Wind Loads

2^{nd} Part
Conversion of wind speeds to wind loads, aerodynamical coefficients, wind pressure and suction, and practical design methods

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Drag forces on a moving object

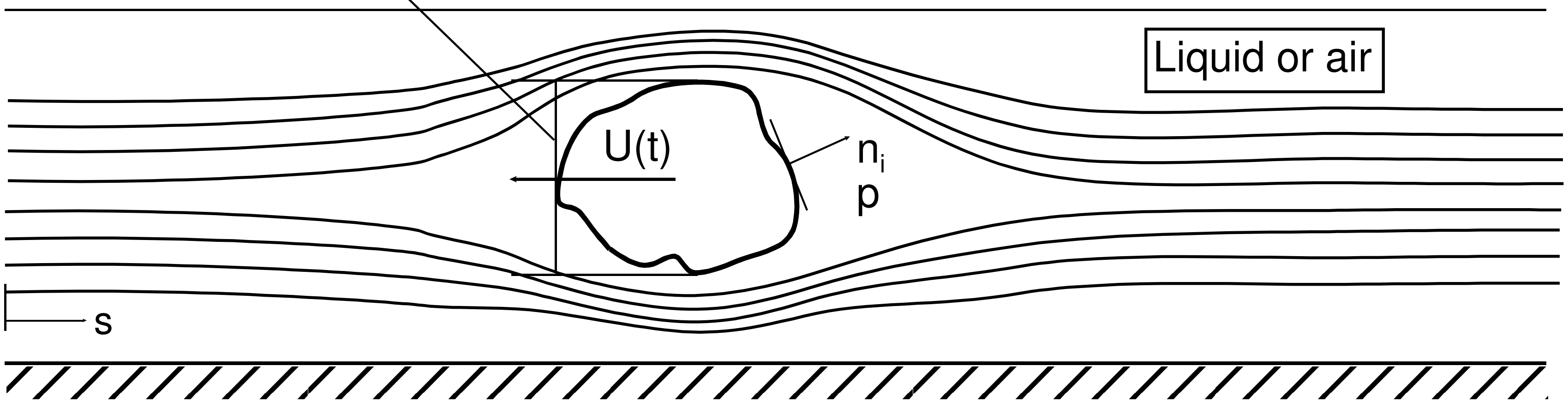
A bluff body is dragged through a liquid with a speed U(t)=U

$$\frac{dU(s,t)}{dt}(=0) + U(s)\frac{dU(s)}{ds} = -\frac{1}{\rho}\frac{dp}{ds} + \frac{1}{\rho}\frac{d\tau_{sz}}{ds}(\approx 0)$$

$$U(s)\frac{dU(s)}{ds} + \frac{1}{\rho}\frac{dp}{ds} = 0 \quad , \; p_0 + \tfrac{1}{2}\rho U_0^2 = p + \tfrac{1}{2}\rho U^2 = constant$$

Surface area of object is projected on a plane perpendicular to flow direction, A_p

A_p: the projection area,
U_0: undisturbed air speeds



Drag forces on a moving object

A bluff body is dragged through a liquid with a speed U(t)=U

Force exerted on the plane:

$$F_1 = \tfrac{1}{2}\rho \cdot U_0^2 \cdot C_p A_p$$

Force exerted on the body:

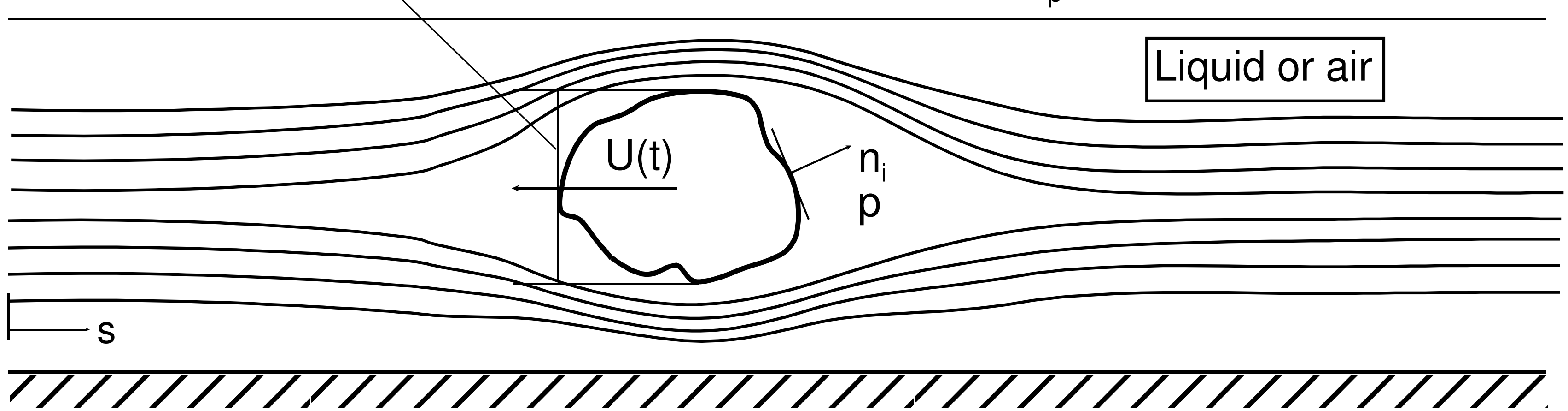
$$F_i = \int_A n_i p dA$$

The form coefficient or aerodynamic pressure coefficient

$$C_p = \frac{F_1}{\tfrac{1}{2}\rho \cdot A_p U^2} = \frac{\left[\int_A n_i p dA\right]_1}{\tfrac{1}{2}\rho \cdot A_p U^2}$$

Surface area of object is projected on a plane perpendicular to flow direction, A_p

U_0: undisturbed air speed
C_p: a form coefficient



Pressure coefficients

The pressure/suction per unit area (m^2)

$$q = \tfrac{1}{2}\rho \cdot U_0^2 \cdot C_p$$

For direct pressure/suction C_p=C_D "the Drag Coefficient"

The pressure coefficients can be measured directly with pressure cells. They will show marked dynamical behaviour (rapid oscillations)

The 10 minute average value will give the static pressure coefficient

Measurement of aerodynamic coefficients

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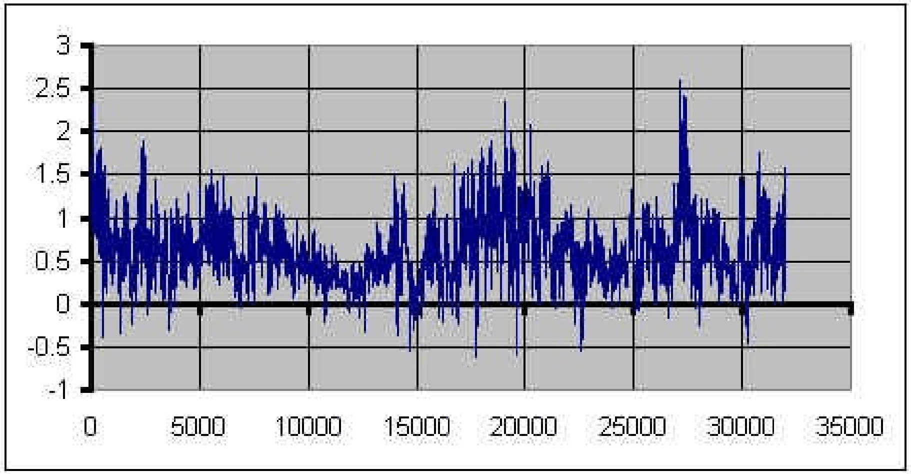

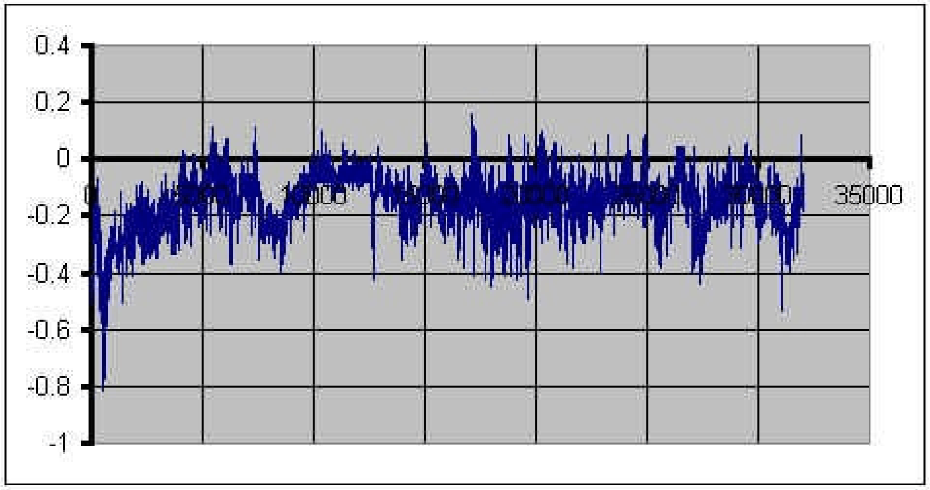

Wind tunnel measurements of the Landsvirkjun office building

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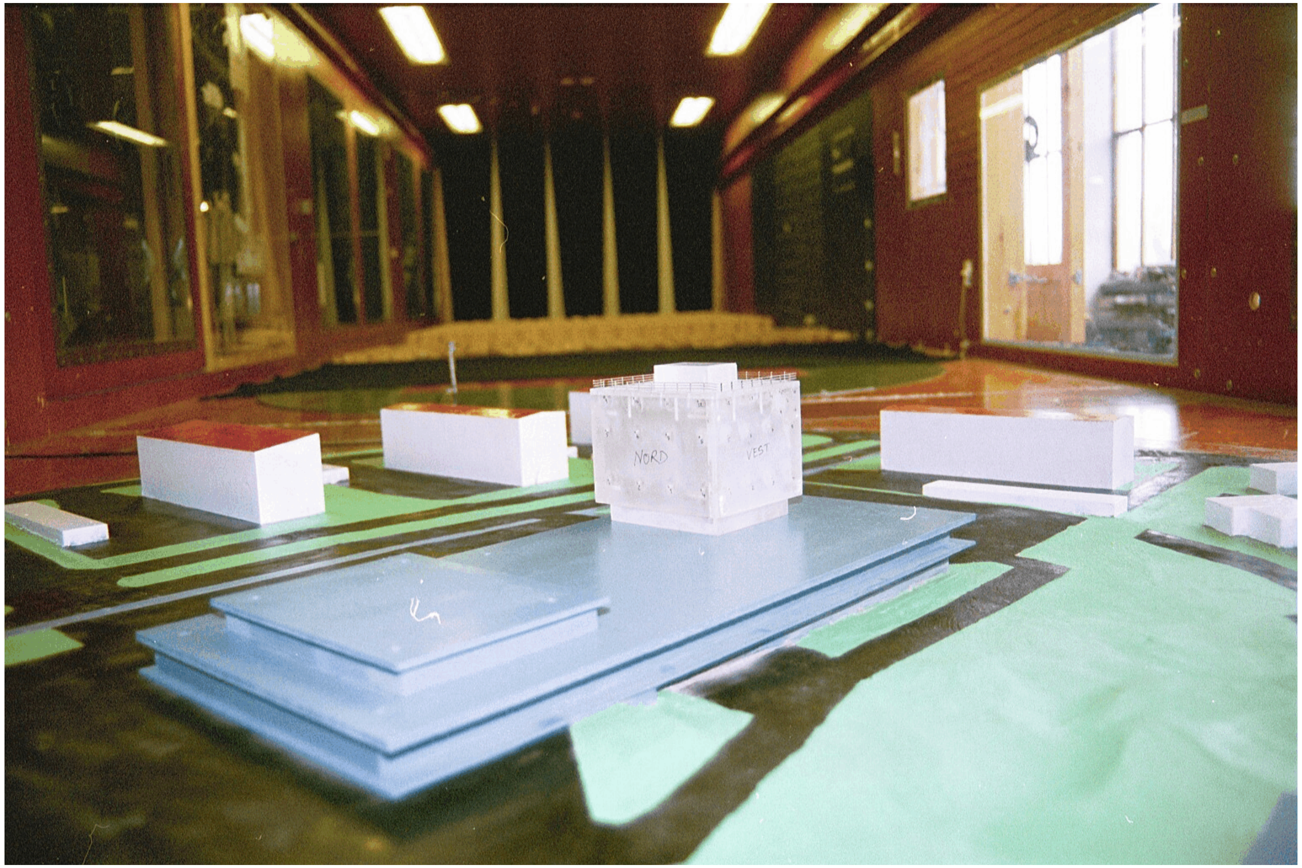
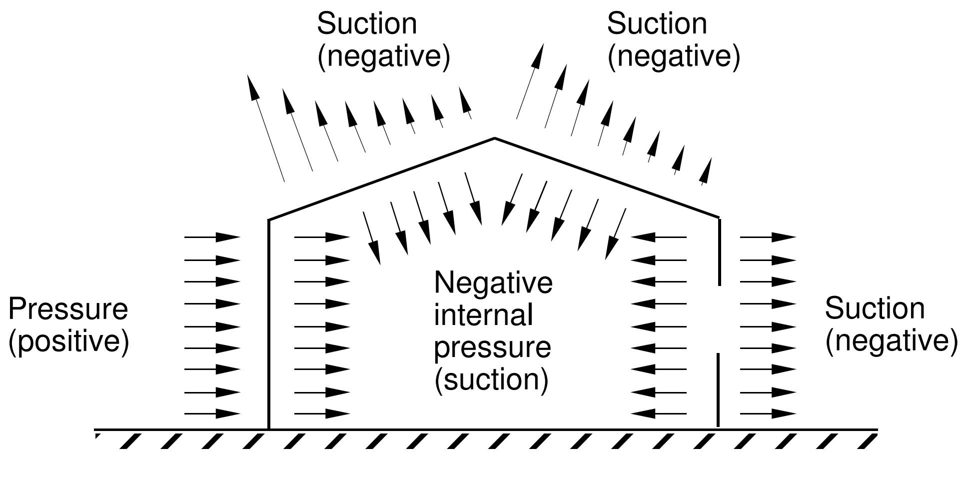
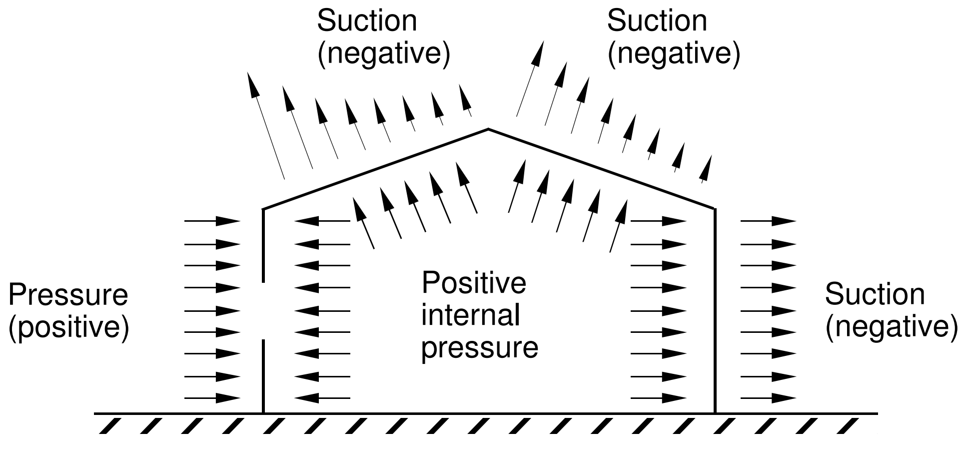


Aerodynamical/Pressure coefficients

Aerodynamical/pressure coefficients are used to interpret wind pressure/suction on bodies or surfaces. Mostly they are based on in-situ or wind tunnel measurements.





Wind blows into the gable

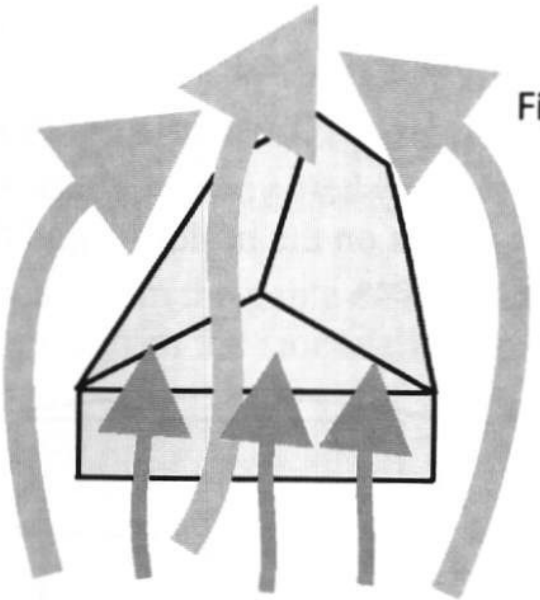


Fig. 6

3-D of Airflow when wind direction is into gable end wall.

Figure 3

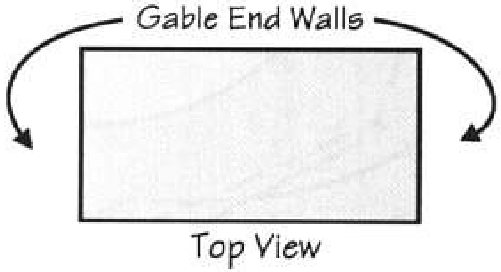
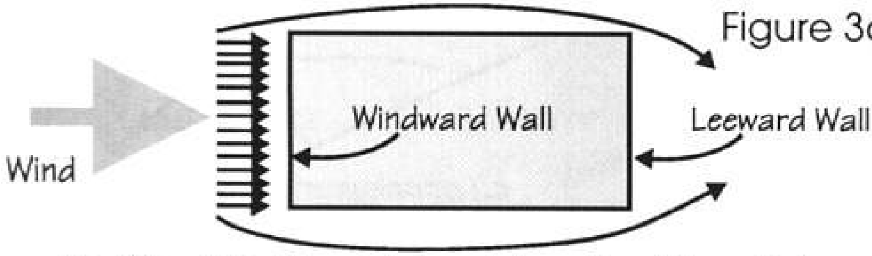
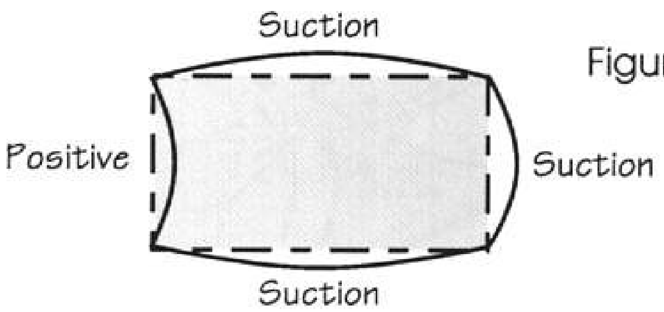


Figure 3a



Top View. Wind blows into gable end wall (parallel to ridge). Air also flows around the building walls.

Figure 3b



Top view. Reaction (deflection) of walls to the dynamic forces. Dotted line denotes undeflected surface. Solid line denotes deflected surface reacting to the forces from the wind flow around the object. Positive pressure exerts a force toward the surface. Suction exerts a force pulling away from a surface.

Wind blows into a main wall

The wind loads depend on openings in the structure and can create very different results

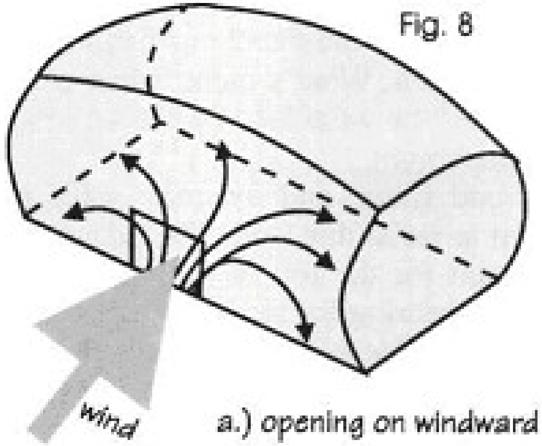


Fig. 8

a.) opening on windward wall

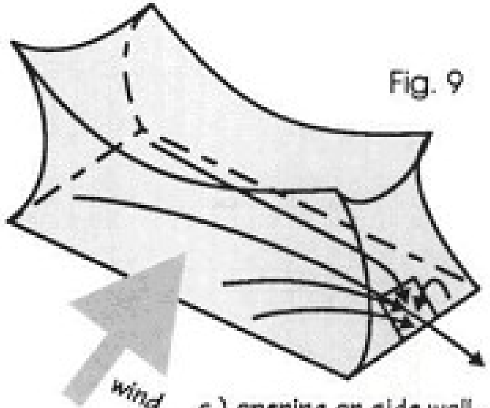


Fig. 9

c.) opening on side wall.

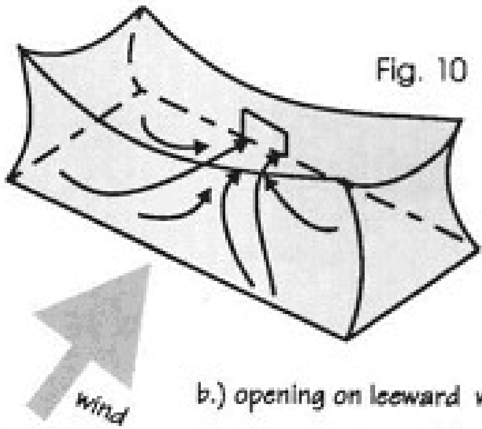
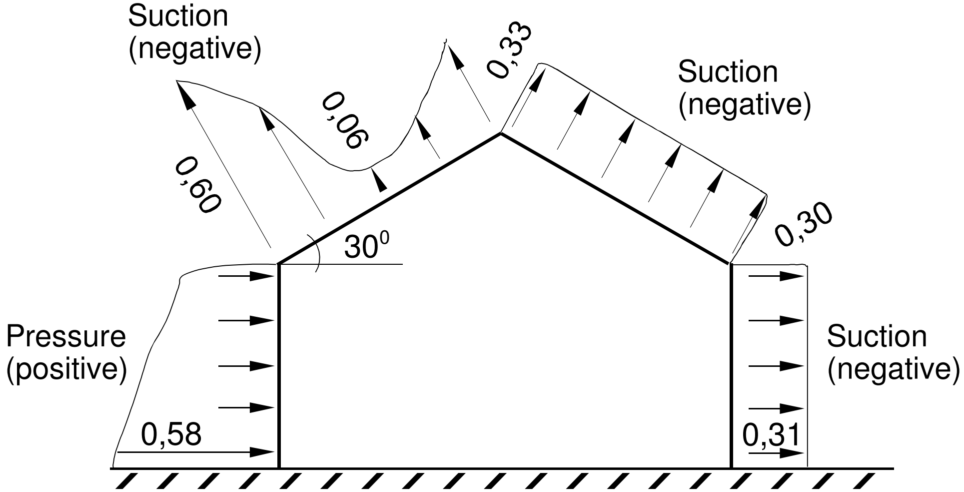


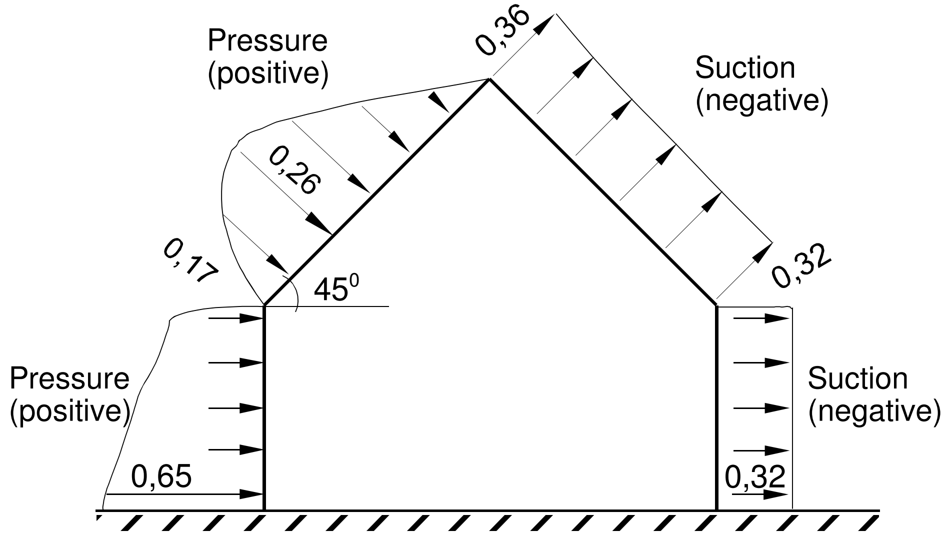
Fig. 10

b.) opening on leeward wall.

Aerodynamical/Pressure coefficients for a typical shed building

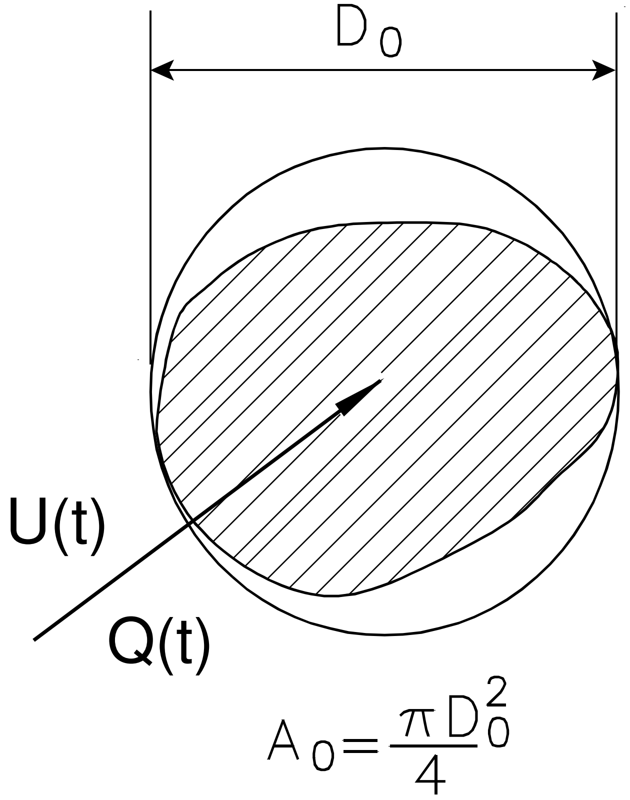
The wind has to blow over the building creating eddies due to turbulence at the edges and roof top. This causes non-uniform distribution of the pressure/suction forces





The Morrison equation

For wave forces on harbour structures as proposed by Morrison 1932



When structural response is affected by dynamical behaviour the acceleration can not be disregarded. The force exerted on the object can then be written as

$$Q(t) = [\tfrac{1}{2}\rho C_D U(t)\left|U(t)\right| + \rho C_M \frac{A_0}{D_0}\dot{U}(t)]A_p$$

D_0 and A_0 are the diameter and area of a circle, which circumscribes the projection area of the object, A_p.

C_D: the “drag” coefficient

C_M: the “added mass” coefficient

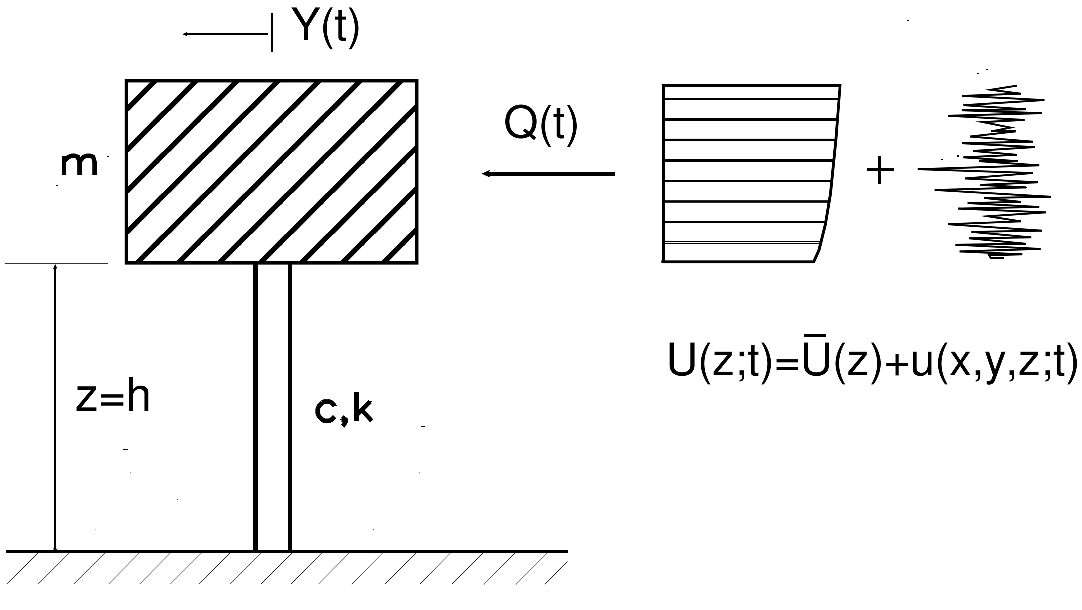
Dynamical response of a simple structure

The drag force depends on the relative velocity and acceleration

$$m\ddot{Y}(t) + c\dot{Y}(t) + kY(t) = [\tfrac{1}{2}\rho C_D (U(t) - \dot{Y}(t))^2 + \rho C_M \frac{A_0}{D_0}(\dot{U}(t) - \ddot{Y}(t))]A_p$$

Insert $U(t)=\bar{U}+u(x,y,z;t)$, discard all second order terms ($u^2(t)$, $u(t)\dot{Y}(t)$, $\dot{Y}^2(t)$) and rearrange to get:

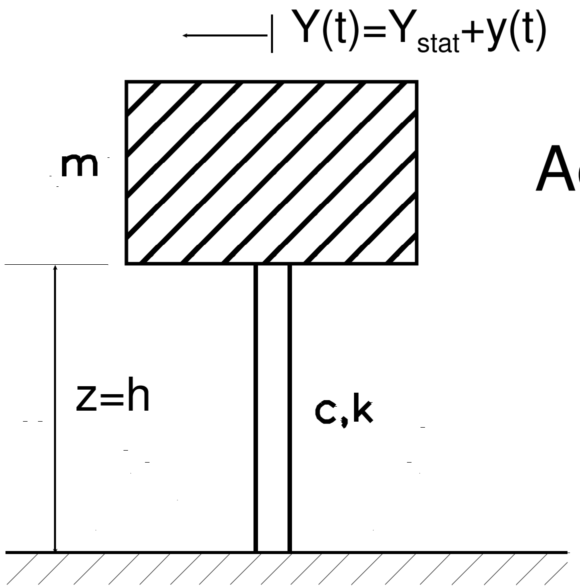
$$\ddot{Y}(t)[m + m_A] + \dot{Y}(t)[c + c_A] + kY(t) = Q_{stat} + Q_{dyn}$$



Dynamical response of a simple structure

The response is composed of a static part (deflection caused by the mean wind speed) and a dynamical part caused by the wind gusts

$$\ddot{Y}(t)[m+m_A]+\dot{Y}(t)[c+c_A]+kY(t)=Q_{stat}+Q_{dyn}$$



Added mass:

$$m_A = \rho\cdot C_M \frac{A_0}{D_0} A_p$$

Aerodynamical damping (sometimes negative!):

$$c_A=\rho C_D \overline{U} \quad , c=\lambda\cdot c_{cr} \quad , c_{cr}=2\sqrt{km}$$

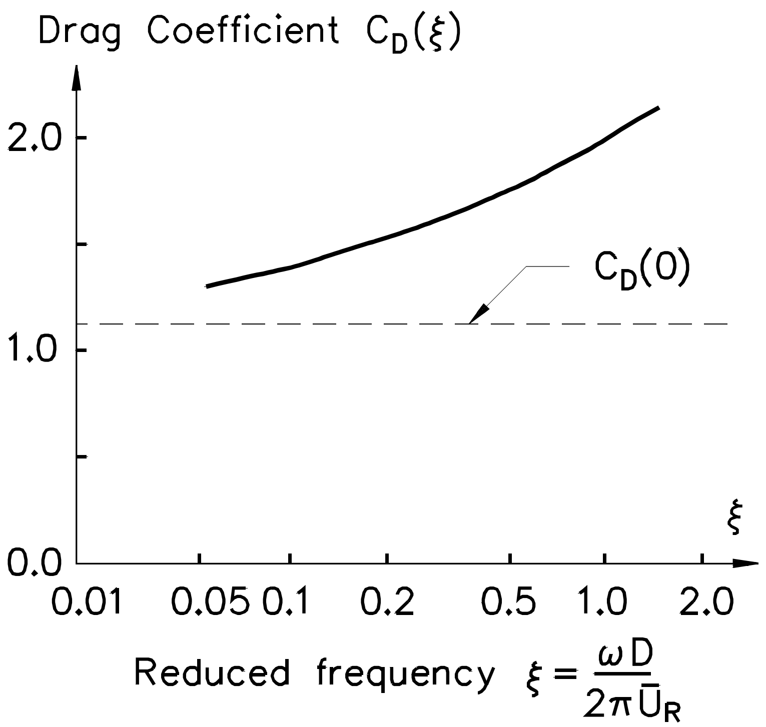
$$Q_{stat}=\tfrac{1}{2}\rho C_D \overline{U}^2 A_p \quad , Y_{stat}=\frac{Q_{stat}}{k}$$

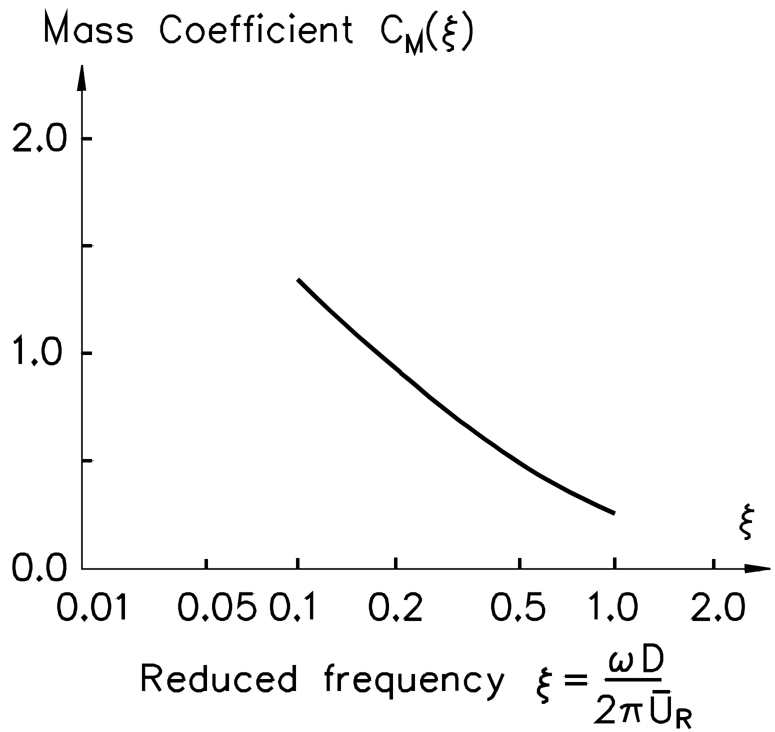
$$Q_{dyn}=[\rho C_D \overline{U} u(t)+\rho C_M \frac{A_0}{D_0}\dot{u}(t)]A_p$$

The form coefficients

It is convenient to introduce the reduced frequency of the wind gusts (turbulent part u(x,y,z;t)), i.e. $\xi = fD/U_R$ where $T=1/f$ is the period of the wind gusts (0,5-30 sec.), D is a reference diameter and U_R the reference wind speed

The drag and mass coefficients are sensitive to the characteristics of turbulence (A.G. Davenport)

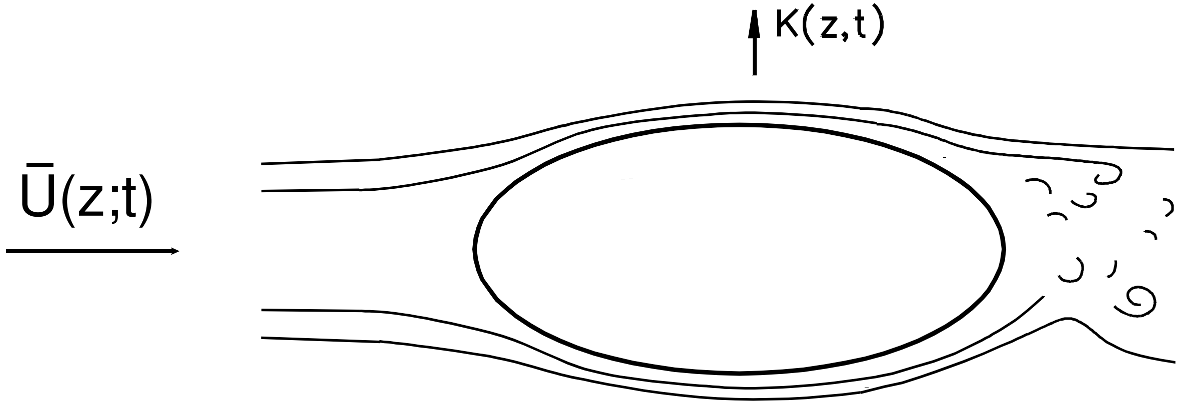




Other form coefficients

The drag coefficient C_D is mostly related to the turbulent x-component u(x,y,z;t). The other components give rise to different kind of excitations or actions

- The lift coefficient C_L is heavily dependent on the Reynold's number Re=U(z;t)D/v
  - z direction (vertical); makes airoplanes fly
  - y direction (horizontal) produces a Strouhal effect (cross wind vibration) in towers and chimneys



(b) SUPERCRITICAL RANGE
$10^5 < R_e < 3.5 \cdot 10^6$

Loss of marked periodicity–random wake pattern. Laminar separations, turbulent reattachments occur.

Stochastic processes and random vibrations

The random nature of the wind gusts, that is, the turbulent part of the wind speed, makes it difficult to interpret unless reverting to the theory of stochastic processes. The wind gusts can be treated as a stochastic Gaussian process $X(t)$ with a power spectral density $S_X(\omega)$ and an autocorrelation function $R_X(\tau)=E[X(t)X(t+\tau)]$

The stochastic character of the wind gusts

σ_U^2=E[u(t)u(t)], μ_U=E[u(t)]=0

The autocorrelation function of the wind gusts is

$$R_U(\tau) = E\left[u(t)u(t+\tau)\right]$$

The power spectral density of the wind gusts is

$$S_U(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_U(\tau)e^{-i\omega\tau}d\tau \leftrightarrow R_U(\tau) = \int_{-\infty}^{\infty} S_U(\omega)e^{i\omega\tau}d\omega$$

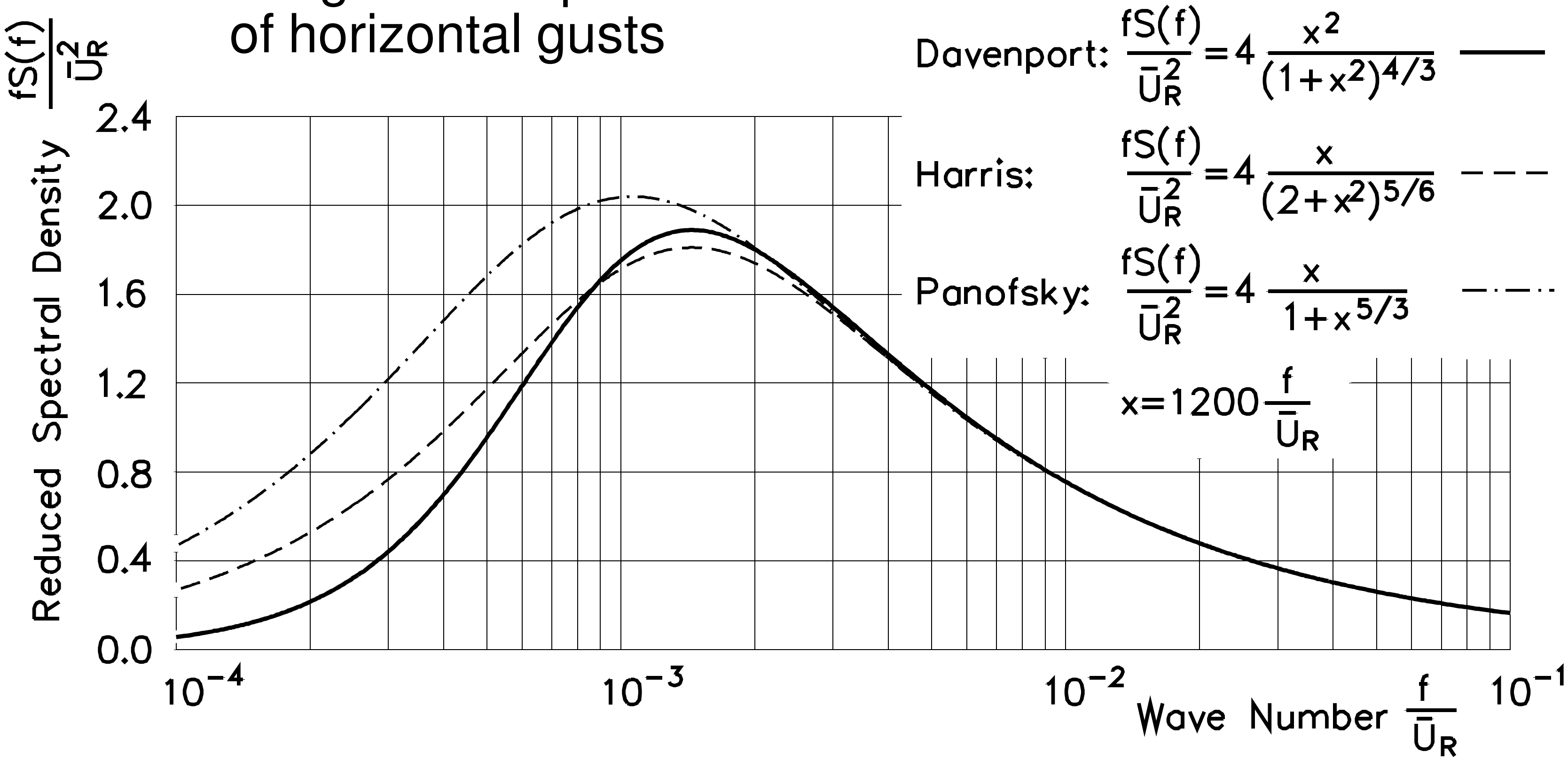
The variance of the wind gusts is therefore given by

$$\sigma_U^2 = E\left[u^2(t)\right] = R_U(0) = \int_{-\infty}^{\infty} S_U(\omega)d\omega$$

The longitudinal spectrum of horizontal wind gusts

The turbulence intensity is defined as $I_U=\sigma_U/U_R=V_U$ (the coefficient of variation). At height z m, $I_U(z)=\sigma_U/v_m(z)$.
$I_U=\sqrt{6\kappa}\approx k_r$ at 10 metre reference height, $I_U(z)=k_r/(c_r(z)\cdot c_o)$ at z m

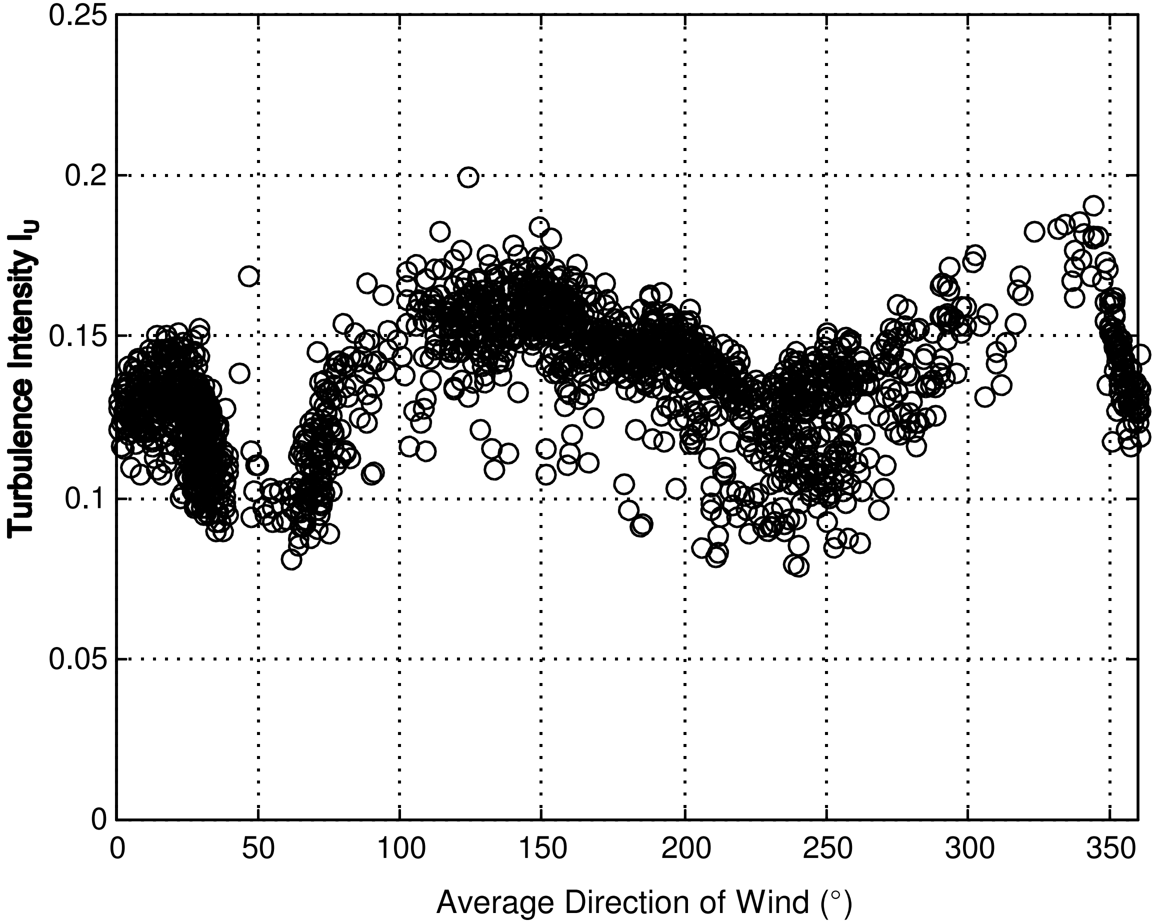
Longitudinal spectrum of horizontal gusts



Turbulence Intensity I_U

Measurements at Keilisnes (z_R=10 m), Iceland

$I_U \approx k_r$=0.15 (terrain category between I and II)



The cross spectrum 1

Consider two stochastic processes X(t) and Y(t)

A spectrum for the cross-correlation function

$R_{XY}(\tau)=E[X(t)Y(t+\tau)]$ can be defined:

$$S_{XY}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} R_{XY}(\tau)e^{-i\omega\tau}d\omega$$

$S_{XY}(\omega)=Co_{XY}(\omega)-iQu_{XY}(\omega)$, where $Co_{XY}(\omega)$ is the co-spectrum and the imaginary part $Qu_{XY}(\omega)$ is the quadrature spectrum. The coherence function is defined as

$$Coh_{XY}(\omega)=\frac{\left|S_{XY}(\omega)\right|^2}{S_X(\omega)S_Y(\omega)} \quad , \quad -1\le Coh_{XY}(\omega)\le 1$$

The cross spectrum 2

The phase angle Φ

The phase angle is proportional to the reduced frequency ξ

$$\Phi_{XY}(\omega) = Arc\tan\left(\frac{Qu_{XY}(\omega)}{Co_{XY}(\omega)}\right) \approx c\,\frac{\omega r_{mn}}{\overline{U}_R} = c \cdot \xi$$

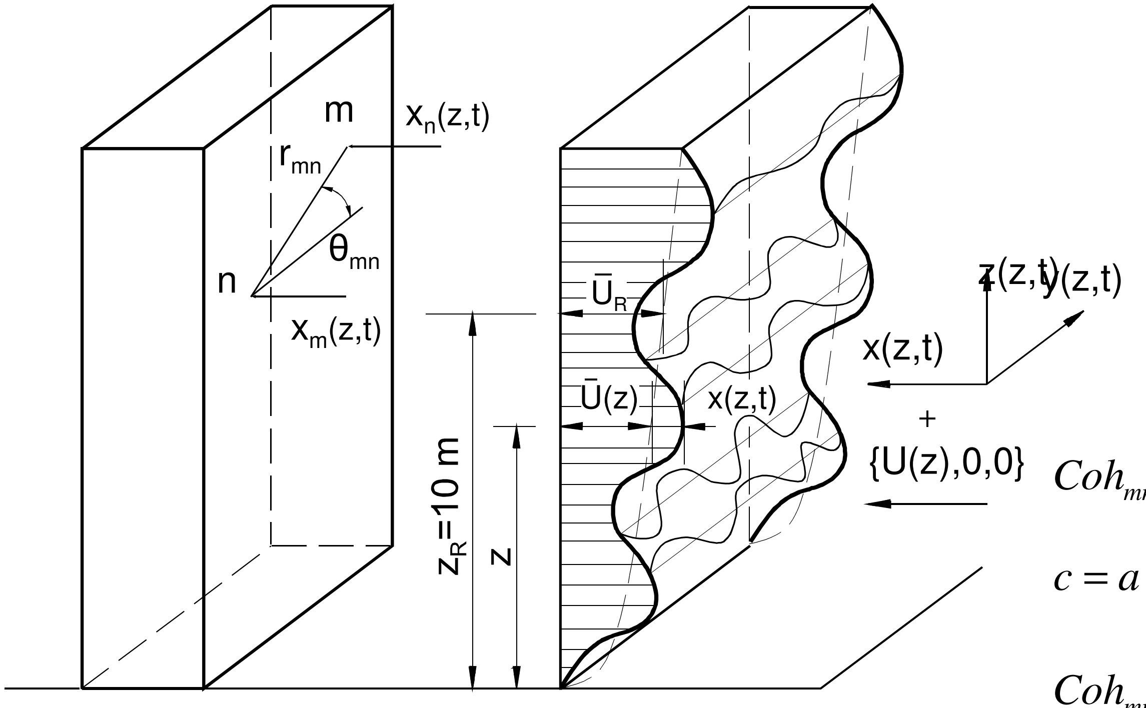
where c is of the order 0.8-1.3. Thus the real and imaginary parts of the cross spectrum can be written as

$$Co_{XY}(\omega) = \sqrt{Coh_{XY}(\omega) S_X(\omega) S_Y(\omega)} \cdot \cos\Phi_{XY}(\omega)$$

$$Qu_{XY}(\omega) = \sqrt{Coh_{XY}(\omega) S_X(\omega) S_Y(\omega)} \cdot \sin\Phi_{XY}(\omega)$$

The Coherence: $Coh_{mn}(\omega)$

The coherence describes the correlation between two pressure points on the building facade in the frequency domain
($-1 \leq Coh_{mn}(\omega) \leq 1$)



Loss of coherence between immediate wind velocities at two points on the facade (m,n) is exponentially related to the reduced frequency ξ

$$Coh_{mn}(\xi) = \exp[-2c\xi]\,,\ \xi = \frac{\omega \cdot r_{mn}}{\overline{U}_R}$$

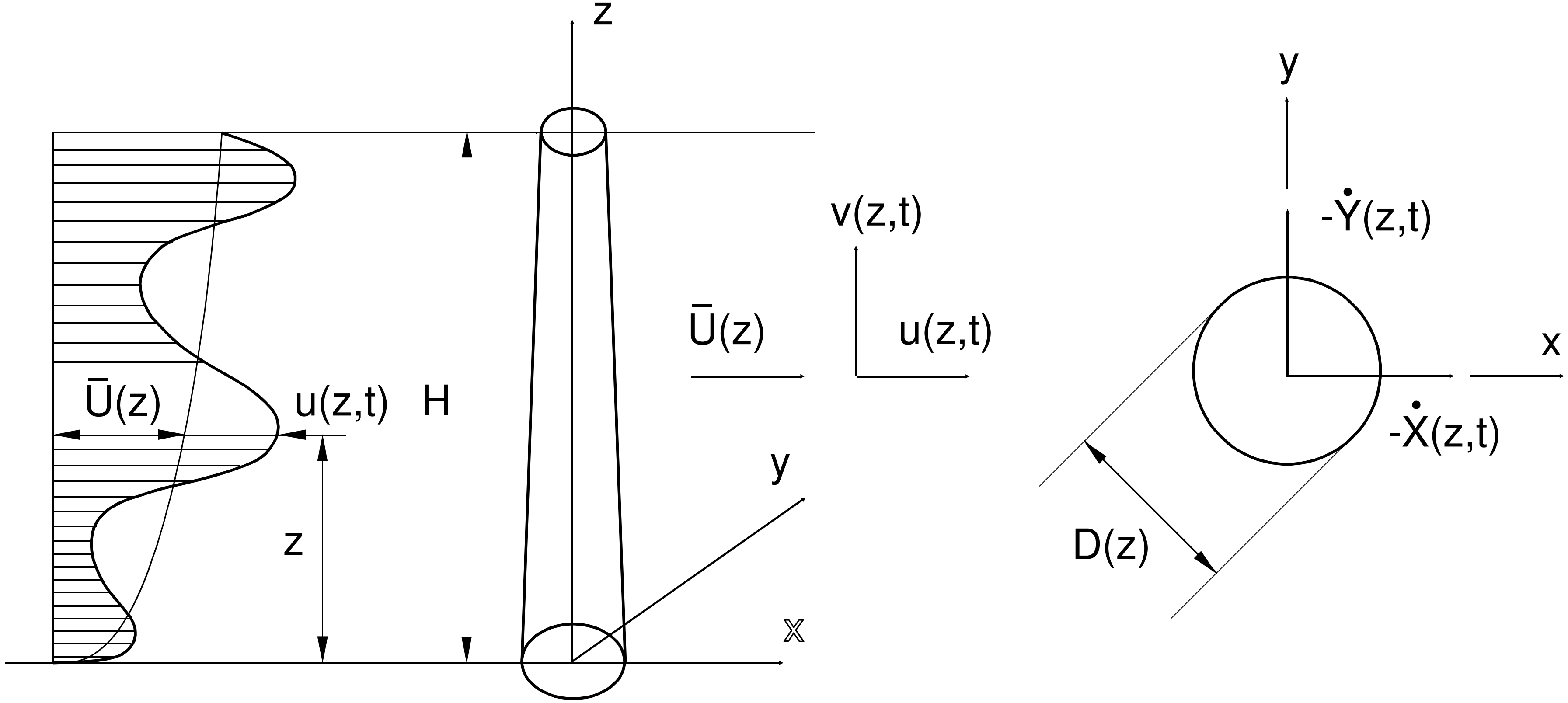
$$c = a\,(horizontal\ separation)\,,\ c = b\ (vertical)$$

$$Coh_{mn}(\omega) = \exp\left[-\frac{2ab}{\overline{U}_R} \cdot \frac{\omega \cdot r_{mn}}{\sqrt{a^2 \sin^2\theta_{mn} + b^2\cos^2\theta_{mn}}}\right]$$

$$a \approx 3b\ in\ the\ northern\ hemisphere\ (a \approx 3.8\,,\ b \approx 1.3)$$

A tapered chimney subjected to wind loads

X(z,t) is the lateral deflection in the along wind direction



Loading functions and response of the chimney

$$\frac{\partial}{\partial z^2}\left(EI(z)\frac{\partial X^2(z,t)}{\partial z^2}\right) + m(z)\frac{\partial X^2(z,t)}{\partial t^2} = P^*(z,t) - C^{aero}(z)\dot{X}(z,t) - M^{aero}(z)\ddot{X}(z,t)$$

$$P^*(z,t) = \overline{P}(z) + P(z,t)$$

$$\overline{P}(z) = \frac{1}{2}\rho C_D(z,0)D(z)\overline{U}^2(z)$$

$$P(z,t) = \rho C_D(z,\xi)D(z)\overline{U}(z)U(z,t) + \rho\frac{\pi}{4}C_M(z,\xi)D^2(z)\dot{U}(z,t)$$

$$C^{aero}(z) = \rho C_D(z,\xi)D(z)\overline{U}(z)$$

$$M^{aero}(z) = \rho\frac{\pi}{4}C_M(z,\xi)D^2(z)$$

$$X(z,t) = \sum_{i=1}^{\infty}\phi_i(z)Q_i(t) \quad , \quad \phi_i(z)\ is\ the\ i-th\ normal\ mode\ shape$$

$$Q_i(t) = \frac{1}{M_i}\int_{-\infty}^{\infty} L_i(u)h_i(t-u)du\ ,\ M_i = \frac{1}{M_i}\int_0^H m(z)\phi_i^2(z)dz\ ,\ L_i(t) = \frac{1}{M_i}\int_0^H P(z,t)\phi_i^2(z)dz$$

Mean response and gust response

The gust response has to be treated as random processes

$$L^*[X(z,t)] = \overline{P}(z) + P(z,t)$$

$$\overline{X}(z) = L^{*^{-1}}[\overline{P}(z)] \quad , \quad L^*[X(z,t)] = (z,t)$$

$$E[X(z,t)X(z,t+\tau)] = R_X(\tau) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\phi_i(z)\phi_j(z)E[Q_i(t)Q_i(t+\tau)]$$

$$S_X(\omega, z) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\phi_i(z)\phi_j(z)S_{Q_iQ_j}(\omega)$$

$$S_{Q_iQ_j}(\omega) = \frac{1}{M_i}\frac{1}{M_j}H_i(\omega)S_{L_iL_j}(\omega)H_j^*(\omega)\ , \ H_i(\omega) = \frac{1}{(\omega^2 - \omega_i^2) + 2i\lambda_i\omega_i}$$

$\textit{The cross spectrum for the generalized force } L_i$

$$S_{L_iL_j}(\omega) = \int_0^H\int_0^H \phi_i(z_m)\phi_j(z_n)S_{P_mP_n}(\omega)dz_m dz_n$$

Stochastic gust response

Having obtained the cross spectrum for the generalized forces L_i , the cross spectrum for the direct wind loads P_i can be obtained The power law $(z/z_R)^\alpha$ is used instead of the logarithm wind speed profile $k_r \cdot \ln(z/z_0)$

$$S_{P_m P_n}(\omega, z_m, z_n) = \rho^2 \overline{U}_R^2 \left(\frac{z_m}{z_R} \cdot \frac{z_n}{z_R} \right)^\alpha [C_D(z_m, \xi_m) C_D(z_n, \xi_n)$$

$$+ \frac{\pi^4}{4} \xi_m \xi_n C_M(z_m, \xi_m) C_M(z_n, \xi_n)]$$

$$+ D(z_m) D(z_n) S_{U_m U_n}(\omega) \quad , \quad \xi_m = \frac{\omega D(z_m)}{2\pi \overline{U}_R}$$

The aerodynamical admittance F(ξ)

Introducing the coherence function $Coh_{mn}(\omega)$ and the phase angle $\Phi_{mn}(\omega)$, the real part of the cross spectrum for the generalized forces L_i can be directly related to the simple wind velocity spectrum $S_U(\omega)$. Thus an aerodynamical admittance converts wind speeds into wind loads. Spatial separation $r_{mn}=(z_m-z_n)$.

$$\frac{S_{L_m L_n}(\omega, z_m, z_n)}{(\frac{1}{2}\rho A U_R^2)} = 4\left|F_{ij}^U(\omega)\right|\frac{S_U(\omega)}{U_R^2} \quad , \quad A = \int_0^H D(z)\,dz$$

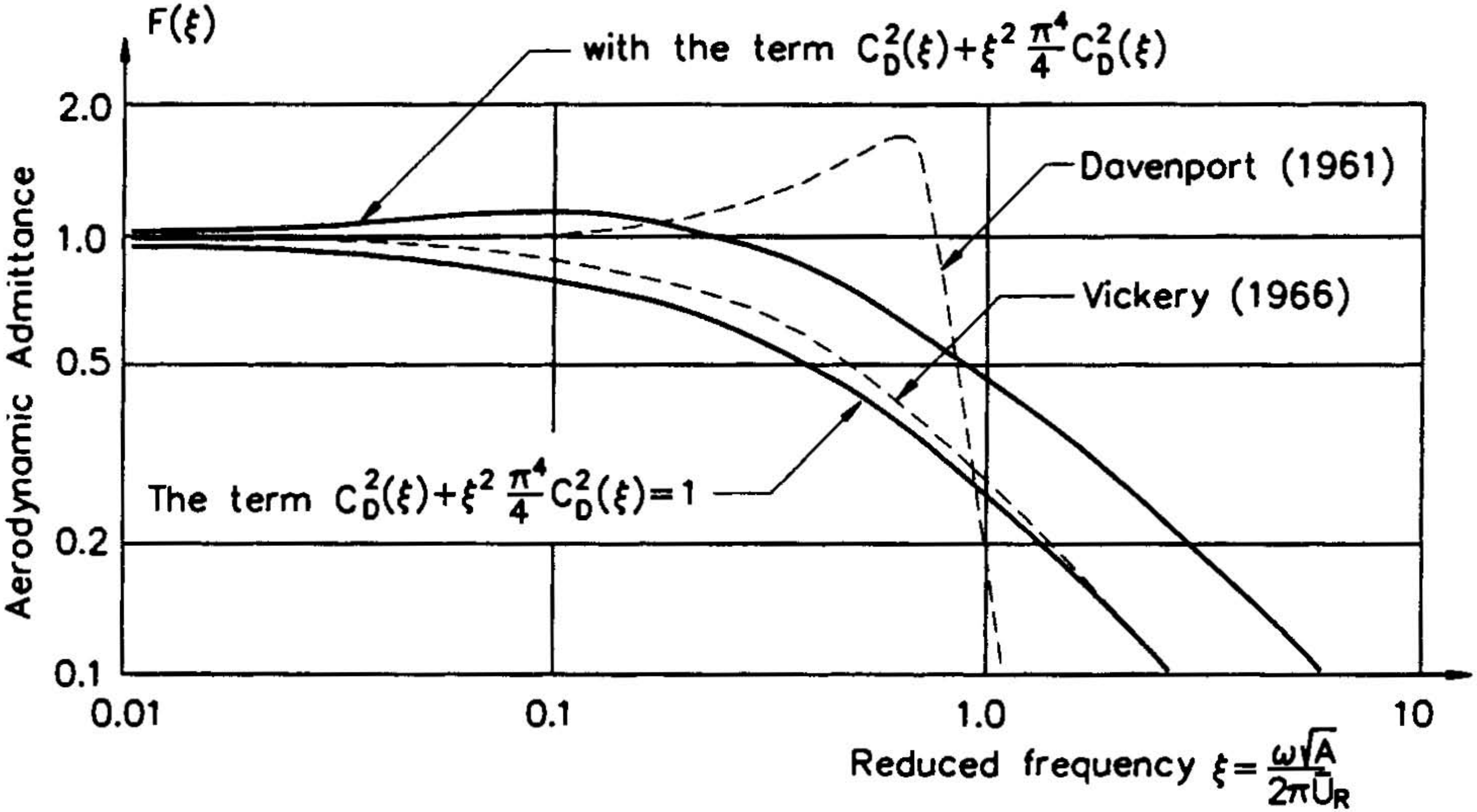
$$\left|F_{ij}^U(\omega)\right|^2 = \frac{1}{A^2}\int_0^H\int_0^H \left[\frac{z_m z_n}{z_R}\right]^2 [C_D(z_m,\xi_m)C_D(z_n,\xi_n)$$

$$+\frac{\pi^4}{4}C_M(z_m,\xi_m)C_M(z_n,\xi_n)]D(z_m)D(z_n)$$

$$+\exp\left[-\frac{b\,\omega\, r_{mn}}{U_R}\right]\cos\left[\frac{c\,\omega\, r_{mn}}{U_R}\right]\phi_i(z_m)\phi_j(z_n)\,dz_m\,dz_n$$

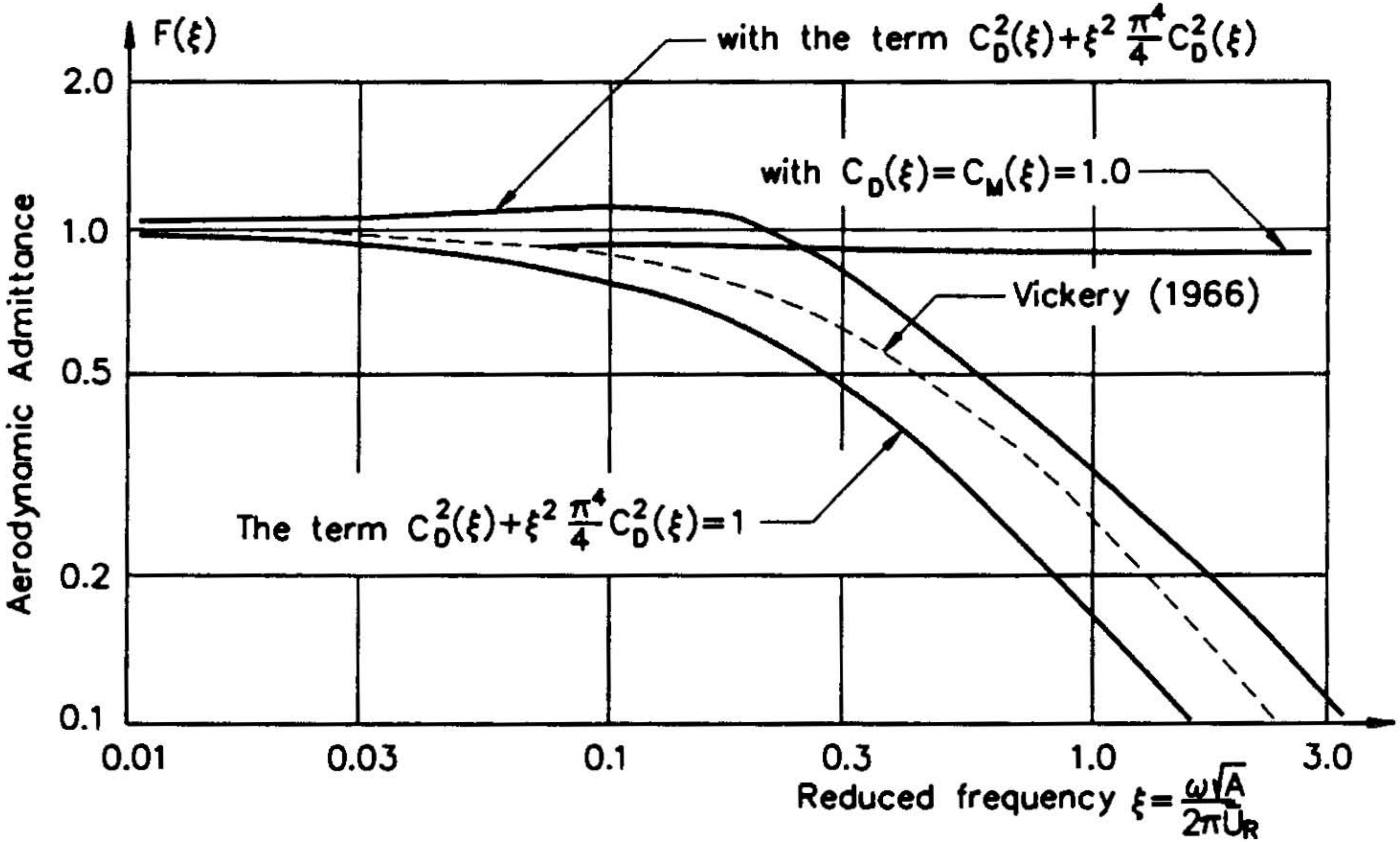
Aerodynamical Admittance

Homogeneous isotropic turbulence, a=b=1.27, c=0.0



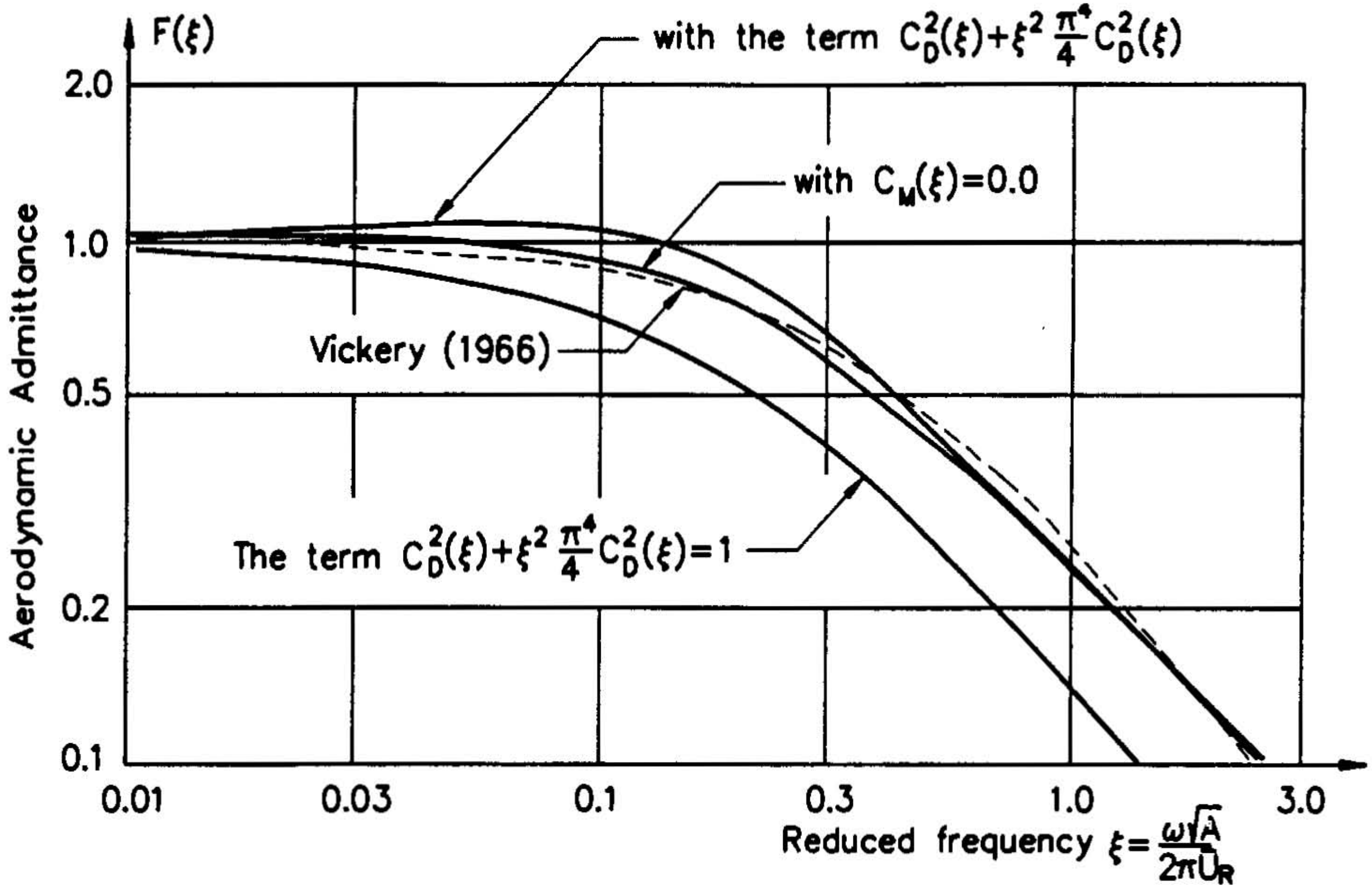
Aerodynamical Admittance

Homogeneous isotropic turbulence, a=b=1.27, c=0.8



Aerodynamical Admittance

Homogeneous isotropic turbulence, a=3.8, b=1.27, c=0.8



Overload design factors

Consider the response process X(t) (deflection)

$X_{max}=\bar{X}+(X_{peak}-\bar{X})$

Introducing the relative extreme peaks

$\Xi_X=(X_{peak}-\bar{X})/\sigma_X$

Having accounted for the static mean response $\bar{X}$, the probability distribution of extreme peaks Ξ_X is fairly narrow and a sensible design value can be taken as the average extreme peak value $E[\Xi_X]$. Thus

$X_{max}=\bar{X}+\sigma_X\, E[\Xi_X]$ or $X_{max}=\bar{X}(1+(\sigma_X/\bar{X})\cdot E[\Xi_X])=GF\cdot\bar{X}$

Numerical analysis of two chimney stacks

Open country environment

	Symbol	Stack 1	Stack 2
Height (m)	H	200	80
Base diameter (m)	D_0	14.0	12.0
Tip diameter (m)	D_1	5.0	9.3
Natural frequency (rps)	ω_1	2.3	6.28
Critical damping ratio	λ_1	0.02	0.02
Drag coefficient	C_D	1.0	1.0
Added mass coefficient	C_M	0	0
Mean wind velocity (m/s)	U_{10}	30	30
Decay constant	b	1.27	1.27
Phase constant	c	0.8	0.8
Wind profile exponent	α	0.16	0.16
Roughness parameter	κ	0.005	0.005
Maximum Response values			
Alongwind response	σ/Ξ	0.162	0.162
Gust factor	GF	1.56	1.6
Across wind response: gusts	σ/Ξ	0.094	0.092
wake	σ/Ξ	0.029	0.914
Gust factor	GF	1.35	3.44
Resulting dynamic response factor	GF	1.60	3.70

End

2^{nd} Part