 0.8 (RC)

 $\zeta_{min} = 0.8$ (Steel), 1.6 (RC)

 ζ_a : Aerodynamic damping ratio

 ζ_d : Damping ratio due to vibration control devices

ÖNORM B4014

Teil 1, Code for Austria

Wind (Actual Wind Load Code for Austria)

• 1st mode damping ratio ζ_1 (%)

$$\zeta_1 = \zeta_m + \zeta_c + \zeta_f$$

 ζ_m : Structural damping ratio due to materials (%)

0.72 (RC with cracks), 0.4 (RC without cracks, PSRC)

 ζ_c : Structural damping ratio due to constructions (%)

0.32 (Steel tall buildings),

0.32 (RC tall buildings, Panel systems) 0.64 (RC tall buildings, Frame systems)

 ζ_i : Structural damping ratio due to foundations (%)

0.08 (Support with hinges)
0.24 (Support with sliding bearings)
0.16 (Fixed support of frame structures)

US	Atomic	Energy egulatory G	Comm	ission
	"Re	gulatory G	uide"	

Structures	Damping Ratio (%)		
Structures	OBE or ½ SSE	SSE	
Welded Steel Bolted Steel Prestressed C Reinforced C	2 4 2 4	4 7 5 7	

OBE: Operating Basis Earthquake SSE: Safe Shutdown Earthquake

ISO

■ ISO4354 (Wind Actions on Structures, 1997)

1st mode damping ratio

 $\zeta_I = 1.0 \%$ (Steel Buildings)

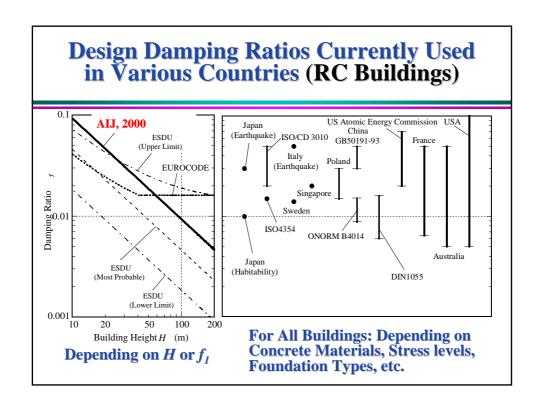
 $\zeta_I = 1.5 \%$ (RC Buildings)

■ ISO/CD3010 (Seismic Actions on Structures, 1999)

1st mode damping ratio

$$\zeta_1 = 2 - 5 \%$$

Design Damping Ratios Currently Used in Various Countries (Steel Buildings) US Atomic Energy Commission Japan (Earthquake) AIJ, 2000 ESDU Australia (Upper Limit) Italy (Earthquake) ISO/CD 3010 Damping Ratio Poland EUROCODE Singapore ISO4354 ONORM B4014 France China ESDU Japan (Habitability) ESDU DIN1055 (Lower Limit) 0.001 For All Buildings: Depending on Building Height H (m) **Connection Types, Stress Levels,** Depending on H or f_1 Foundation Types, etc.



Damping Data & Predictors

- **■** Penzen, J. (1972), U.C. Berkley
- Haviland, R. (1976), MIT
- Cook, N.J. (1985) 'The designer's guide to wind loading of building structures'
- Davenport, A.G. & Hill-Carrol, p. (1986), ASCE
- **Jeary, A.P.** (1986), *JEESD*
- Lagomarsino, S. (1993), JWEIA
- Ellis, B.R. (1998)
- etc.

Desirable Damping Database

- Enough Data
- **■** Enough Building Types
- High-Quality & Accurate
- Information in Detail
 - Building & Soil
 - Measuring Conditions
 - Evaluation Techniques
 - Amplitudes
 - Stationarity

Japanese Damping Database

Research Committee on Damping Data organized by

Architectural Institute of Japan (1993-2000)

Sources of Damping Data

- Original data from Members of the Research Committee
- Research Committee Report on Evaluation of Damping of Buildings, Building Center of Japan, 1993
- Summary Papers presented at the Annual Meeting of Architectural Institute of Japan (AIJ) 1970 -
- Journal of Structural and Construction Engineering (Transactions of AIJ), 1970 -
- Proc. Annual Meeting of Kanto Branch of Architectural Institute of Japan, 1970 -
- Proceedings of Annual Meeting of Kinki Branch of Architectural Institute of Japan, 1970-
- Proc. National Symposium on Wind Engineering, 1970 -
- Proc. National Symposium on Earthquake Engineering, 1970 -
- Proc. International Conference on Earthquake Engineering, 1974 -
- Vibration Tests of Buildings, Architectural Institute of Japan, 1978
- Technical Reports published by Research Institute of Construction Companies, 1974 -

Accuracy and Quality of Damping Data

Questionnaire Studies to Designers and Owners

- **■** Confirmation of Values
 - Dynamic Properties in Literature
- **■** Collection of Necessary Data
 - Building Information
 - Measurement Methods
 - Evaluation Techniques
 - Amplitudes
- **Exclusion of Unreliable Data**
- **■** Approval for World-Wide Distribution
- Many original non-published data and additional information were collected.

Japanese Damping Database

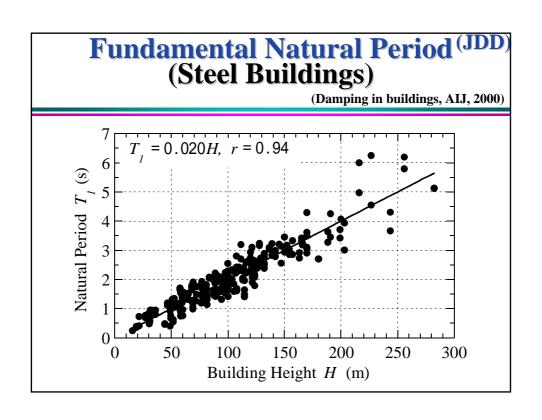
(Damping in buildings, AIJ, 2000)

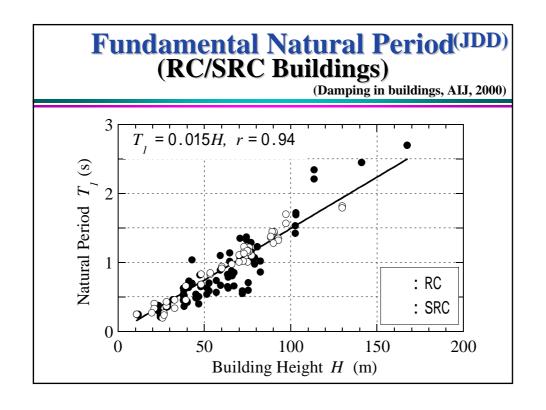
Number of Buildings and Structures									
	285								
Steel Buildings (Steel)	Steel Encased Reinforced Concrete Buildings (SRC)	Reinforced Concrete Buildings (RC)	Tower-Like Non-Building Structures						
137	43	25	80						
$H_{Ave.}$ = 101m	$H_{Ave.} = 60$ m		$H_{Ave.}$ = 124m						
15.5m ~ 282.3m	11.6m ~ 167.4m	10.8m ~ 129.8m	9.1m ~ 226.0m						
Office: 99	Apartment: 35		Chimney: 26						
Hotel: 25	Office	: 20	Lattice: 24						
Others: 13	School	Tower: 23							
	Others	: 9	Others: 6						

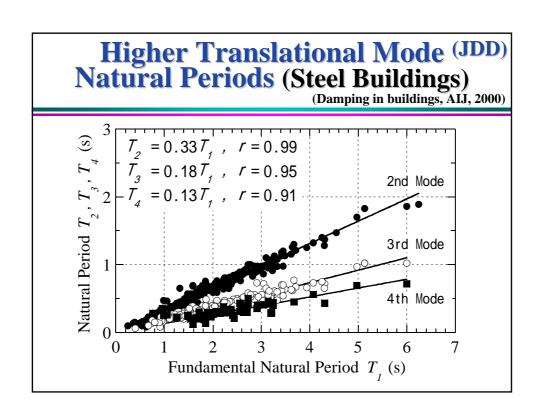
Japanese Damping Database (JDD)

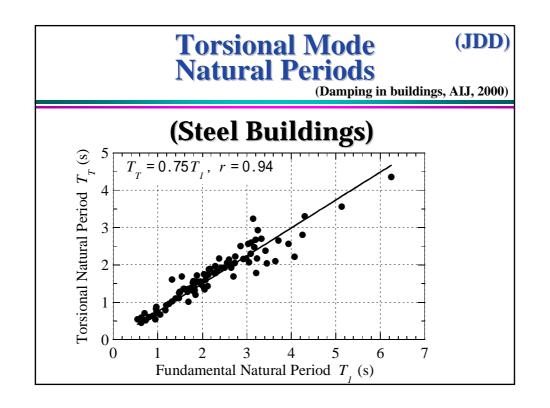
(Damping in buildings, AIJ, 2000)

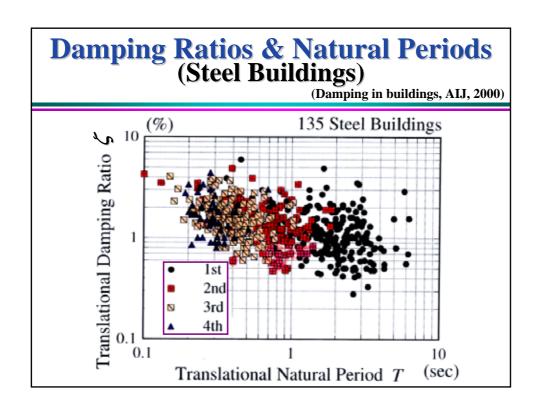
	Contained Information								
	Location	Structural Type							
	Time of Completion Building Usage	Cladding Type Foundation Type							
Building Information	Shape	Embedment Depth							
inioi mation	Height	Length of Foundation Piles							
	Dimensions	Soil Conditions							
	Number of Stories	Reference							
Dynamic Properties	Damping Ratio (up to the 6th mode) Natural Frequency (up to the 6th mode)	Excitation Type Experimental & Measurement Method Evaluation Technique Amplitude							
	Time of Measurement	etc.							





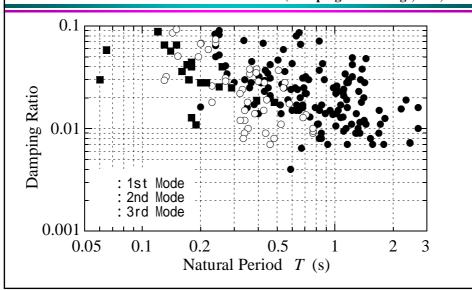






Damping Ratios & Natural Periods (RC/SRC Buildings)

(Damping in buildings, AIJ, 2000)



Damping Predictors

• Jeary (1986) :

$$\zeta_I = 0.01 f_I + 10^{\sqrt{D/2}} (x_H/H)$$

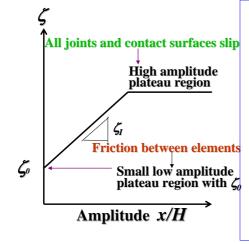
Lagomarsino (1993) :

$$\zeta_I = \alpha / f_I + \beta f_I + \gamma (x_H / H)$$

D: Building Dimension along Vibration Direction

 x_H/H : Tip Drift Ratio

Jeary's Damping Predictor



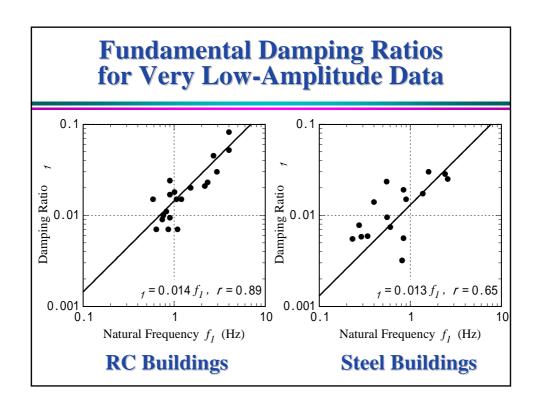
$$\zeta_1 = \zeta_0 + \zeta_I \cdot x/H$$

$$\zeta_0 = fI/100$$

 $\zeta_I = 10^{\sqrt{D}/2}$

f1: Lowest Natural Frequency (Hz)

D: Width of Building Base in Vibration Direction (m)



Very Low-Amplitude Data

Frequency Dependent Term

RC buildings:

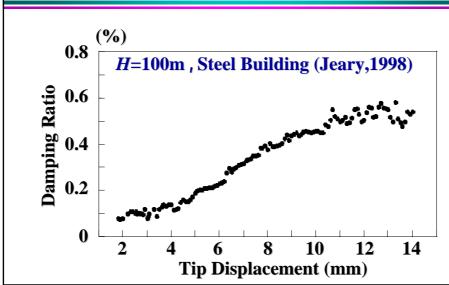
$$\zeta_I = 0.0143 f_I$$
 $(r = 0.89)$

SRC buildings:
$$\zeta_1 = 0.0231 f_1$$
 ($r = 0.32$)

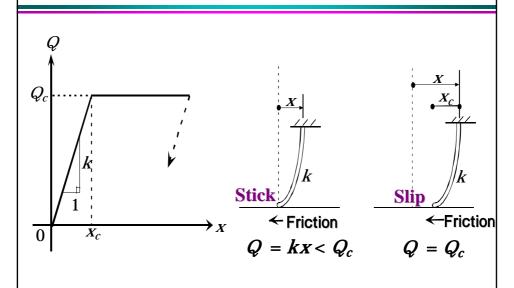
Steel buildings:
$$\zeta_I = 0.013 f_I$$
 ($r = 0.65$)

 f_I : Fundamental Natural Frequency (Hz)

Variation of Damping Ratio with Amplitude



Stick-slip Model for Damping in Buildings



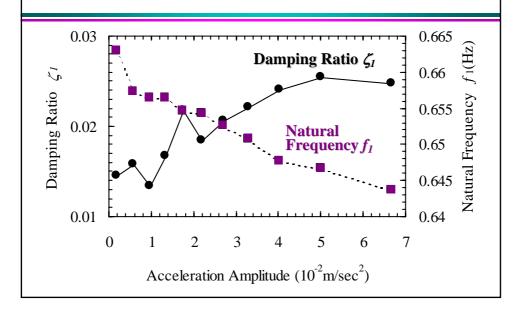
Stick-slip Model for Damping in Buildings

Increase of amplitude

- → Increase of number of slipping joints
- → Increase of friction damping
 - & Decrease of stiffness

Sum of a lot of frictional damping effects





Amplitude Dependence of Damping Ratio

(Damping in buildings, AIJ, 2000)

■ Steel Buildings

$$\zeta_I = A + B \frac{x_H}{H}$$

Tall Office Buildings:

$$B = 400$$
, Upper Limit $x_H/H = 2 \times 10^{-5}$
 $\Delta \zeta_1 (x_H/H) = 0.8\%$

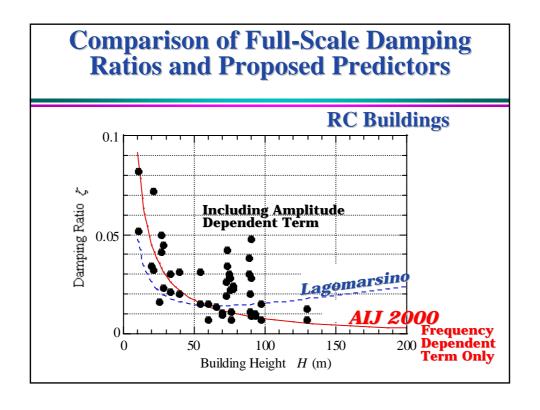
Tall Towers:

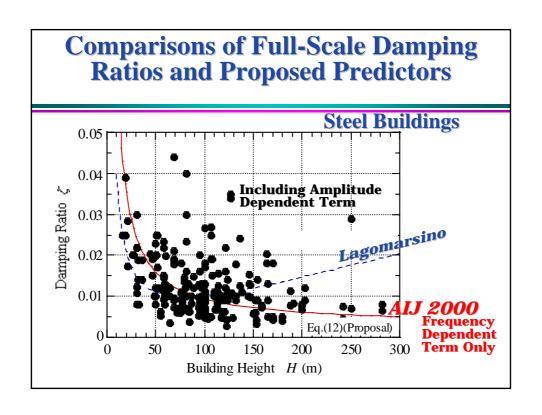
$$B = 3000$$
, Upper Limit $x_H/H = 5 \times 10^{-6}$
 $\Delta \zeta_1 (x_H/H) = 1.5\%$

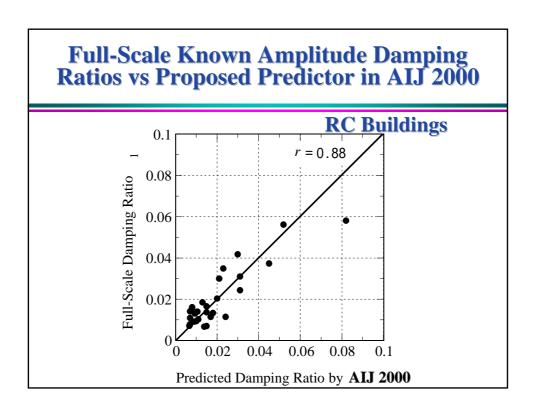
Proposed Damping Predictor in AIJ 2000

Natural Frequency Dependent Term

- RC buildings: ←Height Dependent ←Soil-Structure-Interaction $\zeta_I = 0.0143 f_I + 470(x_H/H) 0.0018$ $x_H/H < 2 \times 10^{-5}, 30m < H < 100m$ Large in Low-rise Buildings ←
- Steel buildings: $\zeta_I = 0.013 f_I + 400(x_H/H) + 0.0029$ $x_H/H < 2 \times 10^{-5}, 30 \text{m} < H < 200 \text{m}$







(JDD) **Damping Ratio** for Structural Design (AIJ, 2000)

- **■Damping Ratio for Habitability**
 - Human Comfort
 - Vibration Perception Threshold
 - H-3 Level (AIJ Guidelines, 1991)
- **■Damping Ratio for Structural Safety**
 - Elastic Region

Fundamental Natural Periods T_1 (sec)

(Damping in buildings, AIJ, 2000)

■ RC/SRC Buildings:

$$T_1 = 0.015 \ \dot{H} \qquad (f_1 = 67/H)$$

Steel Buildings :

cel Buildings :
$$T_1 = 0.020 \ H \qquad (f_1 = 50/H)$$

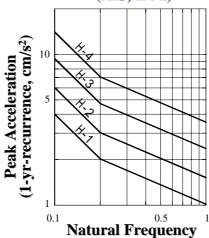
■ Ellis (1980) S/SRC/RC buildings:

$$T_1 = 0.022 \ H \qquad (f_1 = 46/H)$$

H: Building Height (m)

Performance Evaluation of Habitability to Building Vibration

Guidelines for the evaluation of habitability to building vibration (AIJ, 1991)



Performance Evaluation of Habitability to Building Vibration

■ 1-year-recurrence Peak Acceleration

$$A = 2.3 f_1^{-0.431}$$

Level H-3: Guidelines for the evaluation of habitability to building vibration (AIJ, 1991)

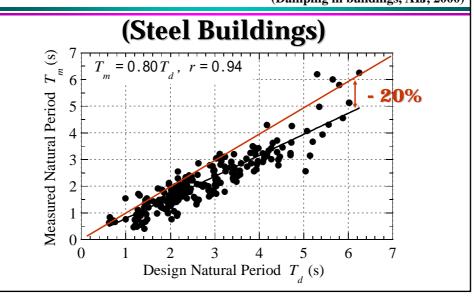
■ Foundamental natural Frequency

$$f_1 = 1 / 0.015H$$
 (RC Buildings)

 $f_1 = 1 / 0.020H$ (Steel Buildings)

Full-scale Fundamental Natural Periods & Their Design Values

(Damping in buildings, AIJ, 2000)



Full-scale Natural Period $T_m^{(\mathrm{JDD})}$ and Design Natural Period T_d

 $T_m = 0.80 T_d$

Steel Buildings: Satake et al. (1997) RC Buildings: Shioya et al. (1993)

Contributions of Secondary Members to Stiffness

Design Damping Ratio for Structural Safety

(JDD)

Tip Drift Ratio $x_H/H = 2 \times 10^{-5}$

RC Buildings

$$f_1 = 1 / 0.018H$$

Steel Buildings

$$f_1 = 1 / 0.024H$$

(JDD)

AIJ 2000 (RC Buildings)

(Damping in buildings, AIJ, 2000)

	Habitability			Safety				
Height H (m)	Natural Frequency	Damping Ratio ζ_I (%)		Frequency ζ_1		Natural Frequency	Damp	oing Ratio
	f_{l} (Hz)	Rec.	Standard	$f_I(Hz)$	Rec.	Standard		
30	2.2	2.5	3	1.9	3	3.5		
40	1.7	1.5	2	1.4	2	2.5		
50	1.3	1.2	1.5	1.1	2	2.5		
60	1.1	1.2	1.5	0.93	1.5	2		
70	0.95	0.8	1	0.79	1.5	2		
80	0.83	0.8	1	0.69	1.2	1.5		
90	0.74	0.8	1	0.62	1.2	1.5		
100	0.67	0.8	1	0.56	1.2	1.5		

• "Rec." : "Recommended" values.

• $f_I = 1 / 0.015H$ (Habitability), $f_I = 1 / 0.018H$ (Safety)

• Safety : Elastic Range

(JDD)

AIJ 2000 (Steel Buildings)

(Damping in buildings, AIJ, 2000)

	Habitability			Safety		
Height H (m)	Natural Frequency	Damping Ratio ζ_I (%)		Natural Frequency	_	oing Ratio
	f_{I} (Hz)	Rec.	Standard	$f_I(Hz)$	Rec.	Standard
30	1.7	1.8	2.5	1.4	2	3
40	1.3	1.5	2	1.0	1.8	2.5
50	1.0	1	1.5	0.83	1.5	2
60	0.83	1	1.5	0.69	1.5	2
70	0.71	0.7	1	0.60	1.5	2
80	0.63	0.7	1	0.52	1	1.5
90	0.56	0.7	1	0.46	1	1.5
100	0.50	0.7	1	0.42	1	1.5
150	0.33	0.7	1	0.28	1	1.5
200	0.25	0.7	1	0.21	1	1.5

- "Rec.": "Recommended" values. $f_I = 1 / 0.020H$ (Habitability), $f_I = 1 / 0.024H$ (Safety) Safety: Elastic Range

(JDD)

Effects of Building Use

(Damping in buildings, AIJ, 2000)

Steel Buildings

Office Buildings

$$\zeta_{AVE} = 1.15 \% (H_{AVE} = 112.6 \text{m})$$

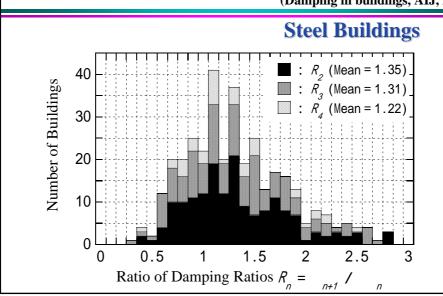
Hotels and Residential Buildings

$$\zeta_{AVE} = 1.45 \% (H_{AVE} = 100.4 \text{m})$$

25% Increase due to interior walls

Ratio of Higher Mode Damping to Next Lower Mode Damping

(Damping in buildings, AIJ, 2000)



Higher Mode Damping Ratio

(Damping in buildings, AIJ, 2000)

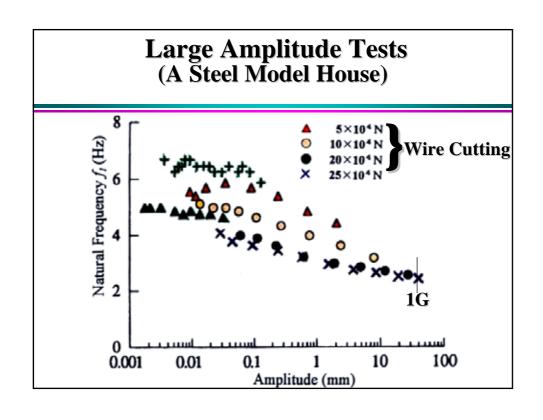
(AIJ2000)

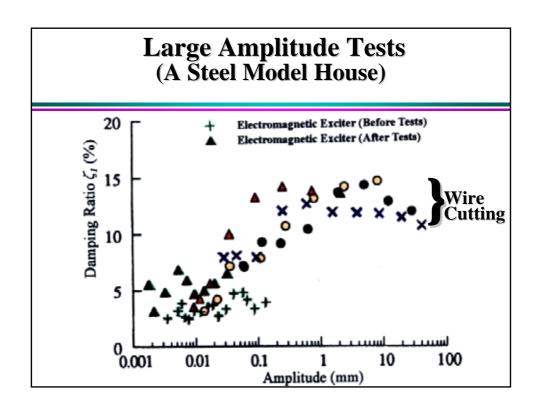
RC Buildings

Equality
$$\zeta_{n+1} = 1.4 \zeta_n$$
, $(n=1,2)$ el Buildings

Steel Buildings

$$\zeta_{n+1} = 1.3 \zeta_n$$
, $(n=1,2)$





on including Policies Co **Damping Ratio** for Ultimate Limit State

- Damage to Secondary Members
- Development of Micro Cracks
- **∠Larger Damping Values**

Almost No Quantitative Evidence

■Effects of Hysteretic Response of **Frames**

Evaluation of Damping Ratio from Randomly Excited Motion

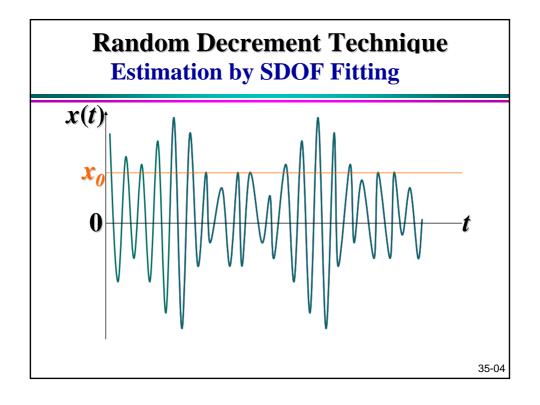
Output Information

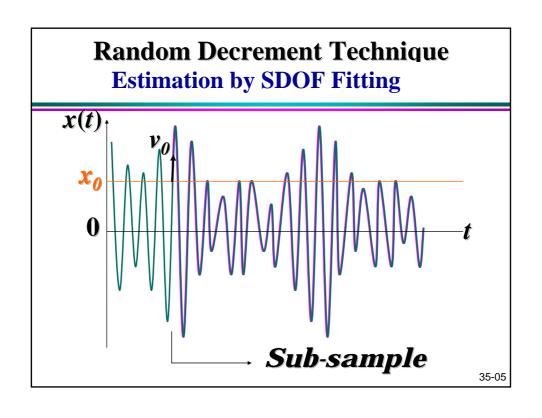
Spectral Methods

- Hal-Power Method
- Auto-Correlation Method Stationarity is strictly required. Random Decrement Technique Stationarity is not necessarily required.

Appropriate for amplitude dependent phenomena

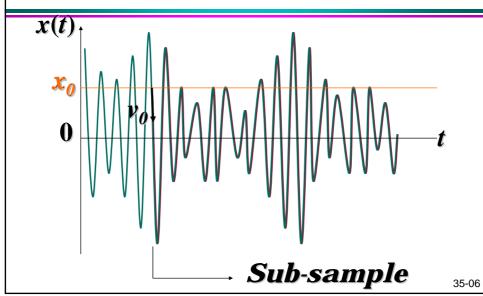
Each mode should be clearly separated. Frequency Domain Decomposition Each mode does not have to be well separated.





Random Decrement Technique

Estimation by SDOF Fitting



Random Decrement Technique

Estimation by SDOF Fitting

General Solution of SDOF

$$M\ddot{x} + C\dot{x} + Kx = f(t)$$

zero-mean random excitation

$$x(t) = D(t) + R(t)$$

D(t): Damped Free Component Depending

on Initial Condition (x_0, v_0)

R(t): Randomly Excited Component

$$= \int_0^t f(\tau)h(t-\tau)d\tau$$

Superimposition of Sub-samples (Ensemble Averaging)

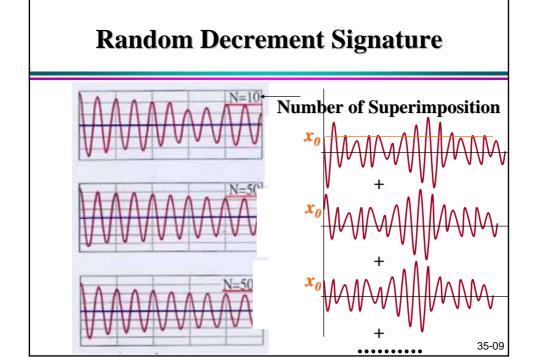
Random Decrement Technique

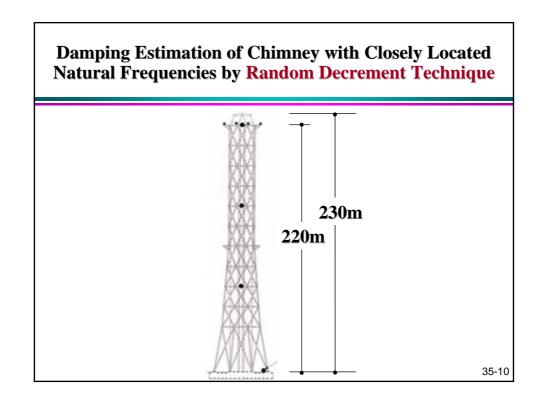
Estimation by SDOF Fitting

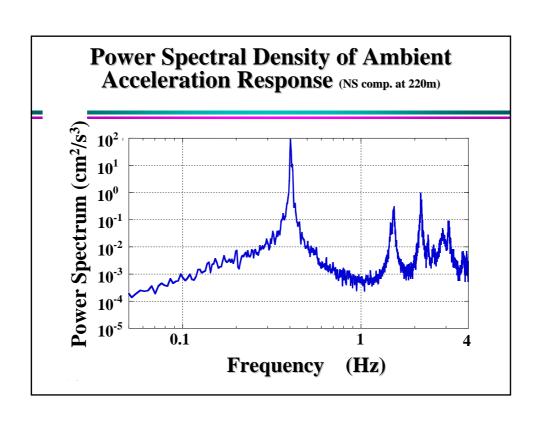
Superimposition of Sub-samples (Ensemble Averaging)

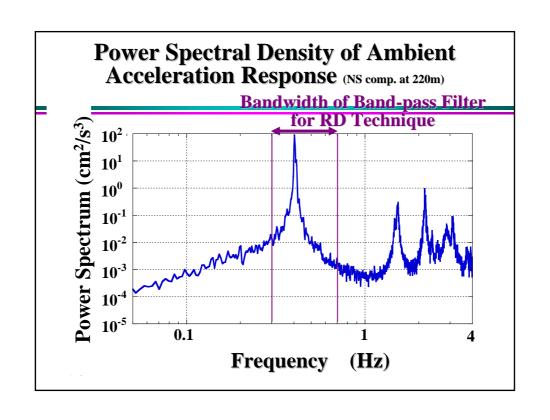
- = Random Decrement Signature
- **∞ Auto-correlation Function**
- \approx Damped Free Component with Initial Amplitude x_0

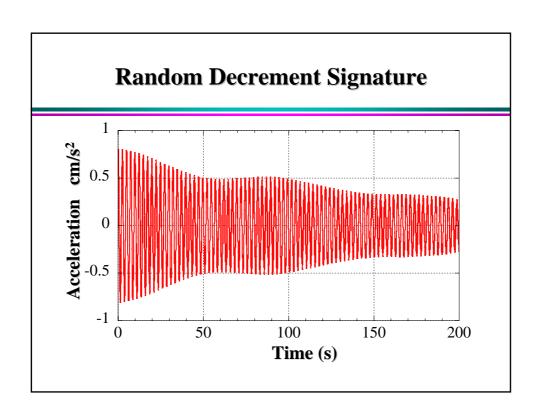
$$\propto \exp{(-\zeta\omega_0\tau)}(\cos{\sqrt{1-\zeta^2}}\omega_0\tau + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin{\sqrt{1-\zeta^2}}\omega_0\tau)$$

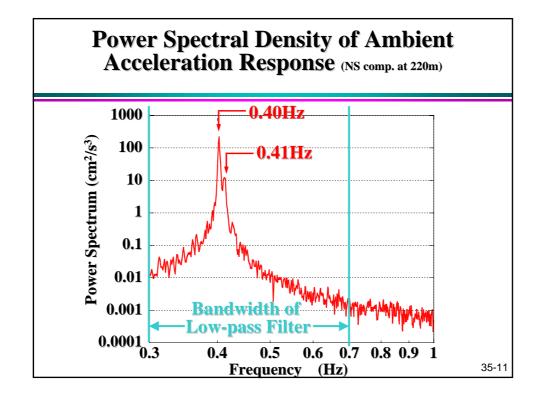








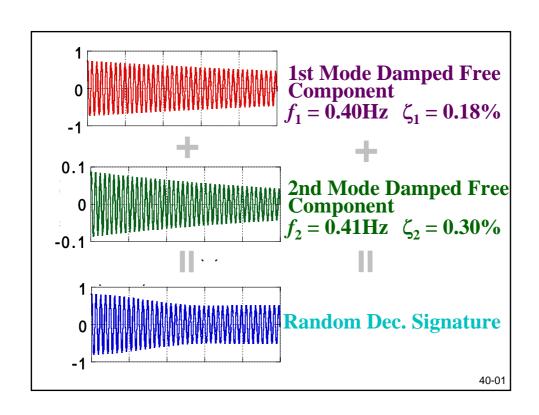




Least Square Approximation of MDOF Random Decrement Signature
$$x_{01} = \frac{x_{01}}{\sqrt{1-\zeta_1^2}} e^{-\zeta_1\omega_1 t} \cos{(\sqrt{1-\zeta_1^2}\omega_1 t - \phi_1)} + x_2 = \frac{x_{02}}{\sqrt{1-\zeta_2^2}} e^{-\zeta_2\omega_2 t} \cos{(\sqrt{1-\zeta_2^2}\omega_2 t - \phi_2)} + x_N = \frac{x_{0N}}{\sqrt{1-\zeta_N^2}} e^{-\zeta_N\omega_N t} \cos{(\sqrt{1-\zeta_N^2}\omega_N t - \phi_N)}$$

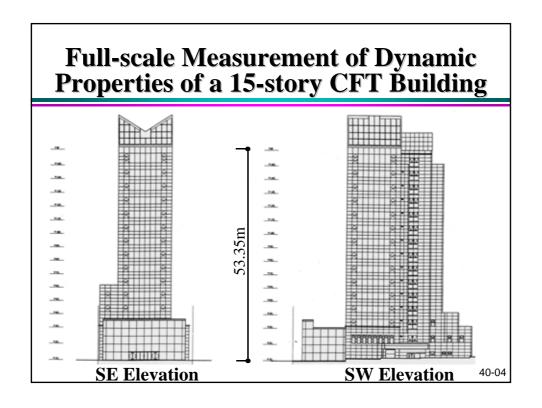
$$x_N = \frac{x_{0N}}{\sqrt{1-\zeta_N^2}} e^{-\zeta_N\omega_N t} \cos{(\sqrt{1-\zeta_N^2}\omega_N t - \phi_N)}$$

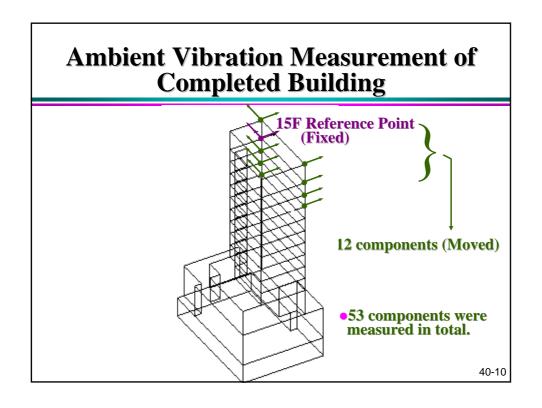
$$x = x_1 + x_2 + \cdots + x_N + m$$
35



Estimated Dynamic Characteristics of a 230m-high
Chimney by 2DOF RD technique and FDD

Mode #	Natural Frequency (Hz)		Damping Ratio (%)	
Mode #	RD	FDD	RD	FDD
1	0.40	0.40	0.18	0.24
2	0.41	0.41	0.30	0.39
3	1.47	1.47	0.83	0.30
4	1.53	1.52	0.85	0.91
5	2.17	2.17	0.55	0.65
6	2.38	2.38	0.42	0.39
7	_	2.87	_	_
8	_	3.10	_	0.77 40-02





Frequency Domain Decomposition (FDD)

Spectral Density Matrix of Measured Responses

 $G_{vv}(j\omega)$



Singular Value Decomposition

$$G_{yy}(j\omega_k) = U_k S_k V_k^H$$

- Singular Value $(\omega_k \approx \omega_i)$ becomes large
- → has a peak equivalent to SDOF-PSD function
- Left Singular Vector u_r associated with a peak \approx Mode Shape

Natural Frequencies, Damping Ratios, Mode Shapes

40-11

FDD: Basic Formulations

PSD: Input/Output relation

$$G_{vv}(j\omega) = H(j\omega)^* G_{xx}(j\omega)H(j\omega)^T$$

■ PSD: Modal Decomposition

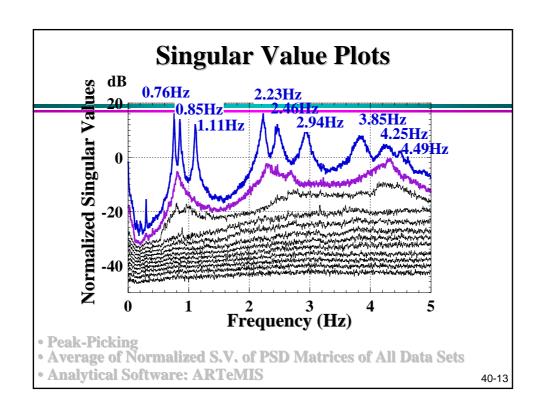
$$G_{yy}(j\omega) = \sum_{r=1}^{N} \frac{d_r \phi_r \phi_r^H}{j\omega - \lambda_r} + \frac{d_r \phi_r \phi_r^H}{j\omega - \lambda_r^*}$$

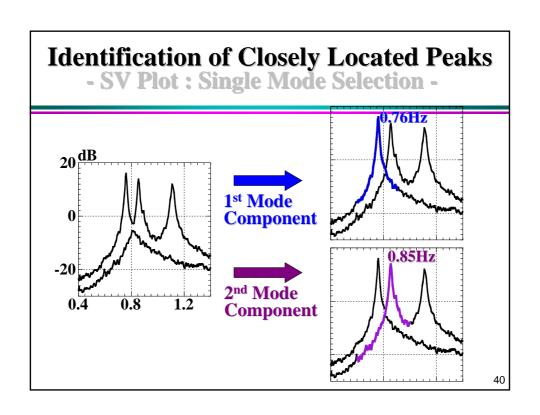
■ PSD: Singular Value Decomposition

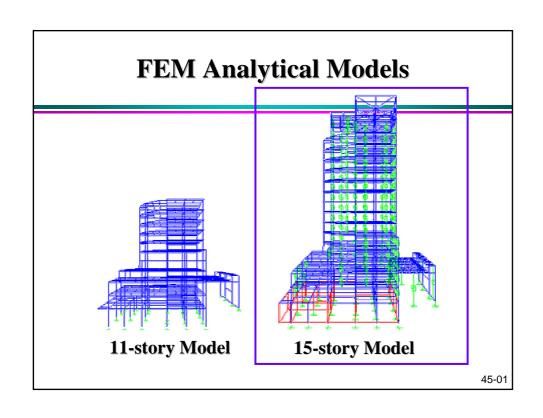
$$G_{yy}(j\omega_k) = U_k S_k V_k^H$$

■ Mode Shape Estimation

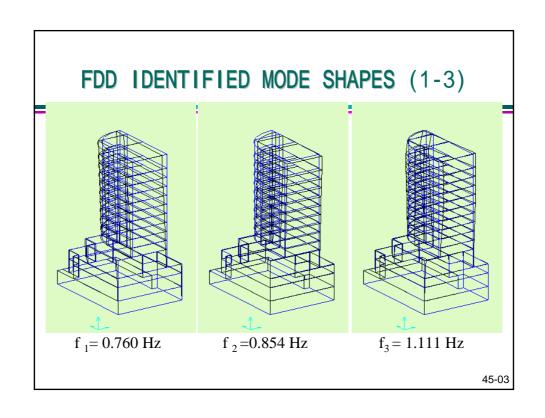
$$\hat{\phi}_r = u_r$$

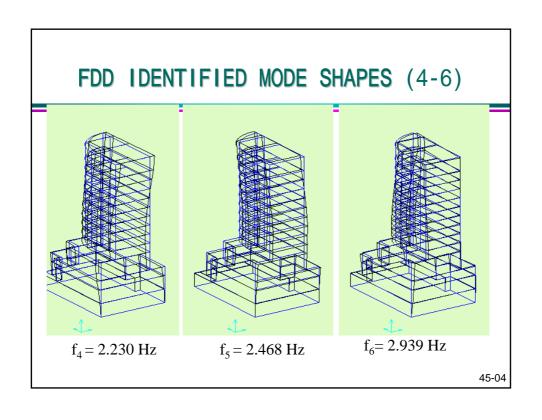


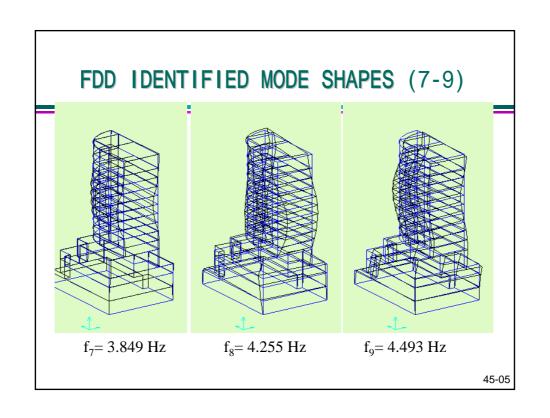


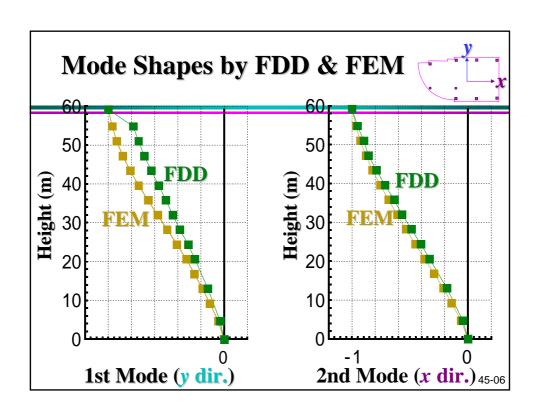


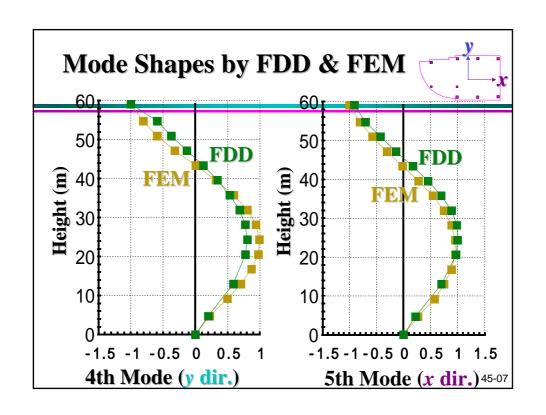
Natural Frequencies of 15-Story CFT Building Field Data				
Mode	FEM (Hz)	AMB (Hz)	Error (%)	
1	, 0.76	0.76	0	
2	0.87	0.86	2.22	
3 Adjust by Add	ed litional 1.15	1.11	3.51	
4Štiffne	2.14	2,23	-3.99	
5	2.53	2.47	2,59	
6	3.02	2.94	2.82	
7	3.85	3.85	0	
8	4.26	4.26	0	
9	4.67	4.47	4.29 45-02	

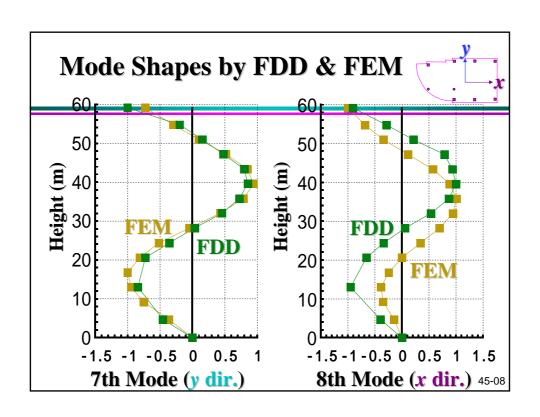






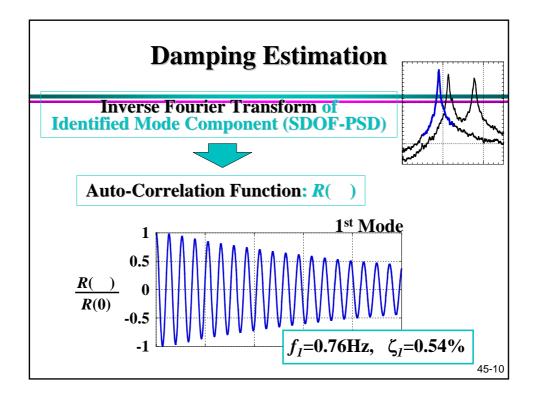


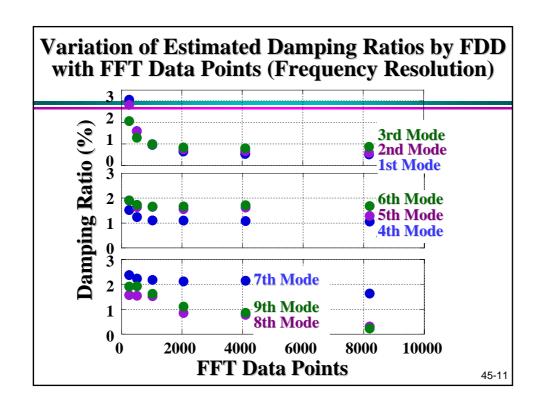


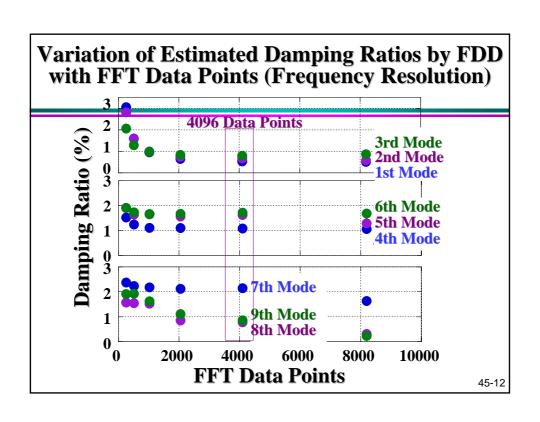


Basic Idea & Procedure of Damping Estimation

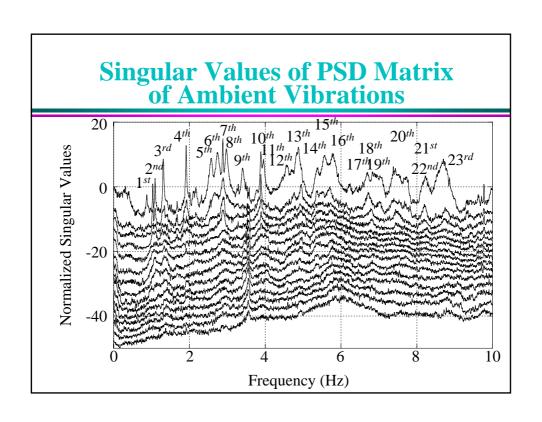
- Select SDOF approximation of the "PSD Bell" based on using MAC
- Calculate SDOF correlation function via Inverse FFT of the selected "PSD Bell"
- Estimate damping ration by Logarithmic Decrement Technique





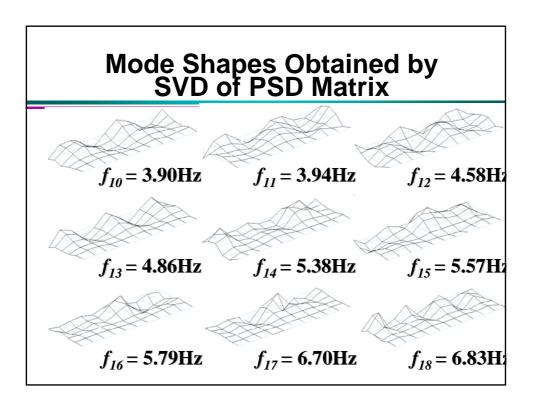


Estimated Damping Ratios and FFT Data Points						
— Data— Points	256	512	1024	2048	4096	8192
1st	3.05	1.60	0.95	0.65	0.54	0.51
2nd	2.81	1.58	0.99	0.74	0.67	0.58
3rd	2.06	1.29	0.98	0.84	0.80	0.87
4th	1.52	1.24	1.11	1.10	1.08	1.06
5th	1.91	1.64	1.65	1.56	1.62	1.29
6th	1.90	1.73	1.66	1.67	1.72	1.68
7th	2.37	2.23	2.18	2.15	2.11	1.63
8th	1.57	1.60	1.38	0.85	0.78	n/a
9th	1.91	1.92	1.62	1.25	0.86	n/a

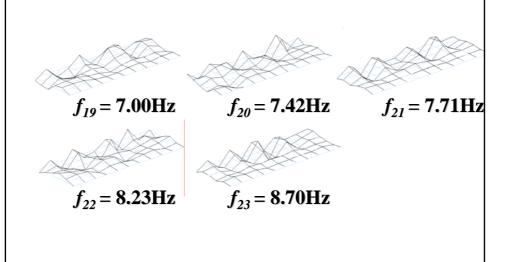


Mode Shapes Obtained by SVD of PSD Matrix
$$f_1 = 1.03$$
Hz $f_2 = 1.09$ Hz $f_3 = 1.31$ Hz

$$f_4$$
 = 1.93Hz f_5 = 2.58Hz f_6 = 2.74Hz f_7 = 2.88Hz f_8 = 2.97Hz f_9 = 3.30Hz



Mode Shapes Obtained by SVD of PSD Matrix



Damping Ratios Obtained by SVD of PSD Matrix					
Mode	ζ (%)	Mode	ζ (%)	Mode	ζ (%)
1	0.69	9	0.91	17	0.73
2	0.59	10	1.44	18	0.75
3	0.56	11	0.66	19	0.50
4	0.21	12	0.98	20	0.51
5	2.17	13	1.01	21	1.04
6	1.38	14	0.83	22	0.72
7	1.47	15	0.85	23	0.50
8	0.27	16	0.61	-	-