THE EFFECT OF FIXTURE IMPEDANCE IN VIBRATION TESTING Revision A

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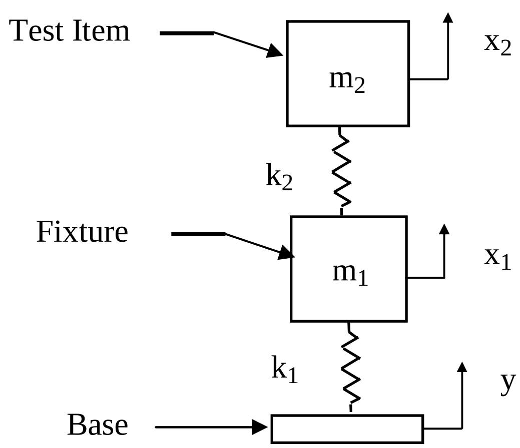

Figure 1. Two-degree-of-freedom Model of Test Item and Fixture

Introduction

The purpose of this tutorial is to show by examples the effect of fixture impedance on the response of a test item.

The test item is considered to be a single-degree-of-freedom system. The test fixture is also considered to be a single-degree-of-freedom system. The test item is mounted to the fixture as shown in Figure 1. The combined systems form a two-degree-of-freedom system.

For simplicity, the vibration excitation in this report is specified as a sine sweep, from 10 Hz to 2000 Hz.

The principles developed in this report also apply to random vibration and shock testing.

Three cases are considered as shown in Table 1.

Table 1. Case Description

Case	Test Item Mass (lbm)	Fixture Mass (lbm)	Test Item Stiffness (lbf/in)	Fixture Stiffness (lbf/in)
1. Compliant Fixture	1	5	10,000	5000
2. Intermediate Fixture	1	5	10,000	50,000
3. Stiff Fixture	1	5	10,000	500,000

Notes:

1. The test item by itself has a constant natural frequency of 312.8 Hz.
2. The stiffness and natural frequency of the fixture varies per case.
3. The fixture natural frequency equals the test item natural frequency for Case 2.
4. The amplification factor for each mode and for each case is Q=10.

Also, note that mechanical impedance is equal to the applied force divided by the response velocity, as shown in Reference 1. For a constant mass, higher stiffness yields higher impedance.

Natural Response

The natural response results for the cases in Table 1 are calculated using the method in Reference 2. The eigenvalues and vectors for the three cases are shown in the appendices.

Transfer Function Test, Simulated via Calculation

The response to base excitation is also calculated using the formulas in Reference 2. The response of each mass for each of the threes cases is shown in Figures 2 through 4, respectively. The base excitation is a 1 G sine sweep for each case. The test represented in each of these plots has the purpose of identifying the transfer functions.

Specified Test, Simulated via Calculation

The test specification requires that the 1 G sine sweep must occur on the fixture rather than the base. Thus the base input must be tailored so that this requirement is met. The base motion itself is essentially the “drive signal.” Assume an ideal control system that can compensate for any fixture resonance whatsoever.

The acceleration of the test item for each case for the specified test is shown in Figure 5.

The corresponding vibrational power curves are shown in Figure 6. These curves show the power flow into the test item. The power flow is sensitive to the fixture stiffness at the test’s items natural frequency.

Note that vibrational power is the RMS of the dot product of force and velocity. Thus, the calculation accounts for the phase angle between the force and velocity at each frequency. The force is calculated indirectly for the base excitation problem. Specifically, the force acting upon the test item is taken as the item’s mass times its acceleration.

As an aside, time histories for Case 3 for the 312 Hz excitation frequency are shown in Figures 7 and 8.

Conclusion

Again, the principles developed in this report apply to all types of shock and vibration testing where the specification is an acceleration level on a fixture.

A concern is that the test item’s damage potential may correlate more closely to the injected vibrational power than to the response acceleration. Alternatively, the damage potential may correlate to both the transmitted force and the response acceleration rather than to the acceleration alone.

Vibration tests are commonly performed using very stiff fixtures to avoid control instabilities at the fixture’s natural frequency. Ideally, the fixture’s natural frequency is well above the maximum test frequency.

A related concern is overtesting at the test item’s natural frequency. This concern can be dealt with by force limited testing, as discussed in Reference 3 and in numerous papers by Terry Scharton.

On the other hand, shock testing is performed using a wide variety of test methods and fixtures. A given shock spectrum might be achieved using a drop tower, shaker, mechanical impact method, or explosive attached to a plate. The impedance of the mounting fixture would likely vary considerably between these methods. Thus, the test item might pass or fail depending on the test method even if the same acceleration specification could be achieved in each case. Luhrs discusses this problem in Reference 4.

Further consideration of mechanical impedance and vibrational power flow is needed, particularly with respect to shock testing. One issue is to determine whether force limiting can used in shock testing.

References

1. T. Irvine, An Introduction to Frequency Response Functions, Vibrationdata, 2000.

2. T. Irvine, The Generalized Coordinate Method for Discrete Systems Subject to Base Excitation, Vibrationdata, 2002.

3. W. Fackler, Equivalence Techniques for Vibration Testing, SVM-9, The Shock and Vibration Information Center, Naval Research Laboratory, United States Department of Defense, Washington D.C., 1972. (See chapter 5).

4. H. Luhrs, “Designing Electronics for Pyrotechnic Shock,” Pyrotechnic Shock Workshop, Shock and Vibration Bulletin 57, Shock and Vibration Information Center, Naval Research Laboratory, Washington D.C., 1987.

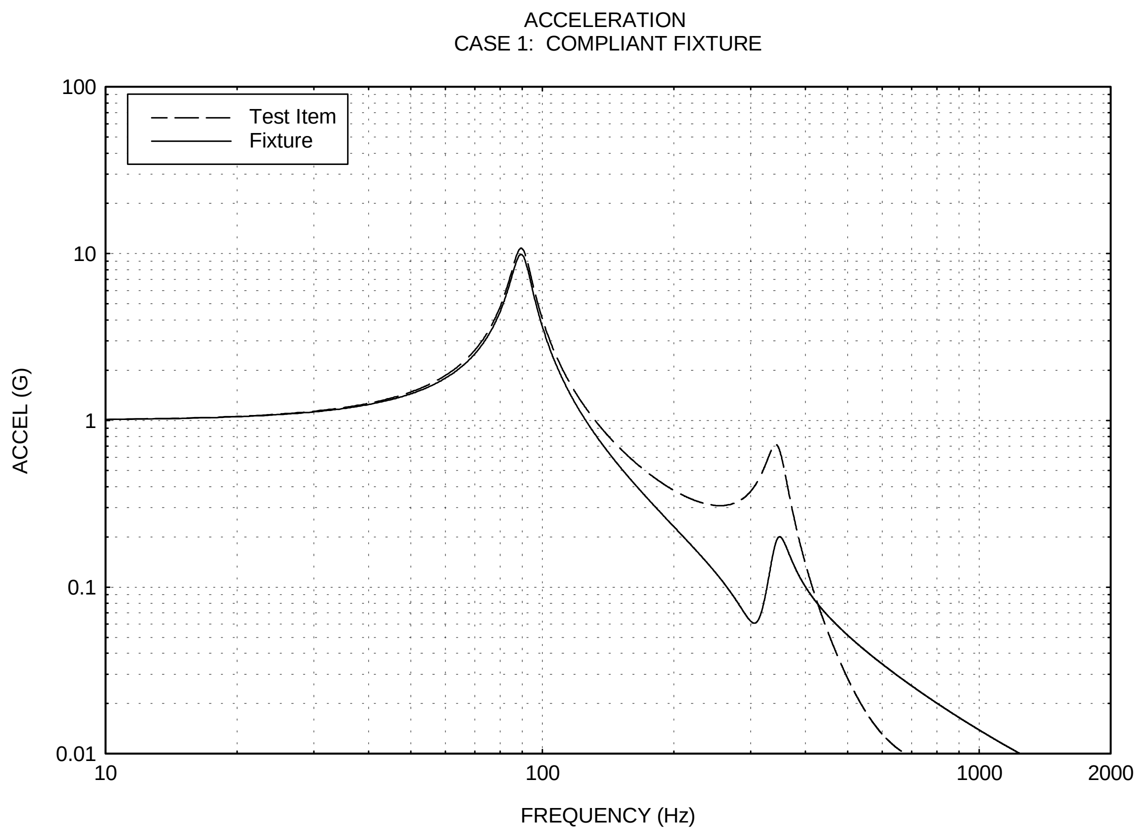

Figure 2.

The sinusoidal base input is 1 G across the entire frequency domain.

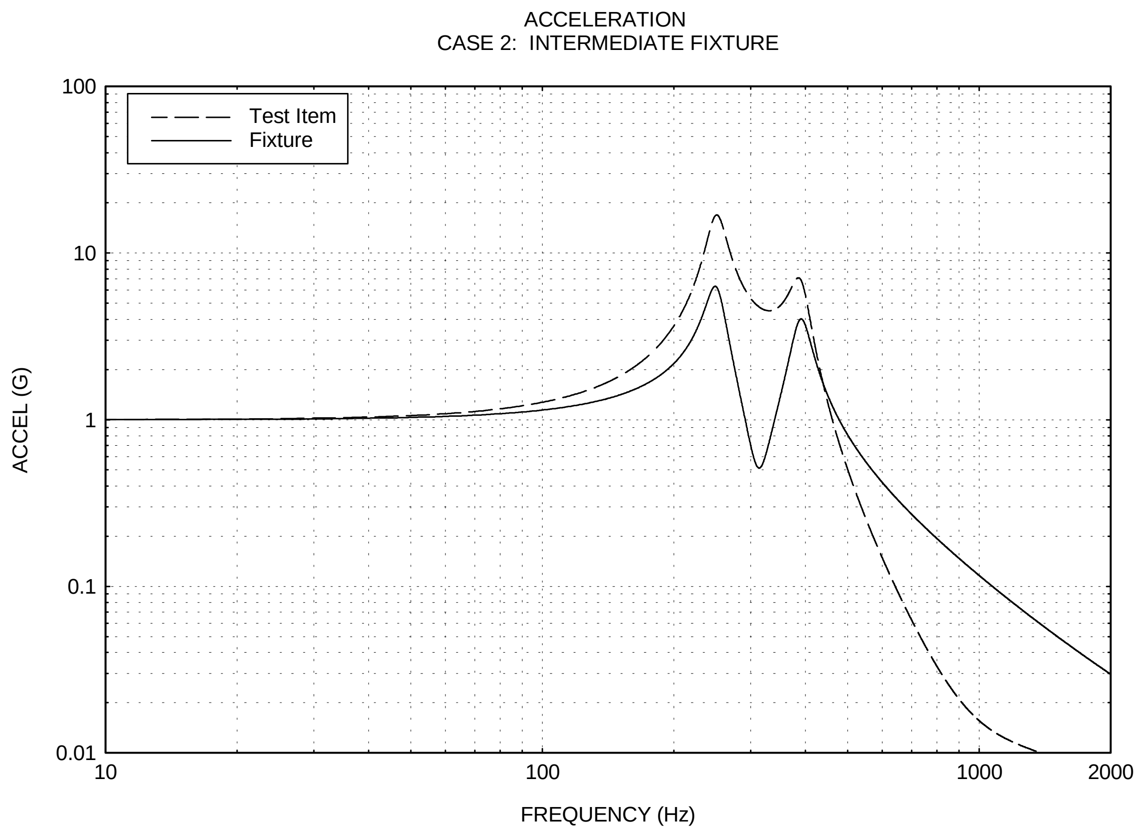

Figure 3.

The sinusoidal base input is 1 G across the entire frequency domain.

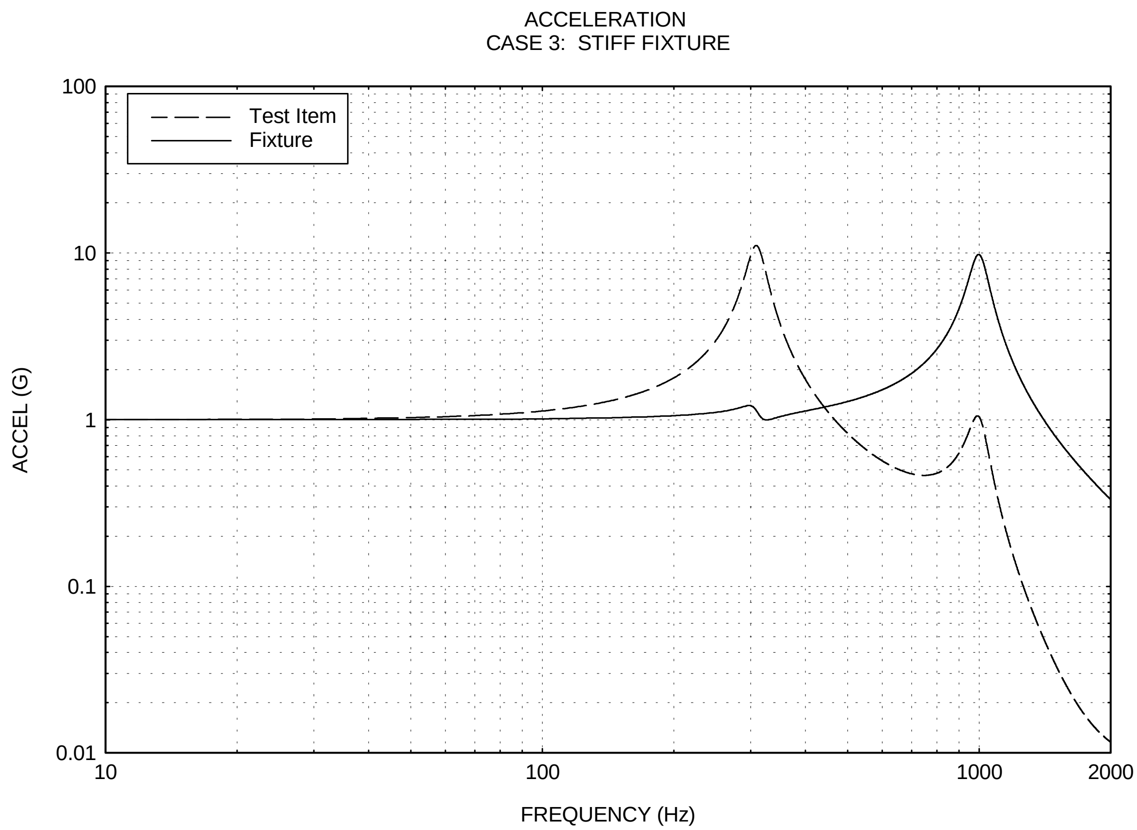

Figure 4.

The sinusoidal base input is 1 G across the entire frequency domain.

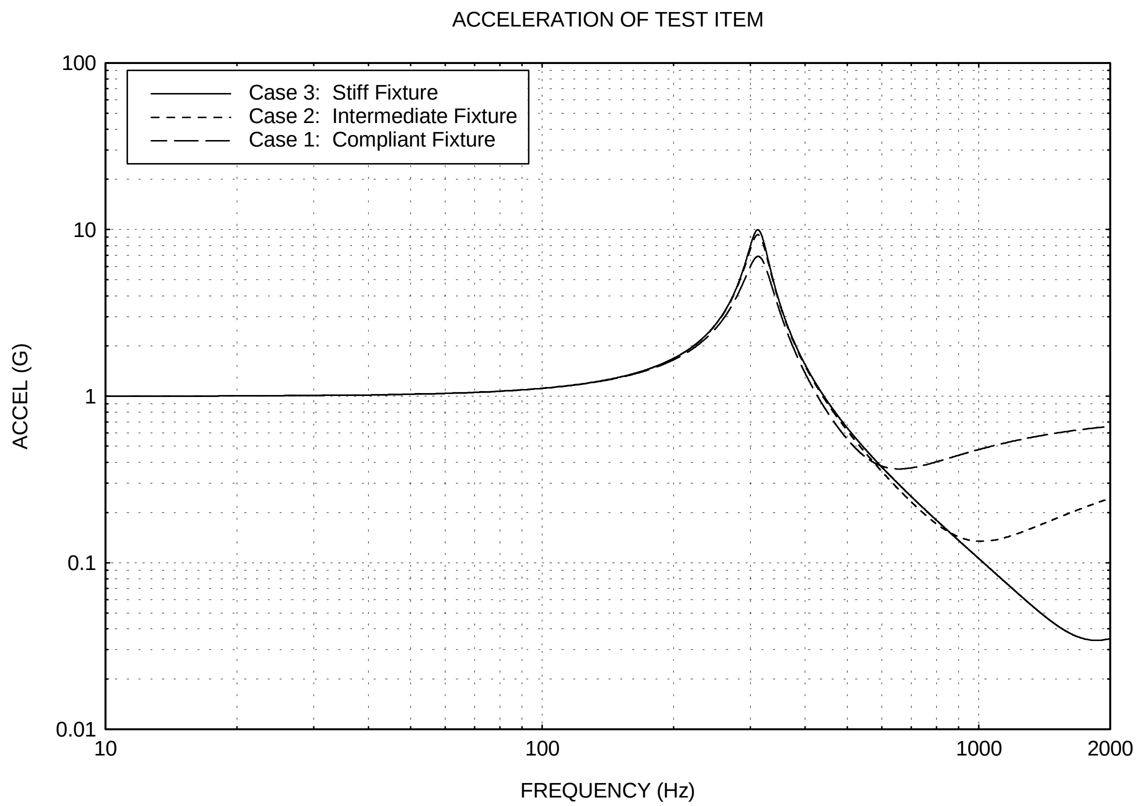

Figure 5.

The sinusoidal base input level is adjusted so that the acceleration of the fixture is 1 G across the entire frequency domain.

The comparison shows that the test item's response varies per the fixture stiffness. The test item's response at its natural frequency is the highest for the stiff fixture case.

The difference between the highest and lowest responses at 312 Hz is 3 dB

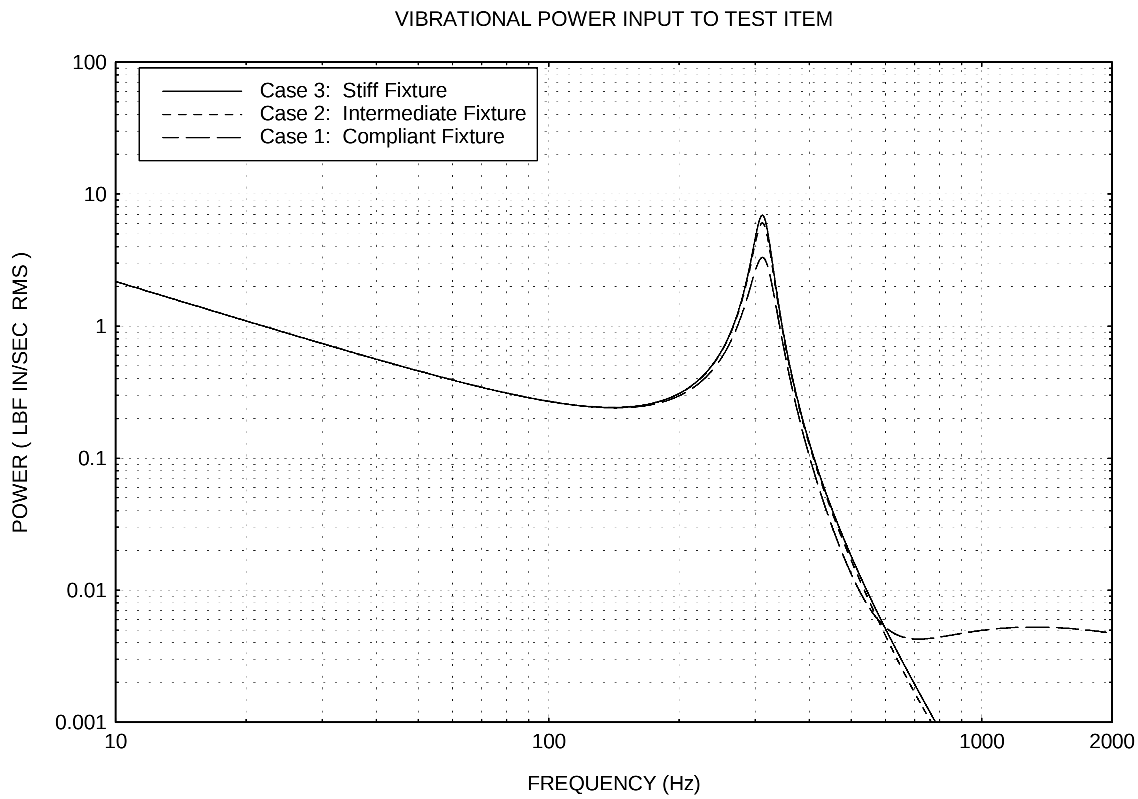

Figure 6.

The sinusoidal base input level is adjusted so that the acceleration of the fixture is 1 G across the entire frequency domain.

The difference between the highest and lowest power inputs at 312 Hz is 6 dB.

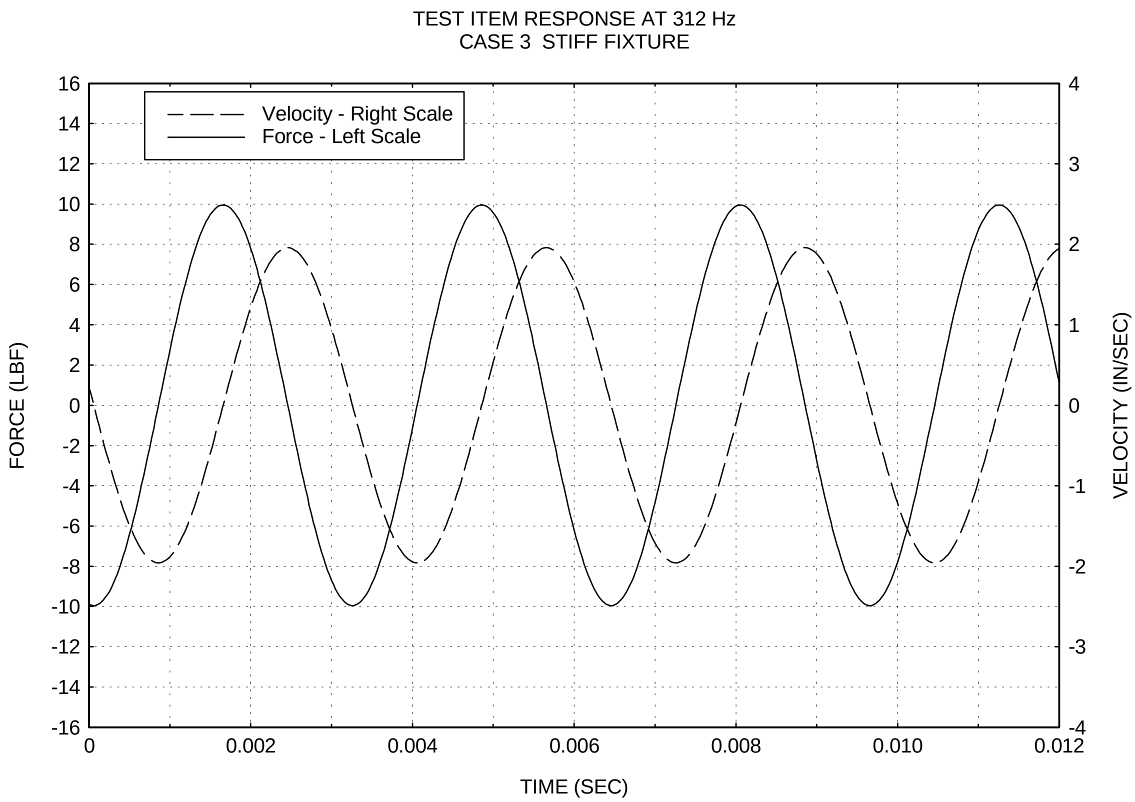

Figure 7.

The force leads velocity by 90 degrees.

The sinusoidal base input level is adjusted so that the acceleration of the fixture is 1 G across the entire frequency domain.

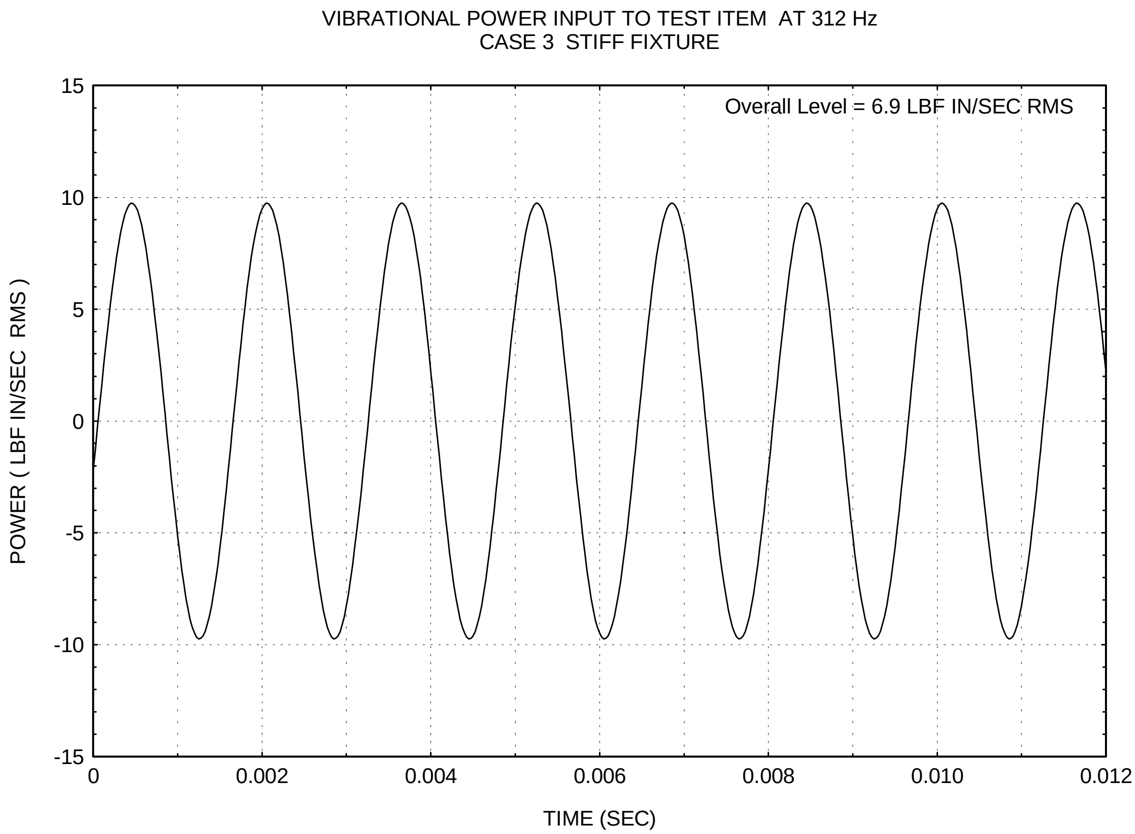

Figure 8.

Note that the power cycle frequency is twice 312 Hz.

The sinusoidal base input level is adjusted so that the acceleration of the fixture is 1 G across the entire frequency domain.

APPENDIX A

Case 1 Compliant Fixture Output File

```
Mass matrix
     0.0130        0.0000
     0.0000        0.0026


Stiffness matrix
 15000.0000   -10000.0000
-10000.0000    10000.0000

eig1 =    316941.3785
eig2 =   4701058.6215

omega1 =       562.9755 rad/sec
omega2 =      2168.1925 rad/sec


fn1 =        89.6003 Hz
fn2 =       345.0786 Hz


Eigenvectors (column format)

     1             2

     0.9179        -0.2179
     1.0000         1.0000


Let Q = eigenvector matrix.


QTMQ
     0.0135        0.0000
     0.0000        0.0032


scale1 =       8.6053
scale2 =      17.6621


Normalized Eigenvectors (column format)

     1             2

     7.8987        -3.8484
     8.6053        17.6621

Let P = normalized eigenvector matrix.
```

```
Verify PTMP
     1.0000        0.0000
     0.0000        1.0000


Verify PTKP
316941.3785       -0.0000
    -0.0000  4701058.6215
```

APPENDIX B

Case 2 Intermediate Fixture Output File

```
Mass matrix
     0.0130        0.0000
     0.0000        0.0026


Stiffness matrix
 60000.0000   -10000.0000
-10000.0000    10000.0000

eig1 =   2477125.7817
eig2 =   6014874.2183

omega1 =      1573.8887 rad/sec
omega2 =      2452.5241 rad/sec


fn1 =       250.4922 Hz
fn2 =       390.3313 Hz


Eigenvectors (column format)

     1             2

     0.3583        -0.5583
     1.0000         1.0000


Let Q = eigenvector matrix.


QTMQ
     0.0043        0.0000
     0.0000        0.0066


scale1 =      15.3335
scale2 =      12.2835


Normalized Eigenvectors (column format)

     1             2

     5.4933        -6.8573
    15.3335        12.2835

Let P = normalized eigenvector matrix.
```

```
 Verify PTMP
      1.0000        0.0000
      0.0000        1.0000


 Verify PTKP
2477125.7817       -0.0000
     -0.0000  6014874.2183
```

APPENDIX C

Case 3 Stiff Fixture Output File

```
Mass matrix
     0.0130        0.0000
     0.0000        0.0026


Stiffness matrix
510000.0000   -10000.0000
-10000.0000    10000.0000

eig1 =   3776284.3072
eig2 =  39455715.6928

omega1 =      1943.2664 rad/sec
omega2 =      6281.3785 rad/sec


fn1 =       309.2805 Hz
fn2 =       999.7124 Hz


Eigenvectors (column format)

     1             2

     0.0217        -9.2217
     1.0000         1.0000


Let Q = eigenvector matrix.


QTMQ
     0.0026        0.0000
     0.0000        1.1041


scale1 =      19.6238
scale2 =       0.9517


Normalized Eigenvectors (column format)

     1             2

     0.4256        -8.7760
    19.6238         0.9517

Let P = normalized eigenvector matrix.
```

```
 Verify PTMP
     1.0000        0.0000
     0.0000        1.0000


 Verify PTKP
3776284.3072        0.0000
         0.0    39455715.6928
```