STEADY-STATE VIBRATION RESPONSE OF A PLATE FIXED ON ALL SIDES SUBJECTED TO A UNIFORM PRESSURE

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July 26, 2012

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The fixed-fixed-fixed-fixed plate in Figure 1 is subjected to a uniform pressure.

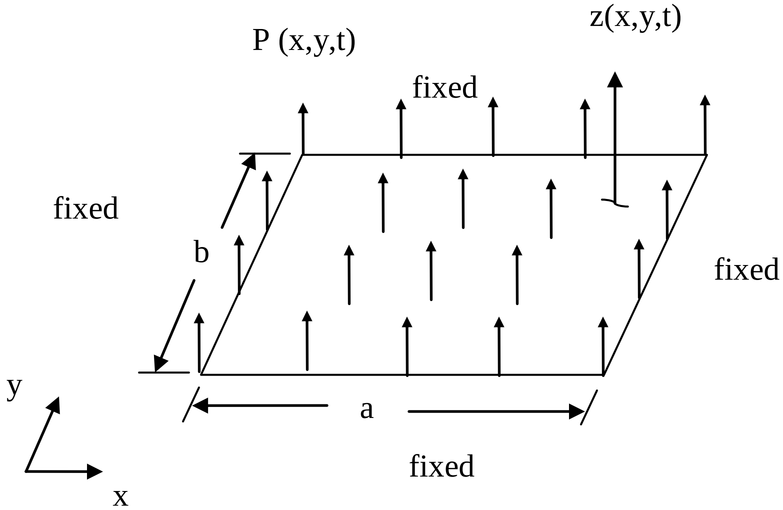


Figure 1.

The following equations are taken from Reference 1.

The governing differential equation is

$$D\left(\frac{\partial^4 z}{\partial x^4} + 2\frac{\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4}\right) + \rho h \frac{\partial^2 z}{\partial t^2} = P(x, y, t) \tag{1}$$

The plate stiffness factor D is given by

$$D = \frac{Eh^3}{12(1-\mu^2)} \quad (2)$$

where

E	is the modulus of elasticity
μ	Poisson's ratio
h	is the thickness
ρ	is the mass density (mass/area)
P	is the applied pressure

Now assume that the pressure field is uniform such that

$$W(t) = P(x, y, t) \quad (2)$$

The differential equation becomes

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho h \frac{\partial^2 w}{\partial t^2} = W(t) \quad (3)$$

The mass-normalized mode shape and its derivatives are

$$Z(x, y) = \frac{1}{\sqrt{\rho abh}}\left\{[\cosh(\beta_x x) - \cos(\beta_x x)] - \sigma[\sinh(\beta_x x) + \sin(\beta_x x)]\right\}$$

$$\bullet \left\{[\cosh(\beta_y y) - \cos(\beta_y y)] - \sigma[\sinh(\beta_y y) + \sin(\beta_y y)]\right\} \quad (4)$$

where

$$\sigma = \left[\frac{\sinh(\beta L) + \sin(\beta L)}{\cosh(\beta L) - \cos(\beta L)} \right] \tag{5}$$

$$\beta L = 4.73004 \tag{6}$$

$$\beta_x = 4.73004 / a \tag{7}$$

$$\beta_y = 4.73004 / b \tag{8}$$

$$\frac{\partial}{\partial x} Z(x,y) = \frac{\beta_x}{\sqrt{\rho a b h}} \left\{ \left[\sinh(\beta_x x) + \sin(\beta_x x) \right] - \sigma \left[\cosh(\beta_x x) + \cos(\beta_x x) \right] \right\}$$
$$\bullet \left\{ \left[\cosh(\beta_y y) - \cos(\beta_y y) \right] - \sigma \left[\sinh(\beta_y y) + \sin(\beta_y y) \right] \right\} \tag{9}$$

$$\frac{\partial^2}{\partial x^2} Z(x,y) = \frac{\beta_x^{\ 2}}{\sqrt{\rho a b h}} \left\{ \left[\cosh(\beta_x x) + \cos(\beta_x x) \right] - \sigma \left[\sinh(\beta_x x) - \sin(\beta_x x) \right] \right\}$$
$$\bullet \left\{ \left[\cosh(\beta_y y) - \cos(\beta_y y) \right] - \sigma \left[\sinh(\beta_y y) + \sin(\beta_y y) \right] \right\} \tag{10}$$

$$\frac{\partial}{\partial y} Z(x,y) = \frac{\beta_y}{\sqrt{\rho a b h}} \left\{ \left[\cosh(\beta_x x) - \cos(\beta_x x) \right] - \sigma \left[\sinh(\beta_x x) + \sin(\beta_x x) \right] \right\}$$
$$\bullet \left\{ \left[\sinh(\beta_y y) + \sin(\beta_y y) \right] - \sigma \left[\cosh(\beta_y y) + \cos(\beta_y y) \right] \right\} \tag{11}$$

$$\frac{\partial^2}{\partial y^2} Z(x,y) = \frac{\beta_y{}^2}{\sqrt{\rho abh}} \{ [\cosh(\beta_x x) - \cos(\beta_x x)] - \sigma[\sinh(\beta_x x) + \sin(\beta_x x)] \}$$

$$\bullet \{ [\cosh(\beta_y y) + \cos(\beta_y y)] - \sigma[\sinh(\beta_y y) - \sin(\beta_y y)] \} \tag{12}$$

Assume that only the fundamental mode is excited by the pressure field.

The fundamental frequency ω_{11} is calculated using either the Blevins or Rayleigh method in Reference 2.

The participation factor for constant mass density from Reference 2 is

$$\Gamma_{11} = 0.690 \sqrt{\rho abh} \tag{13}$$

The displacement response Z(x, y, ω) to the applied force is

$$Z(x,y,\omega) = \frac{1}{\rho h} W(\omega) \frac{\Gamma_{11} Z_{11}(x,y)}{\left(\omega_{11}^2 - \omega^2\right) + j2\xi_{11}\omega\omega_{11}} \tag{14}$$

The bending moments are

$$M_{xx}(x,y,\omega) = -D\left(\frac{\partial^2}{\partial x^2} + \mu\frac{\partial^2}{\partial y^2}\right) Z(x,y,\omega) \tag{15}$$

$$M_{yy}(x,y,\omega) = -D\left(\frac{\partial^2}{\partial y^2} + \mu\frac{\partial^2}{\partial x^2}\right) Z(x,y,\omega) \tag{16}$$

The bending stresses from Reference 3 are

$$\sigma_{xx}(x,y,\omega) = -\frac{E\,\hat{z}}{1-\mu^2}\left(\frac{\partial^2}{\partial x^2} + \mu\frac{\partial^2}{\partial y^2}\right) Z(x,y,\omega) \tag{17}$$

$$\sigma_{yy}(x,y,\omega) = -\frac{E\,\hat{z}}{1-\mu^2}\left(\frac{\partial^2}{\partial y^2} + \mu\frac{\partial^2}{\partial x^2}\right) Z(x,y,\omega) \tag{18}$$

$$\tau_{xy}(x,y,\omega) = -\frac{E\,\hat{z}}{1+\mu}\left(\frac{\partial^2}{\partial x \partial y} Z(x,y,\omega)\right) \tag{19}$$

$\hat{z}$ is the distance from the centerline in the vertical axis

References

1. T. Irvine, Natural Frequencies of Rectangular Plate Bending Modes, Revision B, Vibrationdata, 2011.

2. T. Irvine, The Natural Frequency of a Rectangular Plate with Fixed-Fixed-Fixed-Fixed Boundary Conditions, Vibrationdata, 2012.

3. J.S. Rao, Dynamics of Plates, Narosa, New Delhi, 1999.

4. http://www.efunda.com/formulae/solid_mechanics/mat_mechanics/plane_stress_principal.cfm

5. D. Segalman, C. Flucher, G. Reese, R Field; An Efficient Method for Calculating RMS von Mises Stress in a Random Vibration Environment, Sandia Report: SAND98-0260, UC-705, 1998.

APPENDIX A

Example

Consider a rectangular plate with the following properties:

A fixed-fixed-fixed-fixed aluminum plate has dimensions:

Length	=	10 in
Width	=	8 in
Thickness	=	0.125 in

The elastic modulus is 1.0e+07 lbf/in^2. The mass density is 0.1 lbm/in^3.

The normal mode and frequency response function analysis are performed via a Matlab script.

The resulting displacement and transfer functions magnitudes are shown in Figures A-1 and A-2, respectively.

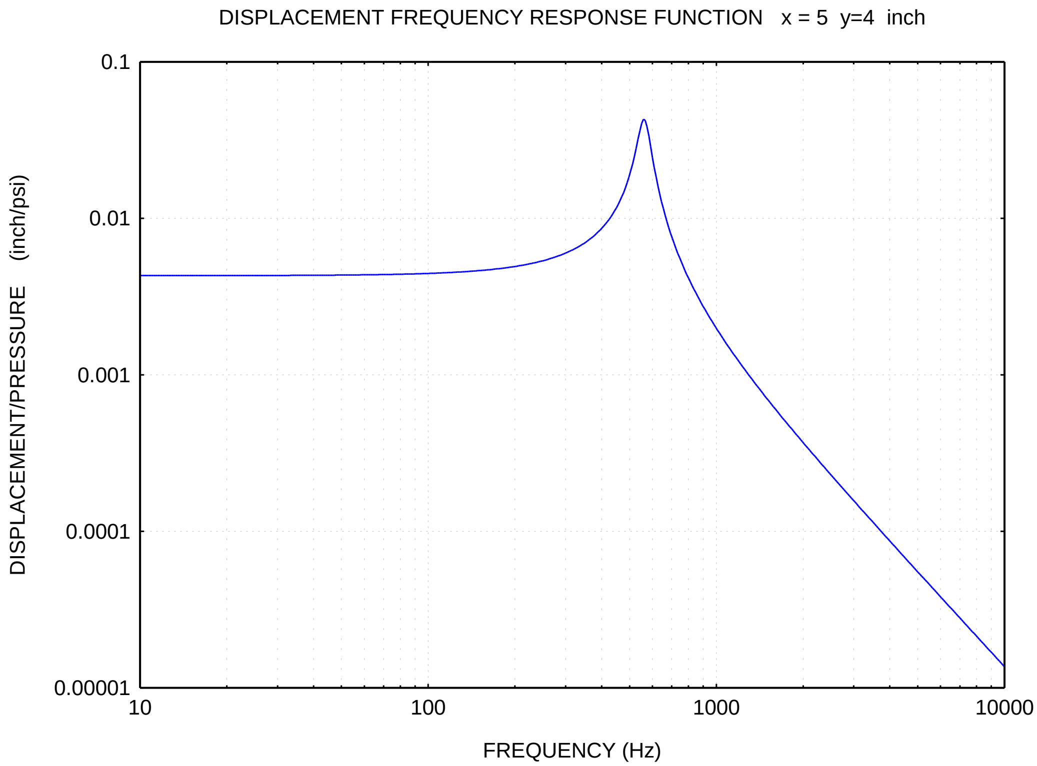


Figure A-1.

The maximum displacement response is: 0.0429 in/psi at 558.3 Hz

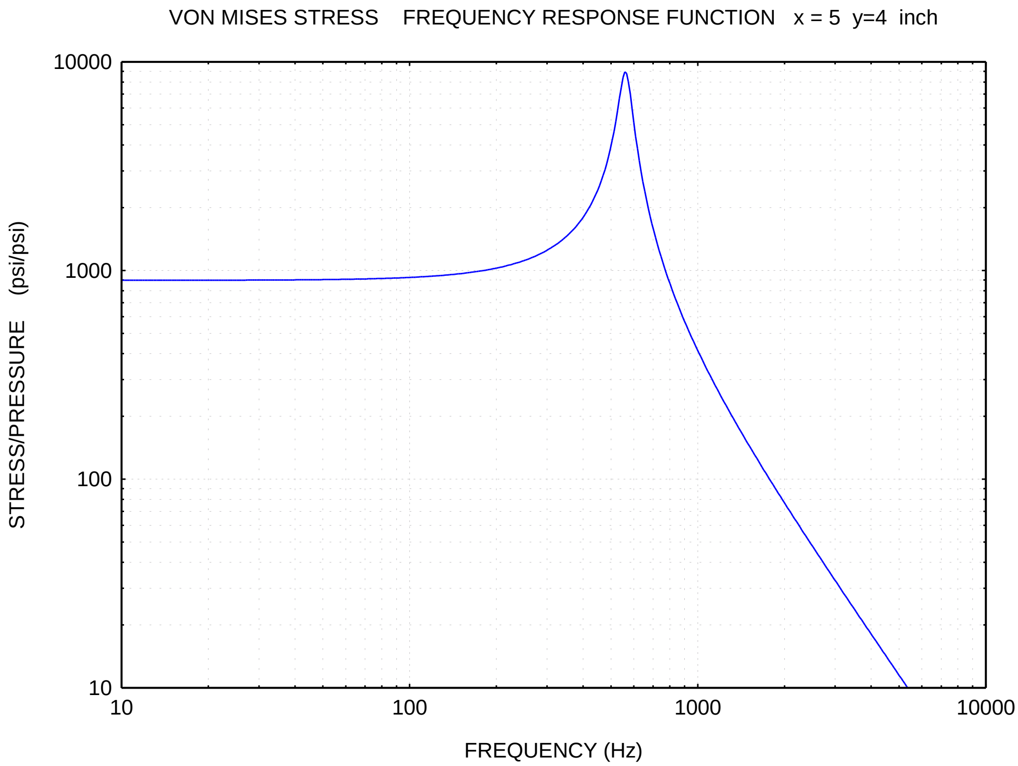
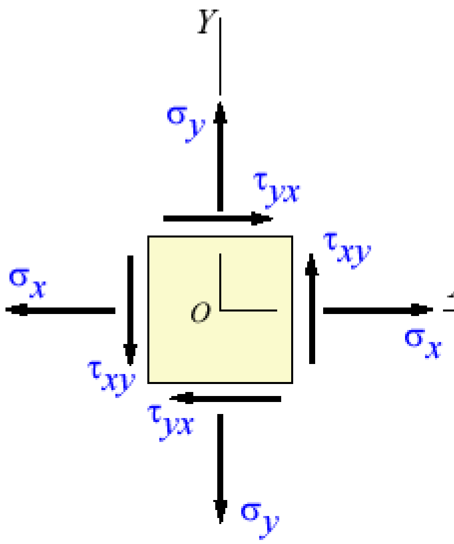


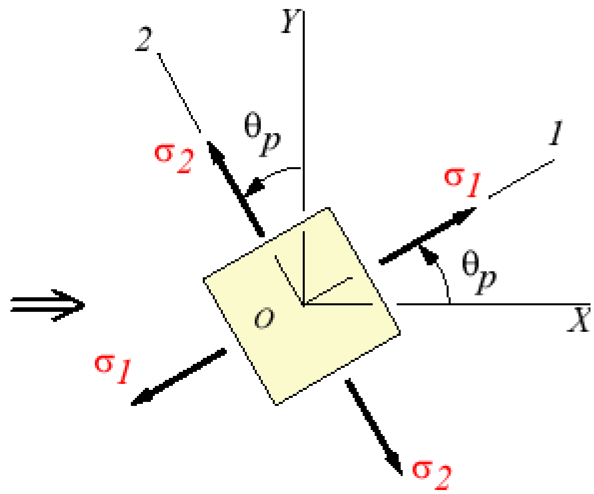
Figure A-2.

The maximum von Mises stress response is: 8949 (psi/psi) at 558.3 Hz

Principal Stress



Stresses in Given Coordinate System



Principal Stresses

The diagrams are taken from Reference 4.

The principle stresses are

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} \qquad \text{(B-1)}$$

The angle at which the shear stress becomes zero is

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \text{(B-2)}$$

The von Mises stress σ_e is

$$\sigma_e = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} \qquad \text{(B-3)}$$

The von Mises stress is used to predict yielding of materials under any loading condition from results of simple uniaxial tensile tests. The von Mises stress satisfies the property that two stress states with equal distortion energy have equal von Mises stress.

An alternate formula from Reference 5 is

$$\sigma_e = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3\tau_{xy}^2} \qquad \text{(B-4)}$$